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Revealed Preferences, Choices, and Psychological Indexes

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Abstract

This paper develops a model of choice that embeds some psychological aspects affecting decision maker's behaviour. In the model, the decision maker attaches an unobservable psychological index—representing, e.g., the level of perceived availability or the level of salience—to each alternative in a universal collection. Choice behaviour of the decision maker is then conditioned by the indexes attached to the alternatives. With this paper we show that, if the conditional choice behaviour satisfies two intuitively appealing properties—namely Monotonicity and Conditional IIA—then the observable part of the choice behaviour, i.e., the unconditional choices, can be interpreted as the product of the maximization of a preference relation. The paper discusses also some welfare consideration regarding the choice model and finally some interpretations of the indexes are provided.

JEL Keywords: Revealed preferences, choice with frame, salience, scarcity bias, bandwagon and snob effect

JEL Class: D11

1 Introduction

The standard model of choice (see Richter, 1966; Sen, 1971) considers a universal collection of alternatives X that are the possible objects of choice and defines a choice problem A as a non–empty subset of the universal collection X. In this model, both the environmental information that is not–necessary for describing and identifying the elements of the choice problem—i.e., ancillary information—and the psychological aspects involving the alternatives—

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e.g., salience and emotive states—are assumed to be irrelevant for choices made by economic agents. Choices are formalized by means of a correspondence $C()$ that attaches to each choice problem a non–empty subset of alternatives $C(A)$—$C(A) \subseteq A \; \forall A \in D$, where D is the class of all choice problems—and choices are independent of ancillary information and the psychological state of the agent. Therefore the choice is assumed to be unaffected by, for example, the order of the alternatives, the description of the situation, the number of times an alternative is repeated in the choice set, or various levels of salience that in different situations are attached to the alternatives. The only thing that matters is the composition of the set A, and once the elements of A have been defined then there are no ways to change the elements chosen from A.

This paper borrows the idea developed by Bernheim and Rangel (2008) and Salant and Rubinstein (2008) that additional components should be attached to the choice problem in order to capture the effect on choices of unobservable information and psychological states. That is, the authors allow choices to be conditioned by abstract entities they call "frames", and consequently, the decision maker can choose different elements from the same set of alternatives based on the frame that is attached to the set. Using the same approach this paper proposes a more specialized conceptualization of frame. We link each alternative in the choice problem with an index that represents an unobservable physical or psychological measure that the decision maker attaches to each alternative in the set. This index can be interpreted in various ways. For instance it can be interpreted as a measure of perceived availability of the alternative, or as the level of salience that the alternative possesses in the specific situation, or as the level of perceived popularity of that alternative in a reference group. All these interpretations are examined in detail below.

The paper shows that if choices from these sets of indexed alternatives satisfy some intuitively appealing properties, then observing the unconditional behaviour, one cannot distinguish between these choices and choices produced by the maximization of a preference relation that is complete and quasi–transitive[1]. The paper then examines in depth the connection between the proposed model and the more general model of "choice with frames" developed by Salant and Rubinstein (2008). Afterwards some welfare considerations are discussed, and finally some examples of how these indexes can be interpreted are discussed in more detail.

2 Model

As mentioned before, the main idea of this paper is to attach to each alternative in a finite universal collection X an index that can reflect some unobservable physical or psychological feature that the alternatives possess. Hence, instead of defining a choice problem simply as a subset A of the collection X, we consider an *indexed choice problem* A_f that is a non–empty subset A of X along with a function f from X into $\mathbb{R}$, that we will call *index function.* That is, an index function f attaches a real number to each alternative present in X. As we will see, the indexes attached to the alternatives can be interpreted in various ways. For instance, these indexes can be interpreted as perceived availability of the alternative by the decision maker, as an index of salience of the alternative, or as the number of people already possessing that alternative. In this model a set of alternatives A is part of many choice problems. Indeed, each set A coupled with distinct index functions define distinct choice problems and hence the set of all the indexed choice problem D^* becomes $D^* = (P(X) - \emptyset) \times \mathbb{R}^X$.

Having defined the concept of choice problem used here, we will use standard definitions concerning choice functions and choice correspondences. We use the term *choice function* for a function $c()$ that attaches to each choice problem $A \in D$ a single element in A, while we use the term *choice correspondence* for a correspondence $C()$ that attaches to each choice problem $A \in D$ a non–empty subset of A. Therefore, given a choice problem, a choice function selects only one chosen element among the available alternatives while a choice correspondence can select many chosen elements among the

[1]A quasi–transitive preference relation is a preference relation for which the strict preference part is transitive and the indifference part is not necessarily transitive.

available alternatives.

In what follows we assume that a decision maker has a choice function $c()$—called *indexed choice function*—that is defined over the set of all the indexed choice problems D^* and selects a single chosen item from the set of available alternatives. It has to be noted that in this framework one can allow for changes in the chosen alternatives according to the index function attached to the choice set. Indeed, a decision maker facing a set of alternatives A can choose an alternative x from A when the choice problem is A_f while he can choose an alternative y when the choice problem is A_g. Moreover, an indexed choice function $c()$ defined over D^* induces a choice correspondence $C()$ defined over $D = P(X) - \emptyset$ such that $C(A) = \bigcup_{f \in \mathbb{R}^X} c(A_f)$. The induced choice correspondence just defined includes all the elements that are chosen from a set A for some function f. That is, given a set A, x belongs to the induced choice correspondence $C(A)$ if and only if there is a function $f \in \mathbb{R}^X$ such that $c(A_f) = x$.

The reason motivating the introduction of an induced choice correspondence is the fact that in many cases the index that is attached to each alternative is unobservable by an external observer while it can be perfectly known by the decision maker—e.g., interpreting the indexes as levels of perceived availability of the alternative or as levels of salience. In such cases an observer of the choice behaviour of the decision maker will be unable to distinguish the circumstances under which the decision maker chooses the alternative x over the alternative y and under which he chooses x over y. Hence the observer can simply record that the individual have chosen both. There are also cases in which the indexes attached to the alternatives can be known—e.g., interpreting the value as the number of people that already possess the alternative or as the supply of that alternative in the market. However, in such cases it may be difficult to observe the indexes and interesting to disregard the information about the values attached to each alternative by focusing on the unconditional behaviour of the decision maker.

So far we have imposed no restriction on the indexed choice function, that is there are no limitations in what the decision maker can choose from a set A under different indexes attached to the alternatives. Obviously without limitations one can obtain every type of choice behaviour and hence we should introduce some restrictions on the behaviour of the indexed choice function $c()$ under a given index function. This is done by introducing two properties, the first binds the behaviour of the indexed choice function when the indexes of the alternatives are fixed and the choice problem can vary, while the second binds the behaviour of the indexed choice function when the choice problem is fixed and the indexes attached to the alternatives can vary.

The first property of the choices over the set of all the indexed choice problems is called *Conditional IIA*.

Def (Conditional IIA)**.** If $x \in B \subset A$ and $x = c(A_i)$ for some $i \in \mathbb{R}^X$, then $x = c(B_i)$.

This property says that having fixed an index function i, the indexed choice function $c()$ satisfies the standard Independence from Irrelevant Alternatives property (IIA). Conditional IIA captures the idea that if the index function has not been changed, the elimination of a non–chosen alternative from the set has no effect on the chosen alternative that must remain the same as chosen before the elimination occurred. In other words this property says that once the decision maker has a complete psychological picture of the alternatives in X—i.e., he/she has attached to each alternative in the universal collection a psychological value—and has decided to choose one alternative from a choice problem A according to this picture, then removing a non-chosen alternative and holding fixed the psychological situation does not change his/her mind.

In the next section, we will show that Conditional IIA will imply that the induced choice correspondence satisfies Sen's property α[2], but for the moment it has to be noted that Conditional IIA with the fact that the indexed choice function is single valued imply that there exists a linear ordering $\succ_i$ such that its maximization describes the choices of $c()$ over D whenever the index function is i, i.e., $c(A_i) = \{x \in A \mid \forall y \in A, x \succ_i y\}$ for all $A_i \in D^*$. Hence the decision maker can be considered as completely rational when the index function is kept constant and moreover he cannot be indifferent

[2]see the Appendix for the definition of this property.

between two alternatives. This means that indifference emerges only because of the unobservability of the indexes attached to the alternatives and that all the irrationality in the decision maker's behaviour is captured by indexes' movements. In other words, the decision maker is never indifferent between two alternatives, but he/she can reverse the preferences according to the particular index function attached to the choice set. Indifference is only in the eyes of the external observer, but we will discuss afterward the interpretation of indifference in this model.

