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**Working Paper**

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CESifo Working Paper, No. 3732

**Provided in Cooperation with:**

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*Suggested Citation:* Balduzzi, Paolo; Graziano, Clara; Luporini, Annalisa (2012) : Voting in small committees, CESifo Working Paper, No. 3732, Center for Economic Studies and ifo Institute (CESifo), Munich

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## Voting in Small Committees

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CESIFO WORKING PAPER NO. 3732

CATEGORY 2: PUBLIC CHOICE

FEBRUARY 2012

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# Voting in Small Committees

## Abstract

A small committee has to approve/reject a project with uncertain return. Members have different preferences: some are value-maximizers, others are biased towards approval. We focus on the efficient use of scarce information when communication is not guaranteed, and we provide insights on the optimal committee composition. We show that the presence of biased members can improve the voting outcome by simplifying the strategies of unbiased members. Thus, heterogeneous committees perform at least as well as homogeneous committees. In particular, when value-maximizers outnumber biased members by one vote, the optimal equilibrium becomes unique. Finally, allowing members to communicate brings no improvement.

JEL-Code: D710, D720.

Keywords: voting, small committees, committees composition, communication in committees.

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# 1 Introduction

In many committees, members are nominated by (and thus represent the interests of) different institutions and this reflects in different voting behaviors. Consider, for instance, boards of directors whose objective, in principle, is to maximize firm value. Directors represent different stakeholders, (majority and minority shareholders, investors, workers, etc.) whose objectives may not be aligned. Another example<sup>1</sup> is provided by monetary policy committees, where some members are chosen within the staff of the central bank while other members are appointed by external bodies, such as the Government (the Bank of England Monetary Policy Committee is a typical example). In this case, internal members are usually more concerned about inflation while external members are more concerned about unemployment. In general, empirical studies show that members belonging to different groups have significant differences in their voting behaviors and that these differences can be explained by factors such as political pressure or the channel of appointment, especially when committee members face retention decisions (see, for example, Sheperd [2009], and Harris, Levine and Spencer [2011]).

The present paper analyzes the effect of member heterogeneity by studying the voting behavior of a small committee that has to approve or reject a project. We consider two types of players: expected value maximizers and biased members who always vote in favor of the project, even disregarding their private information. Then, the following question arises: why should biased members be allowed on this committee? Our model shows that their presence is never detrimental, and is beneficial in the absence of communication among members because it ensures uniqueness and optimality of the equilibrium strategy profile. The intuition is that the bias provides certainty about some members' strategies thus simplifying the responses of the others, and therefore reducing the number of (otherwise) multiple equilibria. In particular, we explore the behavior of uninformed value-maximizing members. Given that they want to maximize the probability that the committee makes the correct decision, they face the question of how to avoid influencing the decision and let informed members determine it. The equilibrium voting strategies prescribe that uninformed unbiased members systematically contrast the vote of biased members. Indeed, in many small committees dissent voting is commonly observed (Spencer [2006]). On the basis of the equilibrium voting strategies, we determine the optimal composition of the committee, consisting in letting unbiased members outnumber biased members by just one vote. Our result is consistent with the actual composition of some committees such as the Bank of England Monetary Policy Committee or the Italian Constitutional Court. Finally, our model provides some suggestions on the role

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<sup>1</sup>Additional examples are special juries as Supreme or Constitutional Courts, and technical committees, where politicians, bureaucrats and experts meet to provide advice.

of communication in the voting process.

The paper is organized as follows. Section 2 reviews the main literature. Section 3 presents the basic model. Section 4 examines, as a benchmark, the voting game in a committee composed only of value-maximizing members. Section 5 introduces biased members and analyzes if and how results change when members have different objectives. Then, in Section 6 we allow members to communicate. Finally, Section 7 concludes. All proofs are collected in the Appendix.

## 2 Related literature

Since Condorcet’s seminal contribution, namely his Jury Theorem, the literature about voting has been constantly growing. A lot of papers have generalized the Jury Theorem<sup>2</sup>, and many others have extended voting games to include both naive and strategic voting<sup>3</sup>.

Traditionally, in this literature the aim of voters has been to aggregate information, with the assumption that taking the right decision (that is, guessing the correct state of the world) was the common objective of all the players. In fact, we believe this is not the case, as heterogeneity of preferences is well documented both in large and in small elections<sup>4</sup>. Feddersen and Pesendorfer [1996; 1997] have focused on heterogeneity in large elections, showing that full information aggregation is still possible. In particular, they show that the probability of electing the “wrong” candidate asymptotically goes to zero.

In small committees, the presence of heterogeneity and (possible) resulting conflicts of interests appear to be a more relevant problem, as information aggregation may be severely limited by strategic voting induced by divergent interests. One possibility is to look for optimal voting rules to minimize information losses. In a standard Condorcet Jury Theorem framework, Chwe [1999] suggests to provide minority members with optimal incentives to participate in voting, thus not wasting their information. But things become more complicated when the relevant issue is not to find optimal voting rules but rather an optimal way to aggregate “relevant” information<sup>5</sup>. Li, Rosen and Suen [2001] examine a two-person committee where each member receives a private signal and reports his information. Since members

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<sup>2</sup>See, for instance, Duggan and Martinelli [2001], Myerson [1998] and McLennan [1998]. See also Piketty [1999] for a brief review of recent contributions about the information-aggregation role of political institutions.

<sup>3</sup>See Austen-Smith and Banks [1996] and an experiment on the use of strategic voting by Eckel and Holt [1989].

<sup>4</sup>See for instance Blinder [2006], Jung [2011], Riboni and Ruge-Murcia [2007] for the case on Monetary Policy Committees.

<sup>5</sup>Berk and Bierut [2004] suggest that often in small committees it is technologically or politically unfeasible to implement optimal voting rules, thus binding voting to be based on simple majority.

have conflicting interests, strategic considerations induce information misreporting and there is no truth-telling equilibrium. Conflict of interest prevents full information aggregation also in larger committees, as shown by Maug and Yilmaz [2002]. These authors suggest to group voters into two separate classes because such a voting mechanism may alleviate the incentive to withhold information when voters have strong conflicts of interests and individual information differs. Thus, voting decisions become more informative. Examining a committee of experts, Wolinsky [2002] suggests to solve the problem in a similar way, by partitioning members in different groups<sup>6</sup>.

In most of the above-mentioned studies there is some communication among committee members. Indeed, since the seminal contribution by Austen-Smith [1990], information sharing and voting in small committees is often analyzed along with the possibility of communication<sup>7</sup>. This strand of literature has been recently and very elegantly reviewed by Austen-Smith and Feddersen [2009]. In particular, they focus on the role of communication, along with voting rules and voting protocols, to foster full information aggregation in small committees.

In our model of simultaneous voting, however, the optimal voting strategy already nests information sharing, and the role of communication, though important, is very limited. In fact, we take a different approach to the problem of information sharing among committee members by examining if the voting outcome can be optimal even if some members do not communicate any information at all. In our model, there is preference heterogeneity in the sense that some members genuinely want to aggregate information while others have a strong bias. Without imposing any explicit revelation mechanism and without looking for optimal voting rules or protocols, we suggest an optimal *composition* rule as a sufficient device to provide the highest possible level of information aggregation, even when biased members do not use their private information. The positive role of the latter members in our model does not rely on their superior information with respect to value maximizers, but on the fact that their presence on the committee eliminates multiple (and possibly suboptimal) equilibria. Communication, when it is possible, serves as a coordination device but does not add value to the decision making.

### 3 The model

A committee is composed of  $2n + 1$ ,  $n \geq 1$ , members who have to decide by majority vote whether to approve a project (voting “yes”) or reject it (voting “no”). If the proposal is

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<sup>6</sup>Committee members have heterogeneous preferences also in the model of Cai [2009]. The focus of Cai, however, is the optimal size of the committee rather than its composition.

