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Michele Lombardi and Roberto Veneziani

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Liberal Principles for Social Welfare Relations in Infinitely-Lived Societies*

Michele LOMBARDI[†]
University of Surrey

Roberto VENEZIANI[‡]
Queen Mary University of London

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Abstract

This paper extends the analysis of liberal principles in social choice recently proposed by Mariotti and Veneziani (2009a) to infinitely-lived societies. First, a novel characterisation of the inegalitarian leximax social welfare relation is provided based on the Individual Benefit Principle, which incorporates a liberal, non-interfering view of society. This result is surprising because the **IBP** has no obvious inegalitarian content. Second, it is shown that there exists no weakly complete social welfare relation that satisfies simultaneously the standard axioms of Finite Anonymity, Strong Pareto, and Weak Preference Continuity, and a liberal principle of Non-Interference that generalises **IBP**.

JEL classification. D63; D70; Q01.

Keywords. Infinite utility streams, Individual Benefit Principle, leximax, Non-Interference, impossibility.

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[†]Department of Economics, University of Surrey, GU2 7XH Guildford, Surrey, United Kingdom. E-mail: M.Lombardi@surrey.ac.uk.

[‡]Corresponding Author: Department of Economics, Queen Mary University of London, London, E1 4NS, United Kingdom. E-mail: R.Veneziani@qmul.ac.uk

Introduction

Liberal principles in philosophy and social choice tend to express some notion of individual autonomy or freedom. In a recent contribution, Mariotti and Veneziani (2009a) have proposed a new axiom - called the Harm Principle (**HP**) - suited for Social Welfare Orderings (SWOs), which captures a liberal view of non-interference inspired by J.S. Mill's conception of liberty. The basic content of **HP** can be illustrated as follows: consider two welfare allocations u and v such that u is socially preferred to v , and two different allocations u' and v' such that agent i is *worse off* at these than at the corresponding starting allocations, the other agents are equally well off, and agent i prefers u' to v' . Whatever the origin of the decrease in agent i 's welfare, **HP** requires that society's preference over u' and v' should agree with person i 's preferences: having already suffered a welfare loss in both allocations, and given that nobody else is affected, agent i should not be punished in the SWO by changing social preferences against her.

Although **HP** incorporates no egalitarian content, Mariotti and Veneziani (2009a) have shown that, together with the standard axioms of Anonymity and Strong Pareto, it characterises the leximin SWO in societies with a finite number of agents. Lombardi and Veneziani (2009) have generalised this counterintuitive result by weakening **HP** and, based on the weak **HP**, they have provided novel characterisations of various SWOs related to Rawls's difference principle. They have also used the weak **HP** to characterise the leximin social welfare *relations* (SWRS) proposed by Asheim and Tungodden (2004) and by Bossert et al. (2007) in infinitely-lived economies. The latter results are particularly interesting because the analysis of infinitely-lived societies is crucial, especially in the discussion of intergenerational justice, and the definition of suitable anonymous and paretian SWRS is still an open question in the infinite context (see Asheim, 2010).

This paper extends the analysis of liberal, non-interfering views in infinitely-lived societies in two main directions. First, the Individual Benefit Principle (**IBP**) - proposed by Mariotti and Veneziani (2009b) in economies with a finite number of agents - is analysed. The **IBP** also incorporates a liberal, non-interfering view of society and it can be taken as the theoretical complement of **HP**, for it requires society not to switch its preferences when agent i 's welfare in both allocations u and v *increases*. It is shown that a weaker version of **IBP** suitable for SWRS in infinitely-lived societies, together with other standard axioms, characterises the inegalitarian *leximax* SWR. This counterintuitive result is interesting per se, because it provides the first characterisation of the *leximax* SWR, and because the **IBP** has no obvious inegalitarian content. But it also has relevant implications for liberal approaches to social choice.

