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Behavioral Dividend Policy

by Marc Gürtler* and Nora Hartmann**

Abstract. In this paper we develop an optimal dividend policy in the presence of limited rational investors. Concretely, investors with mental accounts for dividends and stock prices as well as emotions like disappointment and elation embody the limited rationality. Furthermore, investors evaluate changes in wealth instead of final wealth. A management maximizing investors' 'modified' utility results in the optimality of dividend payments as well as dividend smoothing, which both have long been puzzles to financial theorists. Moreover, a model specification leads to a gradual dividend adjustment to changes in net earnings as described by *Lintner* (1956).

Key words: dividend policy, dividend smoothing, behavioral finance

JEL-Classification: G35

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Behavioral Dividend Policy

1. Introduction

Although dividend policy represents an intensely researched field of modern finance, it is still a challenge to financial economists to develop a framework of optimal dividend policy that is consistent with empirical observations. Especially two questions have almost remained unacknowledged. Due to the tax disadvantage of dividends, the question immediately comes up why firms distribute dividends at all.¹ In 1979 Black already defined this question as ‘dividend puzzle’ and nowadays Baker, Powell, and Veit (2002) show that it is still unsolved. Secondly, financial economists puzzle about the question why the nominal dividend per share fluctuates less than earnings per share for long time periods.^{2, 3} Below, both questions are answered in a general setting with regard to a behavioral decision theory, i.e. ‘disappointment theory’⁴. Furthermore, Baker, Veit, and Powell (2001) empirically show past dividends and current earnings to be the most important factors influencing the current dividend decision.⁵ We place this result on a firm theoretical footing. At last, a framework concretion can reason gradual dividend adjustment to sudden unexpected changes in earnings as observed by Lintner (1956).⁶

The disappointment theory belongs to a research field of descriptive decision theories emerged at the end of the seventies. Such models are developed as a result of an increasing

¹ See Copeland and Weston (1988, p. 480), or Brealey and Myers (2000, pp. 455).

² See for an empirical observation Lintner (1956). For more recent evidence, see Baker and Powell (1999), Garrett and Priestley (2000) or Allen and Michaely (2001).

³ Kumar (1988), Fudenberg and Tirole (1995) or Allen, Bernardo and Welch (2000) may also explain the smoothing of dividends, but their analysis is very complex in terms of signaling and based on restrictive assumptions. See furthermore Lease, John, Kalay, Loewenstein, and Sarig (1999), who review the existing literature about dividend policy and therewith theoretical attempts of solving the dividend puzzle.

⁴ See Bell (1985) or Loomes and Sugden (1986), (1987) and, in addition, Zeelenberg, van Dijk, Manstead, and van der Pligt (2000) for an overview.

⁵ See Lintner (1956) as well.

⁶ Fama and Blacomin (1968) approve Lintner (1956).

number of observations of anomalies in decision-making,⁷ and are targeted on explaining actual behavior.⁸ In particular, the economic theories, which are based on actual behavior, have to apply an accurate identified descriptive theory as the basis for modeling decisions. Thus, since the eighties a field of research arose in the range of capital market theory integrating certain anomalies of investors in existing literature with the objective to gain new insights, especially in terms of pricing. This field of investigation is meanwhile named ‘behavioral finance’.⁹ By the involvement of investor anomalies, findings remain almost descriptive in the field of capital market theory; whereas in the area of corporate finance normative recommendations can even be given: in making financial decisions a (fully) rational management should be considerate of investor anomalies, and adjust its behavior to it to minimize welfare losses. Precisely, the following analysis is based on investors maximizing expected (modified) utility with a utility function according to Bell’s (1985) disappointment theory. Pursuant to that theory, an investor will feel disappointment if his chosen option results in an outcome that is worse than expected. By anticipating these emotions the limited rational investor’s utility of a risky alternative consists of a disappointment function in addition to the straight *von Neumann/Morgenstern* utility:¹⁰ utility does not only depend on the consequence actually arrived, but also on the utility deviation from a reference point. On condition that the utility is higher than the reference point, an investor feels elation, otherwise disappointment.

Within the context of corporate dividend policy, we assume mental accounting of investors in a way that the stock price and the dividend of the evaluated stock are booked in separated mental accounts without the consideration of their mutual connection.¹¹ Such preference structures can express themselves as follows: if the reference point corresponds to the previ-

⁷ See for a particularly prominent experiment of decisions against the expected utility theory Allais (1953).

⁸ See for an overview of such new decision theories Weber and Camerer (1987).

⁹ See for a collection of essays about behavioral finance for example Thaler (1993), and for a literature overview for instance Barberis and Thaler (2001).

¹⁰ See von Neumann and Morgenstern (1947).

¹¹ See Thaler (1985) for mental accounting as well as the further implementations in section 2.

ously paid dividend, a dividend drop by one point will cause disappointment. Given an unchanged profitability, the stock price will increase by one point. With complete and perfect capital markets investors can swap that stock price growth for dividends by selling shares. Nevertheless, this transformation does not remove the loss in utility provoked by a dividend policy against investors' preferences, because emotions already arose. With limited rational investors, different kinds of payments are no longer perfect substitutes.

Even though, Thaler includes behavioral corporate finance on his wish list for further investigations in 1999, the following analysis is not the first trying to answer the questions above by involving limited rationality. Against the background of behavioral decision-making, Shefrin and Statman already deal with the question why corporations pay dividends at all.¹² At first, they argue dividends to be a mechanism of control for weak-willed investors, who tend to spend dividends for periodic consumption, and increases in stock quotation for retirement.¹³ Furthermore, in case of mental accounting, dividend payments can result in a higher utility instead of share repurchases. Finally, different effects of regret can have an effect on the spending of funds.¹⁴ All aspects can lead to a preference for dividends instead of capital appreciation despite of the dividend tax disadvantage. Miller (1986) reviewed these approaches to be nice and interesting stories, but special and not formally founded. As the following approach is deduced from expected (modified) utility maximizing investors, here Miller's criticism is not suitable.

¹² See Shefrin and Statman (1984) as well as DeBondt and Thaler (1995, p. 400).

¹³ This idea bases on the theory of self-control by Thaler and Shefrin (1981).

¹⁴ Like disappointment theory, regret theory – at the same time developed by Bell (1982) and Loomes and Sugden (1982) – includes emotions in the valuation of alternatives. However, emotions in regret theory appear due to a comparison of the arrived result with the result of other alternatives not being chosen.