The second property of choices over the set of all the indexed choice problems is the *Monotonicity* property which is composed of two parts.

Def (Monotonicity)**.** For each $x \in X$ either

$x \uparrow$**.** for all $B_j \in D^*$: $x = c(B_j) \Rightarrow \forall j'$ such that $j'(x) > j(x)$ and $j'(a) = j(a)\ \forall a \in X - \{x\}$, $x = c(B_{j'})$ and; $x \neq c(B_j) = y \Rightarrow \forall j'$ such that $j'(x) < j(x)$ and $j'(a) = j(a)\ \forall a \in X - \{x\}$, $y = c(B_{j'})$ or,

$x \downarrow$**.** for all $B_j \in D^*$: $x = c(B_j) \Rightarrow \forall j'$ such that $j'(x) < j(x)$ and $j'(a) = j(a)\ \forall a \in X - \{x\}$, $x = c(B_{j'})$ and; $x \neq c(B_j) = y \Rightarrow \forall j'$ such that $j'(x) > j(x)$ and $j'(a) = j(a)\ \forall a \in X - \{x\}$, $y = c(B_{j'})$.

Monotonicity means that, if an item is chosen under some circumstances, it has to be chosen when its index is altered in a given direction. More precisely, in case of alternatives of type '$\uparrow$', whenever the alternative is chosen, an increase in its index cannot affect choices. While in case of alternatives of type '$\downarrow$', whenever the alternative is chosen, a decrease in its index cannot affect choices. Obviously there can be alternatives for which both '$\uparrow$' and '$\downarrow$' hold true. In this case we use the notation '$\updownarrow$' and we have that, whenever those alternatives are chosen, both decreasing and increasing the index attached to them cannot alter the choice. Monotonicity has also implications for movements of the indexes of non–chosen alternatives. In particular, Monotonicity implies that if an alternative that is chosen continues to be chosen increasing (decreasing) its index, then when non–chosen a decrease (increase) in its index does not alter the chosen alternative.

It has to be pointed out that Monotonicity is defined for all the alternatives in X, hence it applies also to the alternatives that do not belong to the current choice problem. This means that, for instance, if $x \notin A$ and x is of type '$\uparrow$', then reducing the index attached to x the alternative chosen from A does not change. We are aware that conditioning the behaviour of the indexed choice function to the index of an alternative that is not under consideration may appear quite counterintuitive. However, since an alternative that is not in the current choice problem cannot be chosen, it turns out that Monotonicity implies that the indexes of alternatives outside the current choice problem do not have any effect on choices. In order to see this we introduce the following lemma.

Lemma 1. *If $c()$ is an indexed choice function satisfying Monotoniticy, $x \neq c(A_i)$ for all $i \in \mathbb{R}^X$, and $y = c(A_j)$ for an index function $j \in \mathbb{R}^X$ then for all k such that $k(x) \neq j(x)$ and $k(a) = j(a)$ $\forall a \in X - \{x\}$ we have that $y = c(A_k)$[3].*

Proof. Let $c()$ be an indexed choice function satisfying Monotoniticy, $x \neq c(A_i)$ for all $i \in \mathbb{R}^X$, and $y = c(A_j)$ for an index function $j \in \mathbb{R}^X$. Suppose now that there is an index function k such that $k(x) \neq j(x)$ and $k(a) = j(a)$ $\forall a \in X - \{x\}$ and $y \neq c(A_t) = z$. So suppose that w.l.o.g. $k(x) > j(x)$. In this case if $x \uparrow$ we have $y = c(A_k)$ contradicting $z = c(A_k)$, and if $x \downarrow$ we have $z = c(A_j)$ contradicting $a = c(A_j)$. □

Notice that Lemma 1 holds not only for those alternatives that do not belong to the current choice problem, but also for those alternative that are in the current choice problem but are non–chosen for all the index function. This means that the indexes of the alternatives that are non–chosen from a set A have no effect on the choice of the decision maker. To say it in other words, only the indexes attached to alternatives that are chosen for some index function i matter for determining the choice of the decision maker. A

[3]Notice that in the light of this lemma, the Monotonicity property can be restated using two separate properties: a Monotonicity property restricted to the elements belonging to the current choice problem plus an Invariance property that excludes the effect of a change in the indexes of alternatives outside the choice problem.

final implication of Lemma 1 is for those alternatives that are never chosen in all the choice problems. Indeed Lemma 1 implies that such alternatives can be only of type '$\updownarrow$'.

In order to have a better understanding of how the indexes of the alternatives may affect choices it is worth underlining the implications of Monotonicity. How already shown with Lemma 1 the only indexes that may have effect on choices from a set A are the ones that belong to those alternatives that are chosen from A at some point. But there is another interesting observation about the effect of a change of the indexes of such alternatives. Indeed, if we have that $x \updownarrow$ is a chosen alternative from A, this implies that the index of x does not affect the choice of the decision maker in any way. Both when x is chosen and x is non–chosen from A, an alteration of the index of x cannot lead to changes in the choice.

If instead we have an alternative x such that $x \uparrow$ and not $x \downarrow$ then the index of x has effect on the choice of the decision maker. $x \uparrow$ and not $x \downarrow$ implies that: either there is a situations A_f in which x is chosen and there is a number $k < f(x)$ for which x is not chosen from A, or there is a situation B_j in which $y \neq x$ is chosen and there is a number $t > j(x)$ for which y in not chosen from B. Hence there is at least a set in which a movement in the index of x produces a change in the choice. An interesting observation is that, in the former case, a reduction in the index of x can change the choice from x to an element y, but, $x \uparrow$ implies that further reductions of the index do not affect the choice anymore—i.e., once the choice has switched from x to y a further reduction of the index of x has no consequence for the choice. In the latter case we have that increasing the index of x up to t an element different from y is chosen, but the element chosen cannot be different from x. Indeed if we suppose that z different from x is chosen, we have that z is chosen from B_j contradicting $y = c(B_j)$. Hence also in case x is not chosen there can be only a single change due to the effect of the index of x. Combining the two cases, we have that the situation $x \uparrow$ and not $x \downarrow$ implies that there is at least a choice set A and a combination of indexes for the alternatives different from x for which there exists a value k such that x is chosen when its index is above and $y \neq x$ is chosen when the index is below that threshold[4].

A similar situation arises when considering alternatives of type $x \downarrow$ and not $x \uparrow$. That is, there is at least a choice set A and a combination of indexes for the alternatives different from x for which there exists a threshold k such that x is chosen when its index is below and $y \neq x$ is chosen when the index is above that threshold.

After having introduced and explained the two properties that constrain the behaviour of the indexed choice function we define the concept of *constrained indexed choice function*.

Def (Constrained indexed choice function). A *constrained indexed choice function* is a choice function $c()$ on D^* that satisfies Conditional IIA and Monotonicity.

The main result of the paper is that the choice correspondence induced by a constrained indexed choice function can be rationalized by a quasi–transitive preference relation—i.e., there is a preference relation whose maximization produces the same choices produced by the induced choice correspondence—and, moreover, the choices produced by the maximization of a quasi–transitive preference relation can be produced by a choice correspondence induced by a constrained indexed choice function.

