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Business Cycle Persistence in Developing Countries: How Successful is a DSGE Model with a Vertical Production Chain and Sticky Prices?

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Abstract

It is well documented that business cycles of developed countries are characterised by persistent output fluctuations, and this has been the subject of much theoretical interest. However, the case for developing countries has been somewhat neglected in the literature. This paper addresses this imbalance, revealing that whilst both developed and developing countries exhibit persistent output fluctuations, there is a significant positive relationship between output persistence and level of economic development. This relationship was successfully modelled using a vertical production chain DSGE model (Huang and Liu, 2001). This model lends itself to such an analysis, as by altering the number of production stages (N) it is possible to represent economies at different levels of development. However, calibration of low input-output (γ) parameter values for the US and UK effectively inhibited the model from generating enough persistence to match that observed in these countries. Nonetheless, after abstracting from the US and UK results, there was found to be a strong significant positive relationship between the magnitude of output persistence generated by the model and economic development. A final very significant finding of this analysis is that the model overestimates output persistence in high inflation countries and underestimates output persistence in low inflation countries. This has important implications not only for this model, but also for any economist attempting to construct a business cycle model capable of replicating the observed patterns of output persistence.

JEL Classification: E31, E32, E52
Keywords: Output persistence; Vertical production chain; Staggered price contracts; Economic Development; Inflation.

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1 Introduction

One of the central issues concerning macroeconomists in recent years has been the construction of dynamic general equilibrium models in which monetary policy shocks generate persistent output fluctuations without prices that are set for exogenously long periods. However whilst much work has been carried out on modelling this empirical feature for the industrialised countries, little, if any, theoretical work has examined this in the context of developing country business cycles. Thus, this paper aims to address this balance, by firstly examining the degree of output persistence in developing countries, and its relation to economic development, and secondly through the use of a dynamic general equilibrium model in which monetary policy shocks are able to generate persistent output fluctuations in line with those observed for the developing countries.

Theoretical work on the issue of output persistence originates from the seminal papers of Taylor (1980) and Blanchard (1983) who examine output persistence in the context of staggered price and wage contracts. Their intuition is extended to a general equilibrium model in the influential work of Chari, Kehoe and McGrattan (2000). However, rather surprisingly, they find that a staggered price mechanism is, by itself, incapable of generating persistent output fluctuations beyond the exogenously imposed contract rigidity.

Thus, the need for an alternative specification of the sticky price model became apparent and a burgeoning literature emerged expressing the importance of input-output structures[1] in the transmission of business cycle shock. For example, Bergin and Feenstra (2000) combine the use of translog preferences, rather than the usual CES preferences, and a simple input-output production structure, as proposed by Basu (1995), where an aggregate of differentiated products serves as both the final consumption good and as an input into the production function of each firm. These two features interact in a positive way and generate significant endogenous output persistence, although this level remains considerably below that observed in the data.

A significant advancement then arises from the vertical input-output mechanism of Huang and Liu (2001). This addresses not only the output persistence issue but also another interesting issue, which is not considered by the aforementioned papers, namely that, in response to a monetary policy shock:

> "*prices at early stages of production fall more rapidly and by a larger amount than prices at subsequent stages of production*" Clark (1999, pp.424-425)

In the Huang and Liu model, the production of a final consumption good involves multiple stages of processing and, in order to generate real effects of a monetary shock, prices are staggered among firms within each stage. The input-output structure is fashioned through producers, at all but the initial stage, requiring inputs of labour and a composite of goods produced at earlier stages. Through the input-output relations across stages and the staggered prices within stages, the model is capable of generating persistence output fluctuations in response to monetary policy shocks as well as replicating the observed pattern of dampening price adjustment, as documented by Clark (1999).

[1] Among other suggestions in the literature, including: the application of translog, rather than CES, preferences, e.g. Bergin and Feenstra (2000), the importance of wage staggering, e.g. Huang and Liu (2002), and the inclusion of firm specific capital, see Nolan and Thoenissen (2005) for example.

The intuition behind the model is as follows: if there is only one stage of production, so that labour is the only input, then the real effect of the monetary shock will not last beyond the initial contract duration; following the shock, wages and hence marginal cost increase immediately and consequently firms increase prices as soon as they can renew their contracts. In contrast, if there are several stages of production, the effect of the shock is extenuated through the production chain. Stage one firms experience a full rise in their marginal costs, as in the single stage model, and thus increase prices as soon as they can renew their contracts. However, due to the staggered nature of the stage one firms' price increases, the firms at stage two do not immediately endure a full marginal cost increase. Thus, stage two firms that are able to renew contracts during the initial period will not choose to raise prices fully. At the end of the initial period, when all stage one firms have renewed their contracts, the marginal cost at stage two will also fully adjust and the stage two firms will now choose to fully increase prices when it is time to renew their contracts. Correspondingly, firms at higher stages will face even smaller changes in their marginal cost and have even less of an incentive to adjust prices. Thus, as production chain length increases, movements in the price level decrease and fluctuations in aggregate output become increasingly persistent.

The vertical input-output structure of the Huang and Liu (2001) model lends itself to the examination of economies at different levels of development. It is possible to represent countries at different levels of economic development simply by altering the number of stages of production involved. For example, the world's least economically developed countries, such as Malawi, rely very heavily on exports of agriculture and raw materials, whilst having very little industrial production. As such, these countries can be represented by a very simple input-output structure with just one or two stages of production. On the other hand, an emerging market economy, such as Malaysia, will have a much more developed multi-sector economy. Accordingly, more stages can be incorporated in the input-output structure to represent this.

Thus, the Huang and Liu (2001) model is employed in this paper to examine output persistence in economies at different levels of economic development. However, firstly it is necessary to examine whether the model is actually capable of generating empirically plausible degrees of output persistence. This is because whilst Huang and Liu (2001) demonstrate that the model is theoretically capable of generating significantly persistent output fluctuations in response to monetary policy shocks, they do not consider whether the extent of the persistence generated is quantitatively similar to that observed in the data. This is particularly important because many business cycle models appear theoretically pleasing but are not capable of matching the observed output persistence. Having established the empirical plausibility of the model, the relationship between output persistence and economic development can be further examined.

The subsequent section examines the relationship between output persistence, as measured by its half-life, and economic development[2]. Section 3 describes the Huang and Liu (2001) model to be used in the analysis. Section 4 calibrates the model for a sample of developing countries and assigns the number of stages to be included in the input-output structures. Section 5 examines the sensitivity of the model to the key parameters. Section 6 presents the impulse response functions and associated half-lives for the calibrated countries,

[2] For the purposes of this paper, level of economic development is measured by GDP per capita and Energy Use per capita.

and discusses the success of the model in capturing the patterns of output persistence revealed in section 2. Finally, section 7 concludes.

2 Output Persistence

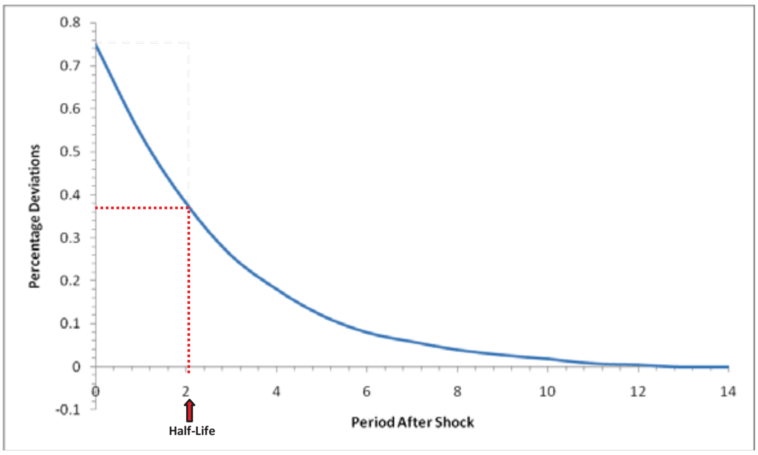
The central aim of this paper is to model the persistence of output fluctuations in developing economies. However, it is first necessary to establish the nature of output persistence in these economies. Moreover, it is of particular interest to establish whether there is any relationship between output persistence and economic development.

As a measure of output persistence, Male (2010) reports the autocorrelations of the cyclical components of either industrial or manufacturing production, providing evidence of significant output persistence in a wide spectrum of economies. However, the magnitude of this persistence appeared to be highest in the industrialised countries. In light of these findings, this paper proceeds to examine the degree of output persistence of a large sample of developing countries and furthermore to examine the relationship between output persistence and economic development. Assessment of the impact of the input-output structure on the persistence of output in the theoretical model is carried out by examining the half-life of the impulse response of output to a monetary shock. Thus, in order to compare the results of the theoretical model to the data, it is necessary to measure the persistence of output as its half-life.

2.1. Half-Life Measurement

For the theoretical model, the half-life of output is defined as the length of time required for the response of output to a shock to halve. In this case, the half-life is clearly evident from observing the resultant impulse response function; for example Figure 1.

Figure 1 **Measuring the Half-Life of an Impulse Response Function**



To calculate the half-life of output from the data, however, is not quite so straightforward. The procedure is simple and accurate where the data can be represented by a stationary AR(1) model, however for models of higher orders {AR(p), p>1 and ARMA(p,q)} the correct procedure has faced much theoretical debate in the literature, especially researchers interested in the puzzles associated with purchasing power parity (PPP); see Chortareas and Kapetanios (2007), Seong *et al.,* (2006) and Choi *et al.,* (2006), amongst many others, for a discussion of the limitations of half-life measures. For the purposes of this paper, a standard approximation for the derivation of the half-life of a stationary AR(p) process is applied. The estimators for the AR(1) process and the AR(p) process are outlined below.

The AR(1) model

Define an AR(1) process:

$$y_t = \rho y_{t-1} + \varepsilon_t \qquad (1)$$

where, ε_t denotes a white noise innovation.

Then, the half-life is correctly estimated by

$$h = \frac{ln(1/2)}{ln(\hat{\rho})} \qquad (2)$$

where, $\hat{\rho}$ is an estimate of ρ .

The AR(p) model

Define an AR(p) process:

$$y_t = \sum_{j=1}^{\rho} \gamma_j y_{t-j} + \varepsilon_t \qquad (3)$$

Take differences to obtain the error correction representation:

$$\Delta y_t = \gamma^* y_{t-1} + \sum_{j=1}^{\rho-1} \phi_j \Delta y_{t-j} + \varepsilon_t \qquad (4)$$

where, $\phi_j = -\sum_{k=j+1}^{p} \gamma_k$ and $\gamma^* = \left(\sum_{i=1}^{p} \gamma_i \right) - 1$

Then for a stationary time series, the half-life can be approximated by:

$$h = \frac{ln(1/2)}{ln(1+\gamma^*)} \qquad (5)$$

It is interesting to note that this reduces to the same formula as (2) for an AR(1) model.

2.2. Output Persistence

Table 1 reports the half-life of output (in months) for the US, UK and Japan and 28 developing countries.

As reliable quarterly real GDP data is not available for a large number of developing countries, indexes of industrial production are used as a proxy for the estimation of the half-life of output[3]. The data is from the IMF International Financial Statistics (IMF) database; manufacturing production (IMF IFS series 66EY) and industrial production (IMF IFS series 66).

The data is deseasonalized using the Census Bureau's X12 ARIMA seasonal adjustment procedure and filtered using the Hoddrick Prescott Filter (λ=1600) to extract the stationary (cyclical) component. An ARMA(p,q) process, as selected by the maximisation of the Akaike information criterion, is then fitted to the cyclical components of output and the half-life calculated using method (5).

The Ljung-Box Q test, which tests for the serial correlation of the residuals, indicates that in most cases there is little evidence of serial correlation of the residuals in the selected model. The exceptions to this are the Côte d'Ivoire, Malawi, and Senegal; for these countries the half-

[3] As suggested by Agénor *et al.* (2000).

life for the AR(1) model are reported as the higher order models {AR(2), AR(3), ARMA(1,1), ARMA(1,2), ARMA(2,1) and ARMA (2,2)} also showed significant residual serial correlation.

Table 1 **Estimated Half-Lives of Output**

Country	GDP per Capita Ranking (2003)	Sample Period	Model	Half-Life (in months)	Q value
US	5	1980:1 – 2005:1	AR(1)	16.6	38.34
UK	20	1980:1 – 2005:1	AR(1)	9.9	55.44
Japan	21	1980:1 – 2005:1	AR(2)	11.1	38.94
Argentina	70	1994:1 – 2004:2	ARMA(1,1)	4.5	9.36
Bangladesh	176	1980:1 – 2004:4	AR(2)	2.2	37.56
Brazil	93	1991:1 – 2005:1	AR(2)	2.7	34.91
Chile	81	1980:1 – 2005:1	AR(2)	7.8	48.86
Colombia	110	1980:1 – 2005:1	AR(1)	3.7	44.84
Côte d'Ivoire	196	1980:1 – 2004:1	AR(1)	2.7	74.24**
Hungary	62	1980:1 – 2005:1	ARMA(1,2)	9.8	26.55
India	152	1980:1 – 2004:4	ARMA(1,1)	4.4	53.01
Israel	44	1980:1 – 2004:4	AR(2)	6.3	45.07
Jordan	139	1980:1 – 2004:4	AR(1)	2.4	38.92
Korea, South	51	1980:1 – 2004:4	AR(1)	9.3	48.56
Lithuania	69	1993:1 – 2005:1	AR(1)	3.3	26.15
Macedonia	105	1993:1 – 2004:4	AR(1)	2.2	15.91
Malawi	230	1980:1 – 2004:2	AR(1)	2.1	59.09**
Malaysia	84	1980:1 – 2004:4	AR(2)	7.5	54.04
Morocco	143	1980:1 – 2003:3	AR(2)	2.3	34.34
Mexico	86	1980:1 – 2005:1	ARMA(1,2)	5.8	51.21
Nigeria	211	1980:1 – 2003:4	AR(1)	3.7	47.10
Pakistan	170	1980:1 – 2004:3	AR(1)	1.1	22.97
Peru	122	1980:1 – 2005:1	ARMA(1,2)	4.6	46.14
Philippines	133	1980:1 – 2005:1	AR(2)	5.8	21.19
Senegal	192	1985:4 – 2003:4	AR(1)	2.2	62.42**
Slovak Republic	65	1993:1 – 2005:1	AR(1)	4.9	24.98
Slovenia	49	1992:1 – 2005:1	ARMA(1,2)	10.6	34.08
South Africa	78	1980:1 – 2005:1	AR(2)	9.4	51.22
Trinidad & Tobago	75	1980:1 – 2003:4	AR(2)	2.8	39.05
Turkey	102	1980:1 – 2005:1	AR(1)	4.3	46.98
Uruguay	64	1980:1 – 2002:3	AR(2)	7.2	44.08

Significance is denoted by * if $p<0.05$ and ** if $p<0.01$

It is clear from Table 1 that the persistence of output (as measured by its half-life) is greater in the industrialised countries than in the majority of the developing countries. However, there are a few exceptions. Given their GDP per capita rankings, both South Africa and the Philippines have remarkably large half-lives of output. Conversely, given its GDP per capita ranking Brazil has a rather short output half-life. A possible explanation for the low persistence of output fluctuations in Brazil and the high degree of persistence experienced in the Philippines and South Africa relates to inflation; whilst South Africa and the Philippines exhibit relatively low inflation rates[4], Brazil experienced a period of hyperinflation during the late 1980s and early 1990s with an average annual inflation rate of 326% for the period 1991-2005. Thus, output appears to be more persistent in low inflation economies, which is consistent with the findings of Kiley (2000). Finally, Hungary and Slovenia both display high output persistence; however this is to be expected both from their relatively high GDP per capita rankings.

