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Abstract. The aim of this paper is to extend existing literature on carbon allowance allocation, investigating the impact of uncertainty and ambiguity, due to the lack of future Environmental policy, on the total production in the market. Specifically, we show that an increase in uncertainty has no effect on the total output, whereas an increase in ambiguity leads to a decrease in the total output. An output-based allocation model in Cournot Oligopoly will be used. We will adopt the National Allocation Plan (NAP) of UK for the Second Phase (2005-07) as a case study.

JEL Classification: D2, D8, Q4

Key Words: Carbon allowances, Permits allocation, EU ETS, Uncertainty.

1. Introduction

The aim of the EU environmental policy is to gradually reduce the total of emissions below the emissions levels of 1990. Setting up EU Emissions Trading Scheme (EU ETS) is aimed to reach the goal in the most economically efficient manner. The essence of the EU ETS is to cap the total emissions of the economy and assign individual installations with allowances, such that the total of permits does not exceed the cap. To create incentives to reduce the emissions the ETS allows a free trade of the emissions permits. In most of the EU members individual allocation is based on historical emissions, practice known as *grandfathering.* In order to reduce the possible Ratchet effect, where the installations have incentive to increase their current production to gain larger share in free future allocation, historical emissions from the period prior to the implementation of the EU ETS is taken. For instance, in the National Allocation Plan (NAP) of the UK it is said

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explicitly that historical emissions prior the ETS implementing is taken, otherwise, it might create incentives in some sectors to emit more.[1]

Post Kyoto negotiation is currently taking place. However no agreement that can give a guideline for a future Environmental Policy, beyond the commitment period of 2012, has been established yet. The situation of policy uncertainty contributes to already tough challenge for the policy maker to reduce the emissions level in a constantly growing global economy where demand for energy is rising. Ignoring the behavioral aspect of the problem might make this challenge even tougher and sometimes unachievable

Despite the interaction between companies strategy and the market uncertainty, very little discussion has been dedicated to the analysis of companies behaviour under policy uncertainty. The aim of my paper is to extend existing literature on allocation of carbon permits. I investigate the impact of uncertainty and ambiguity, due to the lack in future Environmental Policy, on the production. I show that there is a clear distinction between the impacts of uncertainty and ambiguity on the total production in the market. Moreover, I demonstrate that ignoring the lack of information in the carbon market might lead to incorrect policy design.

In order to conduct the analysis, first, I present the literature that deals with allocation of permits and uncertainty. Then, I lay down the model which is based on the UK model of NAP of carbon permits. Later, I investigate the impact of uncertainty in future Environmental Policy on the total output of the market. Finally, I extend the basic theoretical framework by incorporating ambiguity into the model and compare the results obtained by ambiguity and uncertainty driven productions.

In order to investigate the effect of uncertainty I adopt a mean-preserving spread technique. By comparison, I use an accepted method of Choquet integral to study the impact of ambiguity. To conduct this analysis I use an output-based allocation model in Cournot Oligopoly. To get an empirically related analysis I use the UK NAP for the years 2008-12 (Second Phase) which has been approved by the EU Commission on 29 November 2006, as a case study.

It will suggest some policy implications in the concluding section of the paper.

[1] Section 3.5 in the UK NAP for the second phase (2008-2012).

2. Literature Review

The optimal allocation rule of carbon permits has been one of the main issues for a debate in the Policy design for the first (2005-2007) and the second (2008-2012) Phases in the implementation of the Kyoto protocol. There has been extensive research on the efficiency of different methods of allocation. Several alternative policies have been analyzed: auctioning (Cramton and Kerr 2002), pollution taxes (Baldursson *et al.* 2004, Haucap *et al.* 2003), free and output based allocation (see Fischer 2001; Haucap and Kirstein 2003; Neuhoff, Grubb and Keats 2005).[2]

Most of the EU member states choose to distribute their permits based on historic output and/or emissions levels, method that is labeled as *garndfathering*. In the NAP of the UK, Germany and Austria[3] allocation of permits to the existing companies is determined according to their share in the historic emissions prior to the first phase. One of the main justifications for using historical data on emissions/output, beside the practical difficulties of collecting updated data, is that this method eliminates company's strategic behavior. Otherwise it may encourage high productivity and reward less efficient firms for continuing emitting at higher levels allocating bigger share of permits in the future.(Ahman *et al.*, 2005; Fischer 2003).

Two additional issues that are of concern to policy maker are allocation to new entrants, and closure of existing companies. These issues are beyond the scope of this paper. However, it is worth mentioning that most of the EU members choose to set aside some permits for new entrants at the New Entrant Reserve(NER)[4]. New entrants receive their permits from the NER according to a benchmark level of emissions, which is the estimated emissions projection for each sector[5].

Despite a rigorous analysis on the optimal allocation of carbon permits, most of the research conducted in this area ignores the substantial fact that there is no policy after

[2] Most of the EU member (Germany , Austria, Netherlands, Poland etc.) choose to allocate majority of emission permits and auction only small part of them.

[3] These are the only NAP that could be found in their English version.

[4] We refer the readers to the UK NAP section 1.15 for Second Phase for detailed view on the methods and incetives behind the allocation plan. It can be found on the following website: http://www.defra.gov.uk/environment/climatechange/trading/eu/index.htm

[5] Appendix D1 UK NAP. The same benchmark spreadsheet is used to determine the relevant emission for incumbent firms.