[4]Suppose the case $x \uparrow$ and not $x \downarrow$. Then not $x \downarrow$ implies either that: (1) $\exists A_f$ s.t. $x = c(A_i)$ and $\exists k < i(x)$ s.t. $y = c(A_{i'})$ where $i'(a) = i(a) \forall a \in X$ and $i'(x) = k$; or (2) $\exists B_j$ s.t. $z = c(B_j)$ and $\exists k' > j(x)$ s.t. $x = c(B_{j'})$ where $j'(a) = j(a) \forall a \in X$ and $j'(x) = k'$. Notice that (1) if and only if (2). Hence take (1). First we show that no elements different from x and y can be chosen from A for all the index function f such that $f(a) = i(a) \forall a \in X - x$. Indeed if $z = c(A_f)$ for some f such that $f(a) = i(a) \forall a \in X - x$ we have that: $f(a) \geq i(x)$ implies that $z = c(A_i)$ contradicting $x = c(A_i)$, and $f(a) < i(x)$ this implies either $z = c(A_{i'})$ (if $f(x) \geq i'(x)$) or $y = c(A_f)$ (if $f(x) < i'(x)$) producing a contradiction in both cases. Hence no elements different from x and y can be chosen from A for all the index function f such that $f(a) = i(a) \forall a \in X - x$. Now we want to show that there exists a number r for which $i(x) > r$ implies that x is chosen and $i(x) < r$ implies that y is chosen. In order to do this suppose not, suppose that there are two numbers a and b such that $a > b$ and y is chosen from A when $i(x) = a$ and x is chosen from A when $i(x) = b$. Using again the fact that $x \uparrow$ we have that x is chosen from A when $i(x) = b$ with $a > b$ implies that x is chosen from A when $i(x) = a$ a contradiction. Hence there has to be a number r for which $i(x) > r$ implies that x is chosen from A and $i(x) < r$ implies that y is chosen from A. Notice that we cannot say anything about the behaviour of the choice function when $i(x) = r$. We just know that one between x and y must be chosen in r but we do not know which one.

Theorem. *A choice correspondence $C()$ is induced by a constrained indexed choice function $c()$ if and only if $C()$ is rationalizable by a complete and quasi–transitive preference relation $\succsim$.*

In order to prove the main statement we need to prove some preliminary result. In what follows we show that a choice correspondence $C()$ induced by an indexed choice function $c()$ satisfies Sen's properties α, γ, and δ (Sen, 1971), and hence the revealed preference relation R defined as xRy if and only if $\exists A \subseteq X$ such that $x \in C(A)$ and $y \in A$ is a complete and quasi–transitive binary relation whose optimization produces the same choices as $C()$[5].

We start proving that the induced choice correspondence $C()$ satisfies properties α and γ.

Lemma 2. *The choice correspondence $C()$ induced by a constrained indexed choice function $c()$ satisfies Sen's properties α and γ.*

Proof. Concerning property α suppose that $x \in C(A)$, $x \in B$, and $B \subseteq A$, then for the definition of induced choice correspondence, i.e. $C(A) = \bigcup_{f \in \mathbb{R}^X} c(A_f)$, there is an index function i such that $x = c(A_i)$. Then, if $x = c(A_i)$, for Conditional IIA $x = c(B_i)$ and by definition of induced choice correspondence we have that $x \in C(B)$. Thus property α is satisfied.

Moving to property γ suppose that $x \in C(A)$ and $x \in C(B)$ then, for the definition of induced choice correspondence, there exists an index function f such that $x = c(A_f)$ and an index function i such that $x = c(B_i)$. Now what is needed in order to have property γ satisfied is the existence of an index function u for which $x = c((A \cup B)_u)$. The proof is by construction of u and consists of 2 steps.

Step 1: We start proving that, given $x = c(A_f)$ and $x = c(B_i)$, there is an index function u such that $x = c(A_u)$ and $x = c(B_u)$. Consider the index function u on X defined as follows:

[5]For the definitions of Sen's properties α, γ, and δ see the section 7.
For the proof that a Choice Correspondence $C()$ defined over $D = P(X) - \emptyset$ satisfies Sen's properties α, γ, and δ if and only if there exists a quasi–transitive and complete preference relation R such that $C(A) = \{x \in A \mid \forall y \in A, xRy\}$ for all $A \in D$ see Sen (1971)

$$u(z) = \begin{cases} \min\left(f(z), i(z)\right), & \text{if } z\uparrow \wedge \neg z\downarrow \text{ and } z \neq x; \\ \max\left(f(z), i(z)\right), & \text{if } z\downarrow \wedge \neg z\uparrow \text{ and } z \neq x; \\ \max\left(f(z), i(z)\right), & \text{if } z\uparrow \wedge \neg z\downarrow \text{ and } z = x; \\ \min\left(f(z), i(z)\right), & \text{if } z\downarrow \wedge \neg z\uparrow \text{ and } z = x; \\ f(z), & \text{otherwise.} \end{cases}$$

First we prove that $x = c(A_u)$. Notice that, by construction of u, for all the non–chosen alternatives y in $X - \{x\}$, we have that: if $y\uparrow$, $u(y) \leq f(y)$; if $y\downarrow$, $u(y) \geq f(y)$; and if $y\updownarrow$, $u(y) = f(y)$. Hence by Monotonicity the indexes of those alternatives cannot affect the choice. Moreover, concerning $u(x)$ we have that: if $x\uparrow$, $u(x) \geq f(x)$; if $x\downarrow$, $u(x) \leq f(x)$; and if $x\updownarrow$, $u(x) = f(x)$. Thus, also in this case Monotonicity prevent that the index of x affect choices and hence $x = c(A_u)$ by Monotonicity.

Now we prove that $x = c(B_u)$. Applying the same reasoning as before we have that, for all the alternatives in $y \in X$ such that $y\uparrow \wedge \neg y\downarrow$ or $y\downarrow \wedge \neg z\uparrow$, the change of the index from i to u does not affect the choice. Considering the alternatives y such that $y\updownarrow$ notice that $u(y) = f(y)$ that can be different from $i(y)$. But we already know that Monotonicity implies that a change in the index of an alternative of type '$y\updownarrow$' does not have effect on the chosen alternative. Hence Monotonicity implies that $x = c(B_u)$.

Step 2: Now we show that $x = c(A_u) = c(B_u)$ implies $c((A \cup B)_u) = x$. Suppose $c((A \cup B)_u) = z \neq x$, then we have two cases either $z \in A$ or $z \in B - A$. *CASE 1:* Suppose $z \in A$. In this case $z = c(A_u)$ by Conditional IIA and this contradicts Step 1. *CASE 2:* Suppose $z \in B - A$. In this case $z = c(B_u)$ by Conditional IIA, and again a contradiction with Step 1. Therefore, for non-emptiness of $c()$, we conclude that $c((A \cup B)_u) = x$.

In order to complete the proof one needs to note that, by definition of induced choice correspondence, $x = c((A \cup B)_u)$ belongs to $\bigcup_{f \in \mathbb{R}^X} c((A \cup B)_f)$ and therefore $x \in C(A \cup B)$. Thus the induced choice correspondence satisfies Sen's Property γ. □

Since the induced choice correspondence satisfies both Sen's property α

and γ it is Normal (Sen, 1971). This means that $C(A) = R$–gr(A) where the binary relation R is the revealed preference relation—i.e., xRy if and only if $\exists A \subseteq X$ such that $x \in C(A)$ and $y \in A$—and R–gr(A) is the set of the greatest elements in A according to R—i.e., $R\text{–gr}(A) = \{x \in A \mid xRy\ \forall y \in A\}$.

The fact that the revealed preference relation R rationalizes the choice correspondence induced by a constrained indexed choice function implies that the behaviour of the agent can be interpreted as a maximizing one; that is, one can retain the classical assumption about the rationality of the agent: he/she chooses what is the best for him/her according to a complete and acyclic preference relation. In the light of this result, an interesting interpretation of the effect of psychological indexes can be given. If the agent behaves accordingly to a constrained indexed choice function his/her behaviour can be interpreted "as if" he/she maximizes a weak preference relation R obtaining a set of preferred items, and then he/she uses a tie breaking rule based on the "indexes" he/she attaches to the alternatives to choose one item among the preferred items. Notice that the decision maker tie–breaking rule is based on regions of the space of the indexes and that these regions are convex. Indeed if you choose the element x from the set A both in the situation i and in the situation f, a convex combination of the indexes is such that $\min\left(f(a), i(a)\right) \leq \sigma i(a) + (1-\sigma)f(a) \leq \max\left(f(a), i(a)\right)$ for all the a in A and for all the $\sigma \in [0, 1]$, hence Monotonicity guarantees that x is chosen for all the index function that are convex combinations of i and f. Thus the decision maker breaks the ties according to the region to which the current index function belongs to.

