

Department of Economics

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Andrea Carriero, George Kapetanios and Massimiliano Marcellino

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Queen Mary
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A. Carriero

Queen Mary, University of London

G. Kapetanios

Queen Mary, University of London

M. Marcellino

Bocconi University and EUI

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Abstract

The paper provides a proof of consistency of the ridge estimator for regressions where the number of regressors tends to infinity. Such result is obtained without assuming a factor structure. A Monte Carlo study suggests that shrinkage autoregressive models can lead to very substantial advantages compared to standard autoregressive models. An empirical application focusing on forecasting inflation and GDP growth in a panel of countries confirms this finding.

Keywords: Shrinkage, forecasting.

J.E.L. Classification: C13, C22, C53.

1 Introduction

In recent years there has been increasing interest in forecasting methods that utilize large data sets. There is an awareness that there is a huge quantity of information available in the economic arena which might be useful for forecasting, but standard econometric techniques are not well suited to extract this in a useful form.

As a result, a large number of methods which are either new or new to econometrics has been proposed to deal with forecasting using large data sets. The most popular group of methods consists of estimation strategies that allow estimation of a single equation model

that utilizes the whole of the available dataset. This group ranges from factor-based methods (see e.g. the overview in Stock and Watson (2006)) to Bayesian regression (see e.g. the overview in Geweke and Whiteman (2006)). Bayesian regression (Shrinkage), in particular, has proven very popular recently for forecasting with large datasets as work by Banburra, Giannone, and Reichlin (2007) and De Mol, Giannone, and Reichlin (2007) suggests.

This paper considers shrinkage estimation within a single equation framework, suggests a new application of the technique with a clear focus on forecasting, and provides some relevant theoretical, Monte Carlo and empirical results. Our work starts from the standard benchmark for forecasting using dynamic models: the autoregressive model. This model is well known to provide surprisingly good forecasts. A resounding confirmation of its enduring popularity as a forecasting model comes from the fact that most evaluations of new forecasting models are considered to be successful if the new model can beat this benchmark. A problem with autoregressive models is that as the lag order increases these models become less parsimonious and their forecasting performance suffers. This leads to the standard practise of using few lags. This contradicts the well known theoretical result which states that autoregressive models envelop a large class of model including most nonlinear models as long as the lag order is allowed to grow possibly to infinity. Then, autoregressive models become, under certain regularity conditions, equivalent to infinite order moving average model and via the Wold decomposition can approximate alternative models such as nonlinear models.

In this paper we suggest that estimating autoregressive models using shrinkage promotes parsimony while allowing the use of large lag orders. Using recent results from the statistical literature (Huang, Horowitz, and Ma (2008)), we prove consistency for the parameter estimates of such an autoregressive model. In fact, we prove a more general result which shows consistency in the context of ridge regression for regressions where the number of regressors tends to infinity without assuming a factor structure for those regressors as in De Mol, Giannone, and Reichlin (2007). Our extensive Monte Carlo study suggests that our suggested estimation strategy can have very substantial advantages compared to standard autoregressive models in the presence or long and possibly highly persistent lag structures.

We apply the new methods to forecasting a set of macroeconomic series. We focus on a panel of countries and forecast inflation and GDP growth series. Our findings suggest that using shrinkage autoregressive models can provide very substantial advantages compared to the already competitive forecasts produced by standard autoregressive models.

The paper is structured as follows: Section 2 presents the theoretical results of the paper. Section 3 presents an extensive Monte Carlo study. Section 4 presents our empirical results while Section 5 concludes. The appendix contains the proofs of the theoretical results.

2 Theory

The first model we will focus on is a standard regression model of the form

$$y_t = x_t' \beta + \epsilon_t, \quad t = 1, \dots, T \quad (1)$$

where $x_t = (x_{1t}, \dots, x_{pt})'$ is a $p \times 1$ vector of regressors, $\beta = (\beta_1, \dots, \beta_p)'$ is a $p \times 1$ vector of coefficients and ϵ_t is a zero mean error term. The first aim of this section is to consider shrinkage estimation of this model when the number of regressors $p \equiv p_T$ increases with T . We focus on ridge regression whereby the estimator for β takes the form

$$\hat{\beta} = (X_p' X_p + \Lambda_T)^{-1} X_p' y \quad (2)$$

where $X_p = (\tilde{x}_1, \dots, \tilde{x}_p)$, $\tilde{x}_j = (x_{j1}, \dots, x_{jT})'$, $j = 1, \dots, p$ and $y = (y_1, \dots, y_T)'$. We specify Λ_T to be a diagonal matrix where the main diagonal is made up of p penalty terms denoted by λ_{jT} , $j = 1, \dots, p$. A closely related estimation problem has been recently analysed by Huang, Horowitz, and Ma (2008), who provided consistency and distributional results for a general bridge estimator. That bridge estimator corresponds to our specific shrinkage estimator when the parameter γ of the general bridge estimator takes the value 2. In the first instance, we extend in a straightforward way this work. Our extensions are as follows: Firstly, we allow the regressors to be stochastic. Secondly we relax the independence assumption for ϵ_t . Thirdly, we allow for different penalty terms, λ_{jT} to be associated with different regressors. This is crucial for the extension of this methodology to the dynamic case as we will see below. A trivial extension is to note that whereas Huang, Horowitz, and Ma (2008) assume that the parameter of the bridge estimator is less than one, this is not needed for their consistency

and rate result and so their theory is applicable to our framework.

We make the following assumptions

Assumption 1 x_t is a stationary process with finite second moments and a covariance matrix Σ_T whose smallest eigenvalue is bounded away from zero for all T .

Assumption 2 ϵ_t is a martingale difference sequence with zero mean and finite second moment, σ^2 .

Theorem 1 Under assumptions 1 and 2, we have that

$$\left\| \hat{\beta}_T - \beta_0 \right\| = O_p \left(\left(\frac{p_T + \kappa_T}{T} \right)^{1/2} \right) \quad (3)$$

where κ_T is defined in (8).