2012, when the Kyoto protocol expires. This fact creates uncertainty in which companies that are subject to cap and trade of permits will have to consider variety of future policies. Although they can anticipate what possible scenarios are, it is highly unlikely that they can anticipate their exact probability distribution. Therefore, their lack of information creates a special sort of uncertainty often referred as *ambiguity.* In the presence of ambiguity, no mater how much more information companies gather to calculate their optimal behavior, they will remain uncertain as to what is the right probability distribution of possible policy scenarios. In the organizational context, where management faces dispersed knowledge, the distinction between ambiguity and uncertainty is of a great importance (Becker 2001).

The distinction between uncertainty and ambiguity in the decision maker (DM) state of mind is well defined in the economic literature (see for example Ellsberg 1961, Mukerji 1997). Schmidler and Gimlboa (1989) have developed an axiomatic representation of decision where they distinguish between situations where the DM is aware of the objective probabilities of underlying scenarios and where he/she is not. The former is regarded as *uncertainty* and the latter as *ambiguity.* Whereas in the case of uncertainty we may use a standard approach of expected values,we cannot do so in the case of ambiguity. The main reason for that is that ambiguity cannot be represented by an additive probability distribution. In the presence of ambiguity the DM subjective beliefs are represented by convex non-additive probability k sometimes referred as Knightian probability or capacity. To deal with this special case of uncertainty, Choquet integral is accepted as a main tool of evaluating the expected value (Scmeidler 1989, Sarin and Wakker 1992). Schmidler and Gimlboa (1989) show that given a convex non additive probability k, the Choquet integral is a simple minimum of all possible values. Doing so we find the most pessimistic expected value.[6]

Application of Knightian uncertainty is recently made to company's decision making for irreversible investment (Nishimura *et al.* 2007). In that context the authors find that the effect of Knightian uncertainty is drastically different from that of traditional uncertainty. Given these results it seems natural to extend existing literature on emissions allocation incorporating ambiguity in the production decision of companies. It would be interesting to compare companies decision under these two sorts of uncertainties. In order

[6] For an example of how to use a Choquet integral see Dow *et al*, (1992)

to make the proposed model empirically related we choose to base it on one of the EU states NAP. Due to relative simplicity of the UK NAP method of allocation, we adopt it as our case study.

3. The Model

3.1. Preliminaries of the Model. According to the UK NAP carbon permits allocated on the sectorial level. Namely, permits are allocated first to the whole market and afterwords divided among sectors of the market. Therefore, we choose to focus our analysis at each sector individually. First, we derive results assuming that companies know the exact probability distribution of potential future policies (*Uncertainty Case*). Then, we release the later assumption. We analyse a scenario in which companies do not know the exact distribution but rather hold a set of possible probability distributions of potential future policies (*Ambiguity Case*).

T- defined as a time horizon of the model. $t \in T$ can take any natural number between $(0, \infty)$.We restrict our model to three periods only, $t \leq 3$. First, we derive results from two-periods model. Next, we extend two- periods model to incorporate also the third period. In the three-periods model we assume that companies are not aware of the allocation method that governs in period $t = 3$.

N-is the total number of companies in the sector, s.t. $i \in N = (0, \infty)$.

K- is the number of new entrant companies in the sector, s.t. $K = (0, \infty)$ and $K < N$. As a result, total number of incumbent companies in the sector is $(N - K)$.[7].

q_i^t- is an output that each company i produces in period t. We assume that companies choose their level of production at the beginning of each period t.

Q^t- is a total output in the sector in period t where

$$Q^t \equiv \sum^{N} q_i^t \tag{1}$$

E^t- is a total of issued permits for distribution in the sector in period t. Policy maker (in our case, UK government) sets the cap of total permits to emit Green House Gases

[7] It is also possible to account for different amount of firm at each period, thus considering more general cases. However, the assumption that at each period there is an identical amount of firms, will not affect the qualitative result. By assuming that the total number of companies is identical in each period, we impose that number of new entrants and closers are identical.

(GHG) in order to comply with its obligation to reduce its national emissions level. Each permit allows to emit one metric tonne of CO_2.

E^t_{NER}- New Entrant Reserve (NER) is a set aside of permits for new installations in period t. The UK government, as many of the EU states, has decided to create the NER of permits for new companies.[8] According to the UK NAP: '(c)ontribution to the NER in each sector are deducted from the total allocation to that sector before distributing the remaining to existing installations'[9]. As a result, permits that are left for incumbent installations equal

$$(E^t - E^t_{NER})$$

e^t_i - is verified emissions of company i in period t.We assume that actual emissions are expressed as a linear function of companies output, where $\delta^t = (0, \infty)$ is the marginal rate of emissions in period t. For simplicity we assume that companies in the same sector have identical marginal rate of emissions δ^t[10]. Therefore, actual emissions level of a company in the sector can be expressed as

$$e^t_i = \delta^t q^t_i$$

$\overline{q^t}$- is a projected output for new entrant in period t.

m- is a market price to buy or sell permits to emit 1 tonne of GHG. The price of permits is established in the permits market. One of the largest trading platforms for carbon permits is the European Energy Exchange. Due to the large number of participants in the daily trade the price of permits is assumed to be exogenous to the companies.[11]

d- is a discount factor between two adjusting periods. We assume that $d = (0, 1)$

3.2. Allocation rule. G*randfathering* is an allocation rule where permits are distributed based on an historical emission levels. The allocation to each incumbent installation is done according to the following formula[12]:

[8] See section 2.4 in the UK NAP.