<sup>7</sup>See also Adams and Ferreira [2007], Harris and Raviv [2008], and Raheja [2005] for communication in boards of directors.

rejected, a value of 0 is realized. When accepted, the project yields value  $v = -1$  if the state of the world is low ( $L$ ), and  $v = 1$  if the state of the world is high ( $H$ ). Thus,  $v : \{-1, 1\}$ .

Each state, and thus each value, has the same prior (i.e.,  $\frac{1}{2}$ ). This implies that when members have no information on the state of the world there is no one choice that dominates the other. Given these probabilities, the highest expected value that can be achieved by voting correctly (rejecting the project in  $L$  and approving it in  $H$ ) is  $\frac{1}{2}$ . Note that a single uninformed decision maker would always obtain an expected value of 0.

We consider a simple information structure where any member of the committee learns the true state with probability  $\alpha \in [\frac{1}{2}, 1)$  and learns nothing with probability  $1 - \alpha$ .<sup>8</sup> As a consequence, the information set of a generic member  $i$  is  $\Omega_i = \{\omega_i\}$ , with  $\omega_i \in \{H; L\}$ , when  $i$  is informed, while it is  $\Omega_i = \{H, L\}$ , when  $i$  does not know the true state of the world. Furthermore, we consider the case in which committee members can become informed at no cost. As we will point out at the end of section 3, assuming a fixed cost for acquiring information would leave our results on the voting outcome and committee composition unchanged. Introducing such a cost would however set an upper bound to the optimal size of the committee.

We initially assume that members cannot communicate. Then, in Section 6, we relax this assumption, showing that even if we allow for communication, the performance of the committee does not improve. In addition, as usual in the literature on committee voting, we do not consider abstention<sup>9</sup>. Given that abstention is not allowed, the action set of each player has only two elements: vote “yes” to accept the project and “no” to reject it. A strategy  $s_i$  is a member  $i$ ’s voting behavior, conditional on his information set. A mixed strategy is defined as the probability that a member votes “yes”.

The committee is composed of unbiased members who want to maximize the expected value of the project,  $E(v)$ , and of biased members who always want to approve the project independently of the state of the world. We assume that all members are risk neutral and that their types are common knowledge. Let  $M$  denote value-maximizing members, and  $B$  members with a bias. Then, we call  $m$  and  $b$  the probabilities of voting “yes” for an

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<sup>8</sup>Alternatively, we can assume that every member observes a signal that is perfectly informative with probability  $\alpha$  and with probability  $1 - \alpha$  is totally noisy. Referring to this interpretation, it can be shown that our results would not qualitatively change if the signal was only partially noisy (Balduzzi [2005]).

<sup>9</sup>Of course we acknowledge some exceptions, such as Morton and Tyran (2008). Nonetheless, for our heterogeneous committee this assumption is made without loss of generality, because members would never abstain in equilibrium. Furthermore, allowing abstention would bring other issues and restrictions into the picture, driving the attention away from the scope of the paper. First of all, it is not clear what should happen when everyone abstains. An *ad hoc* rule should be applied that however might affect the final result. Second, it is not clear what the general decision rule should be: simple majority of members or simple majority of actual votes? Note also that in many committees abstention is explicitly or implicitly ruled out (juries, the European Courts of Human Rights and the Italian Constitutional Court are some examples).

*uninformed* member of type  $M$  and  $B$  respectively.

The utility function of a  $M$  type positively depends on the expected value of the project; in particular, we assume that it actually corresponds to its expected value:  $u_M(E(v)) = E(v)$ . Given this utility function, a  $M$  member will choose the strategy that maximizes  $E(v)$ . Notice that, given the values the project can take, maximizing  $E(v)$  is equivalent to maximizing the probability that the committee takes the correct decision. Indeed, the latter is given by the sum of the probabilities that “yes” wins when the actual value of the alternative is 1 and that “no” wins when the actual value of the alternative is  $-1$ :

$$\frac{1}{2} \{Y(\cdot | v = 1) + [1 - Y(\cdot | v = -1)]\},$$

where the function  $Y(\cdot | \cdot)$  is the conditional probability that the board as a whole votes “yes”. The expected value of the project is:

$$E(v) = \frac{1}{2}[1Y(\cdot | v = 1) - 1Y(\cdot | v = -1)]$$

and it is straightforward to notice that the two expressions are strategically equivalent.

In order to maximize  $E(v)$ , a  $M$  member will condition his strategy on being pivotal, because that is the only case where he can actually influence the outcome of the voting process and therefore his own utility. Since any strategy is optimal when the player is not pivotal, we concentrate on weakly dominant strategies without loss of generality.

The utility function of a  $B$  member positively depends on the approval of the project<sup>10</sup>. A  $B$  member always supports the project, regardless of the value which is *ex post* realized. His utility  $u_B$  therefore depends on the final decision of the committee and can take the following two values:  $u_B = 1$  if the project is approved and  $u_B = 0$  if the project is rejected. This clearly implies that always voting “yes” is a dominant strategy for a  $B$  member. For simplicity, we abstract from additional problems, such as a  $B$  member’s potential loss of reputation when the approval of the project creates a loss.

Finally, before analyzing the voting behavior of the committee, we introduce the definition of compensating strategy that will be useful in the following sections.

**Definition 1 (Compensating strategy)** *Two members are playing compensating strategies when the following conditions are jointly satisfied: i) they are both uninformed; ii) they play “yes” with probabilities whose sum is equal to 1. When these probabilities take extreme values (0 and 1), we have compensation in pure strategies.*

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<sup>10</sup>Obviously, nothing substantial in our results would change if a biased member always supported rejection.



We concentrate on equilibria where members of the same type follow the same type of strategy, i. e. either they all play pure strategies or they all play mixed strategies.

## 4 The benchmark

We define our benchmark as a committee only composed of members who want to maximize  $E(v)$ , i.e. members of type  $M$ . Clearly, whenever a  $M$  member is informed, he votes according to his information. The issue is to define what an uninformed  $M$  member should do. Intuitively, any uninformed member has an incentive to leave the final decision to the others, who may be informed. It can then be shown that there are only two types of equilibria differing as to the behavior of uninformed members.

The first is an (asymmetric) equilibrium in pure strategies, where all but one member compensate for each other when uninformed, while the remaining member plays indifferently either “yes” or “no” (when uninformed). As the identity of those members who vote “yes” in equilibrium and those who vote “no” is interchangeable, there exists in fact a multiplicity of such equilibria all of which yield the same expected value. No symmetric equilibrium in pure strategies is possible. The second is a (symmetric) equilibrium where all the members compensate for each other in mixed strategies, voting “yes” with probability  $1/2$ .

In order to better understand the nature of the asymmetric equilibrium, consider a simple example with  $\alpha = \frac{1}{2}$  and  $n = 2$ , so that there are five members  $M_i$ ,  $i = 1, 2, 3, 4, 5$ . Suppose that four members vote “yes” when uninformed. The remaining member knows that he is pivotal only if two members vote “no”. But, given the above strategies, this happens only if the two members who vote “no” are in fact informed. Then, the remaining member should vote “no”. This tells us that there cannot exist a symmetric equilibrium in pure strategies where everybody votes “yes” (neither, by the same argument, an equilibrium where everybody votes “no”). Moreover, it immediately appears that a situation where four members vote “yes” (or “no”), when uninformed, cannot be an equilibrium: voting “yes” (“no”) is not the best response for an uninformed individual when there are already three members following the “yes” strategy. His best response is to vote the opposite of another uninformed member, thus giving rise to the asymmetric equilibrium. Once there are three members voting “yes” and two members voting “no”, nobody has an incentive to change his strategy. Consider one of those member voting “yes”. The probability that he is pivotal is the same in both states so he is indifferent between voting “yes” or “no”.

