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Mobility and conflict

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Mobility and Conflict

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Mobility and Conflict

Abstract

We study the role of inter-group differences in the emergence of conflict. In our setting, two groups compete for the right to allocate society's resources, and we allow for costly intergroup mobility. The winning group offers an allocation, that the opposition can either accept, or reject and wage conflict. Expropriating a large share of resources increases political strength by attracting opposition members, but such economic exclusion implies lower per capita shares and higher risk of conflict. In equilibrium, allocations are non-monotonic in the cost of mobility. Moreover, limited commitment with respect to mobility gives rise to inefficient conflict in equilibrium.

JEL-Code: D720, D740, D780, P480.

Keywords: conflict, inter-group mobility, political competition, resource allocation.

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1 Introduction

Politics in divided societies often revolves around the line of social division. Group identities form the basis of political coalitions, and the state identity belongs more to one group than another. Frequently, the group in power engages in accumulating rent, and the opposition group members mobilize themselves in conflict to alter the balance of political power. Much of the existing literature recognizes that the presence of inter-group differences significantly affects the nature and frequency of political conflict. This claim derives its support from evidence of conflict along various social cleavages, such as race, ethnicity, religion, caste, language, geography or ideology. Moreover, the relationship between conflict and the presence of inter-group differences does not seem to be straightforward: For instance, there are examples where two groups are in violent conflict in some society, while groups divided along exactly the same lines co-exist peacefully in another (see for instance, Fearon and Laitin (1996), Posner (2004)). There also are examples of very dissimilar groups coexisting peacefully, while more similar groups engage in conflict. This leads to the central question we ask in this paper: When and how do inter-group differences become salient in political conflict?

We study divided societies in which political power involves gaining the decision rights over allocation of society’s resources. When one group gains power, it can allocate more surplus to itself by restricting the other group’s access. If resources are limited, the ruling group has a strong incentive to engage in such economic exclusion. Examples of group-based resource allocation are ubiquitous. A prime example is India, where different religious, caste-based groups compete for group-based reservation of limited resources, such as government jobs or access to higher education (See Chandra (2004)). In addition, there are examples of language being used as a basis of distributing economic resources (See Laitin (2007)). The main thesis of this paper is that the extent of inter-group differences affects the ruling group’s ability to practice group-based resource allocation, and these factors, in turn, determine the propensity of the groups to engage in conflict.

How do inter-group differences affect the ruling group’s ability to practice economic exclusion? In many contexts, group membership is an endogenous choice, and we measure the extent of inter-group differences by the cost to an individual of moving from one group to another. If a ruling group allocates resources based on group identities, its decision affects which group people in soci-


\[2\] Other examples include group transfers based on ethnicity, profession, geographic location or even party allegiance.

\[3\] Cost of mobility may be endogenous. For instance, groups can build very strong identities that make it hard for outsiders to penetrate, or impose a social cost on members who are likely to switch (Laitin (2007)). An example of the second type of behavior is the “acting white” phenomenon among African American and Hispanic students. Fryer and Torelli (2010) describe it as “a set of social interactions in which some minorities incur costs for investing in behavior characteristics of whites (e.g., raising their hand in class, making good grades, or having an interest in ballet).” Such peer-group effects go beyond the context of the black-white division and can be found along many other cleavages, including ethnicity or class (Fryer (2007)). This, in effect, increases the cost of mobility. In this paper, we focus on the cost of mobility in a given context, and so treat it as fixed.
ety want to belong to. For example, the allocation of jobs based on party allegiance may influence individuals’ choices of switching membership between parties. Redistribution of resources based on geography can affect the incentives for people to migrate. However, the ease or cost of mobility varies widely depending on the basis of social cleavage. For instance, with racially dissimilar ethnic groups, switching identities is intrinsically hard. In contrast, changing one’s allegiance to a political party is much easier. The ruling group can increase (decrease) its group size by retaining a disproportionately large (small) share of resources. On the one hand, the ruling group wants to increase its size in order to increase its political strength, and remain in power. But, on the other hand, an increased group size implies a smaller per capita share for the members. The trade-off between these two effects determines an optimal size that the ruling group wants to have. The feasibility of reaching this optimal size necessarily depends on the cost of mobility. This is how, in our framework, inter-group differences limit the extent of economic exclusion.

The extent of economic exclusion and inter-group mobility together affect the propensity of the opposition to engage in conflict. If the ruling group leaves a very low share for the opposition, this reduces the opposition’s opportunity cost of engaging in conflict. Thus, the opposition now is more inclined to engage in conflict to try and change the balance of power. If the groups’ sizes are such that the opposition has a high chance of overthrowing the incumbent through conflict, then the threat of conflict constrains the extent of economic exclusion. While the extent of economic exclusion is endogenous in our model, agents in society have two costly response mechanisms to improve their own payoffs: moving across groups and waging conflict. The substitutability between these two mechanisms is akin to the “exit and voice” mechanisms that have been studied in different socio-political contexts.

We develop a simple two-period model to analyze the resource allocation problem in a divided society in which the ruling group can allocate resources based on group identities. Society is divided into two groups that compete for political power. In each period, the ruling group gets elected either through a default political process or as a result of conflict. The ruling group earns the right to decide how society’s resources are divided between the two groups. At the start of each period, the ruling group proposes an allocation of resources. The opposition can choose to either accept its share or collectively engage in conflict. The opposition’s cost of conflict is an opportunity cost—it gives up

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4 Other examples include sectoral redistribution of resources between the agricultural and industrial sector affecting the opportunity costs of individuals and their decision to work in their respective sectors.

5 Mobility across ethnic groups can be by inter-racial/inter-ethnic marriages (Caselli and Coleman (2006)).

6 Bates (1983) emphasized this trade-off in his argument for the political salience of ethnicity: “Ethnic Groups are, in short, a form of minimum willing coalition, large enough to secure benefits in the competition for spoils but also small enough to maximize the per capita value of these benefits.”

7 Conflict is modeled as the opposition’s collective action to increase its own chance of gaining power compared to the default political process. In reality, the nature of collective action can be varied—ranging from peaceful political mobilization within the limits of accepted institutional norms to violent resistance. To draw examples from South Asia, the Dravidian movement, in which the backward castes organized electorally against the Brahminical control of the Indian National Congress by forming a party called DK (Dravidar Kazhagham) under Periyar E.V. Ramaswamy, is a case of peaceful mobilization in a democratic setup. At the other extreme, the Jaffna Tamils in Sri Lanka attempted to use
the opportunity to enjoy its share of surplus in the current period. For the incumbent, conflict implies a lower probability of retaining power and a potential loss of economic resources. If the opposition decides to accept the share offered by the ruling group and no conflict occurs, individuals (in both groups) can still choose whether they want to stay in their respective group or switch at an individual cost. If an agent switches groups, she gets a share of the new group’s resources. We characterize the resource allocations, group membership decisions and conflict decisions that arise in equilibrium.

We find that sharing does occur in equilibrium. The two mechanisms of conflict and mobility act as constraints to expropriation, and the optimal sharing is dictated by whether and which constraint binds. In the unique equilibrium of this model, three different regimes can arise. The first type of regime, which we call no-conflict regime, is one in which the opposition does not engage in conflict, and the ruling group allocates resources to induce the optimal amount of switching. The second possible regime is called open-conflict regime, and here, the ruling group keeps everything for itself. The opposition responds by engaging in conflict. Finally, there may be a peaceful-belligerence regime, in which the opposition does not engage in conflict, and the incumbent shares just enough resources with the opposition to prevent them from engaging in conflict.

Switching can occur in equilibrium in both the no-conflict and peaceful-belligerence regimes. The conflict constraint plays a role in the open-conflict and peaceful-belligerence regimes. On the one hand, in the open-conflict regime, both the ruler and the opposition get a higher payoff from conflict, and, therefore, conflict emerges. In the peaceful-belligerence regime, on the other hand, the ruler strictly prefers to avoid conflict, and so shares enough to make the opposition indifferent between conflict and no conflict. Our results also imply that the extent of sharing is non-monotonic in the cost of mobility. The share of resources that the incumbent retains is increasing, decreasing and constant with respect to the cost of mobility in the no-conflict, peaceful-belligerence and open-conflict regimes, respectively.

In our framework, inefficient conflict arises in equilibrium. There are two sources of conflict. One is limited commitment with respect to transfers: The ruling group cannot credibly commit today about the resource allocation it will offer in the next period. This is, in fact, a well-known reason for conflict to arise in standard models. However, one of the main contributions of this paper is to highlight a second independent explanation for conflict: limited commitment with respect to inter-group mobility. In other words, agents cannot credibly commit to not switching group membership after they see the proposed allocation, thus constraining the set of allocations that can be implemented. In particular, certain allocations that Pareto dominate the conflict outcome would require the incumbent group to retain its original size, and this cannot be guaranteed in equilibrium due to the lack of commitment with respect to switching. Thus, we show that endogenous mobility across

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8This mechanism is well studied in explaining democratic transition, coups (Acemoglu and Robinson (2000), Acemoglu and Robinson (2001b)) and civil wars (Fearon (1998)).
groups can increase the likelihood of conflict in society. This finding has two key implications.

On the one hand, when the possibility of endogeneous mobility is low, the incumbent may be able to implement an allocation rule that Pareto dominates the conflict outcome. Indeed, we find that open conflict does not necessarily emerge when the cost of mobility is high. If conflict is too costly for the incumbent, peaceful belligerence occurs in equilibrium. In other words, the ruling group prefers to share resources with the opposition to avoid conflict. It turns out that peaceful belligerence is more likely to occur when a majority rules. This result explains a documented feature of politics in divided societies that existing theory does not explain. Empirical evidence suggests many examples of societies divided along lines of ethnicity or race (in which cost of mobility is naturally high), where there is no conflict over resources, and, indeed, resource sharing occurs. To illustrate, one example is democratic politics in India, where there is a wide range of reservation policies for backward castes and religious minorities (by which economic resources are shared), that have mitigated the threat of conflict. Padró i Miquel (2007) also cites examples of some autocratic regimes (such as Houphouet-Boigny in Ivory Coast) where, somewhat surprisingly, rulers even from majority ethnic groups transfer resources to the opposition.

On the other hand, as the possibility of endogenous mobility increases, the incumbent is constrained in its ability to implement allocations that Pareto-dominate conflict. We find that open conflict can occur at an intermediate cost of mobility. This is an interesting result because, while the existing literature does explain why conflict can arise in ethnically divided societies (high cost of mobility), there is no theory about why we observe conflict in societies divided along factors such as language. In our framework, a high cost of mobility implies that the premium from gaining power in the future is high. This means that the opposition’s incentive to engage in conflict is high when the cost of mobility is high, and the ruling group’s incentive to induce conflict is high when cost of mobility is low. Therefore, open conflict occurs when the cost of mobility is in an intermediate range. We also show that a small group would be more prone to instigate conflict as its short-term per capita gain from full appropriation is high.

When the cost of mobility is sufficiently low, the opposition’s opportunity cost of conflict becomes high, as its members can switch their group identity at low cost. The model predicts that no conflict occurs when groups have a low cost of mobility and when the ruling group is more likely to retain power in conflict. In such situations, the mobility constraint dictates the optimal sharing rule. The group in power aims to maintain an optimal size, large enough to increase the probability of staying in power, but small enough to still have a high per capita share of resources. This optimal group size is endogenously determined, and if the initial size of the ruling group is below the optimal group size, we observe switching in equilibrium. Examples of switching towards the powerful group is not uncommon in history. Post-Reform Europe witnessed a series of religious switching (back and forth between Catholicism and Protestantism), depending on which denomination had the stronger political alliance. Caselli and Coleman (2006) obtain a result that is similar in spirit.
Our framework allows us to ask how much mobility across groups an incumbent would ideally permit, if this were an endogenous choice. For instance, people in society may differ in ethnicity and language, and the ruling group may be able to choose the dimension along which resources will be split. Since the cost of mobility effectively increases a group’s premium from being in power, we should expect ruling groups to always prefer a maximal cost of mobility. However, we find that incumbents may prefer a social division with an intermediate cost of mobility: This happens when conflict sufficiently reduces the chances of the incumbent retaining power.

1.1 Related Literature

This paper contributes to the large literature on conflict in divided societies. The existing literature argues that inter-group differences can matter in political coalition formation and, thereby, in political conflict. Fearon (2006) argues that inter-group heterogeneity and intra-group homogeneity help political entrepreneurs mobilize people based on group identities. Bates (1983) suggests that group identities matter for forming coalitions in distributional conflict over political goods. While this line of argument highlights the role of inter-group differences, it does not explain why certain group divisions matter more than others. Closer to our analysis are Fearon (1999) and Caselli and Coleman (2006), who consider the possibility of inter-group mobility. Fearon suggests that distributive politics favors coalitions based on unchangeable characteristics “because it makes excluding losers from the winning coalition relatively easy.” Caselli and Coleman (2006) are the first to develop a formal model that allows inter-group mobility. In their model, one group can exclude another from enjoying a public good, and the members of the excluded group may switch to the other group. Such exclusion is synonymous with intergroup conflict. Since switching reduces the spoils from exclusion, the authors find that the likelihood of conflict increases with the cost of mobility. In our model, economic exclusion and conflict are separate phenomena determined endogenously in equilibrium. We allow the ruling group to decide how to allocate resources in order to balance the probability of retention of power with increased current period payoffs. This helps us to understand how economic exclusion is linked to the risk of conflict and to the optimal group size. Here, the cost of mobility reflects the premium from gaining the authority to allocate resources: the prize that the groups are fighting for. Specifically, in a situation with a high cost of mobility, while the opposition has a strong incentive to engage in conflict to seize power, the incumbent wants to share resources to mitigate conflict. This tension can result in a peaceful-belligerence equilibrium—an aspect consistent with empirical observation, but not captured in previous work.