The proof of Theorem 1 is given in the appendix. This result is novel in the sense that there is no work on the consistency and rate of convergence for the coefficients of a ridge regression model with stochastic regressors. The only other directly relevant work is De Mol, Giannone, and Reichlin (2007), where a consistency and rate result is obtained but only under the assumption that x_t follows a factor model. Having established this result for the general regression we wish to provide an explicit analysis for an autoregressive model where the number of lags tends to infinity. Therefore, we focus on the following dynamic model

$$y_t = \sum_{i=1}^{\infty} \psi_i y_{t-i} + \epsilon_t \quad t = 1, \dots, T \quad (4)$$

This model cannot be estimated since the lag order is infinite but can be approximated by

$$y_t = \sum_{i=1}^{p_T} \psi_i y_{t-i} + \epsilon_t \quad t = 1, \dots, T \quad (5)$$

when $p_T \rightarrow \infty$. We make the following assumption.

Assumption 3 (a) ϵ_t are i.i.d. random variables such that $E(\epsilon_t) = 0$ and $E(\epsilon_t^2) = \sigma^2$, (b) $\phi(z) \neq 0$ for all $|z| \leq 1$, and $\sum_{k=0}^{\infty} |k|^s |\phi_k| < \infty$ for some $s \geq 1$, (c) The smallest eigenvalue of the covariance matrix of (y_t, \dots, y_{t-p_t}) is bounded away from zero, (d) $p_T \rightarrow \infty$, $p_T = o(T^{1/2})$

The above assumption implies that y_t is stationary with finite second moment.

Theorem 2 *Under assumption 3, the results of Theorem 1 hold for (5).*

The proof of Theorem 2 is given in the appendix.

3 Monte Carlo Study

In this Section, we present a Monte Carlo study of the properties of the shrinkage autoregressive estimator. We consider the following model

$$y_t = \sum_{i=1}^{100} \psi_i y_{t-i} + \epsilon_t \quad t = 1, \dots, T \quad (6)$$

where $\psi_i = \frac{\phi \phi_i}{\sum_{i=1}^{100} \phi_i}$ and $\phi_i = \phi^i$. The parameter ϕ controls the persistence of the autoregressive process and is set to 0.5, 0.9, 0.95 and 0.99. $\epsilon_t \sim N(0, 1)$. The sample size is set to $T = 60, 80, 100, 200, 400$ and 1000 observations. For every sample, $T + 200$ observations are generated. The first 100 observations are disregarded to remove the effect of initial conditions which are set to zero. The last 100 observations are retained for our out-of-sample analysis.

Next, we discuss the implementation of the shrinkage estimation. As discussed in Section 2, the estimator is given by $\hat{\psi} = (Y_p' Y_p + \Lambda_p)^{-1} Y_p' y_p$, where $y = (y_{p_T+1}, \dots, y_T)'$, $Y_p = (\tilde{y}_1, \dots, \tilde{y}_{p_T})$, $\tilde{y}_j = (y_{p_T+1-j}, \dots, y_{T-j})'$, $j = 1, \dots, p_T$. The shrinkage component, Λ_p , is a $p_T \times p_T$ diagonal matrix whose i -th diagonal element is given by $v_1 p_T (0.1 + v_2 * i)$. Higher values of v_1 imply higher overall shrinkage. Higher values of v_2 imply increasingly higher shrinkage for higher lags. We set v_1 to 0.01, 0.1, 0.5, 2 and 5 and v_2 to 0.1, 0.5, 1, 2 and 5. We set p_T to $\max([0.25T], [\ln(T)^2])$. We also estimate a standard autoregressive model for comparative reasons. For this, we set p_T to $[\ln(T)^2]$.

We are interested in the out-of-sample forecasting performance of the shrinkage autoregressive model when compared with the standard autoregressive model. We produce one-step-ahead forecasts from both models and calculated the relative root-mean square forecast error (RMSFE) given by $(RMSFE_{ARS} - 1)/(RMSFE_{AR} - 1)$, where ARS denotes the

autoregressive shrinkage estimator and AR denotes the standard autoregressive estimator. We subtract 1 from both the numerator and the denominator since we know that the lower bound for the RMSFE for all methods is the standard deviation of the error term, ϵ_t , which in our case is set to 1. Results for the cases $\phi = 0.5$ and $\phi = 0.95$ are reported in Table 1¹.

The results make interesting reading. The degree of persistence seems to have little effect on the relative performance of the shrinkage autoregressive model. Overall, the shrinkage autoregressive estimator seems to perform much better than the standard autoregressive estimator. There are instances where the shrinkage estimator gives a reduction of 80% in the RMSFE compared to the standard estimator. The performance is sensitive to the choice of shrinkage parameters as expected. Relatively large values for v_1 and v_2 give the best performance although the largest values for v_1 and v_2 perform invariably badly.

Overall, it is clear that the shrinkage autoregressive estimator is a really competitive alternative and is worth of further investigation in empirical settings. We undertake this task in the next Section.

4 Empirical Application

In this Section we shall examine the performance of the ARS model in predicting actual economic time series. We focus on predicting inflation and (real) GDP growth. The data on inflation are monthly and range from April 1960 to July 2008, while the data on GDP growth are quarterly and range from 1970Q2 to 2008Q2. We consider ten large OECD countries: Canada (CN), France (FR), Germany (GE), Italy (IT), Japan (JP), Netherlands (NL), Spain (SP), Switzerland (SW), United Kingdom (UK), and United States (US). All data are taken from OECD through the SourceOECD web-based database. In the analysis on the GDP growth we have excluded Germany and Switzerland as the available time series were too short.

¹To save space we do not provide the results for the cases $\phi = 0.9$ and $\phi = 0.99$. Such results are similar to those obtained with $\phi = 0.95$ and are available upon request.

Beyond considering the performance of the *ARS* model against the standard autoregressive estimator *AR* we also consider the performance against a simple *AR*(1), as the latter model is often used as a benchmark. The lag orders of the *ARS* and the *AR* are set in exactly the same way as in the Monte Carlo study, so the *ARS* has lags equal to $\max([0.25T], [\ln(T)^2])$ while the *AR* has $[\ln(T)^2]$ lags. The forecasts are produced in pseudo real time, using a rolling estimation window of 120 months for inflation and of 60 quarters for GDP growth.