What is left to prove is the fact that the revealed preference relation R is indeed a quasi–transitive preference relation. This is assured by the following lemma.

Lemma 3. *The choice correspondence $C()$ induced by a constrained indexed choice function $c()$ satisfies Sen's property δ.*

Proof. Suppose $A \subset B$, $x, y \in C(A)$ and $\{x\} = C(B)$. $\{x\} = C(B)$ implies that $x = c(B_i)\ \forall i \in \mathbb{R}^X$. Therefore by Conditional IIA one gets that

$x = c(A_i)\ \forall i \in \mathbb{R}^X$ that contradicts $y \in C(A)$. So the induced choice correspondence satisfies Sen's property δ. $\square$

Since the induced choice correspondence satisfies Sen's properties α, γ and δ, it is Normal and the revealed preference relation R is quasi–transitive (Sen, 1971). That is, the strict preference relation P—i.e., the asymmetric part of R—is transitive but the indifference relation I—i.e., the symmetric part of R—is not necessarily transitive[6].

A specification regarding indifference is due. As already mentioned, indifference is in the eyes of the external observer. When two alternatives x and y are revealed indifferent this means that there is an index function in which x is chosen over y and another in which y is chosen over x, and that the decision maker is not willing to change his choice. That is in the first case he truly prefers x over y and in the second case he truly prefers y over x. This implies that the interpretation of indifference in this model is slightly different from the standard one: indifference should be thought as the absence of an unambiguous strict preference under all the possible psychological situations.

So far we have shown that the induced choice correspondence satisfies Sen's properties α, γ and δ, and hence it is quasi–transitive. Now we show that it may not be a Weak-order, i.e., it may not satisfy Sen's property β. In other to show this consider the following situation, suppose $X = \{x, y, z\}$ and suppose that:

1. $x = c(\{x, y, z\}_i)$, $x = c(\{x, z\}_i)$, $x = c(\{x, y\}_i)$, and $z = c(\{z, y\}_i)$ for all the index functions where $i(x) \leq k \in \mathbb{R}$;

2. $z = c(\{x, y, z\}_i)$, $z = c(\{x, z\}_i)$, $y = c(\{x, y\}_i)$, and $z = c(\{z, y\}_i)$ for all the index functions where $i(x) > k$

This indexed choice function satisfies the Monotonicity and Conditional IIA

[6]The non transitivity of indifference implies that we can have situations in which $xIy \wedge yIz$ but zPx. Notice that this situation is not so unnatural. Indeed, there are no compelling philosophical reasons for requiring transitivity of indifference. See Luce (1956) for a discussion about this issue.

properties but the induced choice correspondence does not satisfy Sen's property β. In fact we have that $\{x, y\} = C(\{x, y\})$ and that $y \notin C(\{x, y, z\})$.

The implications of Lemmas 2 and 3 are that, given a choice procedure that satisfies Monotonicity and Conditional IIA, one can interpret the observable pattern of choices of the agent "as if" he/she is maximizing a quasi–transitive preference relation. But given an agent that maximizes a quasi–transitive preference relation it is possible to interpret his behaviour "as if" he is adopting a choice procedure that satisfies Monotonicity and Conditional IIA? We show this with lemma 4.

Lemma 4. *If choices are determined by the maximization of a complete and quasi–transitive preference relation* $\succsim$ *on the finite set* X*, then there exists a constrained indexed choice function* $c()$ *defined over* $D^* = (P(X) - \emptyset) \times \mathbb{R}^X$ *that induces a choice correspondence* $C()$ *defined over* $D = (P(X) - \emptyset)$ *such that* $C(A) = \succsim$ *–gr*(A) *for all the non–empty subsets* A *of* X[7].

Proof. We define the indexed choice function $c()$ explicitly, and then we prove that $c()$ satisfies Conditional IIA and Monotonicity. Let $<_O$ be an arbitrary linear order on X—i.e., $<_O$ is a complete, transitive and antisymmetric binary relation on X. Define $c(A_i) = x \ \forall i \in \mathbb{R}^X$ such that: $x \in \succsim$ –gr(A) and, $\forall y \in \succsim$ –gr(A), either $i(x) < i(y)$ or $i(x) = i(y) \wedge x <_O y$. First we show that the indexed choice function $c()$ just defined is indeed a choice function—i.e., that is non–empty and single–valued for all the sets $A_i \in D^*$. Considering that the set $\succsim$ –gr(A) is finite and not–empty for each non–empty subset A of X, the non-emptiness and the single-valuedness of $c()$ for all $i \in \mathbb{R}$ follow directly by the fact that a finite set of numbers in $\mathbb{R}$ has always a minimal element and that $<_O$ is a linear order—i.e., the set $<_O$ –gr(A) is a singleton for each subset A of X.

Now we show that $c()$ is a constrained indexed choice function, that is, $c()$ satisfies Conditional IIA and Monotonicity.

[7] The relation $\succsim$ is complete and the asymmetric part $\succ$ is transitive, thus the set $\succsim$ –gr(A) is non–empty for each non–empty subset A of X because of finiteness of X. Suppose not, then for all x in A there exists y such that $\neg x \succsim y$ that implies $y \succ x$ and since the set A is finite, this lead to a contradiction with transitivity of $\succ$.

Conditional IIA: suppose $x = c(A_i)$. Then consider the set $B \subseteq A$ such that $x \in B$. We show that $x = c(B_i)$. The first consideration is that for each $z \in B$ if $z \in \succsim$–gr(A) then it belongs also to $\succsim$–gr(B). Indeed if z is a $\succsim$–greatest element of A it means that $z \succsim k$ for all $k \in A$ and hence $z \succsim k$ for all $k \in B$ that in turn implies $z \in \succsim$–gr(B). The second consideration is that, by definition of $c()$, $x = c(A_i)$ implies that $x \in \succsim$–gr(A) and $\forall z \in \succsim$–gr(A), $i(x) < i(z)$ or $i(x) = i(z) \wedge x <_O z$. Hence combining the two considerations we have that for all $z \in \succsim -gr(B)$, $i(x) < i(z)$ or $i(x) = i(z) \wedge x <_O z$. That implies that $x = c(B_i)$.

Monotonicity: The proof that $c()$ satisfies Monotonicity is based on the fact that increasing the index of a non–chosen alternative y or reducing the index of the chosen one x does not alter the fact that $i(x) \leq i(y)$ $\forall y \in \succsim$–gr(A) and hence does not alter the chosen alternative. Suppose $x = c(A_i)$, then $\forall y \in \succsim$–gr(A), $i(x) < i(y)$ or $i(x) = i(y) \wedge x <_O y$. Take an index function f, $f(z) = i(z)$ $\forall z \in X - \{x\}$ and $f(x) < i(x)$. In this case $f(x) < f(y)$ $\forall y \in \succsim$–gr$(A) - \{x\}$ and hence, since $f(x) = f(x) \wedge x <_O x$, we have $x = c(A_f)$. Take now and index function f', $f'(z) = i(z) \forall z \in X - \{y\}$, $y \neq x$, and $f'(y) > i(y)$. In this case $f(x) \leq f(y)$ $\forall y \in \succsim$–gr(A) and, since $x <_O z \forall z \in \succsim$–gr$(A)$ such that $i(z) = i(x)$, then $x <_O z \forall z \in \succsim$–gr$(A)$ such that $f(z) = f(x)$. Thus we have $x = c(A_{f'})$. Therefore each element x in X satisfies the condition $x \downarrow$ and hence Monotonicity is satisfied.