Concerning the smoothing of dividends, Lintner's model from 1956, which is meanwhile assigned to the field 'behavioral corporate finance', is up to now referred to.¹⁵ According to his survey, management decides about changes in dividends each period instead of the absolute dividend level. In addition, management aims at dividend smoothing in a way that the nominal dividend per share fluctuates less than net earnings per share. Though Lintner develops a model fitting his empirical observations, it is not based on managers maximizing firm profits nor investors maximizing utility. As Miller (1986, p. S467) says: "I assume it [Lintner's model] to be a behavioral model, not only from its form, but because no one has yet been able to derive it as the solution to a maximization problem, despite 30 years of trying it!" By maximizing expected utility of certain preference structures, which will lead to the optimality of stable dividends, the following approach is more fundamental on that score. In addition and as aforementioned, the dividend adjustment process described by Lintner (1956) can be exemplified with this approach. Thus, we place dividend smoothing as well as dividend adjustment on a firm theoretical footing.

The remainder of the paper is organized as follows: in section 2.1. the 'entrepreneurial' framework is concretized. Section 2.2. is concerned with the description of investor anomalies giving a rationale for the assumed utility function. From this, in section 2.3., optimal dividend policy in the presence of limited rational investors is deduced. Section 2.4. analyses dividend volatility over time. Section 2.5. addresses to Lintner's dividend adjustment. Section 2.6. contains discussion, and finally, section 3. concludes.

¹⁵ See Miller (1986), Thaler (1999) or Barberis and Thaler (2001) as well. Beyond, Benartzi, Michaely and Thaler (1997) regard Lintner's model as the best description of the choice of dividend policy available.

2. The model

2.1. The 'entrepreneurial' framework

Consider a firm with an infinite time horizon where time is discrete with a set of dates indexed t . At each date t , the firm generates uncertain net earnings \tilde{x}_t , which are identically and independently distributed over time. The realization x_{tq} of the random variable \tilde{x}_t is determined by the state $q \in \{1, \dots, J\}$ realized at date t . Thereby, x_{tq} is the minimum of all possible outcomes of \tilde{x}_t and x_{tq} is close to zero, but positive (i.e. $x_{tq} > 0$, $x_{tq} \rightarrow 0$) to guarantee that net earnings are not negative. Before state revelation, state $q \in \{1, \dots, J\}$ occurs with probability p_q . Furthermore, we name the time-independent expected net earnings $y := \sum_{j=1}^J p_j \cdot x_{tj}$ and we set $\tilde{\varepsilon}_t := \tilde{x}_t - y$.

At date t income is distributed in terms of a dividend payment D_t to the n_{t-1} stocks existing at the beginning of t whereby the dividend per share D_t / n_{t-1} is termed by d_t .¹⁶ Management can enhance or cut distributable income by external financing on complete and perfect capital markets. Thus, distributable income at state q is defined by net earnings x_{tq} plus cash inflow generated by external financing and marked by F_{tq} . Formally, at each state $q \in \{1, \dots, J\}$, the corporate budget constraint is given by

$$x_{tq} + F_{tq} = D_{tq}. \quad (1)$$

Hence, the conditional equation of the dividend per share at date t and at state q directly results

$$d_{tq}(F_{tq}) = \frac{y + \varepsilon_{tq} + F_{tq}}{n_{t-1}}. \quad (2)$$

¹⁶ The number of stocks n_{t-1} entitled to dividend payments can no longer be influenced at date t .

As financial possibilities external equity financing by a variation of total shares outstanding n_{tq} and debt financing by raising funds B_{tq} at the risk-free rate i are available. Cash inflow using bonds $F_{tq}^{(B)}(B_{tq})$ is embodied by the difference between the cash inflow B_{tq} by raising funds at date t and the cash outflow due to the repayment of debt financing done at date $t-1$:¹⁷

$$F_{tq}^{(B)}(B_{tq}) = B_{tq} - (1+i) \cdot B_{t-1}. \quad (3)$$

If the sign of B_{tq} is negative, there will be no raising of funds, but investment at the risk-free rate i . In case of external equity financing, the cash inflow $F_{tq}^{(n)}(n_{tq})$ is given by multiplying the current stock price S_{tq} with the alteration of the number of total shares outstanding

$$n_{tq} - n_{t-1} :$$

$$F_{tq}^{(n)}(n_{tq}) = S_{tq} \cdot (n_{tq} - n_{t-1}). \quad (4)$$

In doing so, a positive sign of $(n_{tq} - n_{t-1})$ denotes a capital increase; a negative sign characterizes a share repurchase. The stock price S_{tq} (ex dividend) is given by the ratio of the current equity value P_{tq} to the total number of shares outstanding n_{tq} at the end of date t :

$$S_{tq} = \frac{P_{tq}}{n_{tq}}. \quad (5)$$

The current value P_{tq} , in turn, equates the firm value Φ_{tq} reduced by the amount of external debt financing B_{tq} :^{18, 19}

$$P_{tq} = \Phi_{tq} - B_{tq}. \quad (6)$$

¹⁷ Liability is assumed to be unlimited.

¹⁸ The amount of external debt financing just equates the current value of debt.

¹⁹ From $P_{tq} \geq 0$ and the following equation (6), Φ_{tq} serves as an upper limit for B_{tq} .

The firm value (ex dividend) is given by the sum of all expected future net earnings discounted by the cost of capital $r := r' + i$.²⁰

$$\begin{aligned}\Phi_{tq} &= \sum_{\lambda=1}^{\infty} E(\tilde{x}_{t+\lambda}) \cdot \left(\frac{1}{1+r}\right)^{\lambda} = -y + \sum_{\lambda=0}^{\infty} y \cdot \left(\frac{1}{1+r}\right)^{\lambda} \\ &= -y + y \cdot \frac{1}{1-1/(1+r)} = \frac{y}{r} = \Phi.\end{aligned}\tag{7}$$

The entire cash inflow is then given by

$$F_{tq} = F_{tq}^{(n)}(n_{tq}) + F_{tq}^{(B)}(B_{tq}) = P_{tq} \cdot (n_{tq} - n_{t-1}) / n_{tq} + B_{tq} - (1+i) \cdot B_{t-1}.\tag{8}$$

From equation (2) and with regard to the validity of $y + \Phi \stackrel{(7)}{=} (1+r) \cdot \Phi$, at state q the dividend

per share can be expressed by

$$\begin{aligned}d_{tq}(n_{tq}, B_{tq}) &= \frac{y + \varepsilon_{tq}}{n_{t-1}} + \frac{P_{tq} \cdot (n_{tq} - n_{t-1})}{n_{tq} \cdot n_{t-1}} + \frac{B_{tq} - (1+i) \cdot B_{t-1}}{n_{t-1}} \\ &= \frac{\varepsilon_{tq} + (1+r) \cdot \Phi - (1+i) \cdot B_{t-1}}{n_{t-1}} - S_{tq}(n_{tq}, B_{tq}) \\ &= \frac{\varepsilon_{tq} + r' \cdot \Phi}{n_{t-1}} + (1+i) \cdot S_{t-1} - S_{tq}(n_{tq}, B_{tq}).\end{aligned}\tag{9}$$

Financial decisions at date t only affect dividends by the variation of current stock price S_t , which can be managed by debt issues as well as equity issues. In this context, the current dividend level is not influenced by the particular kind of financing, so that the actual choice between debt and equity is irrelevant in accordance with Modigliani and Miller (1958). To simplify the model, we abstract from the concrete financing decision and assume pure debt financing (i.e. $n_t = n_{t-1} =: n$ for all t).²¹

²⁰ The cost of capital r is given in such a way that the stock demand of limited rational investors is positive. Concerning the determination of such a cost of capital as well as an endogenous stock demand by limited rational investors we refer to further literature such as Shefrin and Statman (1994), because in the above approach these aspects are circumstantial.