Tables 2-3 display the root-mean squared forecast error (RMSFE) of the *ARS* model against the *AR* (in Panels A) and the *AR*(1) (in Panel B), for each of the values of v_1 and v_2 in the grid. In particular, Table 2 display results for GDP growth, while Table 3 displays results for inflation. Note that the RMSFE reported in Tables 2-3 are computed as $RMSFE_{ARS}/RMSFE_{AR}$ and $RMSFE_{ARS}/RMSFE_{AR(1)}$, i.e. we do not subtract 1 from the numerator and denominator as in Table 1. Several conclusions can be drawn from the tables.

First, the *ARS* model outperforms both the *AR* and the *AR*(1) in forecasting both inflation and GDP growth. In particular, the *ARS* outperforms its competitors in most of the cases, and in several instances there are combinations of the shrinkage parameters such that the *ARS* is the best forecasting model for all the countries at hand.

Second, the *ARS* model seems to work better when a relatively loose overall shrinkage and relatively high shrinkage for higher lags is used (i.e. low v_1 and high v_2). This is in line with the a-priori belief that lags closer in time are more useful in forecasting. Still, the fact that the *ARS* outperforms the *AR*(1) shows that the higher order lags do contain useful information. Such information needs to be extracted in an efficient way in order to avoid to pay the costs of overparameterization. The fact that the *ARS* outperforms the *AR* shows that shrinkage is able to efficiently extract such information, whereas simply including all the lags in the model would likely result in a loss in prediction performance.

Third, as for the magnitude of the forecasting gains, for the GDP growth (Tables 2) the

gains against the AR(1) model can go up to 28% for the US, 40% for the UK, 18% for Japan. Similarly, the gains against the AR can go up to 25% for the US, 18% for the UK, and 36% for Japan. For inflation (Tables 3), the gains against the AR(1) can go up to 22% for the US, 36% for the UK, and 30% for Japan, while the gains against the AR can go up to 21% for the US, 12% for the UK, 17% for Japan.

Finally, in Tables 4 and 5 we report results for forecasting GDP and inflation 12-step ahead. It turns out that the results are in line with those obtained for the 1 step-ahead case, and the forecasting gains can be sizeable also at the longer horizon.

5 Conclusion

Autoregressive models have been used extensively for forecasting macroeconomic time series. In this paper we propose a new autoregressive model, based on shrinkage estimation, that is especially designed to promote parsimony and thereby provide superior forecasts. In the course of proposing the new model we provide some interesting theoretical extensions of independent interest for ridge estimation both within standard regression models and autoregressive models in particular.

We have carried out an extensive Monte Carlo study, which shows the considerable potential of the new method, as well as an empirical application to inflation and GDP growth rates. Our results are encouraging and suggest that the new method has wide applicability to macroeconomic forecasting.

Appendix

Proof of Theorem 1

We will show that

$$\left\| \hat{\beta}_T - \beta_0 \right\| = O_p \left(\left(\frac{p_T + \kappa_T}{T} \right)^{1/2} \right) \quad (7)$$

where κ_T is defined in (8). By the definition of $\hat{\beta}_T$, we have that

$$\sum_{i=1}^T \left(y_t - \mathbf{x}'_t \hat{\beta}_T \right)^2 + \sum_{j=1}^{p_T} \lambda_{jT} \left| \hat{\beta}_{jT} \right|^2 \leq \sum_{i=1}^T \left(y_t - \mathbf{x}'_t \beta_0 \right)^2 + \sum_{j=1}^{p_T} \lambda_{jT} \left| \beta_{j0} \right|^2$$

or

$$\sum_{i=1}^T \left(y_t - \mathbf{x}'_t \hat{\beta}_T \right)^2 \leq \sum_{i=1}^T \left(y_t - \mathbf{x}'_t \beta_0 \right)^2 + \sum_{j=1}^{p_T} \lambda_{jT} \left| \beta_{j0} \right|^2$$

Define

$$\kappa_T = \sum_{j=1}^{p_T} \lambda_{jT} \left| \beta_{j0} \right|^2, \quad (8)$$

then

$$\begin{aligned} \kappa_T &\geq \sum_{i=1}^T \left(y_t - \mathbf{x}'_t \hat{\beta}_T \right)^2 - \sum_{i=1}^T \left(y_t - \mathbf{x}'_t \beta_0 \right)^2 = \\ &\sum_{i=1}^T \left[\mathbf{x}'_t \left(\hat{\beta}_T - \beta_0 \right) \right]^2 + 2 \sum_{i=1}^T \epsilon_t \mathbf{x}'_t \left(\hat{\beta}_T - \beta_0 \right) \end{aligned}$$

Define $\delta_T = T^{1/2} \Sigma_T^{-1/2} \left(\hat{\beta}_T - \beta_0 \right)$, $\mathbf{D}_T = T^{1/2} \Sigma_T^{-1/2} \mathbf{X}'$, $\Sigma_T = \frac{1}{T} \mathbf{X}' \mathbf{X}$ and $\epsilon_T = (\epsilon_1, \dots, \epsilon_T)'$.