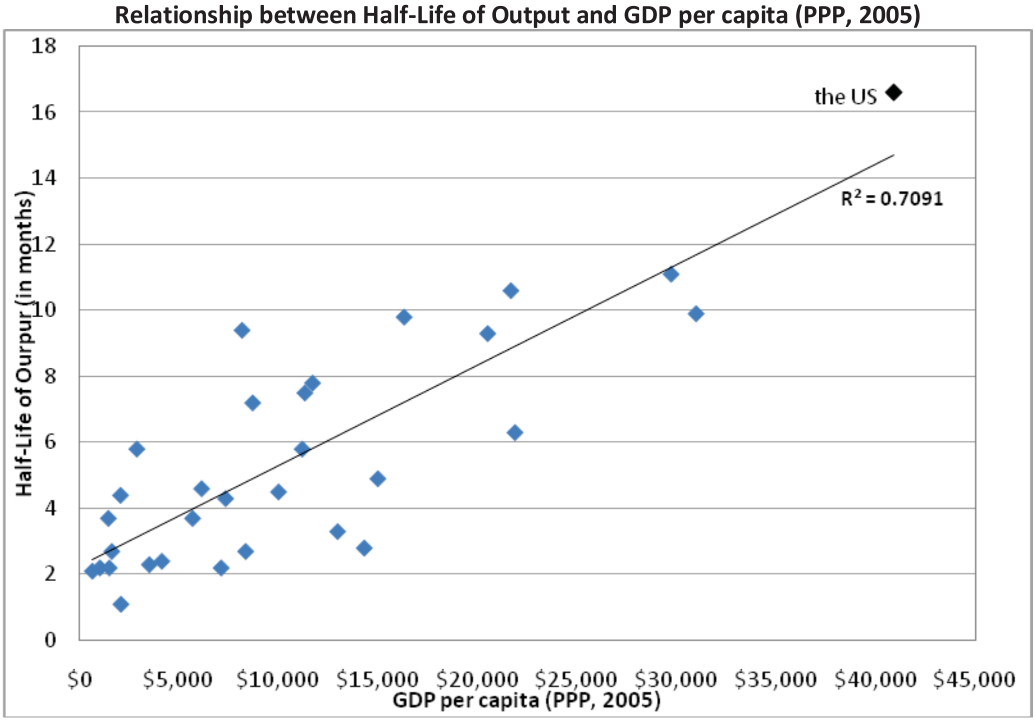
2.3. Relationship between Economic Development and Output Persistence

Economic development is measured both in terms of GDP per capita, and energy use per capita. Intuitively, energy use per capita is a good indicator of economic development. As economies develop, industrial production increases and urbanisation occurs, both of which significantly increase an economy's demand for energy. Consequently there is a close link between energy use per capita and economic growth, which is well documented in the literature (Yoo, 2006; Lee and Chang, 2007; and Zachariadis, 2007). Thus, it is employed here as an additional measure of economic development, in order to enhance the analysis.

The measure of GDP per capita is GDP per Capita, (PPP prices, constant 2005 international $) and Energy Use is Energy Use per Capita (kg of oil equivalent per capita, 2004); source World Bank World Development Indicators. Figure 2a plots the relationship between the half-life of output and GDP per capita, whilst Figure 3a plots the relationship between half-life and energy use per capita.

Figure 2a

Relationship between Half-Life of Output and GDP per capita (PPP, 2005)



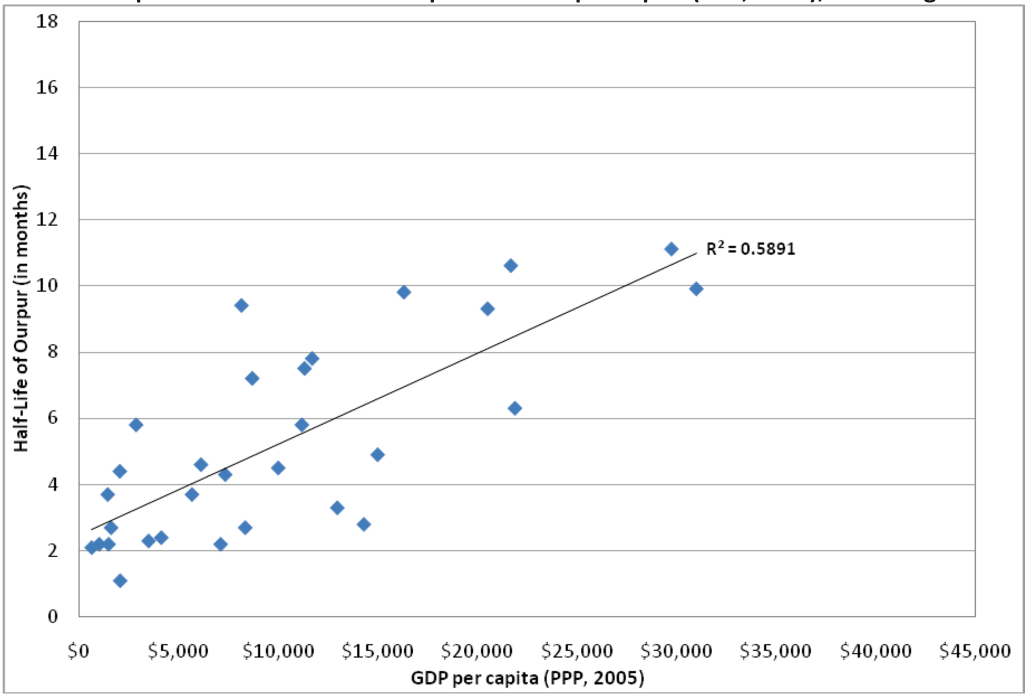
Note that each point of the graph represents an individual country's GDP per capita, for the year 2005, versus the country's estimated output half-life.

[4] The average annual inflation rate for the period 1980-2005 was below 10% in both the Philippines and South Africa.

Examination of Figure 2a reveals that, as expected, there is a positive relationship between per capita GDP and output persistence. Consequently, this can be used to convey that there is a strong positive relationship between output persistence and economic development. However, it is evident that the US displays both much higher output persistence and significantly greater GDP per Capita than the other countries in this sample. Thus, Figure 2b plots the same relationship but with the exclusion of the US, to check whether this potential outlier does not significantly influence the results.

Figure 2b

Relationship between Half-Life of Output and GDP per capita (PPP, 2005); excluding the US



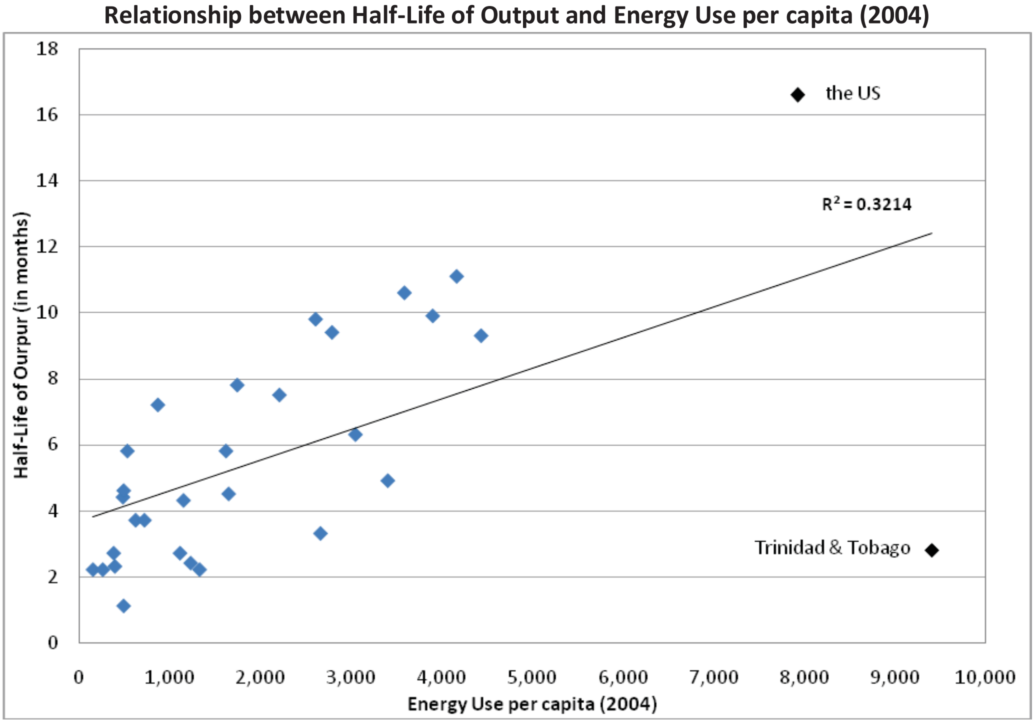
See Figure 2a for notes.

Whilst the exclusion of the US weakens the relationship slightly, with R^2 decreasing from 0.7091 to 0.5891, there is still evidence of a strong positive relationship between GDP per capita and output persistence. Thus, this preliminary analysis suggests that there is indeed a positive relationship between economic development and output persistence.

Subsequently, it necessary to see whether this relationship is also consistent with the alternative measure of economic development, energy use per capita. Figure 3a details the relationship between output persistence and energy use per capita. This figure shows that there is a positive relationship between per capita energy use and output persistence, as expected. This relationship is, however, somewhat weaker than the relationship between output per capita and output persistence. Further examination of the data points reveals two outliers in Trinidad and Tobago and the United States. Both countries have, what appear to be, excessive values for energy use per capita. However, the United States is a highly developed economy with an accordingly high half-life of output, and thus should be expected to have a high level of energy use. Conversely, whilst Trinidad and Tobago fall into the World Bank's upper middle income category, the country exhibits both relatively little output persistence (with a half-life of just 2.8 months) and excessively high energy consumption. The lack of output persistence in Trinidad and Tobago shall, for the moment, remain unexplained. However, a possible explanation for the surprisingly high per capita energy use in Trinidad and

Tobago is the fact that the economy is largely based on petroleum and natural gas production and processing (this accounts for 40% of GDP) and there is evidence that oil-rich economies have higher energy consumption.[5]

Figure 3a



Each point on the graph represents an individual country's energy use per capita, for the year 2004, versus the country's estimated output half-life.

To examine the importance of these outliers in determining the relationship between energy use and output persistence, Figure 3b plots the relationship with the exclusion of Trinidad and Tobago, whilst Figure 3c plots the relationship with the exclusion of both the United States and Trinidad and Tobago.

Excluding Trinidad and Tobago immediately reveals a much stronger positive relationship between energy use per capita and output persistence; see Figure 5.3b and the regression results in Table 2. However, the exclusion of the US once again weakens the relationship slightly. Despite this, there is still evidence of a strong positive relationship between energy use per capita and output persistence amongst the remaining twenty-nine countries.

To examine the relationship between output persistence and economic development in more detail, a simple linear regression between the half-life of output and GDP per capita (PPP, 2005) and energy use per capita (2004) is performed. To satisfy the necessary assumptions for the least squares regression, it was necessary to take logs of GDP per capta and Energy use per capita. The resulting regression equation is given by:

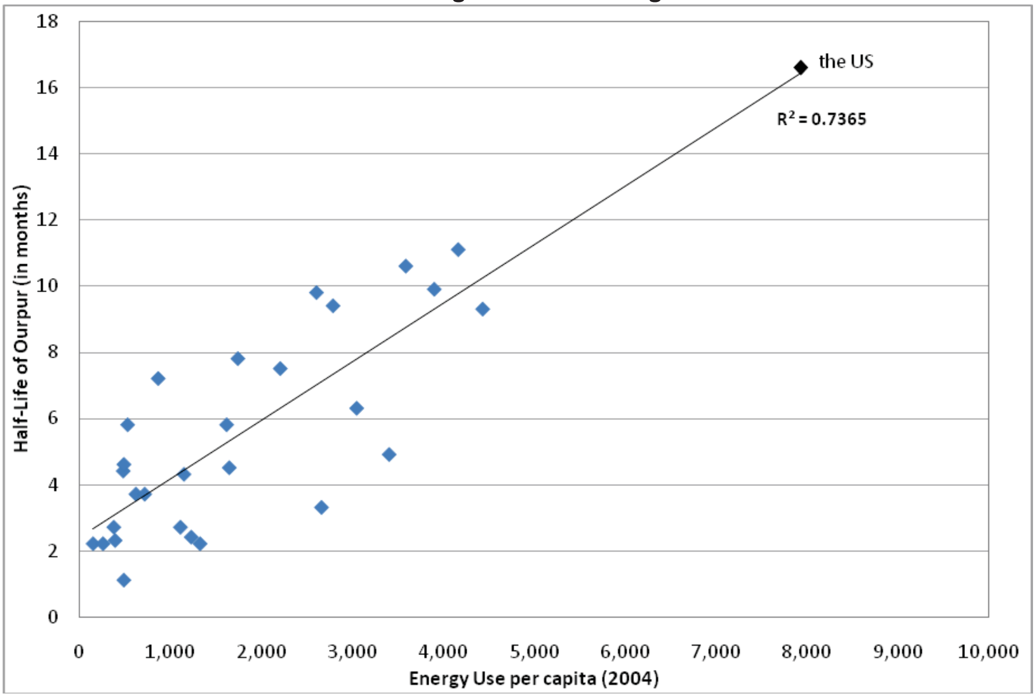
$$HL_i = \beta_0 + \beta_1 Ln(GDP_i) + \beta_2 Ln(ENERGY_i) + \varepsilon_i \qquad (0.6)$$

where, HL_i is the half-life of output, GDP_i is GDP per capita, $ENERGY_i$ is energy use per capita and $\varepsilon_i \sim iid(0,\sigma^2)$; i=1,..,n.

[5] See the article Krauss, Clifford (2007) "Oil-Rich Nations Use More Energy, Cutting Exports" The New York Times; in print December 9, 2007.

Figure 3b

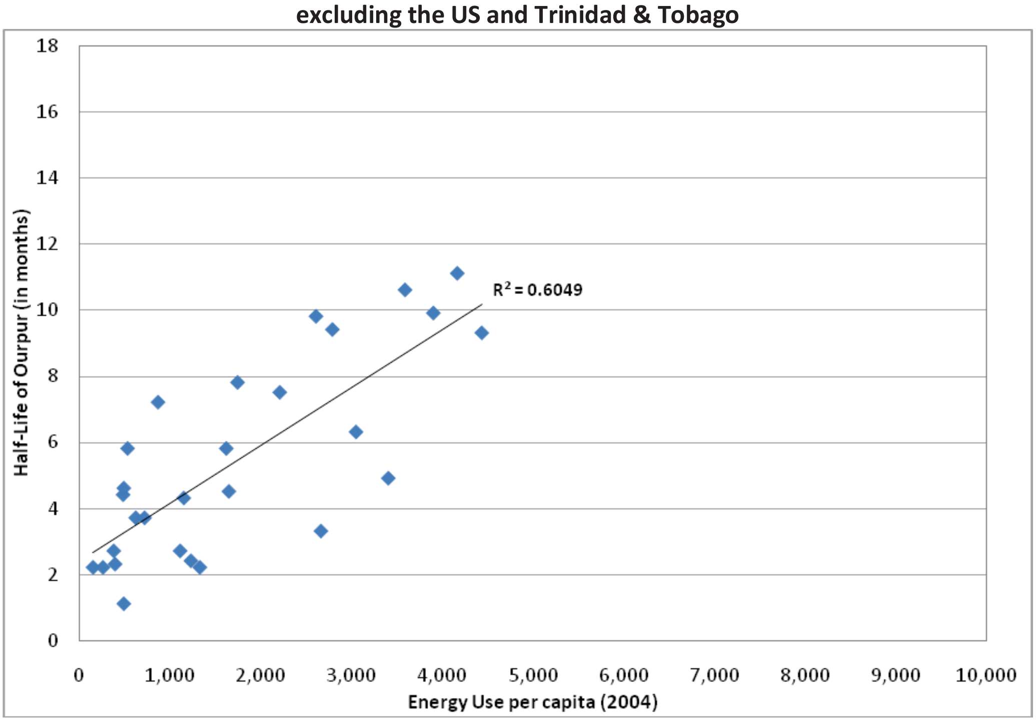
Relationship between Half-Life of Output and Energy Use per capita (2004); excluding Trinidad & Tobago



See Figure 3a for notes.