²¹ A formal proof is available by the authors.

Wealth v_{tq} at state q generated by holding one share for one period can be expressed by the sum of the dividend per share plus the stock price at date t minus the opportunity cost of holding that share. The latter is calculated by the stock price at date $t-1$ added by accrued interest, because the share was bought at that price and funds could be invested risk-freely on the capital market. Then, that wealth v_{tq} is given by

$$v_{tq} = d_{tq} + S_{tq} - (1+i) \cdot S_{t-1}. \quad (10)$$

This can be rewritten as

$$v_{tq} \stackrel{(9),(10)}{=} \frac{r' \cdot \Phi + \varepsilon_{tq}}{n}. \quad (11)$$

2.2. Investors' anomalies and the objective function of management

Rational investors evaluate wealth v_{tq} from equation (11) by a von Neumann and Morgenstern utility function $U(v_{tq})$. The value of this utility is not affected by current dividend policy or otherwise financial policy. Altogether, in this approach the conclusion of Miller and Modigliani (1961) is still valid with regard to only rational investors: a firm's dividend policy does not affect its value nor the utility of investors. With regard to empirical observations about dividend policy, we now attempt to reply to the two questions asked at the beginning of the paper. At first, the impact of limited rational investors to dividend policy is analyzed. Secondly, the optimality of dividend smoothing is verified in the presence of investors with anomalies. Thirdly and finally, the process of dividend adjustment is concretized.

Henceforth, we assume $\hat{n} > 0$ limited rational investors, each of them holding one share for one period, to exist besides $n - \hat{n}$ rational investors.²² As rational investors regard dividend

²² Rational investors could be taken by institutional investors, and limited rational investors by individual investors.

policy as irrelevant, there is no conflict of aims between rational and limited rational investors concerning dividend policy. Therefore, management can completely align dividend policy with the preferences of limited rational investors. To simplify, we assume all existing limited rational investors to have the same preferences. Moreover, the number of shares \hat{n} held by them is assumed to be exogenous, so that management maximizes investors' welfare by optimizing utility of one representative limited rational investor.

In the following, investor anomalies are specified. In accordance with Lintner (1956) as well as with Kahneman and Tversky (1979), we assume limited rational investors to evaluate dividend growth and stock price appreciation instead of the absolute level of wealth.²³ Regarding to Thaler (1985), we further assume investors to administer mental accounts for the dividend and stock price growth of the evaluated stock, which is expressed by two disjoint, separated utility functions $U_t^{(m,d)}$ and $U_t^{(m,S)}$. Therefore, dividend and stock price growth is separately evaluated ignoring mutual connections. Beyond, we abut on disappointment theory by Bell (1986). Thus, investors evaluate dividend and stock price growth with a utility function modified by disappointment instead of a conventional von Neumann and Morgenstern utility function. This modified utility does not only depend on actual dividend and stock price growth, but also on the expectation about the corresponding growth.

The modified, separated utility functions are concretely given by the sum of the von Neumann and Morgenstern utility $U(\cdot)$ over dividend, respectively stock price growth, and the felt emotions. The disappointment function $U^{(D,d)}$ of the dividend account depends on the difference of the actual dividend growth $(1 + \delta_{tq})$ evaluated by the utility function $U^{(d)}(\cdot)$ and a refer-

²³ Lintner (1956) investigated that management does not determine the absolute level of dividends, but rather dividend changes. As management decides in investors' interest, we assume the above mentioned. In addition, this assumption corresponds to the observation that investors evaluate changes in wealth rather than final states. See hereunto Kahneman and Tversky (1979).

ence point. As in Loomes and Sugden (1986) this reference point is the expectation of the assessed value. In this context, it is the dividend growth $(1 + \delta_t^{(R)})$ expected before realization of state, which is given by²⁴

$$1 + \delta_t^{(R)} = 1 + E(\tilde{\delta}_t) = 1 + \sum_{j=1}^J p_j \cdot \delta_{tj} . \quad (12)$$

Also corresponding to Loomes and Sugden (1986), the disappointment function is strictly monotonic increasing [i.e. $U^{(D,d)'(\cdot)} > 0$], so that the degree of emotion rises with increasing distance from the reference point. But, as opposed to Loomes and Sugden (1986), we do not assume a function being symmetric to the origin, but a concave function with $U^{(D,d)''(\cdot)} < 0$. This assumption accords c. p. to the empirical observation that investors stronger evaluate unexpected changes in dividend payments than in stock prices.^{25, 26}

The stock price disappointment function $U^{(D,S)}$ satisfies the above conditions as well and the stock price growth rate σ_{tq} is given by:

$$\sigma_{tq} = \frac{S_{tq}}{S_{t-1}} - 1 \stackrel{(9)}{=} \frac{\varepsilon_{tq} + r' \cdot \Phi + i \cdot P_{t-1}}{P_{t-1}} - (1 + \delta_{tq}) \cdot \frac{d_{t-1}}{S_{t-1}} . \quad (13)$$

Consequentially, the expected growth rate $E(\tilde{\sigma}_t)$ is

$$E(\tilde{\sigma}_t) = \frac{r' \cdot \Phi + i \cdot P_{t-1}}{P_{t-1}} - \left(1 + \sum_{j=1}^J p_j \cdot \delta_{tj} \right) \cdot \frac{d_{t-1}}{S_{t-1}} . \quad (14)$$

In addition, we assume the following relation between stock price and dividend disappointment function

$$U^{(D,S)}(g) = (1/h) \cdot U^{(D,d)}(h \cdot g) \quad \text{with } h > 0. \quad (15)$$

²⁴ $E(\cdot)$ denotes the expectation operator.

²⁵ Aharony and Swary (1980) show that markets positively react to dividend enhancements and strongly negatively to dividend decreases. See also Kao and Wu (1994) or Benartzi, Michaely and Thaler (1997).