Therefore the choice function $c()$ on D^* is a constrained indexed choice function. What we have left to prove is that the induced choice correspondence $C()$ is such that $C(A) = \succsim$–gr(A) for all the non–empty subsets A of X. Suppose $x \in C(A) = \bigcup_{f \in \mathbb{R}^X} c(A_f)$ then it exists an index function i such that $x = c(A_i)$ and hence $x \in \succsim$–gr(A) by construction of $c()$. Suppose instead that $x \in \succsim$–gr(A) and consider an index function i in which $i(x) < i(y)$ for all y in $\succsim$–gr(A). In this case $x = c(A_i)$ and hence $x \in C(A)$. Therefore the constrained indexed choice function $c()$ is such that $C(A) = \succsim$–gr(A) for all the non–empty subsets A of X. □

A comment about Lemma 4. Indeed in the proof of Lemma 4 we build a constrained indexed choice function for which all the alternatives are of type

'$\downarrow$' and hence, either the index of an alternative has no effect on choices, or the effect is in one particular direction. But this is just one of the possible choice functions that one can construct in order to prove Lemma 4. For instance another possibility is to build $c()$ such that it satisfies condition $x \uparrow$ for all the alternatives in X—i.e. defining $c(A_i) = x \; \forall i \in \mathbb{R}^X$ such that: $x \in \succsim$ –gr(A); and $\forall y \in \succsim$ –gr(A), $i(x) > i(y)$ or $i(x) = i(y) \wedge x <_O y$[8]. The interpretation of Lemma 4 is that, if an agent chooses maximizing a complete and quasi–transitive preference relation, then his/her behaviour can be seen "as if" he chooses according to a constrained indexed choice function.

After having obtained the results in Lemmas 2, 3, and 4, it is straightforward to prove the main proposition of the paper. In fact Lemmas 2 and 3 prove that a choice correspondence $C()$ is induced by a constrained indexed choice function $c()$ is rationalized by the revealed preference relation R that is complete and quasi–transitive, while Lemma 4 shows that if there is a complete and quasi–transitive preference relation $\succsim$ whose maximization determines the choice correspondence $C()$, then there exists a constrained indexed choice function $c()$ that induces $C()$.

What is left to prove is the independence of the axioms and in order to do this we provide two examples. In the first example we provide a case in which Monotonicity is satisfied and Conditional IIA is not while in the second example Conditional IIA is satisfied and Monotonicity is not.

EXAMPLE 1. Consider the following situation, suppose $X = \{x, y, z\}$ and suppose that, for all the index function $i \in \mathbb{R}^X$, $x = c(\{x, y, z\}_i)$, $x = c(\{x, z\}_i)$, $y = c(\{x, y\}_i)$, and $y = c(\{z, y\}_i)$. This indexed choice function satisfies the Monotonicity property but not the Conditional IIA property. Indeed if Monotonicity had been satisfied we would have that $x = c(\{x, y, z\}_i)$ implying $x = c(\{x, y\}_i)$ while we have that $y = c(\{x, y\}_i)$.

EXAMPLE 2. Consider this different situation, suppose $X = \{x, y, z\}$ and suppose that: $x = c(\{x, y, z\}_i)$ for all $i \in \mathbb{R}^X$ such that $i(x) \neq 0$ and

[8] A third different possibility is to choose a number $k \in \mathbb{R}$ and to define $c(A_i) = x$ for all the $i \in \mathbb{R}^X$ such that: $x \in \succsim$ –gr(A); and either (1) $\forall y \in \succsim$ –gr(A) such that $\neg x <_O y$, $i(y) < k \wedge i(x) \geq k$; or (2) $\forall y \in \succsim$ –gr(A), $i(y) < k \wedge y <_O x$.

$y = c(\{x, y, z\}_i)$ for all $i \in \mathbb{R}^X$ such that $i(x) = 0$; $x = c(\{x, z\}_i)$ for all $i \in \mathbb{R}^X$; $x = c(\{x, y\}_i)$ for all $i \in \mathbb{R}^X$ such that $i(x) \neq 0$ and $y = c(\{x, y\}_i)$ for all $i \in \mathbb{R}^X$ such that $i(x) = 0$; and $y = c(\{z, y\}_i)$ for all $i \in \mathbb{R}^X$. In this case it is easy to verify that Conditional IIA is satisfied while Monotonicity is not. Indeed taking an index function such that $i(x) > 0$ we have not $x \downarrow$ since $y = c(\{x, y, z\}_i)$ for all $i \in \mathbb{R}^X$ such that $i(x) = 0$, and taking an index function such that $i(x) < 0$ we have not $x \uparrow$ for the same reason. Hence this indexed choice function does not satisfy Monotonicity.

3 Connections with other models

In this section we review two works closely related to our model. The first model was developed by Salant and Rubinstein (2008) and proposes a more abstract framework of choices conditioned to unobservable information. The second model is a model of social choice developed by Sen (1969) in which the author shows that, under some conditions, the aggregation of individual preference relations produces a quasi–transitive social preference relation, which is a result similar to the aggregation of constrained indexed choice function.

The idea to attach an unobservable component to the choice set is not new. Indeed, Bernheim and Rangel (2008) and Salant and Rubinstein (2008) have developed a framework of choices with frames. The main intuition of Salant and Rubinstein is to attach to the class of choice problems (D) a class of frames called F. According to Salant and Rubinstein (2008), a *frame* is an abstract object that can reflect both observable ancillary information, such as the position of alternatives, and unobservable internal manipulation used by an agent in the choice process. Formally, an extended choice problem is defined as a pair (A, f) where $A \in D$ and $f \in F$ is the abstract object called *frame*. Accordingly, the extended choice function c^* is a function that assigns an element of A to every extended choice problem (A, f) and a standard choice correspondence induced by the extended choice function C_{c^*} where $C_{c^*}(A)$ is the set of elements chosen from the set A for some frame f

(Salant and Rubinstein, 2008).

Salant and Rubinstein (2008) show that, if an extended choice function is a *Salient Consideration* function and it satisfies property γ–extended, then the standard choice correspondence induced by the extended choice function is indistinguishable from the choice produced by the maximization of an asymmetric and transitive binary relation. In their model, an extended choice function is *Salient Consideration* if for every frame $f \in F$, there exists a corresponding ordering $\succ_f$ such that $c(A, f)$ is the $\succ_f$ –max(A). Moreover they say that an extended choice function satisfies property γ–extended if $c(A, f) = x$ and $c(B, g) = x$, implies that there exists a frame h such that $c(A \cup B, h) = x$. Notice that there is a manifest relationship between the definitions proposed by Salant and Rubinstein (2008) and our definition and indeed, as we point out in the next paragraph, the two models are equivalent from a formal point of view.

In order to see the close relationship of the two models, note that a Salient Consideration function also satisfies Independence of Irrelevant Alternatives and a choice function that satisfies IIA is a Salient Consideration function. Hence, since a constrained indexed choice function satisfies Conditional IIA, then it is a Salient Consideration Function. Moreover, a constrained indexed choice function satisfies also property γ–extended. In order to see this it is enough to look at the proof lemma 2, where we show that the choice correspondence induced by an indexed choice function satisfies Sen's property γ. Therefore, the model of choice with psychological index can be seen as a member of the bigger family of models defined by Salant and Rubinstein (2008). Indeed, the index function attached to the choice set can be thought as a *frame* in the model of choice with frames.

Although our model can be seen like a specification of the Salant and Rubinstein's model, it is worth considering that, if it is impossible to distinguish choices produced by the maximization of an asymmetric and transitive relation—i.e., the optimization of a complete and quasi–transitive relation—and choices induced by a Salient Consideration function satisfying property γ–extended, then it is also impossible to distinguish them from choices induced by a constrained indexed choice function. In this situation, whenever the frame is unobservable, one can interpret choices “as if” there was a measure attached to each alternative in the choice set that can affect choices according to the Monotonicity property. That is, if choices can be interpreted by using Salant and Rubinstein (2008)’s model you can always interpret the same choices in light of our model. Notice, however, that this is just an interpretation of the choice behaviour; the decision maker does not necessarily chooses according to a measure attached to the alternatives, but he chooses “as if” that was the case. Moreover, even if a constrained indexed choice function which induces a choice correspondence compatible with the observed behaviour may exists, it may be the case that is impossible to provide a meaningful interpretation to the indexes attached to the alternatives. On the other hand, there are psychological models in the literature that indeed use unobservable psychological measures in order to explain behaviour, e.g., availability or salience, and hence if it is supposed that these measures have a monotonic effect on choices, then the model of choices with psychological index is well-suited for showing that, such behaviour is not far from rationality.