Then,

$$\sum_{i=1}^T \left[\mathbf{x}'_t \left(\hat{\beta}_T - \beta_0 \right) \right]^2 + 2 \sum_{i=1}^T \epsilon_t \mathbf{x}'_t \left(\hat{\beta}_T - \beta_0 \right) = \delta_T' \delta_T - 2 \left(\mathbf{D}_T \epsilon_T \right)' \delta_T.$$

It follows that

$$\delta_T' \delta_T - 2 \left(\mathbf{D}_T \epsilon_T \right)' \delta_T - \kappa_T \leq 0$$

which implies that

$$\left\| \delta_T - \mathbf{D}_T \epsilon_T \right\|^2 - \left\| \mathbf{D}_T \epsilon_T \right\|^2 - \kappa_T \leq 0$$

or

$$\left\| \delta_T - \mathbf{D}_T \epsilon_T \right\| \leq \left\| \mathbf{D}_T \epsilon_T \right\| + \kappa_T^{1/2}$$

But then,

$$\left\| \delta_T \right\| \leq \left\| \delta_T - \mathbf{D}_T \epsilon_T \right\| + \left\| \mathbf{D}_T \epsilon_T \right\| \leq 2 \left\| \mathbf{D}_T \epsilon_T \right\| + \kappa_T^{1/2}$$

which implies

$$\left\| \delta_T \right\|^2 \leq 6 \left\| \mathbf{D}_T \epsilon_T \right\|^2 + 3 \kappa_T$$

Rewrite $\mathbf{D}_T \boldsymbol{\epsilon}_T = \sum_{t=1}^T \mathbf{d}_{tT} \epsilon_t$ where \mathbf{d}_{tT} is the t -th column of \mathbf{D}_T . By the martingale difference assumption on ϵ_t , we have $E(\epsilon_t \epsilon_s) = 0$ for $s \neq t$. Then, by the independence between ϵ_t and \mathbf{x}_t , it follows that

$$E \|\mathbf{D}_T \boldsymbol{\epsilon}_T\|^2 = \sum_{t=1}^T E \|\mathbf{d}_{tT}\|^2 E \epsilon_t^2 = \sigma^2 \sum_{t=1}^T E \|\mathbf{d}_{tT}\|^2 = \sigma^2 E \left(\sum_{t=1}^T \|\mathbf{d}_{tT}\|^2 \right) = \quad (9)$$

$$\sigma^2 E(\text{tr}(\mathbf{D}_T \mathbf{D}'_T)) = \sigma^2 p_T$$

Thus,

$$E \|\boldsymbol{\delta}_T\|^2 \leq \mathbf{6} \sigma^2 p_T + 3 \kappa_T$$

or

$$TE \left[\left(\hat{\boldsymbol{\beta}}_T - \boldsymbol{\beta}_0 \right)' \Sigma_T \left(\hat{\boldsymbol{\beta}}_T - \boldsymbol{\beta}_0 \right) \right] \leq \mathbf{6} \sigma^2 p_T + 3 \kappa_T$$

Finally, noting that by assumption the smallest eigenvalue of Σ_T is bounded away from zero for all T gives (7).

Proof of Theorem 2

In order to prove this theorem we only need to modify parts of the proof of Theorem 1. We reexamine (9) and in particular the step that uses the independence between ϵ_t and \mathbf{x}_t . In the case of the sieve autoregression

$$\epsilon_t = \epsilon_{Tt} = \varepsilon_t + \sum_{j=1}^{\infty} (\phi_j^{p_T} - \phi_j) y_{t-j} = \varepsilon_t + v_{Tt}$$

Then, we have

$$\sum_{t=1}^T E (\|\mathbf{d}_{tT}\| \epsilon_{Tt})^2 \leq \sum_{t=1}^T E (\|\mathbf{d}_{tT}\| \varepsilon_t)^2 + \sum_{t=1}^T E (\|\mathbf{d}_{tT}\| v_{Tt})^2$$

As in the proof of Theorem 1

$$\sum_{t=1}^T E (\|\mathbf{d}_{tT}\| \varepsilon_t)^2 = O_p(p_T + \kappa_T)$$

Then,

$$\sum_{t=1}^T E (\|\mathbf{d}_{tT}\| v_{Tt})^2 \leq \sum_{t=1}^T E (\|\mathbf{d}_{tT}\|^2) E(v_{Tt}^2)$$

But by Theorem 7.6.6. of Anderson (1971) v_{Tt}^2 converges to zero in the L_2 norm and so

$$\sum_{t=1}^T E(\|\mathbf{d}_{tT}\|^2) E(v_{Tt}^2) = o_p(p_T + \kappa_T)$$

thereby proving the result.

References

- BANBURRA, M., D. GIANNONE, AND L. REICHLIN (2007): “Bayesian VARS with Large Panels,” *CEPR Discussion Paper No. 6326*.
- DE MOL, C., D. GIANNONE, AND L. REICHLIN (2007): “Forecasting using a large number of predictors: is Bayesian regression a valid alternative to principal components?,” *Mimeo, European Central Bank*.
- GEWEKE, J., AND C. WHITEMAN (2006): “Bayesian Forecasting,” in *Handbook of Economic Forecasting*, ed. by G. Elliott, C. W. J. Granger, and A. Timmermann. North Holland.
- HUANG, J., J. L. HOROWITZ, AND S. MA (2008): “Asymptotic Properties of Bridge Estimators in Sparse High-Dimensional Regression Models,” *Annals of Statistics*, 36, 587–613.
- STOCK, J. H., AND M. W. WATSON (2006): “Forecasting with many predictors,” in *Handbook of Economic Forecasting*, ed. by G. Elliott, C. W. J. Granger, and A. Timmermann. North Holland.

Table 1: Relative RMSFE (ARS/AR)