This paper is also connected to the literature on the relationship between conflict and measures of fragmentation in societies. One class of such measures depends on the distribution of group size alone. For example, the Hirschman-Herfindahl fractionalization index (Hirschman (1964)) is widely used in empirical studies on conflict.\(^9\) Subsequent work introduced polarization indices that

\(^9\) Though widely used, the empirical connection is not always strong (Collier and Hoeffler (2004), Fearon and Laitin
incorporate inter-group heterogeneity through a notion of inter-group distance (Esteban and Ray (1994)). Recent work by (Esteban and Ray (2011)) argues that fractionalization measures that do not depend on variations in inter-group differences cannot really capture the extent of division in societies, and find that the polarization measure is significant in predicting social conflict. We view our work as complementary to this literature. Our model suggests that measures of division in societies, as a predictor of conflict, must incorporate information on both group sizes and inter-group differences. In addition, we provide an explanation of why the contested prize may be increasing in inter-group differences. Specifically, if the winning group can exclude people from accessing economic resources based on group identities, the cost of inter-group mobility can provide us with a measure of the rent that the winning group can extract.

We also contribute to the literature on models of conflict and rent seeking (see Grossman (1991), Hirshleifer (1995), Azam (1995), Azam (2001), Esteban and Ray (1999), Esteban and Ray (2008) and others). However, our paper is substantively different in that we are interested in relating inter-group mobility to conflict. In a similar framework, Acemoglu and Robinson (2001a) develop a model in which two groups share resources and engage in two different kinds of economic activity. They find that the incumbent, even when engaged in a relatively inefficient mode of production, keeps more resources to itself to increase its political strength by attracting new entrants because of limited commitment. In our framework, we consider a symmetric production functions across groups. If we had considered an asymmetric production function, inefficient redistribution would have taken place in our model whenever the incumbent is engaged in less-efficient productive activities.

Finally, our work is also related to a vast empirical literature on inter-group conflict. Collier (2001) and Alesina and La Ferrara (2005) provide useful surveys of this literature. In our framework, conflict and economic rent seeking are simultaneously determined, and the equilibrium amount of rent seeking varies non-monotonically with respect to inter-group differences. These results have testable implications, and a systematic empirical analysis would be very interesting.

The rest of the paper is organized as follows. Section 2 contains the model. In Section 3, we characterize the resource allocations and the regimes that arise in equilibrium. In Section 4, we discuss the key implications and empirical predictions of our paper. Section 5 concludes. Most proofs are in the Appendix.

(2003), Miguel et al. (2004).


11Garfinkel and Skaperdas (2007) provide a comprehensive survey of this literature.

12Alesina et al. (1999) provide some evidence of a positive relationship between ethnic fragmentation and ethnically-based patronage. Guiso et al. (2009) and Spolaore and Wacziarg (2009) look at economic consequences of genetic distance. Though genetic distance is not necessarily a measure of inter-group mobility cost, it can reflect inter-group differences to an extent.
2 Model

Consider the following two-period game. There is a continuum of agents of measure 1. Members of society are divided into two groups $A$ and $B$. In each period, a fixed amount of resources (normalized to 1) must be divided between the two groups.\textsuperscript{13} Agents can participate in some economic activity, and the resources are productive inputs that agents can use to enhance their payoffs from economic activity.

Each period ($t = 1, 2$) starts with a ruling group $W_t$. (We use the terms ruling group, winning group and incumbent interchangeably). At the start of period 1, suppose that the size of the winning group is $\pi_0$. Without loss of generality, we assume that the group with political power in period 1 is group $A$. The winning group proposes a sharing rule $\alpha_t$, where $\alpha_t$ is the fraction of resources to be retained by the ruling group. Once the ruling group announces the split $\alpha_t$, the losing group (opposition) $L_t$ can choose to either accept its share or reject it.

If the sharing rule is accepted, each individual (in $W_t$ and $L_t$) decides whether to remain in his own group or to switch to the other group. Individuals can change groups at a cost $\phi \in [0, 1]$. The parameter $\phi$ measures how difficult it is to assimilate into a different group. The exact nature of the cost depends on the specific context. For example, $\phi$ may represent the cost associated with entry barriers such as language-based discrimination. In other contexts, $\phi$ may measure the extent to which groups are able to discriminate; for instance, it is easy to discriminate based on skin color or racial identity, making such groups hard to infiltrate (high $\phi$).\textsuperscript{14} Here, while switching groups is costly, the cost is bounded. In particular, $\phi \leq 1$ implies that if the ruling group keeps all resources for itself, the members of the other group would find it profitable to switch over. Since we are interested in isolating the effect of inter-group differences, agents are assumed to be homogeneous except for their initial group membership.

Clearly, if individuals switch group membership, this changes the size of the groups. Let $\pi_t$ and $1 - \pi_t$ denote the sizes of the groups at the end of period $t$. If a group of size $\pi_t$ gets fraction $\alpha_t$ of society’s resources, the per capita payoff that its members get from economic activity is given by $\frac{\alpha_t}{\pi_t}$ (the assumption of linear payoff from resources is made here for simplicity).\textsuperscript{15} At the end of the period, one group is chosen as the ruler for the next period through a default political process. We abstract from the institutional details of the political contest, and simply assume that the ruler $W_t$ remains in power with the probability $p_d(\pi_t)$. We assume that the political contest success function $p_d(\cdot)$ is increasing in group size $\pi \in [0, 1]$, and is continuous and twice differentiable. For tractability, we also assume that $p_d(\pi)(1 - \pi)$ is single-peaked, and the maximum is attained at

\textsuperscript{13}Our results are unchanged as long as the size of resources in each period is independent of the group sizes.

\textsuperscript{14}As mentioned before, in reality, $\phi$ may be endogenous: A group can decide to discriminate against members who have infiltrated from a different group and effectively increase the cost of mobility. In this paper, we take $\phi$ as exogenous.

\textsuperscript{15}We assume that a group’s resources are evenly divided among its members. In many contexts, it may be more reasonable to assume that resources are shared unequally, based on some hierarchy within the group. We do not address this issue here.
If the sharing rule is rejected, the ruling group retains all the resources, and the opposition engages in conflict. In terms of current-period payoffs, conflict is socially wasteful: A fraction \((1 - k)\) of the entire surplus gets destroyed. The opposition group gets zero economic payoff in the current period, and the incumbent group enjoys the remaining surplus. Conflict in our model can be interpreted as any kind of political activism undertaken by the opposition group that is costly to them in the short-run—such as violent protests, demonstrations, or mobilization of voters—but increases the probability of their becoming the ruler in the next period.

In case of conflict, individuals do not have the opportunity to switch groups, and so the size of the groups remains unchanged \((\pi_t = \pi_{t-1})\). At the end of the period, one group is chosen as the ruler for the next period. We assume that the ruling group stays in power with probability \(p_c(\pi_t)\). Conflict implies that the ruling group has a lower chance of getting elected relative to the default political process—i.e., \(p_c(\cdot) \leq p_d(\cdot)\). Engaging in conflict is a group decision taken by the

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16Our assumptions on \(p_d(\cdot)\) allow for many common political contest functions such as S-shaped contest functions and proportional representation functions. “First-past-the-post” functions are a limit case of the class of functions considered here.
opposition.\textsuperscript{17} Figure 1 gives a pictorial representation of the game.

The solution concept is sub-game perfect Nash equilibrium. Note that there are two kinds of decisions being made: The winning group makes a collective decision on the allocation rule, and the opposition makes a collective decision on whether or not to accept the proposed allocation. When groups make collective decisions, they seek to maximize the expected long-run payoff of their members.\textsuperscript{18} Since we consider a finite number of periods, we assume that the long-run payoff is simply the sum of per-period payoffs. However, group members make individual switching decisions (in case of acceptance), which are based on maximizing their short-term payoffs.\textsuperscript{19} We make the tie-breaking assumption that when the opposition is indifferent between accepting and rejecting an offer, it accepts.

3 Analysis

We solve the two-stage game by backward induction.

3.1 Equilibrium play in period 2

Consider play in period 2, after a ruling group has been chosen. Any subgame is now described by the identity and size of the group in power. Let $W_2 \in \{A, B\}$ denote the ruling group and let $\pi_1^{W_2}$ denote its size. To characterize equilibrium play, we proceed in three steps. We first characterize the switching rule in period 2 (and resulting group sizes) as a function of the announced allocation. Next, we show that conflict never arises in period 2. Finally, we characterize the optimal equilibrium allocation for the ruling group, and show that it induces no switching by either group in the second period.

First, consider the node where an allocation $\alpha_2^{W_2}$ proposed by the ruling group $W_2$ has been accepted by the opposition $L_2$. We want to characterize the group compositions on and off equilibrium path. It is easy to see that it is impossible to have a situation where members of both groups want to switch to the other group. Further, two conditions must be true: First, in equilibrium, members of neither group can have a strict incentive to switch to the other group, and second, if the group compositions are such that members of one group have a strict incentive to switch to the other group, the size of that group continues to decrease until the incentive to switch no longer exists.\textsuperscript{20} Notice

\textsuperscript{17}We ignore the collective-action problem here. Think of a leader being able to coordinate the decision to wage conflict.
\textsuperscript{18}In order to focus on the key issue, we ignore any collective-action problems despite assuming a continuum of agents. In our context, this is a reasonable assumption since individuals in a group are identical, and so decisions can be unanimous.
\textsuperscript{19}We interpret periods as generations and, hence, treat individual members as myopic and the groups as long-lived. The qualitative results are unchanged if we considered non-myopic agents. Please refer to Section 4.1 for a detailed discussion.
\textsuperscript{20}This description of equilibrium group sizes is similar to the long-run entry and exit conditions for firms in a perfectly competitive market.
that since the share of surplus remains unchanged, as individuals switch from, say, group A to group B, the per capita payoff of the members of group A increases and that of members of group B decreases. The two above conditions together imply that if there is switching (say, from A to B), the size of group A reduces to the point where the members are indifferent between switching and not switching.

The following lemma characterizes the group compositions that obtain in equilibrium at the end of period 2 (as a result of potential switching), for any given allocation $\alpha_2^W$.

**Lemma 1 (Group Switching Decisions in Period 2).** Suppose that the ruling group $W_2$ is of size $\pi_1^W$ at the start of period 2, and offers an allocation $\alpha_2^W$. Define functions $f(\pi) \equiv \pi + \phi\pi(1 - \pi)$ and $g(\pi) \equiv \pi - \phi\pi(1 - \pi)$. The following describes the resulting group size $\pi_2^W$ at the end of period 2, given that the offer of an allocation $\alpha_2^W$ is accepted.

- If $\alpha_2^W < g(\pi_1^W)$, then $\pi_2^W = g^{-1}(\alpha_2^W)$
- If $\alpha_2^W \in [g(\pi_1^W), f(\pi_1^W)]$, then $\pi_2^W = \pi_1^W$
- If $\alpha_2^W > f(\pi_1^W)$, then $\pi_2^W = f^{-1}(\alpha_2^W)$

**Proof.** It is straightforward to check that the functions $f(\cdot)$ and $g(\cdot)$ are strictly increasing on $[0, 1]$, and so, their inverses are well-defined. Consider an allocation $\alpha_2^W > f(\pi_1^W)$. In this range, we have

$$\alpha_2^W > f(\pi_1^W) \iff \frac{\alpha_2^W}{\pi_1^W} - \phi > \frac{1 - \alpha_2^W}{1 - \pi_1^W}.$$  

In other words, for a given incumbent group size $\pi_1^W$, the per capita payoff of members of $W_2$ exceeds that of members of $L_2$ by more than $\phi$. Group $W_2$ retains such a large share of the resources that it will attract switchers from the opposition. The size of group $W_2$ would now increase to ensure that

$$\frac{\alpha_2^W}{\pi_2^W} - \phi = \frac{1 - \alpha_2^W}{1 - \pi_2^W} \iff \alpha_2^W = f(\pi_2^W).$$

In the inequality above, the left-hand side is the second-period payoff of agents who switch from $L_2$ to $W_2$, and the right-hand side is the same for those who stay back in $L_2$. Switching would occur so that the group size adjusts to ensure that the two are the same. In the same way, if the ruling group leaves too little for itself ($\alpha_2^W < g(\pi_1^W)$), there is an incentive for its own members to switch to the opposition:

$$\alpha_2^W < g(\pi_1^W) \iff \frac{\alpha_2^W}{\pi_1^W} < \frac{1 - \alpha_2^W}{1 - \pi_1^W} - \phi,$$

and the size of group $W_2$ decreases to ensure indifference between those who switch and those who do not. In this case, we have $\alpha_2^W = g(\pi_2^W)$. Finally, there is an intermediate range, $\alpha_2^W \in [g(\pi_1^W), f(\pi_1^W)]$, where members of neither group has an incentive to switch. $\alpha_2^W \leq f(\pi_1^W) \iff \frac{\alpha_2^W}{\pi_1^W} - \phi \leq \frac{1 - \alpha_2^W}{1 - \pi_1^W}$ and $\alpha_2^W \geq g(\pi_1^W) \iff \frac{\alpha_2^W}{\pi_1^W} \geq \frac{1 - \alpha_2^W}{1 - \pi_1^W} - \phi$. In this case, no switching would occur.
and $\pi_2^W = \pi_1^W$.

Lemma 1 determines the resulting group sizes (and payoff) of members of group $L_2$ in the event that an allocation $\alpha_2^W$ is accepted. We now ask what range of offers by the incumbent would be accepted by group $L_2$. Since there is no gain from conflict in the second (terminal) period, any offer $\alpha_2^W > 0$ would be accepted by group $L_2$.