In fact, second, following Mariotti and Veneziani (2009b), **HP** and **IBP** are taken as two parts of a single liberal view and a weaker version of their principle of Non-Interference is proposed, which is suitable for SWRS in infinitely-lived societies. An interesting impossibility result for liberal approaches is derived, according to which there exists no weakly complete SWR that satisfies the standard axioms of Finite Anonymity, Strong Pareto Optimality, and Weak Preference Continuity,

and the principle of Non-Interference.

The framework

Let $X \equiv \mathbb{R}^{\mathbb{N}}$ be the set of countably infinite utility streams, where \mathbb{R} is the set of real numbers and \mathbb{N} is the set of natural numbers. An element of X is ${}_1u = (u_1, u_2, \dots)$ and u_t is the utility of a representative member of generation $t \in \mathbb{N}$. For $T \in \mathbb{N}$, ${}_1u_T = (u_1, \dots, u_T)$ denotes the T -head of ${}_1u$ and ${}_{T+1}u = (u_{T+1}, u_{T+2}, \dots)$ denotes its T -tail, so that ${}_1u = ({}_1u_T, {}_{T+1}u)$. A permutation π is a bijective mapping of \mathbb{N} onto itself. A permutation π of \mathbb{N} is finite whenever there is $T \in \mathbb{N}$ such that $\pi(t) = t, \forall t > T$. For any ${}_1u \in X$ and any permutation π , let $\pi({}_1u) = (u_{\pi(t)})_{t \in \mathbb{N}}$ be a permutation of ${}_1u$. For any $T \in \mathbb{N}$ and ${}_1u \in X$, ${}_1\bar{u}_T$ is a permutation of ${}_1u_T$ such that the components are ranked in ascending order.

For any ${}_1u, {}_1v \in X$, we write ${}_1u \geq {}_1v$ to mean $u_t \geq v_t, \forall t \in \mathbb{N}$; ${}_1u > {}_1v$ to mean ${}_1u \geq {}_1v$ and ${}_1u \neq {}_1v$; and ${}_1u \gg {}_1v$ to mean $u_t > v_t, \forall t \in \mathbb{N}$.

Let \succsim be a binary relation over X . For any ${}_1u, {}_1v \in X$, we write ${}_1u \succsim {}_1v$ for $({}_1u, {}_1v) \in \succsim$ and ${}_1u \not\sucsim {}_1v$ for $({}_1u, {}_1v) \notin \succsim$; \succsim stands for ‘‘at least as good as’’. For any ${}_1u, {}_1v \in X$, the asymmetric factor \succ of \succsim is defined by ${}_1u \succ {}_1v$ if and only if ${}_1u \succsim {}_1v$ and ${}_1v \not\sucsim {}_1u$, and the symmetric part \sim of \succsim is defined by ${}_1u \sim {}_1v$ if and only if ${}_1u \succsim {}_1v$ and ${}_1v \succsim {}_1u$. Throughout the paper \succsim on X is assumed to be reflexive and transitive, but not necessarily complete. Let \succsim and \succsim' be relations on X : \succsim' is an extension of \succsim if $\succsim \subseteq \succsim'$ and $\succ \subseteq \succ'$.

The Harm Principle and the Leximin SWR

A standard definition of the leximin SWR used in the literature to compare countably infinite utility streams is due to Asheim and Tungodden.

Definition 1 (Asheim and Tungodden, 2004, Definition 2, p.224) $\forall {}_1u, {}_1v \in X, {}_1u \sim^{LM} {}_1v \Leftrightarrow \exists \tilde{T} \geq 1$ such that $\forall T \geq \tilde{T}: {}_1\bar{u}_T = {}_1\bar{v}_T$, and ${}_1u \succ^{LM} {}_1v \Leftrightarrow \exists \tilde{T} \geq 1$ such that, $\forall T \geq \tilde{T}, \exists t \in \{1, \dots, T\}$ with $\bar{u}_s = \bar{v}_s$ ($\forall 1 \leq s < t$) and $\bar{u}_t > \bar{v}_t$.

The characterisation of the latter leximin SWR by Lombardi and Veneziani (2009) is based on the following axioms.