The model about social choices by Sen (1969) is closely linked with both our and Salant and Rubinstein (2008) models. The author shows that it is possible to find an aggregation of complete and transitive individual preference relations that results into a quasi–transitive social preference relation. In this model Sen considers a complete and transitive relation R_i for each subject i in the community and then he proposes the following aggregation rule in order to build the social preference relation S:

$$xSy \Leftrightarrow \neg \left[(\forall i \; yR_i x) \wedge (\exists i \mid yP_i x) \right]$$

where R_i is the weak preference relation of agent i, P_i is the strict preference of agent i, and S is the social preference relation. Notice that the meaning of this aggregation procedures is that x is socially excluded from being chosen by y—i.e., x is not socially weakly preferred to y—only if all the members of the community weakly prefers y to x and there is someone in the community that strictly prefers y to x. Using this aggregation procedure, the author shows that the social preference relation S is a complete and quasi–transitive relation whose maximization produces a social choice satisfying Arrow's conditions (see Sen, 1969, p. 386).

Concerning our model, the most interesting aspect of Sen (1969) is that there is an aggregation of complete and transitive individual preference that produces a complete and quasi–transitive social choice relation. Indeed, since the Conditional IIA property implies that fixing the index function i one has a linear ordering $\succ_i$ representing preferences, one can interpret the final choice as the aggregation of preferences of a community of "multiple-selves" composed by the index functions. However, our aggregation procedure is different from the one used by Sen. Our procedure is motivated by the unobservability of the indexes—i.e., the unobservability of the voter that is in charge of taking the decision—while the aggregation used by Sen is a social decision function—i.e., a voting rule—used to produce a collective choice.

Notwithstanding, it may be interesting to compare the two aggregation procedures. Conditional IIA implies that for each index function i (i.e., for each voter) we have a linear ordering $\succ_i$, that is a complete, transitive, and antisymmetric preference relation. Antisymmetry implies that given $x, y \in X$ such that $x \neq y$, $x \succ_i y$ implies $\neg y \succ_i x$, thus $x \succ_i y$ implies yP_ix. Hence, by using the aggregation rule proposed by Sen we have that if $x \succ_i y$ for some index function i, then xSy where S is the social preference defined as $xSy \Leftrightarrow \neg\left[(\forall i\ yR_ix) \wedge (\exists i \mid yP_ix)\right]$. Consider now the revealed preference relation R constructed by using a choice correspondence $C()$ induced by an indexed choice function $c()$ that satisfies Conditional IIA, i.e., $xRy \Leftrightarrow x \in C(A) = \bigcup_{f\in\mathbb{R}^X} c(A_f) \wedge y \in A$. On the one hand, we have that if xRy then there exists a set $A \in D$ such that $x \in C(A) \wedge y \in A$ that means that there is an index function i such that $x \succ_i y$ and hence xSy. On the other hand, suppose that xSy, then there is an index function i such that $x \succ_i y$ and hence $x = c(\{x, y\}_i)$. Thus $x \in C(\{x, y\})$ that implies xRy. Hence, if the indexed choice function satisfies the Conditional IIA property—that is equivalent to have a population of linear orderings—it implies that the revealed preference relation R constructed using the induced choice correspondence is equivalent to the Sen's social preference relation S, and hence it is complete and quasi–

transitive (see Sen, 1969, p. 287, Theorem V).

Thus it seems that the Conditional IIA is sufficient in order to get the quasi–transitivity of our revealed preference relation R, but notice that without the Monotonicity property we cannot show that $R - gr(A) = C(A) = \bigcup_{f \in \mathbb{R}^X} c(A_f)$. For this purpose, consider the following example satisfying Conditional IIA but not Monotonicity.

EXAMPLE 3. Consider the following situation, suppose $X = \{x, y, z\}$ and suppose that: (1) for all the index function $i \in \mathbb{R}^X$ such that $i(x) > 0$: $z = c(\{x, y, z\}_i)$, $z = c(\{x, z\}_i)$, $x = c(\{x, y\}_i)$, and $z = c(\{z, y\}_i)$; while (2) for all the index function $i \in \mathbb{R}^X$ such that $i(x) \leq 0$: $y = c(\{x, y, z\}_i)$, $x = c(\{x, z\}_i)$, $y = c(\{x, y\}_i)$, and $y = c(\{z, y\}_i)$. This indexed choice function satisfies the Conditional IIA property but not the Monotonicity property and it is easy to verify that in case (1) choices are produced by the maximization of the following linear order $z \succ_i y$, $y \succ_i x$, $z \succ_i x$, $x \succ_i x$, $y \succ_i y$, and $z \succ_i z$; while in case (2) choices are produced by the maximization of this linear order $y \succ_i x$, $x \succ_i z$, $y \succ_i z$, $x \succ_i x$, $y \succ_i y$, and $z \succ_i z$. Thus if we consider the induced choice correspondence $C()$ we have that $C(X) = \{z, y\}$ while the set $R - gr(X)$ is equal to $\{x, y, z\}$.

Example 3 shows that the two models are different in the sense that, even if we can interpret our choice procedure like an aggregation of Multiple Selves preferences, we require that the alternatives chosen by the maximization of the social preference relation are equal to the union of the alternatives chosen by each "self" present in the decision maker. Moreover, notice that is not true that all the alternatives chosen by the Social Preference $S = R$ in a situation A are chosen by the decision maker. Indeed, we have a rule that decide which self is responsible for the decision according to the indexes attached to the alternatives. More explicitly, we have that, according to the psychological situation, there is a "self" i that is the dictator of the decision maker, and only the unconditional behaviour can be interpreted as if an aggregation of preferences satisfying Arrow's conditions[9].

[9]Notice that with Monotonicity property we indeed constrain the possible combination

A final remark about the relationship between our model and models of social choices has to be made. Even if the multiple self interpretation looks appealing, we would point out that it is not the main interpretation of our model. Indeed, if we follow this interpretation, then the Monotonicity property will impose restrictions on the behaviour of the selves in the population. For instance, an implication of the Monotonicity property is that, if there are two selves i and j that choose the same alternative x from respectively the sets A and B, then there must exist a self u that chooses the alternative x from the set $A \cup B$. Clearly, this feature of the model is at odd with the idea of having a population of multiple selves. Indeed the idea of multiple selves implicitly assumes that the behaviour of one self is independent from the behaviour of another self.

4 Some welfare consideration

This section of the paper discusses the issue of welfare analysis using behavioural models. Indeed, one of the main weaknesses of behavioural models is that they are well suited for a positive description of choice behaviour while they usually fail to provide normative guidance for welfare decisions.

One of the main difficulties of behavioural models is that they usually allow for preference reversal, i.e., the decision maker is willing to choose an alternative x from a set A under some circumstances while he is willing to choose y from A under some others—e.g., endowment effect, framing effect, status quo bias, etc.— These circumstances are usually not observable or are at least difficult to observe externally. Hence, the external observer cannot unambiguously determine if the individual will be better–off or not by switching from the alternative x to the alternative y. For instance, a social planner that has to decide between x and y for the individual cannot determine which alternative is preferred. The problem is even more severe in

of individual orderings, hence not all the Arrow's conditions are satisfied. In particular condition of Unrestricted Domain of the aggregation function (see condition "U" Sen, 1969, p. 386) turns out to be violated. The Monotonicity property implies that if there are two individuals i and j that choose the same alternative x from respectively the sets A and B then there exists an individual u that chooses the alternative x from the set $A \cup B$.

cases where the change in choices depends upon the intervention of the social planner. Indeed, also by assuming that the social planner knows the subject is willing to choose x over y, the social planner probably has no elements to determine how the taste of the decision maker is going to be altered by an intervention. It could be the case that the decision maker has changed his mind and is now willing to choose y over x.

These considerations cast doubts on the possibility of developing normative welfare analysis in behavioural models. However, some authors (Bernheim and Rangel, 2008; Green and Hojman, 2007) have recently tried to overcome these shortcomings by providing a generalization of welfare concepts that would allow welfare policies to be developed on the basis of behavioural models. Starting from the consideration that also standard welfare analysis is based on choices and not on utility, Bernheim and Rangel (2008) propose a revealed preference framework for welfare analysis which, in theory, is able to include all behavioural models. These authors define the notion of ancillary condition as "a feature of the choice environment that may affect behavior, but that is not taken as relevant to a social planner's choice once the decision has been delegated to him" (Bernheim and Rangel, 2008, p. 4). Then they model a generalized choice situation as a subset of the universal collection of alternative X coupled with an ancillary condition—i.e., a pair (A, d) where A is a subset of X and d is an ancillary condition—and they let choices be dependent upon different ancillary conditions. Using this approach, Bernheim and Rangel (2008) are able to build individual welfare relations and to define the concept of individual welfare optima, thus showing that is possible to make basic welfare comparisons also without well–behaved preference relations or utility functions[10].