²⁶ See Anderson and Sullivan (1993) as well as Inman, Dyer and Jia (1997) who show that, ex post, disappointment has a larger impact on utility than elation.

The parameter h characterizes the different degree of ‘absolute disappointment aversion’ felt by the investor in the dividend and stock price account. This is analogous to the Arrow/Pratt measure of absolute risk aversion.²⁷ According to the latter, the measure of absolute disappointment aversion for dividends can be expressed by $R_A^{(d)}(g) = -U^{(D,d)''}(g)/U^{(D,d)'}(g)$, and the measure of absolute disappointment aversion for the stock price by $R_A^{(S)}(g) = -U^{(D,S)''}(g)/U^{(D,S)'}(g) = h \cdot R_A^{(d)}(h \cdot g)$. Particularly, h equates the ratio of $R_A^{(S)}(0)/R_A^{(d)}(0)$, i.e. the ratio of disappointment aversion in case of neither elation nor disappointment. In the special case of constant absolute dividend disappointment aversion, i.e. $U^{(D,d)}(g) = -\exp(-a^{(d)} \cdot g)$, we can further generalize the parameter h , because the absolute stock price disappointment aversion is then also constant [$h = a^{(S)} / a^{(d)}$]. Thus, in that special scenario, h describes the ratio of the measured values of absolute stock price and dividend aversion independently of the concrete stock price and dividend value g .

Corresponding to the modified utility by Loomes and Sugden (1986), the modified separated utility functions for dividend and stock price are given by²⁸

$$U_t^{(m,d)}(1 + \delta_{tq}) = U^{(d)}(1 + \delta_{tq}) + U^{(D,d)}[U^{(d)}(1 + \delta_{tq}) - (1 + \delta_t^{(R)})] \quad \text{and} \quad (16)$$

$$U_t^{(m,S)}(1 + \sigma_{tq}) = U^{(S)}(1 + \sigma_{tq}) + U^{(D,S)}[U^{(S)}(1 + \sigma_{tq}) - (1 + \sigma_t^{(R)})] \quad (17)$$

at date t and at state q .

The total modified utility $U_t^{(m)}$ is composed by the weighted sum of modified dividend and stock price utility. The weights are given by the ratio of the corresponding previous values and the sum of the last dividend and the last stock price. This leads to

²⁷ See for the Arrow/Pratt measure Arrow (1971) and Pratt (1964).

²⁸ However, Loomes and Sugden (1986) do not consider mental accounts, nor do they apply disappointment theory to corporate finance.

$$\hat{U}_t^{(m)} = \frac{d_{t-1}}{d_{t-1} + S_{t-1}} \cdot U_t^{(m,d)}(1 + \delta_{tq}) + \frac{S_{t-1}}{d_{t-1} + S_{t-1}} \cdot U_t^{(m,S)}(1 + \sigma_{tq}). \quad (18)$$

For simplification of the later derivations, $\hat{U}_t^{(m)}$ is positively and linearly transformed as follows²⁹

$$\begin{aligned} U_t^{(m)} &= (d_{t-1} + S_{t-1}) \cdot \hat{U}_t^{(m)} - (1+i) \cdot S_{t-1} \\ &= d_{t-1} \cdot U_t^{(m,d)}(1 + \delta_{tq}) + S_{t-1} \cdot U_t^{(m,S)}(1 + \sigma_{tq}) - (1+i) \cdot S_{t-1}. \end{aligned} \quad (19)$$

2.3. Optimal dividend policy in the case of limited rational investors

In the following, we assume the von Neumann and Morgenstern utility for the stock price and the dividend to be linear.³⁰ This is not synonymous to risk neutral investors, because in the present context, risk is measured in terms of disappointment.³¹ It directly follows from (19) at state q and with $U^{(i)}(g) = g$ for $i = d, S$ as well as $U^{(D)}(\cdot) := U^{(D,d)}(\cdot)$ and in consideration of (12) to (17)

$$\begin{aligned} U_{tq}^{(m)} &= \frac{r' \cdot \Phi + \varepsilon_{tq}}{n} + d_{t-1} \cdot U^{(D)}[\delta_{tq} - E(\tilde{\delta}_t)] + \frac{S_{t-1}}{h} \cdot U^{(D)}[h \cdot (\sigma_{tq} - E(\tilde{\sigma}_t))] \\ &\stackrel{(11),(13),(14)}{=} v_{tq} + d_{t-1} \cdot U^{(D)} \left[\delta_{tq} - \sum_{j=1}^J p_j \cdot \delta_{tj} \right] \\ &\quad + \frac{S_{t-1}}{h} \cdot U^{(D)} \left[h \cdot \left(\frac{\varepsilon_{tq}}{P_{t-1}} - \left(\delta_{tq} - \sum_{j=1}^J p_j \cdot \delta_{tj} \right) \cdot \frac{d_{t-1}}{S_{t-1}} \right) \right]. \end{aligned} \quad (20)$$

Choosing the dividend growth rate δ_{tq} management's objective is to maximize the modified utility of a representative investor. In doing so, management only influences the disappointment extend; the first part of equation (20) will remain constant. At each date t , the subsequent J simultaneous optimization problems follow

$$U_{tq}^{(m)} \rightarrow \max_{\delta_{tq}}! \quad \text{for all } q \in \{1, \dots, J\} \quad (21)$$

²⁹ Such a transformation has no impact on the ranking of alternatives.

³⁰ Bell (1985) and Loomes and Sugden (1986) also assume a linear von Neumann and Morgenstern utility function.

³¹ See for similar considerations Jia and Dyer (1996).

subject to a positive stock price as well as a positive dividend:³²

$$0 < d_{tq} < \frac{\varepsilon_{tq} + r' \cdot \Phi}{n} + (1+i) \cdot S_{t-1} \quad (22)$$

$$\Leftrightarrow -1 < \delta_{tq} < \frac{\varepsilon_{tq} + (1+r) \cdot \Phi - (1+i) \cdot B_{t-1} - D_{t-1}}{D_{t-1}} \text{ for all } q \in \{1, \dots, J\}.$$

While in Loomes and Sudgen (1986) only lotteries are evaluated, in this approach investor's modified utility is affected by an actual decision besides a random variable: via dividend policy, management has the possibility to minimize totally felt disappointment at each state $q \in \{1, \dots, J\}$, because the state is revealed before the choice of dividend policy, but after investors built expectations about the state and the concrete dividend policy. Insofar, this model is an extension of disappointment theory.

