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## Accuracy of premium calculation models for CAT bonds: An empirical analysis

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## Working Paper Series

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## an Empirical Analysis

by Marcello Galeotti, Marc Gürtler, and Christine Winkelvos

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# Accuracy of Premium Calculation Models for CAT Bonds

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## an Empirical Analysis<sup>+</sup>

by Marcello Galeotti\*, Marc Gürtler\*\*, and Christine Winkelvos\*\*\*

**Abstract.** CAT bonds are of significant importance in the field of alternative risk transfer. Since the market of CAT bonds is not complete, the application of an appropriate pricing model is of high relevance. We apply different premium calculation models in order to compare them with regard to their predictive power. Without taking the financial crisis into account, a version of the Wang transformation model and the linear model are the most accurate ones. In contrast, under consideration of the financial crisis, all analyzed models are approximately equivalent. Furthermore, we find that CAT bond specific information does not improve out-of-sample results.

**Keywords:** CAT Bonds, Alternative Risk Transfer, Premium Calculation Models, Empirical Analysis

**JEL classification:** G13, G22

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## Introduction

Since both the trend of insured losses and the trend of numbers of catastrophes are positive, (re-)insurance companies have to consider new ways of coping with the risk.<sup>1</sup> One possibility is to transfer the risk from reinsurance markets to financial markets. Important financial instruments which are used for the transfer are (CAT-)catastrophe bonds.<sup>2</sup> The volume of CAT bond principal outstanding rose to USD 13.8 billion in 2007.<sup>3</sup> After a collapsing market has been observed in 2008, the market regained strength in 2009. The main idea of catastrophe securitization by a CAT bond transaction is that a sponsor – usually a (re-)insurer – enters into an alternative reinsurance contract with a Special Purpose Vehicle (SPV). Thus, the sponsor is protected against high losses due to a specified catastrophe up to a certain limit. In order to guarantee insurance coverage up to the limit, the SPV issues CAT bonds to investors. Investors buy the bonds to diversify their portfolios and to receive high yields resulting from the covered peril.<sup>4</sup> A challenging question for the trading of CAT bonds is how CAT bond transactions can be priced best. The objective of this paper is the identification of the most accurate pricing model. Therefore, we compare different selected premium calculation models and include pricing determining factors.

In order to describe these models, figure 1 presents the basic structure of

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<sup>1</sup>See (Munich Re, 2010).

<sup>2</sup>Generally, bonds whose payment structure is linked to insurance risk are called Insurance-Linked Securities (ILS). If catastrophe risk is securitized, one refers to CAT bonds.

<sup>3</sup>See (Carpenter, 2008, p. 13).

<sup>4</sup>For a more precise description of the functionality of CAT bonds see, for instance, (Carpenter, 2006).

a CAT bond transaction.

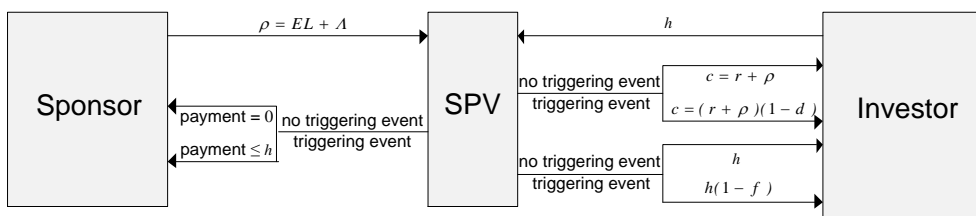


Figure 1: CAT Bond Transaction

Within the framework of the basic structure, the sponsor pays premiums  $\rho$  to the SPV to receive insurance coverage up to the limit  $h$ . The premium  $\rho$  consists of the expected value of loss  $EL$  plus a load for risk margin and expenses  $\Lambda$ . In order to guarantee insurance coverage for the sponsor, the SPV in turn issues CAT bonds to an investor<sup>5</sup> who pays the par amount  $h$  at issue date. If no triggering event occurs, the investor receives at maturity the par amount  $h$  and a coupon  $c$  consisting of the risk-free interest rate  $r$  and the premium  $\rho$ . In case of a triggering event, the coupon to the investor is reduced by  $d$ ,  $0 \leq d \leq 1$ . Furthermore, the par amount at maturity  $h$  might be reduced by  $f$ ,  $0 \leq f \leq 1$ . However, the sponsor receives insurance coverage according to the reinsurance contract between the sponsor and the SPV up to the limit  $h$ .

Obviously, the key parameter of a CAT bond transaction and thus of the CAT bond price is the premium  $\rho$ . The premiums are usually determined on the basis of premium calculation models which use the relationship between  $\rho$  and  $EL$ . For instance, (Lane, 2000) takes the relationship of figure 1 as

<sup>5</sup>Typically, there are more than one investors. For simplicity, we assume only one investor within the basic transaction scheme.

a basis and models  $\Lambda$  by applying a Cobb-Douglas production function on the probability of first loss  $PFL$  and the conditional expected loss  $CEL$ . Instead, (Lane and Beckwith, 2008) and (Lane and Mahul, 2008) suggest a multiple linear relationship between  $\rho$ ,  $EL$ , and an additional factor that covers cycle effects. Other multiple linear approaches have been established by (Berge, 2005) and (Dieckmann, 2008). Both analyses identify further premium determining factors to characterize the risk load  $\Lambda$ . In contrast, (Major and Kreps, 2003) use a loglinear relationship between  $\rho$  and  $EL$  in order to describe catastrophe risk premiums. However, they do not consider catastrophe risk premiums of CAT bonds, but of traditional treaties.<sup>6</sup> Finally, (Wang, 2000) develops a distortion operator, which transforms a probability of loss into an empirical one. It can be used for deriving the CAT bond premiums and fulfills the requirements of a coherent risk measure as defined by (Artzner et al., 1999).

To our knowledge, the literature lacks a comparison of different premium calculation models. Since the determination of the premium is of high relevance for the price of the CAT bond, it is an important question which model is most accurate in order to describe and forecast CAT bond premiums. Thus, the main focus of this paper lies in the comparison of selected premium calculation models from the literature. Therefore, fundamentals of premium calculation models are provided, before the selected models are described in more detail within the presentation of the empirical methodology. All models are based on the parameter  $EL$ . We also include pricing determining factors in the linear and the loglinear models. We examine the

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<sup>6</sup>In comparison, we will apply the loglinear approach to CAT bonds.

influence on pricing of both macroeconomic factors like cyclic, seasonal and business cyclic effects and CAT bond specific factors as the type of trigger mechanism or the peril. The empirical analysis is established on the basis of CAT bond premiums within the period from 1999 to 2006. The predictive power of the analysis is examined by two out-of-sample analyses for CAT bonds issued between 2006 and 2008 as well as 2006 and 2009. In the first out-of-sample analysis, we do not consider bonds that are issued during the financial crisis, whereas the second analysis also takes the financial crisis into account.

