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Abstract

Detection of structural change is a critical empirical activity, but continuous ‘monitoring’ of series, for structural changes in real time, raises well-known econometric issues that have been explored in a single series context. If multiple series co-break then it is possible that simultaneous examination of a set of series helps identify changes with higher probability or more rapidly than when series are examined on a case-by-case basis. Some asymptotic theory is developed for maximum and average CUSUM detection tests. Monte Carlo experiments suggest that these both provide an improvement in detection relative to a univariate detector over a wide range of experimental parameters, given a sufficiently large number of co-breaking series. This is robust to a cross-sectional correlation in the errors (a factor structure) and heterogeneity in the break dates. We apply the test to a panel of UK price indices.

Key Words: monitoring, structural change, panel, CUSUM, fluctuation test

JEL Classification: C100, C590

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1 Introduction

Detection of structural change is a critical empirical activity, for the obvious reason that if such changes are ignored then econometric relations are misspecified, from which numerous problems may flow. An area where it may be particularly important is forecasting. Clements and Hendry argue forcefully (in, *e.g.* 1998a,b) that the main source of forecast error is structural change; Hendry (2000) argues that the dominant cause of these failures is the presence of deterministic shifts. Stock and Watson (1996) looked at many forecasting models of a large number of US time series, and found evidence for parameter instability in a significant proportion of the relations. Groen, Kapetanios, and Price (2009) examine the Bank of England record for output growth and inflation forecasts, assessed against some statistical benchmarks. They suggest that the critical factor explaining the good inflation forecast performance reported in that paper is an ability to detect mean-shifts in the series.

Break detection has a long history - the seminal paper testing for a break at a known point was Chow (1960). Andrews (1993) introduced a methodology that allowed for unknown break-points: one influential paper is Bai and Perron (1998). All these tests test for breaks against specific alternatives. While effective in that case, they are ineffective when the break is not covered by the particular alternative. In addition, by their nature they require some trimming of observations towards the end of the sample in order to be able to perform the test.¹ Therefore, a more effective real-time break testing strategy requires that one augments these class of break tests with a specific end-of-sample test, as in Andrews (2003). Nonetheless, real-time detection remains hard with these approaches as we do not have many observations to help us with it.

An alternative to this general methodology was the CUSUM² approach of Brown, Durbin, and Evans (1975).³ The advantage of the CUSUM test flows from the fact that there are many ways to reject the hypothesis of no structural change. While Wald, LM and LR tests are efficient against specific alternatives, the CUSUM test's usefulness lies partly in the fact that it offers a graphical view of deviations from constancy. But formal significance tests based on boundary conditions can be constructed for hypotheses likely to be observed in

¹In particular, when the errors of the regression model of interest are serially correlated or heteroscedastic, Bai and Perron (1998) suggest that a large trimming of end-of-sample observations might be necessary for proper break testing - typically 10 to 20%.

²An example of a fluctuation test. While some authors (*e.g.* Chu, Stinchcombe, and White (1996)) reserve the term for tests based on variation in parameters, others (*e.g.* Zeileis, Leisch, Kleiber, and Hornik (2005)) apply it more generally.

³Extended to dynamic models by Krämer, Ploberger, and Alt (1988).

practice. Thus the method is more likely to be robust under different break scenarios. Moreover, there is no sample-trimming problem. On the negative side, however, after detection it is hard to detect the cause of the break. Further, Monte Carlo studies reveal that in practice CUSUM tests have low power, which often results in a long delay before a break is detected.

Moreover, all these tests are ‘retrospective’, in the sense that they are designed to test for change in particular datasets - a one-off experiment. The problem that is usually faced in practice is the continual monitoring of a series via repeated tests, whereby tests are applied in successive periods or at intervals. It is not hard to see that as the monitoring period increases, the probability of rejecting a true null hypothesis of no break will eventually approach unity, following the law of iterated logarithm. It is possible this will occur quite rapidly, as, *e.g.*, Chu, Stinchcombe, and White (1996) show with some simulations. The challenge, then, is to find suitable boundary conditions to obviate this. Consequently Chu, Stinchcombe, and White (1996) introduced a sequential testing procedure using fluctuation tests with asymptotically correct size. Zeileis, Leisch, Kleiber, and Hornik (2005) subsequently explored some extensions in dynamic models, and Leisch, Hornik, and Kuan (2000) generalised the class, extending Kuan and Hornik (1995).

But there may be a way to extend the tests to obtain more effective monitoring procedures. The idea is that there are common breaks, also known as ‘co-breaks’,⁴ in multiple time series. This has a natural appeal. For example, changes in monetary regime may affect steady state inflation, and we would expect this to be reflected in disaggregate inflation measures. Or shifts in total factor productivity growth may occur simultaneously in different industries or countries. If the series under consideration are components of an aggregate, such as inflation, it follows that if a component breaks, or if components co-break, then a break also occurs in the aggregate. This can be important for policymakers - for example, detecting changed steady-state inflation after a new monetary regime. An obvious question to ask, therefore, is whether detection of such a common break is easier with multiple series.

Bai, Lumsdaine, and Stock (1998) develop some asymptotic distribution theory for maximum likelihood detection of a break in a multivariate model, but theirs is the multivariate equivalent of the fixed-sample tests. Our approach is instead to extend the monitoring framework of Chu, Stinchcombe, and White (1996) to a multivariate setting. Chu, Stinchcombe,

⁴Clements and Hendry (1999) are mainly concerned with co-breaks in non-stationary systems, analogous to cointegrating relations. In our paper co-breaking signifies that some fraction of the series considered experience a break at approximately the same point in time.

and White (1996) establish that a limiting distribution applies to the univariate CUSUM test statistic such that the critical boundary condition is an increasing function of time.

We assume that there is a set of variables which are generated by structurally stable processes over some initial period. There is then a subset of these variables that co-break at or close to some point. The task of the econometrician is to detect that co-breaking point by a monitoring process that starts after the initial sample. To do this we propose multivariate detectors that take the residuals from a set of equations recursively estimated over a monitoring period. The null is that there are no breaks in any series: the alternative is that at least one series breaks, and if more than one series breaks, their respective break points are temporally close (co-breaks). We construct CUSUM statistics from the normalised residuals purged of cross equation correlation, and examine the asymptotic behaviour of the maximum absolute and average cumulative sum. A version of the Chu, Stinchcombe, and White (1996) result is shown to apply asymptotically to both the supremum and average statistics. Using Monte Carlo methods, we explore the small sample properties of the detectors under different configurations of the proportion of series co-breaking at different dates, under differing monitoring periods, sample lengths and numbers of series. To anticipate, the result is that provided the proportion of series is sufficiently large, the multivariate detectors increase the probability of detection relative to a single series with a break in the majority of cases examined. Similarly, the speed with which a break is detected is also improved. The broad pattern of the results is preserved whether or not there is cross-sectional correlation in the data or heterogeneity in the break dates. Gains from considering the multivariate detector are apparent even for a relatively small proportion of co-breaking series.

In Section 2 we set out the theory underlying our proposed tests. In Section 3 we perform some Monte Carlo experiments. Section 4 applies the multivariate detector to UK RPI annual inflation data, monitoring for a break after 2000. The method does indeed pick up potential breaks that univariate methods fail to capture. Section 5 concludes. All proofs are relegated to an Appendix.

2 Theoretical Considerations

Our interest focuses on a seemingly unrelated set of regressions given by

$$Y_{j,t} = X'_{j,t}\beta_{j,t} + \epsilon_{j,t}, \quad t = 1, \dots \quad j = 1, \dots, p \quad (1)$$

where for each equation j $X_{j,i}$ is a $k \times 1$ random vector and $\beta_{j,t}$ is a $k \times 1$ non-stochastic vector as well as

$$\epsilon_t = \begin{pmatrix} \epsilon_{1,t} \\ \vdots \\ \epsilon_{p,t} \end{pmatrix} \sim (\mathbf{0}, \Sigma)$$

for some positive definite matrix Σ where $\Sigma[j, j] \equiv \sigma_j^2$. Throughout the analysis the following non-contamination assumption is made:

Assumption 1 $\beta_{j,t} = \beta_j$, for $t = 1, 2, \dots, m$, $j = 1, \dots, p$.

Non-contamination therefore means that the parameters in (1) are assumed to be stable for sure in the period up to $t = m$ for each equation j .

The entertained null hypothesis is $H_0 : \beta_{j,t} = \beta_j$, for $t = m + 1, \dots$. The regressions in (1) are potentially related in two ways. The first is standard, i.e., through a non-diagonal disturbance covariance matrix Σ . The second way in which these regressions are related provides the motive for considering multivariate monitoring schemes. We consider alternative hypotheses of the form $H_1 : \beta_{j,t}$ changes at some $T_{0,j} \geq m + 1$ for some $j \in J_{p_1} = \{j_1, \dots, j_{p_1}\}$ where $p_1 > 1$ and $T_{0,j_k}/T_{0,j_l} = 1 + o(1)$ for all $1 \leq k, l \leq p_1$. In words, a subset of the processes under consideration roughly co-breaks, in the sense that these processes exhibit breaks in relatively close temporal proximity. As we will also see in our Monte Carlo study, whether the series break at exactly the same point in time, or at different points in time that are reasonably close, does not seem to make any difference, at least in the context of our Monte Carlo experiments.

The univariate CUSUM detector is specified as follows. Let in (1)

$$\hat{\beta}_{j,t} = \left(\sum_{i=1}^t X_{j,i} X_{j,i}' \right)^{-1} \left(\sum_{i=1}^t X_{j,i} Y_{j,i} \right) \quad (2)$$

be the OLS estimator for the j -th equation at time t . Define recursive residuals as

$$\omega_{j,t} = \begin{cases} 0 & \text{for } t = k \\ \hat{\epsilon}_{j,t} / \nu_{j,t}^{1/2} & \text{for } t = k + 1, \dots, m, m + 1, \dots \end{cases} \quad (3)$$

with

$$\begin{aligned} \hat{\epsilon}_{j,t} &= Y_{j,t} - X_{j,t}' \hat{\beta}_{j,t-1}, \\ \nu_{j,t} &= 1 + X_{j,t}' \left(\sum_{i=1}^{t-1} X_{j,i} X_{j,i}' \right)^{-1} X_{j,t}. \end{aligned}$$

The t -th cumulated sum of recursive residuals is

$$Q_{j,t}^m = \sigma_j^{-1} \sum_{i=k}^n \omega_{j,i} = \sigma_j^{-1} \sum_{i=k}^{k+[(m-k)t]} \omega_{j,i}, \quad n = k + 1, \dots, m, m + 1, \dots \quad (4)$$

for $(n - k)/(m - k) \leq t \leq (n - k + 1)/(m - k)$, where $[\cdot]$ denotes integer part and $\hat{\sigma}_j^2$ is some consistent estimate of σ_j^2 . An obvious choice for this is the estimate of σ_j^2 based on the OLS estimate of β_j obtained in the non-contamination period $t = 1, \dots, m$.

It is well known (see, e.g., Krämer, Ploberger, and Alt (1988)) that under our entertained null hypothesis, $H_0 : \beta_{j,t} = \beta_j$, for $t = m + 1, \dots$,

$$\{t \rightarrow m^{-1/2}Q_{j,t}^m, \quad t \in [0, \infty)\} \Rightarrow \{t \rightarrow W(t), \quad t \in (0, \infty)\}, \quad (5)$$

where $Q_{j,t}^m$ is defined in (4), \Rightarrow denotes the weak convergence of the associated probability measures and $W(t)$ is a standard Brownian motion. This result can be used to motivate the following monitoring scheme for each equation in (1) separately

$$\lim_{m \rightarrow \infty} \Pr \{|Q_{j,t}^m| \geq \sqrt{m}g(n/m), \quad \text{for some } n \geq m\} = \Pr (|W_j(t)| \geq g(t), \quad \text{for some } t \geq 1) \quad (6)$$

where $W_j(t)$ is again a standard Brownian motion. In general, the probability on the right hand side (6) does not have a closed form solution for any arbitrary $g(t)$, but there are some specific instances where such a closed form solution is viable. To keep the discussion general we will parameterize $g(t)$ as follows: $g(t) = g(t, a)$ such that

$$\Pr (|W_j(t)| \geq g(t), \quad \text{for some } t \geq 1) = f_g(a) = \alpha \quad (7)$$

where there is a unique mapping $f_g(a) = \alpha$ for all $\alpha \in (0, 1)$. Admissible functions $g(t)$ are discussed in detail in Chu, Stinchcombe, and White (1996). In particular, in what follows and in common with Chu, Stinchcombe, and White (1996), we assume that g belongs to the class of regular functions as defined in (5)-(6) of Chu, Stinchcombe, and White (1996).

As a natural extension of the above univariate framework we explore multivariate detectors. The advantages of using systems of equations to detect breaks are intuitively clear. Economic data are likely to undergo structural change in tandem if major changes occur in the economic system. While the timing of such changes may not be perfectly synchronised across series, we will argue that relatively minor variations in the timing across series make little difference to the overall performance of multivariate detectors. If series experience structural changes jointly, multivariate methods will clearly have an inferential advantage.

The first step to construct our multivariate detectors is to construct $\hat{\Sigma}$, which is a consistent estimator of Σ in (1). We suggest estimating this from the residuals of univariate OLS estimations for each of the p equations in (1) over the non-contamination period $t = 1, \dots, m$,

since we would like to consider relatively large values of p . Collect the individual recursive residuals in $\omega_t = (\omega_{1,t}, \dots, \omega_{p,t})'$ and purge these from cross-equation correlation, i.e.,

$$\tilde{\omega}_t = \hat{\Sigma}^{-1/2} \omega_t. \quad (8)$$

The t -th cumulated sum of the recursive residuals in (1) based on (8) now is equal to

$$\tilde{Q}_{j,t}^m = \sum_{i=k}^{t+k} \tilde{\omega}_{j,i}. \quad (9)$$

We need a way to aggregate the information contained in $\tilde{Q}_{j,t}^m$ in the form of a summary statistic. Two main statistics are used in a number of literatures for this.⁵ These are the maximum and average of $\tilde{Q}_{j,t}^m$ over j .⁶ We will consider the maximum first as it requires a slightly more complicated theoretical analysis, performs better overall in small samples and is more sensitive, by nature, to cases where only a small proportion of the series experience a break while remaining well behaved in the case where many or all series break.