The model proposed in this paper is similar to that of Bernheim and Rangel (2008) in interpreting the index function as an ancillary condition. Hence, following the same line of reasoning of Bernheim and Rangel (2008), it can be used to determine whether the decision maker is better–off by switching

[10]In the article Bernheim and Rangel (2008) go further than this. They provide a generalization for the concepts of equivalent and compensating variation and they also suggest a generalization of the first welfare theorem. But this it is outside the scope of this section.

from one alternative to another. The model assumes that the external observer cannot know the psychological indexes the decision maker attaches to the alternatives and that choices are conditioned upon the psychological situation the decision maker perceives. As shown above, the observable part of the decision maker's behaviour is given by the induced choice correspondence, which simply records all the alternatives chosen by the agent from a set A for all psychological situations. This implies that, for instance, when the external observer knows that x and y are chosen from $\{x, y\}$, he is unable to say whether the subject is currently willing to choose x over y or y over x. The external observer only knows that there are situations—i.e., some index function i—in which the subject is willing to choose x and other situations—another index function j—in which the subject is willing to choose y.

Despite this difficulty, the revealed preference relation R provides insights into the welfare of the decision maker. The relation R—defined as xRy iff $\exists A \subseteq X$ such that $x \in C(A)$ and $y \in A$, where $C(A) = \bigcup_{f \in \mathbb{R}^X} c(A_f)$—is based on the observable part of choices and thus it can be inferred by the external observer. In the model's section, it was shown that the revealed preference relation has the property of rationalizing the observable part of the choices, but it can also be used for deriving welfare implications. If we consider the asymmetric part P of the relation, i.e., xPy iff $xRy \wedge \neg yRx$, having xPy for some alternative x and y expresses the fact that the agent is always willing to choose x over y, independently from the psychological situation he faces. Thus, whenever the external observer can record that xPy, he can unambiguously conclude that the decision maker is better–off when he is given x instead of y. Hence, P can be used as an individual welfare relation, and it is also possible to define the concept of individual welfare optimum. Borrowing the definition of individual welfare optima by Bernheim and Rangel (2008), we can say that an alternative $x \in A$ is improvable when there is an alternative $y \in A$ such that yPx, and whenever this is not the case we can say that x is an individual welfare optimum in A[11].

[11]Note that the asymmetric part of our revealed preference relation R is equivalent to

Under these circumstances some basic welfare consideration can be drawn, but it is necessary to underline that, whenever two alternatives x and y are not ranked by the asymmetric part P of R, it is impossible for the external observer to decide whether the decision maker is willing to choose x over y or y over x. More precisely, the observer knows that there are psychological situations in which the decision maker is willing to choose x over y and other situations in which the subject is willing to choose y over x.

This consideration also helps in interpreting the symmetric part of the revealed preference relation R. Indeed, the symmetric part I of the revealed preference relation R is usually thought as revealing indifference between two alternatives, but in our model I can be given a different and more precise interpretation. Recalling the definition of I—i.e., xIy iff $xRy \wedge yRx$—xIy implies that there are situations in which the agent chooses x over y and situations in which he chooses y over x, but given a specific situation he is never indifferent between the two. thus we can say that instead of revealing indifference, the relation I reveals the absence of an observable unambiguous preference ordering between the two alternatives. That is, I can be thought not to capture the indifference of the decision maker but the impossibility for the external observer to observe some relevant part of information.

Further insights can be gained by interpreting the model as a multiple–self model. This interpretation clarifies the concept of individual welfare optimum. Considering each index function as a distinct self we find that if two alternatives $x, y \in A$ belong to the items chosen from A—i.e., $x, y \in C(A)$—then they are not ranked by P and hence are both individual welfare optima in A. Nevertheless, these alternatives are also two Pareto equilibria for the society of multiple selves. Indeed, if we fix x, we have xIy for all the $y \in C(A)$, and is impossible to find an alternative in A that is preferred by all the selves; hence, it is also impossible to improve the well–being of one self with-

the welfare relation P^*—xP^*y iff, for all (A, d) such that $x, y \in A$, $y \in C(A, d)$ implies $x \in C(X, d)$—proposed by Bernheim and Rangel (2008). Note also that while the authors define the relation R'—$xR'y$ iff, for all (A, d) such that $x, y \in A$, $y \in C(A, d)$ implies $x \in C(X, d)$—and they use its asymmetric part when defining the concept of weak improvement, such relation is based on the observability of the ancillary conditions, which is ruled out in our setting. Consequently we cannot speak about weak and strict welfare optima.

out reducing the well-being of another self by moving from x. Consequently, without additional information about the indexes, the social planner cannot do better than randomly assigning one of the individual Pareto optima to the agent[12].

5 Interpretation of the Indexes

As mentioned in the previous sections the indexes attached to the alternatives have various interpretations. Among the possible interpretations of the indexes, the more interesting ones are the following:

- **Perceived Availability.** This interpretation of the indexes is suggested by the psychological studies about the so called Commodity Theory developed by Brock (1968) and others (Brock and Brannon, 1992). For this theory the more an alternative is perceived hard to obtain, i.e., it is perceived "scarce", the more the alternative become attractive for the subjects. Indeed, there are experiments, mainly in experimental psychology, which tested the effect of the perceived availability of goods on preferences (Verhallen, 1982; Lynn, 1989, 1991; Lynn and Bogert, 1996; Mittone et al., 2005; Mittone and Savadori, 2008). Manipulating the information about the easiness of obtaining the goods or the numerosity of each good available for the choice, these experiments have provided evidence supporting the sensitivity of preferences to the perceived availability of the good. In particular they have shown how a reduction (increment) of the manipulated availability (scarcity) of the goods produces an increase (decrease) of the likelihood of choice of that good.

  In the light of this evidence one can interpret the indexes of our model like a measure of the perceived availability of the alternative. More precisely, the index attached to an alternative can be thought like a

[12]Note that this interpretation has to be taken with some reservations, we have already pointed out the difficulties of this interpretation at the end of the previous section and we want to stress the fact that this interpretation has to be considered a good tool of analysis but is not the main interpretation of the model.

psychological measure of the easiness of obtaining the alternative. Using this interpretation of the indexes and considering the fact that increasing the scarcity of the good will increase its attractiveness, it seems sensible to assume that, if an alternative is chosen when it has a given level of perceived availability, reducing its availability the decision maker will continue to choose the same alternative. Similarly if a non chosen alternative is perceived as more easy to obtain, i.e. an increase in the index, this will not have effect on choices since the relative scarcity of the chosen alternative will increase. Hence, with this interpretation, it is sensible to assume that Monotonicity works only in the '$\downarrow$' direction.

- **Snob and Bandwagon effects.** The second interpretation of the indexes builds on the work of Leibenstein (1950), in which the author pointed out that the individual demand of some good can be influenced by the overall level of its market demand. Considering the direction of the relation between individual and market demand, Leibenstein (1950) defines two types of effects: if the individual demand increases with the market demand we have the so called "bandwagon effect"; while if the individual demand decreases with the market demand of the good we have the so called "snob effect". Therefore, reinterpreting the two effects from a choice perspective, we can look at the index attached to the alternative like the level of popularity (diffusion) of the alternative in a reference group, e.g., we can see the indexes like the number of decision maker's friends that already possess the alternative. Notice that this interpretation of the indexes can be also motivated by the psychological phenomena regarding the "need for uniqueness" and the "need for conformity" (Hornsey and Jetten, 2004). According to these theories people compare themselves to others to assess their similarity to and distinctiveness from these others, this because of the opposing needs to be included in social groups and to be distinctive from others.

  Using this interpretation of the indexes we can model the snob and bandwagon effect from a choice perspective. Indeed with this interpretation, there is a natural interpretation of the meaning of the Monotonicity property. That is we have a snob effect for those alternatives that are of type '$\downarrow$'—if I chose an alternative when it has a given level of popularity, I continue to choose it when it becomes less popular—while we have a bandwagon effect for those alternatives that are of type '$\uparrow$'—if I chose an alternative when it has a given level of popularity, I continue to choose it when it becomes more popular. Finally there is neither bandwagon nor snob effect for the alternatives of type '$\updownarrow$'.