The expected value of these equilibria can be easily computed. In the case where two members vote “no” when uninformed, e.g.  $m_1 = m_2 = 0$ , and  $m_3 = m_4 = m_5 = 1$ , the

expected value is

$$E(v)_{PS} = \frac{1}{2}[Y(\cdot|v = 1) - Y(\cdot|v = -1)] = \frac{1}{2} \left[ 1 - \frac{1}{8} \right] = \frac{7}{16} < \frac{1}{2}$$

which is clearly equal to the value that obtains if three members vote “yes” when uninformed, e.g.  $m_1 = m_2 = m_3 = 0$ ; and  $m_4 = m_5 = 1$

$$E(v)_{PS} = \frac{1}{2}[Y(\cdot|v = 1) - Y(\cdot|v = -1)] = \frac{1}{2} \left[ \frac{7}{8} - 0 \right] = \frac{7}{16} < \frac{1}{2}$$

Notice that, in the spirit of Condorcet, the expected value is bigger than the value obtained by a single decision maker (0). Still, this committee does not provide full aggregation of information (yielding  $E(v)_{FI} = \frac{1}{2}$ ) In other words, the committee does not always take the correct decision. When all the members are uninformed, the decision (whatever it is) is correct with probability 1/2. Moreover the decision is wrong with probability 1/2 if the only informed members are those voting according to the actual state even if uninformed (those choosing  $m_i = 1$  if uninformed when the actual state is  $v = 1$ , or those choosing  $m_i = 0$  if uninformed when the actual state is  $v = 0$ ).

Consider now the second type of equilibrium where all the uninformed members compensate for each other in (symmetric) mixed strategies. Intuitively, in this equilibrium any member randomizes as long as he is pivotal in both states of the world with the same probability, given the other members’ strategies. But this is simultaneously true for any single member only if all the members are pivotal in each state of the world with the same probability. The only profile which is compatible with this logic is then the one where all the members compensate for each other in mixed strategies, voting “yes” with probability 1/2. This argument rules out any other possible equilibria in mixed strategies: whenever a member is not indifferent between voting “yes” or “no” (because the probability of that he is pivotal in a state is higher than the probability that he is pivotal in another state), he plays a pure strategy. But then, other members will have an incentive to deviate from any mixed strategy to compensate for his pure strategy. Again, we can easily compute the expected value for the five member case with  $m_1 = m_2 = m_3 = m_4 = m_5 = \frac{1}{2}$ , obtaining

$$E(v)_{MS} = \frac{1}{2}[Y(\cdot|v = 1) - Y(\cdot|v = -1)] = \frac{1}{2} \left[ 1 - \frac{53}{256} \right] = \frac{203}{512} < \frac{7}{16}.$$

Thus, the expected value falls short of that obtainable in the pure strategy equilibrium.

Recall that we focus on equilibria where all the members follow the same type of strategy, i. e. either they all play pure strategies or they all play mixed strategies, then Proposition 1

generalizes our findings.

**Proposition 1 (Benchmark)** *In a voting game with  $2n + 1$  members of type  $M$ , informed members always play according to their information. There exist two types of equilibria differing as to the behavior of uninformed players : i)  $n$  players always vote “no”,  $n$  players always vote “yes”, and one player chooses  $m \in \{0, 1\}$ ; ii)  $2n + 1$  players randomize with probability  $\frac{1}{2}$ . Equilibria of type i) yield expected value  $E(v)^* = \frac{1}{2}[1 - (1 - \alpha)^{n+1}]$ . Equilibria of type ii) yield a lower expected value.*

The intuition for this result is the same as in the five-member case: uninformed members do not want to influence the outcome of the voting process, so they compensate for each other, and leave the final decision to possibly informed members. This is also what happens in the second kind of equilibria where uninformed players compensate for each other in mixed strategies. Compensation is more effective when played in pure strategies, as it is realized with probability one:  $E(v)$  is higher in the asymmetric equilibria in pure strategies than in symmetric equilibrium in mixed strategies. Then, in what follows we take the level of  $E(v)^*$  as our benchmark.

**Definition 2** *Any equilibrium that yields  $E(v)^* = \frac{1}{2}[1 - (1 - \alpha)^{n+1}]$  is defined optimal.*

Notice that  $(1 - \alpha)^{n+1}$  is the probability that the decision is wrong, given by the probability that all the members are uninformed, plus the probability that the only informed members are those voting according to the actual state even if uninformed (those choosing  $m_i = 1$  if uninformed when the actual state is  $v = 1$ , or those choosing  $m_i = 0$  if uninformed when the actual state is  $v = -1$ ).<sup>11</sup>

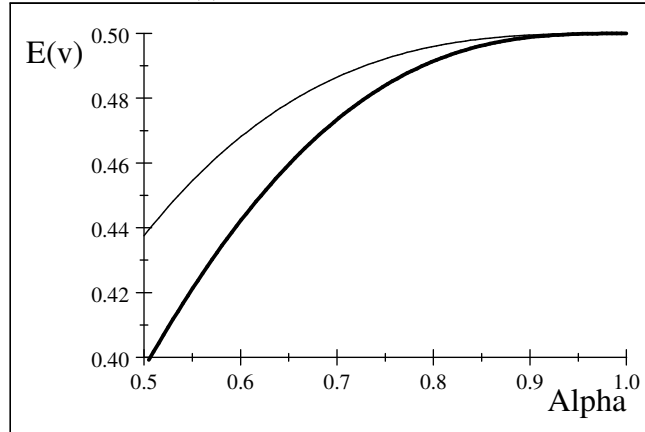
Just for illustration, we draw in Graph 1 and 2 the relationship between  $E(v)$  and the probability of having informed members ( $\alpha$ ), in committees with five and nine members. In both graphs, we compare the optimal pure strategy equilibrium outcome (thin line) with the

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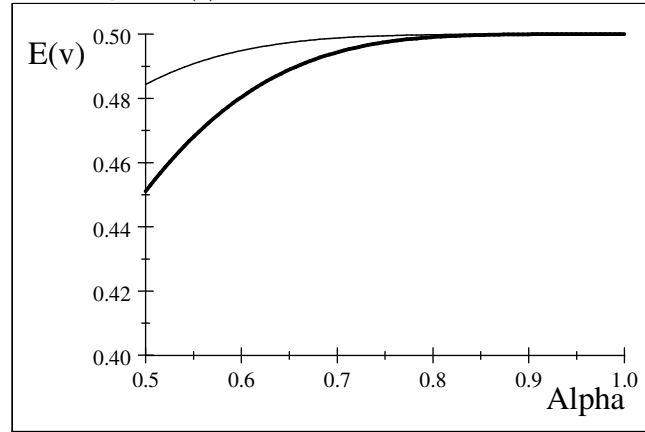
<sup>11</sup>That  $(1 - \alpha)^{n+1}$  represents the probability of making the wrong decision is shown in the proof of Proposition 1, point iii).

symmetric mixed strategy equilibrium outcome (thick line).

Graph 1:  $E(v)$  and  $\alpha$  in the five-member committee



Graph 2:  $E(v)$  and  $\alpha$  in the nine-member committee



The graphs show a positive relationship between  $E(v)$  and  $\alpha$  in both equilibria. They also show that the mixed strategy equilibrium never yields a higher expected value than the pure strategy one. When all the members are informed ( $\alpha = 1$ ), there is no difference between the two equilibria and  $E(v)$  is the same. It is also clear that  $E(v)$  is growing in  $n$ . These relations are formalized by Corollary 1 which immediately follows from  $E(v)^* = \frac{1}{2}[1 - (1 - \alpha)^{n+1}]$

**Corollary 1** *When an optimal equilibrium is played,  $E(v)^*$  is increasing both in  $n$  and in  $\alpha$  at a decreasing rate.*

The positive relation between  $E(v)^*$  and  $n$  may induce into thinking that the optimal size of the committee is unbounded. In the present paper we do not address the issue of the optimal committee size. We take the size as exogenous since it is likely to be determined on the ground of other criteria than the optimality of the voting behavior of the committee<sup>12</sup>.

<sup>12</sup>For example, the need to represent different stakeholders or to balance different powers or just plain political criteria.