Define the maximum absolute cumulative sum as

$$\tilde{Q}_{\max,t}^m = \max_{j=1,\dots,p} \left| \tilde{Q}_{j,t}^m \right|, \quad (10)$$

where $\tilde{Q}_{j,t}^m$ is computed as in (9). We make the following assumption:

Assumption 2 (i) For all j , $m^{-1} \sum_{t=1}^m X_{j,t} \xrightarrow{p} b_j$ and $m^{-1} \sum_{t=1}^m X_{j,t} X'_{j,t} \xrightarrow{p} M_j$ where b_j and $M_j = E(X_{j,t} X'_{j,t})$ are non-stochastic $k_j \times 1$ vectors and non-stochastic full-rank $k_j \times k_j$ matrices respectively. (ii) $E(\epsilon_t \epsilon'_t) = \Sigma < \infty$ where Σ is a positive definite symmetric matrix. (iii) For all j , $\epsilon_{j,t}$ satisfies a functional central limit theorem, i.e.,

$$m^{-1/2} (\sigma_{0j} M_j)^{-1/2} \sum_{t=1}^{[m\ell]} X_{j,t} \epsilon_{j,t} \Rightarrow W(\ell), \quad \ell \in (0, 1), \quad (11)$$

where $E(\epsilon_{j,t}^2) = \sigma_{0j}$.

This assumption is quite mild. In particular, only the validity of a functional central limit theorem and existence of second moments is assumed about the error terms, $\epsilon_{j,t}$, allowing a wide variety of data generating processes to be accommodated. Then we have the following Theorem stating the asymptotic behaviour of the multivariate detector (10) under the null hypothesis for a fixed number of series.

⁵These include testing in the presence of underidentified nuisance parameters and panel unit root tests. (see respectively, e.g., Davies (1977) or Kapetanios (2007).)

⁶There have been other, less popular, summary statistics proposed such as the exponential average of Andrews and Ploberger (1994).

Theorem 1 *Let Assumptions 1 and 2 hold. Let g be regular and such that $t^{1/2}g(t)$ is eventually nondecreasing. Further, assume that the null hypothesis holds, i.e., that, for all j , $Y_{j,t} = X'_{j,t}\beta_j + \epsilon_{j,t}$, $t = 1, \dots, m, m+1, \dots$. Then,*

$$\lim_{m \rightarrow \infty} \Pr \left\{ \tilde{Q}_{\max,t}^m \geq \sqrt{m}g(n/m, a), \quad \text{for some } n \geq m \right\} = 1 - (1 - f_g(a))^p \quad (12)$$

Proof: See Appendix A.1 for details on the proof of this theorem.

In practice one wants to consider a relatively large number of series to detect an aggregate break. For example, in an effort to detect a break in an economy's inflation rate based on our multivariate detector one can consider using all its subcomponents, which can easily lead to using many dozens of series. Therefore, the next theorem generalises the asymptotic behaviour of (10) under the null hypothesis in Theorem 1 by allowing the number of series to go to infinity ($p \rightarrow \infty$).

Theorem 2 *Let Assumptions 1 and 2 as well as $p = o(T^{1/2})$ hold, where the latter indicates that $p \rightarrow \infty$ is allowed for but at a slower rate than the time series dimension. Let g be regular and such that $t^{1/2}g(t)$ is eventually nondecreasing. Further, assume that the null hypothesis holds, i.e., that, for all j , $Y_{j,t} = X'_{j,t}\beta_j + \epsilon_{j,t}$, $t = 1, \dots, m, m+1, \dots$. Then*

$$\lim_{m, p \rightarrow \infty} \Pr \left\{ \tilde{Q}_{\max,t}^m \geq \sqrt{m}g(n/m, a_p(\alpha)), \quad \text{for some } n \geq m \right\} = \alpha \quad (13)$$

where $a_p(\alpha)$ is chosen so that $\lim_{p \rightarrow \infty} 1 - (1 - f_g(a_p(\alpha)))^p = \alpha$

Proof: See Appendix A.2 for details on the proof of this theorem.

Theorems 1 and 2 prove that there is a certain bound for the multivariate statistic (10). We now have to make this bound explicit, so that we can compute critical values for (10). Theorem 3 provides an example of such a boundary function g and a sequence $a(p)$ that satisfies Theorem 2.

Theorem 3 *Let Assumptions 1 and 2 as well as $p = o(m^{1/2})$ hold, where the latter indicates that $p \rightarrow \infty$ is allowed for but at a slower rate than the time series dimension. Further, assume that the null hypothesis holds, i.e., that, for all j , $Y_{j,t} = X'_{j,t}\beta_j + \epsilon_{j,t}$, $t = 1, \dots, m, m+1, \dots$. For $g(t) = ((t+1)[a^2 + \ln(t+1)])^{1/2}$, the sequence $a_p = C \ln(p)^{1/2}$ is admissible for Theorem 2.*

Proof: See Appendix A.3 for details on the proof of this theorem.

Next, we provide some local power results for the multivariate detector in case of the simple location model. We focus on the simple location model for simplicity and because

the results we will obtain for this case provide clear implications for more general models. Based on the simple model

$$Y_{j,t} = \beta_{j,t} + \epsilon_{j,t}, \quad t = 1, \dots, \quad j = 1, \dots, p \quad (14)$$

we consider the following local alternative

$$H_{1,T} : \beta_{j,t} = \beta_j, t \leq T_{T,j,0}, \quad \beta_{j,t} = \beta_j + \beta_1/\sqrt{T}, t \geq T_{T,j,0} + 1, \quad \text{for all } j \quad (15)$$

where $T_{T,j,0}/T_{T,i,0} = 1 + o(1)$. We now have the following Theorem regarding the local power of the standard univariate CUSUM detector as well as the multivariate CUSUM detector (10) in this simple case.

Theorem 4 *Let Assumptions 1 and 2 hold. Let g be regular and such that $t^{1/2}g(t)$ is eventually nondecreasing. Then, under the local alternative $H_{1,T}$, we have that for the univariate detector procedure with significance level α ,*

$$\lim_{m \rightarrow \infty} \Pr \left\{ |Q_{j,t}^m| \geq \sqrt{m}g(n/m, a^*), \quad \text{for some } n \geq m \right\} = f_1(a^*, \tau) \quad (16)$$

where $f(a^*) = \alpha$, $\tau = \lim_{T \rightarrow \infty} \frac{T_{T,0}}{T}$, and f_1 is defined in (A.17) in the Appendix. For the multivariate detector procedure with significance level α , we have

$$\lim_{m \rightarrow \infty} \Pr \left\{ \tilde{Q}_{\max,t}^m \geq \sqrt{m}g(n/m, a^{**}), \quad \text{for some } n \geq m \right\} = 1 - (1 - f_1(a^{**}, \tau))^p \quad (17)$$

where $1 - (1 - f(a^{**}))^p = \alpha$.

Proof: See Appendix A.4 for details on the proof of this theorem.

As we discussed above, there are also alternatives to the use of the maximum for the multivariate detector as in (10). The most natural one can be based on the cross-sectional average of the absolute individual cumulative sums given by

$$\tilde{Q}_{ave,t}^m = \frac{1}{p} \sum_{j=1}^p \left| \tilde{Q}_{j,t}^m \right|. \quad (18)$$

In the following corollaries we present results for the multivariate detector (10) that correspond to the results expressed in Theorems 1 and 2, respectively. So, keeping the number of series fixed the asymptotic behaviour of the multivariate detector (18) under the null hypothesis can be typified as

Corollary 1 *Let Assumptions 1 and 2 hold. Let g be regular and such that $t^{1/2}g(t)$ is eventually nondecreasing. Further, assume that the null hypothesis holds, i.e., that, for all j , $Y_{j,t} = X'_{j,t}\beta_j + \epsilon_{j,t}$, $t = 1, \dots, m, m+1, \dots$. Then,*

$$\lim_{m \rightarrow \infty} \Pr \left\{ \tilde{Q}_{ave,t}^m \geq \sqrt{\frac{m}{p}}g(n/m, a), \quad \text{for some } n \geq m \right\} = f_g(a) \quad (19)$$

Proof: See Appendix A.5 for details on the proof of this corollary.

Similarly, when we let the number of series grow large, to make (18) more useful in practice, we can generalise the result in Corollary 1:

Corollary 2 *Let Assumptions 1 and 2 as well as $p = o(T^{1/2})$ hold, where the latter indicates that $p \rightarrow \infty$ is allowed for but at a slower rate than the time series dimension. Let g be regular and such that $t^{1/2}g(t)$ is eventually nondecreasing. Further, assume that the null hypothesis holds, i.e., that, for all j , $Y_{j,t} = X'_{j,t}\beta_j + \epsilon_{j,t}$, $t = 1, \dots, m, m + 1, \dots$. Then*

$$\lim_{m,p \rightarrow \infty} \Pr \left\{ \tilde{Q}_{ave,t}^m \geq \sqrt{\frac{m}{p}}g(n/m, a(\alpha)), \quad \text{for some } n \geq m \right\} = \alpha \quad (20)$$

where $a(\alpha)$ is chosen so that $f_g(a(\alpha)) = \alpha$.

Proof: See Appendix A.5 for details on the proof of this corollary.

Note that given Corollary 2 there is no need for an result analogous to Theorem 3 for the multivariate detector (18), whereas the equivalent of the second part of Theorem 4 can be obtained trivially for this cross-sectional average-based multivariate detector.

It is perhaps instructive to comparatively discuss the properties of the multivariate and univariate detector when breaks occur during monitoring. Theorem 4 provides some theoretical results but since these are specific to the location model they may not be very informative for general regression models. It is of course difficult to compare the univariate and multivariate detectors since they work under different premises: The univariate detector focuses on a single series while the multivariate look at panels. It is clear that the two kinds of detector focus on different datasets as a result of economic, as opposed, to econometric motivations. Therefore, a comparison of the two kinds of detectors may be most fruitful when very few series, or even just one, break and these include the one that the univariate detector considers. In such circumstances it is important to show that the multivariate detectors are still able to detect breaks, even though the evidence for breaks is diluted by the presence of many series with no breaks. Looking at the extreme example whereby only one series breaks we know the following from the preceding theoretical analysis. If a series breaks, the associated statistic which is used in the construction of the multivariate statistic will diverge to infinity at rate $(n - m)^{1/2}$ where $n \rightarrow \text{infy}$, $m \rightarrow \text{infy}$, $n - m \rightarrow \text{infy}$. So the supremum statistic will diverge at the same rate. This rate is the same as that of the statistic underlying the univariate detector. The average multivariate detector will diverge at rate $(n - m)^{1/2}/p$. This of course is not the whole picture. The boundary condition for the supremum multivariate detector also increases with p , and as shown in theorem 3, this

increase is of order $\ln(p)^{1/2}$. The boundary conditions for the average multivariate detector and the univariate detector do not depend on p . The above suggest that if one series break, the univariate detector is best placed to detect this. But, both the supremum and average multivariate detector will also detect it with probability approaching one as long as $p = o(m^{1/2})$, in the case of the average, which is an assumption we make anyway. Further, it is also clear that the detector based on the supremum statistic will be best placed to detect such breaks compared to the average as we also find through our extensive Monte Carlo study. Overall, we conclude that even in the worst case scenario for the multivariate detectors, they retain the attractive property of detecting breaks with probability approaching one.

The preceding discussion has been focused on a particular monitoring scheme based on the work of Chu, Stinchcombe, and White (1996) and a particular multivariate extension. Both these choices are mainly made for expositional clarity and to illustrate the potential of exploiting a multivariate dataset in the context of monitoring structural breaks. Our work can be generalised in both these dimensions. In particular, rather than focusing on the recursive residual approach of Chu, Stinchcombe, and White (1996), we can consider other fluctuation processes which can be generalised in a similar fashion to the multivariate setting. These include processes based on estimates as in Leisch, Hornik, and Kuan (2000) or Zeileis, Leisch, Kleiber, and Hornik (2005), or on OLS residuals as in Zeileis, Leisch, Kleiber, and Hornik (2005), or on scores as in Zeileis (2005) and Zeileis and Hornik (2007).⁷ We leave all of this to future research and instead we will focus in the next section on the finite sample performance of our two multivariate monitoring statistics (10) and (18).

3 Monte Carlo Study

In this section we investigate the small sample properties of our multivariate detectors (10) and 18 introduced in Section 2 through an extensive Monte Carlo study. The set-up of the different aspects of our simulation experiments are outlined in Section 3.1, and we discuss the Monte Carlo results in Section 3.2.

⁷Note that the last paper provides a very general framework for monitoring structural change in single equation settings as it applies to generalised linear models estimated using M-estimation.

3.1 Set-Up

We begin by considering the simple location model⁸

$$\begin{aligned}
 Y_{j,t} &= \beta_{j,t} + \epsilon_{j,t}, \quad t = 1, \dots, T, \quad j = 1, \dots, p \\
 \text{for } j &= 1, \dots, [p_b p] \quad \beta_{j,t} = \begin{cases} 1 & \text{if } t \leq T_{T,0} = m + (T - m)b \\ 1 + \beta_j & \text{if } t \geq T_{T,0} + 1 \end{cases} \\
 \text{else } & \beta_{j,t} = 1.
 \end{aligned} \tag{21}$$

In (21) $(\epsilon_{1,t} \dots \epsilon_{p,t})' \sim IIDN(\mathbf{0}, I_p)$, $\beta_j \sim N(1, 1)$, p_b is the proportion of series that experience a break, m is the length of the non-contamination period and b is the proportion of the monitoring period at which the break occurs. Thus the variables of interest are generated by a process with an idiosyncratic constant and uncorrelated error. We will use the data generating process (DGP) outlined in (21) to generate data for our baseline experiments for $T = 100, 200, 400$, $j = 10, 20, 40$, $m = [p_m T]$ and $p_m = 0.25, 0.5, 0.75$, $b = 0.25, 0.5, 0.75$ and $p_b = 0.1, 0.2, 0.4, 0.6, 0.8, 1$. The respective Monte Carlo experiments are based on 1000 replications and we report for each break detection statistic under consideration the average probability of break detection across the replications.