## Premium Calculation Models

Premium calculation models determine the premiums  $\rho$ , which need to be paid by the sponsor in order to receive protection against predefined losses. As mentioned above, the coupon payments to investors then result in  $c = r + \rho$ . In order to introduce some relevant variables and to understand the models under consideration, we start with a short introduction to insurance pricing. In this context, we assume risk to be characterized by a non-negative random loss variable  $X$ . Although the range of  $X$  is  $[0, \infty)$ , the insured risk is limited and refers to an interval  $(0, \bar{X}]$  with  $\bar{X}$  defining the maximum insured loss. In addition, the insured risk is usually divided into so-called layers  $(a_i, a_i + h_i]$  ( $i = 1, \dots, n$ ), i.e.  $\bigcup_{i=1}^n (a_i, a_i + h_i] = (0, \bar{X}]$ .<sup>7</sup> A layer, in turn, characterizes a risk range which a specific insurance product refers to. For instance, CAT bonds typically refer to the “last” layer  $(a_n, a_n + h_n]$ . With these assumptions, we are able to characterize the loss that is connected

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<sup>7</sup>See, for an example, (Froot, 2001, pp. 542).

with a layer with attachment point (point of first loss)  $a$  and exhaustion point (point of last loss)  $a + h$ . This so-called layer loss is defined as<sup>8</sup>

$$X_{(a,a+h]} = \begin{cases} 0, & \text{if } X \leq a, \\ X - a, & \text{if } a < X \leq a + h, \\ h, & \text{if } X > a + h. \end{cases} \quad (1)$$

This means that if the loss is less than or equal to the attachment point, then there occurs no loss to the layer  $(a, a + h]$ . If the loss  $X$  lies between the attachment point and the exhaustion point, then the layer loss is given by  $X - a$ . If the loss exceeds the exhaustion point, then the loss charged to the layer is the exhaustion point minus  $a$ . In order to calculate the expected layer loss, we first introduce the cumulative distribution function  $F_X(x) = P(X \leq x)$  of the loss variable  $X$  and the decumulative distribution function  $S_X(x) = 1 - F_X(x) = P(X > x)$ .<sup>9</sup> In addition, we assume the existence of the density function  $f_X(x)$  and, thus, of  $s_X(x) = S'_X(x) = -f_X(x)$ .

In order to determine the decumulative distribution function and the consequent exposure of their assets, reinsurance companies usually use geophysical commercial models.<sup>10</sup> However, under the assumption of an existing decumulative distribution function of  $X$ , it is easy to present the decumula-

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<sup>8</sup>See (Wang, 2004, pp. 19).

<sup>9</sup> $P(A)$  denotes the probability that event  $A$  will occur.

<sup>10</sup>The main geophysical commercial models are provided by Applied Insurance Research Worldwide (AIR), Risk Management Solutions (RMS), and Equecat (EQE). A detailed description of these models can be found in (Nguyen, 2007, pp. 287) and (Strassburger, 2006, pp. 31). For a description and discussion of different types of decumulative distribution functions see (Strassburger, 2006, pp. 75).



tive distribution function of the layer loss  $X_{(a,a+h]}$  as follows:<sup>11</sup>

$$S_{X_{(a,a+h]}}(y) = \begin{cases} S_X(a+y) = P(X > a+y), & \text{if } 0 \leq y < h, \\ 0, & \text{if } y \geq h. \end{cases} \quad (2)$$

By the use of  $S_{X_{(a,a+h]}}$ , it is possible to determine the premium  $\rho$  which consists of the expected layer loss (rate)  $EL$ <sup>12</sup> of an insurance product and the additional absolute risk load  $\Lambda$ . The consideration of a risk load is necessary since risk neutral valuation is not possible because of market incompleteness.<sup>13</sup> Consequently, risk neutral probabilities are not uniquely defined and real probabilities are needed for the determination of the expected loss. Furthermore, empirical studies indicate that the risk load  $\Lambda$  is significantly positive in connection with CAT derivatives premiums.<sup>14</sup>

The determination of the expected loss of an insurance product corresponds to the calculation of expected loss of the associated layer. Since the expected absolute loss for an arbitrary loss variable  $X$  (with minimum value 0) is given by<sup>15</sup>

$$E(X) = \int_0^{\infty} S_X(x) dx, \quad (3)$$

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<sup>11</sup>See (Wang, 2004, pp. 20).

<sup>12</sup>In the following  $EL$  always denotes an expected loss rate. However, we use the abbreviation “expected loss”.

<sup>13</sup>See (Froot, 2001, pp. 537) for a discussion of problems while using the expected loss as a pricing approach.

<sup>14</sup>See (Wang, 2004) and (Lane, 2000, p. 269).

<sup>15</sup>See, for instance, (Furman and Zitikis, 2008, p. 459).

the expected value of the absolute layer loss  $X_{(a,a+h]}$  immediately results from

$$E(X_{(a,a+h]}) = \int_0^{\infty} S_{X_{(a,a+h]}}(y)dy = \int_0^h S_X(a+y)dy = \int_a^{a+h} S_X(x)dx. \quad (4)$$

In addition, it is possible to characterize the expected layer loss  $EL$  by the probability of first loss  $PFL = S_X(a) = P(X > a)$  and the conditional expected loss (rate)  $CEL = E(X_{(a,a+h]}|X > a)/h$ , since<sup>16</sup>

$$EL = \frac{E(X_{(a,a+h]})}{h} = P(X > a) \cdot \frac{E(X_{(a,a+h]}|X > a)}{h} = PFL \cdot CEL. \quad (5)$$

Finally, we introduce the so-called probability of last loss  $PLL = S_X(a+h) = P(X \geq a+h)$ .

Consequently, the premium  $\rho(X)$  for layer  $(a, a+h]$  becomes

$$\rho(X) = EL + \Lambda = PFL \cdot CEL + \Lambda. \quad (6)$$

Besides the linear relationship as described in equation (6), there exist several alternative approaches in the literature which model the dependency between  $\rho(X)$  and  $EL$ . In fact, such a relationship can generally be described as follows:

$$\rho(X) = f(EL, y_1, \dots, y_N), \quad (7)$$

with  $f$  a real function and  $y_1, \dots, y_N$  additional risk load determining parameters. Since this relationship is quite unspecific, we give a short overview of

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<sup>16</sup>See, for instance, (Lane and Beckwith, 2009b).

the different approaches.