[9] Section 2.14 in the UK NAP.

[10] It is a reasonable assumption, as we are dealing with companies at the same sector. Similar assumption is taken by Hepburn et al. (2006)

[11] Website of European Energy Exchange:http://www.eex.com/en/

[12] Section 3.2 in the UK NAP

$$\text{Total Incumbent allocation}=\frac{\text{Incumbent's relevant emissions}}{\text{Sum of relevant emissions of all incumbents in the sector}}(E^t - E^t_{NER}) \quad (2)$$

New entrants receive permits according to their projected emissions using current marginal rate of emissions δ^t. We set it equal to

$$\overline{e^t} = \delta^t \overline{q^t}$$

we have to bear in mind that $\sum \delta^t \overline{q^t} \leq E^t_{NER}$, as the number of permits allocated to new entrants cannot exceed the total of permits that are distributed among new entrants.

First, we are introducing a two period model. In the section 3.3 we will extend the model to three periods.

3.3. Two period Oligopoly-Benchmark case. We denote the inverse demand that companies face in the market for their product

$$P(Q^t) = \alpha - bQ^t$$

c_i- is the marginal cost of company i to produce one additional unit of output q^t_i.

π^t_i-is the profit function of company i in period t.

In the UK NAP, historic emissions is a relevant to the allocation of permits. The policy which is adopted in the second phase of the UK NAP (2008-2012) is that of a rollovering an historic emissions. That is to say, emissions of years 1998-1999 is relevant for permits allocation in the first phase , in the second phase relevant emissions level is rolled over to years 2000-2003. To follow this methodology we say that emissions in period $(t-2)$ determines the allocation for the incumbent in period t. For example, if we want to determine what is the relevant allocation for the incumbent in period $t = 2$,we would take its historic emissions level in period $t = 0$.

The profit function π^t_i can be expressed:

$$\pi^t_i = [\alpha - bQ^t]q^t_i - c_i q^t_i - m[\delta^t q^t_i - \frac{q^{t-2}_i}{Q^{t-2}}(E^t - E^t_{NER})]$$

The expression $\delta^t q^t_i - \frac{q^{t-2}_i}{Q^{t-2}}(E^t - E^t_{NER})$ is the difference between the allocated permits $\frac{q^{t-2}_i}{Q^{t-2}}(E^t - E^t_{NER})$ and company's emissions to produce q^t_i.

The difference between actual an allocation and actual emissions of the company result in surplus/deficit of its permits. On the one hand if a company has a surplus of permits it will sell them for the price of m. The revenue from the selling permits is a subsidy to the company that outperforms and reduces its emissions level below the initial allocation. On the other hand if the company does not hold enough permits to cover its actual emissions, it can purchase additional permits for the price of m. The costs of purchasing additional permits is a 'tax' to the company that emit more that its initial allocation.

Profit function π_i^t for the new entrant:

$$\pi_i^t = [\alpha - bQ^t]q_i^t - c_i q_i^t - m[\delta^t q_i^t - \delta^t \overline{q^t}]$$

The expression $\delta^t q_i^t - \delta \overline{q^t}$ is the difference between the allocated permits $\delta^t \overline{q^t}$ and new entrant's emissions to produce q_i^t.

Π_i^t- is the total profit of company i in period t. We can express the total profit Π_i^t as a discounted sum of all its profit from period $t = 1$ to $t = T$

$$\Pi_i^t \equiv \sum_1 d^{t-1} \pi_i^t$$

We analyse our problem as a game between N companies. Technically we solve optimization problem in Cournot Oligopoly with two periods. We use a standard method of backward induction to find an optimal output in each period t.

Proposition 1. *It two periods oligopoly with grandfathering rule of permits allocation total output is not affected by future rule of permits allocation .*

Second period. Let's denote $t = 0$ as a relevant period for incumbent's allocation of permits in period $t = 2$. E^2 is the total of permits to be distributed among the companies.

The maximization of *incumbent*

$$\arg\max_{q_i^2} \pi_i^2 = [\alpha - bQ^2]q_i^2 - c_i q_i^2 - m[\delta^2 q_i^2 - \frac{q_i^0}{Q^0}(E^2 - E_{NER}^2)]. \tag{3}$$

Furthermore, the maximization of *new entrant*

$$\arg\max_{q_i^2} \pi_i^2 = [\alpha - bQ^2]q_i^2 - c_i q_i^2 - m[\delta^2 q_i^2 - \delta \overline{q^2}] \tag{4}$$

The setup of the profit function of new entrant is similar to the profit function of incumbent represented by Eq.(3). First Order Condition of both new entrant and incumbent installations is represented by Eq. (5).