In addition, we examine two extensions to the baseline experiments based on DGP (21). The first is to allow for cross-sectional correlation among the variables. A natural way to handle this is to assume that a factor structure exists in the disturbance terms of the p equations. In this case (21) is adapted so that it incorporates such a common factor, i.e.,

$$\begin{aligned}
 Y_{j,t} &= \beta_{j,t} + u_{j,t}, \quad t = 1, \dots, T, \quad j = 1, \dots, p \\
 u_{j,t} &= f_t + \epsilon_{j,t}, \\
 \text{for } j &= 1, \dots, [p_b p] \quad \beta_{j,t} = \begin{cases} 1 & \text{if } t \leq T_{T,0} \\ 1 + \beta_j & \text{if } t \geq T_{T,0} + 1 \end{cases} \\
 \text{else } & \beta_{j,t} = 1,
 \end{aligned} \tag{22}$$

where f_t is an i.i.d. common factor variable, $f_t \sim IIDN(0, 1)$.

The second modification is to randomise the break dates, so that breaks occur only approximately at the same time. Specifically, in either DGP (21) or (22) we replace the common break date $T_{T,0}$ with the series-specific break date $T_{j,T,0}$, which is stochastic according to

$$T_{j,T,0} = m + (T - m)b + v_j \tag{23}$$

where v_j is an integer valued random variable distributed uniformly over $[-5, -4, \dots, 4, 5]$, and parameters T, m, b have the same interpretation and values as in (21) and (22).

⁸Note that in the following $[.]$ denotes integer part and I_p symbolises a p -dimensional identity matrix.

Table 1: Probability of break detection - cross-sectionally uncorrelated: Univariate detector

Univariate detector (single series with break)										
		p=10			p=20			p=40		
$T = 100$										
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	0.650	0.585	0.320	0.667	0.595	0.384	-	-	-
	0.2	0.664	0.579	0.350	0.679	0.576	0.400	-	-	-
	0.4	0.653	0.541	0.310	0.695	0.612	0.355	-	-	-
	0.6	0.674	0.577	0.319	0.672	0.615	0.364	-	-	-
	0.8	0.642	0.579	0.338	0.712	0.622	0.384	-	-	-
	1	0.670	0.562	0.331	0.716	0.615	0.371	-	-	-
0.50	0.1	0.594	0.474	0.184	0.609	0.473	0.239	0.635	0.511	0.232
	0.2	0.617	0.488	0.185	0.594	0.512	0.209	0.612	0.506	0.221
	0.4	0.594	0.504	0.197	0.599	0.484	0.204	0.630	0.482	0.228
	0.6	0.595	0.462	0.196	0.617	0.487	0.209	0.639	0.496	0.236
	0.8	0.595	0.445	0.182	0.629	0.481	0.204	0.635	0.479	0.240
	1	0.618	0.459	0.172	0.605	0.490	0.180	0.589	0.498	0.237
0.75	0.1	0.461	0.265	0.112	0.434	0.229	0.117	0.443	0.246	0.092
	0.2	0.438	0.305	0.104	0.430	0.280	0.096	0.404	0.210	0.086
	0.4	0.417	0.269	0.113	0.432	0.265	0.116	0.407	0.208	0.088
	0.6	0.449	0.275	0.122	0.431	0.278	0.114	0.412	0.225	0.100
	0.8	0.450	0.274	0.127	0.424	0.243	0.116	0.398	0.257	0.096
	1	0.439	0.264	0.133	0.419	0.271	0.099	0.394	0.252	0.106
$T = 200$										
0.25	0.1	0.783	0.724	0.511	0.763	0.702	0.489	0.762	0.690	0.496
	0.2	0.775	0.721	0.484	0.758	0.706	0.505	0.744	0.704	0.493
	0.4	0.794	0.719	0.500	0.796	0.723	0.494	0.769	0.701	0.462
	0.6	0.755	0.697	0.516	0.757	0.704	0.462	0.785	0.706	0.485
	0.8	0.777	0.709	0.474	0.770	0.709	0.497	0.758	0.674	0.494
	1	0.763	0.730	0.512	0.781	0.703	0.475	0.747	0.686	0.469
0.50	0.1	0.737	0.636	0.363	0.718	0.653	0.376	0.754	0.618	0.383
	0.2	0.693	0.608	0.361	0.761	0.626	0.345	0.715	0.606	0.392
	0.4	0.724	0.589	0.342	0.721	0.626	0.369	0.731	0.618	0.382
	0.6	0.710	0.648	0.338	0.714	0.646	0.354	0.722	0.617	0.403
	0.8	0.729	0.630	0.362	0.762	0.645	0.345	0.730	0.653	0.368
	1	0.729	0.640	0.364	0.725	0.634	0.350	0.720	0.603	0.369
0.75	0.1	0.556	0.419	0.179	0.558	0.424	0.152	0.595	0.418	0.190
	0.2	0.558	0.427	0.179	0.569	0.425	0.149	0.563	0.450	0.171
	0.4	0.577	0.406	0.165	0.559	0.400	0.143	0.565	0.397	0.167
	0.6	0.568	0.441	0.171	0.537	0.401	0.153	0.589	0.408	0.170
	0.8	0.582	0.436	0.161	0.553	0.423	0.159	0.587	0.421	0.156
	1	0.591	0.414	0.168	0.559	0.411	0.140	0.552	0.422	0.166
$T = 400$										
0.25	0.1	0.836	0.770	0.625	0.813	0.805	0.655	0.826	0.780	0.601
	0.2	0.823	0.791	0.603	0.850	0.791	0.623	0.828	0.751	0.603
	0.4	0.829	0.796	0.614	0.835	0.788	0.628	0.839	0.777	0.604
	0.6	0.828	0.761	0.623	0.841	0.768	0.620	0.804	0.764	0.601
	0.8	0.837	0.795	0.590	0.827	0.803	0.633	0.811	0.767	0.604
	1	0.826	0.753	0.622	0.834	0.785	0.622	0.822	0.767	0.600
0.50	0.1	0.805	0.734	0.520	0.798	0.725	0.469	0.787	0.757	0.567
	0.2	0.806	0.750	0.559	0.800	0.701	0.484	0.836	0.749	0.517
	0.4	0.798	0.730	0.525	0.807	0.709	0.498	0.784	0.724	0.558
	0.6	0.830	0.720	0.515	0.803	0.705	0.467	0.821	0.767	0.514
	0.8	0.801	0.755	0.534	0.788	0.727	0.496	0.816	0.756	0.528
	1	0.808	0.745	0.510	0.818	0.729	0.488	0.819	0.750	0.579
0.75	0.1	0.719	0.594	0.333	0.644	0.537	0.255	0.677	0.553	0.259
	0.2	0.720	0.595	0.312	0.676	0.555	0.251	0.690	0.574	0.265
	0.4	0.718	0.582	0.299	0.666	0.542	0.263	0.695	0.556	0.281
	0.6	0.726	0.568	0.307	0.705	0.508	0.256	0.697	0.565	0.299
	0.8	0.693	0.591	0.284	0.642	0.550	0.267	0.670	0.569	0.288
	1	0.724	0.585	0.296	0.687	0.547	0.250	0.717	0.559	0.305

Notes: The table reports break detection probabilities for the standard univariate CUSUM-based detector (see (4)) across different time series dimensions T and different number of series p - the univariate detector is applied on an individual series randomly selected from one of the series that experienced a break. The probabilities are computed across 1,000 repetitions from DGP (21) for different parameter values. For reference: m/T is the proportion of the entire sample at which monitoring starts; p_b is the proportion of the series which exhibit a break; b is the proportion of the monitoring period at which the break occurs.

Table 2: Probability of break detection - cross-sectionally uncorrelated: Maximum multivariate detector

Multivariate detector (10)										
		p=10			p=20			p=40		
$T = 100$										
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	0.473	0.362	0.142	0.226	0.182	0.113	-	-	-
	0.2	0.706	0.585	0.266	0.353	0.270	0.120	-	-	-
	0.4	0.923	0.830	0.439	0.531	0.409	0.182	-	-	-
	0.6	0.977	0.939	0.606	0.664	0.516	0.247	-	-	-
	0.8	0.993	0.977	0.734	0.800	0.614	0.327	-	-	-
	1	0.999	0.991	0.821	0.875	0.757	0.401	-	-	-
0.50	0.1	0.495	0.348	0.098	0.627	0.425	0.119	0.313	0.183	0.077
	0.2	0.720	0.552	0.184	0.856	0.705	0.172	0.520	0.290	0.096
	0.4	0.921	0.809	0.291	0.985	0.902	0.370	0.789	0.586	0.177
	0.6	0.975	0.910	0.424	0.998	0.979	0.504	0.919	0.710	0.258
	0.8	0.996	0.956	0.553	1.000	0.994	0.628	0.975	0.857	0.359
	1	0.999	0.976	0.626	1.000	0.999	0.781	0.997	0.950	0.487
0.75	0.1	0.271	0.133	0.058	0.378	0.178	0.067	0.492	0.175	0.065
	0.2	0.454	0.197	0.070	0.623	0.273	0.076	0.723	0.351	0.075
	0.4	0.694	0.394	0.084	0.860	0.514	0.107	0.965	0.611	0.145
	0.6	0.809	0.484	0.140	0.960	0.699	0.162	0.993	0.849	0.219
	0.8	0.925	0.608	0.152	0.986	0.821	0.207	1.000	0.928	0.354
	1	0.956	0.698	0.201	0.997	0.893	0.281	1.000	0.979	0.461
$T = 200$										
0.25	0.1	0.661	0.598	0.319	0.807	0.734	0.325	0.469	0.330	0.104
	0.2	0.890	0.825	0.527	0.971	0.936	0.591	0.762	0.555	0.238
	0.4	0.983	0.968	0.768	0.999	0.995	0.849	0.955	0.845	0.441
	0.6	0.999	0.995	0.894	1.000	0.999	0.950	0.990	0.945	0.578
	0.8	1.000	1.000	0.950	1.000	1.000	0.986	1.000	0.987	0.710
	1	1.000	1.000	0.974	1.000	1.000	0.998	1.000	0.996	0.857
0.50	0.1	0.660	0.510	0.238	0.822	0.719	0.341	0.954	0.839	0.385
	0.2	0.841	0.739	0.367	0.981	0.912	0.539	0.998	0.973	0.633
	0.4	0.986	0.921	0.593	1.000	0.995	0.785	1.000	1.000	0.875
	0.6	0.996	0.982	0.747	1.000	1.000	0.904	1.000	1.000	0.968
	0.8	1.000	0.999	0.841	1.000	1.000	0.968	1.000	1.000	0.992
	1	1.000	0.999	0.912	1.000	1.000	0.989	1.000	1.000	0.999
0.75	0.1	0.418	0.282	0.081	0.591	0.372	0.090	0.814	0.513	0.114
	0.2	0.671	0.467	0.139	0.822	0.579	0.133	0.969	0.797	0.201
	0.4	0.870	0.678	0.207	0.967	0.839	0.208	0.998	0.958	0.339
	0.6	0.958	0.815	0.265	0.997	0.939	0.319	1.000	0.995	0.504
	0.8	0.978	0.904	0.336	1.000	0.985	0.407	1.000	1.000	0.640
	1	0.995	0.951	0.407	1.000	0.993	0.493	1.000	1.000	0.730
$T = 400$										
0.25	0.1	0.779	0.707	0.507	0.938	0.907	0.743	0.994	0.978	0.837
	0.2	0.944	0.920	0.740	0.996	0.995	0.934	1.000	1.000	0.975
	0.4	0.997	0.995	0.933	1.000	1.000	0.991	1.000	1.000	1.000
	0.6	0.999	0.999	0.984	1.000	1.000	1.000	1.000	1.000	1.000
	0.8	1.000	1.000	0.997	1.000	1.000	1.000	1.000	1.000	1.000
	1	1.000	1.000	0.998	1.000	1.000	1.000	1.000	1.000	1.000
0.50	0.1	0.709	0.649	0.390	0.916	0.861	0.548	0.990	0.971	0.752
	0.2	0.930	0.864	0.603	0.995	0.977	0.822	1.000	1.000	0.936
	0.4	0.997	0.977	0.827	1.000	1.000	0.963	1.000	1.000	0.996
	0.6	0.998	0.998	0.932	1.000	1.000	0.991	1.000	1.000	1.000
	0.8	1.000	0.999	0.973	1.000	1.000	0.998	1.000	1.000	1.000
	1	1.000	1.000	0.987	1.000	1.000	1.000	1.000	1.000	1.000
0.75	0.1	0.624	0.432	0.161	0.788	0.625	0.244	0.939	0.823	0.271
	0.2	0.816	0.665	0.260	0.956	0.850	0.343	0.998	0.945	0.430
	0.4	0.959	0.877	0.404	0.999	0.976	0.584	1.000	0.997	0.696
	0.6	0.992	0.967	0.554	1.000	0.999	0.727	1.000	1.000	0.853
	0.8	1.000	0.989	0.644	1.000	1.000	0.827	1.000	1.000	0.939
	1	1.000	0.995	0.735	1.000	1.000	0.890	1.000	1.000	0.967

Notes: The table reports break detection probabilities for the maximum multivariate detector (10) across different time series dimensions T and different number of series p - the detector is applied and evaluated across series jointly. The probabilities are computed across 1,000 repetitions from DGP (21) for different parameter values. For reference: m/T is the proportion of the entire sample at which monitoring starts; p_b is the proportion of the series which exhibit a break; b is the proportion of the monitoring period at which the break occurs.