At state q the optimal dividend growth rate results from maximization problem (21). Thus³³

$$\delta_{tq}^* = \delta_{tj}^* + \frac{h \cdot (\varepsilon_{tq} - \varepsilon_{tj})}{n \cdot (h \cdot d_{t-1} + S_{t-1})} \text{ with}^{34} \quad (23)$$

$$-1 < \delta_{tj}^* < \frac{\varepsilon_{tj} + (1+r) \cdot \Phi - (1+i) \cdot B_{t-1} - D_{t-1}}{D_{t-1}}. \quad (24)$$

The optimal dividend growth rate δ_{tq}^* at state q equates the growth rate δ_{tj}^* chosen at the worst state, plus a fraction of net earnings enhancement between the current and the worst state. Consequentially, in this approach earnings as well as past dividends are crucial determinants of any change in dividends as Lintner (1956) and more recently Baker, Veit, and Powell (2001) empirically observed.³⁵ Calculating $(\sigma_{tq} - \sigma_{tj})$ from equation (13) and replacing

³² A zero dividend or stock price would lead to zero dividends and stock prices in all subsequent periods, because the parameters to choose are growth rates. Therefore positive instead of 'non-negative' values are required.

³³ See for proof appendix 1.

³⁴ If the inequalities (24) hold, the border conditions (22) will also hold at each other state $q \in \{1, \dots, J-1\}$. See for proof appendix 2.

³⁵ See also Pruitt and Gitman (1991).

$(\delta_{tq}^* - \delta_{tJ}^*)$ with $h \cdot (\varepsilon_{tq} - \varepsilon_{tJ}) / (n \cdot (h \cdot d_{t-1} + S_{t-1}))$ in accordance to (23) leads to the optimal stock price growth rate at state $q \in \{1, \dots, J-1\}$

$$\sigma_{tq}^* = \sigma_{tJ}^* + \frac{\varepsilon_{tq} - \varepsilon_{tJ}}{n \cdot (h \cdot d_{t-1} + S_{t-1})}. \quad (25)$$

Again, the optimal growth rate equates the sum of the optimal growth rate at the worst state plus a fraction of net earnings enhancement between the current and the worst state. The lower h the more does the stock price participate c. p. in changes in net earnings. Furthermore, as dividends are spent for current consumption, but stock price enhancements serve for retirement savings,³⁶ a more pronounced dividend than stock price disappointment aversion (and thus $h \leq 1$) is plausible. Over the long run, risk becomes less important, and therefore, chances for stock price enhancement take center stage.

At state q , given an optimal dividend policy as indicated by (23), the utility of a representative limited rational investor can be shown as³⁷

$$U_{tq}^{(m)*} = v_{tq} + \frac{(S_{t-1} + h \cdot d_{t-1})}{h} \cdot U^{(D)} \left(\frac{h}{(S_{t-1} + h \cdot d_{t-1})} \cdot \frac{\varepsilon_{tq}}{n} \right). \quad (26)$$

Instead, stocks without dividend payments lead to a lower utility amounting to

$$\check{U}_{tq}^{(m)} = v_{tq} + \frac{S_{t-1}}{h} \cdot U^{(D)} \left(\frac{h}{S_{t-1}} \cdot \frac{\varepsilon_{tq}}{n} \right). \quad (27)$$

So, the first question can already be answered: dividend payments are optimal; and stocks without dividend payments result in welfare losses. Thus, the inclusion of disappointment and mental accounts can solve Black's dividend puzzle, i.e. why firms distribute dividends at all.³⁸

In the following, we deal with the second question: the puzzle of dividend smoothing.

³⁶ See Shefrin and Statman (1984).

³⁷ To obtain (26) substitute δ_{tq}^* according to (23) and insert the result in (20).

³⁸ Introducing taxes in the above framework, dividend payments will still be favored, if welfare losses due to limited rationality are higher than losses due to taxes.

2.4. Dividend volatility over time

To solve the remaining second puzzle, we need a measure to quantify fluctuations of dividends. Therefore, we use the variance of the dividend growth rate given an optimal dividend policy. This variance can be expressed by³⁹

$$\text{var}(\tilde{\delta}_t) = \text{var}(\tilde{\varepsilon}_t) \cdot \mathbb{E} \left(\frac{1}{(\tilde{S}_{t-1}/h + \tilde{d}_{t-1})^2 \cdot n^2} \right). \quad (28)$$

Provided that $h \leq 1$,⁴⁰ the following particularly holds⁴¹

$$\text{var}(\tilde{\delta}_t) < \text{var}(\tilde{\varphi}_t), \quad (29)$$

with $\varphi_{tq} := (x_{tq} - x_{t-1})/x_{t-1}$ as the net earnings' growth rate at state q . Hence, it is shown that the variance of the dividend growth rate is less than that of net earnings per existing shares. Therefore, the consideration of investor anomalies within the dividend decision leads besides the relevance of dividends to the optimality of stable dividends. Especially, this result is independent from the concrete choice of δ_{U}^* . However, the concretion of δ_{U}^* is an open question, which will be answered in the following section.

2.5. Target payout and dividend adjustment

Interviewing firms about periodic choice of dividend levels, Lintner (1956) arrives at the conclusion that management decides about changes in dividends instead of the absolute dividend payout. In doing so, the management aims at realizing a target payout ratio, and gradually adjusts the firm's dividend to it. In 2001 Baker, Veit, and Powell asked managers how they determine dividends, too. About half of their respondents replied that they set an explicit target payout ratio. Pan (2001) also confirms a partial adjustment policy with a long-term dividend payout target in management's mind, so that Lintner's results still seem valid for a big-

³⁹ See for proof appendix 3.

⁴⁰ See for a rationale section 2.3.

⁴¹ See appendix 3.

ger part of firms.⁴² Thus, we can conclude, if net earnings suddenly increase, management will gradually adjust dividend payments to the modified profitability, so that it will achieve c. p. the target payout ratio in the long run.

In the context of our approach, management has to set a target dividend ratio as well, because only the ‘relative’ (referring to states) optimal dividend policy is defined. However, the absolute dividend and stock price level is arbitrary to a certain extent due to δ_{U} as a degree of freedom.⁴³ Following constraint (24), management has to choose δ_{U}^* in the interval $(-1; (\Phi - (1+i) \cdot B_{t-1})/D_{t-1} - 1)$, which is never empty at a date t , since $\Phi - (1+i) \cdot B_{t-1} \underset{(A12)}{>} 0$.

Therefore, at each date t management sets

$$\delta_{\text{U}}^* = \alpha_t \cdot (-1) + (1 - \alpha_t) \cdot \left(\frac{\Phi - (1+i) \cdot B_{t-1}}{D_{t-1}} - 1 \right) \text{ with } 0 < \alpha_t < 1, \quad (30)$$

whereby α_t can be considered as a firm-specific parameter and may be deduced from empirical observations.