- **Salience.** A third interpretation of the indexes is to assume that they represent the salience of the alternatives for the decision maker in that particular choice task. The concept of salience is very broad and the term "salient" has often been used as a synonymous of noticeable, prominent but it has also been interpreted differently in different fields. For instance, in marketing literature the concept of brand salience has been often equated with "the prominence or level of activation of a brand in memory" (Alba and Chattopadhyay, 1986) while in cognitive science—in particular in vision research—the concept of salience is closely related to a measure of the ability of different stimuli to attract the visual attention of the decision maker in different situations (Huang and Pashler, 2005; Itti, 2006, 2007). For our purposes we consider salience as the level of importance or relevance that the decision maker attaches to the alternative with respect to the particular choice task he is facing.

  Using this definition of salience it is sensible to assume that if an alternative is chosen when it has a given level of salience, then it has to be chosen when the salience of that alternative becomes higher, and that if an alternative is non–chosen when it has a given level of salience, then it continues to be non–chosen when the salience of that alternative becomes lower. Hence also in this case we restrict the behaviour of the Monotonicity property to the '$\uparrow$' direction.

- **Reason–Based Choices.** The fourth and the last interpretation of the indexes is based on the psychological research about reason based choices. As Shafir et al. (1993) pointed out, the making of a decision is often difficult because of uncertainty and conflict. People usually considers reasons for and against each option in order to choose and the decisions depends on the weights attached to the option's pros and cons (e.g., the famous list of pros and cons written by Charles Darwin deciding to get married or not). Considering a simplified version of this approach one can interpret the index of an alternative like the number of reasons supporting the choice of that alternative, or like the net number of pros and cons of the alternative or also like the net sum of the weighted pros and cons[13].

Obviously interpreting the indexes like the number of supporting reasons for choosing each alternative it is reasonable to restrict the Monotonicity property only to the '$\uparrow$' direction. That means that it is sensible to assume that: on the one hand, if I choose an alternative when I have a given number of pros, incrementing its pros I should continue to choose that alternative; and, on the other hand, if I reject an alternative when I have a given number of pros I still reject it when the number of pros is lower.

After having introduced some possible interpretation of the indexes, we want to point out a couple of important aspects. First of all it is not necessarily true that the chosen alternative is the scarcest one or the most salient one or the one that has the higher number of pros. Indeed the choice depends not just on the indexes but also on the different alternatives. The choice is the product of the interplay between the indexes and the alternatives to which the indexes are attached. But, even if this feature of the model sounds a little bit odd, we want to underline the fact that both perceived availability and salience do not require that the scarcer or more salient alternative in a set that has to be chosen. Indeed if we consider salience as prominence the decision maker can be immediately attracted by the most prominent alternative

[13] See Bettman et al. (1998) for procedures that attach to each alternative a weighted sum of psychological evaluations of the alternative's attributes and see Alba and Marmorstein (1987) for evidence of the use of frequencies of pros like a choice heuristic.

but then he can become aware of the presence of less salient alternatives and he may finally choose one of these alternatives. However, notice that also the procedure that selects the alternative with the highest (lowest) index is compatible with the properties of our model as the proof of Lemma 4 has shown.

A second aspect of these interpretations is that, in the light of the main proposition of the paper, the overall choice can be also interpreted as a two stage procedure in which at the first stage the decision maker deems choosable a subset of the available options according to a quasi–transitive preference relation and consequently he chooses according to the "numbers" (salience level, perceived availability, etc.) attached to them.

A third point is that, in some of the previous interpretations, it seems reasonable to restrict the domain of the indexes. For instance it seems natural to think salience as a positive real and the number of decision maker's friends possessing the alternative is obviously restricted to the domain of natural numbers. However these restrictions of the domain of the indexes do not alter the results of the model. Indeed, the proofs of the propositions are still valid considering indexes that belong to the set of the natural numbers, or to convex subsets of the set of reals, or also to closed intervals of natural numbers.

A final remark about the interpretations of the indexes is that they can be always considered also as a measure of distance between the alternatives and a reference point in a psychological space in which the decision maker may encode the alternatives. An example may be a manager that has to hire a new secretary. In this case the alternatives are the candidates for the job, and the indexes may represent the unobservable information about the social distance between the manager and the candidates. Notice that interpreting the indexes as distances requires a minor modification about the domain of the indexes, that in this case does not span over the entire real line, but only on the non–negative half–line.

6 Concluding Remarks

This paper has developed a model in which choices are affected by some psychological elements assumed to be an unobservable measure that the decision maker attaches to each alternative in a universal collection. It has been shown that if choices conditioned by these measures satisfy two intuitively appealing properties, namely Monotonicity and Conditional IIA, then the observable part of the choice behaviour, i.e., the unconditional choices, can be interpreted as the product of the maximization of a preference relation. Two related models were then examined and four interpretations of the measure attached to the alternatives were provided.

Two concluding remarks about the implications of the model are due. The first remark is related to the rationality issue, while the second one regards the issue of the experimental testability of the model.

Starting with the first issue, the main result of the paper is supporting the idea that the decision maker's unconditional behaviour can be interpreted "as if" rational. However, the spirit of the paper is not to pursuit the idea that all the psychological phenomena can and should be rationalized, but to show that some of these phenomena can be treated by means of standard economic tools. The paper is aimed to show that, whenever it is possible to have a sensible explanation of the indexes, e.g., in case of salience or in case of perceived availability, it is possible to give the choice behaviour an economic interpretation in terms of underlying preferences and to use the revealed preference relation to derive some basic welfare considerations.

Concerning the testability of the theory this is not an easy task. The unobservability of the indexes raises a major issue about testability. Usually choice axioms provide simple statements about choices that can be tested experimentally, but in our case choice behaviour is conditional to an unobservable component that cannot be controlled in an experimental setting. To make more clear the issue consider the following example: suppose an experimenter wants to test Conditional IIA. This person should first let the decision maker choose from a set of available alternatives and then let him choose from a smaller set in order to check if the subject sticks to the same choice or not. Suppose then that the experimenter observes a change in the chosen alternative, in this case he is unable to determine whether the change is due to a change in the psychological situation or to a violation of the Conditional IIA. The only way the experimenter can test Conditional IIA is to be sure to observe the choices from the smaller set for all the psychologichal situations. The previous example raises important concerns about testability. In order to test the model one have to make a strong assumption about the independence of indexes and sets of alternatives.

Another issue concerns the testability of Monotonicity, in this case the set of alternatives is kept fixed but one have to assume that the experimenter has a form of control of the indexes. Consider the perceived availability interpretation of the indexes. In this case one has to assume that manipulating the experimental situation it is possible to, e.g., reduce the perceived availability of the chosen alternative in order to check whether the decision maker sticks to that alternative. Thus also in this case we have to assume that the experimental manipulations change the psychological situation in the desired direction.

Notice however that the need of additional assumption—although not strong as our assumptions—is not a peculiarity of our model, it pertains also to other models. Testing the standard theory of intertemporal choices, for instance, requires some additional requirements about the stability of preferences and income level in time.

7 Appendix: Definitions

Def (Sen's Property α)**.** A choice correspondence $C()$ satisfies Sen's Property α if $x \in C(A)$ and $x \in B \subseteq A$ implies that $x \in C(B)$.

Def (Sen's Property γ)**.** A choice correspondence $C()$ satisfies Sen's Property γ if $x \in C(A)$ and $x \in C(B)$ implies that $x \in C(A \cup B)$.

Def (Sen's Property δ)**.** A choice correspondence $C()$ satisfies Sen's Property δ if for any pair of sets $A, B \in D$ such that $A \subset B$ and $x, y \in C(A)$. then $C(B) \neq \{x\}$.

Def (Sen's Property β)**.** A choice correspondence $C()$ satisfies Sen's Property β if for any pair of sets $A, B \in D$ such that $A \subset B$, $x, y \in C(A)$ and $x \in C(B)$ then $y \in C(B)$.

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