

Department of Economics

Evaluation with Dynamic Reference: Sustainable Investment

Leon Vinokur

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Leon Vinokur*

Queen Mary, University of London

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Abstract

The Prospect Theory proposes to assess outcomes relative to a reference point (or benchmark). Although the literature recognises the relevance of dynamic benchmarks, most of the applications of Prospect Theory employ static reference points (or a status quo). This paper aims to develop a Prospect Theory framework for investment under uncertainty subject to a dynamic reference point, within the context of environmental policy making, where the distinction between a dynamic and a static frameworks is crucial. I evince that, in contrast to the static framework, in a dynamic framework the investor measures not only the absolute but also the relative risk premium (Sharpe ratio) of the investment opportunity, incorporating the risks and returns of a reference portfolio. I propose that there exists a relation between static and dynamic frameworks. Using the dynamic framework, I argue that in the environmental context international co-operation is the key to a successful environmental policy.

JEL Classification: D46, G18, Q58

Key words: Prospect theory, dynamic reference, sustainable development.

*Department of Economics, Queen Mary, University of London, Mile End Road, London E1 4NS, United Kingdom, l.vinokur@qmul.ac.uk. I wish to thank Marcelo Fernandes and Marco Mariotti for their insightful comments and observations. All errors and omissions are my own.

1 Introduction

The introduction of a new environmental investment opportunity such as the Clean Development Mechanism poses a new challenge to investors and policy makers. Such an investment opportunity enables entrepreneurs to receive additional carbon credits by reducing emissions in developing countries as a cheaper alternative to domestic emissions reductions. These opportunities are, however, characterised by high risks (Springer, 2003) in comparison to the riskiness of the carbon credits price. Therefore, the issue of the valuation method of this investment opportunity becomes relevant.

The importance of the valuation of an outcome relative to a single reference point, as it is proposed by Prospect Theory, has been well established in the economic literature since Kahneman and Tversky (1979). However, the authors argue that there may be situations in which gains and losses are measured in relation to levels different from the status quo. In such situations that inability to adapt to another reference level may encourage agents to take risks which would not otherwise be acceptable. The following example demonstrates the main idea behind the proposition.

Let's take as an example a decision maker who is holding or running a sustainable project. The new reference of the project is assumed to be below the purchase price which is assumed to be the status quo. The adjustment to a new reference level may be due to unpredictable losses the agent has recently incurred to sustain the project. In addition, the decision maker is assumed to exhibit the Disposition effect.¹ The agent realises the gains by liquidating the project and postpones the realisation of losses by keeping the project. I denote the new reference as Y' and the status quo as Y , where $Y' < Y$. When the value of the project is above Y (gain) the risk compensation is not enough for a Disposition-prone agent to keep the project. The agent, therefore, chooses to liquidate it as soon as possible. On the other hand, if the value of the project is below Y' , which is also below Y (loss) by definition, the agent chooses to keep the project rather than liquidating it. This holds even if the agent fails to adjust the reference level. However, if the project's value is in between Y and Y' ,

¹In the empirical literature there is extensive evidence of the Disposition effect (see, among others Dyl 1977, Ferris *et al.* 1988, Goetzmann *et al.* 2008, Grinblatt *et al.* 2005 and more)

there is a clear difference between agents who accept the new reference Y' and those who fail to do so. In the case where the agent fails to adjust the reference to Y' , the value of the project is negative, and the agent prefers to postpone the realisation of losses rather than to liquidate the project. However, if the agent adjusts the reference to Y' , the value of the project becomes positive, and the agent chooses to liquidate the project.

Recent empirical evidence recognises that the notion of a dynamic reference and its location are important factors in predicting the behaviour of an agent exhibiting risk aversion as suggested by the Prospect Theory. Gooding *et al.* (1996) evince that in the context of firms, their risk-taking behaviour depends upon the location of the reference point rather than merely the relative position of a firm to the reference point. They also find that the reference point moves in both directions, as a function of some sample estimate of the population properties (e.g. mean, max, median). Heath *et al.* (1999) show that the agent's decision to exercise a stock option depends on its maximum level attained during the past year. Ivkovic *et al.* (2007) find that an investment made by individual investors, both returns relative to the market and absolute performance matter in determining stock sales.

The profitability of investment opportunities such as the Clean Development Mechanism and Joint Implementation are evaluated against the price of carbon credits (Schneider, 2008). New information is constantly arriving in the carbon market, thus changing the price of carbon credits. The distinction, therefore, between static and dynamic evaluation is crucial. In this sense, an application of the dynamic reference in predicting the behaviour of a decision maker who holds a sustainable project seems natural. Failure to incorporate the dynamic reference may entail the wrong prediction of an agent's behaviour and in turn an inadequate policy design.

Prospect Theory value function applied to investment decisions helps to explain an anomaly such as the Disposition effect (Kyle *et al.*, 2006). The authors, however, restrict their analytical framework only to one static reference level. The aim of this paper is to extend the static framework into a dynamic one and suggest a generalised prediction of the behaviour of an agent under any reference, both static and dynamic. Unlike the static model, I assume that the reference level is an additional stochastic process. I define a new 'reference sensitive' project value Q , which is sensitive to the value of the reference project. I also show that

there is a relation between the dynamic and static valuation frameworks. Specifically, the extended model detects that a decision maker who evaluates absolute performance can also evaluate the relative performance of the investment opportunity. This can explain different findings in the empirical literature regarding the location of a reference. As expected, in the extended model the liquidation decision depends on the relative risk and the returns of the reference portfolio with respect to risk and returns of the investment opportunity. Application of the extended framework to sustainable investments evinces that international co-operation and structured policy are the key to an efficient environmental policy.

The rest of the paper is organised as follows: Section 2 describes the dynamic model and its solution. The first part shows the evolution of the model and three behavioural strategies of a decision maker with dynamic reference, where one of the strategies is consistent with the relative Disposition effect. The second part presents a link between relative and absolute Disposition effects. In section 3, I demonstrate an application of the extended model to a risk averse agent in the environmental framework and discuss the implications of the model on policy analysis. Section 4 offers some conclusions; the appendix contains the proofs and developments.