We can now characterize the optimal offer $\alpha_2^* = f(\pi_1^W) \equiv \pi_1^W + \phi \pi_1^W (1 - \pi_1^W)$ made by group $W_2$ in period 2. Given an initial group size $\pi_1^W$, the ruling group $W_2$ chooses $\alpha_2^*$ to maximize the per capita payoff $\frac{\alpha_1^W}{\pi_2^W(\alpha_2^*)}$ of its current members. Recall that if $\alpha_2^W$ is above a threshold, there will be switchers from group $L_2$, and $\pi_2^W(\alpha_2^W)$ will increase. Similarly, if $\alpha_2^W$ is below a threshold, players will induce a switch away from $W_2$. So, it is unclear a priori how the per capita payoffs change with $\alpha_2^W$. The following lemma establishes that the per capita payoff of the ruling group attains a maximum at the point where switching is just prevented.

**Lemma 2.** Suppose that the size of group $W_2$ at the beginning of period 2 is $\pi_1^W$. The per capita payoff of members of group $W_2$ is maximized at $\alpha_2^* = f(\pi_1^W) \equiv \pi_1^W + \phi \pi_1^W (1 - \pi_1^W)$.

The proof of the lemma is in the appendix. To see the intuition, notice that for switching to occur, the group that attracts new members must offer a higher per capita payoff. In particular, the group attracting members should have a payoff higher than 1, while the other group must have a payoff lower than 1.²¹ Therefore, any allocation where the incumbent induces its own members to switch to the opposition is strictly dominated by the allocation $\alpha_1^W = \pi_1^W$. However, the incumbent may attract members by increasing its own allocation, but in this case, switching ensures that the group size of the incumbent increases at a rate faster than the increase in its share of surplus. This decreases the per capita share. Since there is no political benefit from an increased group size in the terminal period, inducing switching is not attractive in the terminal period. The discussion above directly yields the following proposition that fully characterizes equilibrium play in the second period.

**Proposition 1 (Equilibrium Behavior in Period 2).** Suppose that the ruling group is of size $\pi_1^W$ at the start of period 2.

i) The ruling group allocates a fraction $\alpha_2^* = \pi_1^W + \phi \pi_1^W (1 - \pi_1^W)$ to itself and the remainder $(1 - \alpha_2^*)$ to the opposition.

ii) The opposition does not engage in conflict.

iii) No switching occurs across groups. In particular, members of the ruling group strictly prefer to remain in the group, and members of the opposition are indifferent between switching and not switching.

²¹Since $\pi_1^W \left( \frac{\alpha_1^W}{\pi_1^W} \right) + (1 - \pi_1^W) \left( \frac{1 - \alpha_1^W}{1 - \pi_1^W} \right) = 1$. 

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iv) The per capita payoff of the ruling group in period 2 is given by \(1 + \phi(1 - \pi^W_1)\) and that of the opposition is \(1 - \phi\pi^W_1\).

The crux of the result is that even though there is no threat of conflict in the last period, the incumbent still leaves some surplus for the opposition. The amount of sharing is driven by the “switching constraint.” The ruling group shares just enough resources to make the opposition indifferent between switching and not. Endogenous inter-group mobility acts as a disciplining device for the incumbent and prevents total expropriation of resources. In equilibrium, there is no switching.\(^{22}\) This result is related to the second period being the last. A ruling group would induce switching only if it helps it to gain political strength. However, in the last period, there is no incentive to increase political strength.

Proposition 1 says that for a group of size \(\pi_1\) at the end of period 1, the per capita payoff in period 2 is \(1 + \phi(1 - \pi_1)\) if it wins political power in period 2, and \(1 - \phi(1 - \pi_1)\) if the other group wins political power. Notice that if mobility across groups were costless, then all members of society would enjoy an equal payoff of 1 regardless of which group was in power. With a positive cost of mobility, there is a premium from being in power. In particular, for a group with size \(\pi_1\), the per capita payoff premium from winning political power is \(2\phi(1 - \pi_1)\), which is increasing in the cost of mobility and decreasing in group size. This has two important implications. First, as the cost of mobility increases, the opposition in period 1 has a higher propensity to reject the incumbent’s offer and launch conflict, while the incumbent has a stronger incentive to avoid conflict. Thus, for a high cost of mobility, the society will be more conflict-prone: Either there will be actual conflict in equilibrium, or the allocation of surplus will be driven by the necessity to prevent conflict. Second, while an increase in group size increases the probability of winning power in the next period, it also reduces the value of political power by diluting the per capita premium earned. The decision to attract switchers in period 1 then involves a tradeoff between an increased probability of winning and a loss in per capita payoffs.

### 3.2 Equilibrium play in the first period

Next, we characterize equilibrium behavior in period 1. Without loss of generality, suppose that group \(A\) is the winning group at the start of the game—i.e., \(W_1 = A\). Recall that the initial size of group \(A\) is \(\pi^A_0\). Let \(\pi^A_1\) denote the size of group \(A\) realized at the end of period 1 after switching decisions are made.

Group \(A\) must choose an optimal allocation of resources \(\alpha^A_1\). Once the allocation is announced, the opposition can either accept it or reject it. If the allocation is accepted, we say that play proceeds along the “economic path,” or the path of economic activity (in which switching can take place). If the allocation is rejected, we say that play proceeds along the “conflict path.” Let \(E_A(\alpha^A_1, \pi^A_1)\) and

\(^{22}\)If we were to introduce some heterogeneity in switching costs, switching would occur in equilibrium. We make the assumption of uniform costs of mobility just for simplicity.
$E_B(\alpha_1^A, \pi_1^A)$ denote the per capita payoffs to members in group $A$ and $B$, respectively, when play proceeds along the economic path, given allocation $\alpha_1^A$ and induced new group size $\pi_1^A$. Similarly, let $P_A$ and $P_B$ denote the per capita payoffs, when play proceeds along the path of conflict, given $\alpha_1^A$ and $\pi_0^A$. It is easy to derive expressions for the payoffs along the economic and conflict paths, respectively.

$$
E_A(\alpha_1^A, \pi_1^A) = \frac{\alpha_1^A}{\pi_1^A} + p_d(\pi_1^A)[1 + \phi(1 - \pi_1^A)] + [1 - p_d(\pi_1^A)][1 - \phi(1 - \pi_1^A)]
$$

$$
P_A = \frac{\pi_0^A}{\pi_0^A} + 1 + \phi(1 - \pi_0^A)(2p_c(\pi_0^A) - 1)
$$

$$
P_B = 1 + \phi\pi_0^A(1 - 2p_c(\pi_0^A)).
$$

3.2.1 Play along economic path in period 1

Consider the node in period 1, where the ruling group $A$ offers an allocation $\alpha_1^A$ that group $B$ accepts. By offering different allocations, the ruling group can induce switching activity and change the group size. The following lemma characterizes the new group size $\pi_1^A$ as a function of the offered allocation $\alpha_1^A$, for any given incumbent size $\pi_0^A$.

**Lemma 3.** [Group Switching Decisions in Period 1] Suppose that $A$ is the incumbent group in period 1 with initial size $\pi_0^A$. If the announced allocation $\alpha_1^A$ is accepted, then the new size of group $A$ is given by

$$
\pi_1^A(\alpha_1^A) = \begin{cases} 
\pi_0^A & \text{if } \alpha_1^A \in [g(\pi_0^A), f(\pi_0^A)] \\
 f^{-1}(\alpha_1^A) & \text{if } \alpha_2^A \in f(\pi_0^A) \\
 g^{-1}(\alpha_1^A) & \text{if } \alpha_2^A \in g(\pi_0^A),
\end{cases}
$$

where $f$ and $g$ are defined as before: $f(\pi) \equiv \pi + \phi\pi(1 - \pi)$ and $g(\pi) \equiv \pi - \phi\pi(1 - \pi)$.

Since switching decisions are based only on current-period payoffs, Lemma 3 is a replica of Lemma 1, and, hence, we omit the proof. The lemma shows that if the incumbent retains a very high (very low) share of the resources, this induces switching from the opposition (incumbent) group to the other group. If the allocation is close to the proportional allocation, then no switching occurs. Along the economic path, the incumbent will choose an allocation that induces its most-preferred group size.

The next lemma characterizes this optimal group size $\pi_1$ and the corresponding allocation (denoted by $\alpha^e$). It turns out that the incumbent’s payoff on the economic path is maximized at an intermediate group size. To see why, recall that increasing group size has two opposing effects: It increases the incumbent’s probability of retaining power on the economic path, and it reduces the
per capita payoff. For low $\pi_1$, the first effect dominates, and so, economic payoff is increasing in $\pi_1$. For values of $\pi_1$ close to 1, the opposite effect dominates. Since we assume $p_d(\pi)(1 - \pi)$ is single-peaked, the unique maximum payoff is attained at $\pi_1^A = \hat{\pi}$. In particular, Lemma 4 shows that if $\pi_0^A < \hat{\pi}$, then the incumbent shares more to induce some switching so that the new group size $\pi_1^A = \hat{\pi}$. If the initial group size $\pi_0^A$ is already larger than $\hat{\pi}$, then the maximal payoff on the economic path is reached when the opposition members are indifferent between switching and not switching—i.e., at $\alpha_1^A = f(\pi_0^A)$. The lemma also shows that the payoff on the economic path for group $B$ is single-peaked in the share of surplus.

Lemma 4 (Maximal Payoff on Economic Path). Suppose that $A$ is the incumbent group in period 1, and its offered allocation $\alpha_1^A$ is accepted by $B$. Then, the payoffs along the economic path to each group $E_A(\alpha_1^A, \pi_1(\alpha_1^A))$ and $E_B(\alpha_1^A, \pi_1(\alpha_1^A))$ are single-peaked in $\alpha_1^A$. The payoff for group $A$ is maximized at $\alpha_1^A = \alpha^e$, given by

$$\alpha^e = f(\overline{\pi}^A), \text{ where } \overline{\pi}^A = \max\{\pi_0^A, \hat{\pi}\}$$

The proof of the lemma, in the appendix, builds on an intuition similar to Lemma 2’s.

3.2.2 Opposition’s preference for conflict in period 1

We have characterized group compositions induced by each allocation conditional on acceptance and the corresponding payoffs for each group on the economic path. Next, in order to determine which path of play will be chosen in equilibrium, we analyze each group’s preferences over going down the path of conflict. Consider, first, the preferences of the opposition.

Lemma 5 (Opposition’s Conflict Threshold). For any $\pi_0^A$, there is a threshold $\bar{\alpha} \in [0, 1]$ such that the opposition (group $B$) accepts an allocation $\alpha_1^A$ proposed by the incumbent (group $A$) if and only if the allocation $\alpha_1^A$ satisfies $\alpha_1^A \leq \bar{\alpha}$. The threshold $\bar{\alpha}$ is decreasing in the cost of mobility, and there exists a threshold $\phi_1 > 0$ given by

$$\phi_1 = \frac{1}{\pi_0^A \left(1 + 2p_d(1)\frac{1}{\pi_0^A} - 2p_c(\pi_0^A)\right)}$$

such that $\bar{\alpha} = 1$ if $\phi \leq \phi_1$. Thus, all allocations are accepted if the cost of mobility is below $\phi_1$.

The interested reader may refer to the Appendix for the proof. The logic of the proof is as follows: On the one hand, we know from Lemma 4, that group $B$’s payoff on the economic path first increases and then decreases with $\alpha_1^A$. On the other hand, its payoff on the conflict path is constant. It is easy to check that, when $\alpha_1^A = 0$, its payoff on the economic path is higher than that from conflict. This implies that two cases can arise: (i) $B$’s payoff along the economic path is higher than that on the conflict path for all allocations $\alpha_1^A$; or (ii) $B$’s payoff along the economic path is
higher for low enough $\alpha_1^A$ (high enough share for $B$). Since the payoff from conflict is increasing in the cost of mobility, the former case obtains when the cost of mobility is low enough.

The two thresholds $\phi_1$ and $\bar{\alpha}$ completely describe the opposition’s preferences over conflict. The decision to reject the incumbent’s offer and launch conflict may be thought of as an investment. By rejecting an offer, the opposition gives up its payoff in the current period, but raises the probability of winning power in the next period. If the cost of intergroup mobility is below the threshold $\phi_1$, then even if the incumbent group offers nothing to the opposition, the opposition finds it more profitable to simply switch sides and share the incumbent’s surplus rather than launch conflict. However, if the cost is above $\phi_1$, the premium from winning power is large enough so that the current-period benefit must be high enough for the allocation to be accepted.

### 3.2.3 Incumbent’s preference for conflict in period 1

Lemma 5 tells us that $E := [0, \bar{\alpha}]$ is the set of allocations that induces the opposition to follow the economic path, and the complement (which we denote by $P$) is the set of allocations that induces the opposition to engage in conflict. To understand which path of play the incumbent would prefer, we need to compare the incumbent’s payoff along the path of conflict with its maximum possible payoff along the economic path—i.e., we compare $P_A$ with $\max_{\alpha_1^A \in E} E_A(\alpha_1^A)$. We show in the following lemma that there is a threshold such that the incumbent’s maximal payoff on the economic path is higher than that on the conflict path if and only if the cost of mobility is above the threshold.