FINITE ANONYMITY, **FA**: $\forall {}_1u \in X$ and \forall finite permutation π of \mathbb{N} , $\pi({}_1u) \sim {}_1u$.

STRONG PARETO OPTIMALITY, **SPO**: $\forall {}_1u, {}_1v \in X: {}_1u > {}_1v \Rightarrow {}_1u \succ {}_1v$.

WEAK PREFERENCE CONTINUITY, **WPC**: $\forall {}_1u, {}_1v \in X: \exists \tilde{T} \geq 1$ such that $({}_1u_T, {}_{T+1}v) \succ {}_1v \forall T \geq \tilde{T} \Rightarrow {}_1u \succ {}_1v$.

WEAK COMPLETENESS, **WC**: $\forall {}_1u, {}_1v \in X, \exists T \geq 1$ $({}_1u_T, {}_{T+1}v) \neq {}_1v \Rightarrow ({}_1u_T, {}_{T+1}v) \succsim {}_1v$ or ${}_1v \succsim ({}_1u_T, {}_{T+1}v)$.

HARM PRINCIPLE, **HP**: $\forall_{1u,1v,1u',1v'} \in X : \exists T \geq 1 \ 1u = (1u_{T,T+1} v) \succ 1v$, and $1u', 1v'$ are such that, $\exists i \leq T$,

$$\begin{aligned} u'_i &< u_i \\ v'_i &< v_i \\ u'_j &= u_j \ \forall j \neq i \\ v'_j &= v_j \ \forall j \neq i \end{aligned}$$

implies $1v' \not\succeq 1u'$ whenever $u'_i > v'_i$.

FA and **SPO** are standard and need no further comment. **WPC** has been proposed by Asheim and Tungodden (2004, p.223) and it represents a mainly technical, weak requirement to deal with infinite-dimensional vectors. **WC** states that a SWR should be able to compare vectors with the same tail: this seems an obviously desirable property, as it imposes a minimum requirement of completeness. Finally, **HP** formalises the Harm Principle in infinitely-lived societies. It is weaker than the version analysed by Mariotti and Veneziani (2009a), because it does not require that $1u' \succ 1v'$ and instead it emphasises the negative prescription of the Harm Principle by simply ruling out the reversal of strict preferences. Moreover it only holds for vectors with the same tail.¹

The following Theorem can be proved.

Theorem 2 (Lombardi and Veneziani, 2009, Theorem 3.4) \succsim is an extension of \succsim^{LM} if and only if \succsim satisfies **FA**, **SPO**, **HP**, **WPC**, and **WC**.

As argued by Mariotti and Veneziani (2009a), a characterisation of the leximin based on **HP** is surprising, because **HP** has no obvious egalitarian content, unlike the standard axiom of Hammond Equity (see, e.g., Hammond, 1976; Asheim and Tungodden, 2004). Equally surprising is that, by a germane perturbation of **HP**, it is possible to characterise the strongly *inegalitarian* leximax SWR.

The Benefit Principle and the Leximax SWR

According to the leximax, that society is best which (lexicographically) maximises the welfare of its best-off members. In infinitely-lived economies, this intuition can be formalised as follows.

Definition 3 $\forall_{1u,1v} \in X, 1u \sim^{LX} 1v \Leftrightarrow \exists \tilde{T} \geq 1$ such that $\forall T \geq \tilde{T}: 1\bar{u}_T = 1\bar{v}_T$, and $1u \succ^{LX} 1v \Leftrightarrow \exists \tilde{T} \geq 1$ such that, $\forall T \geq \tilde{T}, \exists t \in \{1, \dots, T\}$ with $\bar{u}_s = \bar{v}_s$ ($\forall t < s \leq T$) and $\bar{u}_t > \bar{v}_t$.

In order to characterise the leximax SWR, the same axioms as for the leximin are used, except for **HP**, which is substituted with the Individual Benefit Principle. The **IBP** also captures a liberal requirement of non-interference and can be formalised as follows.