3.2 Results

The aim of our Monte Carlo experiments is to uncover the finite sample performance of three break detecting statistics within a monitoring framework: the standard absolute CUSUM statistic, the maximum multivariate statistic (10) and the average multivariate statistic (18). We do that by presenting results on the average probability of break detection across the Monte Carlo replications based on the DGPs described in Section 3.1.⁹

Let us start by examining the uncorrelated case, i.e., DGP (21). Table 1 reports the probability of detecting a break for the univariate case in the absence on any cross-sectional correlation, which execute by picking in each Monte Carlo one of the series that exhibits a break applying the univariate detector on this series. As a consequence of this cross-correlation and variations in the proportions of series breaking (p_b) cases are irrelevant as there is only one series tested, but we structure the table in this way to facilitate comparison with the multivariate results later on.¹⁰ Tables 2 and 3 report the multivariate cases based on, respectively, the maximum detector (10) and the average detector (18). The messages are straightforward and accord with our intuition. First, it is clear that as b (measuring the lateness of the break within the monitoring period) increases, performance falls in both the univariate and multivariate cases. Similarly, as the proportionate length of the monitoring period (m/T) rises, performance falls. When only a small number of series co-break, as measured by the proportion p_b , the multivariate methods are inferior to the univariate detector. But the performance of the multivariate detectors becomes rapidly superior when that proportion increases, so that in most cases examined performance is superior at $p_b \geq 0.2$. The increase in detection probability is very marked in many cases. For example, for the realistic case where $T = 100$ and $p = 20$, where there is a relatively long sample before monitoring starts ($m/T = 0.75$) and where 80% of the series co-break, the probability *vis-à-vis* the univariate detector roughly trebles for $b = 0.50$, and rises from a detection probability of about 40% to almost 100% for $b = 0.25$. The results are all the more striking, given they are compared to univariate detection based on a series known to exhibit a break.

When we compare the performance of the maximum multivariate detector in Table 2 with that of the average multivariate detector in Table 3, the results have an easily understood interpretation. The maximum detector (10) preforms relatively better when only a small subset of the series under investigation undergo structural change. On the other hand, the

⁹An alternative metric that we explored is the relative mean delay of break detection, but as this leads to very similar conclusions we do not report results here

¹⁰Variation within each column for differing p_b is due to random variation in draws.

Table 3: Probability of break detection - cross-sectionally uncorrelated: Average multivariate detector

Multivariate detector (18)										
		p=10			p=20			p=40		
T = 100										
m/T	p _b	b = 0.25	b = 0.50	b = 0.75	b = 0.25	b = 0.50	b = 0.75	b = 0.25	b = 0.50	b = 0.75
0.25	0.1	0.392	0.318	0.146	0.410	0.269	0.095	-	-	-
	0.2	0.693	0.539	0.217	0.672	0.536	0.184	-	-	-
	0.4	0.930	0.863	0.471	0.922	0.791	0.387	-	-	-
	0.6	0.985	0.947	0.683	0.982	0.933	0.556	-	-	-
	0.8	0.997	0.986	0.810	0.999	0.971	0.651	-	-	-
	1	0.999	1.000	0.906	0.998	0.991	0.715	-	-	-
0.50	0.1	0.319	0.204	0.065	0.552	0.385	0.108	0.820	0.586	0.176
	0.2	0.591	0.376	0.112	0.881	0.664	0.198	0.982	0.864	0.354
	0.4	0.900	0.732	0.217	0.992	0.951	0.479	1.000	0.996	0.661
	0.6	0.977	0.909	0.388	1.000	0.993	0.716	1.000	1.000	0.851
	0.8	0.997	0.969	0.576	1.000	0.999	0.844	1.000	1.000	0.944
	1	1.000	0.989	0.706	1.000	1.000	0.929	1.000	1.000	0.980
0.75	0.1	0.162	0.079	0.045	0.279	0.123	0.065	0.627	0.343	0.107
	0.2	0.361	0.150	0.060	0.555	0.260	0.062	0.940	0.667	0.169
	0.4	0.643	0.327	0.084	0.902	0.570	0.128	0.999	0.929	0.379
	0.6	0.821	0.494	0.135	0.980	0.794	0.216	1.000	0.992	0.585
	0.8	0.946	0.699	0.149	0.999	0.923	0.286	1.000	1.000	0.735
	1	0.976	0.788	0.232	1.000	0.969	0.391	1.000	1.000	0.863
T = 200										
0.25	0.1	0.510	0.351	0.150	0.758	0.603	0.286	0.935	0.855	0.451
	0.2	0.766	0.691	0.307	0.952	0.909	0.569	0.996	0.987	0.781
	0.4	0.974	0.940	0.649	1.000	0.998	0.909	1.000	1.000	0.985
	0.6	0.997	0.994	0.870	1.000	1.000	0.984	1.000	1.000	0.999
	0.8	0.999	0.998	0.956	1.000	1.000	0.999	1.000	1.000	1.000
	1	1.000	1.000	0.985	1.000	1.000	1.000	1.000	1.000	1.000
0.50	0.1	0.424	0.263	0.080	0.626	0.425	0.111	0.915	0.762	0.252
	0.2	0.723	0.534	0.169	0.932	0.799	0.271	1.000	0.979	0.610
	0.4	0.957	0.874	0.422	1.000	0.987	0.685	1.000	1.000	0.966
	0.6	0.995	0.976	0.674	1.000	0.999	0.901	1.000	1.000	0.998
	0.8	0.999	0.996	0.830	1.000	1.000	0.978	1.000	1.000	0.999
	1	1.000	1.000	0.922	1.000	1.000	0.996	1.000	1.000	1.000
0.75	0.1	0.243	0.118	0.062	0.399	0.182	0.070	0.609	0.289	0.056
	0.2	0.480	0.249	0.093	0.725	0.418	0.089	0.948	0.717	0.121
	0.4	0.851	0.558	0.151	0.963	0.830	0.215	1.000	0.984	0.388
	0.6	0.959	0.774	0.219	0.998	0.966	0.368	1.000	0.997	0.636
	0.8	0.990	0.917	0.347	1.000	0.991	0.584	1.000	1.000	0.839
	1	0.998	0.969	0.454	1.000	1.000	0.714	1.000	1.000	0.949
T = 400										
0.25	0.1	0.582	0.463	0.200	0.787	0.697	0.308	0.971	0.943	0.649
	0.2	0.855	0.797	0.437	0.979	0.946	0.672	1.000	0.999	0.947
	0.4	0.988	0.982	0.840	1.000	1.000	0.975	1.000	1.000	1.000
	0.6	0.999	0.999	0.964	1.000	1.000	1.000	1.000	1.000	1.000
	0.8	1.000	1.000	0.996	1.000	1.000	1.000	1.000	1.000	1.000
	1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.50	0.1	0.514	0.360	0.130	0.725	0.517	0.137	0.936	0.787	0.266
	0.2	0.831	0.695	0.306	0.965	0.868	0.425	0.997	0.989	0.724
	0.4	0.993	0.957	0.703	1.000	0.999	0.904	1.000	1.000	0.993
	0.6	0.999	0.995	0.899	1.000	1.000	0.993	1.000	1.000	1.000
	0.8	0.999	0.999	0.966	1.000	1.000	0.999	1.000	1.000	1.000
	1	1.000	1.000	0.994	1.000	1.000	1.000	1.000	1.000	1.000
0.75	0.1	0.363	0.196	0.067	0.500	0.302	0.091	0.694	0.410	0.086
	0.2	0.667	0.400	0.126	0.859	0.632	0.161	0.980	0.847	0.214
	0.4	0.931	0.784	0.312	0.998	0.951	0.471	1.000	0.998	0.687
	0.6	0.995	0.946	0.517	1.000	0.998	0.754	1.000	1.000	0.939
	0.8	0.998	0.991	0.696	1.000	0.999	0.908	1.000	1.000	0.992
	1	1.000	0.997	0.824	1.000	1.000	0.961	1.000	1.000	1.000

Notes: The table reports break detection probabilities for the average multivariate detector (18) across different time series dimensions T and different number of series p - the detector is applied and evaluated across series jointly. The probabilities are computed across 1,000 repetitions from DGP (21) for different parameter values. For reference: m/T is the proportion of the entire sample at which monitoring starts; p_b is the proportion of the series which exhibit a break; b is the proportion of the monitoring period at which the break occurs.

Table 4: Probability of break detection - common factor error structure: Univariate detector

		Univariate detector								
		p=10			p=20			p=40		
		$T = 100$								
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	0.668	0.578	0.343	0.657	0.598	0.357	-	-	-
	0.2	0.670	0.615	0.341	0.693	0.620	0.350	-	-	-
	0.4	0.653	0.590	0.345	0.651	0.580	0.368	-	-	-
	0.6	0.639	0.611	0.339	0.651	0.585	0.361	-	-	-
	0.8	0.678	0.616	0.358	0.668	0.585	0.350	-	-	-
	1	0.688	0.608	0.342	0.691	0.572	0.328	-	-	-
0.50	0.1	0.636	0.490	0.233	0.609	0.516	0.209	0.641	0.471	0.226
	0.2	0.579	0.510	0.213	0.612	0.495	0.213	0.620	0.478	0.209
	0.4	0.580	0.467	0.188	0.586	0.488	0.233	0.600	0.520	0.224
	0.6	0.601	0.500	0.214	0.587	0.470	0.198	0.601	0.477	0.224
	0.8	0.604	0.483	0.234	0.581	0.455	0.231	0.633	0.483	0.207
	1	0.638	0.475	0.212	0.625	0.472	0.223	0.619	0.496	0.208
0.75	0.1	0.406	0.259	0.080	0.435	0.253	0.100	0.414	0.244	0.103
	0.2	0.396	0.230	0.089	0.442	0.229	0.094	0.449	0.277	0.122
	0.4	0.436	0.237	0.066	0.428	0.252	0.096	0.444	0.286	0.119
	0.6	0.420	0.234	0.093	0.430	0.240	0.080	0.434	0.289	0.117
	0.8	0.408	0.225	0.101	0.417	0.234	0.104	0.442	0.279	0.124
	1	0.391	0.228	0.105	0.394	0.275	0.092	0.436	0.277	0.123
		$T = 200$								
0.25	0.1	0.768	0.693	0.520	0.779	0.696	0.505	0.768	0.691	0.492
	0.2	0.769	0.702	0.495	0.763	0.687	0.496	0.759	0.692	0.493
	0.4	0.765	0.680	0.519	0.772	0.704	0.514	0.745	0.700	0.504
	0.6	0.763	0.696	0.524	0.786	0.718	0.469	0.767	0.709	0.522
	0.8	0.757	0.704	0.522	0.772	0.686	0.491	0.761	0.677	0.508
	1	0.748	0.712	0.491	0.757	0.693	0.487	0.766	0.687	0.502
0.50	0.1	0.724	0.605	0.360	0.738	0.662	0.357	0.751	0.619	0.388
	0.2	0.717	0.633	0.372	0.720	0.611	0.364	0.706	0.627	0.359
	0.4	0.684	0.640	0.360	0.727	0.613	0.365	0.738	0.623	0.364
	0.6	0.745	0.619	0.374	0.727	0.624	0.354	0.728	0.634	0.377
	0.8	0.755	0.614	0.370	0.715	0.622	0.361	0.734	0.640	0.395
	1	0.724	0.635	0.355	0.730	0.628	0.386	0.718	0.609	0.404
0.75	0.1	0.581	0.440	0.149	0.571	0.426	0.162	0.562	0.401	0.159
	0.2	0.549	0.384	0.173	0.575	0.419	0.161	0.539	0.395	0.134
	0.4	0.584	0.388	0.136	0.584	0.446	0.170	0.587	0.406	0.162
	0.6	0.554	0.378	0.131	0.540	0.417	0.172	0.562	0.417	0.140
	0.8	0.602	0.371	0.142	0.565	0.396	0.157	0.567	0.397	0.153
	1	0.536	0.420	0.123	0.573	0.433	0.150	0.567	0.427	0.146
		$T = 400$								
0.25	0.1	0.816	0.800	0.616	0.855	0.803	0.634	0.832	0.808	0.638
	0.2	0.827	0.800	0.620	0.828	0.802	0.625	0.825	0.813	0.637
	0.4	0.819	0.786	0.660	0.838	0.801	0.638	0.829	0.789	0.607
	0.6	0.832	0.795	0.601	0.838	0.792	0.627	0.852	0.791	0.608
	0.8	0.835	0.787	0.635	0.854	0.798	0.644	0.842	0.790	0.651
	1	0.819	0.777	0.633	0.830	0.793	0.628	0.847	0.788	0.672
0.50	0.1	0.815	0.731	0.491	0.788	0.733	0.506	0.824	0.712	0.476
	0.2	0.794	0.732	0.497	0.810	0.763	0.533	0.788	0.720	0.503
	0.4	0.786	0.714	0.501	0.799	0.739	0.508	0.790	0.722	0.519
	0.6	0.811	0.731	0.526	0.827	0.741	0.503	0.781	0.732	0.514
	0.8	0.808	0.712	0.494	0.801	0.717	0.504	0.804	0.731	0.497
	1	0.815	0.730	0.518	0.806	0.734	0.502	0.806	0.725	0.508
0.75	0.1	0.686	0.547	0.245	0.674	0.518	0.248	0.686	0.587	0.318
	0.2	0.684	0.535	0.273	0.670	0.554	0.264	0.692	0.602	0.321
	0.4	0.685	0.528	0.239	0.662	0.538	0.240	0.690	0.589	0.301
	0.6	0.678	0.550	0.269	0.655	0.523	0.273	0.687	0.572	0.303
	0.8	0.686	0.552	0.269	0.675	0.559	0.236	0.696	0.591	0.312
	1	0.686	0.555	0.258	0.686	0.560	0.272	0.674	0.580	0.306

Notes: See the notes for Table 1, but now based on DGP (22).

average detector (18) works better when a large proportion of the series undergo structural change. As a result we view these two summaries of the individual cumulative sums as complementary, each having the upper hand in naturally interpretable circumstances. We also note that the reduction in detection ability for small numbers of co-breaking series for the average multivariate measure is typically greater than the increase in detection for large numbers. Thus the maximum multivariate statistic may be considered to be more robust, especially if there is an *a priori* belief that only a minority of series are co-breaking.

The results for the common factor DGP (22) are reported in Tables 4-6, reflecting the performances of, respectively, the standard univariate detector, the maximum multivariate detector and the average multivariate statistic. Qualitatively and quantitatively, the results are similar to the uncorrelated case in Tables 1-3. Once again, for the multivariate detectors the probability of detection tends to rise with fraction of series that experience a break, p_b , and p . In general, all patterns reported for the case of uncorrelated errors, remain for the case of correlated errors, albeit that in the common factor case the performance of the average statistic (18) improves to a certain extent relative to the maximum statistic (10). This is particularly so when in combination with a fairly large number of series, the monitoring period is relatively short and the breaks occur relatively late in this monitoring period.