Notice however that a limit to the optimal size of the board is immediately imposed if we remove our simplifying assumption that information is costless. Consider for example the case where each member has to pay a cost  $c$  in order to obtain information with probability  $\alpha$ . Given that the expected value is increasing in  $n$  at a decreasing rate, the size of the committee must not be too large in order for all the members to acquire information<sup>13</sup>.

## 5 Heterogeneous preferences

Having defined the optimal equilibrium, we compare this benchmark to the outcome of a committee composed of members with heterogeneous preferences. Consider again a committee with five members ( $n = 2$ ), in the case where  $\alpha = \frac{1}{2}$ , but let now members be either of type  $M$  or of  $B$ . If  $B$  members hold the majority, the case is trivial because the committee always approves the project and the  $M$  members are never pivotal. Then, we concentrate on the remaining two interesting cases in which the committee is composed of : i) four value-maximizing members  $M_i$ ,  $i = 1, 2, 3, 4$ , and one biased member  $B_5$ ; and ii) three value-maximizing members  $M_i$ ,  $i = 1, 2, 3$ , and two biased members  $B_j$ ,  $j = 4, 5$ .

We start from the latter case and we show that there exists a unique equilibrium. Given that  $B$  members always vote “yes”, independently of their information,  $M$  members vote according to their information whenever informed, and vote “no” when they are uninformed. Such rejection is optimal, because the probability that an uninformed member is pivotal is higher when the state of the world is  $L$ . In this case, we can easily compute:

$$E(v)_{M=B+1} = \frac{1}{2}[Y(\cdot|v = 1) - Y(\cdot|v = -1)] = \frac{1}{2} \left[ \frac{7}{8} - 0 \right] = \frac{7}{16}$$

Quite surprisingly, the performance of this committee is the same as the optimal performance of the committee composed of unbiased members only. In addition, this equilibrium is unique. Thus, we can say that the heterogeneous committee *ensures* the optimal outcome, provided that the  $M$  members outnumber  $B$  members by just one vote.

Consider now the case where the committee is composed of  $M_i$ ,  $i = 1, 2, 3, 4$ , and  $B_5$ . Recall that the dominant strategy of an informed  $M$  member is to vote according to his information, and that of the unique  $B$  member is to always approve the project. Then, two types of equilibria emerge, differing as to the behavior of uninformed  $M$  members. In the equilibria of the first type, two of the  $M$  members compensate for each other in pure

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<sup>13</sup>Since we know from the proof of Corollary 1 that the increase in the expected value is equal to  $\frac{\alpha}{2}(1-\alpha)^{n+1}$ , it immediately appears that the size of the committee must not exceed  $2\bar{n} + 1$  where  $\bar{n}$  is the smallest integer such that:  $\frac{\alpha}{2}(1-\alpha)^{n+1} \geq c$ . See Balduzzi, Graziano, Luporini [2011]. The idea that information costs may set a limit to the size of a committee is not new in the literature. See for example Harris and Raviv (2008).

strategies when uninformed, and the other two  $M$  members vote “no”. In the equilibrium of the second type, all of the four  $M$  members play the same mixed strategy, when uninformed.

While the equilibrium of the first type is optimal, yielding expected value

$$E(v)_{PS_{M>B+1}} = \frac{1}{2}[Y(\cdot|v = 1) - Y(\cdot|v = -1)] = \frac{1}{2} \left[ \frac{7}{8} - 0 \right] = \frac{7}{16}$$

the equilibrium of the second type is suboptimal. We find in fact the following solution:  $m_1 = m_2 = m_3 = m_4 = \frac{5-\sqrt{13}}{6}$ , and

$$E(v)_{MS_{M>B+1}} = \frac{1}{2}[Y(\cdot|v = 1) - Y(\cdot|v = -1)] \simeq \frac{197}{512} < \frac{7}{16}$$

Intuitively, when the  $M$  members outnumber the  $B$  members by more than one, the former have two possible choices. Either one single member offsets the bias of  $B_5$  in pure strategies and the remaining members compensate for each other in pure strategies, or all of the  $M$  members play the same mixed strategy with the aim to collectively contrast the bias of  $B_5$ . This is the reason why the symmetric mixed strategy of the  $M$  members in the second type of equilibrium is now biased towards rejection,  $m_i > 1/2$ ,  $i = 1, 2, 3, 4$ .

Proposition 2 generalizes these results.

**Proposition 2** *Consider a committee with  $2n + 1$  members where members of type  $B$  always approve the project and informed members of type  $M$  always vote according to their information. We can distinguish two cases:*

*i) if there are  $n$  members of type  $B$  and  $n + 1$  members of type  $M$  the game has a unique equilibrium in which all the members of type  $M$  always vote “no” when uninformed. This unique equilibrium is optimal;*

*ii) if there are  $n - k$  members of type  $B$  ( $n > k > 0$ ) and  $n + 1 + k$  members of type  $M$ , the voting game may have multiple equilibria. There always exists an optimal equilibrium where  $2k$  members of type  $M$  compensate for each other in pure strategies and the remaining  $n - k + 1$  members of type  $M$  vote “no” when uninformed.*

From the above proposition, Corollary 2 immediately follows.

**Corollary 2** *The expected value  $E(v)^*$  is not increased by increasing the proportion of value-maximizing members above  $\frac{n + 1}{2n + 1}$ .*

For a given size of the committee, increasing the proportion of the  $M$  members is not profitable, provided they already hold the majority. By increasing the proportion of the  $M$  members, optimal equilibria can still be obtained but there may also exist other kind of

equilibria. Thus, if value-maximizing members outnumber biased members by only one vote, the situation is greatly simplified with respect to our benchmark case because the optimal equilibrium is unique.

We know from Corollary 1 that increasing the size of the committee (increasing  $n$ ), increases  $E(v)^*$ , when the optimal equilibrium is played. Since an increase in  $n$  may now result in an increase in the number of the  $B$  members, this point deserves some attention. Consider the case with  $n$  members of type  $B$  and  $n + 1$  members of type  $M$  and recall that

$$E(v) = \frac{1}{2}[Y(\cdot|v = 1) - Y(\cdot|v = -1)].$$

The probability of approving the project when the state of nature is unfavorable  $Y(\cdot|v = -1)$  is still equal to zero after an increase in  $n$ , because the effect of additional biased members is compensated for by the  $M$  members voting “no” when uninformed. On the contrary, the probability of approving the project when it is profitable,  $Y(\cdot|v = 1)$ , increases with  $n$ , because it is equal to the probability that at least one of the  $n + 1$  members of type  $M$  is informed. Hence adding new members (including biased ones) is profitable<sup>14</sup>.

## 6 Voting outcome with communication

So far we have assumed that the members of the committee do not communicate prior to voting. On the one hand, this is clearly not crucial for biased members; on the other hand, unbiased members may have a strong incentive to share their information. In this section, we examine the effect of communication in a committee with both types of members.

Communication is introduced as a pre-voting stage where members of type  $M$  send costless messages about their information sets. Recall that the information set of a generic member  $i$  is  $\Omega_i = \{\omega_i\}$ , with  $\omega_i \in \{H; L\}$ , when  $i$  is informed, and  $\Omega_i = \{H, L\}$  when  $i$  is uninformed. Consequently, member  $i$  can send a message  $\sigma_i \in \{\omega_i; 0\}$ , where  $\sigma_i = 0$  means that  $i$  sends no information. Messages are simultaneously exchanged among unbiased members and update their information sets<sup>15</sup>. Finally, we assume that an informed member, whenever indifferent, sends a truthful message.

When communication is possible, equilibrium strategies are as follows.

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<sup>14</sup>Clearly our comment on the optimal size of the committee following Corollary 1 still applies.