Notice that, if $\phi \leq \phi_1$, then $P$ is an empty set. In this case, the incumbent is restricted to the economic path, and must choose $\alpha^e$ even if conflict provides a higher payoff than the maximal payoff on the economic path. Notice, also, that if $P$ is non-empty, all choices of $\alpha_1^A \in P$ lead to the same payoff along the path of conflict. We assume in this case that the incumbent chooses $\alpha_1^P = 1$. This assumption is consistent with the interpretation that if an offer is rejected, all the surplus remains with the incumbent, and further note that if $P$ is non-empty, $\alpha_1^P = 1$ always lies in $P$.

**Lemma 6 (Incumbent’s Conflict Threshold).** There exists a threshold $\phi_2$ given by

$$
\phi_2 = \left( \frac{k - \pi_0^A}{1 - \pi_0^A} \right) \left[ \frac{1}{\pi_0^A \left( 1 + 2p_d(\pi_0^A) \frac{\pi_0^A}{1 - \pi_0^A} - 2p_c(\pi_0^A) \right)} \right]
$$

---

$^{23}$We could have an alternative specification of the model in which the incumbent’s payoff under conflict is $\frac{kA}{\pi_1}$ rather than simply $\frac{kA}{\pi_1}$. Here, the interpretation is that after the incumbent decides the allocation, the opposition chooses to either consume its share of resources in productive economic activity or to invest it to mobilize conflict. In this case, $\alpha_1^P = 1$ is the strictly optimal allocation for the incumbent. To see why, note that the incumbent’s payoff $P_A(\alpha_1^A)$ is linearly increasing in $\alpha_1^P$ and it chooses $\alpha_1^P$ to maximize $\max_{\alpha_1^A \in P} P_A(\alpha_1^A), \max_{\alpha_1^A \in E} E_A(\alpha_1^A)$. It is easy to see that if $P$ is non-empty, $\alpha_1^P = 1 \in P$. 

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such that $E_A(\alpha^e, \pi^A_1(\alpha^e, \pi^A_0)) \geq P_A$ if and only if the cost of mobility $\phi$ is weakly greater than the threshold $\phi_2$.

The proof of the above lemma is in the Appendix. The intuition is straightforward. By inducing the path of conflict, the incumbent can enjoy the entire surplus in the current period, but there is a reduction in the probability of retaining power in the next period. Therefore, inducing conflict is worthwhile only if the premium from winning power in the next period is low—i.e., the cost of mobility is below a threshold.

Note that $\phi_2$ can lie outside $[0, 1]$. Since the attractiveness of the conflict path is increasing in $k$, the threshold $\phi_2$ is strictly increasing in $k$. If $k > \pi^A_0$, it is possible that $\phi_2 > 1$—i.e., for any cost of mobility, the incumbent prefers the conflict path over its maximum payoff on the economic path. This happens when conflict does not sufficiently reduce the incumbent’s probability of retaining power; for example, if $k = 1$, $\bar{\pi} = \pi^A_0$ and $p_d(\pi^A_0) - p_c(\pi^A_0) < \frac{1-\pi^A_0}{2\pi^A_0}$. However, if conflict is very destructive, (if $k < \pi^A_0$), then $\phi_2 < 0$. In this case, the incumbent does not want conflict, if the opposition will accept allocation $\alpha^e$. Next, we characterize the conditions under which the opposition does, indeed, accept $\alpha^e$.

We show in the lemma below, that there is a threshold $\phi_3$, above which $\alpha^e$ is not feasible along the economic path. If $\phi$ is very high ($\phi > \phi_3$), then there is a high premium from power in the second period. This increases the propensity of the opposition to engage in conflict. In this case, a split of $\alpha^e$ leaves too little for the opposition to accept and is, therefore, not feasible on the economic path. To induce the opposition to follow the economic path, the incumbent needs to offer a higher share to the opposition. The “best” allocation for the incumbent that still induces economic activity is then $\bar{\alpha}$, where the opposition is given just enough to make it indifferent between the economic path and conflict.

**Lemma 7 (Feasibility of $\alpha^e$ on economic path).** There exists a threshold $\phi_3 > 0$ given by

$$\phi_3 = \frac{1}{\pi^A_0 \left(1 + \frac{2p_d(\bar{\pi}^A)\bar{\pi}^A_{\pi^A} - 2p_c(\pi^A_0)}{\pi^A_0}\right)},$$

such that $\alpha^e$ induces economic activity—i.e., $\alpha^e \in E$ if and only if the cost of mobility $\phi$ is weakly less than the threshold $\phi_3$. Whenever $\phi > \phi_3$, the incumbent’s payoff from economic activity $E_A(\alpha, \pi^A_1(\alpha, \pi^A_0))$ is increasing in $\alpha$ in the set of allocations $E = [0, \bar{\pi}]$ that induce economic activity.

The interested reader may refer to the Appendix for the proof of the lemma. This lemma implies that if $\alpha^e$ will not induce the opposition to follow the economic path, then the incumbent must choose between inducing conflict and offering allocation $\bar{\alpha}$ and inducing the economic path: It must compare $E_A(\bar{\alpha}, \pi^A_1(\bar{\alpha}, \pi^A_0))$ and $P_A$. Recall, that as the cost of mobility increases, there are two opposing effects: On the one hand, there is a large premium from gaining power in the next period,
and so the incumbent would prefer to induce economic activity. On the other hand, as \( \phi \) increases, the incumbent has to offer more to the opposition in the current period to induce economic activity. The incumbent’s choice is driven by this tradeoff across periods. It turns out that for large enough \( \phi \), the first effect dominates the second. In other words, there is a threshold cost of mobility \( \phi_4 \) above which the incumbent prefers \( E_A(\pi, \pi_A^1(\pi, \pi_0^A)) \) to \( P_A \). The following lemma states this formally.

**Lemma 8 (Sharing to prevent conflict).** There exists a threshold \( \phi_4 \geq \max\{\phi_2, \phi_3\} \) given by

\[
\phi_4 = \frac{1}{\pi_0^A \left( 1 + 2p_d(\pi_0^A) - 2p_c(\pi_0^A) \right)},
\]

such that the incumbent prefers to offer \( \pi \) rather than \( \alpha^P \) whenever \( \phi \geq \phi_4 \).

The proof of the lemma is in the Appendix.

### 3.2.4 Incumbent’s optimal allocation choice in period 1

Now, we can fully characterize the resource allocations that arise in equilibrium. There are two factors that determine how the incumbent decides to allocate resources. First, if the incumbent keeps too much surplus for itself, it may attract switchers from the opposition, which would increase its political strength, but reduce the per capita share for the original members of the group. Thus, the incumbent will decide its allocation so as to achieve its optimal group size. Second, the ruling group might also want to share resources with the opposition so that the economic path is sufficiently attractive for the opposition, and they do not engage in conflict. These two constraints on expropriation—the switching constraint and the conflict constraint—together determine how resources are shared on the economic path. In the unique equilibrium, three different regimes arise depending on parameter values.

- **No-Conflict regime**: In this regime, play proceeds on the economic path, and the switching constraint determines the allocation. The optimal allocation choice is \( \alpha_1^* = \alpha^e \). If \( \pi_0^A < \bar{\pi} \), the incumbent induces opposition members to switch and achieve the target group size \( \bar{\pi} \). If \( \pi_0^A > \bar{\pi} \), then there is no switching, and the incumbent shares enough to keep the opposition indifferent between switching and not switching.

- **Peaceful-Belligerence regime**: In this regime also, play proceeds along the economic path, but the extent of sharing is driven by the imperative to prevent the opposition from engaging in conflict. Here, \( \alpha_1^* = \pi \). The incumbent shares just enough resources to make the opposition indifferent between the economic path and conflict. If \( \pi_0^A < \pi_1^A(\pi) \leq \bar{\pi} \), then there is some switching, and otherwise, there is no switching.

- **Open-Conflict regime**: In this regime, play proceeds along the conflict path. The incumbent implements conflict through full exploitation of resources—i.e., \( \alpha_1^* = \alpha^P = 1 \). Neither
conflict constraint nor the switching constraint binds, and the incumbent prefers to allow conflict.

The next proposition is the main result of the paper and characterizes equilibrium play in the first period.

**Proposition 2 (Equilibrium Allocation Choice in Period 1).** Suppose that \( A \) is the incumbent group in period 1 with size \( \pi_0^A \). The equilibrium choice of allocation \( \alpha_1^* \) in period 1 is as follows.

- If \( \phi \leq \phi_1 \), then the no-conflict regime prevails.
- If \( \phi \in (\phi_1, \phi_2) \), then the open-conflict regime occurs.
- If \( \phi \in (\max\{\phi_1, \phi_2\}, \phi_3) \), then the no-conflict regime prevails.
- If \( \phi \in (\max\{\phi_2, \phi_3\}, \phi_4) \) then peaceful-belligerence regime occurs if \( k \) is lower than a certain threshold and open conflict prevails otherwise.
- If \( \phi \geq \phi_4 \), then peaceful-belligerence prevails.

The proof of this proposition is in the Appendix. The intuition is as follows. When the cost of mobility is low, the incumbent wants to induce conflict by retaining the entire surplus in the current period. However, its ability to induce conflict is limited by the opposition’s preference for conflict. When the cost of mobility is sufficiently low, even if the incumbent retains a very high share, the opposition finds it more profitable to switch groups. However, at an intermediate range of \( \phi \), the opposition does respond by engaging in conflict. When the cost of mobility is high, the premium from gaining power in the second period is high. So, the incumbent wants to avoid conflict to retain power, while the opposition wants to engage in conflict. Ideally, the incumbent wants to induce economic activity by retaining \( \alpha^e \). But, when the cost of mobility is sufficiently high, the incumbent needs to offer more to the opposition to prevent conflict. To illustrate the equilibrium, we present a specific example below.

**Example 1.** Suppose that the contest success functions are

\[ p_d(\pi) = \pi \left( \pi + d \left( 1 - \pi \right) \right) , \]  

\[ p_c(\pi) = \pi \left( \pi + c \left( 1 - \pi \right) \right) \]

Both functions increase in \( \pi \) and satisfy our concavity condition for all \( d \geq 0 \). Also, \( d \geq c \Rightarrow p_d(\pi) \geq p_c(\pi) \). If \( d = 1 \), \( p_d(\pi) = \pi \)–i.e., the success probability is measured by the group size. If \( d > 1 \), the ruling group enjoys an incumbency advantage, in addition to the size effect, along the economic path. Figure 2 plots the success probabilities and the equilibrium regimes for any \( \phi \) and \( \pi_0 \) (for \( d = 2, c = 0.5 \) and \( k = 0.9 \)). Notice that open conflict does not necessarily occur at a high cost of mobility. Further, peaceful belligerence occurs for high values of \( \pi_0 \) and \( \phi \). The dotted line shows the optimal group size \( \tilde{\pi} \) (which, in this example, is 0.42). If the initial incumbent group size is below \( \tilde{\pi} \), switching happens in the no-conflict regime. These observations hold quite generally. See Section 4 for a discussion.\(^{24}\)

\(^{24}\)We have also worked out examples with \( S \)-shaped success functions and find similar results.
4 Implications and Empirical Predictions

In this section, we highlight some important implications and empirical predictions of our framework.

4.1 Inefficiency in equilibrium

Both conflict and switching are socially inefficient. Conflict reduces surplus directly. Switching is costly, but aggregate surplus is fixed. So, any outcome that involves conflict or switching is dominated by an outcome with the same allocation but without conflict or switching. The only efficient equilibria are those played on the economic path with no switching. Why do inefficient outcomes arise in equilibrium?

4.1.1 Inefficient conflict

One of the main contributions of this paper is to provide new insight into why we observe inefficient conflict. The standard rational explanation for observing conflict appeals to asymmetric information and limited commitment with the use of power (see Fearon (1995), Garfinkel and Skaperdas (2007), Powell (2004)). In our model, while there is no asymmetric information, the lack of credible commitment with respect to future transfers does restrict the allocation choices that can be implemented on the economic path. However, our framework identifies a second new source of conflict: the inability of agents to commit to not switch to the incumbent group once an allocation is offered. In particular, an allocation that can Pareto improve upon the conflict outcome may require groups
to retain their original sizes. But the lack of commitment with respect to switching leaves the incumbent with fewer allocation choices that are implementable. To see why, note that the highest allocation that the incumbent can retain in the first period, while avoiding conflict, is \( \alpha \). However, if the cost of mobility is not very high, then the allocation \( \alpha \) induces too much switching from the opposition, thus reducing the incumbent’s per capita share so much that the expected payoff on the economic path is no longer worth avoiding conflict. Therefore, there is an intermediate range (denoted hereafter by \( C \)) where the incumbent actually prefers to induce conflict. \(^{25}\)

To better understand how the lack of commitment with respect to switching gives rise to conflict, it is useful to consider a hypothetical game where, in the first period, the opposition can choose to commit to not switching after observing the allocation. In this “new game,” first, nature chooses the incumbent; then, the opposition decides whether or not to commit; and then, the original game is played. \(^{26}\)

Consider the situation in this new game where the opposition does not commit to not switch. Clearly, this subgame is the original game, and so, if \( \phi \in C \), open conflict prevails, and the payoffs are

\[
P_A = \frac{k}{\pi_0^4} + 1 + \phi(1 - \pi_0^A)(2p_c(\pi_0^A) - 1) \quad \text{and} \quad P_B = 1 + \phi\pi_0^A(1 - 2p_c(\pi_0^A)).
\]

Now, suppose that the opposition commits to not switch after any allocation \( \alpha \) is announced. Then, the payoffs of the groups on the economic path are

\[
E_{NS}^A(\alpha) = \frac{\alpha}{\pi_0^4} + 1 + \phi(1 - \pi_0^A)(2p_d(\pi_0^A) - 1) \quad \text{and} \quad E_{NS}^B(\alpha) = \frac{1 - \alpha}{1 - \pi_0^A} + 1 + \phi\pi_0^A(1 - 2p_d(\pi_0^A)).
\]

Notice that group A’s (B’s) payoff is strictly increasing (decreasing) in \( \alpha \). For \( \phi \in C \), if the opposition commits to not switch, the incumbent will offer \( \alpha^* \), where \( \alpha^* \) is the maximum share that it can retain without inducing conflict (\( E_{NS}^B(\alpha^*) = P_B \)). A simple comparison of the expressions for \( E_{NS}^A, P_A, E_{NS}^B \) and \( P_B \) then yields the result that the allocation \( \alpha^* \) Pareto strictly dominates the conflict outcome. In particular, at allocation \( \alpha^* \), the opposition is at least as well off as under conflict, and the incumbent is strictly better off. So, in this new game, where the opposition has the choice to commit to not switching, conflict does not arise in equilibrium. Further, it is easy to check that \( \alpha^* > f(\pi_0^4) \). This implies that in the original game with no commitment, \( \alpha^* \) would induce switching, thus reducing the incumbent’s per capita payoff so much that it would not be optimal to propose \( \alpha^* \). We state this formally in the proposition below. The details of the proof are in the Appendix.