Figure 3c

Relationship between Half-Life of Output and Energy Use per capita (2004); excluding the US and Trinidad & Tobago



See Figure 3a for notes.

Table 2 details the simple linear regression results for the relationship between output persistence.

Table 2

Regression Results: Relationship between Output Persistence and Economic Development

	Model 1				Model 2				Model 3			
	A	B	C	D	A	B	C	D	A	B	C	D
Ln[GDP]	2.396**	2.008**	2.494**	2.112**					2.681*	2.477*	1.278	1.341
	(0.421)	(0.380)	(0.405)	(0.364)					(1.109)	(0.959)	(1.185)	(1.032)
Ln[Energy]					2.261**	1.806**	2.797**	2.341**	-0.111	-0.362	1.568	1.046
					(0.496)	(0.459)	(0.454)	(0.428)	(1.083)	(0.938)	(1.227)	(1.082)
Constant	-15.597**	-12.414**	-16.312**	-13.186**	-10.617**	-7.599*	-14.206**	-11.175**	-17.419**	-14.035**	-16.779**	-13.850**
	(3.749)	(3.359)	(3.600)	(3.212)	(3.618)	(3.319)	(3.278)	(3.057)	(4.367)	(3.914)	(4.047)	(3.652)
R^2	0.528	0.500	0.575	0.555	0.426	0.364	0.584	0.535	0.528	0.494	0.602	0.565
F	32.39**	27.94**	37.87**	33.61**	20.76**	15.45**	37.94**	29.96**	15.09**	12.68**	19.67**	16.22**

Significance is denoted by: * if $p<0.05$ and ** if $p<0.01$
Standard errors are reported in brackets.

A: All countries are included in the regression
B: The US is excluded from the regression
C: Trinidad & Tobago are excluded from the regression.
D: Both the US and Trinidad & Tobago are excluded from the regression.

The results for models one and two supports the graphical findings of a strong positive relationship between the half-life of output and economic development, as measured by GDP per capita (Model 1) and energy use per capita (Model 2). In models one and two, the coefficients on GDP per capita and energy use per capita, respectively, are all positive and significant. This indicates that each of these measures of economic development plays a statistically significant role in explaining the half-life of output. Thus, it can be said that output persistence is positively related to economic development; or that the more economically developed an economy the greater the persistence of output. Unfortunately, the joint effects of energy use per capita and GDP per capita on the half-life cannot be explored meaningfully due to the collinearity of GDP per Capita and energy use.[6]

This analysis has revealed that there is a strong positive relationship between economic development and output persistence. Thus, through the application of the Huang and Liu (2001) vertical production chain dynamic stochastic general equilibrium (DSGE) model, this chapter shall procede to attempt to model this relationship. The model is well suited to the task, as alteration of the number of production stages in the vertical chain will allow the representation of economies at different stages of economic development. A more economically developed economy, for example, will tend to have a more sophisticated input-output structure,[7] which can be modelled by increasing the number of production stages accordingly. Likewise, a very low income economy is likely to have a very simple input-output structure; thus, this could be modelled by introducing just two or three production stages. Furthermore, given the intuition behind the model, namely that: as the number of production stages increases, movements in the price level in response to a monetary shock decrease and

[6] The relationship between GDP per capita and Energy Use per capita yields an R^2 value of 0.893 (when all countries are included in the regression).
[7] As documented by Leontief (1963).

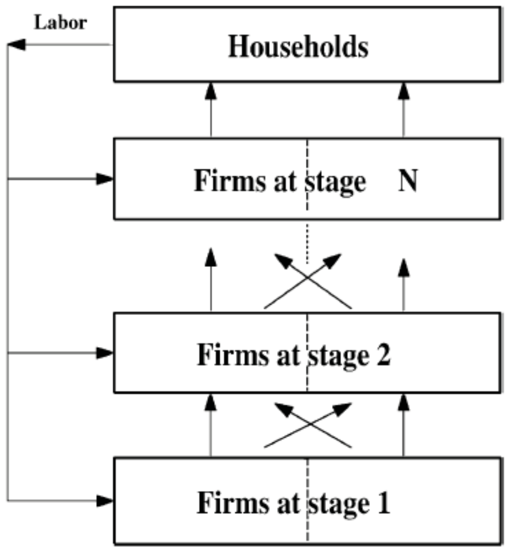
thus output fluctuations become increasingly persistent. This should be able to reproduce the observed pattern of greater output persistence in more economically developed copuntries.

3 The Model

The model follows that of Huang and Liu (2001), which features a vertical input-output structure, as detailed in Figure 4, where the production of a final consumption good requires multiple stages of processing. At each stage, there is a continuum of monopolistically competitive firms, indexed $i \in [0,1]$, producing differentiated goods and setting prices in a staggered fashion. Firms at stage 1, require only labour input from a representative household, whilst firms at stages $n \in [2,...,N]$ require labour input and goods produced at stage n-1.

In each period t, the economy experiences one of many events (monetary shocks) s_t. The history of events up to and including period t is given by $s^t = (s_{0,\dots,}s_t)$ and the probability of any particular history occurring is $\pi(s^t)$.

Figure 4 **The Input-Output Structure of the Economy**



Huang and Liu (2001, p.442; Fig.1)

3.1. The Representative Household

There is an infinitely lived representative household with preferences given by the discounted utility function:

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t \pi(s^t)\left[\ln C(s^t)+\Phi\ln\left(\frac{M(s^t)}{\bar{P}_N(s^t)}\right)-\Psi L(s^t)\right] \qquad (7)$$

where, $\beta \in [0,1]$ is the subjective discount factor, $C(s^t)$ is consumption, $M(s^t)$ is nominal money balances, $L(s^t)$ is labour hours and $\bar{P}_N(s^t)$ is a price index for goods produced at the final stage.

The consumption good, $C(s^t)$, is a Dixit and Stigilitz (1977) composite of the final-stage goods:

$$C(s^t) = \left[\int_0^1 Y_N\left(i, s^t\right)^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)} \equiv Y_N(s^t) \quad (8)$$

where, $Y_N\left(i, s^t\right)$ is a type i good produced at stage N and θ is the elasticity of substitution between these goods. $Y_N\left(s^t\right)$ can be interpreted as aggregate output corresponding to real GDP in the data.

Households choose their period t allocations of consumption, labour hours, nominal money balances and one-period bonds, $B\left(s^{t+1}\right)$, after the realisation of event s_t, in order to maximise their utility function (7) subject to a budget constraint (9) and a borrowing constraint (10):

$$\begin{aligned} &\int_0^1 P_N\left(i, s^t\right) Y_N\left(i, s^t\right) di + \sum\nolimits_{s^{t+1}} D\left(s^{t+1} \middle| s^t\right) B\left(s^{t+1}\right) + M\left(s^t\right) \\ &\leq W\left(s^t\right) L\left(s^t\right) + \Pi\left(s^t\right) + B\left(s^t\right) + M\left(s^{t-1}\right) + T\left(s^t\right) \end{aligned} \quad (9)$$

where, $P_N\left(i, s^t\right)$ is the price of a type i consumption good, $W\left(s^t\right)$ is the nominal wage rate, $\Pi\left(s^t\right)$ are nominal profits distributed to the household and $T\left(s^t\right)$ are nominal lump-sum transfers from the monetary authority.

Each of the nominal bonds $B\left(s^{t+1}\right)$ is a claim to one dollar in the next period if event s^{t+1} occurs. The bonds cost $D\left(s^{t+1} \middle| s^t\right)$ dollars at s^t. The household faces the following borrowing constraint

$$B\left(s^t\right) \geq -\overline{B} \quad (10)$$

for some large positive $\overline{B}$.

Utility maximisation yields the demand for money:

$$\phi \frac{Y_N\left(s^t\right)}{m\left(s^t\right)} = \frac{r\left(s^t\right)}{1 + r\left(s^t\right)} \quad (11)$$

where, $m\left(s^t\right)$ is real money balances and $r\left(s^t\right)$ is the real interest rate.

And the demand for a type i good produced at stage N:

$$Y_N^d\left(i, s^t\right) = \left[\frac{P_N\left(i, s^t\right)}{\overline{P}_N\left(s^t\right)} \right]^{-\theta} Y_N\left(s^t\right) \quad (12)$$

3.2. The Firms

At each stage, there is a continuum of monopolistically competitive firms, indexed $i \in [0,1]$, producing differentiated goods. Firms at stage 1, simply require labour input from a representative household, whilst firms at stages $n \in [2, ..., N]$ require labour input and a combination of goods produced at stage n-1. Firms are price-takers in their input-markets and price setters in their output markets. Assuming two-period staggered pricing, half of the firms at each stage can set new prices for their outputs in each period and this price remains effective for two periods.

Firm i at stage $n \in [1,...,N]$ that is able to set a new price at time t, will choose $P_n(i,s^t)$, after the realisation of s[t], to maximise:

$$Max \sum_{\tau=t}^{t+1} \sum_{s^t} D(s^\tau | s^t) \left[P_n(i,s^t) - V_n(i,s^\tau) \right] Y_n^D(i,s^\tau) \tag{13}$$

taking unit cost $V_n(i,s^\tau)$ and the demand schedule $Y_n^D(i,s^\tau)$ as given.

(a) Production at Stage 1

Production by firms at stage 1 requires only labour input, $L_1(i,s^t)$, from a representative household. Production has constant returns to scale, and is described by the following function:

$$Y_1(i,s^t) = L_1(i,s^t) \tag{14}$$

where, $Y_1(i,s^t)$ is the output of a stage 1 firm of type i.

Since firms at stage 1 only employ labour as an input, the unit cost is simply the nominal wage rate:

$$V_1(s^t) \equiv V_1(i,s^t) = W(s^t) \tag{15}$$

(b) Production at Stage n; $n \in [2,...,N]$

Production by firms at stage n, $n \in [2,...,N]$, requires labour input from the representative household and a composite of the goods produced at stage n-1, with production function:

$$Y_n(i,s^t) = \left[\int_0^1 Y_{n-1}(i,j,s^t)^{(\theta-1)/\theta} dj \right]^{\theta\gamma/(\theta-1)} L_n(i,s^t)^{1-\gamma} \tag{16}$$

where, $Y_n(i,s^t)$ is the output of a stage n firm of type i, $Y_{n-1}(i,j,s^t)$ is the output of a stage n-1 firm of type j supplied as an input to i, $L_n(i,s^t)$ is labour input and $\gamma \in [0,1]$ is the share of stage n-1 goods in i's production.

In this case, cost minimisation yields the following unit cost:

$$V_n(s^t) \equiv V_n(i,s^t) = \tilde{\gamma} \overline{P}_{n-1}(s^t)^\gamma W(s^t)^{1-\gamma} \tag{17}$$

where, $\tilde{\gamma} = \gamma^{-\gamma}(1-\gamma)^{\gamma-1}$ and $\overline{P}_{n-1}(s^t) \equiv \left[\int_0^1 P_{n-1}(j,s^t)^{1-\theta} dj \right]^{1/(1-\theta)}$ is a price index of goods produced at stage n-1. Assuming constant returns to scale in the production function, unit cost equals marginal cost and is firm independent.

Firms at stage $n \in [2,...,N]$ demand inputs of labour and goods produced at stage n-1. Solving the cost minimisation problem for firms at stage n+1 yields the input demand function for the intermediate goods $(n \in [1,...,N-1])$:

$$Y_n^d(i,s^t) = \left[\frac{\gamma}{1-\gamma} \right]^{1-\gamma} \left[\frac{P_n(i,s^t)}{\overline{P}_n(s^t)} \right]^{-\theta} \left[\frac{\overline{P}_n(s^t)}{W(s^t)} \right]^{\gamma-1} \tilde{Y}_{n+1}(s^t) \tag{18}$$

Finally, need to solve for the optimal pricing decision rule for firms at all stages, $n \in [1,...,N]$. Taking unit cost and the demand schedule as given, solving the profit maximisation problem (13) provides:

$$P_n(i,s^t) = \frac{\theta}{(\theta-1)} \frac{\sum_{\tau=t}^{t+1} \sum_{s^\tau} D(s^\tau | s^t) Y_n^d(i,s^\tau) V_n(s^\tau)}{\sum_{\tau=t}^{t+1} \sum_{s^\tau} D(s^\tau | s^t) Y_n^d(i,s^\tau)} \quad (19)$$

This implies optimal price is simply a constant mark-up over marginal costs.

3.3. The Monetary Authority

The nominal money supply process is given by:

$$M^S(s^t) = \mu(s^t) M^S(s^{t-1}) \quad (20)$$

where, $\mu(s^t)$ is a stochastic process.

The new money balances are distributed to the economy via lump-sum nominal transfers to the household:

$$T(s^t) = M^S(s^t) - M^S(s^{t-1}) \quad (21)$$

3.4. Equilibrium

An equilibrium for this economy, consists of allocations for the households and firms at all stages $(n \in [1,...,N])$ together with a wage rate $W(s^t)$, bond prices $D(s^{t+1} | s^t)$ and price indices $\{\bar{P}_n(s^t)\}_{n\in[1,...,N]}$ that satisfy:

i. taking prices and wages as given, the household's allocations solve the utility maximisation problem (7)
ii. taking all prices but its own and wages as constant, each firm's price solves its profit maximisation problem (13)
iii. markets for labour, money and bonds clear

3.5. Model Solution

(a) Log-Linearized Model

Following Huang and Liu (2001), the analysis focuses on a symmetric equilibrium where firms in the same cohort make identical pricing decisions. As such, firms can be identified simply by the stage at which they produce and the time at which they are able to change prices. Accordingly, $P_n(t)$ denotes prices set at time t for goods produced at stage $n \in [1,...,N]$ and the identifying i and j indices are dropped.

The equilibrium conditions are reduced to a system of 2N + 2 equations: N pricing equations, N price level equations, a labour supply equation and a money demand equation. These simplified equilibrium conditions are log-linearized around a steady-state yielding the following log-linearized equations:

i. The linearized pricing rule for firms at stage $n \in [1,...,N]$

$$0=\frac{1}{1+\beta}\left[\gamma \overline{p}_{n-1}(t)+\left(1-\gamma\right)w(t)\right]-p_N(t)$$
$$+\frac{\beta}{1+\beta}\mathrm{E}_t\left[\gamma \overline{p}_{n-1}(t+1)+\left(1-\gamma\right)w(t+1)\right] \tag{22}$$

ii. The price index for goods produced at stage $n\in\left[1,\ldots,N\right]$

$$\overline{p}_n(t)=\frac{1}{2}\left[p_n(t-1)+p_n(t)\right] \tag{23}$$

iii. The labour supply decision of the household

$$0=y_N\left(t\right)+\overline{p}_N\left(t\right)-w\left(t\right) \tag{24}$$

iv. The money demand equation

$$0=\left(1-\beta\right)m(t)-y_N(t)-\overline{p}_N(t)+\beta\mathrm{E}_t\left[\overline{p}_N(t+1)+y_N(t+1)\right] \tag{25}$$

v. The money supply equation

$$m(t)=m(t-1)+\varepsilon(t) \tag{26}$$

Lowercase letters are used to indicate log-deviations of the corresponding variable from its steady-state in the log-linearized equations.