<sup>15</sup>Alternatively we can assume that messages are exchanged among all the members and enter everybody's information set. Notice however that biased members cannot commit to send truthful messages because of their strong bias. Thus, they would never be believed. This is equivalent to assuming that they do not send any message, i.e.  $\sigma_B = 0$ . On the other hand members of type  $B$ , given their preferences, would not change their strategies even if they received a message revealing that the state of nature is  $L$ . For these reasons we focus on the message strategies of the  $M$  members.

**Proposition 3** *In a committee with  $n$  members of type  $B$  and  $n + 1$  members of type  $M$  who can communicate, there exist multiple equilibria all of which yield the optimal outcome  $E(v)^* = \frac{1}{2}[1 - (1 - \alpha)^{n+1}]$ . Each informed  $M$  member sends a truthful message and then votes according to his information; each uninformed  $M$  member receiving at least one  $\sigma_i = L$  votes according to the received message(s), and indifferently votes either “yes” or “no” otherwise.*

Quite surprisingly, communication does not improve the outcome of the voting game as long as the number of unbiased members is  $n + 1$  and the number of biased members is  $n$ . Propositions 2 and 3 imply that the expected value of the project reaches the same level  $E(v)^* = \frac{1}{2}[1 - (1 - \alpha)^{n+1}]$  independently of communication.

If the number of unbiased members is kept constant, the effect of allowing communication is null because the right decision is made with probability 1 when at least one  $M$  member is informed and with probability  $\frac{1}{2}$  when no  $M$  member is informed. But this is precisely what happens in the case without communication. Indeed, the voting strategies of the  $M$  members in the case with no communication (contrasting biased members and leaving the decision to possibly informed members) minimize the information required to reach the best possible outcome,  $E(v)^*$ .

The introduction of communication results in the expansion of the set of equilibria. When no information is revealed,  $M$  members now know that nobody is actually informed and thus have no strategic reason to contrast biased members and make other unbiased members pivotal. They can indifferently cast any vote and thus multiple equilibria arise: there now also exist equilibria where some unbiased members vote “no” after observing  $\sigma_i = H$ . These additional equilibria may entail an unconvincing behavior on the part of the  $M$  members, nonetheless they all yield<sup>16</sup>  $E(v)^* = \frac{1}{2}[1 - (1 - \alpha)^{n+1}]$ .

Another difference with the no communication case lies in the fact that the proportion of  $B$  and  $M$  members is now undetermined: as long as the latter hold the majority, the proportion is irrelevant for the optimal equilibrium to be reached. When communication is not allowed, biased members play the role of a coordination device for uninformed  $M$  members: as all  $M$  members vote “no” when uninformed,  $E(v)^*$  is reached. To this end, it is crucial that  $M$  members outnumber  $B$  members exactly by one. When communication is possible, instead, there is no need of a coordination device. It suffices that one member of type  $M$  observes the state of nature, for are all the unbiased members to vote correctly. We can then conclude that, for a given number of  $M$  members, communication cannot improve on already optimal equilibria but rules out sub-optimal ones. As there is a positive relation

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<sup>16</sup>Notice that, contrary to what happens in the type ii) equilibria of proposition 1, this multiplicity does not entail any coordination problem: whatever the choice of the  $M$  members receiving message  $\sigma_i = H$ , an equilibrium with expected value  $E(v)^*$  is reached.



between  $n$  and  $E(v)^*$ , it is now profitable to raise the proportion of unbiased members as much as possible.

## 7 Conclusions

We have analyzed the voting behavior of a small committee that has to approve or reject a project whose return is uncertain. Members have heterogeneous preferences: some members want to maximize the expected value while others have a bias towards project approval and disregard their private information. More precisely, we have shown that, in the absence of communication among members, heterogeneous committees can function at least as well as committees with homogeneous value-maximizing members. In particular, when value maximizers outnumber biased members by just one vote, the presence of biased members can improve the voting outcome by simplifying the strategies of the value maximizers: the equilibrium becomes unique and yields the optimal outcome. Increasing the number of value-maximizing members above the minimum that ensures majority does not increase the expected value and gives rise to additional suboptimal equilibria. For a given number of value maximizers, even allowing for communication among members does not improve the outcome. With communication, however, the number of biased members becomes irrelevant, provided they still are the minority. Thus, our model suggests that committee composition is particularly important when communication among members is limited.

Despite being quite simple, we believe our framework can be easily applied to explain voting behaviors in a number of different small decisional bodies such as monetary policy committees, juries, boards of directors, and so on. In all of these committees, it is not uncommon to observe dissent voting. We explain such dissent as the result of optimal voting strategies, given an optimal composition rule of the committee itself. Furthermore, our result shows that the composition actually used in some small committees (for instance, in the Bank of England Monetary Policy Committee or the Italian Constitutional Court) is optimal if members are diverse and communication is limited.

## 8 Appendix

### 8.1 Proof of Proposition 1

First, each informed member votes according to his information, as this maximizes the probability of making the correct decision. Thus, in what follows we only focus on the voting strategies of uninformed members. Second, recall that value-maximizing members choose

their strategies as if they were pivotal, as what they do when they are not pivotal is irrelevant for the voting outcome. Thus, we concentrate on equilibria in weakly dominant strategies. Third, we focus on equilibria where either all the members play pure strategies or all the members play mixed strategies.

Considering a committee composed of  $2n + 1$  members of type  $M$ , we prove that there only exist i) multiple equilibria with compensation in pure strategies ii) a unique equilibrium with compensation in mixed strategies. Finally, we prove that equilibria of type i) maximize  $E(v)$ .

**i) There exist multiple equilibria where  $n$  members choose  $m_j = 1$ ,  $n$  members choose  $m_z = 0$ , and one member, denoted by  $M_i$   $i \neq j, z$ , chooses  $m_i \in \{0, 1\}$ .**

We prove the existence of these equilibria in four steps. First we prove that player  $i$  is voting optimally, given the strategies of the other  $2n$  players; then we prove that the other  $2n$  members are voting optimally as well (steps 2 and 3). Finally, we prove that these are the only equilibria in pure strategies of the game.

1. *If  $n$  members choose  $m_j = 1$ ,  $n$  members choose  $m_z = 0$ , the best response of  $M_i$ ,  $i \neq j, z$ , is to choose  $m_i \in \{0, 1\}$ .*

When  $n$  members are voting “yes” and  $n$  members are voting “no”  $M_i$  is pivotal in both states of the world with the same probability. Indeed, when  $v = 1$ ,  $M_i$  is pivotal when everybody else is uninformed or when the only informed members are those  $n$  members who would vote “yes” even if uninformed (thus not changing their votes whether informed or not). As the former case (everybody is uninformed) is a subcase of the latter, the probability that  $M_i$  is pivotal is

$$(1 - \alpha)^n \left[ \sum_{j=0}^n \frac{n!}{j!(n-j)!} \alpha^{n-j} (1 - \alpha)^j \right] = (1 - \alpha)^n.$$

where  $\frac{n!}{j!(n-j)!}$  represents the number of combination with  $n - j$  informed members among the  $n$  members who vote “yes” if uninformed, and the term in brackets is equal to 1 from the binomial theorem. When  $v = -1$ ,  $M_i$  is pivotal when everybody else is uninformed or when the only informed members are those who would vote “no” even if uninformed. Then the probability that  $M_i$  is pivotal is again  $(1 - \alpha)^n$ .

Hence,  $M_i$  is indifferent between the two possible values of  $m_i \in \{0, 1\}$ .

2. *If  $n$  members choose  $m_j = 1$ ,  $n - 1$  members choose  $m_z = 0$ , and member  $M_i$ ,  $i \neq j, z$ , chooses  $m_i = 0$  the best response of the remaining member (denoted by  $k$ ,  $k \neq i, j, z$ ) is to choose  $m_k \in \{0, 1\}$ ; if  $M_i$  chooses  $m_i = 1$  the best response of  $M_k$  is to choose  $m_k = 0$ .*

If  $M_i$  chooses  $m_i = 0$ , we are back to point 1. So the optimal response of the remaining  $M_k$  is  $m_k \in \{0, 1\}$ . If instead  $M_i$  chooses  $m_i = 1$ , then  $M_k$  is pivotal only when  $v = -1$ . Consequently, choosing  $m_k = 0$  is a weakly dominant strategy for  $M_k$ .