Panel A: Results for $\phi = 0.5$										
	$T = 60$					$T = 80$				
v_1/v_2	0.1	0.5	1	2	5	0.1	0.5	1	2	5
0.01	0.979	0.915	0.853	0.757	0.601	1.061	1.001	0.944	0.842	0.671
0.1	0.843	0.580	0.444	0.310	0.186	0.934	0.649	0.510	0.370	0.232
0.5	0.567	0.270	0.194	0.139	0.099	0.631	0.307	0.212	0.148	0.110
2	0.284	0.120	0.092	0.085	0.088	0.328	0.138	0.100	0.115	0.084
5	0.162	0.090	0.076	0.081	0.093	0.186	0.104	0.086	0.109	0.116
	$T = 100$					$T = 200$				
0.01	1.229	1.142	1.057	0.937	0.719	1.898	1.729	1.562	1.245	0.834
0.1	1.049	0.710	0.526	0.361	0.218	1.520	0.824	0.559	0.371	0.215
0.5	0.702	0.300	0.202	0.108	0.109	0.828	0.301	0.188	0.128	0.103
2	0.333	0.129	0.098	0.109	0.124	0.359	0.137	0.117	0.095	0.138
5	0.187	0.108	0.089	0.097	0.126	0.164	0.128	0.126	0.118	0.123
	$T = 400$					$T = 1000$				
0.01	3.130	2.514	2.027	1.529	0.946	6.155	3.768	2.830	1.873	0.986
0.1	2.112	0.928	0.558	0.317	0.184	2.721	0.941	0.566	0.305	0.141
0.5	0.903	0.282	0.185	0.113	0.136	0.976	0.270	0.167	0.200	0.215
2	0.318	0.101	0.078	0.109	0.177	0.318	0.019	0.204	0.186	0.343
5	0.221	0.037	0.167	0.170	0.224	0.207	0.119	0.174	0.272	0.410
Panel B: Results for $\phi = 0.95$										
	$T = 60$					$T = 80$				
v_1/v_2	0.1	0.5	1	2	5	0.1	0.5	1	2	5
0.01	0.978	0.925	0.862	0.765	0.608	1.050	0.991	0.944	0.843	0.684
0.1	0.835	0.604	0.462	0.344	0.257	0.938	0.686	0.526	0.398	0.277
0.5	0.580	0.306	0.235	0.198	0.209	0.656	0.356	0.271	0.223	0.238
2	0.313	0.212	0.212	0.223	0.291	0.359	0.234	0.239	0.278	0.365
5	0.206	0.204	0.236	0.283	0.324	0.244	0.252	0.241	0.351	0.432
	$T = 100$					$T = 200$				
0.01	1.195	1.139	1.043	0.931	0.717	1.960	1.717	1.481	1.222	0.853
0.1	1.047	0.723	0.529	0.382	0.256	1.439	0.832	0.580	0.377	0.261
0.5	0.691	0.359	0.284	0.219	0.298	0.794	0.334	0.253	0.289	0.362
2	0.369	0.229	0.241	0.305	0.386	0.371	0.291	0.351	0.412	0.571
5	0.240	0.276	0.303	0.386	0.504	0.254	0.344	0.448	0.599	0.798
	$T = 400$					$T = 1000$				
0.01	3.123	2.354	2.096	1.550	0.919	5.735	3.618	2.415	1.993	0.943
0.1	1.992	0.906	0.597	0.364	0.315	2.874	0.955	0.530	0.420	0.347
0.5	0.906	0.329	0.271	0.334	0.475	0.904	0.323	0.377	0.369	0.778
2	0.382	0.323	0.391	0.531	0.961	0.420	0.349	0.677	0.961	2.004
5	0.297	0.477	0.683	1.012	1.455	0.297	0.743	1.290	1.792	2.631

Table 2: Relative RMSFE in predicting GDP

Panel A: ARS/AR									
v_1	v_2	CN	FR	IT	JP	NL	SP	UK	US
0.01	0.1	0.979	0.971	0.99	0.989	0.996	0.962	0.987	0.981
	0.5	0.879	0.847	0.93	0.917	0.97	0.808	0.929	0.894
	1	0.751	0.745	0.844	0.787	0.919	0.658	0.859	0.801
	2	0.648	0.701	0.785	0.683	0.87	0.567	0.826	0.75
	5	0.614	0.707	0.77	0.644	0.867	0.543	0.854	0.744
0.1	0.1	0.927	0.903	0.96	0.956	0.984	0.878	0.957	0.935
	0.5	0.763	0.752	0.851	0.797	0.924	0.669	0.864	0.812
	1	0.646	0.697	0.784	0.681	0.871	0.565	0.82	0.761
	2	0.603	0.696	0.763	0.64	0.895	0.547	0.864	0.77
	5	0.61	0.748	0.78	0.642	0.976	0.588	0.992	0.808
0.5	0.1	0.887	0.856	0.935	0.924	0.973	0.819	0.933	0.902
	0.5	0.709	0.724	0.818	0.742	0.899	0.618	0.839	0.784
	1	0.619	0.692	0.77	0.655	0.871	0.547	0.826	0.765
	2	0.602	0.715	0.77	0.643	0.952	0.574	0.928	0.805
	5	0.624	0.787	0.801	0.663	1.062	0.646	1.107	0.866
2	0.1	0.838	0.808	0.901	0.877	0.956	0.755	0.905	0.864
	0.5	0.663	0.704	0.793	0.696	0.877	0.577	0.823	0.768
	1	0.606	0.699	0.767	0.643	0.897	0.548	0.855	0.784
	2	0.611	0.743	0.786	0.661	1.033	0.627	1.024	0.857
	5	0.641	0.823	0.823	0.693	1.149	0.714	1.232	0.929
5	0.1	0.766	0.754	0.853	0.8	0.925	0.672	0.866	0.814
	0.5	0.622	0.695	0.772	0.657	0.872	0.548	0.826	0.769
	1	0.606	0.725	0.778	0.651	0.978	0.591	0.941	0.836
	2	0.627	0.776	0.811	0.701	1.146	0.716	1.172	0.931
	5	0.66	0.856	0.846	0.731	1.237	0.791	1.364	0.994