(Berge, 2005) accomplishes a multivariate linear regression analysis in order to determine factors explaining the risk loads of CAT bonds in addition to the  $EL$ . He finds that factors as the peril and the applied trigger mechanism determine the premium. The relationship between the premium  $\rho(X)$  and the  $EL$  is the following:

$$\rho(X) = \alpha + \beta \cdot EL + \gamma_1 \cdot y_{peril} + \sum_{i=2}^N \gamma_i \cdot y_i, \quad (8)$$

where  $\alpha, \beta, \gamma_1, \dots, \gamma_N$  are coefficients,  $y_{peril}$  refers to the peril and  $y_2, \dots, y_N$  refer to further determining factors. (Dieckmann, 2008) establishes a similar multiple linear regression in order to analyze the impact of hurricane Katrina. However, the database is quite small since he considers 61 CAT bond premiums only between 3/31/2005 and 3/31/2006. Like (Berge, 2005), (Dieckmann, 2008) identifies CAT bond specific factors as the peril and the applied trigger mechanism as pricing determining factors. Furthermore, he finds that premiums were significantly higher after hurricane Katrina than before.

(Lane, 2000) only focuses on the risk load  $\Lambda$ . He states that the risk load  $\Lambda$  can be represented by a Cobb-Douglas production function of the probability of first loss  $PFL$  and the conditional expected loss  $CEL$ . The model yields

$$\rho(X) = EL + \gamma \cdot (PFL)^\alpha \cdot (CEL)^\beta, \quad (9)$$

where  $\alpha$ ,  $\beta$ , and  $\gamma$  are constants set by fitting the equation to empirical data.<sup>17</sup>

After observing the market for some years, (Lane and Beckwith, 2008) found that there is insufficient variety in CEL. Thus, they could not establish good statistical estimates and abandoned the approach. Instead, they suggest allowing for cyclic adjustments in the linear model in order to explain the risk load. Therefore, they propose to use a cyclic index, which is developed in (Lane et al., 2007). The corresponding linear dependency under consideration of cyclic effects has been tested by (Lane and Mahul, 2008) using a multiple linear regression. The relationship between the premium  $\rho(X)$  and the  $EL$  results in

$$\rho(X) = \alpha + \beta \cdot EL + \gamma \cdot y_{Cycle}, \quad (10)$$

where  $y_{Cycle}$  refers to the cyclic index.<sup>18</sup>

(Major and Kreps, 2003) establish an empirical analysis in order to identify influencing factors on the catastrophe risk premium of traditional treaties. They choose a loglinear relationship between the premium  $\rho$  and the  $EL$ . Apart from that, they consider further determining factors as the geographic location or the lead reinsurer. As already mentioned, we apply this approach to CAT bonds, since in both cases – traditional treaties and CAT bonds –

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<sup>17</sup>See (Lane, 2000, p. 271).

<sup>18</sup>(Lane and Mahul, 2008) also divide the expected loss into different peril-related  $EL$ -factors  $EL_p$ , i.e.  $EL = \sum_p \beta_p \cdot EL_p$ . Since our database does not allow for such detailed analysis, we only refer to expected total loss  $EL$ . Furthermore, we should mention that (Lane and Mahul, 2008) also analyze a linear relationship between  $\rho/y_{Cycle}$  and  $EL$ . However, according to (Lane and Mahul, 2008), this alternative version has a lower explanatory power.

the catastrophe risk premium has the same meaning. On the basis of the loglinear model, the relationship between the premium  $\rho(X)$  and the  $EL$  is given by

$$\ln(\rho(X)) = \alpha + \beta \cdot \ln(EL) + \gamma_1 \cdot y_{geocode} + \sum_{i=2}^N \gamma_i \cdot y_i, \quad (11)$$

where  $y_{geocode}$  refers to the geographic location.

A problem of the presented approaches is the violation of the so-called translation invariance, a requirement of a coherent risk measure which has been introduced by (Artzner et al., 1999). Against this background, (Wang, 1996), (Wang, 2000), and (Wang, 2004) propose a class of distortion functions which fulfill the requirements of coherency and can be regarded as appropriate risk measures. Within this framework, it is not possible to provide a direct relationship between the  $EL$  and the premium  $\rho$ . Instead, the relationship between a transformed version of  $EL$  and the risk premium  $\rho$  is analyzed.<sup>19</sup> Wang proposed a premium calculation model expressed by

$$\rho(X) \cdot h = \int_a^{a+h} g(S_X(x)) dx, \quad (12)$$

where  $S_X(x)$  denotes the decumulative distribution function of the loss variable  $X$  as defined above. Furthermore, the function  $g : [0, 1] \rightarrow [0, 1]$  needs to fulfill four necessary criteria in order to allow for a coherent risk measure. The first one is that  $g$  must be increasing ( $g'(u) \geq 0$ ) to ensure that the

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<sup>19</sup>The procedure is described in the next section in more detail.

<sup>20</sup>In contrast to Wang who applies an absolute risk premium,  $\rho(X)$  refers to a relative risk premium.

transformation retains the properties of a decumulative distribution function. Second,  $g$  must be concave ( $g''(u) \leq 0$ ) in order to generate a non-negative risk load and to guarantee that the relative risk loading increases as the attachment point increases for a fixed limit. Third, in order to define valid probabilities after applying the distortion operator, it is necessary that  $g(0) = 0$ ,  $g(1) = 1$ , and  $0 < g(u) < 1$  for all  $0 < u < 1$ . Finally,  $g'(u) \rightarrow +\infty$  as  $u \rightarrow 0+$  is demanded in order to ensure unbounded relative loading at extremely high layers.<sup>21</sup>

Assuming this theoretical background, Wang proposed the distortion operator  $g_\lambda(u) = \Phi(\Phi^{-1}(u) + \lambda)$ , with  $\Phi$  being the standard normal distribution function and  $\lambda > 0$ . Posing  $u = S_X(x)$  and  $g_\lambda(u) = S_X^+(x)$ , one gets the probability transformation

$$S_X^+(x) = \Phi(\Phi^{-1}(S_X(x)) + \lambda), \quad (13)$$

which transforms the determined probability of attaching the layer into an empirical probability including a risk load. We call this transformation the Wang1 Transformation.