3. *If  $n - 1$  members choose  $m_j = 1$ ,  $n$  members choose  $m_z = 0$ , and member  $M_i$ ,  $i \neq j, z$ ,*

chooses  $m_i = 1$  the best response of the remaining member (denoted by  $k$ ,  $k \neq i, j, z$ ) is to choose  $m_k \in \{0, 1\}$ ; if  $M_i$  chooses  $m_i = 0$  the best response of  $M_k$  is to choose  $m_k = 1$ .

The argument is symmetric to the one used at point 2.

Finally, note that any member can be in the position of  $M_i$ , or in that of an  $M_j$  voting “yes” or also in that of an  $M_z$  voting “no” when uninformed. Thus, there is a multiplicity of equilibria such as the one we are considering.

4. *There are no other equilibria in pure strategies.*

Consider what happens if more than  $n$  members vote “yes”, i.e. suppose  $n + k$  members ( $k \in \{1, 2, 3, \dots, n - 1\}$ ) choose  $m_j = 1$ . Then every remaining member knows that he is pivotal with a higher probability when  $v = -1$ . Hence, the remaining  $n - k - 1$  members choose  $m_z = 0$ . However, this cannot be an equilibrium. Also members voting  $m_j = 1$  know that they are pivotal with a higher probability when  $v = -1$ . Hence, as long as more than  $n$  members still vote “yes” when uninformed ( $k \neq 0$ ), they have an incentive to change their strategy and vote “no” when uninformed.

A symmetric argument can be used to analyze what happens if more than  $n$  members vote “no” when uninformed, i.e. if  $n + k$  members ( $k \in \{1, 2, 3, \dots, n - 1\}$ ) choose  $m_z = 0$  and consequently to rule out the existence of other equilibria in pure strategies.

**ii): There exists a unique equilibrium in mixed strategies where all the  $2n + 1$  members choose  $m_j = \frac{1}{2}$ .**

We prove the existence of this equilibrium in two steps.

1. *If  $2n$  members choose  $m_j = \frac{1}{2}$ , the best response of the remaining member (denoted by  $i$ ,  $i \neq j$ ) is to choose  $m_i = \frac{1}{2}$ .*

Both when  $v = 1$  and when  $v = -1$ ,  $M_i$  is pivotal if a) everybody is uninformed and  $n$  members vote “yes” while the other  $n$  members vote “no”, or b) no more than  $n$  members are informed and vote accordingly, while uninformed members vote in such a way that results in  $n$  members voting “yes” and  $n$  members voting “no”. Given that the other members choose  $m_j = \frac{1}{2}$ ,  $M_i$  is pivotal with the same probability in both states of the world. Since both states are equally possible,  $M_i$  is then indifferent among any  $m_i \in [0, 1]$ .

As this holds true for every member, it immediately follows that  $m_j = \frac{1}{2}$  for  $j = 1, \dots, 2n + 1$ , sustains an equilibrium of the game.

2. *If at least one member chooses  $m_i \neq \frac{1}{2}$ , there is at least one member who has a pure strategy as his best response. Consequently there cannot exist an equilibrium in mixed strategies with  $m_i \neq \frac{1}{2}$  for one or more members*

If  $M_i$  were to choose  $m_i > \frac{1}{2}$  while  $2n - 1$  members choose  $m_j = \frac{1}{2}$ , the best response of the remaining member denoted by  $k$ ,  $k \neq i, j$ , would be  $m_k = 0$ , because  $M_k$  would be pivotal with a higher probability when  $v = -1$  than when  $v = 1$ . With  $2n - 1$  members choosing

$m_j = \frac{1}{2}$  and  $M_k$  choosing  $m_k = 0$ , however the best response of  $M_i$  becomes  $m_i = 1$ , because  $M_i$  would be pivotal with a higher probability in  $v = 1$  than in  $v = -1$ .

Symmetrically if  $M_i$  were to choose  $m_i < \frac{1}{2}$ , while  $2n - 1$  members choose  $m_j = \frac{1}{2}$ , the best response of the remaining member denoted by  $k \neq i, j$ , would be  $m_k = 1$ , and with  $2n - 1$  members choosing  $m_j = \frac{1}{2}$  and  $M_k$  choosing  $m_k = 1$ , the best response of  $M_i$  becomes  $m_i = 0$ .

A similar argument holds true if member  $M_i$  choosing  $m_i > \frac{1}{2}$  were compensated by another member, denoted by  $h$ , choosing  $m_h < \frac{1}{2}$  and such that  $m_i + m_h = 1$ . In this case no other member has an incentive to deviate from  $m_j = \frac{1}{2}$ , but it immediately appears that  $m_h < \frac{1}{2}$  is not a best response. Given that  $M_h$  is pivotal with a higher probability in  $v = -1$  than in  $v = 1$ , his best response is  $m_h = 0$ .

More generally, by applying the same line of reasoning, it can be verified that there cannot exist an equilibrium with  $m_i \neq \frac{1}{2}$  for at least one  $i$ , because as soon as one or more agents choose  $m_i \neq \frac{1}{2}$ , there is at least one agent (possibly one of those choosing  $m_i \neq \frac{1}{2}$ ) who has a pure strategy as his best response. Hence the only equilibrium in pure strategies is the one with  $m_i = \frac{1}{2}$  for  $i = 1, 2, \dots, 2n + 1$ .

### iii) The equilibria in pure strategies maximize $E(v)$

In order to compare the expected value obtained in a pure strategy equilibrium to that obtained in the mixed strategy one, consider that  $E(v)$  can be written as

$$\begin{aligned} E(v) &= \frac{1}{2}[Y(\cdot|v = 1) - Y(\cdot|v = -1)] \\ &= \frac{1}{2}[1 - Y(\cdot|v = -1) - (1 - Y(\cdot|v = 1))]. \end{aligned}$$

where  $Y(\cdot|v = -1) + (1 - Y(\cdot|v = 1))$  is the probability of making the wrong decision.

We first show that in the pure strategy equilibria  $E(v)$  is equal to

$$E(v)_{PS} = \frac{1}{2}[1 - (1 - \alpha)^{n+1}].$$

Consider the specification of the pure strategy equilibrium where  $2n$  members compensate for each other in pure strategies and the remaining member chooses  $m_z = 0$ . In this case  $Y(\cdot|v = -1) = 0$  while  $(1 - Y(\cdot|v = 1)) = (1 - \alpha)^{n+1}$  as the probability of rejecting the project when  $v = 1$  is equal to the probability that those  $n + 1$  members who choose  $m_z = 0$  are uninformed (see point i). Symmetrically, in the pure strategy equilibrium where  $2n$  members compensate for each other in pure strategies and the remaining member chooses  $m_j = 1$ ,  $Y(\cdot|v = -1) = (1 - \alpha)^{n+1}$  while  $(1 - Y(\cdot|v = 1)) = 0$ . Hence  $E(v)_{PS} = \frac{1}{2}[1 - (1 - \alpha)^{n+1}]$ .

In the mixed strategy equilibrium it is instead

$$E(v)_{MS} = \frac{1}{2} \left[ 1 - 2(1 - \alpha)^{n+1} \left( \frac{1}{2} \right)^{n+1} \sum_{i=0}^n \frac{(2n+1)!}{i!(2n+1-i)!} \left( \alpha + \frac{(1-\alpha)}{2} \right)^i \left( \frac{(1-\alpha)}{2} \right)^{n-i} \right]$$

where

$$(1 - \alpha)^{n+1} \left( \frac{1}{2} \right)^{n+1} \sum_{i=0}^n \frac{(2n+1)!}{i!(2n+1-i)!} \left( \alpha + \frac{(1-\alpha)}{2} \right)^i \left( \frac{(1-\alpha)}{2} \right)^{n-i}.$$

is  $Y(\cdot|v = -1) = (1 - Y(\cdot|v = 1))$ , i.e. the probability that at least  $n + 1$  members do not vote correctly in one of the two states of nature.