¹For a detailed discussion of the axioms, see Lombardi and Veneziani (2009).

INDIVIDUAL BENEFIT PRINCIPLE, **IBP**: $\forall_{1u,1v,1u',1v' \in X : \exists T \geq 1 \ 1u = (1u_{T,T+1} v) \succ 1v$, and $1u', 1v'$ are such that, $\exists i \leq T$,

$$\begin{aligned} u'_i &> u_i \\ v'_i &> v_i \\ u'_j &= u_j \ \forall j \neq i \\ v'_j &= v_j \ \forall j \neq i \end{aligned}$$

implies $1v' \not\succeq 1u'$ whenever $u'_i > v'_i$.

IBP stipulates that after an increase in welfare that concerns only agent i , society should *not reverse* the strict preference between $1u$ and $1v$ to a strict preference for $1v'$ over $1u'$, possibly except when i prefers otherwise. Whatever the origin of the increase in welfare for agent i , society should not ‘punish’ i by reversing its strict preferences, given that nobody else is affected and therefore for all other agents the choice between $1u'$ and $1v'$ is the same as between $1u$ and $1v$.

The liberal moral intuition behind **IBP** is similar to the **HP**, and yet Theorem 4 proves that **IBP** leads to a rather different result.

Theorem 4 \succsim is an extension of \succsim^{LX} if and only if \succsim satisfies **FA**, **SPO**, **IBP**, **WPC**, and **WC**.

Proof: (\Rightarrow) Let $\succsim^{LX} \subseteq \succsim$. It is easy to see that \succsim meets **FA**, **SPO**, **WPC**, and **WC**. We show that \succsim satisfies **IBP**. Take any $1u, 1v, 1u', 1v' \in X$ such that $1u = (1u_{T,T+1} v) \succ 1v \ \exists T \geq 1$, and $1u', 1v'$ are such that, $\exists i \leq T, u'_i > u_i, v'_i > v_i, u'_j = u_j \ \forall j \neq i, v'_j = v_j \ \forall j \neq i$. We show that $1u' \succ 1v'$ whenever $u'_i > v'_i$. As $1u, 1v$ have the same tail, $1u \succ^{LX} 1v$. Then, $\exists \tilde{T} \geq 1$ such that, $\forall T' \geq \tilde{T}, \exists t \in \{1, \dots, T'\}$ with $\bar{u}_s = \bar{v}_s \ \forall t < s \leq T'$ and $\bar{u}_t > \bar{v}_t$. Consider any $T' \geq \tilde{T}$. If $\bar{u}_{T'} > \bar{v}_{T'}$, the result follows as $\bar{u}'_{T'} \in \{u'_i, \bar{u}_{T'}\}$ and $\bar{v}'_{T'} \in \{v'_i, \bar{v}_{T'}\}$. Therefore suppose $\bar{u}_{T'} = \bar{v}_{T'}$. If $\bar{v}_k = \bar{v}'_k \ \forall t \leq k \leq T'$, the result follows. Otherwise, let $\bar{v}_k \neq \bar{v}'_k$ for some $t \leq k \leq T'$. We distinguish two cases.

Case 1. $\bar{v}_t < v'_i < \bar{v}_{t+1}$

Then, $\bar{v}'_{t+1} > \bar{v}'_t = v'_i > \bar{v}_t$ and $\bar{v}'_s = \bar{v}_s \ \forall T' \geq s > t$. If $u'_i \in (v'_i, \bar{u}_{t+1}]$, then $\bar{u}'_t > v'_i = \bar{v}'_t$. Otherwise, let $u'_i > \bar{u}_{t+1}$. Thus, there exists $j \geq t+1$ such that $u'_i = \bar{u}'_j > \bar{u}_j$. Let

$$m = \max \{t+1 \leq j \leq T' \mid \bar{u}'_j > \bar{u}_j\}.$$

Since $\bar{v}'_j = \bar{v}_j = \bar{u}_j \ \forall t < j \leq T'$ it follows that $\bar{u}'_m > \bar{v}'_m$. In both cases, there exists $t^* \leq T'$ such that $\bar{u}'_s = \bar{v}'_s \ \forall t^* < s \leq T'$ and $\bar{u}'_{t^*} > \bar{v}'_{t^*}$.