**Proposition 3.** Consider a new game where, in period 1, the opposition (B) has the option to commit

\(^{25}\)Precisely, \( C = (\phi_1, \phi_2) \cup \{\phi : k > \phi\pi_0^A(1 + 2p_d(\pi_1^A(\alpha, \pi_0^A)) - 2p_c(\pi_0^A)) \} \) and \( \phi < \phi_4 \).

\(^{26}\)Here, we allow a commitment decision only in period 1. A similar result holds if we allow commitment in both periods.
not to switch before the incumbent (A) offers the allocation. Open conflict cannot arise in equilibrium in this game. Moreover, for the parameter range in which open conflict arises in the original game—i.e., \( \phi \in C = (\phi_1, \phi_2) \cup \{ \phi : k > \phi \pi^A_1 (1 + 2p_d(\pi^A_1(\bar{\alpha}, \pi^A_0)) - 2p_c(\pi^A_0)) \} \) and \( \phi < \phi_4 \)—the equilibrium in the new game Pareto dominates the open-conflict equilibrium, and the equilibrium share \( \alpha^* \) is strictly greater than \( f(\pi^A_0) \).

4.1.2 Inefficient switching across groups

Next, we ask why inefficient switching arises in equilibrium in the first period. Recall that the only reason why an incumbent wants to induce opposition members to switch over is to increase its chances of retaining power in the future. If there were no uncertainty about the future distribution of power, there would be no motive to induce switching. This is also why we observe no switching in the second period.

It is worthwhile to point out that if agents were not myopic, then also, switching would not arise. The intuition for this is as follows. If agents are non-myopic, any equilibrium allocation that causes switching must leave the switchers and non-switchers in the opposition with the same expected two-period payoff. Therefore, the difference in second-period expected per capita payoffs between the two groups must be exactly equal to the difference in the first-period per capita payoffs plus the cost of mobility. Put differently, there is no net benefit to inducing switching in equilibrium: Any increase in second-period payoff due to increased political strength is exactly offset by an increase in the first-period share that must be given to the non-switchers in the opposition. However, even if there is no actual switching in equilibrium, the threat of switching still restricts the set of implementable allocations. In the no-conflict regime, the switching constraint binds. So, if there were some heterogeneity in \( \phi \) among agents, inefficient switching would again arise. This would be entirely driven by the uncertainty regarding the future distribution of political power.\(^{27}\)

4.2 Conflict may not arise at high cost of mobility

Our framework delivers some important insights about the relationship between conflict and inter-group mobility. Low inter-group mobility is often claimed to be at the root of many of the social conflicts. Fearon (2006) argues that low mobility across groups can provide an attractive basis for coalition formation. Along similar lines, Caselli and Coleman (2006) show that conflict is relatively less likely to occur with high inter-group mobility since it is anticipated that the winning coalition would expand. Their model predicts that intense conflict is expected to arise in societies divided along characteristics that are relatively difficult to change, such as ethnicity, race, color or religion. However, empirical evidence suggests that there is not such a simple causal relationship between

\(^{27}\)A detailed analysis of the setting with non-myopic agents is available from the authors.
mobility and conflict (see Collier and Hoeffler (2004) and Fearon and Laitin (2003)).\footnote{Fearon and Laitin (2003) write “... it appears not to be true that a greater degree of ethnic or religious diversity—or indeed any particular cultural demography—by itself makes a country more prone to civil war. This finding runs contrary to a common view among journalists, policy makers, and academics, which holds “plural” societies to be especially conflict-prone due to ethnic or religious tensions and antagonisms.”} There are examples in which intense conflict arises between groups where the cost of mobility is low (e.g., language-based discrimination), as well as others where cost of mobility is very high, and yet conflict does not arise. Our model yields equilibrium predictions that are consistent with these diverse examples. In particular, we show that open conflict may not arise even when the cost of mobility is very high.

**Proposition 4.** Assume that $A$ is the incumbent group in period 1 with size $\pi_0^A$.

i) Suppose that conflict is sufficiently likely to reduce $A$’s probability of retaining power so that

$$p_d(\pi_0^A) - p_c(\pi_0^A) > \frac{1-\pi_0^A}{2\pi_0^A}.$$  

Then, there will be peaceful belligerence at $\phi = 1$.

ii) Suppose that conflict is less likely to reduce $A$’s probability of retaining power, so that

$$p_d(\pi_0^A) - p_c(\pi_0^A) \leq \frac{1-\pi_0^A}{2\pi_0^A}.$$  

Then, there will be open conflict at $\phi = 1$ if and only if $k$ is sufficiently high.

Details of the analysis are in the Appendix. The intuition is, by now, familiar. When the cost of mobility is maximal, both groups have strong incentives to gain power. But conflict entails two different costs for the incumbent. It reduces the incumbent’s probability of retaining power, and can be wasteful in the first period. When conflict significantly reduces the incumbent’s probability of retaining power, the incumbent can avoid conflict only by sharing resources with the opposition. However, if conflict does not significantly reduce the incumbent’s probability of retaining power, the incumbent induces open conflict in equilibrium unless it is highly wasteful (low $k$).

As mentioned in the introduction, there are examples of societies divided along ethnicity or caste (high cost of mobility) where there is no conflict, and, indeed, resource sharing occurs. For instance, Padró i Miquel (2007) mentions Ivory Coast as an example, where the opposition is strong enough that it needs to be bought off: Houphouet-Boigny’s regime in Ivory Coast was known to actually transfer resources to the minority opposition ethnic groups. Another example is India, where resources are shared with backward castes through a range of reservation policies, which have helped mitigate conflict. Such sharing in the shadow of conflict arises in equilibrium in our model.

The above proposition, together with Proposition 2, shows that there is no direct relationship between conflict and mobility. It is possible for conflict to arise at intermediate costs of mobility even when it may not arise at a very high cost of mobility.
4.3 Destruction as a deterrent to conflict

The possibility of conflict disciplines the incumbent in our framework, by reducing its probability of retaining power and by surplus destruction. Proposition 4 explores the role of the first effect, and now we turn our attention to the second.

In general, open conflict increases as conflict becomes less wasteful (as $k$ increases). A decrease in $k$ moves the conflict threshold of the incumbent, $\phi_2$, to the left. Thus, conflict becomes less attractive to the incumbent, and the possibility of open conflict decreases. For a low cost of mobility, the no-conflict region replaces a part of open-conflict region, and for a high cost of mobility, peaceful belligerence replaces open conflict for some values of $\phi$. Formally, we have the following result.

**Proposition 5.** Suppose that for a given initial incumbent size $\pi_0^A$, open conflict prevails if the cost of mobility $\phi$ lies in the set $\phi \in C$. This set $C$ shrinks (monotonically decreases in the sense of set inclusion) as $k$ decreases. For $k \leq \pi_0^A$, $C$ is an empty set.

The above result suggests that conflict is observed only when it is not very destructive. This is, indeed, a feature of all models where agents have perfect information about the cost of conflict and the success probability. To this extent, our model does not explain why we observe highly destructive conflict such as civil wars. Highly destructive conflict could arise in equilibrium if there were some incomplete information about cost or success parameters.29

4.4 Peaceful belligerence does not arise with small incumbents

Another important prediction of our model is that if the incumbent group is a small minority of elites, then sharing, if any, is driven by the switching constraint.

**Proposition 6.** If the incumbent group size is sufficiently small, then peaceful belligerence does not occur in equilibrium, regardless of the cost of mobility. Formally, there exists a threshold $\pi$ such that if the initial group size is smaller than $\pi$, then $\phi_2 > 1$. In particular, this threshold is increasing in $k$.

The proof of the result is in the Appendix. If the initial group size is low enough, full expropriation leads to a large pie being shared among a small number of individuals, raising the per capita payoff. In such a situation, the incumbent will prefer full expropriation to the maximal payoff obtainable on the economic path for any value of $\phi$. Consequently, if the incumbent’s conflict threshold is above 1, the peaceful belligerence regime does not arise in equilibrium.

Indeed, Propositions 2 and 6 together imply that peaceful belligerence occurs only for high values of both $\pi$ and $\phi$. In other words, in a society with a high cost of mobility, if a majority group assumes power, then it will share spoils with the minority to retain power and prevent conflict, but if the minority is in power, then it will have an incentive to extract all surplus.

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29See, for example, Wärneryd (forthcoming), Collier and Hoeffler (2007), Garfinkel and Skaperdas (2007) for discussion of the role of information in conflict.
4.5 Non-monotonic equilibrium allocations

Our model implies that the equilibrium allocation is non-monotonic in the cost of mobility.

**Proposition 7.** *The equilibrium choice of allocation is increasing in the cost of mobility in the no-conflict regime, decreasing in the peaceful-belligerence regime, and constant in the open-conflict regime.*

The result follows directly from Lemmata 4 and 5. The intuition is straightforward: In the no-conflict regime, the ruling group retains just enough surplus to induce optimal switching. So, as switching becomes more costly, the incumbent can keep more for itself. In the open-conflict regime, the incumbent induces conflict by full expropriation. In the peaceful-belligerence regime, the equilibrium allocation is the maximum that the incumbent can keep without provoking conflict. An increase in the cost of mobility raises the premium from winning political power and, thus, enhances the opposition’s incentive for conflict. The opposition has to be offered more to be prevented from engaging in conflict, and, hence the equilibrium allocation is decreasing.

Together with Propositions 2 and 4, Proposition 7 implies that in societies with easy intergroup mobility, we should expect equilibrium allocations to increase with the cost of mobility. Further, in societies characterized by a high cost of mobility, when the threat of conflict is strong, the equilibrium allocation is decreasing in the cost of mobility. These results have testable implications, and a systematic empirical analysis would be interesting.

4.6 Optimal group size and switching

Our model predicts that the ruling group’s equilibrium choice of allocation rule in the no-conflict regime is determined by its incentive to maintain an optimal group size.

**Proposition 8.** *There exists a unique interior optimal group size for the ruling group. If the ruling group’s initial size is below this optimal size, it induces switching from the opposition in the no-conflict regime. Otherwise, the incumbent does not induce switching in equilibrium.*

The proof is straightforward, and so we omit it here. The ruling group aspires to achieve an ideal size $\bar{\pi}$ where its increased political strength is balanced against the reduced share of per capita surplus. When the ruling group’s size is below the optimal size, it prefers to induce switching to increase its political strength. The only way it can induce switching is by retaining more resources for itself. However, such a strategy also reduces the opposition’s opportunity cost of conflict. In the no-conflict regime, the ruling group can retain enough resources so that the opposition prefers switching to conflict.

For tractability, we assumed that there are only two periods in the game, and that any group size can be achieved in the current period by appropriate choice of allocation. A comprehensive analysis of the multi-period game is beyond the scope of this paper. However, we conjecture that in
the dynamic game, whenever there is no open conflict, the incumbent would increase its size unless already larger than its optimal size. Moreover, as power alternates, group sizes would also swing in opposite directions.\(^{30}\) However, the size of each group would vary within an upper and a lower limit.

### 4.7 Ruling group’s preferred cost of mobility

In our framework, the cost of mobility is exogenous. We can ask what the incumbent’s preferred cost of mobility would be, if he could choose it. Think of two groups that can be distinguished based on more than one characteristic. For example, two ethnic groups living in the same area may develop different professional skills or different religious practices. These different characteristics are associated with different costs of mobility. The group in power can decide the specific characteristic along which resources would be split. In such a setting, which social cleavage would the incumbent choose?\(^{31}\)

Since the premium from power increases with the cost of mobility \(\phi\), we may expect that the incumbent would choose a maximal cost of mobility. However, it turns out that if conflict is sufficiently effective in changing the regime, then the incumbent may prefer an intermediate cost of mobility.

**Proposition 9.** Suppose that \(A\) is the incumbent group in period 1 with size \(\pi_A^0\), and let \(V_A(\phi)\) denote \(A\)’s expected two-period per capita payoff as a function of the cost of mobility \(\phi\).

i) If \(A\)’s success probability in conflict, \(p_c(\pi_A^0)\), is sufficiently high, \(V_A\) always reaches its maximum at \(\phi = 1\), the maximal cost of mobility.

ii) If \(A\)’s success probability in conflict, \(p_c(\pi_A^0)\), is not sufficiently high, there can be an interior cost of mobility at which \(V_A\) attains its maximum.