2 The Model

I extend the Status Quo model to allow for a dynamic reference level in the liquidation decision. In contrast with the model of Kyle et al. (2006), I account for the dynamics in the reference level. I also account for the fact that the same relative gain/loss is evaluated differently at different reference levels. For instance, if the reference is 1 and the gain is 10, the relative gain is $10-1=9$. However, in case the reference is 101 and the absolute gain is 110 the relative gain is also 9 ($110-101$), but the same gain is perceived differently at two reference levels. To account for this, I therefore construct a reference sensitive valuation. I define a new 'reference sensitive' project value $Q_t \equiv \frac{P_t}{R_t}$, where P_t is the value of a project, and R_t is its reference in which the agent breaks even.

The agent receives the value P_t of the project upon its liquidation at time $\tau = \min [\tau_1, \tau_2]$. The liquidation of the project is either voluntary at time τ_2 or at natural liquidation at time

τ_1 . Natural liquidation follows a Poisson distribution with arrival rate λ . For example, a rejection of a CDM project's baseline can be regarded as a natural liquidation event in the context of sustainable investments.²

I assume that the value of a sustainable project follows a Brownian motion $dP_t/P_t = \mu_P dt + \sigma_P dZ_t$ with constant parameters of drift μ_P and variance σ_P . The introduction of a reference requires an additional structure. In the empirical literature there are several alternative measures for a dynamic reference. For instance, Heath *et al.* (1999) and Gooding *et al.* (1996) base the reference on limited past data of the population properties (e.g. mean, max or median). The author suggests that one year is an appropriate time period for the survival of the reference.³ Ivkovic *et al.* (2007), on the other hand, measure the performance of the stock relative to its market performance since the purchase. In all of the mentioned studies, the reference adjusts throughout time. To capture the dynamics of the reference, I assume that its evolution follows a geometric Brownian motion $dR_t/R_t = \mu_R dt + \sigma_R dW_t$ with constant parameters of drift μ_R and variance σ_R . The assumption of the evolution of reference is general enough to nest all of the mentioned alternative measures of the reference. I assume that there exists a correlation between two Wiener processes, specifically, $\text{corr}(dZ_t, dW_t) = \rho$.

Upon the liquidation of the project agent realises his/her utility. Similarly to Kyle *et al.* (2006), I impose a piecewise exponential value function to evaluate the project:

$$u(Q_t) = \begin{cases} (1 - e^{-\gamma_1(\ln Q_t)}) & \text{if } Q_t \geq 1 \\ \phi_2(e^{\gamma_1(\ln Q_t)} - 1) & \text{if } 0 < Q_t < 1 \end{cases} \quad (1)$$

The value function (1) satisfies the following assumptions:

- A1** An index of loss aversion $\phi_2 > 1$;
- A2** Local absolute risk aversion is $\gamma_1 > 1$;
- A3** $u'(1+) < u'(1-)$ so that $\gamma_1 < \phi_2\gamma_1$;⁴

²For more details on CER, please visit:

<http://cdm.unfccc.int/about/index.html>

³In some cases, however, the whole historical data is taken, for example in the case of high watermarks where the hedge fund manager receives an incentive fee when the fund value is above the previous maximum.

⁴See Kobberling and Wakker (2005) for a detailed interpretation of parameter ϕ_2 , which can represent

A4 Value function is smooth in the continuation region.⁵

Assumption A1 ensures that the value function has a steeper curve in the region of gains. Assumptions A2, together with A3, ensure that the value function satisfies the properties of the Prospect Theory value function, with more sensitivity to losses than to gains, in addition to convexity below the reference. Using Ito's Lemma, the dynamic path of Q_t is:

$$dQ = \{\sigma_P dZ - \sigma_R dW + (\mu_P - \mu_R + \sigma_R^2 - \sigma_P \sigma_R \rho) dt\} Q_t \quad (2)$$

The drift of process Q_t is:

$$\mu_Q \equiv \mu_P - \mu_R + \sigma_R^2 - \sigma_P \sigma_R \rho$$

and the variance of process Q_t is:

$$\sigma_Q \equiv (\sigma_P^2 + \sigma_R^2 - 2\sigma_P \sigma_R \rho) \quad (3)$$

To maximise (1), agents choose an optimal liquidation time τ_2 that maximises the expected value function:

$$V(Q_t) = \max_{Q_t} \mathbb{E}[u(Q_t)]$$

Following the static framework of Kyle *et al.* (2006), I set the time discount rate in the value function to zero. The optimal stopping time problem above can be written as:

$$V(Q_t) = \max_Q \{u(Q_t), \lambda dt[u(Q_t) - V(Q_t)] + \mathbb{E}[V(Q_t + dQ) | Q_t]\} \quad (4)$$

The function represented by Equation (4) is the maximisation between the value from liquidating immediately $u(Q_t)$, and the value of postponing the liquidation $\lambda dt[u(Q_t) - V(Q_t)] + \mathbb{E}[V(Q_t + dQ) | Q_t]$. A4 assures that $V(Q_t)$ is a smooth function. I use Ito's Lemma to derive the differential equation of $V(Q_t)$. First, I evaluate the value function $V(Q_t)$ over the period of $(t, t + dt)$ by Taylor expansion:

the risk aversion at the reference and set equal to $\phi_2 = \frac{u'_+(1)}{u'_-(1)}$.