A potentially more attractive, alternative, benchmark that can be used to assess the performance of our multivariate statistics in case of DGP (22) is to extract a common factor from the set of series and use the standard univariate detector to monitor for structural change in this factor. The logic is that a common break explains a high proportion of the variance in the series and will thus be captured. Clearly, this is not in the spirit of our test, where we are looking for breaks in parameters and although there is a common factor in the experimental design, this is not the source of the break. Nevertheless, in Table 7 we report the results of extracting a common factor from the cross correlated data set, which we construct as the dominant principal component extracted from the $Y_{j,t}$'s in (22), and monitoring this common factor for structural change based on the standard univariate CUSUM statistic. We find that the probability of detection increases as the proportion of series co-breaking increases, but in all cases this break detection strategy is clearly dominated by those based on our two multivariate detectors.

So far, the co-breaks in our experimental designs have occurred at a common point. But in practice and in our theoretical structure there is no necessity for breaks to coincide pre-

Table 5: Probability of break detection - common factor error structure: Maximum multivariate detector

		Multivariate detector (10)								
		p=10			p=20			p=40		
		$T = 100$								
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	0.493	0.389	0.142	0.286	0.223	0.102	-	-	-
	0.2	0.753	0.641	0.291	0.442	0.334	0.158	-	-	-
	0.4	0.946	0.876	0.484	0.647	0.494	0.248	-	-	-
	0.6	0.987	0.952	0.642	0.763	0.607	0.295	-	-	-
	0.8	0.997	0.983	0.722	0.821	0.669	0.368	-	-	-
	1	0.997	0.985	0.756	0.849	0.710	0.359	-	-	-
0.50	0.1	0.573	0.397	0.166	0.734	0.536	0.151	0.470	0.327	0.099
	0.2	0.780	0.653	0.253	0.930	0.796	0.297	0.755	0.498	0.188
	0.4	0.960	0.863	0.382	0.997	0.972	0.520	0.939	0.777	0.284
	0.6	0.987	0.944	0.526	0.999	0.995	0.684	0.983	0.897	0.376
	0.8	0.995	0.978	0.607	1.000	0.996	0.790	0.990	0.921	0.435
	1	1.000	0.988	0.666	1.000	1.000	0.802	1.000	0.924	0.510
0.75	0.1	0.350	0.180	0.098	0.480	0.232	0.085	0.652	0.334	0.091
	0.2	0.555	0.283	0.107	0.750	0.419	0.125	0.905	0.581	0.140
	0.4	0.789	0.494	0.158	0.949	0.671	0.201	0.998	0.869	0.256
	0.6	0.911	0.614	0.188	0.980	0.820	0.223	1.000	0.964	0.411
	0.8	0.940	0.693	0.209	0.993	0.871	0.294	1.000	0.982	0.480
	1	0.961	0.730	0.273	0.997	0.875	0.303	1.000	0.973	0.521
		$T = 200$								
0.25	0.1	0.710	0.633	0.388	0.871	0.796	0.412	0.734	0.599	0.260
	0.2	0.933	0.884	0.620	0.980	0.957	0.718	0.943	0.833	0.455
	0.4	0.995	0.980	0.872	1.000	0.999	0.941	0.994	0.978	0.704
	0.6	0.999	1.000	0.951	1.000	1.000	0.990	0.997	0.991	0.846
	0.8	1.000	1.000	0.984	1.000	1.000	0.994	1.000	0.999	0.891
	1	1.000	1.000	0.983	1.000	1.000	0.999	1.000	0.999	0.913
0.50	0.1	0.678	0.553	0.288	0.877	0.798	0.397	0.977	0.901	0.426
	0.2	0.908	0.821	0.449	0.986	0.944	0.646	0.997	0.993	0.733
	0.4	0.986	0.965	0.690	1.000	0.995	0.876	1.000	1.000	0.947
	0.6	0.998	0.994	0.838	1.000	0.999	0.957	1.000	1.000	0.991
	0.8	1.000	0.996	0.904	1.000	1.000	0.980	1.000	1.000	0.996
	1	1.000	0.999	0.924	1.000	1.000	0.986	1.000	1.000	0.998
0.75	0.1	0.504	0.344	0.111	0.665	0.447	0.096	0.897	0.632	0.170
	0.2	0.712	0.492	0.140	0.904	0.647	0.175	0.986	0.891	0.273
	0.4	0.916	0.732	0.216	0.994	0.892	0.314	1.000	0.989	0.471
	0.6	0.985	0.855	0.302	0.999	0.968	0.415	1.000	0.998	0.593
	0.8	0.991	0.921	0.335	1.000	0.984	0.460	1.000	0.999	0.700
	1	0.994	0.930	0.390	1.000	0.991	0.509	1.000	1.000	0.751
		$T = 400$								
0.25	0.1	0.779	0.739	0.535	0.953	0.922	0.745	0.997	0.989	0.886
	0.2	0.969	0.937	0.821	1.000	0.994	0.945	1.000	1.000	0.992
	0.4	0.999	0.994	0.963	1.000	1.000	0.999	1.000	1.000	1.000
	0.6	1.000	1.000	0.992	1.000	1.000	1.000	1.000	1.000	1.000
	0.8	1.000	1.000	0.996	1.000	1.000	1.000	1.000	1.000	1.000
	1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.50	0.1	0.778	0.674	0.401	0.928	0.873	0.591	0.994	0.981	0.788
	0.2	0.932	0.898	0.645	0.999	0.991	0.860	1.000	0.999	0.956
	0.4	0.997	0.986	0.862	1.000	1.000	0.968	1.000	1.000	0.998
	0.6	1.000	0.998	0.957	1.000	1.000	0.997	1.000	1.000	1.000
	0.8	1.000	1.000	0.971	1.000	1.000	0.999	1.000	1.000	1.000
	1	1.000	1.000	0.989	1.000	1.000	1.000	1.000	1.000	1.000
0.75	0.1	0.601	0.467	0.151	0.829	0.656	0.258	0.954	0.846	0.338
	0.2	0.847	0.694	0.291	0.972	0.897	0.411	1.000	0.979	0.576
	0.4	0.980	0.910	0.441	1.000	0.988	0.634	1.000	1.000	0.821
	0.6	0.995	0.962	0.592	1.000	0.999	0.775	1.000	1.000	0.899
	0.8	1.000	0.990	0.694	1.000	1.000	0.824	1.000	1.000	0.957
	1	1.000	0.996	0.720	1.000	0.999	0.869	1.000	1.000	0.972

Notes: See the notes for Table 2, but now based on DGP (22).

Table 6: Probability of break detection - common factor error structure: Average multivariate detector

		Multivariate detector (18)								
		p=10			p=20			p=40		
		$T = 100$								
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	0.590	0.487	0.226	0.627	0.466	0.207	-	-	-
	0.2	0.852	0.735	0.429	0.868	0.760	0.342	-	-	-
	0.4	0.972	0.937	0.688	0.984	0.945	0.576	-	-	-
	0.6	0.997	0.988	0.810	0.999	0.983	0.703	-	-	-
	0.8	1.000	0.996	0.873	1.000	0.996	0.746	-	-	-
	1	0.999	0.997	0.896	1.000	0.998	0.762	-	-	-
0.50	0.1	0.541	0.387	0.142	0.781	0.596	0.227	0.939	0.797	0.302
	0.2	0.835	0.633	0.250	0.953	0.882	0.441	0.998	0.971	0.543
	0.4	0.964	0.922	0.423	1.000	0.994	0.739	1.000	1.000	0.851
	0.6	0.995	0.973	0.583	1.000	1.000	0.853	1.000	1.000	0.938
	0.8	0.998	0.989	0.673	1.000	1.000	0.928	1.000	1.000	0.952
	1	1.000	0.988	0.704	1.000	1.000	0.915	1.000	1.000	0.967
0.75	0.1	0.296	0.139	0.051	0.481	0.265	0.071	0.855	0.590	0.169
	0.2	0.527	0.297	0.068	0.816	0.509	0.108	0.982	0.883	0.327
	0.4	0.813	0.533	0.119	0.983	0.811	0.229	1.000	0.995	0.614
	0.6	0.909	0.640	0.166	0.996	0.921	0.321	1.000	1.000	0.784
	0.8	0.966	0.736	0.188	1.000	0.961	0.345	1.000	0.999	0.802
	1	0.972	0.764	0.205	1.000	0.964	0.399	1.000	1.000	0.819
		$T = 200$								
0.25	0.1	0.660	0.570	0.331	0.889	0.816	0.514	0.976	0.944	0.706
	0.2	0.888	0.840	0.555	0.989	0.975	0.815	1.000	0.999	0.948
	0.4	0.989	0.987	0.843	1.000	1.000	0.980	1.000	1.000	0.998
	0.6	1.000	0.998	0.944	1.000	1.000	1.000	1.000	1.000	1.000
	0.8	1.000	1.000	0.978	1.000	1.000	0.999	1.000	1.000	1.000
	1	1.000	1.000	0.991	1.000	1.000	1.000	1.000	1.000	1.000
0.50	0.1	0.628	0.509	0.239	0.837	0.704	0.353	0.990	0.920	0.608
	0.2	0.880	0.758	0.418	0.982	0.941	0.666	1.000	0.997	0.896
	0.4	0.991	0.958	0.717	0.999	0.999	0.917	1.000	1.000	0.999
	0.6	0.998	0.991	0.864	1.000	1.000	0.981	1.000	1.000	1.000
	0.8	1.000	1.000	0.909	1.000	1.000	0.995	1.000	1.000	1.000
	1	1.000	0.999	0.935	1.000	1.000	0.993	1.000	1.000	1.000
0.75	0.1	0.445	0.266	0.075	0.680	0.434	0.115	0.891	0.660	0.133
	0.2	0.730	0.494	0.120	0.923	0.743	0.206	0.989	0.942	0.301
	0.4	0.925	0.796	0.244	0.996	0.963	0.443	1.000	1.000	0.717
	0.6	0.985	0.910	0.328	1.000	0.995	0.606	1.000	1.000	0.869
	0.8	0.994	0.950	0.426	1.000	1.000	0.679	1.000	1.000	0.913
	1	0.999	0.947	0.415	1.000	0.996	0.711	1.000	1.000	0.931
		$T = 400$								
0.25	0.1	0.741	0.667	0.437	0.916	0.871	0.644	0.995	0.980	0.883
	0.2	0.931	0.904	0.725	0.998	0.982	0.928	1.000	0.999	0.994
	0.4	0.998	0.995	0.935	1.000	1.000	0.999	1.000	1.000	1.000
	0.6	1.000	1.000	0.987	1.000	1.000	1.000	1.000	1.000	1.000
	0.8	1.000	1.000	0.998	1.000	1.000	1.000	1.000	1.000	1.000
	1	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
0.50	0.1	0.763	0.637	0.362	0.879	0.830	0.486	0.986	0.957	0.695
	0.2	0.916	0.861	0.610	0.996	0.970	0.793	1.000	1.000	0.959
	0.4	0.998	0.989	0.889	1.000	1.000	0.988	1.000	1.000	0.999
	0.6	1.000	0.999	0.966	1.000	1.000	0.999	1.000	1.000	1.000
	0.8	1.000	1.000	0.989	1.000	1.000	1.000	1.000	1.000	1.000
	1	1.000	1.000	0.990	1.000	1.000	1.000	1.000	1.000	1.000
0.75	0.1	0.589	0.435	0.145	0.788	0.585	0.191	0.950	0.817	0.289
	0.2	0.842	0.720	0.288	0.969	0.880	0.409	0.997	0.988	0.660
	0.4	0.981	0.929	0.539	0.999	0.995	0.785	1.000	1.000	0.950
	0.6	0.999	0.984	0.673	1.000	0.999	0.903	1.000	1.000	0.992
	0.8	1.000	0.998	0.790	1.000	1.000	0.963	1.000	1.000	0.999
	1	1.000	0.995	0.789	1.000	1.000	0.951	1.000	1.000	0.999

Notes: See the notes for Table 3, but now based on DGP (22).