First Order Condition:

$$\frac{d\pi_i^2}{dq_i^2} = \alpha - b(Q^2) - bq_i^2 - c_i - m\delta^2 = 0 \tag{5}$$

Summing up N First Order Conditions, as the number of companies in the sector, we get:

$$N\alpha - (N+1)b(Q^2) - \Sigma c_i - Nm\delta^2 = 0 \tag{6}$$

Optimal total output in the sector in $t = 2$, we solve Eq.(6) for Q^{2*} :

$$Q^{2*} = \frac{N\alpha - \Sigma c_i - Nm\delta^2}{(N+1)b} \tag{7}$$

We see that optimal total output is not affected by the future allocation of permits. We derive Second Order Condition to verify the optimal condition for Q^{2*}.

Second Order Condition:

$$\frac{d^2\pi_i^2}{d^2q_i^2} = -2b < 0 \tag{8}$$

The Second Order Condition is satisfied insuring that Eq.(7) is the optimal total output that maximizes total profit in period $t = 2$.[13]

First period. We use the method of backward induction to find the optimal output in period $t = 1$. We add the discounted profit π_i^2 to the profit π_i^1 which results in total

[13]As the number of firms in the market increases, the conditions in the market approach the competitive equilibrium. The output in the market approaches competitive output. To find the competitive output we have to assume that $\lim_{N\to\infty}(\frac{\sum C_i}{(N+1)b}) = C$ is a constant. Otherwise the equation explodes and tends to infinity:

$$\lim_{N\to\infty} Q^2 = \frac{\alpha - m\delta}{b} - C \tag{9}$$

profit Π_i^1 of company i in period $t=1$. We denote period $t=-1$ as a relevant period for allocation of permits to incumbent in period $t=1$. The maximization of *incumbent* is

$$\arg\max_{q_i^1} \Pi_i^1 = [\alpha - bQ^1]q_i^1 - c_i q_i^1 - m[\delta^1 q_i^2 - \frac{q_i^{-1}}{Q^{-1}}(E^1 - E_{NER}^1)] + d\pi_i^2 \tag{10}$$

The maximization of *new entrant* is

$$\arg\max_{q_i^1} \Pi_i^1 = [\alpha - bQ^1]q_i^1 - c_i q_i^1 - m[\delta^1 q_i^2 - \delta\overline{q^1}] + d\pi_i^2 \tag{11}$$

Profit function π_i^2 does not depend on q_i^1. Therefor first order condition is

$$\frac{d\Pi_i^1}{dq_i^1} = \alpha - b(Q^1) - bq_i^1 - c_i - m\delta^1 = 0 \tag{12}$$

FOC of new entrant in $t=1$ is similar to Eq. (12). Summing up N first order conditions and solving for the optimal output in period $t=1$, we we get that $Q^1$is identical to Q.

The optimal output in period $t=2$ [14]:

$$Q^{1*} = \frac{N\alpha - \sum c_i - Nm\delta^1}{(N+1)b} \tag{13}$$

Let's denote:

$$\overline{Q^1} \equiv Q^{1*} \tag{14}$$

$$\overline{Q^2} = Q^{2*} \tag{15}$$

To sum up, in two periods framework companies have no incentive to increase their output to receive larger share of permits in the future allocation. Therefore, in the benchmark case, policy of revision of total emissions cap (E^t) to achive the target of reducing emissions level can be implemented. That is to say, as the total production in the sector does not depend on the future policy and future cap, policy maker can revise the cap at each phase individually and set E^t in a way that achieves the desired emissions in the economy in period t. It is done based on the emissions projection in that period which are derived from the total estimated output Q^t. We should note, however, that

[14]For the S.O.C please check the solution for the first Second period-Oligopoly. The condition is represented by Eq. (8).

the former argument is restricted only to myopic companies that consider their action for the nearest future. In our case there are two periods only. For instance, policy maker in period $t = 1$ would ideally set up the cap of E^1 to satisfy the following equality

$$E^1 = \delta^1 \overline{Q^1} = \lambda(\delta^1 Q_{BAU})$$

rearranging we get that

$$\overline{Q^1} = \lambda Q_{BAU}$$

where $\lambda \in (0, 1)$ is the parameter indicating the commitment of policy maker to reduce the emissions level, such that $\lambda = 0$ would represent policy maker which is committed to reduce the emissions by 100 percents. And Q_{BAU} is the production in the business as usual scenario, which stands for production in case there is no cap on the emission. The same rule would apply to the cap of period $t = 2$.

However, companies are not acting myopically and consider their action with respect to longer horizons that are beyond two periods. Two questions naturally arise in this context. First, what happens to total output if we consider a framework of more than two periods. Second, whether the UK NAP still fulfills its purpose of eliminating the incentives of companies to act strategically. In the following section we address these questions.

3.4. Three period Oligopoly- Uncertainty Case. Although there is a general commitment in the Energy White Paper of the UK to continue reducing emission beyond the Kyoto commitment period, there is no clear policy of how it would be done. Therefore, from now on we assume that companies face an uncertain future environmental policy beyond 2012, when the Kyoto protocol expiries. We show the effect of uncertainty on total output. To do so, we first extend the benchmark model to three periods. Next, we state what the most probable allocation policies are for the third period. We also assign probabilities to possible allocation scenarios as they are perceived by companies. To conclude this section we compare total output under uncertainty with total output under the benchmark model.