⁵It is possible due to the fact that the value function is a transformation of smooth functional form, both below and above the reference level.

$$\mathbb{E}(dV) = V_Q Q \mu_Q dt + \frac{1}{2} V_{QQ} Q^2 \sigma_Q dt \quad (5)$$

I denote F_x to be derivative of function F with respect to a variable x . If the project is not liquidated voluntarily, putting together (5) and (4) I get the following expression:

$$V_Q Q \mu_Q + \frac{1}{2} V_{QQ} Q^2 \sigma_Q - \lambda V + \lambda u = 0 \quad (6)$$

I can identify that (6) consists of a *homogeneous* part $V_Q Q \mu_Q + \frac{1}{2} V_{QQ} Q^2 \sigma_Q - \lambda V$ and a *non-homogenous* part λu . First, I solve its homogeneous part.

$$V_Q Q \mu_Q + \frac{1}{2} V_{QQ} Q^2 \sigma_Q - \lambda V = 0 \quad (7)$$

The solution of (7) has the following form:

$$V^+(Q_t) = A_1 Q_t^{\alpha_1} + B_1 Q_t^{\alpha_2} \quad (8)$$

$$V^-(Q_t) = A_2 Q_t^{\alpha_1} + B_2 Q_t^{\alpha_2} \quad (9)$$

where V^+ stands for the value function above and V^- below the reference, and

$$\alpha_1 = \frac{\frac{1}{2}\sigma_Q - \mu_Q + \sqrt{(\mu_Q - \frac{1}{2}\sigma_Q^2)^2 + 2\lambda\sigma_Q}}{\sigma_Q^2} > 0,$$

$$\alpha_2 = \frac{\frac{1}{2}\sigma_Q - \mu_Q - \sqrt{(\mu_Q - \frac{1}{2}\sigma_Q^2)^2 + 2\lambda\sigma_Q}}{\sigma_Q^2} < 0.$$

In order to find the solution to the value function $V(Q_t)$, I proceed to the specific solution of a non-homogenous part.

2.1 Solution of a non-homogeneous part

Proposition 1 Denote by $S \equiv \frac{\mu_Q}{\sigma_Q^2}$ the relative Sharpe ratio of the project. Under (1),

$$(a) \text{ If } Q_t \geq 1, \mathbb{E}(du(Q_t)) = \begin{cases} \geq 0 & \text{if } S > \frac{1}{2}\gamma_1 + \frac{1}{2} \\ < 0 & \text{otherwise.} \end{cases}$$

$$(b) \text{ If } 0 < Q_t < 1, \mathbb{E}(du(Q_t)) = \begin{cases} \geq 0 & \text{if } S > -\frac{1}{2}\gamma_1 + \frac{1}{2} \\ < 0 & \text{otherwise.} \end{cases}$$

The implication of the proposition is that if the value function is expected to increase over time, an agent would never liquidate the project voluntarily. However, if the value function is expected to decrease, an agent would liquidate the project immediately as the value of holding the project is decreasing.

In order to analyse the optimal strategy I adopt a similar method presented by Kyle *et al.* (2006). First, I restrict the value function to be smooth when $Q_t = 1$. Imposing this condition I achieve that at each reference, the agent is indifferent about whether to liquidate or not liquidate the project. This assists me in determining the optimal behaviour of an agent. I set the value function from never liquidating to the expected value function u :

$$V_0(Q_t) = \mathbb{E}[u(Q_{t+\tau_1})] \quad (10)$$

Function V_0 indicates that an agent holding the project would not liquidate it voluntary, unless the expected value from holding the projected is expected to decrease in the future. For tractability of the proof I impose that at each reference point the value function has a value of zero. It is a simplifying assumption to make the calculation easier. Formally I impose

$$\lim u(1) = 0$$

This can be interpreted as the condition where the agent derives no utility from the project. Unlike the static framework, a liquidation decision of a project in a dynamic framework not only depends on the project's statistical values but also on the values of the reference and its correlation with the value of the project ρ . These additional parameters are an important addition to the static framework. It is possible to verify that the dynamic framework works down to the static framework if the reference is set to a constant parameter, specifically, $R = 1$. In the following analysis of an agent's liquidation strategy there are 3 different cases to consider. Prior to each case specification, I summarise it in proposition.

Proposition 2 *If the Sharpe ratio is $S > \frac{1}{2}\gamma_1 + \frac{1}{2}$ and $V_0(1) > u(1) = 0$, then $V(Q_t)$ is*

increasing in Q_t and represented by:

$$V_0(Q_t) = \begin{cases} \frac{\phi_2 C_2 \gamma_1 - C_1 \gamma_1 - \phi_2 (C_2 - 1) \alpha_1 - (C_1 - 1) \alpha_1}{\alpha_2 - \alpha_1} Q_t^{\alpha_2} + 1 - C_1 e^{-\gamma_1 (\ln Q_t)} & \text{if } Q_t \geq 1 \\ \frac{\phi_2 C_2 \gamma_1 - C_1 \gamma_1 - \phi_2 (C_2 - 1) \alpha_2 - (C_1 - 1) \alpha_2}{\alpha_2 - \alpha_1} Q_t^{\alpha_1} + \phi_2 (C_2 e^{\gamma_1 (\ln Q_t)} - 1) & \text{if } 0 < Q_t < 1 \end{cases} \quad (11)$$

where

$$C_1 = \frac{\lambda}{\gamma_1 \mu_Q - \frac{1}{2}(\gamma_1^2 + \gamma_1) \sigma_Q + \lambda}$$

$$C_2 = \frac{\lambda}{\lambda - \{\gamma_1 \mu_Q + \frac{1}{2}(\gamma_1^2 - \gamma_1) \sigma_Q\}}$$

Case 1: According to proposition 1, the relative Sharpe ratio of the project is high enough to compensate the agent for the risk for any relative value Q_t of the project. In addition, the value function (10) is expected to increase over time. In this case an agent has no incentive to liquidate the project voluntarily. When the relative value of the project is $Q_t = 1$ (where the project breaks even), the value of keeping the project ($V_0(1)$) is higher than liquidating it ($u(1)$). Therefore, the agent is better off by keeping the project and (11) \succ) is the value function from never liquidating the project voluntarily.

Proposition 3 *If the Sharpe ratio is $S > \frac{1}{2}\gamma_1 + \frac{1}{2}$ and $V_0(1) < u(1) = 0$, then $V(Q_t)$ is increasing in Q_t and represented by:*

$$V(Q_t) = \begin{cases} -(1 - C_1)Q_t^{\alpha_2} + 1 - C_1 e^{-\gamma_1 (\ln Q_t)} & \text{if } Q_t \geq 1 \\ -\phi_2 (C_2 - 1)Q_t^{\alpha_1} + \phi_2 (C_2 e^{\gamma_1 (\ln Q_t)} - 1) & \text{if } 0 < Q_t < 1. \end{cases} \quad (12)$$

and

$$V_Q^+(1) - V_Q^-(1) = (\alpha_1 - \alpha_2)V_0(1) < 0 \quad (13)$$

Case 2: According to proposition 1, and similarly to case 1, the relative Sharpe ratio is high enough to compensate the agent for the risk for any value of $Q_t \neq 1$ ((the project does not break even). However, if the project's relative value of the project is $Q_t = 1$ the agent would choose to liquidate the project as the sensitivity of the agent toward losses is higher than to gains around the reference. The latter conclusion results from the difference between slopes around reference ($Q_t = 1$) and represented by (13).