(b) Numerical Solution of Log-Linearized System

The model is solved numerically through the application of the Uhlig (1997) toolkit, which uses the method of undetermined coefficients to solve for the recursive equilibrium law of motion. This requires the calibration of the model parameters, and the log-linearization of the necessary equilibrium conditions around the steady-state, as above. The complete log-linearized model must then be summarised in the following system of equations:

$$0=AAx\left(\mathrm{t}\right)+BBx\left(\mathrm{t}\text{-}1\right)+CCy\left(\mathrm{t}\right)+DDz\left(\mathrm{t}\right) \tag{27}$$

$$0=\mathrm{E}_t\left[FFx\left(\mathrm{t}+1\right)+GGx\left(\mathrm{t}\right)+HHx\left(\mathrm{t}\text{-}1\right)+JJy\left(\mathrm{t}+1\right)KKy\left(\mathrm{t}\right)+LLz\left(\mathrm{t}+1\right)+MMz\left(\mathrm{t}\right)\right] \tag{28}$$

$$z\left(\mathrm{t}+1\right)=NNz\left(\mathrm{t}\right)+\varepsilon\left(\mathrm{t}+1\right) \tag{29}$$

where, x(t) = [y_1(t),…, y_N(t), p_1(t),…, p_N(t)] are the endogenous state variables, y(t) = [v_1(t),…, v_N(t), l_1(t),…, l_N(t), w(t)] are the endogenous other variables and z(t) = [m(t)] is the exogenous state variable.

The Uhlig (1997) toolkit then solves for the equilibrium law of motion:

$$x\left(t\right)=PPx\left(t-1\right)+QQz\left(t\right) \tag{30}$$

$$y\left(t\right)=RRx\left(t-1\right)+SSz\left(t\right) \tag{31}$$

Details of the first-order conditions, steady-states, and log-linearizations necessary for the solution of this model, as well as definitions of the required matrices, are available from the author upon request.

4 Calibration

4.1. Parameter Calibration

The parameters for calibration are the subjective discount factor β, the monetary policy parameters ρ_μ and σ_μ, the goods demand elasticity parameter θ, the share of the composite of stage (n-1) goods in i's production γ, and finally the preference parameters Φ and Ψ, which determine the relative weight of real money balances and leisure time, respectively, in the utility function. The sources of data for the calibrations are the IMF International Financial

Statistics (IFS), the OECD Input-Output Tables and the International Labour Organization (ILO) Bureau of Statistics LABORSTA.

The developing countries for which the necessary data are available are Argentina, Brazil, Colombia, Chile, Hungary, India, Israel, South Korea, Lithuania, Malaysia, Mexico, Peru, the Philippines Slovak Republic, Slovenia, South Africa and Turkey. The calibrations were also completed for the US, UK and Japan. The calibrated parameters for each country are summarised in Table 3.

Table 3 **Calibrated Parameters**

Country	β	ρ_μ	σ_μ	Φ	Ψ	θ	γ
Argentina	0.87	0.69	0.06	0.030[a]	1.24	9.24	0.84
Brazil	0.61	0.92	0.03	0.109	1.22	17.52	0.83
Colombia	0.71	0.57	0.07	0.004	1.20	13.38[a]	0.83[a]
Chile	0.74	0.66[a]	0.05[a]	0.002	1.25[a]	13.38[a]	0.83[a]
Mexico	0.72	0.81	0.03	0.003	1.26	13.38[a]	0.83[a]
Peru	0.78	0.31	0.05	0.030[a]	1.31	13.38[a]	0.83[a]
Average	**0.74**	**0.66**	**0.05**	**0.030**	**1.25**	**13.38**	**0.83**
India	0.72	0.04	0.04	0.150[a]	1.29	10.45	0.90
Korea, South	0.86	0.25	0.07	0.012	1.40	19.75	0.89
Malaysia	0.89	0.58	0.06	0.008	1.35[a]	14.03[a]	0.84[a]
Philippines	0.79	0.42	0.05	0.010	1.36	14.03[a]	0.84[a]
Turkey	0.82[a]	0.32[a]	0.06[a]	0.557	1.33	11.90	0.73
Average	**0.82**	**0.32**	**0.06**	**0.150**	**1.35**	**14.03**	**0.84**
Hungary	0.76	0.74	0.08	0.010[a]	1.08[a]	35.00	0.90
Lithuania	0.86	0.66	0.11	0.005	1.08	32.63[a]	0.83
Slovenia	0.80	0.68	0.06	0.004	1.08	32.63[a]	0.91
Slovak Republic	0.85	0.08	0.21	0.009	1.08[a]	30.25	0.95
Average	**0.82**	**0.54**	**0.12**	**0.010**	**1.08**	**32.63**	**0.93**
Israel	0.79	0.13	0.04	0.023	1.12	50.00	0.89
South Africa	0.67	0.58	0.07	0.015	1.23	28.60	0.87
Average	**0.73**	**0.35**	**0.05**	**0.019**	**1.17**	**39.30**	**0.88**
Japan	0.91	0.53	0.05	-0.169	1.14[a]	34.50	0.93
UK	0.79	0.50[a]	0.08[a]	-0.070[a]	1.15	47.00	0.79
US	0.85	0.47	0.10	0.029	1.13	29.50	0.78
Average	**0.85**	**0.50**	**0.08**	**-0.070**	**1.14**	**37.00**	**0.84**

[a] Indicates that the regional average is used.

The Subjective Discount Factor

Using data for the quarterly money market rate (IMF IFS series 60B), the subjective discount factor (β) is calculated from the steady-state Euler equation:

$$1 = \beta\left(1 + r^*\right) \tag{32}$$

where, r* is the real interest rate.

For the US, the average real interest rate for the period 1965:3 – 2003:4 is 0.18, yielding a subjective discount factor of 0.85. Similarly, for India (1965:3 – 2003:1) the average real interest rate is 0.39 yielding β = 0.72 and for Brazil (1994:3 – 2005:2) the average real interest rate is 0.65 yielding a very low β of 0.61.

The Monetary Policy Parameters

These are calculated from a simple AR(1) process on quarterly M1 data (IMF IFS series 34):

$$\log\left(\mu_t\right) = \rho \log\left(\mu_{t-1}\right) + \varepsilon_t \tag{33}$$

where ρ_μ is the AR(1) coefficient in the money growth process and σ_μ is the standard deviation of ε_t.

For the US, the calculated values of ρ_μ and σ_μ are 0.47 and 0.101 respectively for M1 growth over the period 1965:3 – 2003:4. Similarly, for India (1965:3 – 2003:1) ρ_μ is 0.04 and σ_μ is 0.04 and for Brazil (1994:3 – 2005:2) ρ_μ is 0.92 and σ_μ is 0.03.

The Goods Demand Elasticity Parameter

The goods demand elasticity parameter θ determines the steady-state mark-up of price over marginal cost. Following Huang and Liu (1999), θ_n is set equal to θ and a value of θ is assigned such that the model implies a constant steady-state price cost margin (PCM) for each country. For the model, the PCM is defined as:

$$PCM = \frac{\bar{P}_N - \nu \bar{P}_N}{\bar{P}_N} = 1 - \nu \tag{34}$$

where, $\nu = \left((\theta - 1)/\theta\right)^N$ is steady-state unit cost. This relationship is used to determine the value of θ.

The value of the PCM is calculated using data from the OECD Input-Output Tables (2005), using the following definition:

$$PCM = \frac{\text{value added } - \text{ compensation of employees}}{\text{industry output}} \tag{35}$$

For consistency, since output is measured by either manufacturing production (IMF IFS series 66EY) or industrial production (IMF IFS series 66), all of the values calculated from the OECD Input-Output tables are calculated solely from industries contained in Major Division 3 (Manufacturing) of the International Standard Industrial Classification of all Economic Activities (ISIC-Rev.2, 1968). From the 2000 OECD input-output table for the US (currency = million US $), value added is 70134.14, compensation is 45315.77 and industry output is 199395.17; all of the preceding values are averages over all the manufacturing industries. This yields a price-cost margin of 0.13, giving a steady-state unit cost of 0.87, from which theta is calculated to be 27.5. Similarly, for India (1998 input-output table; currency = Rupees in Lakhs), value added is 1139937.62, compensation is 0 and industry output is 4503581.14 yielding a price cost margin of 0.27. Hence, steady-state unit cost for India is 0.74 and theta is 13.5. Finally for Brazil (2000 input-output table; currency = thousand Real), value added is 12347513.92, compensation is 3130746.48 and industry output is 35864106.36, yielding a price cost margin of 0.25. Hence, steady-state unit cost for Brazil is 0.75 and theta is 14.6.

The Share of Composite of Stage (n-1) Goods in i's Production, $\gamma \in (0,1)$

From the steady-state relationships,

$$\frac{1}{1-\eta} = \sum_{n-1}^{N} \frac{\bar{P}_n Y_n}{\bar{P}_N \bar{Y}_N} = \frac{1-(\gamma/\mu)^N}{1-(\gamma/\mu)} \tag{36}$$

where, N is the number of processing stages, γ is the share of composite of stage (n-1) goods in i's production, η is the share of intermediate goods in total manufacturing and $\mu = \theta/(\theta - 1)$ is the steady-state mark-up of price over marginal cost.

The value of the steady-state mark-up of price over marginal cost is determined by the value of θ. The value of η is calculated using the OECD Input-Output Tables (2005) and is defined as:

$$\eta = \frac{\text{industry output } - \text{ value added}}{\text{industry output}} \qquad (37)$$

For the US, the share of intermediate goods in total manufacturing is 0.637, as calculated from the 2000 OECD input-output table. The steady-state mark-up of price over marginal cost is 1.035, given θ = 29.5. These yield γ = 0.787. Similarly for India, the share of intermediate goods in total manufacturing is 0.735, the steady-state mark-up of price over marginal cost is 1.082 (given θ = 14.6), yielding γ = 0.92. Finally for Brazil, the share of intermediate goods in total manufacturing is 0.648, the steady-state mark-up of price over marginal cost is 1.074 (given θ = 14.6), yielding γ = 0.831.

The Relative Weight of Real Money Balances

This is calculated using the implied steady-state money demand equation

$$\Phi = \frac{M*}{\bar{P}_N * C*}\left(\frac{R*-1}{R*}\right) \qquad (38)$$

where, R* is the steady-state nominal interest rate and $\bar{P}_N * C* / M*$ is the steady-state consumption velocity.

Consumption velocity is the ratio of consumption expenditures to real money balances and is calculated here using M1 (IMF IFS series 34), Private Consumption (IMF IFS series 96F) and CPI (IMF IFS series 64). For the US (1965:3 – 2003:4), average consumption velocity is 0.07 and average nominal money market rate is 1.73, giving a real money balances parameter (Φ) of 0.029. Similarly for India (1965:3 – 2003:1), average consumption velocity is 0.07 and average nominal money market rate is 2.025, giving a real money balances parameter (Φ) of 0.033 and for Brazil (1994:3 – 2005:2), average consumption velocity is 0.14 and average nominal money market rate is 4.175, giving a real money balances parameter (Φ) of 0.109.

The Relative Weight of Leisure Time

This is derived from annual data for the hours of work in manufacturing (per week) (ILO LABORSTA series 4B). For the US, the average time devoted to market activity for the period 1970 to 2005 is 40.7 hours per week or ¼; for the model to predict an average share of time allocated to market activity of ¼ then requires Ψ equal to 1.13. Similarly for India, the average time devoted to market activity for the period 1982 to 2004 is 46.4 hours per week or $^2/_7$ requiring Ψ equal to 1.29. Finally, the average time devoted to market activity in Brazil (2000 to 2004) is 43.8 hours per week or ¼ which requires Ψ to equal to 1.22. It is interesting to note that most business cycle models assume the average share of time devoted to market activity is $^1/_3$ which then implies a Ψ of 1.56.

4.2. Number of Stages (N)

The relationship between economic development and the sophistication of an economy's input-output structure is well documented in the literature. In particular, the seminal work of Leontief (1963) demonstrated that the larger and more developed an economy, the more complete is its economic structure. Consequently, it is assumed that the more developed an economy, as measured here as measured by real GDP per capita and energy use per capita, the more sophisticated the input-out structure and thus the greater the number of production stages. It has not been possible to estimate the complexity of each economy's input-output structure and consequently calibrate the number of stages. Therefore, instead, the sophistication of the input-output structure is estimated by the country's relative level of economic development. Consequently, countries are ranked according to a weighted average of real GDP per capita and energy use per capita (2004 values) and grouped with countries of similar weighted averages. Each of these groups is then assigned an N value corresponding to the development ranking. The rankings and N values are shown in Table 4.

Table 4 **Number of Stages (N)**

Country	GDP per Capita Ranking (2003)	Energy Use & GDP	Half Life (in months)	Group	N
Bangladesh	176	6.0	2.2	1	2
Senegal	192	6.4	2.2	1	2
Côte d'Ivoire	196	6.7	2.7	2	3
India	152	6.9	4.4	2	3
Pakistan	170	6.9	1.1	2	3
Nigeria	211	6.9	3.7	2	3
Morocco	143	7.1	2.3	3	5
Philippines	133	7.1	5.8	3	5
Peru	122	7.5	7.4	3	5
Colombia	110	7.5	3.7	3	5
Jordan	139	7.7	2.4	4	8
Uruguay	64	7.9	7.2	4	8
Turkey	102	8.0	4.3	4	8
Brazil	93	8.0	2.7	4	8
Macedonia	105	8.0	2.2	4	8
Argentina	70	8.3	4.5	5	13
Mexico	86	8.4	5.8	5	13
Chile	81	8.4	7.8	5	13
South Africa	78	8.5	9.4	5	13
Malaysia	84	8.5	7.5	5	13
Lithuania	69	8.7	3.3	6	21
Hungary	62	8.8	9.8	6	21
Slovak Republic	65	8.9	4.9	6	21
Israel	44	9.0	6.3	6	21
Slovenia	49	9.1	10.6	7	34
Korea, South	51	9.2	9.3	7	34
UK	20	9.3	9.9	7	34
Japan	21	9.3	11.1	7	34
US	5	9.8	16.6	8	55

As the model demonstrates diminishing returns, in terms of output persistence, for each additional production stage, the N values are assigned according to a Fibonacci sequence (1,1,2,3,5,8,13,21,34,55,…). The least developed countries (namely Bangladesh and Senegal) are assigned a value of N = 2, whilst the most developed country (namely the US) is assigned a value of N = 55.

5 Sensitivity Analysis

The central premise of the model is that through the input-output relations across stages and the staggered prices within stages, it is possible to generate persistent output fluctuations in response to monetary policy shocks. In theory, as the number of stages increase, movements in the price level should decrease, and fluctuations in aggregate output should become increasingly persistent. Thus, this section examines the importance of the number of stages, N, in generating output persistence and the sensitivity of the results to the calibrated parameter values.

From the system of log-linearized equations, equations (21) to (25), it is evident that the key parameters in determining the extent of output persistence are the share of the composite of stage (n-1) goods in i's production (γ) and the subjective discount factor (β). The effect of changing these parameters is examined for three representative countries, Brazil, India and the US.