Third period - Uncertainty Case. To analyse the effect of uncertainty on the total output in the market we assume that companies consider only two policies of allocation. On the one hand, policy maker continues with rollingover the relevant historic emissions. This is a reasonable assumption. In period $t = 3$ emissions level of period $t = 1$ are available and show a more updated measure of historic emissions than emissions level of period $t = 0$. On the other hand, policy maker might adopt more recent emissions level. That is to say, historic emissions of period $t = 2$ is a relevant emissions for allocation of permits in period $t = 3$. A reasonable justification for that can be that policy maker might try to diminish companies incentive to adjust their behavior in period $t = 1$ to receive a larger share of permits in period $t = 3$[15]. Let's denote p^tas a probability that policy maker assigns relevant historic emissions to be in period t such that

$$\sum p^t = 1.$$

The maximization of *incumbent* is

$$\arg\max_{q_i^3} \pi_i^3 = [\alpha - bQ^3]q_i^3 - c_i q_i^3 - m[\delta^3 q_i^3 - \{p^1 \frac{q_i^1}{Q^1} + p^2 \frac{q_i^2}{Q^2}\}(E^3 - E_{NER}^3)] \tag{16}$$

The maximization of *new entrant*:

$$\arg\max_{q_i^3} \pi_i^3 = [\alpha - bQ^3]q_i^3 - c_i q_i^3 - m[\delta^3 q_i^3 - \delta^3 \overline{q^3}] \tag{17}$$

First Order Condition of new entrant's maximization in period $t = 3$ is similar to the First Order Condition of the incumbent and represented by Eq. (18).

First Order Condition:

$$\frac{d\pi_i^3}{dq_i^3} = \alpha - b(Q^3) - bq_i^3 - c_i - m\delta^3 = 0 \tag{18}$$

Summing up N FOC we get that the total output in period $t = 3$ equals:

$$Q^3 = \frac{N\alpha - \sum c_i - Nm\delta^3}{(N+1)b} \tag{19}$$

To find what the total output is in periods $t = 1, 2$ we proceed, as before, in standard method of backward induction.

[15] We can also assume that companies can assign probabilities to policy that assign relevant emissions to $t = 0$. However, it will not affect qualitative result as this option will be discarded in FOC. We therefore ignore this scenario and concentrate on the two mentioned scenarios.

Lemma 2. *Let's denote Q_U^t as total output in period t when companies consider future policy in period $t = 3$ and the policy is uncertain. Total output in periods $t < 3$ increases, so that*

$$Q_U^t = \frac{\overline{Q^t}}{2} + \sqrt{\frac{(\overline{Q^t})^2}{4} + d^{T-t}\frac{m(N-1)(E^3 - E^3_{NER})p^t}{(N+1)b}} > \overline{Q}.$$

Second period-Uncertainty Case. **Proof.** See Appendix A.1.1 ■

First period-Uncertainty Case. **Proof.** See Appendix A.1.2 ■

We conclude that in the presence of uncertainty, total output in the sector increases in both periods $t = 1$ and $t = 2$. This results suggest that in companies which consider longer horizons policies tend to overproduce to receive a larger share in future permits' allocation- *the ratchet effect.* Unlike the benchmark case, the production in current period ($t = 1$) is a positive function future allocation of allowances E^3. Therefore, policy maker which ignores that effect may find it hard to achieve its goal of reducing emissions levels. Only considering short term policy would underestimate the productions levels Q^1 in the economy and consequently miss the emissions reduction targets.

3.5. Ambiguity vs. Uncertainty. In the previous section we assume only two scenarios, where their probabilities are know to the companies and summing up to 1. In reality it is highly unlikely that companies aware of the exact probability distribution of possible future policies. Instead of having one probability distribution, companies might hold a set of probability distributions. This situation of uncertainty is labelled as *ambiguity.* In ambiguity both possible future policies and their expected value are uncertain. In order to understand the effect of ambiguity on total output we have to briefly introduce a notion of a *capacity* and *Choquet expected value.*

Capacity. In contrast to standard assumptions on probabilities, capacities(Knightian probabilities) assign non-additive weights to possible scenarios. Capacity can be formally represented by a real function k which satisfied the following properties (Schmeidler, 1989):

(a) for two events A,$B \in \Omega$ s.t $A \subseteq B \Longrightarrow k(A) \leq k(B)$

(b) $k(\varnothing) = 0$;

(c) $k(\Omega) = 1$.

Capacity is convex if it satisfies: $k(A) + k(B) \leq k(A \cup B) + k(A \cap B)$. In this paper we will concentrate on convex capacities.

According to Scmeidler and Gilboa(1989) a *core 'C'* of k, where $\Delta(\Omega)$ is the set of all additive probability measures on Ω:

$$C(k) = \{p \in \Delta(\Omega) \mid p(A) \geq k(A) \; \forall A \subseteq \Omega\}$$

meaning that $C(k)$ is the set of all the plausible probabilities, that companies may assign to future policy. Therefor, $k(A) = \min_{p \in C(k)} p(A)$.