Panel B: ARS/AR(1)									
v_1	v_2	CN	FR	IT	JP	NL	SP	UK	US
0.01	0.1	1.52	1.228	1.177	1.256	0.759	1.095	0.717	0.947
	0.5	1.366	1.072	1.106	1.165	0.739	0.92	0.674	0.863
	1	1.168	0.942	1.004	0.999	0.7	0.748	0.624	0.773
	2	1.006	0.887	0.933	0.867	0.663	0.645	0.599	0.724
	5	0.954	0.895	0.916	0.817	0.66	0.618	0.62	0.718
0.1	0.1	1.44	1.142	1.142	1.214	0.75	0.999	0.695	0.903
	0.5	1.185	0.951	1.012	1.012	0.704	0.762	0.628	0.784
	1	1.004	0.882	0.932	0.865	0.664	0.643	0.596	0.735
	2	0.936	0.88	0.908	0.812	0.682	0.622	0.627	0.744
	5	0.948	0.946	0.928	0.815	0.743	0.669	0.72	0.78
0.5	0.1	1.378	1.083	1.112	1.173	0.741	0.932	0.678	0.871
	0.5	1.102	0.915	0.973	0.941	0.685	0.703	0.609	0.757
	1	0.962	0.875	0.916	0.832	0.663	0.622	0.6	0.738
	2	0.936	0.904	0.916	0.816	0.725	0.653	0.674	0.778
	5	0.969	0.995	0.952	0.842	0.809	0.735	0.804	0.836
2	0.1	1.303	1.021	1.072	1.113	0.728	0.859	0.657	0.834
	0.5	1.03	0.891	0.942	0.883	0.668	0.657	0.598	0.741
	1	0.942	0.883	0.912	0.816	0.683	0.624	0.621	0.757
	2	0.949	0.94	0.935	0.839	0.787	0.713	0.744	0.828
	5	0.996	1.041	0.979	0.88	0.875	0.812	0.894	0.897
5	0.1	1.19	0.954	1.014	1.016	0.705	0.765	0.629	0.786
	0.5	0.966	0.879	0.919	0.834	0.664	0.623	0.599	0.743
	1	0.942	0.917	0.925	0.826	0.745	0.673	0.684	0.808
	2	0.974	0.982	0.964	0.89	0.873	0.815	0.851	0.898
	5	1.026	1.082	1.006	0.928	0.942	0.9	0.991	0.96

Table 3: Relative RMSFE in predicting Inflation

Panel A: ARS/AR											
v_1	v_2	CN	FR	GE	IT	JP	NL	SP	SW	UK	US
0.01	0.1	1.036	0.987	1.032	1.079	1.006	1.046	1.063	0.997	1.07	0.922
	0.5	0.837	0.875	0.871	0.846	0.925	0.922	0.975	0.836	0.931	0.801
	1	0.764	0.906	0.86	0.739	0.84	0.906	0.889	0.788	0.874	0.792
	2	0.775	0.994	0.935	0.716	0.832	1.078	0.889	0.829	0.932	0.838
	5	0.811	1.088	1.025	0.739	0.878	1.328	0.936	0.892	1.005	0.921
0.1	0.1	0.904	0.898	0.917	0.921	0.963	0.965	1.015	0.888	0.983	0.831
	0.5	0.768	0.9	0.865	0.745	0.845	0.907	0.893	0.792	0.876	0.79
	1	0.782	0.991	0.95	0.713	0.841	1.115	0.899	0.839	0.948	0.821
	2	0.851	1.108	1.055	0.734	0.926	1.504	0.989	0.932	1.059	0.905
	5	0.936	1.292	1.166	0.807	0.999	1.786	1.079	1.013	1.134	1.06
0.5	0.1	0.844	0.877	0.877	0.853	0.929	0.928	0.98	0.842	0.936	0.805
	0.5	0.766	0.935	0.896	0.726	0.829	0.96	0.881	0.802	0.892	0.799
	1	0.81	1.042	1.001	0.718	0.877	1.294	0.939	0.883	1.004	0.848
	2	0.911	1.186	1.106	0.762	0.981	1.7	1.056	0.984	1.116	0.954
	5	1.009	1.411	1.217	0.861	1.052	1.942	1.164	1.061	1.196	1.134
2	0.1	0.799	0.875	0.858	0.797	0.891	0.902	0.939	0.809	0.896	0.792
	0.5	0.777	0.98	0.94	0.719	0.835	1.068	0.892	0.829	0.932	0.816
	1	0.856	1.105	1.052	0.733	0.926	1.5	0.994	0.933	1.063	0.886
	2	0.978	1.267	1.147	0.799	1.036	1.863	1.138	1.03	1.177	1.003
	5	1.073	1.514	1.253	0.917	1.099	2.055	1.256	1.098	1.259	1.194
5	0.1	0.769	0.9	0.867	0.748	0.846	0.908	0.894	0.793	0.876	0.791
	0.5	0.813	1.053	1.009	0.725	0.877	1.296	0.941	0.885	1.006	0.854
	1	0.94	1.195	1.109	0.766	1.000	1.753	1.088	1.000	1.142	0.94
	2	1.054	1.348	1.182	0.851	1.099	2.01	1.254	1.074	1.255	1.051
	5	1.13	1.602	1.28	0.975	1.144	2.144	1.361	1.126	1.326	1.243
Panel B: ARS/AR(1)											
v_1	v_2	CN	FR	GE	IT	JP	NL	SP	SW	UK	US
0.01	0.1	0.92	0.748	0.874	1.229	0.849	0.491	0.733	0.905	0.78	0.907
	0.5	0.743	0.664	0.738	0.963	0.781	0.433	0.672	0.759	0.679	0.789
	1	0.678	0.687	0.729	0.842	0.709	0.425	0.613	0.716	0.637	0.779
	2	0.688	0.754	0.792	0.815	0.702	0.506	0.612	0.753	0.68	0.825
	5	0.72	0.826	0.869	0.842	0.741	0.623	0.645	0.809	0.733	0.907
0.1	0.1	0.803	0.681	0.777	1.049	0.812	0.453	0.7	0.806	0.716	0.818
	0.5	0.682	0.683	0.733	0.849	0.713	0.426	0.615	0.719	0.638	0.778
	1	0.694	0.752	0.805	0.812	0.71	0.523	0.62	0.762	0.691	0.809
	2	0.755	0.841	0.894	0.836	0.781	0.706	0.682	0.846	0.772	0.891
	5	0.831	0.98	0.988	0.92	0.843	0.838	0.743	0.919	0.827	1.043
0.5	0.1	0.749	0.666	0.743	0.972	0.784	0.435	0.675	0.765	0.682	0.793
	0.5	0.68	0.71	0.759	0.827	0.7	0.451	0.607	0.728	0.65	0.787
	1	0.72	0.79	0.848	0.818	0.74	0.607	0.647	0.801	0.732	0.835
	2	0.809	0.9	0.937	0.867	0.828	0.798	0.728	0.893	0.814	0.939
	5	0.896	1.071	1.031	0.981	0.888	0.912	0.802	0.964	0.872	1.116
2	0.1	0.709	0.664	0.727	0.908	0.752	0.423	0.647	0.734	0.653	0.78
	0.5	0.69	0.744	0.797	0.819	0.705	0.501	0.615	0.753	0.68	0.803
	1	0.761	0.839	0.891	0.834	0.782	0.704	0.685	0.847	0.775	0.872
	2	0.869	0.961	0.972	0.91	0.874	0.874	0.784	0.935	0.858	0.988
	5	0.953	1.148	1.062	1.045	0.928	0.965	0.866	0.997	0.917	1.175
5	0.1	0.683	0.683	0.735	0.851	0.714	0.426	0.616	0.72	0.639	0.779
	0.5	0.722	0.799	0.855	0.825	0.741	0.608	0.649	0.803	0.733	0.841
	1	0.835	0.907	0.94	0.873	0.844	0.823	0.75	0.907	0.833	0.925
	2	0.936	1.022	1.002	0.969	0.928	0.943	0.864	0.975	0.914	1.035
	5	1.003	1.215	1.085	1.111	0.966	1.006	0.938	1.022	0.966	1.224