Proposition 4 *If the Sharpe ratio is $-\frac{1}{2}\gamma_1 + \frac{1}{2} < S \leq \frac{1}{2}\gamma_1 + \frac{1}{2}$ and $V_0(1) < u(1) = 0$, then $V(Q_t)$ is increasing in Q_t and represented by:*

$$V(Q_t) = \begin{cases} 1 - e^{-\gamma_1(\ln Q_t)} & \text{if } Q_t \geq 1 \\ -\phi_2(C_2 - 1)Q_t^{\alpha_1} + \phi_2(C_2 e^{\gamma_1(\ln Q_t)} - 1) & \text{if } Q_t < 1. \end{cases} \quad (14)$$

and

$$V_Q^-(1) - V_Q^+(1) = (\alpha_2 - \alpha_1)V_0(1) > 0 \quad (15)$$

Case 3: According to proposition 1, the relative Sharpe ratio of the project is not high enough to compensate the agent for the risk if its relative value is $Q_t \geq 1$. The agent would, therefore, choose to liquidate the project. Otherwise, the agent would choose to keep the project until its relative value is set to $Q_t = 1$. As (15) points out, the sensitivity of the agent towards gains is higher than to losses around the reference, therefore, the agent would liquidate the project when it breaks even.

2.2 Absolute and relative dispositions

Financial literature provides strong evidence supporting the Prospect Theory valuation and the Disposition effect. There is, however, no consensus among scholars regarding what the appropriate reference should be. Odean (1998) presents evidence of the Disposition effect based on absolute returns. Godding *et al.* (1996), on the other hand, find strong evidence for a dynamic reference and reject the existence of any fixed reference in the context of firms' risk taking behaviour. Heath *et al.* (1999), find evidence for a dynamic reference in the context of exercising stock options. Ivkovic *et al.* (2005), however, find that stock purchases made by individual investors are driven by both absolute and relative valuations. In light of this extensive evidence, I argue that there exists a relation between the two frameworks.

To do so, I draw attention to Proposition 4. It states that under certain conditions an agent exhibits a relative Disposition effect. In particular, agents choose to liquidate the project when it has relative gains and postpone the liquidation if it has relative losses. I argue that if agents exhibit an absolute Disposition on the project and its reference, then they would also exhibit a relative Disposition effect. I summarise it in the following proposition:

Proposition 5 Denote by $v_P \equiv \frac{\mu_P}{\sigma_P^2}$ and $v_R \equiv \frac{\mu_R}{\sigma_R^2}$ absolute Sharpe ratios of a project and its reference, respectively. If under (1), $-\frac{1}{2}\gamma_1 + \frac{1}{2} < \nu_P < \frac{1}{2}\gamma_1 + \frac{1}{2}$; $-\frac{1}{2}\gamma_1 + \frac{1}{2} < \nu_R < \frac{1}{2}\gamma_1 + \frac{1}{2}$ and $\rho \leq 0$, then $-\frac{1}{2}\gamma_1 + \frac{1}{2} < S \leq \frac{1}{2}\gamma_1 + \frac{1}{2}$.

The proposition suggests that there might be cases of absolute, relative and combined Disposition effects. It can, therefore, explain the variety of empirical evidence which is not conclusive of what the appropriate reference should be. Although the conditions of Proposition 5 are restricted to the case where projects and their references complement investment opportunities, the result is very important in understanding the risk perception of decision makers under valuation with reference.

3 Disposition and sustainable development

Insley (2003) suggests that the riskiness of sustainable projects plays a vital role in the incentives of undertaking emissions reduction projects. Furthermore, Schneider (2008) argues that the profitability of a sustainable project, such as the Clean Development Mechanism, is evaluated against the profitability of holding carbon allowances. This, together with the fact that the value of carbon allowances and sustainable projects are in constant shift, makes application of the above relative valuation framework suitable for the analysis of such investment opportunities. Among all the results provided in the previous section, Proposition 4 seems the most suitable for such an analysis, as it detects the well-documented Disposition effect. Conducting such analysis could lead policy makers to a better understanding of decision-making dynamics in the context of sustainable development projects. This analysis would improve the efficiency of environmental policies and give a more accurate guidance to policy makers about measures to be taken to create incentives in favour of sustainable projects. In what follows, I discuss firstly the way that the Disposition effect is eliminated using the framework of Proposition 4, where I detect such an effect. Secondly, I apply this framework to the analysis of decision-making dynamics in the context of sustainable development projects, such as the CDM.

3.1 How to eliminate disposition

According to Proposition 4, decision makers who exhibit relative Disposition evaluate the profitability of holding a project according to the following inequality

$$\mu_P - \left(\frac{1}{2}\gamma_1 + \frac{1}{2}\right)\sigma_P^2 \leq \mu_R - \left(-\frac{1}{2}\gamma_1 + \frac{1}{2}\right)\sigma_R^2 - \gamma_1\sigma_P\sigma_R\rho \quad (16)$$

$$\mu_P - \left(-\frac{1}{2}\gamma_1 + \frac{1}{2}\right)\sigma_P^2 > \mu_R - \left(\frac{1}{2}\gamma_1 + \frac{1}{2}\right)\sigma_R^2 + \gamma_1\sigma_P\sigma_R\rho \quad (17)$$

where P and R are the values of a project and its reference portfolio, respectively. By assumption **A2**, $(-\frac{1}{2}\gamma_1 + \frac{1}{2}) < 0$ and $(\frac{1}{2}\gamma_1 + \frac{1}{2}) > 0$. Conditions of (16) therefore, suggest that it is not enough to reduce the risk of the project (σ_P^2) to eliminate the Disposition effect. To do so, one should reduce the risk of the project so that its risk would be relatively lower than the risk of the reference portfolio. Technically, the inequalities (16) should become

$$\mu_P - \left(\frac{1}{2}\gamma_1 + \frac{1}{2}\right)\sigma_P^2 > \mu_R - \left(-\frac{1}{2}\gamma_1 + \frac{1}{2}\right)\sigma_R^2 - \gamma_1\sigma_P\sigma_R\rho \quad (18)$$