The magnitude of persistence is measured using both the half-life of output, as defined in section 2.1, and the contract multiplier. The contract multiplier, as proposed by Chari *et al.* (2000), measures the degree to which the real effect of the monetary policy shock extends beyond the initial contract duration; the higher the ratio, the more persistent the response of output to the monetary shock. With two cohorts of price setters, as in this model, the contract multiplier is defined as the ratio of the output response after 6 months to that at time zero.

In what follows, with the exception of the parameter of interest, all the parameters are as calibrated for the particular country.

5.1. The subjective discount factor (β)

This compares the output persistence generated by the model, in response to a one-percent monetary shock, when β is at its minimum calibrated value (0.61 Brazil), when β is at its maximum calibrated value (0.91 Japan), when β takes on the extreme values of 0.5 and 0.99, and when β is at its actual calibrated value for the country. The impulse response functions are presented in Figure 5 and the peak responses, contract multipliers and half-lives are reported in Table 5.

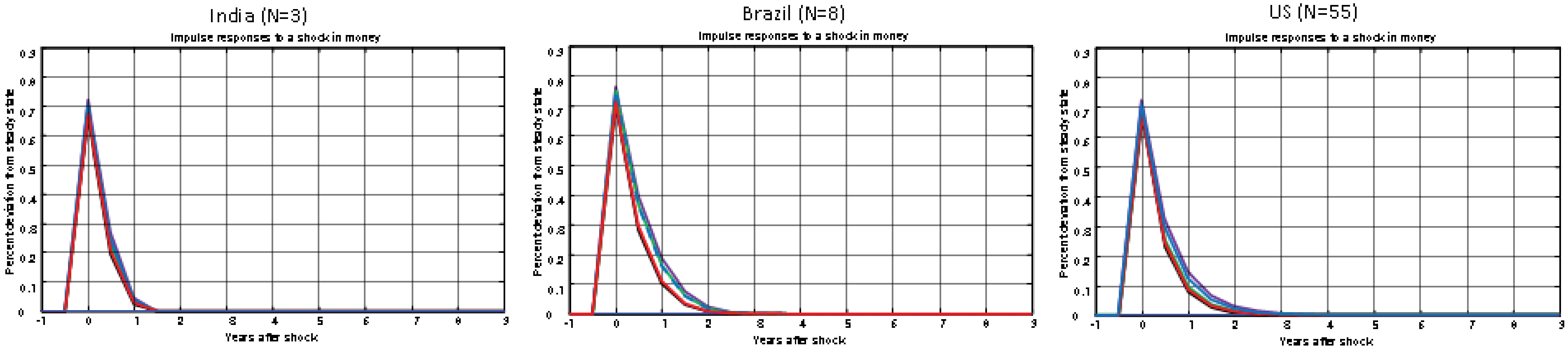
Figure 5 demonstrates that the smaller the value of β, the greater is the degree of output persistence generated by the model. This effect is limited, however it does appear to be magnified slightly when the number of stages of production (N) is larger; for example the impulse response functions are slightly more spread out for the US and Brazil than for India. However, examination of Table 5 reveals that the impact of a change in β for a change in the half-life is limited; all three countries show less than a one month increase in the half-life when β decreases from it maximum calibrated value of 0.91 to its minimum calibrated value of 0.61.

5.2. The share of the composite of stage (n-1) goods in i's production (γ)

This compares the output persistence generated by the model when γ is at its minimum calibrated value (0.73 Turkey), when γ is at its maximum calibrated value (0.95 Slovak Republic), when γ takes on the extreme values of 0.5 and 0.99, and when γ is at its actual calibrated value for the country. The impulse response functions are presented in Figure 6 and the peak responses, contract multipliers and half-lives are reported in Table 6.

From Figure 6, it is clear that the larger the value of γ, the greater is the degree of output persistence generated by the model and that this effect is magnified as the number of stages (N) increases. The spread between the impulse response functions is clearly greater for the US, with N=55, than for either India (N=3) or Brazil (N=8).

Figure 5 **Sensitivity of the Impulse Response of Output to β**

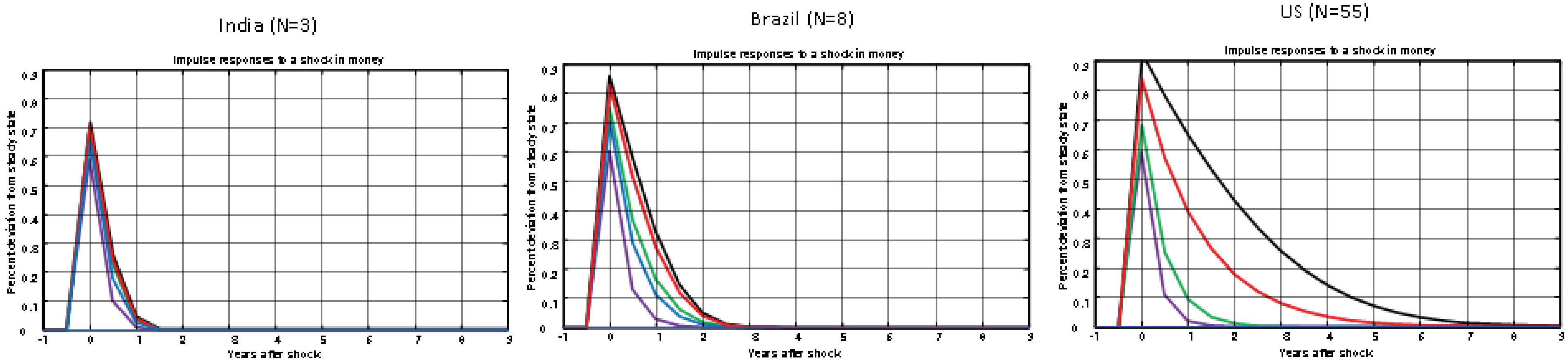


Key: β (as calibrated), β = 0.5, β = 0.61, β = 0.91, β = 0.99

Table 5 **Sensitivity of the Contract Multiplier and Half-Life of Output to β**

	India (N=3)					Brazil (N=8)					US (N=55)				
	Calibrated β = 0.72	β = 0.5	β = 0.61	β = 0.91	β = 0.99	Calibrated β = 0.61	β = 0.5	β = 0.61	β = 0.91	β = 0.99	Calibrated β = 0.85	β = 0.5	β = 0.61	β = 0.91	β = 0.99
t = 0	0.690	0.724	0.714	0.676	0.669	0.752	0.768	0.752	0.714	0.705	0.695	0.725	0.711	0.679	0.672
1	0.224	0.271	0.249	0.205	0.196	0.368	0.401	0.368	0.297	0.282	0.271	0.326	0.300	0.243	0.231
2	0.034	0.047	0.041	0.029	0.027	0.163	0.190	0.163	0.112	0.102	0.106	0.147	0.127	0.087	0.080
3	0.000	0.000	0.000	0.000	0.000	0.062	0.077	0.062	0.036	0.031	0.041	0.066	0.054	0.031	0.027
Contract Multiplier (Y_1 /Y_0)	0.34	**0.37**	0.35	0.30	0.29	0.49	**0.52**	0.49	0.42	0.40	0.39	**0.45**	0.42	0.36	0.34
Half-Life (in months)	4.1	**4.8**	4.7	3.9	3.8	5.9	**6.6**	5.9	5.1	5.0	4.7	**5.7**	5.4	4.7	4.5

Figure 6 **Sensitivity of the Impulse Response of Output to γ**



Key: — γ (as calibrated), — γ = 0.5, — γ = 0.76, — γ = 0.95, — γ = 0.99

Table 6 **Sensitivity of the Contract Multiplier and Half-Life of Output to γ**

	India (N=3)					Brazil (N=8)					US (N=55)				
	Calibrated γ = 0.90	γ = 0.5	γ = 0.73	γ = 0.95	γ = 0.99	Calibrated γ = 0.83	γ = 0.5	γ = 0.73	γ = 0.95	γ = 0.99	Calibrated γ = 0.79	γ = 0.5	γ = 0.73	γ = 0.95	γ = 0.99
t = 0	0.690	0.591	0.652	0.704	0.715	0.752	0.606	0.707	0.829	0.863	0.695	0.593	0.685	0.841	0.924
1	0.224	0.101	0.177	0.242	0.257	0.368	0.129	0.289	0.516	0.584	0.271	0.110	0.253	0.573	0.778
2	0.034	0.011	0.024	0.038	0.041	0.163	0.027	0.110	0.273	0.327	0.106	0.020	0.094	0.390	0.646
3	0.000	0.000	0.000	0.000	0.000	0.062	0.006	0.037	0.117	0.146	0.041	0.004	0.035	0.264	0.527
Contract Multiplier (Y_1 /Y_0)	0.32	0.17	0.27	0.34	**0.36**	0.49	0.21	0.41	0.62	**0.68**	0.39	0.19	0.37	0.68	**0.84**
Half-Life (in months)	4.4	3.6	4.2	4.5	**4.8**	5.8	3.9	5.1	8.7	**9.6**	4.8	3.6	4.8	10.8	**21.9**

Looking at Table 6 the importance of the value of γ on output, and the magnification effect, is clearly demonstrated in the values of the half-lives and the contract multipliers. For the US, the difference is substantial; with the lowest value of γ (0.5) the model generates a half-life of just 3.6 months whilst when γ takes on the largest value of 0.99 the half-life increases to almost 2 years (21.9 months). In the previous analysis, the half-life of output for the US was reveal to be 16.6 months; thus, the model is clearly capable of generating enough persistence to match the data so long as the calibrated value of γ for the economy is large enough.

5.2. The share of the composite of stage (n-1) goods in i's production (γ)

This compares the output persistence generated by the model when γ is at its minimum calibrated value (0.73 Turkey), when γ is at its maximum calibrated value (0.95 Slovak Republic), when γ takes on the extreme values of 0.5 and 0.99, and when γ is at its actual calibrated value for the country. The impulse response functions are presented in Figure 6 and the peak responses, contract multipliers and half-lives are reported in Table 6.

From Figure 6, it is clear that the larger the value of γ, the greater is the degree of output persistence generated by the model and that this effect is magnified as the number of stages (N) increases. The spread between the impulse response functions is clearly greater for the US, with N=55, than for either India (N=3) or Brazil (N=8).

Looking at Table 6 the importance of the value of γ on output, and the magnification effect, is clearly demonstrated in the values of the half-lives and the contract multipliers. For the US, the difference is substantial; with the lowest value of γ (0.5) the model generates a half-life of just 3.6 months whilst when γ takes on the largest value of 0.99 the half-life increases to almost 2 years (21.9 months). In the previous analysis, the half-life of output for the US was reveal to be 16.6 months; thus, the model is clearly capable of generating enough persistence to match the data so long as the calibrated value of γ for the economy is large enough.

5.3. The number of stages (N)

This examines how the persistence of output changes as the number of stages (N) increases, N = {2, 3, 5, 8, 13, 21, 34, 55}, and how this is affected by changing the values of β and γ both individually and simultaneously. The impulse response functions are presented in Figure 7 and the contract multipliers and half-lives are reported in Tables 7 and 8, respectively.

As expected, as the number of stages increases, the degree of output persistence generated by the model also increases. However, this is dependent on the values of γ and β as these limit the degree of persistence generated by the model; looking at Figure 7a it is possible to see that given the calibrated parameters for the US, the model cannot generate any additional persistence beyond N=8. Therefore, the effect of increasing the number of stages is severely limited and it is clearly not enough to simply increase the value of N in order to generate increased persistence.

Figure 7b shows the impulse responses functions as N increases when the minimum calibrated value of β (0.61) is applied, instead of the calibrated value for the US. From this, it is clear that the reduction in the value of β has only a very small impact on the degree of output persistence and that increasing the number of stages only has an effect up to N=13; further increases in N make no difference to the impulse responses of output.

Figure 7c shows the impulse responses functions as N increases when the maximum calibrated value of γ (0.95) is applied, instead of the calibrated value for the US. In this case, it is clear that γ is highly significant for the degree of output persistence and increasing the value of N has a significant effect, which is not limited. This has important implications for the model. In particular, as discussed in Huang and Liu (2001), as γ→1 the persistence of output becomes infinite. Thus, money would have a permanent real effect on output.

Figure 7 Impulse Response Functions for the US: Impact of Changing β, γ and N

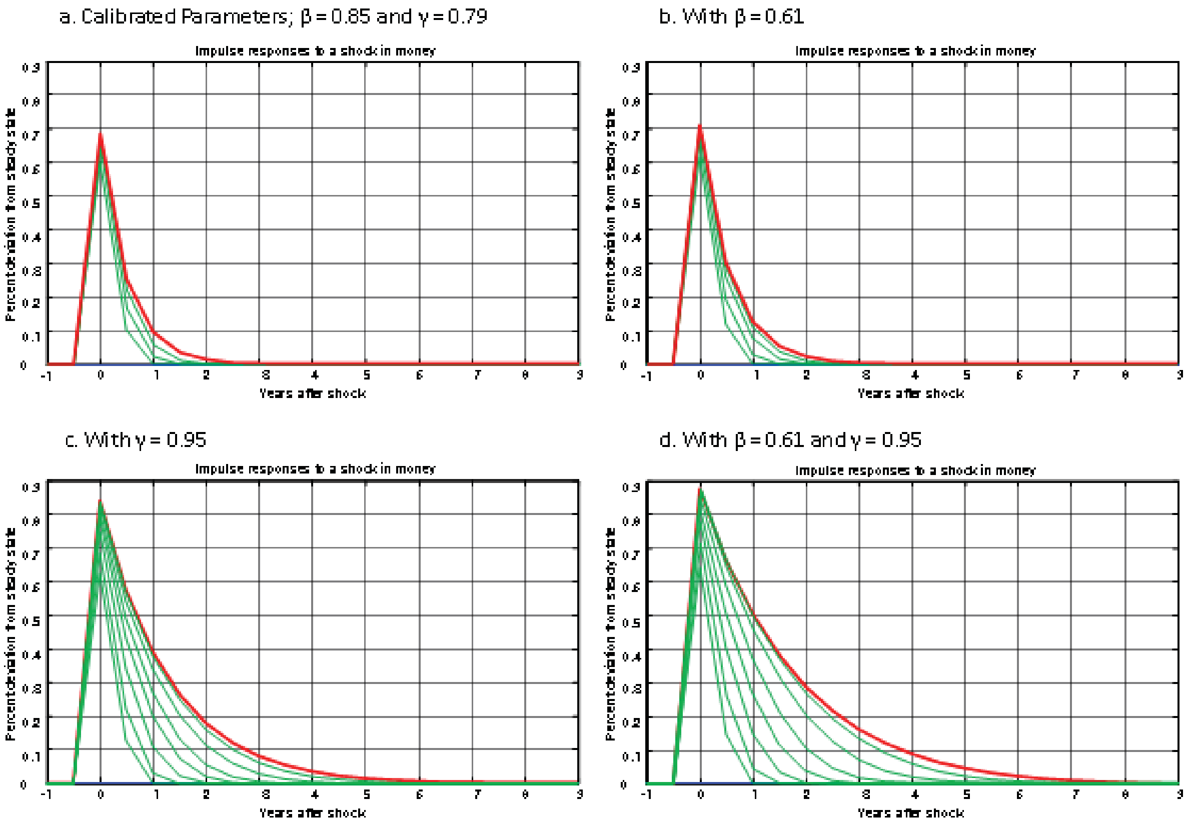


Table 7 Contract Multiplier (Y_1 /Y_0)

N =	2	3	5	8	13	21	34	55
Calibrated Parameters	0.17	0.25	0.33	0.37	0.39	0.39	0.39	**0.39**
With β = 0.61	0.19	0.29	0.37	0.41	0.42	0.42	0.42	**0.42**
With γ = 0.95	0.20	0.32	0.45	0.55	0.61	0.65	0.67	**0.68**
With β = 0.61 and γ = 0.95	0.23	0.36	0.51	0.62	0.69	0.73	0.75	**0.76**

Table 8 Half-Life (in months)

N =	2	3	5	8	13	21	34	55
Calibrated Parameters	3.6	4.1	4.3	4.7	4.8	4.8	4.8	**4.8**
With β = 0.61	3.6	4.2	4.8	5.2	5.4	5.5	5.5	**5.5**
With γ = 0.95	3.6	4.3	5.5	7.2	8.4	9.8	10.6	**10.9**
With β = 0.61 and γ = 0.95	3.9	4.8	6.2	8.4	10.3	12.7	14.4	**15.4**

Figure 7d shows the impulse response functions as N increases, when the maximum calibrated value of γ (0.95) and the minimum calibrated value of β (0.61) are simultaneously applied. It is obvious from this, that simultaneously lowering β and raising γ reinforces the individual effects and significantly increases the output persistence generated by the model. In this case the half-life of output increases to 15.4 months which is just short of that observed in the US economy (16.6 months). Thus, the model is clearly capable of generating empirically plausible output persistence values.