According to an example of Lintner (1956), a firm paid \$2 a share on reasonable stable earnings of \$4 a share. After earnings increased to a \$6 level, an ultimate adjustment to a \$3 dividend rate would be indicated. But, the firm increased the dividend to \$2.25 in the first year, to \$2.50 in the second, and to \$2.65 in the third year.⁴⁴ Up to now, we have generally shown the optimality of dividend payments and smoothing as a result of the presence of limited rational investors. Henceforth, we exemplify the empirically observed dividend adjustment via our framework by a concretion of α_t .

⁴² Admittedly, based on annual data of aggregate earnings and dividends from 1871-1993 Pan (2001) finds that managers over adjust dividends, but he confirms Lintner when only considering post-war data.

⁴³ Of course, the constraint (24) has to be considered.

⁴⁴ See Lintner (1956, p. 103).

Example:

At each date t , a firm generates certain net earnings amounting to $y = \$1,000$. The risk-free interest rate amounts to 1% and the cost of capital r amounts to 5% so that the firm value is given by $\Phi = \$20,000$ at each date. Furthermore, the parameter h is 1. Beyond, the firm has 1,000 shares outstanding and chooses $\alpha_t = \alpha = 0.5$ for all t . To be able to calculate the further data, we assume a starting point ($t = 0$) at which the firm has debt outstanding with $B_{-1} = B_0 = \$19,192$ so that the equity value at $t = 0$ is given by $P_0 = \$808$. The realization of the random variable at $t = 0$ amounts to $\varepsilon_0 = 0$, which results in net earnings amounting to $x_0 = \$1,000$. Moreover, the dividend payment is given by $D_0 = \$1,000 + \$19,192 - 1.01 \cdot \$19,192 \approx \808 .

At first, we analyze the effects of a positive shock in net earnings at date 2 amounting to $\varepsilon_2 = \$600$ and remaining for the next 12 dates. This income shock leads to net earnings of $x_t = \$1,600$ at $t = 2, \dots, 12$, so that the earnings' growth rate at $t = 2$ amounts to 60%, but from $t = 3$ until $t = 12$ to 0%. The further data can be taken from table 1.

*** Table 1 here ***

No shock at date 1 results in constant net earnings ($x_0 = x_1 = \$1000$) as well as constant dividend payments and an unchanged extent of debt financing. After an unexpected increase in earnings by 60% to $\$1,600$ as it is at date $t = 2$, the dividend merely grows by 37% to $\$1,108$. The net earnings remain at a level of $\$1,600$ at the following 12 dates. In contrast, the dividend payout is further raised the following 8 dates, but the amount of modification declines over time until the dividend equates the target payout of $\$1,414$ at date 12 – on condition that net earnings remain at the same level as at date $t = 2$. Recapitulating we can conclude

dividends are gradually adjusted, so that the target payout ratio is achieved in the long run.⁴⁵ That manner corresponds to the management of dividends described by Lintner (1956). In addition, it is evident that the variance of the dividend growth rate in the example is less than the variance of the net earnings growth rate even if the dividend rate more frequently changes. In this connection, it should be highlighted that in the above context the dividend growth rate fluctuates less than the net earnings growth rate over time, and in fact, independently from the concrete choice of α_t as already shown in section 2.4.

Now, we briefly address the issue of a permanent negative net earnings shock at date $t = 2$ which leads to net earnings of $x_t = \$400$ at $t = 2, \dots, 12$, so that the earnings' growth rate at $t = 2$ amounts to -60% , but from $t = 3$ until $t = 12$ to 0% .⁴⁶ The corresponding further data can be taken from table 2.

*** Table 2 here ***

After unexpected decreases in earnings as at $t = 2$, the structure of dividend policy is similar to the results after unexpected increases: dividend payments are gradually adjusted until they reach the target payout amounting to $\$202$ at date $t = 12$.

2.6. Discussion

The presented model points out the possibility of gaining new insights by integrating limited rationality in corporate finance. Moreover, empirical observations can theoretically be described and explained. Concretely, we deduced why firms pay and smooth dividends. Fur-

⁴⁵ The analysis of Garrett and Priestley (2000) suggests the adjustment to be quicker than earlier assumed by Lintner (1956).

⁴⁶ We assume the same starting point as in case of the positive net earnings' shock.

thermore, we exemplarily showed how firms adjust their dividend payments to unexpected changes in earnings. For these purposes, we basically made three plausible assumptions about how limited rational investors evaluate dividends and stock prices. Firstly, investors are assumed to evaluate changes in instead of final wealth. Secondly, we assume that investors mentally divide dividends and stock prices. And thirdly, investors are assumed to feel and anticipate disappointment and elation in evaluating dividends and stock prices.

The presented approach is just a partial view, because the pricing on capital markets is deliberately neglected to keep it simple. Therefore, it remains ambiguous how the cost of capital r is determined in a way that both limited and fully rational investors hold shares. An explicit analysis of equilibrium has certainly to be developed, and will lead to interesting effects of back coupling between dividend policy and firm valuation. However, the partial view already results in important and new insights.

Also, the assumption of the solely debt financing for dividend purposes barely does justice to the empirically observed endless diversity of financial instruments. Nevertheless, the irrelevancy of financial policy tracing back to Modigliani and Miller (1958) still holds. Only dividend policy becomes relevant whereby especially the form of payout (dividends versus capital gains) matters. In particular, the results will not change by equity financing as already shown by equation (9).

3. Concluding remarks

Recently, evidence about anomalies in individuals' decisions and evaluations is mainly integrated in the field of capital market theory for purposes of pricing. Even if the conclusions on this field are interesting, these approaches just serve to describe market behavior. However, the consideration of limited rationality in connection with corporate finance, which rarely is

done, can even result in concrete recommendations for management – in this context about optimal dividend policy. Insofar, we wish to encourage further research in the area of ‘behavioral corporate finance’.