Because of parameter uncertainties in the modeling of catastrophe losses (Wang, 2004) suggests to replace the normal distribution by a Student's t-distribution and to use the distortion operator  $g_{k,\lambda}(u) = Q_k(\Phi^{-1}(u) + \lambda)$ , where  $Q_k$  denotes the Student's t-distribution with  $k$  degrees of freedom. In

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<sup>21</sup>See (Wang, 2000, p. 18).

our paper, the corresponding probability transformation

$$S_X^+(x) = Q_k(\Phi^{-1}(S_X(x)) + \lambda) \quad (14)$$

is called the Wang2 Transformation.

Considering either version of the Wang transformation, the premium  $\rho$ , resulting from the premium calculation model (12), is

$$\rho(X) \cdot h = \int_a^{a+h} S_X^+(x) dx = EL^+, \quad (15)$$

where  $EL^+$  can be interpreted as a transformed expected loss.

## Empirical Methodology

In the literature, the premium calculation models as described above have only been analyzed in isolation. Thus, the literature lacks a comparison of different models. The empirical analysis has the goal of describing and predicting CAT bond premiums of the linear model, the loglinear model, as well as the Wang transformation model. The most accurate model for forecasting CAT bond premiums shall be identified.

Hence, we will analyze three types of models. First, we consider a linear model as proposed, for instance, by (Berge, 2005) and (Lane and Mahul, 2008). We analyze both a linear1 model ( $\rho_{L1M}$ ), where only the  $EL$  is included as a premium determining factor, and a linear2 model ( $\rho_{L2M}$ ), where additional premium determining factors are included. Second, we follow the suggestion by (Major and Kreps, 2003) of describing the relationship between

$EL$  and  $\rho_{LogLM}$  as a loglinear one. Analogous to the linear models, we implement the loglinear1 model ( $\rho_{LogL1M}$ ) and the loglinear2 model ( $\rho_{LogL2M}$ ). Third, we consider the two models that are based on the Wang1 and Wang2 transformations ( $\rho_{W1T}$  and  $\rho_{W2T}$ ) as proposed by (Wang, 2000) and (Wang, 2004). In order to assess the predictive power of the models, we apply an out-of-sample analysis. More precisely, we estimate the model parameters on the basis of the CAT bond contracts that started between April 1999 and May 2006 (in-sample period 1). Afterwards, we apply the parameterized models on the CAT bond contracts that started between June 2006 and June 2008 (out-of-sample period 1)<sup>22</sup> in order to evaluate the deviations between the model-predicted and the real CAT bond premiums  $\rho_{L1M}$ ,  $\rho_{L2M}$ ,  $\rho_{LogL1M}$ ,  $\rho_{LogL2M}$ ,  $\rho_{W1T}$ , and  $\rho_{W2T}$ .<sup>23</sup> The above time periods are examined since, in a first step, we only want to analyze the models within a functioning market environment. In a second step, the model parameters are estimated on the basis of CAT bond contracts issued between April 1999 and June 2006 (in-sample period 2) and the out-of-sample analysis is based on contracts that started between June 2006 and March 2009 (out-of-sample period 2), this way considering contracts issued during the financial crisis as well. In both studies the prediction accuracy of the respective (in-sample) model is evaluated by comparing the coefficients of determination in the out-of-sample analysis on the basis of the premiums. We compute the out-of-sample  $R^2$

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<sup>22</sup>We use June 2008 as the cut-off point because, according to the literature, the market for CAT bonds stopped in mid-2008. See (Priebe, 2009).

<sup>23</sup>In this connection, it should be emphasized that also within the framework of the loglinear models the out-of-sample analysis is always applied to CAT bond premium  $\rho$  and not to the logarithm  $\ln(\rho)$  of the premium to obtain comparability of the coefficients of determination.



according to the following equation<sup>24</sup>

$$R_{OS}^2 = 1 - \frac{\sum_{t=1}^T (\rho_t - \hat{\rho}_t)^2}{\sum_{t=1}^T (\rho_t - \bar{\rho}_t)^2}, \quad (16)$$

where  $\rho_t$  refers to the observation in the out-of-sample period,  $\hat{\rho}_t$  is the fitted value using results from a predictive regression estimated through the in-sample period, and  $\bar{\rho}_t$  is the historical average premium estimated through the in-sample period.

### **Predicting the CAT Bond Premium – the Test Environment**

In this section, we present the regression equations that are applied in the subsequent study. In a first step, we implement the linear and the loglinear model without considering additional premium determining factors apart from the *EL*. Consequently, the linear1 and loglinear1 models yield

$$\rho_{L1M} = \alpha + \beta \cdot EL + \epsilon \quad (17)$$

and

$$\ln(\rho_{LogL1M}) = \alpha + \beta \cdot \ln(EL) + \epsilon, \quad (18)$$

respectively.

In a second step, we include further premium determining factors as proposed in the literature. Since the linear models and the loglinear model use different additional premium determining factors, we standardize the factor

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<sup>24</sup>See (Campbell and Thompson, 2008).

set and denote the factors by  $y_i$  ( $i = 1, \dots, N$ ). The precise descriptions of all factors are presented in the next section. Thus, the linear2 and the loglinear2 models result in

$$\rho_{L2M} = \alpha + \beta \cdot EL + \sum_{i=1}^N \gamma_i \cdot y_i + \epsilon \quad (19)$$

and

$$\ln(\rho_{LogL2M}) = \alpha + \beta \cdot \ln(EL) + \sum_{i=1}^N \gamma_i \cdot y_i + \epsilon, \quad (20)$$

respectively.<sup>25</sup>

Consequently, the linear model assumes the risk premium to consist of a fixed basic premium  $\alpha$  and variable premium components. In this context,  $\beta$  can be interpreted as the “price” per unit (additional) expected loss and  $\gamma_i$  as the “price” per unit factor risk  $y_i$ . An analogous interpretation emerges in the case of the loglinear model, which refers to the logarithm of  $\rho$  and the logarithm of  $EL$ , respectively.