Case 2. $v'_i \geq \bar{v}_{t+1}$

If $v'_i \geq \bar{v}_{T'}$, then $\bar{u}'_{T'} > \bar{v}'_{T'}$ as $u'_i > v'_i$. Otherwise, let $v'_i < \bar{v}_{T'}$. Let

$$\ell = \min \{t+1 < j \leq T' \mid v'_i < \bar{v}_j\}.$$

Then, $\bar{v}_\ell > v'_i = \bar{v}'_{\ell-1} \geq \bar{u}_{\ell-1} = \bar{v}_{\ell-1}$. Since $v'_i < u'_i$, then $u'_i > \bar{u}_{\ell-1}$. If $u'_i \in (\bar{u}_{\ell-1}, \bar{u}_\ell]$, then $\bar{v}'_{\ell-1} < u'_i = \bar{u}'_{\ell-1}$. Otherwise, let $\bar{u}_\ell < u'_i$. Then, there exists $\ell \leq m \leq T'$ such that $\bar{v}'_m = \bar{v}_m <$

$\bar{u}'_m = u'_i$ and if $m < T'$, $\bar{u}'_s = \bar{v}'_s \forall m < s \leq T'$. In both cases, there exists $t^* \leq T'$ such that $\bar{u}'_s = \bar{v}'_s \forall t^* < s \leq T'$ and $\bar{u}'_{t^*} > \bar{v}'_{t^*}$.

Since it holds $\forall T' \geq \tilde{T}$, we have that ${}_1u' \succ {}_1v'$ as $\succ^{LX} \subseteq \succ$.

(\Leftarrow) Suppose that \succ satisfies **FA**, **SPO**, **IBP**, **WPC**, and **WC**. We show that $\sim^{LX^*} \subseteq \sim$ and $\succ^{LX^*} \subseteq \succ$. Take any ${}_1u, {}_1v \in X$. If ${}_1u \sim^{LX} {}_1v$, then ${}_Tv = {}_Tv \forall T > \tilde{T}$, so **FA** implies ${}_1u \sim {}_1v$.

Next, we show that ${}_1u \succ {}_1v$ whenever ${}_1u \succ^{LX} {}_1v$. Thus, suppose that ${}_1u \succ^{LX} {}_1v$. Take any $T \geq \tilde{T}$ and consider the vector ${}_1w \equiv ({}_1u_{T,T+1} v)$. By contradiction, we prove that ${}_1w \succ {}_1v$. By **FA** and transitivity, we can consider ${}_1\bar{w} \equiv ({}_1\bar{u}_{T,T+1} v)$ and ${}_1\bar{v} \equiv ({}_1\bar{v}_{T,T+1} v)$.

Case 1. Suppose that ${}_1\bar{v} \succ {}_1\bar{w}$.

Then, $\bar{v}_l > \bar{w}_l, \exists l < t \leq T$, otherwise **SPO** yields a contradiction. Let