Considering that

$$\begin{aligned} \left( \frac{1}{2} \right)^n \sum_{i=0}^n \frac{(2n+1)!}{i!(2n+1-i)!} \left( \alpha + \frac{(1-\alpha)}{2} \right)^i \left( \frac{(1-\alpha)}{2} \right)^{n-i} &> \\ &> \left( \frac{1}{2} \right)^n \frac{(2n+1)!}{i!(2n+1-i)!} \left( \alpha + \frac{(1-\alpha)}{2} \right)^n > 1, \end{aligned}$$

it follows that  $E(v)_{MS} < E(v)_{PS}$ .

## 8.2 Proof of Corollary 1

The Corollary follows immediately considering that

$$\frac{\partial[E(v)^*]}{\partial\alpha} = \frac{n+1}{2}(1-\alpha)^n > 0, \quad \frac{\partial[E(v)^*]^2}{\partial\alpha^2} = -\frac{n(n+1)}{2}(1-\alpha)^{n-1};$$

and that the marginal expected value when  $n$  increases is

$$\Delta_{2n+1}E(v)_{PS} \equiv \frac{1}{2}[1 - (1 - \alpha)^{(n+1)+1}] - \frac{1}{2}[1 - (1 - \alpha)^{n+1}] = \frac{\alpha}{2}(1 - \alpha)^{n+1}$$

which is clearly decreasing in  $n$ .

## 8.3 Proof of Proposition 2

First, each informed  $M$  member votes according to his information, as this maximizes the probability of making the correct decision. Thus, in what follows we only focus on the voting strategies of uninformed members. Second, recall that  $M$  members choose their strategies as if they were pivotal, as what they do when they are not pivotal is irrelevant for the voting outcome. Thus, we concentrate on equilibria in weakly dominant strategies. Thirdly, we focus on equilibria where either all of the  $M$  members choose pure strategies or all of the  $M$

members choose mixed strategies. The proof is organized as follows; we prove that:

i) if the committee is composed of  $n+1$  value-maximizing members and  $n$  biased members, there exists a unique equilibrium where each  $M$  member votes “no” when uninformed;

ii<sub>1</sub>) if the committee is composed of  $n+1+k$  value-maximizing members and  $n-k$  biased members, there always exist equilibria where  $n-k$  value-maximizing members vote “no” when uninformed and the remaining  $2k$  value-maximizing members compensate for each other in pure strategies. These are the only pure strategy equilibria of the voting game;

ii<sub>2</sub>) if the committee is composed of  $n+1+k$  value-maximizing members and  $n-k$  biased members, there may exist equilibria where all the  $n+1+k$  value-maximizing members play the same mixed strategy;

iii) the equilibria in pure strategies (sub i) and sub ii<sub>1</sub>) are optimal.

**i) In a committee with  $n+1$  value-maximizing members and  $n$  biased members there exists a unique equilibrium where all the  $M$  members vote “no” whenever uninformed** (that is,  $m_i = 0; i = 1, 2, \dots, n+1$ ).

Consider member  $M_{n+1}$ .

When  $v = 1$ ,  $M_{n+1}$  is pivotal only if all the other  $M$  members are uninformed and vote “no”, which happens with probability:

$$(1 - \alpha)^n \prod_{i=1}^n (1 - m_i)$$

When  $v = -1$ ,  $M_{n+1}$  is pivotal if:

- all the other  $M$  members are uninformed and vote “no”, which happens with probability

$$(1 - \alpha)^n \prod_{i=1}^n (1 - m_i),$$

- all the other  $M$  members are informed, which happens with probability

$$\alpha^n,$$

- at least one (but not all) of the other  $M$  members is informed and the others vote “no” when uninformed, which happens with probability

$$\sum_{h=1}^n \frac{n!}{h!(n-h)!} \alpha^{n-h} (1 - \alpha)^h \prod_{i=1}^h (1 - m_i).$$

where  $\frac{n!}{h!(n-h)!}$  represents the number of combination with  $h$  uninformed value-maximizing

members and  $n - h$  informed value-maximizing members. It is straightforward that  $M_{n+1}$  is pivotal with a higher probability when  $v = -1$ . Hence  $M_{n+1}$  chooses  $m_{n+1} = 0$ . As the same reasoning holds for any other value-maximizing member  $i \neq n + 1$ , it follows that every  $M$  member will vote “no” when uninformed.

Finally, note that we have not restricted  $m_i$ ,  $i \neq n + 1$ , to any particular value, so the result also proves that this equilibrium is unique.

**ii<sub>1</sub>) In the case of  $n - k$  biased members ( $n > k > 0$ ) and  $n + 1 + k$  value-maximizing members, there exist multiple equilibria with  $n - k + 1$  value-maximizing members voting against the project and  $2k$  value-maximizing members compensating for each other.**

We prove the existence of these equilibria in three steps. In the first step, we prove that when  $n - k$  value-maximizing members vote against the project and  $2k$  value-maximizing members compensate for each other, the remaining  $M$  member has still an incentive to vote against the project; in the second step, we prove that when  $n - k$  value-maximizing members vote against the project to contrast the  $n - k$  insiders, and a majority of the other value-maximizing members also vote against the project, the remaining  $M$  member has an incentive to compensate, voting “yes”. Finally, we show that there are no other equilibria in pure strategies.

1. If  $n$  value-maximizing members choose  $m_z = 0$ , and  $k$  value-maximizing members choose  $m_j = 1$ , the best response of  $M_i$ ,  $i \neq j, z$ , is to choose  $m_i = 0$ .

When  $v = 1$ ,  $M_i$  is pivotal if all the value-maximizing members are uninformed or if at least one of those  $k$  value-maximizing members who choose  $m_j = 1$  when uninformed, is in fact informed. Thus,  $M_i$  is pivotal with probability

$$(1 - \alpha)^n \left[ \sum_{j=0}^k \frac{k!}{j!(k-j)!} \alpha^{k-j} (1 - \alpha)^j \right] = (1 - \alpha)^n$$

where  $\frac{k!}{j!(k-j)!}$  represents the number of combination with  $j$  uninformed value-maximizing members,  $k - j$  informed  $M$  members and the term in brackets is equal to 1 from the binomial theorem. When  $v = -1$ ,  $M_i$  is pivotal if all the  $M$  members are uninformed or if at least one of those  $n$  value-maximizing members who choose  $m_z = 0$  when uninformed, is in fact informed. Then  $M_i$  is pivotal with probability

$$(1 - \alpha)^k \left[ \sum_{z=0}^n \frac{n!}{z!(n-z)!} \alpha^{n-z} (1 - \alpha)^z \right] = (1 - \alpha)^k$$

Given that  $(1 - \alpha)^k > (1 - \alpha)^n$ , the probability that  $M_i$  is pivotal is higher when  $v = -1$

than when  $v = 1$ . Hence  $M_i$  chooses  $m_i = 0$ .

2. *If  $n + 1$  value-maximizing members choose  $m_z = 0$  and  $k - 1$  value-maximizing members choose  $m_j = 1$ , the best response of  $M_i$ ,  $i \neq j, z$  is to choose  $m_i = 1$*

When  $v = 1$ ,  $M_i$  is pivotal if only one of the  $n + 1$  value-maximizing members choosing  $m_z = 0$  is informed and votes “yes”. This happens with probability

$$(n + 1)(1 - \alpha)^n \alpha.$$

On the contrary,  $M_i$  is never pivotal when  $v = -1$ . Hence, he chooses  $m_i = 1$ .

Finally, note that any  $M$  member can be in the position of  $M_i$  or in that of an  $M_j$  voting “yes”, or also in that of an  $M_z$  voting “no”. Thus, there is a multiplicity of equilibria such as the one we are considering.