In contrast to the linear and the loglinear model, the two versions of the Wang transformation establish a relationship between  $\rho(X)$  and the above described transformation  $EL^+$  of the expected loss. Since the data set characterizing CAT bonds does not contain the transformed expected loss, we have to approximate the  $EL^+$ . Using the trapezian rule, the integral in (15)

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<sup>25</sup>The representations of the loglinear models (18) and (20) are used in the context of in-sample analyses. Since the out-of-sample analyses are based on  $\rho$  and not on  $\ln(\rho)$ , we apply the exponential function on both sides of (18) and (20), respectively.

can be approximated as

$$\begin{aligned}\rho(X) &= \frac{1}{h} \cdot \int_a^{a+h} g(S_X(x)) dx \approx \frac{1}{h} \cdot \frac{1}{2} \cdot h \cdot [g(S_X(a)) + g(S_X(a+h))] \\ \Leftrightarrow \rho(X) &\approx \frac{1}{2} \cdot [g(PFL) + g(PLL)].\end{aligned}\quad (21)$$

Against this background, we test the Wang1 transformation model on the basis of the nonlinear regression

$$\rho_{W1T}(X) = \frac{1}{2} \cdot [\Phi(\Phi^{-1}(PFL) + \lambda) + \Phi(\Phi^{-1}(PLL) + \lambda)] + \epsilon_\lambda, \quad (22)$$

with regression parameter  $\lambda$ . Analogously, the Wang2 transformation model is evaluated by using the nonlinear regression

$$\rho_{W2T}(X) = \frac{1}{2} \cdot [Q_k(\Phi^{-1}(PFL) + \lambda) + Q_k(\Phi^{-1}(PLL) + \lambda)] + \epsilon_{k,\lambda}, \quad (23)$$

with regression parameters  $k(\in \mathbb{N})$  and  $\lambda$ .

According to (Wang, 2000) and (Wang, 2004) the regression parameter  $\lambda$  can be interpreted as the market price of risk, i.e. the “price” per unit (additional) total risk. In this connection, (Wang, 2000) shows that  $\lambda$  corresponds to the Sharpe ratio if the underlying random variable  $X$  is normally distributed and consequently risk is measured by standard deviation. In the case of alternative distributions,  $\lambda$  is a direct extension of the Sharpe ratio (Wang, 2004, p. 21). By this means,  $\lambda$  has a similar interpretation as  $\beta$  in the linear and loglinear models. However,  $\lambda$  refers to total risk and  $\beta$  refers to  $EL$ .

## Description of the Data

The empirical analysis uses original<sup>26</sup> data sets provided by Lane Financial LLC and Standard & Poor's (S&P), where 176 CAT bond transactions between the years 1999 and mid 2009 are specified. The data include, in particular, values of the above mentioned *PFL*, *PLL*, *CEL*, and *EL*. Furthermore, CAT bond specific information is available for all CAT bonds regarding maturity, rating, trigger mechanism, peril, and issue date. We only take into account CAT bonds rated by S&P<sup>27</sup> since only for these CAT bonds we received information on the peril and the applied trigger mechanism. The CAT bond specific information is described in the following.

### *Trigger Mechanisms*

The payout of a CAT bond connected to a specified catastrophe is defined by trigger mechanisms. Basically, there are five different trigger mechanisms. The indemnity trigger uses the height of actual losses of the sponsor, the parametric trigger uses a physical measure like the Richter scale, the index trigger uses a specified index, the modeled loss trigger uses catastrophe modeling software, and the hybrid trigger uses combinations of different triggers in order to define the payout in case of catastrophe.<sup>28</sup> Our data from 1999 to 2009 shows that the issued CAT bonds (rated by S&P) split up into 47%

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<sup>26</sup>Most existing analyses are established by using original data sets, for example, the analyses by (Major and Kreps, 2003), (Lane, 2000), (Lane and Beckwith, 2008), (Berge, 2005), and (Wang, 2004). However, (Lane and Mahul, 2008) as well as (Dieckmann, 2008) establish different analyses using both original and secondary market data.

<sup>27</sup>The rating of CAT bonds is mainly done by S&P, Moody's Investors Service (Moody's) and Fitch Ratings (Fitch).

<sup>28</sup>See (Carpenter, 2007, pp. 27) and (Dubinsky and Laster, 2003) for a detailed description of trigger mechanisms for CAT bonds.

parametric triggered bonds, 20% indemnity triggered bonds, 23% industry index triggered bonds, 7% modeled triggered bonds, and 3% hybrid triggered bonds.<sup>29</sup>

All of the trigger mechanisms are susceptible to basis risk and moral hazard to a certain extent. (Cummins and Weiss, 2009) and (Dubinsky and Laster, 2003) suppose that prices for CAT bonds with an indemnity trigger might be higher compared to CAT bond prices with different trigger mechanisms due to basis risk. They also state that transaction costs for indemnity triggered CAT bonds are very high, since more documentation is needed compared to nonindemnity trigger mechanisms. We will verify these statements in our empirical analysis. For this purpose, we include four dummy variables, where the industry index trigger is given by

$$y_{IndustryIndex} = \begin{cases} 1, & \text{if an Industry Index is used,} \\ 0, & \text{else.} \end{cases} \quad (24)$$

$y_{Parametric}$ ,  $y_{Modeled}$ ,  $y_{Hybrid}$  are designed in an equivalent way for the case of parametric triggers, modeled triggers, and hybrid triggers, respectively. The base variable is represented by  $y_{Indemnity}$ , which belongs to the indemnity trigger.

### *Rating of the bond*

The purpose of a CAT bond rating is to provide independent and professional

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<sup>29</sup>(Cummins and Weiss, 2009) find that the CAT bond volume for 1997-2007 was distributed as follows: 25.9% for parametric triggered bonds, 30% for indemnity triggered bonds, 21.5% for industry index triggered bonds, 8.5% for modeled triggered bonds, and 14% for hybrid triggered bonds.

information for investors. Therefore, rating agencies evaluate the catastrophe risk analysis as established by specialized firms (e.g. AIR, RMS, EQE). (Anders, 2005) objects that rating agencies only have little knowledge in the field of catastrophe risk assessment. Thus, it is questionable if the main driver of a CAT bond rating is evaluated appropriately. Against this background, we want to examine whether the CAT bond rating has an impact on the premiums. Since the data set only contains CAT bonds with rating classes  $A$ ,  $BBB$ ,  $BB$ , and  $B$ , the included dummy variables are

$$y_{BB} = \begin{cases} 1, & \text{if the bond is rated BB,} \\ 0, & \text{else,} \end{cases} \quad (25)$$

for  $BB$  rated CAT bonds and  $y_{A,BBB}$  for the aggregated class of  $A$  or  $BBB$  rated CAT bonds. The base variable is  $y_B$ , which characterizes  $B$  rated CAT bonds.