Inequality (18) satisfies Proposition 3, which gives decision makers an incentive to keep the project rather than liquidating it. Proposition 5, however, points to an additional difficulty. It is possible that decision makers holding the project exhibit both relative and absolute Disposition effects. In this case, it is not enough to reduce the relative riskiness of the project. It is also necessary to ensure that the riskiness of a project is low enough to eliminate both relative and absolute Disposition effects.

3.2 Application

As I mention above, a sustainable project (such as the CDM project) is evaluated against the profitability of holding carbon allowances. In what follows, I, therefore, denote sustainable project and carbon portfolio absolute values to be P and R , respectively. The desired goal of eliminating the Disposition effect in the context of sustainable projects can be achieved in two ways. One is to reduce the absolute risk of the project, σ_P^2 , by the CDM host country. Another is to reduce the riskiness of the reference portfolio, σ_R^2 , by the domestic policy maker. In order to do that, the policy maker should understand the source of such risks.

Schneidler (2008) suggests that agents' incentives of holding sustainable projects lie within the riskiness of these projects. These projects are often characterised by high risks and uncertainties. Springer (2003) outlines the potential sources of the risks involved in such projects. Among them:

Technological risk: The final allocation of credits to the entrepreneur is a function of the actual emission reduction in the host country. Delays or actual output reduction of factories in the host country, therefore, result in fewer emissions reductions than actually planned. As a result, the entrepreneurs receive fewer credits than was originally planned.

Economic risk: In addition to the technological risks, the entrepreneur is exposed to the economic factors that affect any investment opportunity. Interest rates, exchange rates and the price of foreign land are among the factors that can have a major impact on the riskiness of the sustainable project.

Political risk: The Kyoto protocol restricts the CDM mechanism to be implemented in developing countries. These countries are often characterised by restrictive regulations and political instability. These factors contribute to the already existing uncertainty surrounding these projects and have the potential of increasing the riskiness of such projects.

It is worth mentioning that technological risks are inherited in the riskiness of the projects and it is up to the entrepreneurs to deal with these risks. Political and economic risks, however, are the real market failures. And the policy makers should aim at eliminating those unnecessary risks involved in the projects. There are several ways to do so. In what follows I describe the way the policy makers could address these issues.

The carbon concentration in the atmosphere is a global problem and not restricted by geographical borders. It is, therefore, in the global interest to encourage measures that have the potential for reducing emissions in areas where such measures are likely to contribute more on a marginal basis. It is evident that structured regulations are one of the ways of eliminating unnecessary risks. Empirical evidence shows that host countries such as China, Mexico and Brazil which have structured regulations on Kyoto-related institutions, attract most of the investment from the developed countries (Oleschak *et al.*, 2007). Structured regulations can contribute to boosting local sustainable technology and reducing substantively the emissions in those regions. Fast growing economies, such as those of China and

India, are also rapidly increasing their carbon footprint. It is, therefore, not enough to adhere to the targets set up by the developed countries, such as those in the EU, to eliminate the potential threat of global warming. More structured measures, involving international co-operation, should be implemented. Sustainable projects, such as the Clean Development Mechanism, have the direct impact of achieving the efficient reduction of carbon emissions globally through economic incentives. Additionally, CDM projects have a direct impact; they are sustainable in the long term and are changing the course of development in the host countries through technology transfer. This has the potential to reduce the current carbon footprint in the developing world and also to contribute to sustainable development in the future.

Another aspect of the successful emissions reduction is in international co-operation. International co-operation is the key to the successful implementation of any sustainable policy. Separate actions of host and domestic countries of the entrepreneurs require substantial effort from both sides to achieve emissions reduction targets. This important factor could, potentially, create a situation similar to the prisoner's dilemma, where neither side would be willing to reduce domestic emissions. This is very similar to the current situation, where developing countries are not willing to reduce their emissions; whereas developed countries are reluctant to credit the investors for potential emissions reduction in host countries and investors are, therefore, not willing to invest. This scenario is not desirable. International co-operation can eliminate these obstacles more effectively. Co-operative efforts can eliminate the riskiness of the projects. According to my framework above, these steps would diminish Disposition both by incurring lower costs and using the comparative advantage of each economy in tackling this issue. As changes in regulation have an effect on the risk structure in the market (Grout *et al.*, 2006), the transparency of domestic (Nondek *et al.*, 2005) and foreign policies is the way of reducing the riskiness of these investment opportunities (IPCC, 2000).

4 Conclusion

This paper extends the static framework of liquidation decisions, first presented by Kyle *et al.*, 2006. I have constructed a new relative valuation which is reference sensitive. I have incorporated dynamics of reference in the decision and show that in this framework, decision makers can also exhibit the Disposition effect, found in many empirical studies. I have showed that there is a relation between the Disposition decision in static and dynamic frameworks. I have applied a decision making under dynamic reference to the liquidation decision of sustainable projects, such as the CDM. I have showed that the behavioural pattern with dynamic reference can play a major role in the implementation of the environmental policy. I have evinced that dynamic reference can play a role in the investment decisions and thus has to be taken into account by policy makers in implementing policy in constantly changing settings such as environmental projects. I have argued that international co-operation is the key to the successful implementation of sustainable developments.