Table 4 Relative RMSFE in predicting GDP 12-step ahead

Panel A: ARS/AR									
v_1	v_2	CN	FR	IT	JP	NL	SP	UK	US
0.01	0.1	0.993	0.983	0.998	0.99	0.998	0.951	0.997	0.994
	0.5	0.958	0.909	0.984	0.924	0.985	0.72	0.979	0.97
	1	0.914	0.831	0.964	0.792	0.954	0.447	0.95	0.953
	2	0.904	0.782	0.961	0.683	0.917	0.308	0.922	0.959
	5	0.916	0.798	0.984	0.643	0.923	0.279	0.918	0.991
0.1	0.1	0.975	0.944	0.99	0.96	0.992	0.832	0.988	0.981
	0.5	0.915	0.841	0.964	0.801	0.957	0.465	0.951	0.954
	1	0.898	0.788	0.961	0.679	0.917	0.301	0.912	0.965
	2	0.915	0.838	1.012	0.63	0.964	0.277	0.921	1.046
	5	0.975	0.961	1.1	0.626	1.081	0.31	1.042	1.234
0.5	0.1	0.96	0.916	0.984	0.929	0.986	0.739	0.98	0.971
	0.5	0.902	0.814	0.958	0.741	0.938	0.375	0.936	0.955
	1	0.899	0.794	0.975	0.651	0.922	0.278	0.897	0.988
	2	0.943	0.915	1.069	0.624	1.046	0.297	0.981	1.161
	5	1.034	1.046	1.17	0.643	1.174	0.345	1.143	1.425
2	0.1	0.941	0.885	0.976	0.883	0.977	0.624	0.969	0.962
	0.5	0.897	0.795	0.958	0.693	0.921	0.316	0.918	0.961
	1	0.907	0.831	1.007	0.632	0.963	0.276	0.904	1.041
	2	0.993	1.002	1.14	0.635	1.143	0.33	1.073	1.332
	5	1.092	1.106	1.226	0.67	1.242	0.374	1.226	1.601
5	0.1	0.915	0.844	0.964	0.803	0.957	0.469	0.951	0.954
	0.5	0.897	0.795	0.974	0.652	0.922	0.278	0.894	0.988
	1	0.943	0.927	1.086	0.626	1.076	0.305	0.974	1.196
	2	1.064	1.089	1.219	0.668	1.24	0.37	1.183	1.561
	5	1.142	1.147	1.268	0.698	1.288	0.396	1.29	1.75

Panel B: ARS/AR(1)									
v_1	v_2	CN	FR	IT	JP	NL	SP	UK	US
0.01	0.1	0.903	0.87	0.77	1.378	0.759	2.386	0.841	0.563
	0.5	0.871	0.805	0.759	1.286	0.749	1.806	0.826	0.55
	1	0.831	0.735	0.744	1.102	0.726	1.122	0.802	0.54
	2	0.822	0.692	0.742	0.95	0.698	0.772	0.778	0.543
	5	0.833	0.706	0.759	0.895	0.702	0.699	0.775	0.562
0.1	0.1	0.886	0.836	0.765	1.336	0.755	2.087	0.834	0.556
	0.5	0.832	0.745	0.744	1.115	0.728	1.167	0.803	0.541
	1	0.816	0.697	0.742	0.945	0.698	0.756	0.77	0.547
	2	0.832	0.741	0.781	0.877	0.733	0.695	0.778	0.593
	5	0.887	0.851	0.849	0.871	0.822	0.778	0.879	0.699
0.5	0.1	0.872	0.811	0.76	1.293	0.75	1.853	0.827	0.55
	0.5	0.82	0.721	0.74	1.031	0.714	0.94	0.79	0.541
	1	0.817	0.703	0.752	0.906	0.701	0.697	0.757	0.56
	2	0.858	0.81	0.825	0.868	0.796	0.744	0.828	0.658
	5	0.941	0.926	0.903	0.895	0.893	0.865	0.965	0.807
2	0.1	0.855	0.783	0.753	1.229	0.743	1.567	0.818	0.545
	0.5	0.815	0.703	0.74	0.965	0.701	0.792	0.775	0.545
	1	0.824	0.736	0.778	0.879	0.732	0.691	0.763	0.59
	2	0.903	0.887	0.88	0.884	0.869	0.828	0.905	0.755
	5	0.992	0.979	0.946	0.932	0.945	0.939	1.035	0.907
5	0.1	0.832	0.747	0.744	1.118	0.728	1.178	0.803	0.541
	0.5	0.815	0.704	0.752	0.907	0.701	0.698	0.755	0.56
	1	0.857	0.821	0.838	0.87	0.818	0.764	0.822	0.678
	2	0.968	0.964	0.941	0.929	0.943	0.928	0.999	0.884
	5	1.038	1.015	0.979	0.971	0.98	0.994	1.089	0.992