### *Perils*

Most of the perils are hurricane and earthquake perils in the US. (Cummins and Weiss, 2009) find that 61.4% of the CAT bonds issued between 1997 and 2007 covered perils in the US.<sup>30</sup> Apart from that, European windstorms and earthquakes and typhoons in Japan are securitized quite often. Our data show that 25% of the CAT bonds issued between 1999 and 2006 (and rated by S&P) covered earthquake perils, 23% covered hurricane perils, 13% covered the combination hurricane/earthquake, 25% covered combinations of any perils, 12% covered European windstorms, and 2% covered industrial

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<sup>30</sup>Thereby, 29.6% covered US earthquakes and about 31.8% covered hurricanes.

casualties. We include dummy variables

$$y_H = \begin{cases} 1, & \text{if hurricane is the peril,} \\ 0, & \text{else,} \end{cases} \quad (26)$$

for hurricane perils,  $y_{H,EQ}$  for hurricane and earthquake perils,  $y_{Eurwind}$  for European windstorms,  $y_{Comb}$  for combinations of any perils, and  $y_{Ind.Casualty}$  for industrial casualties. The base variable is  $y_{EQ}$  for earthquake perils.

### *Maturity*

There are different maturities of CAT bonds in the markets. In our data set, 42% of the CAT bonds issued between 1999 and 2009 (and rated by S&P) have a maturity of 25 to 36 months, 30% have maturities between 12 and 24 months, and 28% have maturities between 37 and 60 months. It could occur that sponsors prefer CAT bonds with longer maturities in order to avoid price changes on the reinsurance market. This assumption is analyzed by including dummy variables as follows.

$$y_{12m,24m} = \begin{cases} 1, & \text{if the maturity is between 12 and 24 months,} \\ 0, & \text{else,} \end{cases} \quad (27)$$

is included to characterize CAT bonds with maturities between 12 and 24 months,  $y_{25m,36m}$  is included to denote CAT bonds with maturities between 25 and 36 months, and the base variable  $y_{37m,60m}$  refers to CAT bonds with maturities between 37 and 60 months.

Apart from CAT bond specific information, macroeconomic factors, which

will be described in the following, are included in the empirical analysis as well.

### *Cyclic Index*

It is generally accepted in the field of (re-)insurance research that the traditional (re-)insurance market is affected by insurance cycles.<sup>31</sup> Generally, it can be stated that, following a soft market, which can be identified by relatively low prices and new market participants, the market turns into a hard market with relatively high prices. In the special case of catastrophe reinsurance, (Froot, 2001) argues that cyclic effects triggered by catastrophe events can be observed. In the literature, on the one hand it is stated that the CAT bond market is less affected by insurance cyclic effects than the reinsurance market. On the other hand, (Lane and Beckwith, 2007) and (Lane and Beckwith, 2008) assume that cyclic effects have a strong impact on the pricing mechanisms of CAT bonds. As already mentioned, they suggested a cyclic index in order to analyze the impact on pricing. The cyclic index is based on observations of secondary market prices of Insurance-Linked Securities, a pseudo constant expected loss series of original issues from Swiss Re and observations of prices for Industry Loss Warranties. The main problem regarding this index is the fact that the index has not been developed by using statistical methods. Instead, price changes from one year to another have been used. A problem in finding an appropriate index is the unavailability of appropriate data sets. For instance, the insurance cycle lasts about

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<sup>31</sup>See (Lamm-Tennant and Weiss, 1997) and (Cummins and Outreville, 1987) for comprehensive analyses of insurance cycles.



7 years for the United States according to an analysis by (Lamm-Tennant and Weiss, 1997).<sup>32</sup> Assuming that a CAT bond cycle would also last 7 years approximately, we do not have enough CAT bond data to establish a time series analysis. Thus, we will use the cyclic index as proposed by (Lane and Beckwith, 2007) in order to verify whether the adjustment for cyclic effects is improving the model results. We include the cyclic index in our regression analysis ( $y_{cycle}$ ).

### *Seasonal Index*

Apart from cyclic effects, seasonal effects can be observed when examining CAT bonds as well. (Lane and Beckwith, 2007) and (Mocklow et al., 2002) state that before and after hurricane season prices rise and fall due to the expectation of higher losses in this season. We verify whether seasonal effects have any impact on the proposed pricing models. Therefore, the seasonal index proposed by (Lane and Beckwith, 2009a) is used in order to adjust our data for seasonal effects. In fact, they averaged the monthly price shifts over several years. For the analysis, we use the index ( $y_{season}$ ) as another predictor for the regression analysis.

### *Capital Markets*

We use S&P 500 as another predictor in the regression analysis ( $y_{SP}$ ). The purpose is to verify the statement from the literature that developments on capital markets are independent of developments on CAT bond markets. For the assumption of independence, we refer to (Litzenberger et al., 1996) and

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<sup>32</sup>For the analysis, (Lamm-Tennant and Weiss, 1997) used the average loss ratio.

a paper by (Lane and Beckwith, 2009b).

## **Empirical Findings**

The following empirical study is based on two different situations. On the one hand, we consider a reduced data set which does not comprise data after June 2008 since the smooth functioning of markets cannot be taken for granted during the financial crisis. On the other hand, we consider the complete data set including the financial crisis in order to test the predictive power of the models in an arbitrary situation. Within the framework of the linear2 and the loglinear2 model we apply the stepwise linear regression method (by using the PASW Statistics 18 procedure) to obtain the factors that should be included in the model.<sup>33</sup>

### **Reduced Data Set – Stable Market Environment**

As already mentioned, we separate the data set into in-sample data (from April 1999 to May 2006, approximately 2/3 of the data) and out-of-sample data (from June 2006 to June 2008, approximately 1/3 of the data) in order to analyze the predictive power of the linear1,2 models, the loglinear1,2 models, and the Wang1,2 transformation models in a stable market environment without considering the effects of the financial crisis. The results of the regression analyses according to equations (17), (18), (19), (20),<sup>34</sup> (22),

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<sup>33</sup>See (Draper and Smith, 1981) for notes on the stepwise regression procedure.

<sup>34</sup>It should be mentioned again that within the framework of the out-of-sample analysis we apply the exponential function on both sides of (18) and (20), respectively, to make results comparable.

and (23) are shown in table 1. The models are evaluated on the basis of the (adjusted) coefficients of determination referring to the out-of sample analyses.