The proof of the proposition is in the Appendix. The intuition is as follows. Two cases arise: First, with low values of \(\phi\), the switching constraint binds, and the incumbent’s payoff is increasing in the cost of mobility. Second, with high values of \(\phi\), there is either open conflict, or peaceful belligerence (both determined by the conflict constraint). In this case (with high \(\phi\)), the incumbent’s payoff depends on its probability of retaining power in conflict. Notice that in the second period, the winner’s payoff increases and the loser’s payoff decreases in the cost of mobility. If \(p_c(\pi_0)\) is sufficiently low (high), the incumbent is less (more) likely to be the winner in conflict, and its payoff is decreasing (increasing) in \(\phi\). Hence, if \(p_c(\pi_0)\) is sufficiently low, the incumbent may

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\(^{30}\)Such swings can be often observed as a political party in power wins the support of some community with targeted policies.

\(^{31}\)The incumbent may also be able to take measures to change the cost of mobility between the groups. We can ask what its preferred level of mobility would be.
actually choose an interior cost of mobility.\textsuperscript{32} On the contrary, if $p_c(\pi_0)$ is sufficiently high, the incumbent’s payoff is increasing in all equilibrium regimes, and it prefers a maximal $\phi$.

Horowitz (1985) recounts how color provided a more advantageous form of differentiation than religion between the English and the African slaves in seventeenth century North America (as conversion to Christianity become more common).\textsuperscript{33} Such an extreme form of discrimination was possible and remained in effect for a long time, as the English found little threat of losing power in conflict.

The example below illustrates the result of Proposition 9 by plotting the incumbent’s expected aggregate payoff as a function of the cost of mobility, for specific parameter values.

Example 2. We revisit Example 1. We assume that $A$ is the incumbent in period 1. Consider the following parameter specifications: $\pi_{0A} = 0.4$, $k = 0.9$, $d = 3$. Figure 3 plots $A$’s expected two-

![Figure 3: Incumbent’s expected total payoff against cost of mobility](image)

period payoff as a function of the cost of mobility $\phi$. The left panel corresponds to a case with low success probability during conflict ($c = 0.5$), and the right panel corresponds to a case with high success probability during conflict ($c = 2.8$). In the first case, $A$’s payoff is decreasing in the open-conflict regime, and, therefore, we have an interior maximum at the opposition conflict

\textsuperscript{32}It is important to note that at an interior maximal cost of mobility, we may observe peaceful belligerence (if $\phi_4 < 1$) or no conflict (if $\phi_2 < 1 < \phi_3$) in equilibrium.

\textsuperscript{33}Horowitz (1985, p 43) states that “...the English were originally called ‘Christians,’ while the African slaves were described as ‘heathens.’ The initial differentiation of groups relied heavily on religion. After about 1680, however, a new dichotomy of ‘whites’ and ‘blacks’ supplanted the former Christian and heathen categories, for some slaves had become Christians. If reliance had continued to be placed mainly on religion, baptism could have been employed to escape from bondage.”
threshold $\phi_1$, which is 0.46. In the second case, payoff is increasing in the open-conflict regime and maximized at $\phi = 1$.

5 Concluding Remarks

In this paper, we study group-based politics in divided societies, with central objective of developing a coherent model that explains the salience of inter-group differences in conflict. We present a model of political competition between two groups, where political power implies the right to allocate society’s resources and allows the possibility of engaging in economic exclusion based on group identities. We model group membership to be endogenous: Individuals can switch groups by incurring a cost, where this cost of mobility captures the extent of inter-group differences.

The main substance of the analysis is in showing (i) how the extent of inter-group differences determines the level of economic exclusion that a ruling group can exercise; and (ii) how these factors, in turn, determine the emergence of inter-group conflict. We characterize how resources are shared in equilibrium and when conflict arises. We provide a new explanation for why inefficient conflict is observed in equilibrium: limited commitment with respect to mobility across groups. We also derive several predictions that are consistent with stylized facts, and that have not been shown earlier. For instance, we can explain why open conflict does not necessarily arise when the cost of mobility is high. In particular, we can show that in equilibrium, a majority ethnic group may choose to transfer resources to the opposition to avoid conflict. We also show that open conflict can occur at an intermediate cost of mobility.

However, many interesting questions remain unanswered. A simplifying assumption is that all members in a group are treated homogeneously. In many contexts, it is more realistic to allow some within-group hierarchy: For instance, new members and original members may be treated differently. Allowing a richer action space that allows heterogeneous treatment may lead to new insights. Another assumption made for tractability is that the game lasts for two periods. While we conjecture that many of the qualitative insights will carry over to an infinite-horizon model, a fully dynamic model will allow us to analyze the dynamics of regime changes and how group sizes evolve over time. Finally, a promising line of investigation is related to the broader question of what constitutes the basis for group formation in politics. For instance, when do groups form along ethnic lines (with a high cost of mobility) and when do they form along ideological lines (a relatively low cost)? Is there a theory that explains widespread politicization of ethnic or religious identities? We leave these questions for future research.
References


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6 Appendix

6.1 Proof of Lemma 2

Proof. For \( \alpha_2^W < g(\pi_1^W) \), the per capita payoff is given by \( \frac{\alpha_2^W}{\pi_2} = 1 - \phi[1 - \pi_2^W(\alpha_2^W)] \), which is increasing in \( \pi_2^W(\alpha_2^W) \) and, consequently, in \( \alpha_2^W \). In the range \( \alpha_2^W \in [g(\pi_1^W), f(\pi_1^W)] \), \( \frac{\alpha_2^W}{\pi_2^W(\alpha_2^W)} = \frac{\alpha_2^W}{\pi_2} \), which increases linearly in \( \alpha_2^W \). For \( \alpha_2^W > f(\pi_1^W) \), the per capita payoff is \( \frac{\alpha_2^W}{\pi_2^W(\alpha_2^W)} = 1 + \phi[1 - \pi_2^W(\alpha_2^W)] \) which is decreasing in \( \pi_2^W(\alpha_2^W) \) and, therefore, in \( \alpha_2^W \). It follows that the per capita share of surplus \( \frac{\alpha_2^W}{\pi_2^W(\alpha_2^W)} \) for group \( W \) has a unique maximum, which occurs at \( \alpha_2^W = f(\pi_1^W) \). \( \square \)

6.2 Proof of Lemma 4

Proof. We first show that \( E_A(\alpha_1^A, \pi_1^A(\alpha_1^A)) = \frac{\alpha_1^A}{\pi_1^A} + 1 + \phi(1 - \pi_1^A)(2p_d(\pi_1^A) - 1) \) is single-peaked. Consider \( E_A(\alpha_1^A, \pi_1^A(\alpha_1^A)) \) in the range \( \{ \alpha : \alpha \leq g(\pi_0^A) \} \). By Lemma 3, when \( \alpha_1^A < g(\pi_0^A) \), this induces switching from \( A \) to \( B \) and the new size of \( A \) is \( \pi_1^A = g^{-1}(\alpha_1^A) \). Substituting, we have,

\[
E_A(\alpha_1^A, \pi_1^A(\alpha_1^A)) = 2 - 2\phi(1 - \pi_1^A)(1 - p_d(\pi_1^A)),
\]

which is increasing in \( \pi_1^A \). We know that \( g \) is increasing, and so \( \pi_1^A = g^{-1}(\alpha_1^A) \) is increasing in \( \alpha_1^A \).

It follows that \( E_A(\alpha_1^A, \pi_1^A(\alpha_1^A)) \) is increasing in \( \alpha_1^A \).

Now, for \( \alpha_1^A \in [g(\pi_0^A), f(\pi_0^A)] \), we know that no switching occurs and \( \pi_1^A(\alpha_1^A) = \pi_0^A \). Therefore, \( E_A(\alpha_1^A, \pi_1^A(\alpha_1^A)) \) is increasing in \( \alpha \) in this range.

Finally, we show that \( E_A \) first increases and then decreases in \( \alpha_1^A \) over the range \( \{ \alpha_1^A : \alpha_1^A \geq f_1(\pi_0^A) \} \). Consider \( \alpha_1^A > f_1(\pi_0^A) \). We know, again from Lemma 3, that this would induce switching from group \( B \) to group \( A \) and the new size of group \( A \) would be \( \pi_1^A = f^{-1}(\alpha_1^A) \). So, we have,

\[
E_A(\alpha_1^A, \pi_1^A(\alpha_1^A)) = 2 + 2\phi p_d(\pi_1^A)(1 - \pi_1^A),
\]

which decreases in \( \pi_1^A \) above \( \tilde{\pi} \), and so decreasing in \( \alpha_1^A \) above \( \max \{ f(\pi_0^A), f(\tilde{\pi}) \} \) in the range \( \{ \alpha_1^A : \alpha_1^A > f_1(\pi_0^A) \} \). Define \( \max \{ \pi_0^A, \tilde{\pi} \} = \pi_0^A \). It follows immediately that the function \( E_A \) is single-peaked and maximized at \( \alpha_1^A = f(\pi_0^A) \).

Next, consider \( E_B(\alpha_1^A, \pi_1^A(\alpha_1^A)) = \frac{1-\alpha_1^A}{1-\pi_1^A} + 1 + \phi \pi_1^A(1 - p_d(\alpha_1^A)) \). Since \( p_d(\pi)(1 - \pi) \) is single-peaked, this implies that \( \pi(p_d(1 - \pi)) \) is single-peaked. Let \( \tilde{\pi} \) denote the value at which the maximum is attained. Consider the range where \( \alpha_1^A < g(\pi_0^A) \). In this case, switching leads to \( \pi_1^A = g^{-1}(\alpha_1^A) \). Substituting for \( \alpha_1^A = g(\pi_0^A) \), we find \( E_B(\alpha_1^A, \pi_1^A(\alpha_1^A)) = 1 + \phi \pi_1^A(1 - p_d(\pi_1^A)) \), which increases in \( \pi_1^A \) up to \( \tilde{\pi} \), and so increasing in \( \alpha_1^A \) up to \( \min \{ g(\pi_0^A), f(\tilde{\pi}) \} \) in the range \( \{ \alpha_1^A : \alpha_1^A < g(\pi_0^A) \} \). Now consider \( \alpha_1^A \in [g(\pi_0^A), f(\pi_0^A)] \). In this range, no switching occurs \( (\pi_0^A = \pi_1^A) \). So, \( E_B \) is decreasing in \( \alpha_1^A \). Finally, when \( \alpha_1^A > f(\pi_0^A) \), switching occurs along the
economic path, and \( \pi_1^A = f^{-1}(\alpha^A) \). Substituting for \( \alpha^A = f(\pi_1^A) \), we find \( E_B(\alpha^A, \pi_1^A(\pi_0^A)) = 1 + 1 - 2\phi \pi_1^A p_d(\pi_1^A) \), which decreases in \( \pi_1^A \) and, therefore, also in \( \alpha^A \). Thus, \( E_B(\alpha^A, \pi_1^A(\alpha^A)) \) is also single-peaked in \( \alpha^A \) with the peak occurring at \( \alpha^A = \min \{ g(\pi_0^A), g(\tilde{\pi}) \} \).

### 6.3 Proof of Lemma 5

**Proof.** We start by comparing the function \( E_B(\alpha^A, \pi_1^A(\alpha^A)) \) with \( P_B \).

\[
E_B(\alpha^A, \pi_1^A(\alpha^A)) = \begin{cases} 
2 + 2\phi \pi_1^A (1 - p_d(\pi_1^A)) & \text{if } \alpha^A < g(\pi_0^A) \\
\frac{1 - \alpha^A}{1 - \pi_0^A} + 1 + \phi \pi_0^A (1 - 2p_d(\pi_0^A)) & \text{if } \alpha^A \in [g(\pi_0^A), f(\pi_0^A)] \\
2 - 2\phi \pi_1^A p_d(\pi_1^A) & \text{if } \alpha^A > f(\pi_0^A)
\end{cases}
\]

\[
P_B = 1 + \phi \pi_0^A (1 - 2p_c(\pi_0^A))
\]

If \( \alpha^A = 0 \), switching would occur from \( A \) to \( B \) and \( \pi_1^A = g^{-1}(0) = 0 \). Consequently, \( E_B(0, \pi_1^A(0, \pi_0^A)) = 1 + 1 \). At \( \alpha^A = 0 \), \( E_B = 2 > P_B \). Moreover, Lemma 4 shows that the function \( E_B \) first increases and then decreases. This implies that either \( P_B \) intersects \( E_B \) at exactly one point (which is given by \( \tilde{\pi} \)) or \( E_B \) lies entirely above \( P_B \), in which case \( \tilde{\pi} = 1 \).

First consider the case where \( \tilde{\pi} \) is given by the intersection between \( P_B \) and \( E_B \). We know that there cannot be two such intersections. Note, now, that at \( \alpha = g(\pi_0^A) \), \( E_B > 2 > P_B \). Therefore, \( \tilde{\pi} > g(\pi_0^A) \). If \( \tilde{\pi} \in (g(\pi_0^A), f(\pi_0^A)) \), then \( \tilde{\pi} \) is given by

\[
\frac{1 - \alpha^A}{1 - \pi_0^A} + 1 + \phi \pi_0^A (1 - 2p_d(\pi_0^A)) = 1 + \phi \pi_0^A (1 - 2p_c(\pi_0^A))
\]

\[
\tilde{\pi} = 1 - 2\phi \pi_0^A (1 - \pi_0^A)[p_d(\pi_0^A) - p_c(\pi_0^A)],
\]

which is decreasing in \( \phi \) since \( \pi_0^A \in (0, 1) \) and \( p_d(\pi_0^A) \geq p_c(\pi_0^A) \). However, if \( \tilde{\pi} > f(\pi_0^A) \), then \( \tilde{\pi} \) is given implicitly by the group composition \( \tilde{\pi} \) that satisfies

\[
2 - 2\phi \pi_1^A p_d(\pi_1^A) = 1 + \phi \pi_0^A (1 - 2p_c(\pi_0^A))
\]

\[
\pi_1 p_d(\pi_1) = \frac{1}{2} \left[ \frac{1}{\phi} - \pi_0^A (1 - 2p_c(\pi_0^A)) \right]
\]

Since the LHS is strictly increasing in \( \pi_1 \) and the RHS is constant, there is a unique solution to the equation. Also, since \( \pi_1^A(\alpha) \) is increasing in the range \( \alpha > f(\pi_0^A) \), there is a unique \( \tilde{\alpha} \) that corresponds to \( \tilde{\pi} \). Notice that \( \tilde{\pi} \) and, hence, \( \tilde{\alpha} \) is decreasing in \( \phi \). Therefore, whenever \( \tilde{\alpha} < 1 \), it is decreasing in \( \phi \).