A possible extension to the framework would be to incorporate different types of risk aversion at different reference levels. So far, the literature deals with a single reference. However, introducing dynamics to the reference can pose some doubt as to the assumption of unified risk aversion at the reference. Köbberling and Wakker (2005) assumed that risk aversion at the reference set equal to $\frac{u'_+(1)}{u'_-(1)}$, which is the ratio of marginal utilities around reference. However, one could think of different risk aversion at different reference. It is similar to the situation of different valuation of the equal absolute gain/loss. For instance, when considering reference of 10 and 1000 it is not necessary that $\frac{u'_+(10)}{u'_-(10)} = \frac{u'_+(1000)}{u'_-(1000)}$, as the decision maker would be more sensitive to changes in gain/loss around reference of 10 than to the same change around reference of 1000. Although it is an interesting idea, it is beyond the scope of this paper. It shows, however, a promising way of extending the current framework of decision making under dynamic valuation.

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A Appendix

Appendix A. Derivations

A.1 Properties of a piecewise function (1)

First, I derive the shape of the value function when $Q_t \geq 1$:

$$u_Q = \gamma_1 \frac{1}{Q_t} e^{-\gamma_1(\ln Q_t)} > 0$$

$$u_{QQ} = \gamma_1 \left(-\frac{1}{Q_t^2} e^{-\gamma_1(\ln Q_t)} - \frac{1}{Q_t^2} \gamma_1 e^{-\gamma_1(\ln Q_t)} \right) < 0.$$

satisfying the properties of a concave function.

The shape of the value function when $0 < Q_t < 1$:

$$u_Q = \phi_2 \gamma_1 \frac{1}{Q_t} e^{\gamma_1(\ln Q_t)} > 0$$

$$u_{QQ} = \phi_2 \gamma_1 \left(-\frac{1}{Q_t^2} e^{\gamma_1(\ln Q_t)} + \frac{1}{Q_t^2} \gamma_1 e^{\gamma_1(\ln Q_t)} \right) > 0$$

iff $\gamma_1 > 1$.

satisfying the properties of a convex function.

A.2 The dynamics of (2)

First I define the partial derivatives of Q with respect to P_t and R_t :

$$Q_P = \frac{1}{R}; Q_R = -\frac{P}{R^2}; Q_{PR} = -\frac{1}{R^2}; Q_{PP} = 0; Q_{RR} = \frac{2P}{R^3} \quad (A1)$$

using Ito's Lemma, I derive the evolution of Q

$$dQ = Q_P dP + Q_R dR + 1/2 Q_{PP} d^2 P + 1/2 Q_{RR} d^2 R + Q_{PR} dP dR \quad (A2)$$

putting together (A1) and (A2) results in:

$$= Q_P(\mu_P dt + \sigma_P dZ_t) + Q_R(\mu_R dt + \sigma_R dW_t) + 1/2 Q_{PP} P^2 \sigma_P^2 dt + 1/2 Q_{RR} R^2 \sigma_R^2 dt + Q_{PR} P R \sigma_P \sigma_R \rho dt \quad (A3)$$

• putting together (A1) and (A3) results in:

$$= \frac{1}{R} P \sigma_P dZ_t - \frac{P}{R^2} R \sigma_R dW_t + \left(\frac{1}{R} P \mu_P - \frac{P}{R^2} R \mu_R + 1/2 \frac{2P}{R^3} R^2 \sigma_R^2 - \frac{1}{R^2} P R \sigma_P \sigma_R \rho \right) dt$$

rearranging

$$= \{ \sigma_P dZ - \sigma_R dW + (\mu_P - \mu_R + \sigma_R^2 - \sigma_P \sigma_R \rho) dt \} Q$$

A.3 The optimal stopping of (4)

$$\begin{aligned} V(Q_t) &= \max_{Q_t} \{ u(Q_t), \lambda u(Q_t) dt + (1 - \lambda dt) \mathbb{E}[V(Q_t + dQ) | Q_t] \} \\ &= \max_{Q_t} \{ u(Q_t), \lambda u(Q_t) dt - \lambda dt \mathbb{E}[V(Q_t + dQ) | Q_t] + \mathbb{E}[V(Q_t + dQ) | Q_t] \} \\ &= \max_{Q_t} \{ u(Q_t), \lambda dt [u(Q_t) - V(Q_t)] + \mathbb{E}[V(Q_t + dQ) | Q_t] \} \end{aligned}$$

A.4 The Taylor expansion of (5)

$$\begin{aligned} E(dV) &= \mathbb{E} \left\{ V_Q dQ + \frac{1}{2} V_{QQ} (dQ)^2 \right\} \\ &= \mathbb{E} \left\{ V_Q [Q \sigma_P dZ - Q \sigma_R dW + Q \mu_Q dt] + \frac{1}{2} V_{QQ} (Q \sigma_P dZ - Q \sigma_R dW)^2 \right\} \\ &= V_Q Q \mu_Q dt + \frac{1}{2} V_{QQ} Q^2 (\sigma_P^2 + \sigma_R^2 - 2 \sigma_P \sigma_R \rho) dt \end{aligned}$$

A.5 Solution of the homogeneous part of (7)

Noting that (7) has a standard solution of the $V(Q_t) = AQ_t^\alpha$ form, I define its derivatives in the following fashion:

$$\begin{aligned} V_Q &= A\alpha Q^{\alpha-1} \\ V_{QQ} &= A\alpha(\alpha-1)Q^{\alpha-2} \end{aligned} \tag{A4}$$

putting together (A4) and (7) results in:

$$\frac{1}{2}(\sigma_Q^2)Q_t^2 A\alpha(\alpha-1)Q_t^{\alpha-2} + \mu_Q Q_t A\alpha Q_t^{\alpha-1} - \lambda A Q_t^\alpha = 0$$

dividing both sides by AQ^α gives way to:

$$\frac{1}{2}\sigma_Q^2\alpha(\alpha-1) + (\mu_Q)\alpha - \lambda = 0 \tag{A5}$$

The solutions to (A5) are:

$$\begin{aligned} \alpha_1 &= \frac{\frac{1}{2}\sigma_Q^2 - \mu_Q + \sqrt{(\mu_Q - \frac{1}{2}\sigma_Q^2)^2 + 2\lambda\sigma_Q^2}}{\sigma_Q^2} > 0, \\ \alpha_2 &= \frac{\frac{1}{2}\sigma_Q^2 - \mu_Q - \sqrt{(\mu_Q - \frac{1}{2}\sigma_Q^2)^2 + 2\lambda\sigma_Q^2}}{\sigma_Q^2} < 0. \end{aligned}$$