Model equation	Linear1 (17)	Loglinear1 (18)	Linear2 (19)	Loglinear2 (20)	Wang1 (22)	Wang2 (23)
In-sample Analysis						
			Dependent Variable			
	$\rho_{L1M}$	$\ln(\rho_{LogL1M})$	$\rho_{L2M}$	$\ln(\rho_{LogL2M})$	$\rho_{W1T}$	$\rho_{W2T}$
<i>constant</i>	0.027 (0.002)**	-0.764 (0.123)**	0.037 (0.002)**	-0.910 (0.155)**		
<i>EL</i>	2.418 (0.102)**		2.174 (0.112)**			
$\ln(EL)$		0.465 (0.026)**		0.410 (0.032)**		
<i>yParametric</i>			-0.010 (0.003)**	-0.261 (0.047)**		
<i>yComb</i>			0.011 (0.003)**	0.355 (0.050)**		
<i>yA,BBB</i>			-0.017 (0.004)**	-0.263 (0.092)*		
$\lambda$					0.673	
$\{\lambda, k\}$						{0.471, 7}
<i>(adj.) R<sup>2</sup></i>	84.2%	75.6 %	88.1%	85.1%	72.0%	79.7%
F-Test	563.448**	328.565**	198.430**	154.130**	272.535 **	417.576 **
<i>N</i>	108	108	108	108	108	108
Out-of-sample Analysis						
			Dependent Variable			
	$\rho_{L1M}$	$\rho_{LogL1M}$	$\rho_{L2M}$	$\rho_{LogL2M}$	$\rho_{W1T}$	$\rho_{W2T}$
<i>(adj.) R<sup>2</sup><sub>OS</sub></i>	86.3%	83.1%	86.3%	81.5%	81.5%	86.0%
F-Test	358.081**	281.400**	92.860**	64.099**	251.321**	351.561**
<i>N</i>	58	58	58	58	58	58

*Notes:* The results for the linear2 and loglinear2 models have been received by establishing a stepwise regression method. Standard errors are in parentheses.

\*: Significant at the 95 percent confidence level or better.

\*\* : Significant at the 99 percent confidence level or better.

Table 1: Empirical Results – Reduced Data Set

The in-sample results show very high (adjusted) coefficients of determination varying between 72.0% and 88.1%, which indicates that the models

are able to explain the CAT bond premiums very well referring to the in-sample data. The application of the models with respect to the out-of-sample data also leads to quite good results since the (adjusted) coefficients of determination of the models vary between 81.5% and 86.3%. However, the Wang2 model as well as the linear1,2 models seem to dominate the other models. A surprising result is that the loglinear1 model which does not consider additional premium determining factors dominates the loglinear2 model regarding the out-of-sample results. As far as the linear1 model and the linear2 model are concerned, the premium determining factors do not have a particular relevance since the (adjusted) coefficients of determination are nearly identical.

However, the premium influencing factors in the linear2 and loglinear2 models can be explained economically and are discussed briefly.<sup>35</sup> Combinations of any perils show an increasing effect on premiums in both models. Thus, we can assume that multi-peril CAT bonds are imposed by the market with an additional risk load compared to earthquake perils. Possibly, this can be explained by the fact that the most severe losses to CAT bonds were caused by hurricanes which are mostly included in multi-peril bonds. For instance, 15 Property & Casualty (P & C) Insurers have been left insolvent as a consequence of losses due to hurricanes Hugo (1989), Andrew (1992),

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<sup>35</sup>Since the logarithm function is strictly increasing, the signs of the premium influencing factors in the linear and loglinear models may be compared. However, the factor  $\beta$  in the linear model describes the absolute increase of the risk premium in the case of a one unit (ceteris paribus) increase of  $EL$ , whereas the factor  $\beta$  in the loglinear model characterizes the relative increase of the risk premium in the case of a marginal (ceteris paribus) relative increase of  $EL$ . By this means, the factor  $\beta$  in the loglinear model corresponds to the elasticity of  $\rho$  with respect to  $EL$ . For further information see (Wooldridge, 2009, p. 211).

Amber and Iniki (1992).<sup>36</sup>

Considering trigger mechanisms, the parametric trigger has a decreasing effect on pricing compared to the indemnity trigger in both the linear2 model and the loglinear2 model. Different trigger mechanisms refer to a different impact of basis risk and moral hazard on pricing. In the literature it is stated that a parametric trigger significantly reduces moral hazard for investors compared to indemnity triggers.<sup>37</sup> Apart from that, the trigger can quickly be verified by investors. Thus, investors are particularly interested in CAT bonds using a parametric trigger and demand a risk load lower than when an indemnity trigger is applied. Our results support this thesis.

In both models, a negative impact of CAT bonds rated *A* or *BBB* by S&P is observed compared to CAT bonds rated *B*. This results from the intuitive fact that the risk premium is lower for CAT bonds with a better rating.

As mentioned above, we also want to analyze the complete data set taking into consideration the financial crisis in order to find out whether the models are also able to explain CAT bond premiums in situations when the proper functioning of the markets is rather doubtful. The analysis of the complete data set is topic of the next section.

### **Complete Data Set – Consideration of the Financial Crisis**

Analogous to the preceding section, we proceed by separating the whole data set into an in-sample and an out-of-sample data set. The in-sample data set

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<sup>36</sup>See, for instance, (Carpenter, 2006) and (Banks, 2004, pp. 124).

<sup>37</sup>See (Carpenter, 2007).

comprises CAT bond contracts starting between April 1999 and June 2006 (approximately 2/3 of the data). The out-of-sample data set is extended compared to the first analysis and covers the period from June 2006 to March 2009 (approximately 1/3 of the data). However, the differences between the six considered models are less pronounced than in the case of a complete stable market environment. This issue and the influence of the model factors on the CAT bond premiums are presented in table 2.

Similar to the preceding analysis, the in-sample results again show very high (adjusted) coefficients of determination, varying between 75.2% and 89.1%, which indicates that the models are able to explain the CAT bond premiums very well referring to the in-sample data. The application of the models with respect to the out-of-sample data also leads to quite good but similar results except for the Wang1 transformation model and the Loglinear1 model since the (adjusted) coefficients of determination of the four other models vary between 67.5% and 68.7%. In contrast to the first analysis, a slight improvement of out-of-sample results has been achieved when additional premium determining factors have been included in the linear2 model and the loglinear2 model.