Table 7: Probability of break detection - common factor error structure: Univariate monitoring of common factor

Univariate detector applied on estimated factor										
		p=10			p=20			p=40		
$T = 100$										
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	0.167	0.089	0.063	0.139	0.103	0.060	-	-	-
	0.2	0.323	0.201	0.093	0.314	0.171	0.068	-	-	-
	0.4	0.595	0.441	0.143	0.655	0.440	0.144	-	-	-
	0.6	0.825	0.661	0.254	0.875	0.723	0.242	-	-	-
	0.8	0.908	0.820	0.405	0.971	0.874	0.366	-	-	-
	1	0.970	0.910	0.500	0.993	0.954	0.529	-	-	-
0.50	0.1	0.111	0.092	0.055	0.096	0.050	0.038	0.084	0.046	0.050
	0.2	0.203	0.127	0.055	0.196	0.108	0.040	0.209	0.103	0.061
	0.4	0.445	0.227	0.071	0.511	0.270	0.069	0.531	0.264	0.094
	0.6	0.659	0.460	0.136	0.726	0.434	0.105	0.788	0.463	0.114
	0.8	0.829	0.604	0.200	0.880	0.632	0.195	0.941	0.675	0.184
	1	0.921	0.735	0.254	0.963	0.781	0.249	0.980	0.848	0.267
0.75	0.1	0.057	0.048	0.045	0.065	0.049	0.054	0.075	0.058	0.050
	0.2	0.086	0.060	0.049	0.094	0.051	0.048	0.101	0.083	0.057
	0.4	0.203	0.100	0.042	0.189	0.090	0.059	0.235	0.094	0.059
	0.6	0.327	0.137	0.070	0.342	0.156	0.073	0.372	0.169	0.077
	0.8	0.460	0.217	0.079	0.485	0.218	0.086	0.555	0.253	0.098
	1	0.583	0.281	0.092	0.667	0.322	0.096	0.697	0.378	0.129
$T = 200$										
0.25	0.1	0.238	0.130	0.087	0.214	0.128	0.052	0.186	0.118	0.045
	0.2	0.437	0.322	0.113	0.448	0.246	0.106	0.482	0.258	0.081
	0.4	0.747	0.592	0.263	0.835	0.643	0.188	0.911	0.708	0.216
	0.6	0.906	0.822	0.420	0.957	0.883	0.405	0.997	0.924	0.424
	0.8	0.972	0.918	0.612	0.992	0.965	0.600	0.999	0.989	0.665
	1	0.986	0.964	0.734	1.000	0.991	0.811	1.000	0.998	0.834
0.50	0.1	0.167	0.108	0.051	0.141	0.098	0.058	0.143	0.080	0.055
	0.2	0.350	0.185	0.064	0.320	0.156	0.072	0.340	0.163	0.082
	0.4	0.603	0.432	0.155	0.727	0.453	0.128	0.768	0.472	0.130
	0.6	0.824	0.613	0.243	0.922	0.708	0.249	0.960	0.755	0.233
	0.8	0.935	0.818	0.366	0.982	0.884	0.393	0.998	0.923	0.374
	1	0.974	0.929	0.494	0.998	0.955	0.529	1.000	0.982	0.559
0.75	0.1	0.094	0.065	0.064	0.092	0.082	0.062	0.054	0.067	0.057
	0.2	0.151	0.092	0.055	0.141	0.083	0.059	0.126	0.081	0.051
	0.4	0.333	0.156	0.062	0.335	0.182	0.071	0.315	0.149	0.073
	0.6	0.490	0.265	0.096	0.539	0.302	0.082	0.602	0.318	0.093
	0.8	0.703	0.418	0.125	0.767	0.459	0.141	0.796	0.431	0.112
	1	0.819	0.557	0.155	0.904	0.605	0.187	0.935	0.620	0.162
$T = 400$										
0.25	0.1	0.297	0.229	0.086	0.319	0.230	0.068	0.324	0.195	0.076
	0.2	0.596	0.433	0.192	0.671	0.434	0.151	0.744	0.469	0.185
	0.4	0.872	0.764	0.428	0.936	0.833	0.387	0.988	0.908	0.437
	0.6	0.953	0.924	0.603	0.992	0.964	0.686	0.999	0.991	0.754
	0.8	0.989	0.963	0.788	1.000	0.997	0.855	1.000	0.999	0.911
	1	0.996	0.990	0.889	1.000	0.998	0.947	1.000	1.000	0.973
0.50	0.1	0.255	0.152	0.069	0.227	0.128	0.071	0.189	0.087	0.041
	0.2	0.454	0.304	0.102	0.482	0.267	0.110	0.526	0.238	0.061
	0.4	0.764	0.599	0.236	0.867	0.638	0.216	0.910	0.712	0.183
	0.6	0.924	0.811	0.390	0.974	0.887	0.421	0.995	0.935	0.412
	0.8	0.975	0.914	0.556	0.996	0.974	0.658	1.000	0.990	0.641
	1	0.990	0.969	0.738	1.000	0.993	0.810	1.000	0.999	0.815
0.75	0.1	0.118	0.084	0.051	0.086	0.061	0.035	0.113	0.091	0.077
	0.2	0.234	0.120	0.050	0.214	0.105	0.042	0.235	0.140	0.075
	0.4	0.480	0.276	0.100	0.477	0.240	0.075	0.597	0.308	0.120
	0.6	0.712	0.454	0.149	0.756	0.481	0.137	0.846	0.538	0.184
	0.8	0.844	0.659	0.240	0.908	0.659	0.210	0.961	0.777	0.252
	1	0.931	0.771	0.320	0.966	0.826	0.298	0.997	0.904	0.395

Notes: The table reports break detection probabilities for the univariate CUSUM-based detector applied to a common factor extracted from a panel across different time series dimensions T and different number of series p . The probabilities are computed across 1,000 repetitions from DGP (22) for different parameter values and the common factor is estimated as the dominant principal component from the $Y_{j,t}$'s in this DGP. For reference: m/T is the proportion of the entire sample at which monitoring starts; p_b is the proportion of the series which exhibit a break; b is the proportion of the monitoring period at which the break occurs.

Table 8: Probability of break detection - uncorrelated processes with heterogeneous break dates: Maximum multivariate detector

Multivariate detector (10)										
		p=10			p=20			p=40		
$T = 100$										
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	0.434	0.340	0.152	0.183	0.111	0.084	-	-	-
	0.2	0.690	0.557	0.234	0.328	0.206	0.095	-	-	-
	0.4	0.908	0.811	0.398	0.484	0.394	0.157	-	-	-
	0.6	0.969	0.934	0.544	0.630	0.472	0.216	-	-	-
	0.8	0.992	0.979	0.691	0.713	0.580	0.281	-	-	-
	1	1.000	0.990	0.786	0.818	0.714	0.368	-	-	-
0.50	0.1	0.487	0.334	0.123	0.611	0.424	0.117	0.352	0.217	0.108
	0.2	0.730	0.555	0.210	0.881	0.671	0.195	0.587	0.354	0.144
	0.4	0.926	0.794	0.334	0.988	0.910	0.385	0.852	0.621	0.235
	0.6	0.984	0.898	0.427	0.998	0.968	0.518	0.951	0.798	0.311
	0.8	0.995	0.969	0.536	1.000	0.992	0.688	0.984	0.885	0.423
	1	1.000	0.982	0.642	1.000	1.000	0.792	0.997	0.958	0.572
0.75	0.1	0.276	0.132	0.070	0.409	0.192	0.068	0.503	0.213	0.066
	0.2	0.465	0.200	0.079	0.627	0.325	0.112	0.774	0.391	0.105
	0.4	0.697	0.349	0.111	0.886	0.527	0.163	0.968	0.672	0.235
	0.6	0.838	0.498	0.163	0.949	0.694	0.263	0.994	0.846	0.325
	0.8	0.882	0.629	0.203	0.978	0.814	0.313	1.000	0.945	0.481
	1	0.943	0.728	0.238	0.995	0.882	0.418	1.000	0.985	0.614
$T = 200$										
0.25	0.1	0.671	0.584	0.366	0.812	0.736	0.404	0.535	0.334	0.162
	0.2	0.892	0.855	0.584	0.976	0.934	0.621	0.778	0.613	0.265
	0.4	0.987	0.968	0.794	0.999	0.998	0.872	0.953	0.874	0.455
	0.6	0.998	0.993	0.920	1.000	1.000	0.971	0.996	0.972	0.649
	0.8	1.000	1.000	0.967	1.000	1.000	0.989	1.000	0.988	0.757
	1	1.000	1.000	0.991	1.000	1.000	0.999	1.000	0.999	0.888
0.50	0.1	0.645	0.509	0.202	0.827	0.712	0.305	0.957	0.832	0.397
	0.2	0.844	0.729	0.381	0.970	0.929	0.516	0.998	0.975	0.639
	0.4	0.983	0.943	0.611	1.000	0.990	0.787	1.000	1.000	0.877
	0.6	0.994	0.989	0.745	1.000	1.000	0.918	1.000	1.000	0.973
	0.8	0.999	0.995	0.832	1.000	1.000	0.964	1.000	1.000	0.994
	1	1.000	1.000	0.919	1.000	1.000	0.985	1.000	1.000	1.000
0.75	0.1	0.445	0.266	0.097	0.635	0.374	0.086	0.818	0.517	0.121
	0.2	0.677	0.446	0.135	0.851	0.612	0.170	0.961	0.790	0.209
	0.4	0.892	0.693	0.213	0.967	0.835	0.260	1.000	0.962	0.362
	0.6	0.970	0.823	0.290	0.998	0.936	0.353	1.000	0.996	0.513
	0.8	0.988	0.894	0.359	1.000	0.977	0.458	1.000	0.999	0.639
	1	0.995	0.943	0.436	1.000	0.992	0.561	1.000	1.000	0.733
$T = 400$										
0.25	0.1	0.774	0.733	0.499	0.921	0.881	0.691	0.988	0.980	0.812
	0.2	0.952	0.910	0.755	0.997	0.983	0.916	0.999	1.000	0.966
	0.4	0.998	0.996	0.940	1.000	1.000	0.994	1.000	1.000	1.000
	0.6	1.000	0.998	0.987	1.000	1.000	1.000	1.000	1.000	1.000
	0.8	1.000	1.000	0.997	1.000	1.000	1.000	1.000	1.000	1.000
	1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.50	0.1	0.738	0.671	0.404	0.925	0.865	0.571	0.989	0.963	0.710
	0.2	0.928	0.882	0.621	0.997	0.985	0.801	0.999	0.999	0.913
	0.4	0.993	0.984	0.846	0.999	1.000	0.961	1.000	1.000	0.994
	0.6	0.999	0.996	0.955	1.000	1.000	0.990	1.000	1.000	1.000
	0.8	1.000	1.000	0.979	1.000	1.000	1.000	1.000	1.000	1.000
	1	1.000	1.000	0.991	1.000	1.000	0.998	1.000	1.000	1.000
0.75	0.1	0.586	0.399	0.160	0.794	0.612	0.220	0.950	0.808	0.298
	0.2	0.807	0.616	0.242	0.962	0.849	0.322	1.000	0.956	0.468
	0.4	0.973	0.855	0.398	0.996	0.971	0.567	1.000	0.997	0.723
	0.6	0.994	0.948	0.480	1.000	0.999	0.703	1.000	1.000	0.863
	0.8	0.999	0.983	0.635	1.000	1.000	0.807	1.000	1.000	0.938
	1	1.000	0.995	0.692	1.000	0.999	0.887	1.000	1.000	0.968

Notes: See the notes for Table 2, but now based on using break date specification (23) in DGP (21).

Table 9: Probability of break detection - common factor error structure with heterogeneous break dates: Average multivariate detector

Multivariate detector (18)										
		p=10			p=20			p=40		
$T = 100$										
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	0.601	0.508	0.248	0.623	0.502	0.234	-	-	-
	0.2	0.849	0.762	0.448	0.880	0.762	0.366	-	-	-
	0.4	0.971	0.947	0.722	0.981	0.952	0.612	-	-	-
	0.6	0.994	0.988	0.823	0.997	0.982	0.725	-	-	-
	0.8	0.999	0.993	0.872	0.999	0.993	0.784	-	-	-
	1	1.000	0.999	0.897	1.000	0.998	0.793	-	-	-
0.50	0.1	0.543	0.361	0.125	0.793	0.613	0.228	0.925	0.776	0.293
	0.2	0.807	0.636	0.231	0.968	0.871	0.428	0.997	0.968	0.548
	0.4	0.972	0.889	0.449	0.998	0.994	0.725	1.000	0.999	0.860
	0.6	0.995	0.970	0.542	1.000	0.997	0.860	1.000	1.000	0.935
	0.8	0.998	0.980	0.665	1.000	0.999	0.911	1.000	1.000	0.964
	1	0.999	0.989	0.692	1.000	1.000	0.909	1.000	1.000	0.971
0.75	0.1	0.349	0.195	0.078	0.542	0.288	0.087	0.855	0.595	0.170
	0.2	0.571	0.319	0.106	0.832	0.558	0.142	0.982	0.876	0.349
	0.4	0.842	0.540	0.157	0.978	0.847	0.268	1.000	0.986	0.612
	0.6	0.932	0.714	0.226	0.999	0.931	0.355	1.000	1.000	0.728
	0.8	0.981	0.791	0.219	1.000	0.953	0.408	1.000	1.000	0.816
	1	0.973	0.790	0.266	0.999	0.968	0.422	1.000	1.000	0.817
$T = 200$										
0.25	0.1	0.675	0.617	0.348	0.888	0.846	0.521	0.980	0.956	0.697
	0.2	0.918	0.840	0.592	0.979	0.971	0.855	0.999	0.999	0.952
	0.4	0.994	0.981	0.872	1.000	0.999	0.979	1.000	1.000	0.999
	0.6	0.999	1.000	0.953	1.000	1.000	0.999	1.000	1.000	1.000
	0.8	1.000	0.999	0.980	1.000	1.000	1.000	1.000	1.000	1.000
	1	1.000	1.000	0.987	1.000	1.000	1.000	1.000	1.000	1.000
0.50	0.1	0.640	0.500	0.200	0.843	0.696	0.287	0.990	0.924	0.555
	0.2	0.881	0.774	0.410	0.984	0.948	0.615	1.000	0.998	0.910
	0.4	0.993	0.958	0.675	1.000	0.997	0.922	1.000	1.000	0.998
	0.6	1.000	0.994	0.853	1.000	1.000	0.979	1.000	1.000	1.000
	0.8	1.000	0.998	0.916	1.000	1.000	0.994	1.000	1.000	1.000
	1	1.000	0.997	0.918	1.000	1.000	0.994	1.000	1.000	1.000
0.75	0.1	0.478	0.285	0.080	0.660	0.395	0.108	0.878	0.667	0.205
	0.2	0.712	0.487	0.129	0.935	0.725	0.235	0.997	0.942	0.390
	0.4	0.941	0.767	0.252	0.999	0.964	0.440	1.000	0.998	0.786
	0.6	0.985	0.906	0.350	1.000	0.994	0.619	1.000	1.000	0.905
	0.8	0.998	0.959	0.420	1.000	1.000	0.687	1.000	1.000	0.953
	1	0.998	0.962	0.430	1.000	1.000	0.713	1.000	1.000	0.960
$T = 400$										
0.25	0.1	0.751	0.666	0.436	0.912	0.857	0.600	0.992	0.983	0.896
	0.2	0.943	0.910	0.717	0.993	0.987	0.913	1.000	0.999	0.992
	0.4	0.997	0.995	0.947	1.000	1.000	0.998	1.000	1.000	1.000
	0.6	1.000	1.000	0.991	1.000	1.000	1.000	1.000	1.000	1.000
	0.8	1.000	1.000	0.997	1.000	1.000	1.000	1.000	1.000	1.000
	1	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
0.50	0.1	0.721	0.625	0.384	0.893	0.820	0.507	0.988	0.957	0.702
	0.2	0.930	0.887	0.643	0.993	0.980	0.797	1.000	1.000	0.964
	0.4	0.997	0.990	0.904	1.000	1.000	0.987	1.000	1.000	1.000
	0.6	1.000	0.999	0.969	1.000	1.000	1.000	1.000	1.000	1.000
	0.8	1.000	1.000	0.990	1.000	1.000	1.000	1.000	1.000	1.000
	1	1.000	1.000	0.991	1.000	1.000	1.000	1.000	1.000	1.000
0.75	0.1	0.580	0.392	0.168	0.753	0.560	0.157	0.945	0.771	0.276
	0.2	0.832	0.709	0.285	0.950	0.868	0.376	0.999	0.987	0.605
	0.4	0.978	0.918	0.551	1.000	0.992	0.751	1.000	1.000	0.958
	0.6	0.999	0.984	0.648	1.000	1.000	0.875	1.000	1.000	0.997
	0.8	1.000	0.995	0.768	1.000	1.000	0.939	1.000	1.000	0.999
	1	1.000	0.996	0.776	1.000	1.000	0.948	1.000	1.000	1.000

Notes: See the notes for Table 3, but now based on using break date specification (23) in DGP (22).

cisely at the same point in time across the series that exhibit a break. We therefore explore whether indeed the performance of our multivariate detectors (10) and (18) remain unaffected by cross-sectional heterogeneity in break dates. Tables 8 and 9 report probabilities for break detection based on stochastic heterogeneity in the break dates, as specified in (23). Clearly, we can use the break date specification (23) in conjunction with both DGP (21) and (22), but to save space we only report two cases: no cross-correlation with the maximum statistic and the common factor for the average statistic.¹¹ In the latter case, we chose the common factor DGP as the average detector performed relatively better than under the no cross-correlation set-up. Therefore, Table 8 needs to be compared with Table 2 and Table 9 with Table 6. It is evident that introducing heterogeneity in break dates makes little difference for the performance of the maximum detector (10). The average detector (18), however, performs somewhat better under this break date heterogeneity.