Appendix

Appendix 1: Derivation of the optimal dividend growth rate

Under consideration of equation (20) we get the following necessary and sufficient conditions

$$\begin{aligned}
0 &= \frac{\partial U_{tq}^{(m)}}{\partial \delta_{tq}} = d_{t-1} \cdot U^{(D)} \cdot \left[\delta_{tq} - \sum_{j=1}^J p_j \cdot \delta_{tj} \right] \cdot (1 - p_q) \\
&+ S_{t-1} \cdot U^{(D)} \cdot \left(h \cdot \left(\frac{\varepsilon_{tq}}{P_{t-1}} - \left(\delta_{tq} - \sum_{j=1}^J p_j \cdot \delta_{tj} \right) \cdot \frac{d_{t-1}}{S_{t-1}} \right) \right) \cdot \frac{d_{t-1} \cdot (p_q - 1)}{S_{t-1}} \\
&\stackrel{\substack{U^{(D)} > 0 \\ p_q < 1}}{\Leftrightarrow} \delta_{tq} - \sum_{j=1}^J p_j \cdot \delta_{tj} = h \cdot \left(\frac{\varepsilon_{tq}}{P_{t-1}} - \left(\delta_{tq} - \sum_{j=1}^J p_j \cdot \delta_{tj} \right) \cdot \frac{d_{t-1}}{S_{t-1}} \right) \\
&\Leftrightarrow \delta_{tq} - \sum_{j=1}^J p_j \cdot \delta_{tj} = \frac{h \cdot \varepsilon_{tq}}{(S_{t-1} + h \cdot d_{t-1}) \cdot n} \quad \text{for all } q \in \{1, \dots, J\}.
\end{aligned} \tag{A1}$$

In the following, the actual optimal dividend policy is deduced whereby the optimum’s fulfillment of the border constraints (22) is proved in appendix 2. Due to $p_J = 1 - \sum_{j=1}^{J-1} p_j$ and with

$h / ((S_{t-1} + h \cdot d_{t-1}) \cdot n) =: c$ equation (A1) can be rewritten as

$$\sum_{j=1}^{J-1} p_j \cdot (\delta_{tj} - \delta_{tj}) - (\delta_{tq} - \delta_{tq}) = -c \cdot \varepsilon_{tq} \quad \text{for all } q \in \{1, \dots, J-1\}. \tag{A2}$$

Defining

$$\Delta := \begin{pmatrix} \delta_{t1} - \delta_{t1} \\ \vdots \\ \delta_{tJ-1} - \delta_{tJ-1} \end{pmatrix}, \quad P := \begin{pmatrix} p_1 - 1 & \cdots & p_{J-1} \\ \vdots & \ddots & \vdots \\ p_1 & \cdots & p_{J-1} - 1 \end{pmatrix}, \quad E := \begin{pmatrix} \varepsilon_{t1} \\ \vdots \\ \varepsilon_{tJ-1} \end{pmatrix},$$

then, (A2) is equivalent to

$$P \cdot \Delta = -c \cdot E \Leftrightarrow \Delta = -c \cdot P^{-1} \cdot E. \tag{A3}$$

P^{-1} is given as⁴⁷

$$P^{-1} = \begin{pmatrix} \pi_{11} & \cdots & \pi_{1J-1} \\ \vdots & \ddots & \vdots \\ \pi_{J-11} & \cdots & \pi_{J-1J-1} \end{pmatrix} \text{ with } \pi_{jj} = -p_j/p_j - 1 \text{ and } \pi_{qj} = -p_j/p_j \text{ for } j \neq q. \quad (\text{A4})$$

It results from (A3) in consideration of (A4) for all $q \in \{1, \dots, J-1\}$

$$\begin{aligned} \delta_{tq}^* &= \delta_{tj}^* - c \cdot \sum_{j=1}^{J-1} \pi_{qj} \cdot \varepsilon_{tj} = \delta_{tj}^* + c \cdot \left(\frac{1}{p_j} \cdot \sum_{j=1}^{J-1} p_j \cdot \varepsilon_{tj} + \varepsilon_{tq} \right) \\ &= \delta_{tj}^* + c \cdot (\varepsilon_{tq} - \varepsilon_{tj}) = \delta_{tj}^* + \frac{h \cdot (\varepsilon_{tq} - \varepsilon_{tj})}{(S_{t-1} + h \cdot d_{t-1}) \cdot n}, \end{aligned} \quad (\text{A5})$$

whereby δ_{tj}^* is given, and has to satisfy the following constraint

$$-1 < \delta_{tj}^* < \frac{\varepsilon_{tj} + (1+r) \cdot \Phi - (1+i) \cdot B_{t-1} - D_{t-1}}{D_{t-1}}. \quad (\text{A6})$$

Appendix 2: Proof that the border conditions are fulfilled

If the postulated border condition $-1 < \delta_{tq} < (\varepsilon_{tq} + y + \Phi - (1+i) \cdot B_{t-1} - D_{t-1})/D_{t-1}$ is fulfilled for $q = J$, the ‘remaining’ (locally) optimal dividend policy will fulfill the border conditions, too, and thus will be globally optimal as shown in the following

$$\begin{aligned} \delta_{tq}^* &\stackrel{(\text{A5})}{\underset{>-1}{=}} \underbrace{\delta_{tj}^*}_{>-1} + \frac{h}{(S_{t-1} + h \cdot d_{t-1}) \cdot n} \cdot \underbrace{(\varepsilon_{tq} - \varepsilon_{tj})}_{\geq 0} > -1, \\ \delta_{tq}^* &\stackrel{(\text{A5})}{=} \underbrace{\delta_{tj}^*}_{< \frac{(\varepsilon_{tj} + y + \Phi - B_{t-1} \cdot (1+i) - D_{t-1})}{d_{t-1} \cdot n}} + \frac{h}{\underbrace{(S_{t-1} + h \cdot d_{t-1}) \cdot n}_{= 1/((S_{t-1}/h) + d_{t-1}) \cdot n} \leq 1/(d_{t-1} \cdot n)}} \cdot \underbrace{(\varepsilon_{tq} - \varepsilon_{tj})}_{\geq 0} \\ &< \frac{y + \Phi + \varepsilon_{tq} - B_{t-1} \cdot (1+i) - D_{t-1}}{d_{t-1} \cdot n}. \end{aligned} \quad (\text{A7})$$

⁴⁷ It is easy to show that $P \cdot P^{-1} = P^{-1} \cdot P = I$, whereby I is the identity matrix.

Appendix 3: Volatility

It immediately results from (A1) as an optimal variance of dividend growth rate⁴⁸

$$\begin{aligned}
\text{var}(\tilde{\delta}_t) &= \text{var}(\tilde{\delta}_t - E(\tilde{\delta}_t)) \stackrel{(A1)}{=} \text{var}\left(\frac{\tilde{\varepsilon}_t}{(\tilde{S}_{t-1}/h + \tilde{d}_{t-1}) \cdot n}\right) \\
&= \text{var}\left(\mathbb{E}\left(\frac{\tilde{\varepsilon}_t}{(\tilde{S}_{t-1}/h + \tilde{d}_{t-1}) \cdot n} \middle| \tilde{S}_{t-1}, \tilde{d}_{t-1}\right)\right) + \mathbb{E}\left(\text{var}\left(\frac{\tilde{\varepsilon}_t}{(\tilde{S}_{t-1}/h + \tilde{d}_{t-1}) \cdot n} \middle| \tilde{S}_{t-1}, \tilde{d}_{t-1}\right)\right) \\
&= 0 + \text{var}(\tilde{\varepsilon}_t) \cdot \mathbb{E}\left(\frac{1}{((\tilde{S}_{t-1}/h + \tilde{d}_{t-1}) \cdot n)^2}\right) = \text{var}(\tilde{\varepsilon}_t) \cdot \mathbb{E}\left(\frac{1}{(\tilde{P}_{t-1}/h + \tilde{D}_{t-1})^2}\right).
\end{aligned} \tag{A8}$$