In the presence of ambiguity we assume that companies assigns Knightian, non-additive, probabilities $k^1$and k^2 to the same policies as in uncertainty case. For convex capacities, the fact that $\sum k^t < 1$ reflects ambiguity of the DM. The measure of ambiguity aversion can be represented by

$$AA = 1 - k^1 - k^2.$$

The higher the AA, the higher is the ambiguity aversion of the DM (Dow *et al.* 1992). An increase of ambiguity aversion (AA) leads can be shown by introducing the concept of ε-contamination to show it .

ε-contamination. Behavioural foundation for ε-contamination can be found in Nishimura *et al.* (2006) and its application to a discrete time search in Nishimura *et al.* (2004). The concept of ε-contamination is usually used in the context of Bayesian uncertainty. To deal with Bayesian uncertainty a new set of priors is introduces by contaminating one single hypothetical prior (Nishimura *et al.* 2004) This procedure is often referred as ε-contamination. We also follow this technique by contaminating the prior distribution (p^1, p^2) which we assume in the uncertainty case.

In the previous section we defined capacity as $k(A) = \min_{p \in C(k)} p(A)$. We contaminate priors (p^1,p^2) and set the core to be in the range of $C(k) = (p^t - \varepsilon^t, p^t + \varepsilon^t)$, where $\varepsilon^t > 0$ is a small number. An increase in ε^t can be seen as an increase of ambiguity. An increase in ε^t leads to increase of AA as k^t decrease. Companies become more ambiguous regarding the future policy. They therefore they tend to decrease their output.

Choquet Expected value. Choquet integral is accepted as a main tool to evaluate the expected value in case of non-additive probabilities (Scmeidler 1989, Sarin and Wakker

1992). Schmidler and Gimlboa (1989) show that given a convex non additive probability k the Choquet expected value is a simple minimum of all possible values. In our case companies simply choose the lowest possible value for allocation of permits according to the following formula:

$$CE = \begin{pmatrix} \min \\ p \in C(k) \end{pmatrix} \sum_{t=1}^{t=2} p \frac{q_i^t}{Q^t}$$

where $\frac{q_i^t}{Q^t}$ is the relevant output ratio for allocation in period $T = 3$. To find out the explicit expression for the Choquet expected value we consider two cases:

Case 3. *For companies which relative production in period* $t = 1$ *is higher than in period* $t = 2$ $(\frac{q_i^1}{Q^1} > \frac{q_i^2}{Q^2})$, *the Choquet expected value* (CE) *is*

$$CE = k^1 (\frac{q_i^1}{Q^1} - \frac{q_i^2}{Q^2}) + \frac{q_i^2}{Q^2}$$

Case 4. *For companies which relative production in period* $t = 1$ *is lower than in period* $t = 2$ $(\frac{q_i^1}{Q^1} < \frac{q_i^2}{Q^2})$, *the Choquet expected value* (CE) *is*

$$CE = k^2 (\frac{q_i^2}{Q^2} - \frac{q_i^1}{Q^1}) + \frac{q_i^1}{Q^1}$$

We have to note that unless $\sum k^t < 1$, two Choquet expected values above coincide.

Third period - Ambiguity Case. New entrant incur initial costs. Therefore, we assume that companies entering the market period $t = 1$ increase their share in total production in period $t = 2$, s.t.$\frac{q_i^1}{Q^1} < \frac{q_i^2}{Q^2}$. We assume that total of incumbents in period $t = 3$ which satisfies condition $\frac{q_i^1}{Q^1} < \frac{q_i^2}{Q^2}$ equals G. The rest of incumbents satisfy $\frac{q_i^1}{Q^1} < \frac{q_i^2}{Q^2}$. In light of these assumptions we distinguish between two maximisations of incumbent in period $t = 3$.

1) when $\frac{q_i^1}{Q^1} > \frac{q_i^2}{Q^2}$:

$$\arg\max_{q_i^3} \pi_i^3 = [\alpha - bQ^3]q_i^3 - c_i q_i^3 - m[\delta^3 q_i^3 - \{k^1 \frac{q_i^1}{Q_A^1} + (1 - k^1)\frac{q_i^2}{Q_A^2}\}(E^3 - E_{NER}^3)] \quad (20)$$

2) when $\frac{q_i^1}{Q^1} < \frac{q_i^2}{Q^2}$:

$$\arg\max_{q_i^3} \pi_i^3 = [\alpha - bQ^3]q_i^3 - c_i q_i^3 - m[\delta^3 q_i^3 - \{(1-k^2)\frac{q_i^1}{Q_A^1} + k^2 \frac{q_i^2}{Q_A^2}\}(E^3 - E_{NER}^3) \quad (21)$$

The maximization problem of new entrant in period $T = 3$:

$$\arg\max_{q_i^3} \pi_i^3 = [\alpha - bQ^3]q_i^3 - c_i q_i^3 - m[\delta^3 q_i^3 - \delta\overline{q^3}] \quad (22)$$

Total output of incumbent and new entrant in period $t = 3$ equals to Q^3 and is represented by Eq. (7)

Proof. See section 3.2. ■

Using backward induction we calculate total output both in period $t = 2$ and $t = 1$.