At \( \alpha^A = 1 \), \( \pi_1^A = f^{-1}(1) = 1 \). Therefore, \( E_B = 1 + 1 - 2\phi p_d(1) \). By comparing \( P_B \) with \( E_B \) at \( \alpha^A = 1 \), it is easy to see that \( E_B \geq P_B \) for all \( \alpha^A \) with strict equality only at \( \alpha^A = 1 \) if and only
if 
\[ \phi \leq \frac{1}{2p_d(1) + \pi_0^A(1 - 2p_c(\pi_0^A)))} := \phi_1. \]

Since \(p_d(\cdot)\) is increasing and a probability, \(p_d(1) > \pi_0^A\). This implies that \(\phi > 0\). \(\square\)

### 6.4 Proof of Lemma 6

**Proof.** We compare \(E_A(\alpha_1^e, \pi_1^A(\alpha_1^e))\) with \(P_A\). Notice that \(\alpha_1^e = f(\pi_0^A) = \pi_0^A + \phi(1 - \pi_0^A)\) from Lemma 4. Therefore, at the allocation \(\alpha_1^e\), \(E_A\) is given by \(E_A(\alpha_1^e, \pi_1^A(\alpha_1^e)) = 2 + 2\phi p_d(\pi_0^A)(1 - \pi_0^A)\). So, \(E_A\) is greater than \(P_A\) if and only if \(2 + 2\phi p_d(\pi_0^A)(1 - \pi_0^A) \geq \frac{k}{\pi_0^A} + 1 + \phi(1 - \pi_0^A)(2p_c(\pi_0^A) - 1)\). Simplifying, we get

\[
\phi \geq \left[ \frac{\left(\frac{k - \pi_0^A}{1 - \pi_0^A}\right)}{\pi_0^A \left( 1 + 2p_d(\pi_0^A) \frac{1 - \pi_0^A}{1 - \pi_0^A} - 2p_c(\pi_0^A) \right)} \right] := \phi_2.
\]

\(\square\)

### 6.5 Proof of Lemma 7

**Proof.** From Lemma 4, \(\alpha^e = f_1(\pi_0^A)\). Hence, we have

\[
\alpha^e \in E \iff E_B(\alpha^e, \pi_1^A(\alpha^e, \pi_0^A)) \geq P_B \\
\iff \phi \leq \frac{1}{\pi_0^A \left( 1 + 2p_d(\pi_0^A) \frac{1 - \pi_0^A}{\pi_0^A} - 2p_c(\pi_0^A) \right)} := \phi_3.
\]

Since the denominator \(\pi_0^A \left( 1 + 2p_d(\pi_0^A) \frac{1 - \pi_0^A}{\pi_0^A} - 2p_c(\pi_0^A) \right) > \pi_0^A \left( 1 + 2p_d(\pi_0^A) - 2p_c(\pi_0^A) \right) > \pi_0^A \left( 1 + 2p_d(\pi_0^A) - 2p_c(\pi_0^A) \right) > 0\), we must have \(\phi_3 > 0\). Now, if \(\phi > \phi_3\), clearly, \(\alpha^e \notin E\). From Lemma 5, \(\alpha^e > \alpha\). Also, since \(E_A(\alpha, \pi_1^A(\alpha, \pi_0^A))\) is single-peaked in \(\alpha\) with the peak occurring at \(\alpha^e\), we must have \(E_A(\alpha, \pi_1^A(\alpha, \pi_0^A))\) strictly increasing in \(\alpha\) in the range \([0, \pi]\). \(\square\)

### 6.6 Proof of Lemma 8

**Proof.** First, we establish that \(\phi_4 \geq \max\{\phi_2, \phi_3\}\). To see that, notice that

\[
\phi_2 \leq \frac{1}{\pi_0^A \left( 1 + 2p_d(\pi_0^A) \frac{1 - \pi_0^A}{\pi_0^A} - 2p_c(\pi_0^A) \right)} \leq \frac{1}{\pi_0^A \left( 1 + 2p_d(\pi_0^A) \frac{1 - \pi_0^A}{\pi_0^A} - 2p_c(\pi_0^A) \right)} = \phi_4,
\]

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\[ \phi_3 = \frac{1}{\pi_0^A(1 + 2p_d(\pi_0^A)\frac{\pi_1^A}{\pi_0^A} - 2p_c(\pi_0^A))} \leq \frac{1}{\pi_0^A(1 + 2p_d(\pi_0^A)\frac{\pi_1^A}{\pi_0^A} - 2p_c(\pi_0^A))} = \phi_4. \]

Now, if \( \phi \geq \phi_4 \), we must have \( \phi \geq \max\{\phi_2, \phi_3\} \). Thus, the incumbent has to choose between \( \pi \) and \( \alpha^P \). Now, when \( \pi \in (g(\pi_0^A), f(\pi_0^A)) \), then \( \pi \) is given by \( \pi = 1 - 2\phi_0^A(1 - \pi_0^A)p_d(\pi_0^A) - p_c(\pi_0^A) \). Substituting for \( f(\pi_0^A) \), for \( \pi \), we have \( \pi_0^A + \phi_0^A(1 - \pi_0^A) = 1 - 2\phi_0^A(1 - \pi_0^A)p_d(\pi_0^A) - p_c(\pi_0^A) \), or \( \phi = \phi_4 \). Since \( \pi \) is continuous and strictly decreasing in \( \phi \), \( \pi < f(\pi_0^A) \) for \( \phi \geq \phi_4 \). Therefore, \( \pi_1^A(\pi, \pi_0^A) = \pi_0^A \) for \( \phi \geq \phi_4 \). Now, \( E_A(\pi, \pi_1^A(\pi, \pi_0^A)) - P_A \) is equal to

\[ \frac{\alpha_1^A - k}{\pi_0^A} + \phi(1 - \pi_0^A)(2p_d(\pi_0^A) - 2p_c(\pi_0^A)) = \frac{1 - k}{\pi_0^A} > 0 \]

since \( \alpha_1^A = 1 - 2\phi_0^A(1 - \pi_0^A)p_d(\pi_0^A) - p_c(\pi_0^A) \).

### 6.7 Proof of Proposition 2

**Proof.** First, by Lemma 5, if \( \phi \) is below \( \phi_1 \), the opposition will accept any allocation, and, therefore, in this range, the incumbent is forced to choose \( \alpha^e \). The choice of the incumbent matters only when \( \phi > \phi_1 \). Now, as Lemma 6 shows, when \( \phi \leq \phi_2 \), the incumbent actually prefers conflict to any allocation implementable along the economic path. If we have \( \phi \in (\phi_1, \phi_2) \), the incumbent then induces conflict by offering \( \alpha^P = 1 \). When \( \phi > \max\{\phi_1, \phi_2\} \), then the incumbent prefers economic activity if \( \alpha^e \), is accepted. By Lemma 7, \( \alpha^e \) is accepted if and only if \( \phi < \phi_3 \). Therefore, the incumbent offers \( \alpha^e \) and induces economic activity if \( \phi \in (\max\{\phi_1, \phi_2\}, \phi_3) \). For \( \phi > \phi_3 \), the incumbent must make a larger offer \( \bar{\alpha} \) to induce the economic path. For \( \phi > \max\{\phi_2, \phi_3\} \), the incumbent has to choose between \( \bar{\alpha} \) and \( \alpha^P \). If \( \pi_1^A = \pi_0^A \), then it is easy to check that \( \phi_4 = \phi_3 \), and then, by Lemma 4, for \( \phi > \phi_4 \), the incumbent offers \( \bar{\alpha} \), which is just enough to prevent the opposition from launching conflict. However, if \( \pi_1^A < \pi_0^A \), then we have another range \( (\max\{\phi_2, \phi_3\}, \phi_4) \) where the choice between open conflict and peaceful belligerence depends on the cost and benefit of conflict.

Suppose that \( \phi \in (\max\{\phi_2, \phi_3\}, \phi_4) \). Since \( \phi > \max\{\phi_2, \phi_3\} \), the optimal choice is either \( \bar{\alpha} \) or \( \alpha^P \), depending on the sign of \( E_A(\pi, \pi_1^A(\pi, \pi_0^A)) - P_A \). From Lemma 5, \( \pi \) is continuous and strictly decreasing in \( \phi \). From the proof of Lemma 8, we know that when \( \phi = \phi_4 \), \( \pi = f(\pi_0^A) \). Therefore, for \( \phi < \phi_4 \), \( \pi > f(\pi_0^A) \). Moreover, when \( \pi > f(\pi_0^A) \), we know that there is switching, and the consequent group size \( \pi_1^A(\pi, \pi_0^A) \) is strictly increasing in \( \pi \), and, therefore, strictly decreasing in \( \phi \). Now, we express \( E_A(\pi, \pi_1^A(\pi, \pi_0^A)) - P_A \) as \( Z(\phi) \), and examine its sign as a function of
φ. Just for notational convenience, we write π₁⁻(π, π₀⁻) simply as ̂π(φ)

\[
Z(φ) = E_{A}(π, π₁⁻(π, π₀⁻)) - P_A
\]

\[
= \frac{1}{π₀⁻} + φ(1 - 2p_c(π₀⁻)) + 2φp_d(̂π(φ)).
\]

It is easy to see that \(Z(φ) ≥ 0\) if and only if \(k ≤ φπ₀⁻(1 + 2p_d(π₁⁻(π, π₀⁻)) - 2p_c(π₀⁻))\). Open conflict prevails otherwise. When \(k = 0\), \(Z(φ) = φ(1 - 2p_c(π₀⁻)) + 2φp_d(̂π(φ)) > 0\). We now show that \(Z(φ) < 0\) when \(k = 1\). \(Z(φ)\) at \(k = 1\) is

\[
-\frac{1}{π} + φ[1 + 2p_d(̂π) - 2p_c(π)]
\]

\[
= \left(\frac{̂π - π}{π}\right) \left(φ[1 - 2p_c(π)] - \frac{1}{π}\right)
\]

Since \(̂π - π > 0\), if \(1 - 2p_c(π) < 0\), then \(Z(φ)\) is negative. Now, suppose that \(1 - 2p_c(π) > 0\). We have \(φ < φ_4\), implying that \(φ < \frac{1}{π[1 - 2p_c(π)] + 2p_d(π)} \frac{1}{π[1 - 2p_c(π)]} \). This simplifies to \(φ[1 - 2p_c(π)] < \frac{1}{π}\). Again, \(\left(\frac{̂π - π}{π}\right) \left(φ[1 - 2p_c(π)] - \frac{1}{π}\right) < 0\). Therefore, \(Z(φ)\) at \(k = 1\) is negative. \(\square\)

6.8 Proof of Proposition 3

Proof. Consider the subgame where the opposition does not commit not to switch. Clearly, this subgame is precisely the “original game.” So, for \(φ ∈ C\), conflict prevails in equilibrium, and payoffs are

\[
P_A = \frac{k}{π₀⁻} + 1 + φ(1 - π₀⁻)(2p_c(π₀⁻) - 1)
\]

\[
P_B = 1 + φπ₀⁻(1 - 2p_c(π₀⁻)).
\]

Now, consider the subgame where the opposition commits not to switch. The payoffs to each group on the economic path in this subgame are given by

\[
E_A^{NS}(α) = \frac{α}{π₀⁻} + 1 + φ(1 - π₀⁻)(2p_d(π₀⁻) - 1)
\]

\[
E_B^{NS}(α) = \frac{1 - α}{1 - π₀⁻} + 1 + φπ₀⁻(1 - 2p_d(π₀⁻)).
\]

We show that in equilibrium, the incumbent offers \(α^*\), where \(α^*\) is defined as by \(E_B^{NS}(α^*) = P_B\).

First, note that \(α^*\) exists as long as \(φ ∈ (φ_1, φ_2)\). From the definition of \(α^*\), we have

\[
α^* = 1 - 2φπ₀⁻(1 - π₀⁻)(p_d(π₀⁻) - p_c(π₀⁻)).
\]

Since \(p_d(π₀⁻) > p_c(π₀⁻)\), \(α^* < 1\). For \(α^* > 0\), we need \(φ < \frac{1}{2π₀⁻(1 - π₀⁻)(p_d(π₀⁻) - p_c(π₀⁻))} := \overline{φ}\). Now,

\[
\frac{1}{φ_4} - \frac{1}{\overline{φ}} = π₀⁻ + 2(π₀⁻)(p_d(π₀⁻) - p_c(π₀⁻)) > 0 ⇒ \overline{φ} > φ_4.
\]
Since \( \phi_2 < \phi_4 \), we must have \( \phi < \phi_1 \). Therefore, \( \alpha^* \in (0, 1) \)

Any \( \alpha > \alpha^* \) will be rejected, and will result in payoffs \( \{P_A, P_B\} \). We show that \( E^N_{\alpha^*}(\phi^*) > P_A \).

\[
E^N_{\alpha^*}(\phi^*) - P_A = \frac{1-k}{\pi_0^k} - 2\phi(1-\pi_0^A)(p_d(\pi_0^A) - p_c(\pi_0^A)) + 2\phi(1-\pi_0^A)(p_d(\pi_0^A) - p_c(\pi_0^A)) = \frac{1-k}{\pi_0^k} > 0.
\]

Therefore, the incumbent prefers offering \( \alpha^* \) (and inducing the economic path) to conflict. Moreover, \( \alpha^* \) is the maximal share implementable on the economic path.