6 Empirical Results

This section presents results from simulating the model for the calibrated countries, and evaluates how successful the model is in (i) reproducing the observed half-lives for the calibrated countries and (ii) capturing the patterns of output persistence observed in section 2; namely the positive relationship between economic development and output persistence.

6.1. Impulse Response Functions and Estimated Half-Lives

To compute the impulse responses, the value of the innovation term (ε_t) in the money growth process (26) at time zero is set equal to one ($\varepsilon_0 = 1$), so that the money stock rises by one-percent one year after the shock. For all $t \geq 1$, the innovation term is set equal to zero. The impulse response functions for all the calibrated countries are provided in Appendix A. The magnitude of persistence for each of these countries is measured using both the half-life of output and the contract multiplier; these values are detailed in Table 9.

Table 9 **Estimated Half-Lives and Contract Multipliers**

Country	N	Half-Life: model (months)	Contract Multiplier (Y_1/Y_0)
India	3	4.4	0.32
Colombia	5	5.2	0.42
Peru	5	5.0	0.40
Philippines	5	5.2	0.42
Brazil	8	5.8	0.49
Turkey	8	4.4	0.34
Argentina	13	5.4	0.45
Chile	13	6.0	0.50
Malaysia	13	7.2	0.44
Mexico	13	6.1	0.51
South Africa	13	6.7	0.54
Hungary	21	7.6	0.58
Israel	21	7.2	0.55
Lithuania	21	5.9	0.49
Slovak Republic	21	6.0	0.50
Japan	34	8.9	0.61
Korea, South	34	7.0	0.54
Slovenia	34	9.6	0.65
UK	34	5.2	0.41
US	55	4.8	0.39

The central premise of Huang and Liu (2001) is that the greater the number of production stages (N), the more persistent the response of output. However, initial examination of the impulse response functions and half-life estimates suggests that, for these countries, this relationship is weak at best. Figure 8(a) plots the relationship between N and the half-life (in months).

Examination reveals that there are two notable exceptions to such a positive trend, namely the US and the UK; both of these countries are highly economically developed and consequently have high N values, and yet the model generates very little output persistence. Exclusion of the US and UK from the analysis yields a significant strong positive relationship between N and the half-life (in months), as conjectured in Huang and Liu (2001); this is shown in Figure 8(b).

To explain this lack of persistence, it is necessary to turn to the sensitivity analysis of section 5; this revealed that the most important parameter in determining the magnitude of output persistence is the share of the composite of stage n-1 goods in i's production (γ). Thus, the diminutive half-lives can be explained to some extent by the calibration from the data of extremely low gamma values; 0.78 and 0.79 respectively. These values effectively inhibit the model from generating any significant degree of output persistence for either the US or the UK.

In order to further examine the relationship between N and the degree of output persistence generated by the model, the consistency of the gamma values are investigated in light of the associated literature. Basu and Fernald (1997) estimate the average steady state mark-up of price over marginal cost (μ) for US industries to be 1.08, whilst Brandt (2007) estimates the mark-up for a number of OECD countries, from which the average for US industries is 1.23. Combining these mark-up values with the values for the share of intermediate goods in total manufacturing (η) and the steady-state unit cost (υ) for US manufacturing industries as calibrated in section 3, produces a value of γ between 0.816 (when μ=1.08) and 0.929 (1.23); both of which are significantly higher than the values of γ calibrated directly from the input-output tables. Consequently, the average of these two values, namely γ = 0.87, is taken and the simulations are repeated for both the US and UK. As expected, the higher gamma values enable the model to generate a much greater half-life of output for both of these countries; for the UK the half-life of output increases from 5.2 months to 6.6 months, whilst for the US the half-life increases from 4.8 months to 6.4 months.

Further examination of the relationship between N and the half-life (in months) with the new half-life values for the US and UK, yields a considerably stronger positive relationship; this is shown in Figure 8(c). However, the degree of persistence generated for the US still remains an outlier; this is because, given the value of gamma (γ = 0.87), the model is unable to generate any additional persistence beyond N = 34 (the half-life when the model is run with 34 stages and the half-life when the model is run with all 55 stages are identical).

Figure 8(a)

Relationship between Half-Life and Number of Stages; all countries

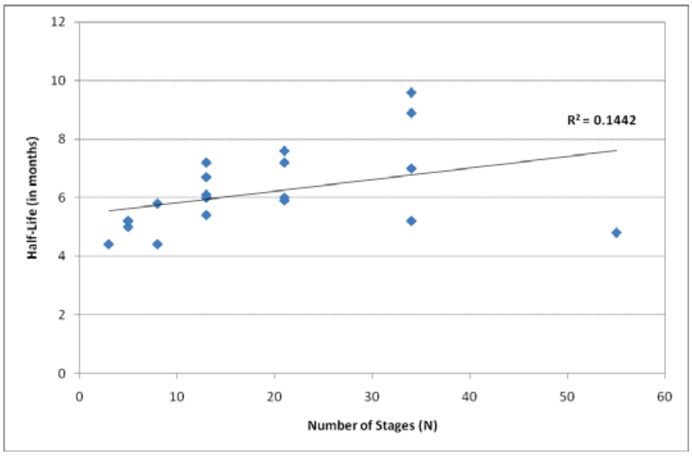


Figure 8 (b)

Relationship between Half-Life and Number of Stages; excluding UK and US

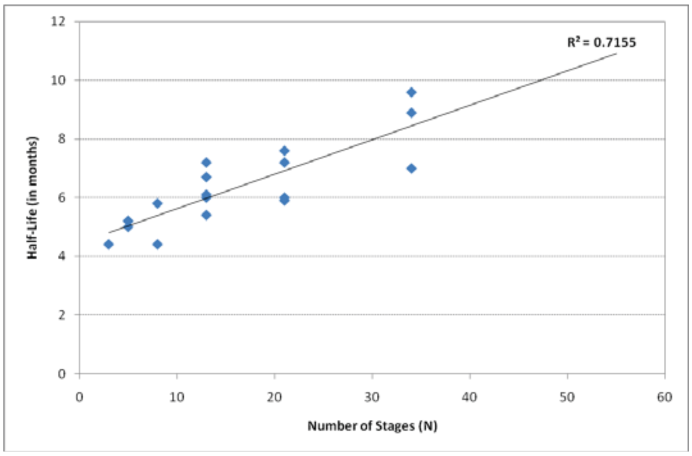
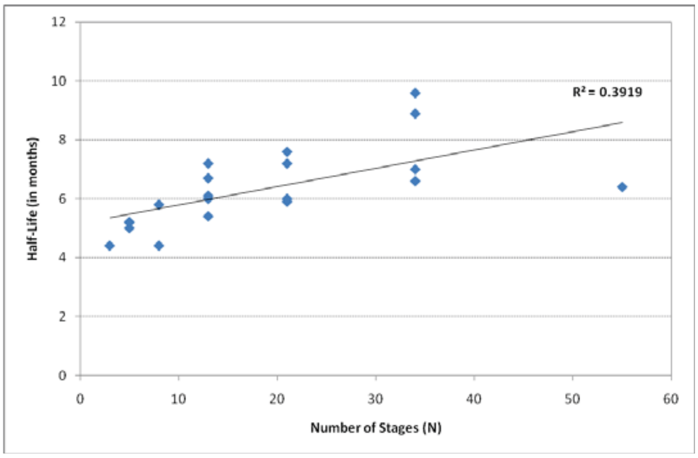


Figure 8 (c)

Relationship between Half-Life and Number of Stages; including UK and US with γ=0.87



6.2. Relationship to Actual Output Persistence

Having established that there is a positive relationship between the number of production stages (N) and the magnitude of output persistence, the next step is to examine how successful the model is in reproducing the observed half-lives for the calibrated countries. Table 10 details the half-life of output (as estimated from the actual data), the half-life of output (as estimated from the impulse response function), and the difference between the two half-lives.

Table 10 Relationship between Model and Actual Output Persistence

Country	N	Half-Life: Data (months)	Half-Life: Model (months)	Difference
US	55	16.6	4.8	11.8
Japan	34	9.9	8.9	1.0
UK	34	11.1	5.2	5.9
Argentina	13	4.5	5.4	-0.9
Brazil	8	2.7	5.8	-3.1
Chile	13	7.8	6.0	1.8
Colombia	5	3.7	5.2	-1.5
Hungary	21	9.8	7.6	2.2
India	3	4.4	4.4	0.0
Israel	21	6.3	7.2	-0.9
Korea, South	34	9.3	7.0	2.3
Lithuania	21	3.3	5.9	-2.6
Malaysia	13	7.5	7.2	0.3
Mexico	13	5.8	6.1	-0.3
Peru	5	4.6	5.0	-0.4
Philippines	5	5.8	5.2	0.6
Slovak Republic	21	4.9	6.0	-1.1
Slovenia	34	10.6	9.6	1.0
South Africa	13	9.4	6.7	2.7
Turkey	8	4.3	4.4	-0.1

Table 10 reveals that, in most cases, the model generates a half-life which is reasonably close to the half-life of the actual data; in fact, for 90% of the countries, the estimated half-life is within one quarter of the actual half-life. The two exceptions, as discussed in the previous section, are the US and the UK. However, at first glance, there is no clear pattern as to whether the model over- or underestimates the degree of output persistence.

Section 2 revealed that there is a strong positive relationship between level of economic development, as measured by GDP per capita and energy use per capita, and the persistence of output fluctuations. This is consistent with the positive relationship between the number of stages of production (N) and the estimated half-life for the model. Furthermore, there is another salient feature of countries' output persistence, namely that output fluctuations are less persistent in high inflation countries; this is notably documented by Kiley (2000). This characteristic provides another angle to examine the relationship between a country's actual output persistent and the model's estimated output persistence. As the model does not account for inflation, it is feasible that the magnitude of output persistence for countries with low inflation rates may be underestimated, whilst the output persistence of high inflation countries may be overestimated.

Table 11, details the relationship between inflation and the difference between real and model half-life. Inflation data is taken from the World Bank World Development Indicators, (series: inflation, consumer prices (annual %)), from which the average annual inflation rate over the period 1980 to 2005 is calculated for each country. Average annual inflation rate is classified as being high when the annual rate exceeds 15% and as low when it below 15%; such a classification is consistent with the literature, for example Kakkar and Ogaki (2002) rank inflation as high when it is greater than 10%, medium between 5 and 10% and low when it is below 5%, whilst Gagnon (2009) classifies high inflation as anything above 10-15% and low inflation as anything below 10-15%.

Table 11

Relationship between Inflation and the Difference between Real and Model Half-Life

Country	**Average Annual Inflation Rate (%)** (1980 – 2005)	Inflation Ranking	**Difference** (Actual Half-Life *minus* Model Half-Life)
Brazil	432.66	HIGH	-3.1
Lithuania	38.41	HIGH	-2.6
Colombia	18.15	HIGH	-1.5
Slovak Republic	7.23	LOW	-1.1
Argentina	294.90	HIGH	-0.9
Israel	50.76	HIGH	-0.9
Peru	461.05	HIGH	-0.4
Mexico	33.44	HIGH	-0.3
Turkey	53.74	HIGH	-0.1
India	7.97	LOW	0
Malaysia	3.18	LOW	0.3
Philippines	9.96	LOW	0.6
Japan	1.24	LOW	1.0
Slovenia	9.47	LOW	1.0
Chile	12.71	LOW	1.8
Hungary	12.95	LOW	2.2
Korea, South	5.90	LOW	2.3
South Africa	10.27	LOW	2.7
United Kingdom	4.78	LOW	5.9
United States	3.85	LOW	11.8

Looking at Table 11, there is a very clear relationship between inflation and the difference between a country's actual half-life and the model's estimated half-life: where the model overestimates a country's half-life, the country has high inflation, whilst where the model underestimates a country's half-life, the country has low inflation. There are just one exception to this; the Slovak Republic, which has low inflation and yet the model overestimates the degree of output persistence. Referring to the calibrations, it is evident that Slovak Republic has an exceedingly high gamma value which, given the model's sensitivity to the value of gamma, may explain the overestimation of output persistence.

Table 11 also reveals that three of the Latin American economies, Argentina, Brazil and Peru, had extremely high average annual inflation rates over the sampling period.

Examining this in the case of Peru, it is evident that the country suffered from very high inflation between 1980 and 1993, reaching a peak of almost 3400% in 1989, whilst from 1994 onwards, the inflation rate was low; average annual inflation rate for 1993 to 2005 was just 5.9%. Thus, this provides an opportunity to further investigate the conjecture that the model overestimates the magnitude of output persistence in high inflation countries whilst underestimating the magnitude of output persistence for low inflation countries. Calculating the half-life of output for Peru for the low inflation period (1994:1 – 2005:1) and comparing this to the model half-life, yields a difference of +0.5[8], whilst for the high inflation period (1980:1 – 1993:4) the difference between model and actual half-life is -0.6[9]. Thus, these results further corroborate the relationship between inflation and the difference between real and model half-life.

Econometric analysis of this relationship was carried out using the least squares dummy variable (LSDV) method. Two dummy variables were created, high and low;

$High_i$ = 1 if the country i's average inflation rate exceeds 15%
= 0 otherwise

Low_i = 1 if the country i's average inflation rate is below 15%
= 0 otherwise

Following the previous discussion, Peru was considered as a low inflation country and the difference between actual half-life for the period 1994:1 – 2005:1 and model half-life was used. The regression is run in STATA using the LSDV1 method, which drops one of the dummy variables, Low_i in this case, to ensure that the model is identified. This method ensures that the R^2 and F statistics obtained from the regression are correct. Table 12 details the regression results and Figure 9 represents this relationship graphically.

Table 12 **Regression Results: Relationship between Difference and Inflation**

	A	B	C	D
HIGH	-3.574* (1.315)	-3.851** (1.313)	-2.370** (0.550)	-2.583** (0.504)
Constant	2.231** (0.778)	2.508** (0.797)	1.027** (0.343)	1.240** (0.324)

R^2	0.291	0.336	0.538	0.636
F	7.38*	8.60**	18.95**	26.24**

Significance is denoted by: * if p<0.05 and ** if p<0.01
Standard errors are reported in brackets.