3. *There cannot exist other equilibria in pure strategies than those characterized at points 1 and 2.*

We must now consider what happens if either a) more than  $n$  value-maximizing members vote “no” and the others vote “yes”, or b) more than  $k$  value-maximizing members vote “yes” and the rest vote “no”.

a) *If  $n - h$  value-maximizing members choose  $m_z = 0$ , and  $k + h$  value-maximizing members choose  $m_j = 1$ ,  $n \geq h > 0$ , the best response of  $M_i$ ,  $i \neq j, z$ , is to choose  $m_i = 0$  because  $M_i$  is never pivotal when  $v = 1$  while he may be pivotal when  $v = -1$ . This happens in the case where  $h$  of those  $n + h$  value-maximizing members who choose  $m_j = 1$  if uninformed, are in fact informed. As this is true for any  $h > 0$ , we are back to the case examined at point 1 above.*

b) *If  $n + h$  value-maximizing members choose  $m_z = 0$ , and  $k - h$  value-maximizing members choose  $m_j = 1$ ,  $k \geq h > 1$ , the best response of  $M_i$ ,  $i \neq j, z$ , is to choose  $m_i = 1$  because  $M_i$  is never pivotal when  $v = -1$  while he may be pivotal when  $v = 1$ . This happens in the case where  $h$  of those  $n + h$  value-maximizing members who choose  $m_z = 0$  if uninformed, are in fact informed. As this is true for any  $h > 1$ , we are back to the case examined at point 2 above.*

**ii<sub>2</sub>) In the case of  $n - k$  biased members ( $n > k > 0$ ) and  $n + 1 + k$  value-maximizing members, there may exist equilibria where all the  $M$  members choose the same mixed strategy.**



In this case, each generic  $M$  member solves the following problem:

$$\begin{aligned} \max_{m_i} E(v) &= \frac{1}{2}[Y(\cdot|v=1) - Y(\cdot|v=-1)] \\ \text{s.t. } m_i &\in (0,1) \\ m_i &= m_j, \forall j \neq i, j = 1, 2, \dots, n+k+1 \\ B_z &\text{ always votes "yes" } z = 1, \dots, n-k \end{aligned}$$

where the first constraint refers to the fact that we are looking for a mixed strategy (i.e., an internal solution), the second constraint imposes the symmetry of this strategy, and the third constraint takes into account that any  $B$  member follows his dominant strategy.

We have solved this problem for a committee of five members, one of type  $B$  (hence,  $B_1$  always votes "yes") and the remaining four of type  $M$ . To make the maximization problem more explicit, we rewrite it as follows where  $\bar{m} \equiv m_1 = m_2 = m_3 = m_4$ :

$$\max_{\bar{m} \in (0,1)} \frac{1}{2}[Y(\cdot|v=1) - Y(\cdot|v=-1)],$$

where

$$Y(\cdot|v=1) = \sum_{i=1}^3 \frac{4!}{i!(5-i)!} [\alpha + (1-\alpha)\bar{m}]^{5-i} [(1-\alpha)(1-\bar{m})]^{i-1}$$

is the probability that the committee votes "yes" when  $v=1$ , member  $B$  always votes "yes", and all of the four  $M$  members choose the same mixed strategy  $\bar{m}$ . Analogously,

$$Y(\cdot|v=-1) = \sum_{i=1}^3 \frac{4!}{i!(5-i)!} [(1-\alpha)\bar{m}]^{5-i} [\alpha + (1-\alpha)(1-\bar{m})]^{i-1}.$$

The solution of the problem, evaluated at  $\alpha = \frac{1}{2}$ , is:

$$m_1 = m_2 = m_3 = m_4 = \frac{5 - \sqrt{13}}{6}.$$

We do not characterize the solution for different values of  $\alpha$ , committee size or composition as it is sufficient to our purpose to show that such an equilibrium may exist.

### iii) The equilibria in pure strategies are optimal

Recall that

$$E(v) = \frac{1}{2}[Y(\cdot|v=1) - Y(\cdot|v=-1)].$$

In the unique equilibrium of the case with  $n$  biased members and  $n+1$  value-maximizing members (point i), as well as in the pure strategy equilibrium of the case with  $n-k$  biased

members ( $n > k > 0$ ) and  $n + 1 + k$  value-maximizing members (point ii<sub>1</sub>),  $Y(\cdot|v = -1) = 0$  and  $Y(\cdot|v = 1)$  is equal to the probability that at least one of the  $n + 1$  members who choose  $m_i = 0$  is informed. Then in both cases  $E(v)$  is equal to

$$\frac{1}{2} \sum_{i=0}^n \frac{(n+1)!}{i!(n+1-i)!} \alpha^{n+1-i} (1-\alpha)^i = \frac{1}{2} [1 - (1-\alpha)^{n+1}]$$

implying that the pure-strategy equilibria are optimal.

## 8.4 Proof of Proposition 3

With probability  $(1-\alpha)^{n+1}$  no  $M_i$  is informed,  $i = 1, 2, \dots, n+1$ , whereas with probability  $1 - (1-\alpha)^{n+1}$ , at least one member, say  $M_j$ , obtains information and consequently sends a truthful message  $\sigma_{M_j} = \omega_{M_j}$ . In the latter case, subsequent voting strategies are straightforward:  $M_i$  votes according to his information and all  $M_{i \neq j}$  vote according to the received information. More precisely, if the revealed information is  $\sigma_{M_j} = L$ , each unbiased member is pivotal and votes “no”. On the contrary, if the revealed information is  $\sigma_{M_j} = H$ , those unbiased members who receive the message are no longer pivotal (as the correct decision has already been made) and, given that the sender votes “yes”, can cast any vote. The argument immediately generalizes to the case where more than one unbiased member obtains information.

When instead no member is informed, any  $M_i$  chooses  $m_i \in \{0, 1\}$ . Given equal priors about the states of the world, utility is independent of  $m_i$ , so any probability  $m_i \in \{0, 1\}$  is utility maximizing for  $M_i$ .

To formally characterize the equilibrium strategy of the generic member  $M_i$ , recall that  $s_{M_i}$  is  $M_i$ 's strategy profile. Thus, it follows that the equilibrium (weakly dominant) strategy (in terms of probability of voting “yes”) for  $M_i$  is:

$$s_{M_i}^* : \left\{ \begin{array}{l} 1 \mid \Omega_{M_i} = \{\omega_{M_i} = H\}; \\ 0 \mid \Omega_{M_i} = \{\omega_{M_i} = L\}; \Omega_{M_i} = \{\sigma_{M_j} = L\}; \\ \forall m_i \in \{0, 1\} \mid \Omega_{M_i} = \{\sigma_{M_j} = H\}; \Omega_{M_i} = \{H, L; 0\} \end{array} \right\}$$

where  $j \neq i$ ,  $i, j = 1, 2, \dots, n+1$ .

In order to show that all the equilibria are optimal, recall that

$$E(v) = \frac{1}{2} [Y(\cdot|v = 1) - Y(\cdot|v = -1)] = \frac{1}{2} [1 - Y(\cdot|v = -1) - (1 - Y(\cdot|v = 1))]$$

where  $Y(\cdot|v = -1) + (1 - Y(\cdot|v = 1))$  is the probability of making the wrong decision.

Considering this is now equal to the probability that no  $M$  member is informed,  $(1 - \alpha)^{n+1}$ , it immediately follows that

$$E(v) = \frac{1}{2} [1 - (1 - \alpha)^{n+1}] .$$

**Acknowledgments** We thank participants from the ASSET Conference 2008, the CEPET Workshop 2009, the EALE Conference 2009, CESifo Seminar in applied Micro 2010 and from seminars held at Stony Brook University, Catholic University of Milan, University of Milan-Bicocca for helpful suggestions. All mistakes are the authors' own responsibility. Financial help from IEF (Catholic University) and PRIN 2008 is gratefully acknowledged.

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