Table 5: Relative RMSFE in predicting Inflation 12-step ahead

Panel A: ARS/AR											
v_1	v_2	CN	FR	GE	IT	JP	NL	SP	SW	UK	US
0.01	0.1	1.085	1.001	1.023	1.089	1.173	1.087	1.073	1.124	1.142	0.898
	0.5	0.918	0.826	0.915	0.92	1.037	1.014	0.986	0.901	1.029	0.819
	1	0.856	0.795	0.953	0.82	0.828	1.05	0.92	0.835	0.964	0.807
	2	0.893	0.84	1.117	0.772	0.723	1.354	0.938	0.891	0.992	0.845
	5	0.98	0.943	1.306	0.755	0.719	1.744	0.995	0.974	1.043	0.942
0.1	0.1	0.975	0.876	0.942	0.983	1.106	1.034	1.022	0.975	1.073	0.837
	0.5	0.857	0.792	0.945	0.829	0.837	1.041	0.919	0.833	0.964	0.806
	1	0.9	0.839	1.13	0.785	0.723	1.401	0.943	0.896	0.998	0.845
	2	1.074	1.062	1.437	0.788	0.738	2.037	1.06	1.041	1.089	1.029
	5	1.304	1.485	1.646	0.926	0.795	2.518	1.25	1.156	1.222	1.255
0.5	0.1	0.922	0.83	0.916	0.926	1.044	1.014	0.988	0.906	1.032	0.822
	0.5	0.861	0.802	1.001	0.811	0.768	1.134	0.915	0.846	0.966	0.81
	1	0.96	0.899	1.261	0.781	0.72	1.687	0.985	0.954	1.034	0.905
	2	1.229	1.303	1.587	0.863	0.776	2.384	1.185	1.125	1.173	1.174
	5	1.492	1.877	1.746	1.114	0.85	2.822	1.474	1.217	1.365	1.416
2	0.1	0.881	0.801	0.91	0.876	0.96	1.005	0.952	0.855	0.994	0.813
	0.5	0.886	0.824	1.088	0.8	0.73	1.314	0.932	0.879	0.988	0.829
	1	1.064	1.018	1.415	0.799	0.738	2.018	1.056	1.033	1.084	1.004
	2	1.414	1.62	1.705	1.001	0.829	2.708	1.381	1.194	1.3	1.331
	5	1.652	2.262	1.807	1.352	0.901	3.044	1.753	1.254	1.531	1.545
5	0.1	0.857	0.792	0.943	0.831	0.839	1.039	0.919	0.832	0.964	0.806
	0.5	0.958	0.889	1.252	0.795	0.721	1.68	0.983	0.951	1.033	0.896
	1	1.273	1.281	1.608	0.886	0.791	2.473	1.233	1.143	1.201	1.183
	2	1.631	2.039	1.799	1.254	0.899	3.024	1.733	1.251	1.508	1.501
	5	1.787	2.638	1.847	1.657	0.947	3.21	2.108	1.279	1.723	1.65
Panel B: ARS/AR(1)											
v_1	v_2	CN	FR	GE	IT	JP	NL	SP	SW	UK	US
0.01	0.1	0.586	0.58	0.554	0.729	1.192	0.329	0.427	0.868	0.609	0.596
	0.5	0.496	0.478	0.495	0.616	1.054	0.306	0.392	0.696	0.548	0.543
	1	0.463	0.46	0.515	0.549	0.841	0.317	0.366	0.645	0.514	0.535
	2	0.483	0.486	0.604	0.517	0.735	0.409	0.373	0.688	0.529	0.561
	5	0.53	0.546	0.707	0.506	0.73	0.527	0.396	0.753	0.556	0.625
0.1	0.1	0.527	0.508	0.51	0.658	1.124	0.313	0.407	0.753	0.572	0.555
	0.5	0.463	0.459	0.511	0.555	0.85	0.315	0.366	0.643	0.514	0.535
	1	0.486	0.486	0.612	0.526	0.734	0.423	0.375	0.692	0.532	0.561
	2	0.58	0.615	0.777	0.527	0.75	0.616	0.422	0.804	0.58	0.683
	5	0.705	0.86	0.891	0.62	0.807	0.761	0.497	0.893	0.651	0.832
0.5	0.1	0.498	0.481	0.496	0.62	1.061	0.306	0.393	0.7	0.55	0.545
	0.5	0.466	0.465	0.542	0.543	0.78	0.343	0.364	0.654	0.515	0.537
	1	0.519	0.52	0.682	0.523	0.732	0.51	0.392	0.737	0.551	0.6
	2	0.664	0.754	0.859	0.578	0.789	0.721	0.471	0.869	0.625	0.779
	5	0.806	1.087	0.945	0.746	0.864	0.853	0.587	0.94	0.728	0.939
2	0.1	0.476	0.464	0.493	0.586	0.975	0.304	0.379	0.661	0.53	0.539
	0.5	0.479	0.477	0.589	0.536	0.741	0.397	0.371	0.679	0.526	0.55
	1	0.575	0.59	0.765	0.535	0.75	0.61	0.42	0.798	0.578	0.666
	2	0.764	0.938	0.923	0.67	0.843	0.819	0.55	0.923	0.693	0.883
	5	0.893	1.31	0.978	0.905	0.916	0.92	0.698	0.969	0.816	1.025
5	0.1	0.463	0.458	0.51	0.557	0.853	0.314	0.366	0.643	0.514	0.535
	0.5	0.518	0.515	0.677	0.532	0.732	0.508	0.391	0.735	0.55	0.595
	1	0.688	0.742	0.87	0.593	0.803	0.748	0.491	0.883	0.64	0.785
	2	0.882	1.181	0.973	0.84	0.913	0.914	0.69	0.967	0.804	0.996
	5	0.966	1.528	0.999	1.11	0.962	0.97	0.839	0.988	0.918	1.095

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**Department of Economics
Queen Mary, University of London
Mile End Road
London E1 4NS
Tel: +44 (0)20 7882 5096
Fax: +44 (0)20 8983 3580
Web: www.econ.qmul.ac.uk/papers/wp.htm**