Proposition 5. *Let's denote Q_A^t as total output in period t when companies consider future policy in period $t = 3$ and the policies are ambiguous. Then*

$$\begin{cases} Q_A^1 < Q_U^1 \text{ if } k^2 = p^2 \\ Q_A^1 > Q_U^1 \text{ if } k^1 = p^1 \end{cases}$$

Second period-Ambiguity Case. **Proof.** See Appendix A.2.1 ■

First period-Ambiguity Case. **Proof.** See appendix A.2.2 ■

We show that the total output depends on the subjective beliefs by the companies, specifically capacities. We make a reasonable assumption in order to show the effect of the ambiguity on the total production. We assume that $k^1 = p^1$. Introducing this assumption we say that $\varepsilon-$contamination of policy that we assign probability p^1 in the *Uncertainty case* is very small and we set it to be $\varepsilon^1 = 0$. This might seems to be a reasonable assumption as the probability of future policy to follow the same method of rolling-over relevant emissions/output is less ambiguous than the other potential policy. Therefore, we say that only ambiguous scenario is policy that we assign probability p^2 in the *Uncertainty case.* Using the proposition above we see that under our assumption the total output in period $t = 1$, when we assume ambiguity, is higher than in the case where we assume uncertainty.

Given these results, it is clear that under ambiguity policy maker might find real difficulties achieving goals of emissions reduction if it does not account for the ambiguity aversion. It seems that under ambiguity case, total output is even higher than in the case of uncertainty and consequently the emissions levels in the economy. Therefore, as suggested in the previous section, only considering short term policy would underestimate the productions levels Q^1 in the economy and consequently miss the emissions reduction targets.

4. Analysis: Increase in Uncertainty vs. Increase in Ambiguity

The effect of output-based allocation of emissions permits has been already examined by Fischer (2001). Fischer, in a simple one period model, finds that an output-based allocation has smaller impact on the output reduction than a fixed allocation. Similar results were also found in the empirical analysis of emissions in the province of Alberta, Canada (Haites, 2003). However, these results are restricted to one period only. Our model presents a more general framework for analysing the impact of output-based allocation on total production. We show in the previous section that ambiguity and uncertainty of future allocation policy increases the total output in the sector in comparison with lump-sum allocation.

Next, we propose an analysis of total output when companies face an increase in ambiguity and/or uncertainty.

Proposition 6. *Whenever companies face an increase of ambiguity of future policy it leads to an* ***increase of*** *total output. Whereas an increase of uncertainty of future policy has* ***no effect*** *on total output.*

Proof. See Appendix A.3. ■

4.1. Increase of Uncertainty. It is accepted to analyse an increase in uncertainty by a mean preserving spread technique. In our context an increase of uncertainty spreads possible future policy, such that the new spread preserves the expected value of the expected policy.

Two alternatives for possible $(\frac{q_i^1}{Q_U^1}, \frac{q_i^2}{Q_U^2})$. The former stand for policy that considers historical emissions of period $t=1$ and the later of period $t=2$. The spread is $(\frac{q_i^1}{Q_U^1} + \theta, \frac{q_i^2}{Q_U^2} - \frac{p^1}{1-p^1}\theta)$. Parameter θ is interpreted as an additional factor to future policies. For instance, policy makers can change the number of total permits to be distributed in the sector, this way adjusting to an updated information on the environmental impacts.

It can also represent an additional tax or subsidy that policy makers can impose on companies that are subject to cap and trade of emissions permits. We can clearly see from Proposition **6** that total output in period $t < 3$ is not affected by an increase of uncertainty. However, an increase of ambiguity has a different effect.

4.2. Increase in Ambiguity. An increase of ambiguity aversion (AA) leads to a increase in total output. Employing the concept of ε-contamination we can show that an increase in ε can be seen as an increase of ambiguity. An increase in ε leads to increase of AA as subsequently k^t decrease. Companies become more ambiguous regarding the future policy.

We employing our previous assumption that ε-contamination of p^1 is such that $\varepsilon^1 = 0$. Next we move to analyse the scenario where ambiguity aversion, AA, increases as a result of higher contamination of probability p^2, such that ε^2 is increasing. This setting indicates that total output in period $t = 1$ increases as a result of high degree of AA.

5. Policy implications

In this section we describe some of the policy implications. We show that whenever there is an uncertainty in the market regarding the future policy it tends to affect the total output in the market. Companies tend to increase their output when they face uncertain future policy. For instance, in the UK alone emissions to cap ratio has risen from 15% in 2005 to 19.5% in 2007[16]. These figures show that the UK industry increases its emissions beyond the initial allocation. Our model can suggest that the rise in the emissions to cap ration is driven by the behavioral biases. And if we are right in our predictions then it seems that the role of the policy maker is to eliminate such behavioural biases. This conclusion corresponds with the Environmental literature. For example, a similar idea is proposed by Baldursson *et al.* (2004). The authors suggest that in the presence of uncertainty in the market policy makers should favour tax regulations on emissions rather than issuing transferable permits as the former regime has a smaller effect on the companies behaviour. In other words, policy makers should favour a regime which diminishes behavioural biases of companies. To conclude, we suggest some policy implications.