The simulation results in this section confirm our intuition regarding the usefulness of our multivariate break detectors (10) and (18) when a relatively large number of series experience a break at or around a similar point in time. ‘Relatively large’ in this context does not necessarily mean ‘a lot’ - our simulations show that when only 20% of the series in a panel are affected by a common break our methods outperform standard approaches to break monitoring. All of this is robust to the inclusion of common factor errors and a certain degree of heterogeneity in break dates.

4 Empirical Application

In this section we present an empirical application that illustrates the potential of the multivariate break detector. As we have noted earlier, one possible way in which the multivariate break detector may be of use is to consider whether disaggregated data may be used to provide information on the presence of breaks in an aggregate series. A natural testbed for this is price inflation data, where there is a large number of easily obtainable and interpretable component series, which aggregate in a relatively straightforward way. We use UK RPI data, for which we have 77 component indices available over our sample. For this empirical application we focus on the multivariate detector based on the maximum of the absolute cumulative sums of residuals.

Monthly data for the UK RPI series and its components is available on a consistent basis

¹¹The results are qualitatively and quantitatively similar in the other two unreported cases and are available upon request.

at this level of aggregation from January 1987. We drop the first fifty periods so that our annual inflation series begins in March 1992 (so forecast errors commence in April 1992) and ending in August 2007. We choose this starting point as 1992 marks the transition to a formal inflation targeting regime in the United Kingdom. In addition to making a natural historical point to choose, the previous period exhibited more inflation volatility than the post-1992 period (King (2002)), so that arguably a potential structural break is excluded from the sample. This is important as in the test framework the series under investigation is assumed not to have undergone structural change prior to the start of monitoring.¹²

Some further discussion of the monetary regime and history may be relevant. The inflation target introduced in October 1992 was a range of 1 to 4% for RPIX¹³ inflation. In 1997, on Bank independence, this was changed to the mid-point, 2.5%. Although this could have been interpreted as a change in regime, in practice the post-1992 period is often considered to be a single monetary regime.¹⁴ In June 2003 it was announced that in November 2003 the target would change to 2.0% Consumer Price Index (CPI)¹⁵ inflation, but this may have had only a small change on the regime. There is a wedge between the two measures of inflation, and although the average varies over time, CPI inflation had largely tended to lie below RPIX inflation. Nevertheless, as CPI inflation is constructed differently from RPIX, an interesting policy question is whether the changed target would manifest itself in a change in the average level of RPI or RPIX inflation. However, other events occurred in the monitoring period. These include a period of volatile house price inflation, which affects both RPIX and, to a greater extent, RPI inflation.¹⁶ There was also a large and continued rise in the price of energy and some other commodities, which may be dated to around 2004.¹⁷ Thus there are several reasons to think that there may have been a structural break in this period. From the point of view of the policymaker, a structural break resulting from the regime change announced in June 2003 would be of particular interest. So to keep this exercise aligned to a practical issue, a timely discovery of that potential break, we restrict our sample to end four years after the announcement.

We set the monitoring period to start in September 2001, roughly three or four years before the change in regime, the beginning of the energy price rises and the decline in house

¹²Or, if they did, then that structural change was properly modeled.

¹³A measure of RPI that excludes most elements of housing costs.

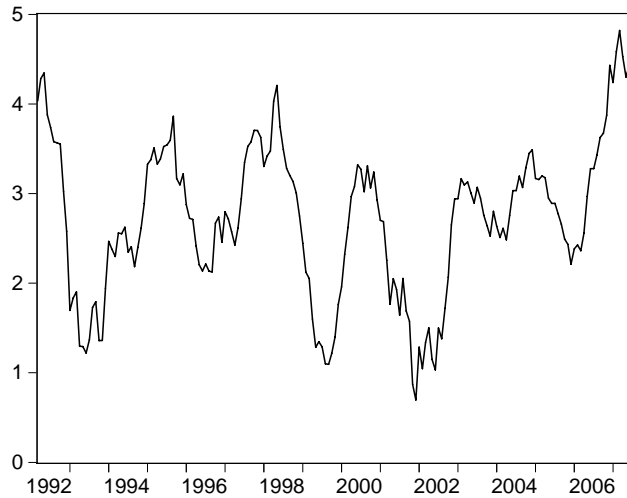
¹⁴Characterised as inflation targeting: see King (2002).

¹⁵The UK CPI is defined equivalently to the euro area HCPI.

¹⁶This alone means that the wedge between RPIX and CPI inflation is itself moderately large and variable. Over our monitoring period annual house price inflation (the average of the Halifax and Nationwide series) peaked at 25.0% in 2003Q1, with a low of 2.9% in 2005Q3.

¹⁷The dollar price of the Brent marker rose from about \$28 in January 2004 to about \$55 in January 2006.

Figure 1: Annual RPI inflation



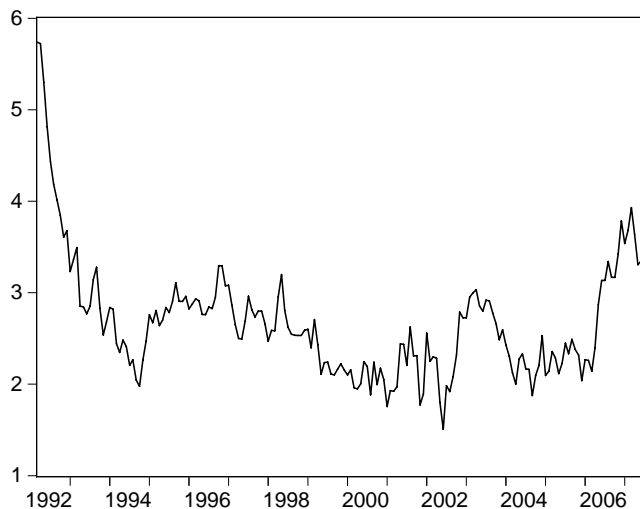
price inflation from its peak. Figures 1 and 2 show the data for RPI and RPIX inflation over the period since 1992.

We monitor the RPI index and its components individually with the univariate break detector, as well as the whole panel of components using the maximum multivariate detector.¹⁸ We consider critical values that correspond to a 95% significance level. We prewhiten each inflation series by fitting an AR(1) model. In the first case we examine, we use all data available. After extracting the residuals they are examined for a mean shift using the simple location model. We consider such a shift following our prior belief that in this particular instance focus should be placed on shifts in the mean of the inflation process, as opposed to the dynamics of the process, which we keep fixed *a priori*.

Our results make interesting reading. Out of 77 RPI components 65 reveal no breaks up to and including August 2007. Of the 12 components where we do find breaks, 9 reveal a break at the very beginning of the monitoring period. The univariate detector cannot find a break in the aggregate RPI series. By contrast, the multivariate detector detects a break at December 2003. This is a remarkable result - just one month after the (pre-announced) target change. However, we consider the timing too close for it to be plausibly capturing the regime shift, rather than other structural breaks. Figure 3 illustrates the detector. There is

¹⁸We know that the average detector is more effective when many series break, but is less powerful when only a few break. As our candidate breaks may not affect all series equally, we consider the maximum detector to be more appropriate.

Figure 2: Annual RPIX inflation



a marked rise in the detector statistic in 2003, just prior to the identified break. The statistic then remains close to the bound, falling below but just exceeding it again in October 2005. By the end of the period the statistic is well above the bound.

The pre-whitening has of course been conducted using whole-sample information. It is important that we maintain a constant dynamic structure, interested as we are in mean shifts. But for robustness we also consider the case where the prewhitening occurs using an AR(1) model estimated using data only prior to the monitoring period. In terms of the variation between the univariate and multivariate procedures, the results are similar. For the univariate detector the vast majority of series report no break; only 7 series report a break, and these do not include the aggregate RPI series. As previously, the multivariate detector again detects a break, in this case at October 2005.¹⁹ The detector is shown in Figure 4.

The detector, based as it is on the CUSUM, does not give a magnitude for the shift in mean. It would be natural to test for structural breaks using standard techniques such as those in Bai and Perron (1998), but the identified dates allow insufficient observations to do so.²⁰ Table 1 therefore reports the results of estimating AR(1) processes with shift dummies

¹⁹The is the date at which the detector statistic crosses the boundary for the second time in Figure 3.

²⁰Standard practice is to trim the start and end of the sample by 15% to perform the Bai and Perron (1998) test. This would lose approximately 28 observations from the end of the sample, which is before the later break identified by the monitoring method.

Figure 3: Maximum CUSUM detector: pre-whitened using whole sample

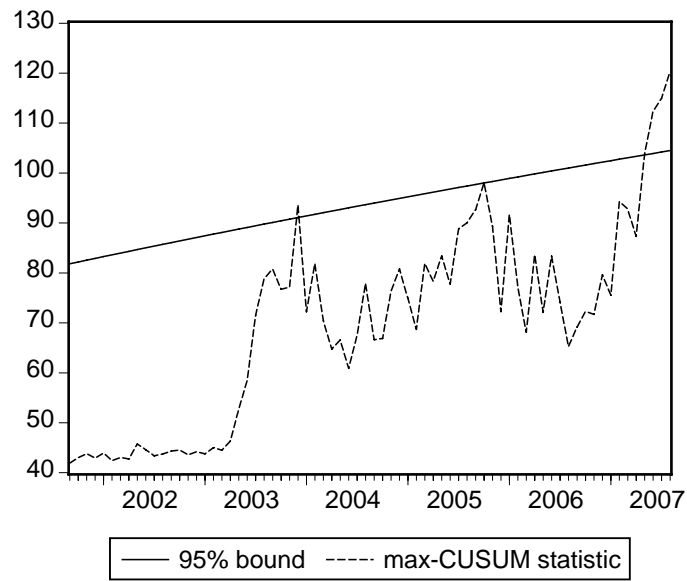


Figure 4: Maximum CUSUM detector: pre-whitened using pre-monitoring sample

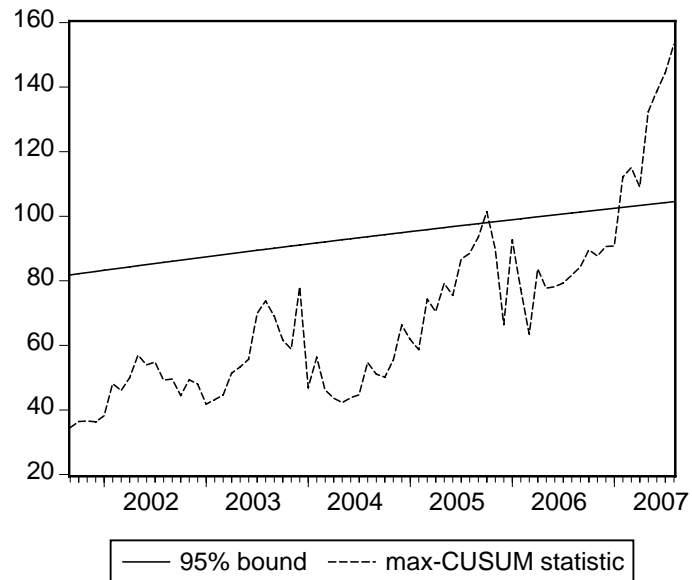


Table 10: Shift-dummies in AR(1) processes; long-run impact

Series	Break	Mean	Shift (se)
RPIX	2003m12	2.50	0.16 (0.30)
	2005m10	2.48	0.56 (0.37)
RPI	2003m12	2.39	1.36 (0.78)*
	2005m10	2.48	1.91 (1.01)*

* indicates significance at the 10% level

Sample April 1992 - August 2007.

in either December 2003 or October 2005, for both RPI and RPIX, which amount to simple Wald tests for the hypotheses in question (mean shifts). In all cases, before the break the estimated long-run inflation rate was close to 2.5%. For RPIX there are insignificant long-run shifts of 0.2 and 0.6 percentage points after the two candidate break points. For RPI, which includes housing costs and was not targeted, the estimated shifts are larger and significant at the 10% level. These amount to weak evidence for a structural break in the long-run mean of RPIX using the full sample, that could not have been detected using standard methods in real-time or by univariate monitoring techniques, but which were indicated by the multivariate monitoring technique we have developed in this paper.²¹

5 Conclusions

‘Monitoring’ of series for structural breaks raises special econometric problems, primarily because classical methods are invalid in repeated experiments. Fortunately, a methodology exists for testing individual relationships which can cope with this, by defining appropriate boundary conditions for various types of test, including the well-known CUSUM. This has the additional advantage that no end-of-sample trimming is required. But an unexplored avenue that may lead to earlier and more reliable detection is to extend the approach to include multiple series. The idea is motivated by the recognition that in many instances - for example, with inflation measures - it is plausible that several series co-break simultaneously. Asymptotic distribution theory is developed for panel CUSUM detection tests, based on the supremum from and average of individual CUSUMs constructed from normalised recursive residuals purged of cross-equation correlation.