Appendix B. Proofs

B.1 Proof of Proposition 1

(a) First and second order derivatives of $u(Q_t)$:

$$u_Q = \gamma_1 \frac{1}{Q_t} e^{-\gamma_1(\ln Q_t)}; u_{QQ} = \left(-\gamma_1^2 \frac{1}{Q_t^2} - \gamma_1 \frac{1}{Q_t^2}\right) e^{-\gamma_1(\ln Q_t)} \tag{B1}$$

using Ito's Lemma:

$$\mathbb{E}(du(Q_t)) = \mathbb{E}\left\{u_Q dQ_t + \frac{1}{2}u_{QQ}d^2Q_t\right\} \tag{B2}$$

putting together (B1) and (B2):

$$= \left\{ \gamma_1 \frac{1}{Q_t} Q_t \mu_Q + \frac{1}{2} \left[-\gamma_1^2 \frac{1}{Q_t^2} - \gamma_1 \frac{1}{Q_t^2} \right] Q_t^2 \sigma_Q^2 \right\} e^{-\gamma_1 (\ln Q_t)} dt$$

then rearranging:

$$\mathbb{E}(du(Q_t)) = \gamma_1 \left\{ \mu_Q - \frac{1}{2} \gamma_1 \sigma_Q^2 - \frac{1}{2} \sigma_Q^2 \right\} e^{-\gamma_1 (\ln Q_t)} dt.$$

(b) In a similar fashion:

$$u_Q = \phi_2 \gamma_1 \frac{1}{Q_t} e^{\gamma_1 (\ln Q_t)}; u_{QQ} = \left(\phi_2 \gamma_1 \frac{1}{Q_t^2} - \phi_2 \gamma_1 \frac{1}{Q_t^2} \right) e^{\gamma_1 (\ln Q_t)} \quad (B3)$$

putting together (B2) and (B3):

$$= \left\{ \phi_2 \gamma_1 \frac{1}{Q_t} Q_t \mu_Q + \frac{1}{2} \left[\phi_2 \gamma_1 \frac{1}{Q_t^2} - \phi_2 \gamma_1 \frac{1}{Q_t^2} \right] Q_t^2 \sigma_Q^2 \right\} e^{\gamma_1 (\ln Q_t)} dt$$

then rearranging:

$$\mathbb{E}(du(Q_t)) = \phi_2 \gamma_1 \left\{ \mu_Q + \frac{1}{2} \gamma_1 \sigma_Q^2 - \frac{1}{2} \sigma_Q^2 \right\} e^{\gamma_1 (\ln Q_t)} dt$$

B.2 Proof of Proposition 2

I impose on the value function $V(Q_t)$ to satisfy the following boundary conditions: **(a)** $\lim_{Q \rightarrow \infty} V(Q_t) = 1$ and **(b)** $\lim_{Q \rightarrow 0} V(Q_t) = -\phi_2$. These conditions assure that the value function is

bounded. I also impose on $V(Q_t)$ to have a smooth and continuous passing at each reference. Mathematically, I impose **(c)** $\lim V^+(1) = \lim V^-(1)$ and **(d)** $\lim V_Q^+(1) = \lim V_Q^-(1)$.

Therefore the following must hold:

$$\begin{aligned} \text{(a)} \quad \lim V(\infty) &= 1 \Rightarrow A_1 = 0 \\ \text{(b)} \quad \lim V(0) &= -\phi_2 \Rightarrow B_2 = 0 \end{aligned} \quad (B4)$$

$$(c) \lim V^+(1) = \lim V^-(1) \quad (B5)$$

$$B_1 Q_t^{\alpha_2} + 1 - C_1 e^{-\gamma_1(\ln Q_t)} = A_2 Q_t^{\alpha_1} + \phi_2 (C_2 e^{\gamma_1(\ln Q_t)} - 1)$$

$$B_1 + 1 - C_1 = A_2 + \phi_2 (C_2 - 1)$$

$$B_1 = A_2 + \phi_2 (C_2 - 1) + C_1 - 1$$

$$(d) \lim V_Q^+(1) = \lim V_Q^-(1)$$

which is equivalent to:

$$\frac{B_1 \alpha_2 Q_t^{\alpha_2} + C_1 \gamma_1 e^{-\gamma_1(\ln Q_t)}}{Q_t} = \frac{A_2 \alpha_1 Q_t^{\alpha_1} + \phi_2 \gamma_1 C_2 e^{\gamma_1(\ln Q_t)}}{Q_t} \quad (B6)$$

when $Q_t = 1$, (B6) becomes:

$$B_1 \alpha_2 + \gamma_1 C_1 = A_2 \alpha_1 + \phi_2 \gamma_1 C_2$$

putting together (B5) and (B6):

$$(A_2 + \phi_2 (C_2 - 1) + C_1 - 1) \alpha_2 + \gamma_1 C_1 = A_2 \alpha_1 + \phi_2 \gamma_1 C_2 \quad (B7)$$

rearranging (B7)

$$A_2 = \frac{\phi_2 C_2 \gamma_1 - C_1 \gamma_1 - \phi_2 (C_2 - 1) \alpha_2 - (C_1 - 1) \alpha_2}{\alpha_2 - \alpha_1}$$

In a similar fashion:

$$B_1 = \frac{\phi_2 C_2 \gamma_1 - C_1 \gamma_1 - \phi_2 (C_2 - 1) \alpha_1 - (C_1 - 1) \alpha_1}{\alpha_2 - \alpha_1}$$

Proof of Proposition 3

I impose on the value function $V(Q_t)$ to satisfy the following boundary conditions: **(a)** $\lim V(\infty) = 1$ and **(b)** $\lim V(0) = -\phi_2$. These conditions assure that the value function is bounded. Therefore, (B4) must hold. I also impose on $V(Q_t)$ to be continuous. Mathematically, I impose **(c)** $\lim V^+(1) = \lim V^-(1) = 0$. Then,

$$V^+(1) = B_1 Q^{\alpha_2} + 1 - C_1 e^{-\gamma_1(\ln Q)} = 0 \quad (B8)$$

$$B_1 = -(1 - C_1)$$

and

$$\begin{aligned} V^-(1) &= B_2 Q^{\alpha_1} + \phi_2 (C_2 e^{\gamma_1 (\ln Q)} - 1) = 0 \\ B_2 &= \phi_2 (1 - C_2); \end{aligned} \tag{B9}$$