$$k = \max\{1 \leq l < t \mid \bar{v}_l > \bar{w}_l\}.$$

By **FA**, let $w_i = \bar{w}_k$ and $v_i = \bar{v}_{k+g} \exists g \in \mathbb{N}$ such that $0 < g \leq t - k$ and $\bar{w}_{k+g} > \bar{v}_{k+g}$. Let $d_1, d_2 > 0$, and consider vectors ${}_1w', {}_1v'$ formed from ${}_1\bar{w}, {}_1\bar{v}$ as follows: \bar{v}_{k+g} is raised to $\bar{v}_{k+g} + d_1$ such that $\bar{w}_{k+g} > \bar{v}_{k+g} + d_1$; \bar{w}_k is raised to $\bar{w}_k + d_2$ such that $\bar{v}_{k+g} + d_1 > \bar{w}_k + d_2 > \bar{v}_k$; and all other entries of ${}_1\bar{w}$ and ${}_1\bar{v}$ are unchanged. By **FA**, consider ${}_1\bar{w}' = ({}_1\bar{w}'_{T,T+1} v)$ and ${}_1\bar{v}' = ({}_1\bar{v}'_{T,T+1} v)$. By construction $\bar{w}'_j \geq \bar{v}'_j \forall j \geq k$, with $\bar{w}'_{k+g} > \bar{v}'_{k+g}$. **IBP** implies ${}_1\bar{w}' \not\succeq {}_1\bar{v}'$ and by **WC** it follows that ${}_1\bar{v}' \succ {}_1\bar{w}'$. Moreover, by **SPO** d_1, d_2 can be chosen so that ${}_1\bar{v}' \succ {}_1\bar{w}'$, without loss of generality. Consider two cases:

a) Suppose that $\bar{v}_k > \bar{w}_k$, but $\bar{w}_l \geq \bar{v}_l \forall l < k$. It follows that ${}_1\bar{w}' > {}_1\bar{v}'$, and so **SPO** implies that ${}_1\bar{w}' \succ {}_1\bar{v}'$, a contradiction.

b) Suppose that $\bar{v}_l > \bar{w}_l \exists l < k$. Note that by construction $\bar{v}'_l = \bar{v}_l$ and $\bar{w}'_l = \bar{w}_l \forall l < k$. Then, let

$$k' = \max\{1 \leq l < k \mid \bar{v}'_l > \bar{w}'_l\}.$$

The above argument can be applied to ${}_1\bar{w}', {}_1\bar{v}'$ to derive vectors ${}_1\bar{w}'', {}_1\bar{v}''$ such that $\bar{w}''_j \geq \bar{v}''_j \forall j \geq k'$, whereas by **IBP**, **WC**, and **SPO** ${}_1\bar{v}'' \succ {}_1\bar{w}''$. And so on. After a finite number of iterations q , two vectors ${}_1\bar{w}^q, {}_1\bar{v}^q$ can be derived such that, by **IBP**, **WC**, and **SPO**, ${}_1\bar{v}^q \succ {}_1\bar{w}^q$ but, by **SPO**, ${}_1\bar{w}^q \succ {}_1\bar{v}^q$, yielding the desired contradiction.

Case 2. Suppose that ${}_1\bar{v} \sim {}_1\bar{w}$.

By assumption, $\bar{v}_t < \bar{u}_t \equiv \bar{w}_t$. Therefore, define ${}_1\bar{w}'$ as follows: $\bar{w}'_\tau = \bar{w}_\tau \forall \tau \in \mathbb{N} \setminus \{t\}$ and $\bar{w}'_t = \bar{w}_t - \epsilon > \bar{v}_t, \exists \epsilon > 0$. By **SPO** and transitivity, it follows that ${}_1\bar{v} \succ {}_1\bar{w}'$ but ${}_1\bar{w}' \succ^{LX} {}_1\bar{v}$. Hence, the argument of *Case 1* above can be applied to ${}_1\bar{v}$ and ${}_1\bar{w}'$, yielding the desired contradiction.

As ${}_1\bar{v} \not\succeq {}_1\bar{w}$ **WC** implies ${}_1\bar{w} \succ {}_1\bar{v}$. **FA** and transitivity imply that $({}_1u_{T,T+1} v) \succ {}_1v$. Since this is true $\forall T \geq \tilde{T}$, **WPC** implies ${}_1u \succ {}_1v$. ■

The axioms in Theorem 4 are independent (see Addendum).

Theorem 4 has an interesting theoretical implication. Consider the next axiom, which incorporates the normative intuitions behind **HP** and **IBP** in a unified liberal framework, and generalises the principle of Non-Interference proposed by Mariotti and Veneziani (2009b) to infinitely-lived

economies.