The results referring to the premium influencing factors in the linear2 and the loglinear2 model are also slightly different from the corresponding results of the first study. However, the results for the variables  $y_{Comb}$  and  $y_{A,BBB}$  remain in the same direction as observed in the first analysis. The parametric trigger does not show any significant influence in the linear2 model. Instead, the index trigger has a decreasing effect on prices in the linear2 model compared to the indemnity trigger. Advantages for the sponsor when

Model equation	Linear1 (17)	Loglinear1 (18)	Linear2 (19)	Loglinear2 (20)	Wang1 (22)	Wang2 (23)
In-sample Analysis						
			Dependent Variable			
	$\rho_{L1M}$	$\ln(\rho_{LogL1M})$	$\rho_{L2M}$	$\ln(\rho_{LogL2M})$	$\rho_{W1T}$	$\rho_{W2T}$
<i>constant</i>	0.027 (0.002)**	-0.703 (0.117)**	0.028 (0.002)**	-0.826 (0.138)**		
<i>EL</i>	2.464 (0.102)**		2.345 (0.105)**			
$\ln(EL)$		0.475 (0.025)**		0.435 (0.030)**		
<i>yParametric</i>				-0.257 (0.044)**		
<i>yComb</i>			0.008 (0.003)*	0.363 (0.050)**		
<i>yA,BBB</i>			-0.016 (0.004)**	-0.192 (0.090)*		
<i>yH</i>			0.008 (0.003)*	0.126 (0.054)*		
<i>yEurwind</i>			-0.010 (0.004)*			
<i>yIndex</i>			-0.010 (0.003)**			
<i>y12m24m</i>			-0.007 (0.003)*			
$\lambda$					0.648	
$\{\lambda, k\}$						{0.499, 8}
<i>(adj.) R<sup>2</sup></i>	83.5%	76.5%	89.1%	85.5%	75.2%	79.8%
F-Test	583.472**	373.698**	136.034**	138.115**	356.913**	467.590**
<i>N</i>	117	117	117	117	117	117
Out-of-sample Analysis						
			Dependent Variable			
	$\rho_{L1M}$	$\rho_{LogL1M}$	$\rho_{L2M}$	$\rho_{LogL2M}$	$\rho_{W1T}$	$\rho_{W2T}$
<i>(adj.) R<sup>2</sup><sub>OS</sub></i>	67.5%	66.0%	68.7%	67.7%	63.1%	68.5%
F-Test	118.177**	110.334**	19.161**	25.328**	97.275**	124.125**
<i>N</i>	59	59	59	59	59	59

*Notes:* The results for the linear2 and loglinear2 models have been received by establishing a stepwise regression method. Standard errors are in parentheses.

\*: Significant at the 95 percent confidence level or better.

\*\* : Significant at the 99 percent confidence level or better.

Table 2: Empirical Results – Complete Data Set

using industry index triggers are that the transaction is simple to execute and that the sponsor does not need to provide confidential information. The resulting disadvantage is the high basis risk to the sponsor, since the actual losses of the sponsor might differ significantly from the industry index. The main advantage for the investor is that the industry index does prevent him from moral hazard, although the transparency for the investor is not as high as when applying the parametric trigger, for instance.<sup>38</sup>

In addition to multi-perils, hurricane perils show an increasing effect on premiums in both models as well. The explanation is the same as for the variable combinations of any perils  $y_{Comb}$  in the first analysis. European windstorms show a decreasing effect on premiums in the linear2 model. This implicates that investors demand a smaller risk load for CAT bonds which insure European windstorms compared to CAT bonds with earthquake perils.

Finally, in the linear2 model CAT bonds with a short maturity have a decreasing effect on premiums compared to CAT bonds with a long maturity. This could result from the fact that investors might fear the long risk period in the latter and thus CAT bonds with a short maturity of 12 to 24 months are imposed by the market with lower risk loads.

It has to be stated that, besides the low impact of CAT bond specific information in both the linear2 and the loglinear2 model, we could not identify any significant influence on the premiums of macroeconomic factors in neither analysis. Cyclic effects have recently been discussed in the literature as a factor which influences premiums for CAT bonds. Both analyses – the one with the reduced data set and the one with the complete data set – do

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<sup>38</sup>See (Dubinsky and Laster, 2003) and (Carpenter, 2007).



not verify this statement, which could implicate either that the implemented index is not appropriate or that CAT bond premiums are not affected by cyclic effects. A further development of such indices with more data available could validate our findings. However, the absence of an influence of business cycle effects on premiums supports the thesis of the independence of CAT bond and capital markets, which is widely assumed in the literature.

## Conclusion

Due to an incomplete market for catastrophe risks and the lack of transparency on the CAT bond market, it is difficult to determine an accurate pricing model for CAT bonds. For the same reason, the comparison of different CAT bond premium calculation models remains a challenging question. This paper presented an overview of existing premium calculation models for CAT bonds. The models suggest a relationship between the CAT bond premium  $\rho$  and the expected loss  $EL$ . We distinguish between models postulating either a linear connection or a loglinear relationship between  $\rho$  and  $EL$ . Furthermore, we consider two versions of the so-called Wang transformation model that lead to a relationship between the premium and a transformed version of expected loss  $EL$ . In compliance with the literature, we integrate pricing determining factors (e.g., macroeconomic factors like cyclic, seasonal and business cyclic effects, and CAT bond specific factors, such as the type of trigger mechanism or the peril) into the linear and loglinear models.

The models are compared on the basis of an out-of-sample analysis which has been carried out as follows. First, the model parameters have been

determined on the basis of CAT bond contracts that were issued during the in-sample periods between April 1999 and May 2006 and April 1999 and June 2006, respectively. Second, the calibrated models have been applied to the contracts issued in the out-of-sample periods. Since we essentially want to assess the quality of the models in times of functioning markets, we initially have considered the time period between June 2006 and June 2008 as the out-of-sample period in order to exclude effects of the financial crisis. Subsequently, we have included the financial crisis data and have analyzed an out-of-sample period between June 2006 and March 2009. Finally, the quality of the models has been determined on the basis of the coefficients of determination of the out-of-sample analysis.

Our results show that the consideration of additional premium determining factors in the linear and loglinear models, as recommended in the literature, improve in-sample results compared to the models without these additional factors. However, the (adjusted) coefficients of determination of the out-of-sample analyses of the extended models linear2 and loglinear2 are not significantly better than the results of the linear1 and loglinear1 models. In the case of the first analysis, where effects of the financial crisis have been excluded, the results of the out-of-sample analysis are even worse for the extended models. Apart from the low impact of CAT bond specific information on premiums, we could not identify any significant influence on the premiums of macroeconomic factors, such as a cyclic, seasonal, and business cyclic index.

Moreover, we have found that the Wang2 transformation model always leads to better in-sample and out-of-sample results than the Wang1 trans-

formation. This results from the fact that the Student's t-distribution is able to fit the data better than the normal distribution. Furthermore, the Wang2 transformation as well as the linear1 model and the linear2 model have been most accurate to predict CAT bond premiums in the first analysis with coefficients of determination varying between 86.0% and 86.3%. In the second analysis, where, in addition, CAT bonds issued after mid 2008 have been taken into consideration, we could not identify any model which was significantly better than the others. Summarizing, we would recommend to either implement the Wang2 transformation model or the linear1 model in order to predict CAT bond premiums. Although the linear2 model has a similar adjusted coefficient of determination as the linear1 and the Wang2 transformation model, it is more costly to be implemented, since it requires data concerning peril, rating and the applied trigger mechanism.

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