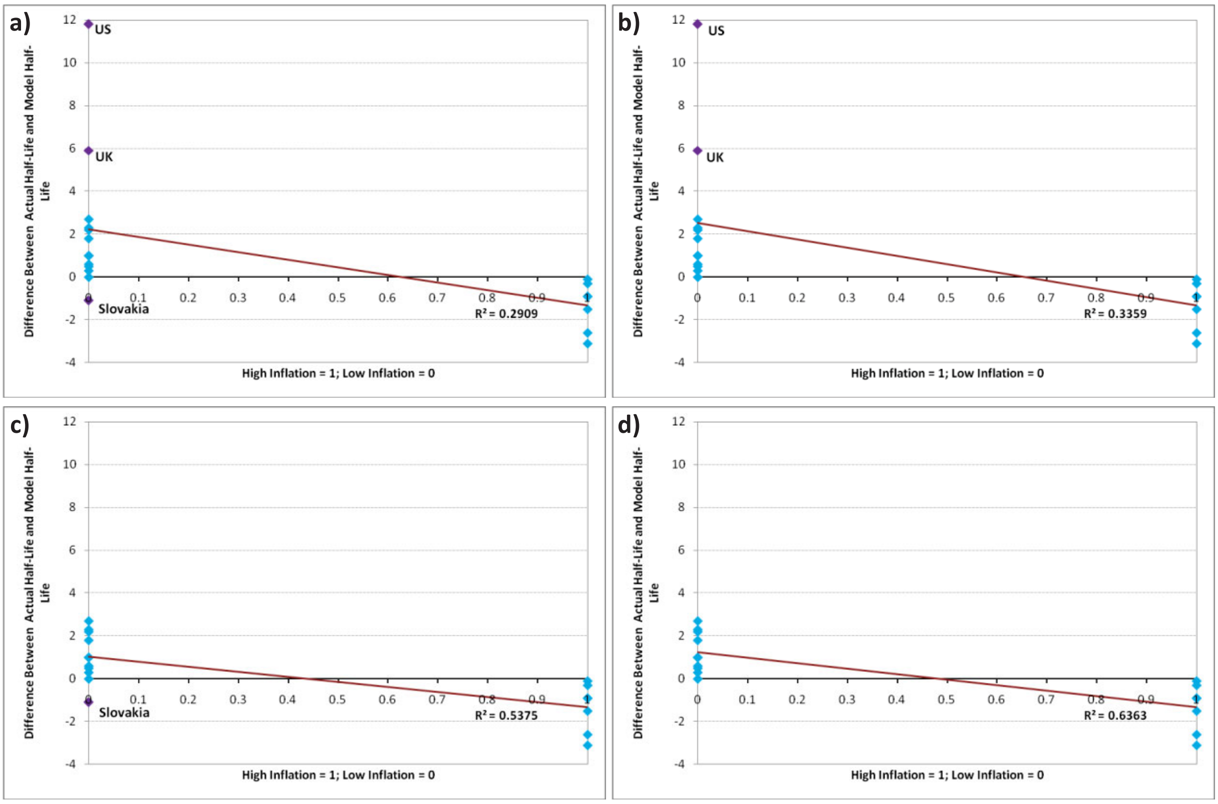
A: All countries are included in the regression
B: Slovak Republic is excluded from the regression
C: The US and UK are excluded from the regression
D: The US and UK and Slovak Republic are excluded from the regression

[8] To estimate the half-life for the period 1994:1–2005:1 an ARMA(1,1) model was fitted to the data, giving a half-life of 5.5 months. The Ljung-Box Q statistic indicated that the residuals were not serially correlated at the 1% level (Q = 28.26).

[9] To estimate the half-life for the period 1980:1–1993:4 an ARMA(1,2) model was fitted to the data, giving a half-life of 4.4 months. The Ljung-Box Q statistic indicated that the residuals were not serially correlated at the 1% level (Q = 24.77).

Figure 9

Graphical Representation of Regression Results; (a) all countries, (b) excluding Slovak Republic, (c) excluding the US and UK, (d) excluding Slovak Republic, the US and UK



With all of the countries included in the regression the relationship is weak, although it is still significant at the 95% level. This weak result can be explained partly by the inclusion of Slovak Republic, as previously discussed the calibration of an extremely high gamma value for Slovak Republic causes the model to overestimate the country's inflation, and partly by the inclusion of the US and the UK. Although these two countries follow the general pattern, they have low inflation and the model underestimates the half-life, they are outliers in that the difference between actual and model half-life is much greater than for any of the other countries. As previously discussed, both the US and the UK are highly economically developed countries and have correspondingly high levels of output persistence, however the calibration of low gamma values for both countries inhibits the model from generating anywhere near the degree of output persistence that is necessary to match the data.

Removing each of the outliers in turn significantly strengthens the relationship between inflation and the over/underestimation of the country's half-life. Figure 9(d) clearly shows that, in the absence of the outliers, the model overestimates output persistence for countries with high inflation and underestimates output persistence for countries with low inflation.

One possible criticism of this analysis is that there could be a systematic difference in the response of output to supply shocks that might explain the difference between actual and model half-life, rather than inflation. However, Kiley (2000) reveals that the lack of persistence in high inflation economies is not the result of less persistent aggregate supply or demand shocks, less-persistent nominal output fluctuations or greater variability of nominal output, greater openness of the economy or inflation crises. Thus, Kiley (2000) concludes that the results are "*supportive of less-persistent output fluctuations in high-inflation economies, as predicted by an endogenous price stickiness model*" (p.51)

The only significant explanatory variable identified in Kiley (2000) is that of income per capita. However, as revealed in this analysis, there is a strong positive relationship between persistence and economic development, and therefore such a relationship is to be expected.

6.3. Relationship to Economic Development

The analysis in section 2.3 demonstrated that there is a clear positive relationship between economic development, as measured by GDP per capita and energy use per capita, and output persistence. Thus, it is now interesting to investigate how successful the model is in replicating this pattern.

It is assumed that the more economically developed an economy, the more sophisticated the input-output structure. Therefore, the more economically developed the countries in the sample, the greater the number of stages in the input-output structure (N) they were assigned. Examination of the relationship between N and the degree of output persistence generated by the model revealed, with the exception of the US and the UK, a significant strong positive relationship. This is consistent with both the finding of greater output persistence in more economically developed countries and the central proposition of Huang and Liu (2001) that the greater the number of production stages (N), the more persistent the response of output.

For completeness, the relationship between the magnitude of output persistence generated by the model and the values of GDP per capita and energy use per capita is examined. Table 13 details the regression results and Figures 10 and 11 demonstrate this graphically. As anticipated from previous analysis in this section, the US and UK are significant outliers and are abstracted from accordingly.

Table 13 **Relationship between Model Half-Life and Economic Development**

	Model 1		Model 2	
	A	B	A	B
Ln[GDP]	0.942* (0.373)	1.611** (0.310)		
Ln[Energy Use]			0.883* (0.353)	1.468** (0.301)
Constant	-2.623 (3.497)	-8.542** (2.871)	-0.491 (2.678)	-4.600 (2.248)
R^2	0.262	0.627	0.258	0.598
F	6.38*	26.91**	6.27*	23.79*

Significance is denoted by: * if p<0.05 and ** if p<0.01
Standard errors are reported in brackets.

A: All countries are included in the regression
B: The US and UK are excluded from the regression

Figure 10 **Relationship between Model Half-Life and GDP per capita (PPP, 2005); (a) all countries, (b) excluding the US and UK**

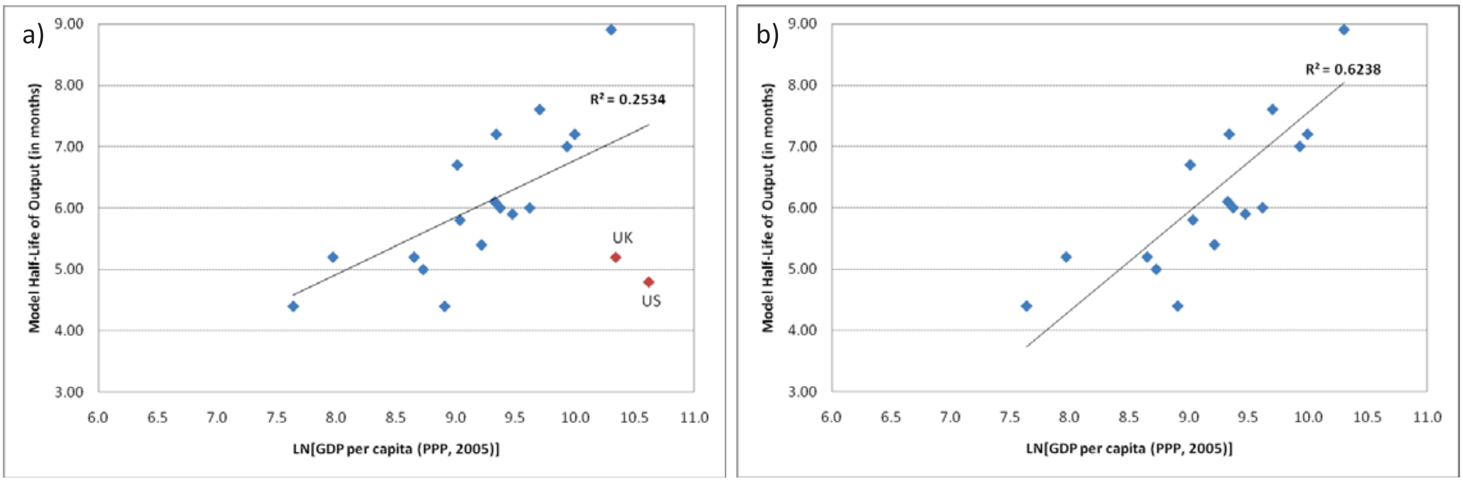
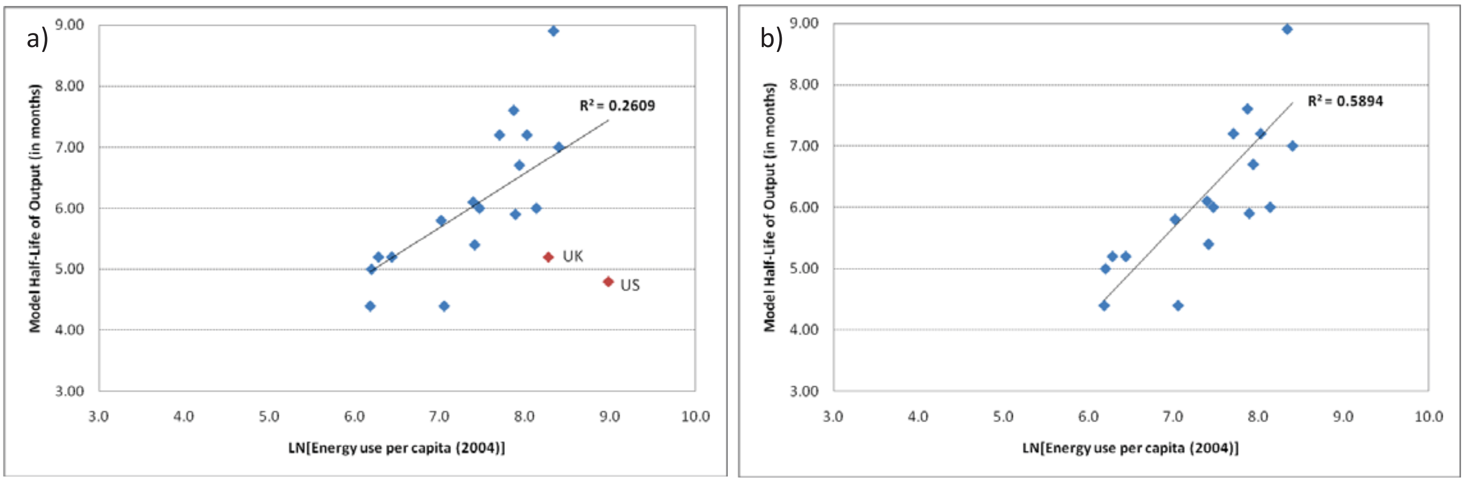


Figure 11 **Relationship between Model Half-Life and Energy Use per capita (2004); (a) all countries, (b) excluding the US and UK**



These results show that, whilst there is only a weak positive relationship between the magnitude of output persistence generated by the model and economic development when all countries are included in the analysis, this becomes a strong significant positive relationship upon the exclusion of the US and the UK. This suggests that, amongst the developing countries at least, the model is successful in replicating the observed patterns of output persistence across countries at different levels of persistence.

7 Conclusion

This paper has shown that there is a close relationship between output persistence and level of economic development, with more economically developed countries exhibiting much higher output persistence than less developed countries. This relationship was explored through the use of the Huang and Liu (2001) model. The vertical input-output structure embedded in this model enabled the representation of countries at various levels of economic development, from India to the US, simply by altering the number of production stages (N).

The model was calibrated for 20 countries at varying levels of economic development, and the results support the key premise of Huang and Liu (2001), namely that there is a strong positive relationship between the number of production stages and the magnitude of output persistence. Furthermore, sensitivity analysis revealed that the model is capable of generating output persistence anywhere between 3.6 months and 15.4 months; thus, it is clearly capable of representing both the most developed of countries, for example the US with a half-life of 16.6 months, and the least developed, for example India with a half-life of 4.4 months.

However, the effect of increasing the number of stages is severely limited by the share of the composite of stage n-1 goods in i's production (γ). This was particularly poignant in the modelling of the US and UK; both countries are highly economically developed and had correspondingly high N values, however calibration gave low values of γ which effectively inhibited the model from generating any significant degree of output persistence for either country. Nonetheless, after abstracting from the US and UK results, there was found to be a strong significant positive relationship between the magnitude of output persistence generated by the model and economic development.

A very significant finding of this analysis is that the model overestimates output persistence in high inflation countries and underestimates output persistence in low inflation countries. This has important implications not only for this model, but also for any economist attempting to construct a business cycle model capable of replicating the observed patterns of output persistence. It may be possible to account for this inflation dichotomy by increasing the degree of price stickiness in low inflation countries, perhaps by increasing the number of price setting cohorts, and conversely by decreasing the degree of price stickiness in high inflation economies. In the context of the Huang and Liu (2001) model, each country would then not only be ranked according to level of economic development and assigned a corresponding N value, but would also be ranked according to whether they have high or low inflation and correspondingly assigned either two or four cohorts of price setters. This should significantly improve the fit of the model to countries' observed output persistence.