²¹As Hendry and his coauthors have emphasised, mean-shifts may often be seen as symptoms of structural breaks, rather than to be taken literally. In this case the obvious cause is the very rate of energy price inflation after 2004.

A Monte Carlo exercise strongly suggests that, given a sufficient number of co-breaking series, the method does increase both the speed with which breaks are detected and the probability of detection in a wide variety of situations. This holds when there is a common factor error structure, and when there is a certain degree of heterogeneity in the break date. Both the maximum and average tests show a marked improvement in detection probability over a univariate detector. The two multivariate detectors perform similarly with very high detection probabilities for both, but the average has the relative edge when a larger number of series break, and *vice versa*. The theoretical and simulated results are supported by tests using UK RPI inflation, where the multivariate method suggests the existence of breaks after 2001, in 2003 or 2005, for which univariate methods provide no evidence at all, which may well have been temporary breaks induced by the large fluctuations in house and energy prices over this period.

So we have demonstrated a technique that appears to improve the power of monitoring tests. This leaves open the question of what to do after a break is detected, for example when forecasting, as standard methods of modeling breaks suffer from a lack of observations for estimation. This is the subject of our future research.

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A Proofs

A.1 Proof of Theorem 1

Using Corollary 3.5 of Chu, Stinchcombe, and White (1996), we have that, for finite p ,

$$\lim_{m \rightarrow \infty} \Pr \left\{ \tilde{Q}_{\max, t}^m \geq \sqrt{m}g(n/m, a), \quad \text{for some } n \geq m \right\} = \quad (\text{A.1})$$

$$\Pr \left(\max_{j=1, \dots, p} |W_j(t)| \geq g(t, a), \quad \text{for some } t \geq 1 \right) \quad (\text{A.2})$$

where $W(t) = (W_1(t), \dots, W_p(t))'$ is a multivariate standard Brownian motion. But

$$\Pr \left(\max_{j=1, \dots, p} |W_j(t)| \geq g(t, a), \quad \text{for some } t \geq 1 \right) = \quad (\text{A.3})$$

$$1 - \Pr \left\{ \max_{j=1, \dots, p} |W_j(t)| \leq g(t, a), \quad \text{for all } n \geq m \right\} \quad (\text{A.4})$$

Then,

$$\Pr \left\{ \max_{j=1, \dots, p} |W_j(t)| \leq g(t, a), \quad \text{for all } n \geq m \right\} = \quad (\text{A.5})$$

$$\Pr (\{|W_1(t)| \leq g(t, a), \quad \text{for all } n \geq m\} \cap \dots \cap \{|W_p(t)| \leq g(t, a), \quad \text{for all } n \geq m\}) \quad (\text{A.6})$$

But, by the independence between $W_1(t), \dots, W_p(t)$, we get

$$\Pr (\{|W_1(t)| \leq g(t, a), \quad \text{for all } n \geq m\} \cup \dots \cup \{|W_p(t)| \leq g(t, a), \quad \text{for all } n \geq m\}) = \quad (\text{A.7})$$

$$\prod_{i=1}^p \Pr \{|W_1(t)| \leq g(t, a), \quad \text{for all } n \geq m\} = (1 - f_g(a))^p \quad (\text{A.8})$$

proving the theorem.

A.2 Proof of Theorem 2

Given the assumptions of Theorem 2, Theorem 1 and Lemma 1 of Appendix A.6, it is sufficient to use sequential asymptotics. To see this simply note that $E \left| \sup_i \tilde{Q}_{i,t}^m \right|^\theta < \infty$, for some $\theta > 1$, follows from $E |\epsilon_{i,t}|^\theta < \infty$ and $E \|X_{i,t}\|^\theta < \infty$ which, in turn follow from the fact that both $\epsilon_{i,t}$ and $X_{i,t}$ are assumed to have finite second moments.

Then, it is sufficient to prove that every element of $\hat{\Sigma}^{-1/2}\omega_t - \Sigma^{-1/2}\omega_t$ is $o_p(1)$. But, by $m^{1/2}$ -consistency of every element of $\hat{\Sigma}$, and continuity of the inverse, it follows that every element of $\hat{\Sigma}^{-1/2}$ is $m^{1/2}$ -consistent for $\Sigma^{-1/2}$. Since every element of $\Sigma^{-1/2}\omega_t$ is a function of p elements of $\hat{\Sigma}^{-1/2}$, it follows that every element of $\hat{\Sigma}^{-1/2}\omega_t - \Sigma^{-1/2}\omega_t$ is $O_p(pm^{-1/2})$ -consistent. Hence the result follows by the assumption that $p = o(m^{1/2})$.

A.3 Proof of Theorem 3

By (8) of Chu, Stinchcombe, and White (1996), we have that $f_g(a) = e^{-a^2/2}$. It is sufficient to show that the limit of $1 - (1 - f_g(a_p))^p$ where $a_p = \ln(p)^{1/2}$ is bounded. Then, an appropriate choice for C gives $\lim_{p \rightarrow \infty} 1 - (1 - f_g(a_p(\alpha)))^p = \alpha$. Setting $1 - (1 - f_g(a))^p = \alpha$ gives $a = [-2 \ln [1 - (1 - \alpha)^{1/p}]]^{1/2}$. It is sufficient to show that

$$\left| \lim_{p \rightarrow \infty} \frac{\ln(1 - C_1^{1/p})}{\ln p} \right| = C_2 < \infty \quad (\text{A.9})$$

for some $0 < C_1 < 1$. By L'Hopital's rule we get that

$$\lim_{p \rightarrow \infty} \frac{\ln(1 - C_1^{1/p})}{\ln p} \approx \lim_{p \rightarrow \infty} \frac{(1 - C_1^{1/p})^k}{p^k} \quad (\text{A.10})$$

for all integer $k > 0$ where \approx is defined so that $a \approx b$ is equivalent to $a/b = O(1)$. This immediately implies that $\lim_{p \rightarrow \infty} \frac{(1 - C_1^{1/p})^k}{p^k} \approx 1$ proving the result.

A.4 Proof of Theorem 4

We simplify our analysis by disregarding the normalisation by

$1 + X'_{j,T} \left(\sum_{i=1}^{T-1} X_{j,i} X'_{j,i} \right)^{-1} X_{j,T}$ to obtain the recursive residuals since this normalisation term converges to 1 almost surely asymptotically. Further, since $T_{T,j,0}/T_{T,i,0} = 1 + o(1)$, it is asymptotically appropriate to simplify the analysis by setting $T_{T,j,0} = T_{T,0}$ for all j . We have that, under the local alternative,

$$\beta_{T-1} = \beta + \frac{1}{T-1} \sum_{i=1}^{T-1} \epsilon_i, \quad (\text{A.11})$$

if $T-1 < T_{T,0} + 1$ and

$$\beta_{T-1} = \beta + \frac{1}{T-1} \sum_{i=1}^{T-1} \epsilon_i + \frac{1}{T-1} \sum_{i=T_{T,0}+1}^{T-1} \frac{\beta_1}{\sqrt{T}} = \beta + \frac{1}{T-1} \sum_{i=1}^{T-1} \epsilon_i + \frac{(T-1 - T_{T,0})\beta_1}{(T-1)\sqrt{T}} \quad (\text{A.12})$$

if $T-1 > T_{T,0}$. Similarly,

$$\hat{\epsilon}_T = \epsilon_T - \sum_{i=1}^{T-1} \epsilon_i, \quad (\text{A.13})$$

if $T-1 < T_{T,0} + 1$ and

$$\hat{\epsilon}_T = \epsilon_T - \sum_{i=1}^{T-1} \epsilon_i - \frac{(T-1 - T_{T,0})\beta_1}{(T-1)\sqrt{T}} + \frac{\beta_1}{\sqrt{T}} = \epsilon_T - \sum_{i=1}^{T-1} \epsilon_i + \frac{T_{T,0}\beta_1}{(T-1)\sqrt{T}} \quad (\text{A.14})$$

if $T - 1 > T_{T,0}$. Then, under the local alternative hypothesis, and for $t > T_{T,0}$,

$$Q_{j,t}^m = Q_{j,t}^{*m} + \sum_{s=T_{T,0}+1}^t \frac{T_{T,0}\beta_1}{(s-1)\sqrt{T}} \quad (\text{A.15})$$

where $Q_{j,t}^{*m}$ is the cumulative sum under the null hypothesis. $\sum_{s=T_{T,0}+1}^t \frac{T_{T,0}\beta_1}{(s-1)\sqrt{T}}$ can be written as a function of t (and $T_{T,0}$) and is asymptotically of the order $\sqrt{t} \ln t$. We write

$$g_1(t, T_{T,0}) = \sum_{s=T_{T,0}+1}^t \frac{T_{T,0}\beta_1}{(s-1)\sqrt{T}} \rightarrow g_1(t, \tau), \quad (\text{A.16})$$

as $T \rightarrow \infty$ where $\tau = \lim_{T \rightarrow \infty} \frac{T_{T,0}}{T}$. Without loss of generality let us assume that $\beta_1 > 0$. Then,

$$\lim_{m \rightarrow \infty} \Pr \left\{ |Q_{j,t}^m| \geq \sqrt{m}g(n/m, a^*), \quad \text{for some } n \geq m \right\} = \quad (\text{A.17})$$

$$\Pr (|W_j(t) + g_1(t, \tau)| \geq g(t, a^*), \quad \text{for some } t \geq 1) \equiv f_1(a^*, \tau) \quad (\text{A.18})$$

Unfortunately, the nature of $g(t, a^*)$ and $g_1(t, \tau)$ implies that a closed form solution for $f_1(a^*, \tau)$ is not readily available (see also Theorem A of Chu, Stinchcombe, and White (1996)). Similarly and using arguments from the proof of Theorem 1, it is straightforward to show that for the multivariate detector

$$\lim_{m \rightarrow \infty} \Pr \left\{ \tilde{Q}_{\max,t}^m \geq \sqrt{m}g(n/m, a^{**}), \quad \text{for some } n \geq m \right\} = 1 - (1 - f_1(a^{**}, \tau))^p \quad (\text{A.19})$$

A.5 Proof of Corollaries 1 and 2

For Corollary 1, the result follows immediately from Corollary 3.5 of Chu, Stinchcombe, and White (1996), the fact that $W_j(t)$ is normally distributed and the fact that the sum of the absolute values of normally distributed random variables is itself distributed as an absolute value of a normal random variable. Similarly for Corollary 2, given Corollary 1 and Lemma 1 of Appendix A.6, the proof goes through similarly to the proof of Theorem 2, with minor obvious modifications.

A.6 Lemma 1

We give a Lemma that is of use in the proof of Theorem 2.

Lemma 1 *Let $Y_{i,T}$, $i = 1, \dots, N$, $T = 1, \dots$, be random scalars. Then, $\sup_i Y_{i,T} \xrightarrow{d} Y$ as $N, T \rightarrow \infty$ sequentially implies that $\sup_i Y_{i,T} \xrightarrow{d} Y$ as $N, T \rightarrow \infty$ jointly, if $E |\sup_i Y_{i,T}|^\theta < \infty$, for some $\theta > 1$.*

Proof Sequential convergence implies that, for all i , there exists Y_i such that $Y_{i,T} \xrightarrow{d} Y_i$ as $T \rightarrow \infty$. Then, from Lemma 6 of PM the result of the lemma follows if we show that

$$\limsup_{N,T} \left| E \left(f \left(\sup_i Y_{i,T} \right) \right) - E \left(f \left(\sup_i Y_i \right) \right) \right| = 0, \quad \forall f \in \mathcal{C} \quad (\text{A.20})$$

where \mathcal{C} is the space of all bounded continuous real functions on \mathbb{R} . Without loss of generality let the functions f be such that $|f^{(k)}(x)| \leq 1$ where $f^{(k)}(x)$ denotes the k -th derivative function of $f(x)$. Fix f . Let

$$g(h) = \sup_x |f(x+h) - f(x) - f'(x)h|$$

Set $x = \sup_i Y_{i,T}$ and $h = \sup_i Y_{i,T} - \sup_i Y_i$. It follows by the triangle inequality that

$$\limsup_{N,T} \left| E \left(f \left(\sup_i Y_{i,T} \right) \right) - E \left(f \left(\sup_i Y_i \right) \right) \right| \leq \quad (\text{A.21})$$

$$\limsup_{N,T} \left| E \left(f' \left(\sup_i Y_{i,T} \right) \left(\sup_i Y_{i,T} - \sup_i Y_i \right) \right) \right| + \limsup_{N,T} \left| E \left(g \left(\sup_i Y_{i,T} - \sup_i Y_i \right) \right) \right|$$

But since $|f^{(k)}(x)| \leq 1$

$$\limsup_{N,T} \left| E \left(f' \left(\sup_i Y_{i,T} \right) \left(\sup_i Y_{i,T} - \sup_i Y_i \right) \right) \right| \leq \limsup_{N,T} \left| E \left(\sup_i Y_{i,T} \right) - E \left(\sup_i Y_i \right) \right| \quad (\text{A.22})$$

Also by the mean value theorem and for some finite M

$$g(h) \leq M \min\{|h|, h^2\}$$

Thus,

$$\limsup_{N,T} \left| E \left(g \left(\sup_i Y_{i,T} - \sup_i Y_i \right) \right) \right| \leq M \limsup_{N,T} E \left| \sup_i Y_{i,T} - \sup_i Y_i \right| \quad (\text{A.23})$$

From (A.22) and (A.23), it follows that the result of the Lemma is true if

$$\limsup_{N,T} E \left| \sup_i Y_{i,T} - \sup_i Y_i \right| = 0 \quad (\text{A.24})$$

However, uniform integrability of $|\sup_i Y_{i,T}|$ implies (A.24). By Theorem 12.10 of Davidson (1994) $E |\sup_i Y_{i,T}|^\theta < \infty$ implies uniform integrability of $|\sup_i Y_{i,T}|$. Hence, the result of the Lemma follows.

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