NON-INTERFERENCE, **NI**: $\forall_{1u, 1v, 1u', 1v'} \in X : \exists T \geq 1 \ 1u = (1u_T, T+1v) \succ 1v$, and $1u', 1v'$ are such that, $\exists i \leq T$,

$$\begin{aligned} (u'_i - u_i) (v'_i - v_i) &> 0 \\ u'_j &= u_j \ \forall j \neq i \\ v'_j &= v_j \ \forall j \neq i \end{aligned}$$

implies $1v' \not\succeq 1u'$ whenever $u'_i > v'_i$.

In other words, in a liberal perspective, changes in one individual's welfare that leave all other agents unaffected should not be a motive for penalising the individual in the social welfare judgement (by switching strict preferences against her), whether the change involves a damage or a benefit for her. Theorems 2 and 4 reveal a fundamental tension between standard axioms in social choice and liberal principles of noninterference in infinitely-lived societies.

Theorem 5 *There exists no SWR on X that satisfies **FA**, **SPO**, **WPC**, **WC**, and **NI**.*

Proof: Let $\nu, \mu \in \mathbb{R}$ with $\nu > \mu$ and consider vectors $1u, 1v \in X$ such that $u_t = \mu, \forall t$, whereas $v_1 < \mu$ and $v_t = \nu, \forall t \geq 2$. By Theorem 2, $1u \succ 1v$, but by Theorem 4, $1v \succ 1u$, a contradiction. ■

Theorem 5 suggests that a fully noninterfering liberal approach cannot provide adequate foundations for social welfare judgments in infinitely-lived societies. Of the appeals of individuals to be protected from interference when nobody else is affected, at least some will have to be overridden. This insight is robust. The incompatibility between the axioms is not due to their clashing in peculiar configurations of welfare levels, but rather to the general tension between two conflicting SWRs which are simultaneously implied by the axioms. And analogous impossibility results can be obtained by using alternative definitions of the leximin SWR, such as those proposed by Asheim and Tungodden (2004, Definition 1, p.224) and by Bossert et al. (2007).

Conclusions

This paper analyses liberal axioms for SWRs in infinitely-lived economies. The leximax SWR is characterised by appealing to the Individual Benefit Principle, which incorporates a liberal, non-interfering view of society. This result is interesting per se, since it provides the first characterisation of the leximax in infinitely-lived economies, and because the **IBP** has no obvious inegalitarian content. It also has relevant implications for liberal approaches to social choice. For it allows us to show that there exists no weakly complete SWR that satisfies the standard axioms of Finite Anonymity, Strong Pareto, Weak Preference Continuity, and the liberal principle of Non-Interference.

References

- [1] Asheim, G.B., 2010. Intergenerational equity. *Annual Review of Economics*, forthcoming.
- [2] Asheim, G.B., Tungodden, B., 2004. Resolving distributional conflicts between generations. *Economic Theory* 24, 221-230.
- [3] Bossert, W., Sprumont, Y., Suzumura, K., 2007. Ordering infinite utility streams. *Journal of Economic Theory* 135, 179-189.
- [4] Hammond, P., 1976. Equity, Arrow's Conditions and Rawls' Difference Principle. *Econometrica* 44, 793-804.
- [5] Lombardi, M., Veneziani, R., 2009. Liberal Egalitarianism and the Harm Principle. IER Discussion Paper 078, Hitotsubashi University.
- [6] Mariotti, M., Veneziani, R., 2009a. Non-Interference Implies Equality. *Social Choice and Welfare* 32, 123-128.
- [7] Mariotti, M., Veneziani, R., 2009b. The Impossibility of Non-Interference. Mimeo, Queen Mary University of London.