Since \( \phi \in C \), if the opposition does not commit, it earns a payoff of \( P_B \). On committing not to switch groups, it earns the same amount. We assumed that the the economic path is chosen when the opposition is indifferent. So, the opposition commits not to switch in equilibrium.

Finally, note that \( \alpha^* - f(\pi_0^A) = (1 - \pi_0^A)[1 - \phi\pi_0 \{2(p_d(\pi_0^A) - p_c(\pi_0^A)) + 1\}] > 0 \), since \( \phi < \phi_4 \). \( \square \)

### 6.9 Proof of Proposition 4

Before we prove Proposition 4, we establish the following lemma:

**Lemma 9.** \( \phi_3 \leq \max_k \phi_2 \leq \phi_4 \). The relationship holds with strict inequalities if \( \pi_0^A < \bar{\pi} \) and with equality otherwise.

**Proof.** We omit the proof, as it follows directly from the definitions of \( \bar{\pi} \) and \( \bar{\pi} \) and by inspection of the expressions for \( \phi_1 \), \( \phi_3 \) and \( \phi_4 \). \( \square \)

**Proof of Proposition 4:**

**Proof.** The first part of the proposition derives the condition for \( \phi_4 < 1 \). To see this,

\[
\phi_4 < 1 \iff \frac{1}{\pi_0^A (1 + 2p_d(\pi_0^A) - 2p_c(\pi_0^A))} < 1
\]

\[
\iff (p_d(\pi_0^A) - 2p_c(\pi_0^A)) > \frac{1}{2} \left( \frac{1}{\pi_0^A} - 1 \right) = \frac{1 - \pi_0^A}{2\pi_0^A}.
\]

On the one hand, if \( \phi_4 < 1 \), by Proposition 2, there will be peaceful belligerence at \( \phi = 1 \).

On the other hand, if \( \phi_4 \geq 1 \), by Lemma 9, we see that \( \max_k \phi_2 \) can also be greater or equal to 1. We split this into two subcases: (i) \( \max_k \phi_2 \geq 1 \) and (ii) \( \max_k \phi_2 < 1 \).

In subcase (i), as \( \phi_2 \) is linearly increasing in \( k \in [0, 1] \), there exists a threshold \( \bar{k}_1 \), such that \( \phi_2 \geq 1 \) if and only if \( k \geq \bar{k}_1 \). As \( \phi_1 \) is always less than 1, we then have \( 1 \in (\phi_1, \phi_2) \). Therefore, by Proposition 2, there is open conflict at \( \phi = 1 \) if and only if \( k \geq \bar{k}_1 \).

In subcase (ii), we have \( \max_k \phi_2 < 1 \) but \( \phi_4 \geq 1 \). By Lemma 9, we see that \( 1 \in (\max \{\phi_2, \phi_3\}, \phi_4) \).

By Proposition 2, it implies that open conflict occurs at \( \phi = 1 \) if \( k \) is above a certain threshold (denote the threshold by \( \bar{k}_2 \)), which is derived in the proof of Proposition 2).
Together, we see that in both cases, open conflict occurs at $\phi = 1$, if $k$ is sufficiently high.  

6.10 Proof of Proposition 6

Proof. Assume that $k > \pi_0^A$. It is easy to see that a sufficient condition for $\phi < 1$ is

$$2p_d(\pi^A)\frac{1 - \pi^A}{1 - \pi_0^A} - 2p_c(\pi_0^A) < \frac{k}{\pi_0} + \pi_0^A - 2.$$

Notice that $2p_d(\pi^A)(1 - \pi^A)$ has a maximum value of 2, and $\frac{k}{\pi_0} + \pi_0^A - 2$ increases unboundedly as $\pi_0^A$ goes down since $k > \pi_0^A > (\pi_0^A)^2$. So, for $\pi_0 < \pi$ where $\frac{k}{\pi_0} + \pi_0^A - 2 = 2$–i.e., $\pi = 2 - \sqrt{4 - k}$. Notice that $\pi < k$ since $k < 1$.  

6.11 Proof of Proposition 5

Proof. For any $\phi \leq \max\{\phi_2, \phi_3\}$, $\phi \in C$ if and only if $\phi \in (\phi_1, \phi_2)$. And for $\phi > \max\{\phi_2, \phi_3\}$, $\phi \in C$ if and only if $k > \phi\pi_0^A (1 + 2p_d(\pi^A(\bar{\alpha}, \pi_0^A)) - 2p_c(\pi_0^A))$ and $\phi < \phi_4$. So, we can define

$$C = (\phi_1, \phi_2) \cup \{\phi : k > \phi\pi_0^A (1 + 2p_d(\pi^A(\bar{\alpha}, \pi_0^A)) - 2p_c(\pi_0^A)) \ and \ \phi < \phi_4\}.$$

Now, as $k$ increases, $\phi_2$ increases, leading to an expansion in $(\phi_1, \phi_2)$. Also, with an increase in $k$, the set $\{\phi : k > \phi\pi_0^A (1 + 2p_d(\pi^A(\bar{\alpha}, \pi_0^A)) - 2p_c(\pi_0^A)) \}$ and, thus, $\{\phi : k > \phi\pi_0^A (1 + 2p_d(\pi^A(\bar{\alpha}, \pi_0^A)) - 2p_c(\pi_0^A)) \} \cap (\phi_4, 1]$ expands. Therefore, $C$ expands with $k$.  

6.12 Proof of Proposition 9

To prove this result, we need the following lemma, which describes how the incumbent’s expected two-period per capita payoff varies with the cost of mobility in the three different equilibrium regimes.

Lemma 10. Suppose that $A$ is the incumbent group in period 1 with size $\pi_0^A$, and let $V_A(\phi)$ denote $A$’s expected two-period per capita payoff as a function of the cost of mobility $\phi$. In the no-conflict equilibrium regime, $V_A$ is increasing in $\phi$. In the open-conflict regime and in the peaceful-belligerence regime with no switching, $V_A$ is increasing in $\phi$ if and only if $p_c(\pi_0^A) \geq \frac{1}{2}$. In the peaceful-belligerence regime with switching, a sufficient condition for $V_A$ to be increasing in $\phi$ is that $p_c(\pi_0^A) \geq \frac{1}{2}$.  

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Proof. \( V_A(\phi) \) denotes \( A \)'s expected two-period per capita payoff as a function of \( \phi \).

\[
V_A(\phi) = \begin{cases} 
E_A(\alpha^e_1, \pi^A_1(\alpha^e_1)) & \text{in the no-conflict regime} \\
\alpha & \text{in the open-conflict regime} \\
E_A(\bar{\pi}, \pi^A_1(\bar{\pi}, \pi^A_0)) & \text{in the peaceful-belligerence regime}
\end{cases}
\]

It is easy to see that \( E_A(\alpha^e_1, \pi^A_1(\alpha^e_1)) \) is strictly increasing in the cost of mobility \( \phi \) and \( \alpha \) is strictly increasing in \( \phi \) if and only if \( p_c(\pi^A_0) > \frac{1}{2} \).

The relationship between the incumbent’s payoff in the peaceful-belligerence regime and the cost of mobility depends on whether or not switching occurs in equilibrium. First, consider peaceful belligerence without switching. Such a case arises if \( \bar{\pi} \in [g(\pi^A_0), f(\pi^A_0)] \). In this case, \( E_A(\bar{\pi}, \pi^A_1(\bar{\pi}, \pi^A_0)) = \frac{1}{\pi^A_0} + \phi(1 - \pi^A_0)(2p_c(\pi^A_0) - 1) \), which is increasing in \( \phi \) if and only if \( p_c(\pi^A_0) > \frac{1}{2} \).

Next, consider the peaceful-belligerence regime with switching. Such a case arises if \( \bar{\pi} > f(\pi^A_0) \). In this case, \( \bar{\pi} \) satisfies \( E_B(\alpha, \pi^A(\alpha)) = P_B \). As derived in the proof of Lemma 5, we see that \( \bar{\pi} \) is given implicitly by the group composition \( \hat{\pi} (= \pi^A_1(\bar{\pi}, \pi^A_0)) \) that satisfies \( \pi_1p_d(\pi_1) = \frac{1}{2} \left[ \frac{1}{\hat{\pi}} - \pi^A_0(1 - 2p_c(\pi^A_0)) \right] \), and \( \hat{\pi} \) is decreasing in \( \phi \). In this case, we have \( \hat{\pi}E_A(\bar{\pi}, \pi^A_1(\bar{\pi}, \pi^A_0)) + (1 - \hat{\pi})E_B(\bar{\pi}, \pi^A_1(\bar{\pi}, \pi^A_0)) = 2 \). Therefore, substituting for \( E_B(\cdot) \) we get

\[
E_A(\bar{\pi}, \pi^A_1(\bar{\pi}, \pi^A_0)) = 1 + \frac{1}{\pi} + \left( \frac{1}{\pi} - 1 \right) \phi\pi^A_0(1 - 2p_c(\pi^A_0)).
\]

As \( \hat{\pi} \) is decreasing in \( \phi \), and if \( p_c(\pi^A_0) > \frac{1}{2} \), all the terms in (1) are positive and increasing in the cost of mobility \( \phi \). Therefore, a sufficient condition for \( E_A(\bar{\pi}, \pi^A_1(\bar{\pi}, \pi^A_0)) \) (in the peaceful-belligerence regime with switching) to be increasing in \( \phi \) is that \( p_c(\pi^A_0) > \frac{1}{2} \).

\[ \mathbf{Proof\ of\ Proposition\ 9} \]

Proof. We can rewrite \( V_A(\phi) \) as follows:

\[
V_A(\phi) = \max\{E_A'(\phi), P_A'(\phi)\}
\]

where

\[
E_A'(\phi) = \begin{cases} 
E_A(\alpha^e, \pi^A_1(\alpha^e)) & \text{for } \phi \in [0, \phi_3] \\
E_A(\bar{\pi}, \pi^A_1(\bar{\pi})) & \text{for } \phi \in (\phi_3, 1]
\end{cases}
\]

and

\[
P_A'(\phi) = \begin{cases} 
0 & \text{for } \phi \in [0, \phi_1] \\
P_A & \text{for } \phi \in (\phi_1, 1]
\end{cases}
\]

For the first part of the proposition, we show that if \( p_c(\pi^A_0) > \frac{1}{2} \), \( V_A(\phi) \) is maximized at \( \phi = 1 \).

As \( E_A(\alpha^e, \pi^A_1(\alpha^e)) = E_A(\bar{\alpha}, \pi^A_1(\bar{\alpha})) \) at \( \phi = \phi_3 \), it follows that \( E_A'(\phi) \) is continuous in \( \phi \in [0, 1] \).
Moreover, by Lemma 10, if $p_c(\pi_0^A) > \frac{1}{2}$, $E_A(\alpha^e, \pi_1^A(\alpha^e))$ is strictly increasing in $\phi \in [0, \phi_3]$, $E_A(\bar{\alpha}, \pi_1^A(\bar{\alpha}))$ is strictly increasing in $\phi \in (\phi_3, 1]$ and $P_A$ is strictly increasing in $\phi$. Therefore, if $p_c(\pi_0^A) > \frac{1}{2}$, the function $E'_A(\phi)$ is strictly increasing in $\phi \in [0, 1]$, and the function $P'_A(\phi)$, by construction, is constant over $[0, \phi_1]$ and strictly increasing over $(\phi_1, 1]$. Now, notice that if there are real valued functions $f$ and $g$ that are strictly (weakly) increasing over the same range, then the function $\max\{f, g\}$ will also be strictly (weakly) increasing over the same range. This indicates that $V_A(\phi)$ is weakly increasing over $[0, \phi_1]$ and strictly increasing over $(\phi_1, 1]$. Moreover, since

$$V_A(\phi) = \max\{E_A(\alpha^e, \pi_1^A(\alpha^e)), 0\} = E_A(\alpha^e, \pi_1^A(\alpha^e))$$

for $\phi \in [0, \phi_1]$, $V_A(\phi)$ is strictly increasing over $[0, \phi_1]$. Therefore, $V_A(\phi)$ is strictly increasing (possibly discontinuously) over the entire range of $\phi$.

To prove the second part of the proposition, we show that there may exist local maxima in $(0, 1)$ if $p_c(\pi_0^A) < \frac{1}{2}$. By Lemma 10, $V_A(\phi)$ is strictly decreasing over $(\phi_1, \phi_2]$. As $V_A(\phi)$ is increasing up to $\phi = \phi_1$, we may have a local maximum at $\phi_1$. A sufficient condition for this local maximum to be a global maximum is that $\phi_2 \geq 1$. Similarly, one can derive other sufficient conditions for $\phi = 1$ not to be a global maximum. For example, if $\phi_4 < 1$, by Proposition 2, we know that peaceful-belligerence regime without switching prevails in $(\phi_4, 1]$. Further, as $p_c(\pi_0^A) < \frac{1}{2}$, by Lemma 10, $V_A(\phi)$ is decreasing in $(\phi_4, 1]$. Therefore, $\phi = 1$ cannot even be a local maximum in this case. \(\square\)