It follows as the variance of the net earnings' growth rate φ_t ⁴⁹

$$\begin{aligned}
\text{var}(\tilde{\varphi}_t) &= \text{var}\left(\frac{\tilde{x}_t}{\tilde{x}_{t-1}} - 1\right) = \text{var}\left(\frac{\tilde{x}_t}{\tilde{x}_{t-1}}\right) \\
&= \text{var}\left(\mathbb{E}\left(\frac{\tilde{x}_t}{\tilde{x}_{t-1}} \middle| \tilde{x}_{t-1}\right)\right) + \mathbb{E}\left(\text{var}\left(\frac{\tilde{x}_t}{\tilde{x}_{t-1}} \middle| \tilde{x}_{t-1}\right)\right) \\
&= \text{var}\left(\frac{y}{\tilde{x}_{t-1}}\right) + \text{var}(\tilde{\varepsilon}_t) \cdot \mathbb{E}\left(\frac{1}{\tilde{x}_{t-1}^2}\right).
\end{aligned} \tag{A9}$$

The volatility of net earnings' growth rate will be higher than that of dividend growth rate, if

$$\begin{aligned}
\text{var}(\tilde{\varphi}_t) - \text{var}(\tilde{\delta}_t) &\stackrel{(A8), (A9)}{=} \text{var}\left(\frac{y}{\tilde{x}_{t-1}}\right) + \text{var}(\tilde{\varepsilon}_t) \cdot \mathbb{E}\left(\frac{1}{\tilde{x}_{t-1}^2} - \frac{1}{(\tilde{P}_{t-1}/h + \tilde{D}_{t-1})^2}\right) \\
&= \text{var}\left(\frac{y}{\tilde{x}_{t-1}}\right) + \text{var}(\tilde{\varepsilon}_t) \cdot \mathbb{E}\left(\frac{(\tilde{P}_{t-1}/h + F_{t-1} + \tilde{x}_{t-1})^2 - \tilde{x}_{t-1}^2}{\tilde{x}_{t-1}^2 \cdot (\tilde{P}_{t-1}/h + \tilde{D}_{t-1})^2}\right) \\
&= \text{var}\left(\frac{y}{\tilde{x}_{t-1}}\right) + \text{var}(\tilde{\varepsilon}_t) \cdot \mathbb{E}\left(\frac{(\tilde{P}_{t-1}/h + F_{t-1})^2 + 2 \cdot \tilde{x}_{t-1} \cdot (\tilde{P}_{t-1}/h + F_{t-1})}{\tilde{x}_{t-1}^2 \cdot (\tilde{P}_{t-1}/h + \tilde{D}_{t-1})^2}\right) > 0.
\end{aligned} \tag{A10}$$

The latter statement will hold, if $0 \leq P_{t-1}/h + F_{t-1}$ for all t. From (22) we have

$$\begin{aligned}
0 < D_{\text{U}} &\stackrel{(1)}{=} x_{\text{U}} + F_{\text{U}} \xrightarrow{(x_{\text{U}} \rightarrow 0)} F_{\text{U}} \\
&\Rightarrow F_{\text{U}} > 0.
\end{aligned} \tag{A11}$$

⁴⁸ With regard to the second transformation see Rohatgi (1976, p. 170). According to him, $\text{var}(\tilde{X}) = \text{var}(\mathbb{E}(\tilde{X} | \tilde{Y})) + \mathbb{E}(\text{var}(\tilde{X} | \tilde{Y}))$ holds for two random variables \tilde{X} and \tilde{Y} . Concerning equation (A8), we concretely set $\tilde{X} := \tilde{\varepsilon}_t / ((\tilde{S}_{t-1}/h + \tilde{d}_{t-1}) \cdot \tilde{n}_{t-1})$ and $\tilde{Y} := (\tilde{S}_{t-1}, \tilde{d}_{t-1})$.

⁴⁹ See the latter footnote and set $\tilde{X} := \tilde{X}_t / \tilde{X}_{t-1}$ and $\tilde{Y} := \tilde{X}_{t-1}$.

As $P_{tj} > 0$ we then have at state J

$$0 < P_{tj} + F_{tj} \stackrel{(3),(6)}{=} \Phi - (1+i) \cdot B_{t-1} \stackrel{(3),(6)}{=} P_{tq} + F_{tq} \tag{A12}$$

$$\leq P_{tq} / h + F_{tq} \quad \text{for all } q \in \{1, \dots, J\} \text{ and all } t.$$

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Table 1:

Positive shock in net earnings¹

The table shows the reactions over time of the dividend (D_t), the dividend growth rate (δ_t), the equity value (P_t), and the amount of external debt financing (B_t) to a permanent positive net earnings shock at $t = 2$ amounting to $\varepsilon_2 = \$600$.

t	D_t	δ_t	P_t	B_t
0	808	0.00	808	19,192
1	808	0.00	808	19,192
2	1,108	0.37	1,108	18,892
3	1,260	0.14	1,260	18,740
4	1,336	0.06	1,336	18,664
5	1,375	0.03	1,375	18,625
6	1,394	0.01	1,394	18,606
7	1,404	0.01	1,404	18,596
8	1,409	0.00	1,409	18,591
9	1,412	0.00	1,412	18,588
10	1,413	0.00	1,413	18,587
11	1,413	0.00	1,413	18,587
12	1,414	0.00	1,414	18,586
13	1,414	0.00	1,414	18,586
14	1,414	0.00	1,414	18,586

¹ Numbers are rounded off.

Table 2:

Negative shock in net earnings²

The table shows the reactions over time of the dividend (D_t), the dividend growth rate (δ_t), the equity value (P_t), and the amount of external debt financing (B_t) to a permanent negative net earnings shock at $t = 2$ amounting to $\varepsilon_2 = -\$400$.

t	D_t	δ_t	P_t	B_t
0	808	0.00	808	19,192
1	808	0.00	808	19,192
2	508	-0.37	508	19,492
3	357	-0.30	357	19,643
4	280	-0.21	280	19,720
5	241	-0.14	241	19,759
6	222	-0.08	222	19,778
7	212	-0.04	212	19,788
8	207	-0.02	207	19,793
9	205	-0.01	205	19,795
10	203	-0.01	203	19,797
11	203	0.00	203	19,797
12	202	0.00	202	19,798
13	202	0.00	202	19,798
14	202	0.00	202	19,798

² Numbers are rounded off.