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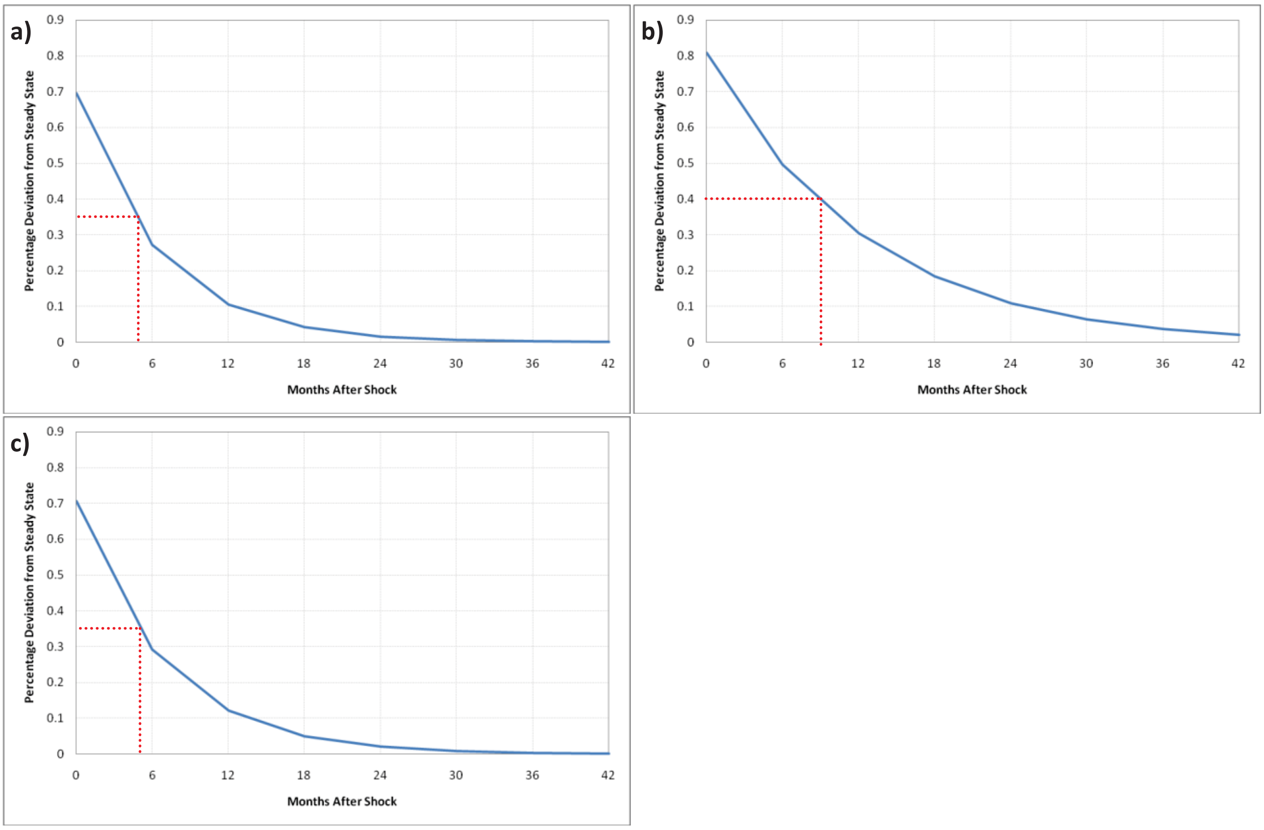
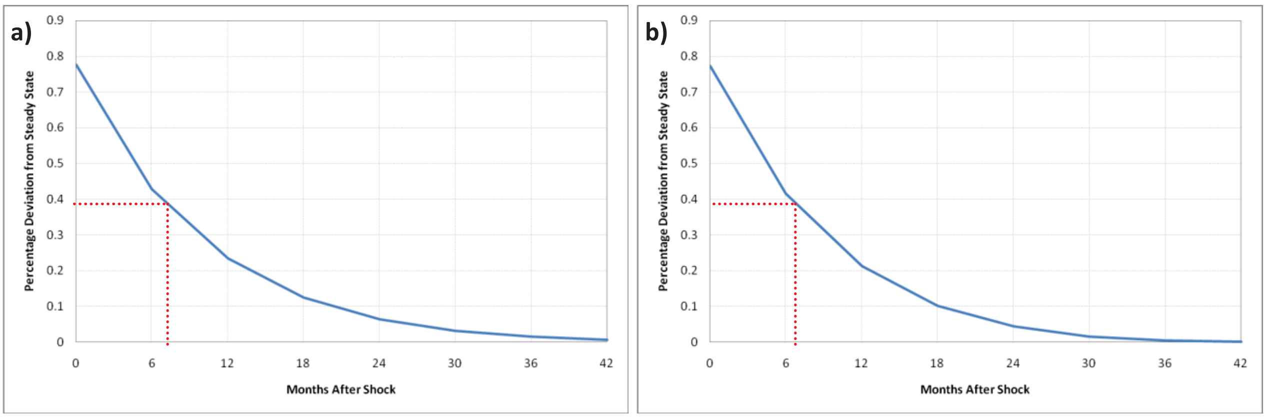
Appendix A

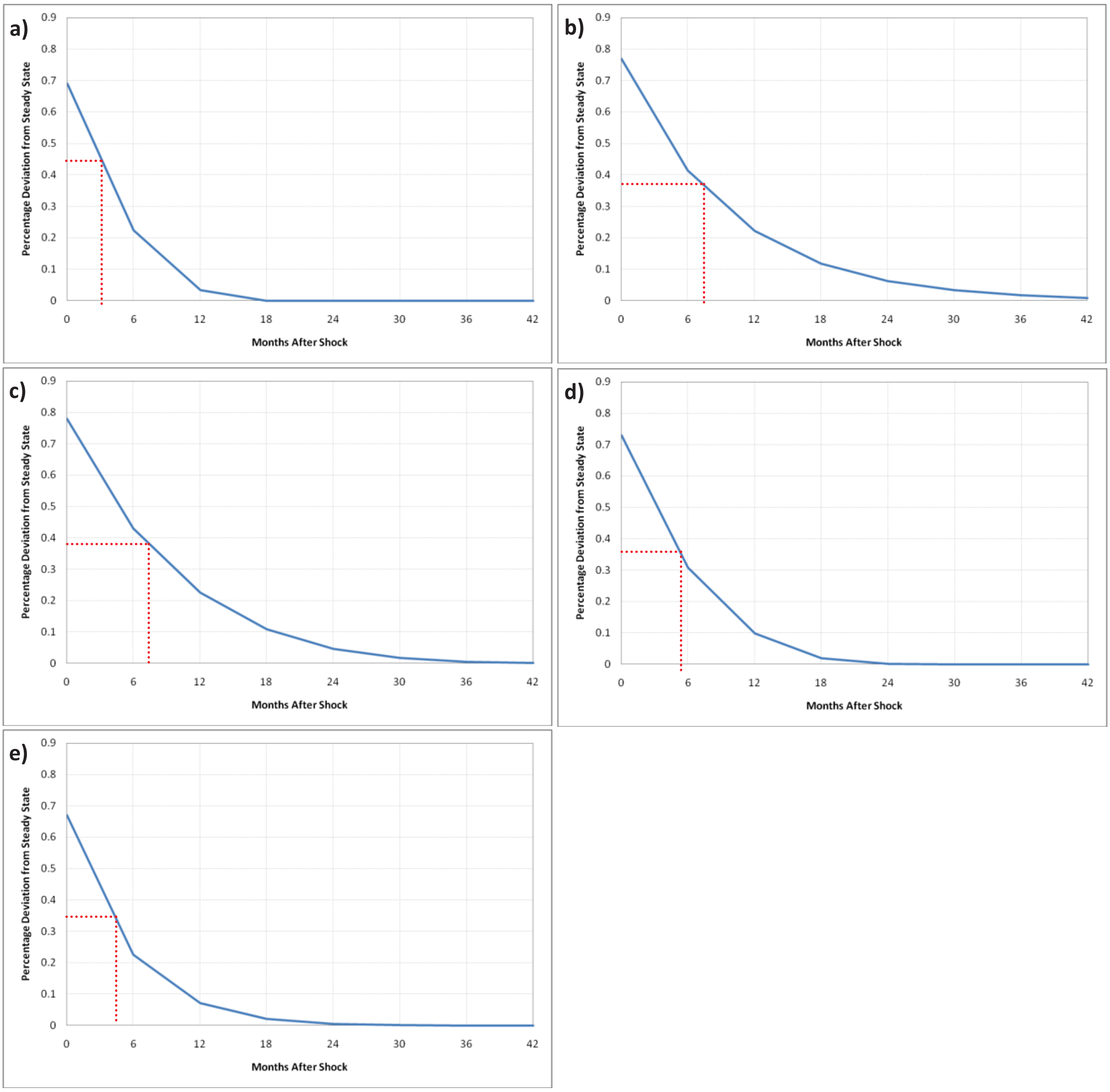
1. **Impulse Responses of Aggregate Output for (a) the US, (b) Japan and (c) the UK**



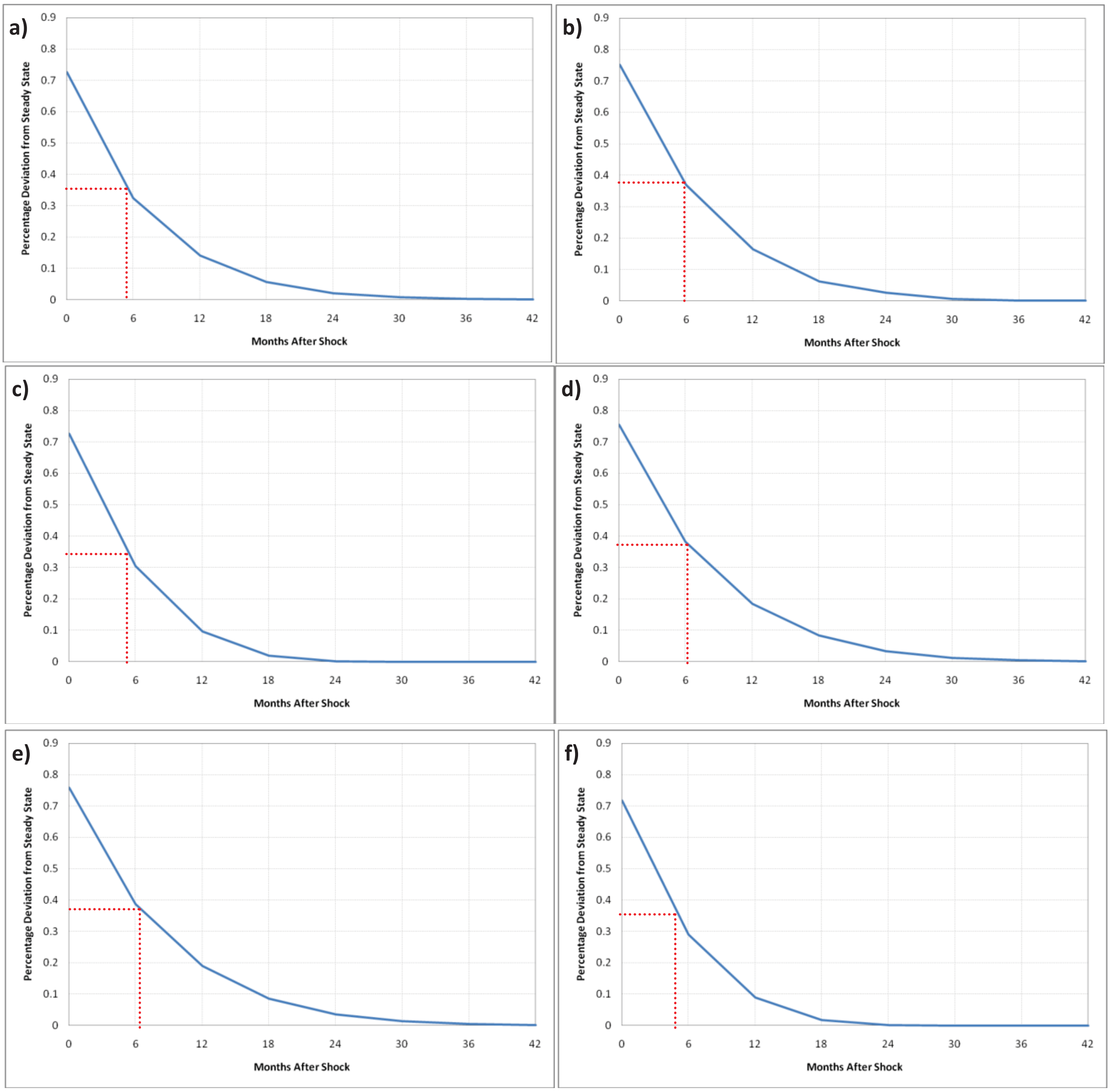
2. **Impulse Responses of Aggregate Output for (a) Israel and (b) South Africa**



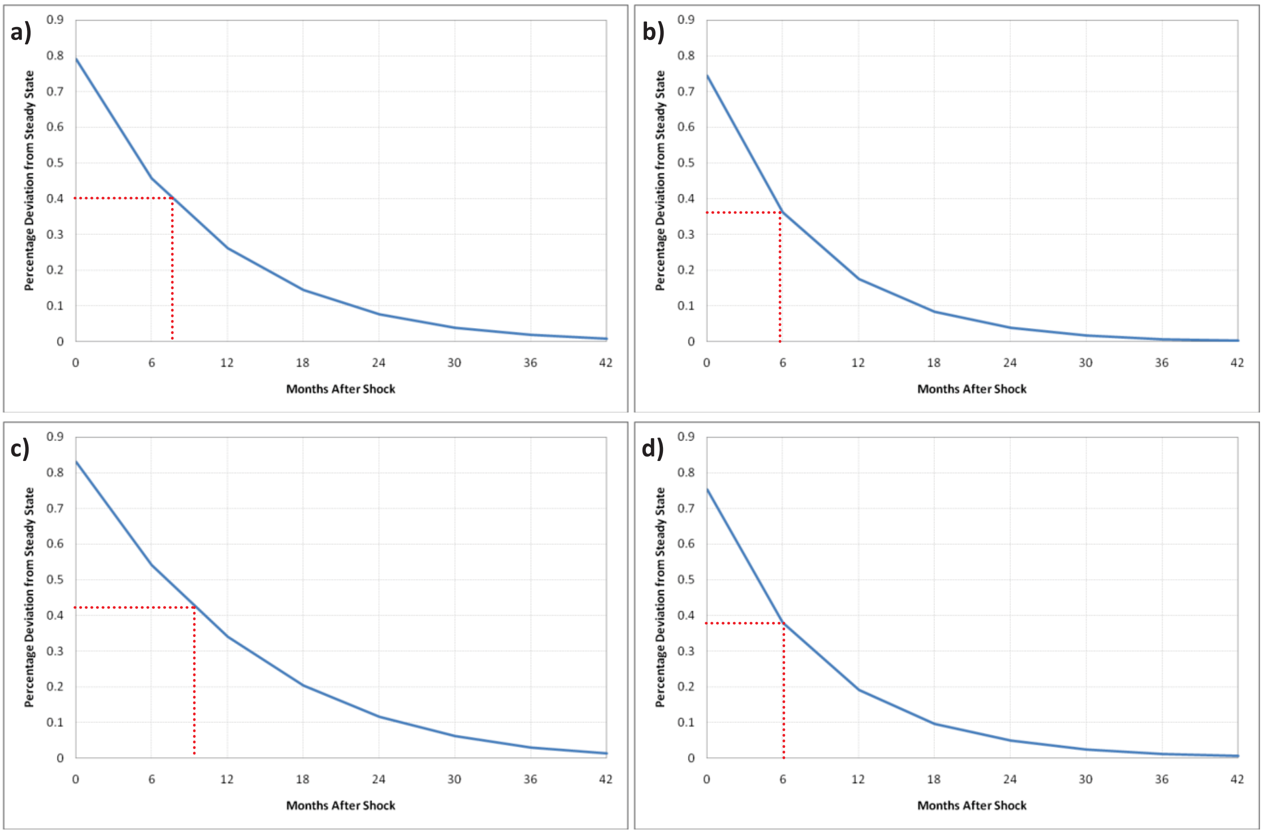
3. Impulse Responses of Aggregate Output for (a) India, (b) South Korea, (c) Malaysia, (d) the Philippines and (e) Turkey



4. Impulse Responses of Aggregate Output for (a) Argentina, (b) Brazil, (c) Colombia, (d) Chile, (e) Mexico and (f) Peru



5. **Impulse Response Functions for (a) Hungary, (b) Lithuania, (c) Slovenia and (d) Slovak Republic**



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