The derivative of $V(Q)$ with respect to Q :

$$\begin{aligned} V_Q^+(1) &= \frac{-(1 - C_1)\alpha_2 Q^{\alpha_2} + C_1 \gamma_1 e^{-\gamma_1 (\ln Q)}}{Q} \\ V_Q^-(1) &= \frac{-\phi_2 (C_2 - 1)\alpha_1 Q^{\alpha_1} + \phi_2 C_2 \gamma_1 e^{-\gamma_1 (\ln Q)}}{Q} \end{aligned} \tag{B10}$$

when $Q = 1$, (B10) becomes

$$\begin{aligned} V_Q^+(1) &= -(1 - C_1)\alpha_2 + C_1 \gamma_1 \\ V_Q^-(1) &= -\phi_2 (C_2 - 1)\alpha_1 + \phi_2 C_2 \gamma_1 \end{aligned}$$

and their difference is

$$V_Q^+(1) - V_Q^-(1) = -(1 - C_1)\alpha_2 + C_1 \gamma_1 + \phi_2 (C_2 - 1)\alpha_1 - \phi_2 C_2 \gamma_1 = (\alpha_1 - \alpha_2)V_0(1) < 0$$

Proof of Proposition 4

I impose on the value function $V(Q)$ to satisfy the following boundary conditions **(a)** $\lim V(0) = -\phi_2$. This condition assures that the value function is bounded.

$$\lim V(0) = -\phi_2 \Rightarrow A_2 = 0;$$

I also impose on $V(Q)$ to be continuous. Mathematically, I impose **(c)** $\lim V^+(1) = \lim V^-(1) = 0$. Therefore, (B9) is satisfied. In addition, the derivative of $V(Q)$ with respect to Q :

$$\begin{aligned} V_Q^+(Q_t) &= \gamma_1 e^{-\gamma_1 (\ln Q_t)} \\ V_Q^-(Q_t) &= \frac{-\phi_2 (C_2 - 1)\alpha_1 Q_t^{\alpha_1} + \phi_2 C_2 \gamma_1 e^{-\gamma_1 (\ln Q_t)}}{Q_t} \end{aligned} \tag{B11}$$

when $Q_t = 1$, (B11) becomes

$$\begin{aligned} V_Q^+(Q_t) &= \gamma_1 \\ V_Q^-(1) &= -\phi_2 (C_2 - 1)\alpha_1 + \phi_2 C_2 \gamma_1 \end{aligned}$$

and their difference is

$$V_Q^+(1) - V_Q^-(1) = -\phi_2(C_2 - 1)\alpha_1 + \phi_2 C_2 \gamma_1 - \gamma_1 = (\alpha_2 - \alpha_1)(V_0) > 0$$

Proof of proposition 5

Under static reference the Disposition conditions of the project P and reference R are:

$$-\frac{1}{2}\gamma_1 + \frac{1}{2} < \nu_P < \frac{1}{2}\gamma_1 + \frac{1}{2} \quad (B12)$$

$$-\frac{1}{2}\gamma_1 + \frac{1}{2} < \nu_P < \frac{1}{2}\gamma_1 + \frac{1}{2} \quad (B13)$$

rearranging (B12):

$$\mu_P - \left(\frac{1}{2}\gamma_1 + \frac{1}{2}\right)\sigma_P^2 < 0 \quad (B14)$$

$$\mu_P - \left(-\frac{1}{2}\gamma_1 + \frac{1}{2}\right)\sigma_P^2 > 0$$

rearranging (B13):

$$\mu_R - \left(\frac{1}{2}\gamma_1 + \frac{1}{2}\right)\sigma_R^2 < 0 \quad (B15)$$

$$\mu_R - \left(-\frac{1}{2}\gamma_1 + \frac{1}{2}\right)\sigma_R^2 > 0$$

taking together (B14) and (B15):

$$\mu_P - \left(\frac{1}{2}\gamma_1 + \frac{1}{2}\right)\sigma_P^2 < \mu_R - \left(-\frac{1}{2}\gamma_1 + \frac{1}{2}\right)\sigma_R^2 \quad (B16)$$

$$\mu_P - \left(-\frac{1}{2}\gamma_1 + \frac{1}{2}\right)\sigma_P^2 > \mu_R - \left(\frac{1}{2}\gamma_1 + \frac{1}{2}\right)\sigma_R^2$$

under the assumption that $\rho \leq 0$ (B16) is equivalent to

$$\mu_P - \left(\frac{1}{2}\gamma_1 + \frac{1}{2}\right)\sigma_P^2 \leq \mu_R - \left(-\frac{1}{2}\gamma_1 + \frac{1}{2}\right)\sigma_R^2 - \gamma_1\sigma_P\sigma_R\rho$$

$$\mu_P - \left(-\frac{1}{2}\gamma_1 + \frac{1}{2}\right)\sigma_P^2 > \mu_R - \left(\frac{1}{2}\gamma_1 + \frac{1}{2}\right)\sigma_R^2 + \gamma_1\sigma_P\sigma_R\rho$$

alternatively, condition (B16) becomes

$$-\frac{1}{2}\gamma_1 + \frac{1}{2} < \frac{\mu_P - \mu_R + \sigma_R^2}{\sigma_P^2 + \sigma_R^2} < \frac{1}{2}\gamma_1 + \frac{1}{2}.$$

which is equivalent to

$$-\frac{1}{2}\gamma_1 + \frac{1}{2} < S \leq \frac{1}{2}\gamma_1 + \frac{1}{2}$$

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**Department of Economics
Queen Mary, University of London
Mile End Road
London E1 4NS
Tel: +44 (0)20 7882 5096
Fax: +44 (0)20 8983 3580
Web: www.econ.qmul.ac.uk/papers/wp.htm**