Addendum to "Liberal Principles for Social Welfare Relations in Infinitely-Lived Societies"

Let Π be the set of all finite permutations. For any ${}_1u \in X$ and any permutation π , let $\pi({}_1u) = (u_{\pi(t)})_{t \in \mathbb{N}}$ be a permutation of ${}_1u$. Let ${}_{con}\epsilon$ denote the stream of constant level of well-being equal to $\epsilon \in \mathbb{R}$. Finally, if there are only a finite set $\{1, \dots, T\} = N \subset \mathbb{N}$ of agents, or generations, X_T denotes the set of utility streams of X truncated at $T = |N|$, where $|N|$ is the cardinality of N . For any $T \in \mathbb{N}$ and ${}_1u \in X$, ${}_1\bar{u}_T$ is a permutation of ${}_1u_T$ such that the components are ranked in ascending order.

For an example violating *only* **FA**, define \succsim on X in the following way: $\forall {}_1u, {}_1v \in X$,

$$\begin{aligned} {}_1u = {}_1v &\Rightarrow {}_1u \sim {}_1v \\ \exists T \in \mathbb{N} : u_t = v_t \ \forall t < T \text{ and } u_T > v_T &\Rightarrow {}_1u \succ {}_1v \end{aligned}$$

The SWR \succsim on X is not an extension of the leximin SWR \succsim^{LX} . The SWR \succsim on X satisfies all properties except **FA**.

For an example violating *only* **SPO**, define \succsim on X in the following way: $\forall {}_1u, {}_1v \in X$, ${}_1u \sim {}_1v$. The SWR \succsim on X is not an extension of the leximax SWR \succsim^{LX} . Clearly, the SWR \succsim on X satisfies all properties except **SPO**.

For an example violating *only* **IBP**, let \succsim on X be the leximin SWR (see Definition 1, p. 3). It follows that the SWR \succsim on X is not an extension of the leximax SWR. The SWR \succsim on X satisfies all properties except **IBP**.

For an example violating *only* **WC**, define \succsim on X in the following way: $\forall {}_1u, {}_1v \in X$,

$${}_1u \geq \pi({}_1v) \ \exists \pi \in \Pi \Rightarrow {}_1u \succsim {}_1v.$$

The relation \succsim on X is the *Suppes-Sen grading principle*. The SWR \succsim on X satisfies all properties except **WC**.

For an example violating *only* **WPC**, $\forall T \in \mathbb{N}$, let the leximax ordering on X_T be denoted as \succsim_T^{LX} , i.e., $\forall {}_1u_T, {}_1v_T \in X_T$:

$${}_1u_T \succsim_T^{LX} {}_1v_T \Leftrightarrow {}_1u_T = \pi({}_1v_T) \ \exists \pi \in \Pi \text{ or } \exists t \leq T : \bar{u}_j = \bar{v}_j \ (\forall j \leq T \text{ s.t. } t < j) \text{ and } \bar{u}_t > \bar{v}_t.$$

Define \succsim_T^L on X by letting, $\forall {}_1u, {}_1v \in X$,

$${}_1u \succsim_T^L {}_1v \Leftrightarrow {}_1u_T \succsim_T^{LX} {}_1v_T \text{ and } {}_{T+1}u \geq {}_{T+1}v.$$

It is easy to see that the relation \succsim_T^L is reflexive and transitive $\forall T \in \mathbb{N}$. Furthermore, \succsim_T^L is also complete for vectors having the same tail. Then, let $\succsim = \bigcup_{T \in \mathbb{N}} \succsim_T^L$. By definition, this relation is reflexive and transitive as well. The SWR \succsim on X satisfies all properties but **WPC**. [To see that **WPC** is violated consider the following vectors, ${}_1u = (3, {}_{con}0)$ and ${}_1v = (2, {}_{con}1)$. Then, $({}_1u, {}_1v) \notin \succsim$ and $(({}_1u_T, {}_{T+1}v), {}_1v) \in \succsim_T^L \ \forall T \in \mathbb{N}$].

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**Department of Economics
Queen Mary, University of London
Mile End Road
London E1 4NS
Tel: +44 (0)20 7882 5096
Fax: +44 (0)20 8983 3580
Web: www.econ.qmul.ac.uk/papers/wp.htm**