[16] Source :http://www.carbonmarketdata.com/pages/Press%20Release%20EU%20ETS%20Data%20-%20April%202008.pdf

5.1. Information Certainty. As we show in our previous analysis ambiguity of future policy increases current total output with comparison to the lump-sum allocation (*Benchmark case*). In addition, we show that an increase in ambiguity tends to increase the total output. One of the possible interpretations of increase in ambiguity is that there is high information uncertainty regrading the future policy. It is to say, the higher the information uncertainty in future policy the higher is the current total output and emissions level. Therefore, policy maker should try and reveal its long-term policy. This way it contribute to decrease of total output in the economy and subsequently make it possible for achieving emission reduction targets.

As we point out companies are not aware of the correct probability distribution of potential allocation policies. However companies hold a set of probabilistic beliefs, rather than one probability distribution, as to what potential policies might be. In other words, companies state of mind is of ambiguity rather than of uncertainty. Therefore policy maker can affect company's state of mind by shrinking the set of their beliefs. This can be done by signalling what the future allocation policy is expected to be. Correct signalling might eliminate strategical behaviour or at least diminish behavioral biases of companies and achieve the desired policy goal.

For instance, if policy makers want to encourage lower output in the market, they could release an information regarding future policy that should decreases $\left(\min_{p^2 \in C(k)}\right)(1-p^2)$, as we have already shown. According to our model such signalling decreases total output in the sector.

$$Q_A^1 < \frac{\overline{Q^1}}{2} + \sqrt{\frac{(\overline{Q^1})^2}{4} + d^2 \frac{m(N-1)(E^3 - E_{NER}^3)\left(\min_{p^2 \in C(k)}\right)(1-p^2)}{2(N+1)b}}.$$

Periodical press releases that indicate what future policies might be affect companies believes. Indeed, EU members states periodically issue press releases regarding future environmental policies. Such releases include NAP, guidelines and goals of future policy. [17] We see that the EU uses signalling as a tool. However, it is not clear whether the use of signalling is aimed towards the modification of beliefs or merely for informative reasons. It is, however, clear that policy makers could use signalling to produce desirable outcomes. It is especially useful, as we see from the analysis above, when one deals with

[17] http://ec.europa.eu/environment/climat/2nd_phase_ep.htm

Environmental policies.

5.2. Diminishing allocation. In the model we show that the total output in the sector is a function of future allocation of permits $(E^3 - E^3_{NER})$. In addition to the information disclosure, which has been discussed earlier, policy maker could diminish the effect of the future allocation on present production by reducing the $(E^3 - E^3_{NER})$ variable. This can be done in two ways. On the one hand, policy makers may gradually diminish the amount of permits that are distributed for free, namely E^3. One can increase the number of permits for an action instead of offering them for free distribution. This way the permits are allocated to companies that values them the most. Moreover, the revenue received from auctioning can be allocated for R&D of environmental friendly technology that can reduce GHG. Similar views are advocated by Bovenberg *et al.*(2005), Quirion(2003) and Hepburn *et al.*(2006)

In reality, this corresponds to the current tendency in the environmental policy in the EU. For instance, UK, Austria, France, Poland and many more gradually increase the number of permits that are auctioned. This way the mentioned EU member states reduce the permits that are distributed for free. However, we should note that E^3_{NER} is a function of E^3. Therefore reducing E^3 not only affects incumbent companies but also new entrants; and it may affect a competition in the market by posing a substantial obstacle for new entrants to enter the market.

On the other hand, policy maker may increase the total of permits of the new entrant reserve, namely E^3_{NER}. Increasing E^3_{NER} decreases the effect of future allocation on present output and diminishes incentives for strategic behaviour. In addition, this step contributes to the competition in the market by reducing entrance barriers. There are no signs that the EU member states undertake the former step to diminish the psychological biases of companies.

5.3. Clean Technology. One of the results above indicates that in the ideal scenario policy maker should set up a cap of the emissions that corresponds to the optimal production and marginal rate of emissions, formally $E^3 = \delta^3 Q^3$. It seems that another way of diminishing the effect of future policy on current production is by diminishing future marginal rate of emissions, δ^3. This way policy maker should achieve to goals. First, expected growth of production to the future level of Q^3. Second, fulfilling its targets of diminishing emissions level.

In order to diminish future marginal rate of emissions policy maker should encourage R&D in cleaner technologies which could potentially provide companies with environmentally friendly process of production.

6. Concluding remarks

In this paper we analyze the effect of ambiguity on total output. We show that in the presence of ambiguity or uncertainty with output based allocation companies tend to increase their production compared to the fixed allocation. Decreasing ambiguity has a diminishing effect on the total output. In the analysis of these results we point out how the former result can be made used of by policy makers. They might achieve both high rates of productivity and emissions abatement. Despite the generality of our model it has few shortcomings which can be seen as potential subjects for future research. One is to account for heterogeneity in ambiguity that is perceived by companies. Some may say that small or new companies are more vulnerable to changes in ambiguity, whereas large companies are less vulnerable. As a portfolio of later is more diversified. To see the effect of the production on the price structure one should endogenise the price of permits. While we assumed an identical number of new entrants and closures at each period, one could think of heterogeneous number of new entrants and closures.

Despite the mentioned shortcoming of our model, it sheds some insights on the output determination in the EU ETS. In the structural terms, our paper solves three-periods Oligopoly model with ambiguity.

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A. APPENDIX

A.1. Proof of Lemma 2.

A.1.1 Second Period Total Output. Assuming that we have the value of the total output which maximises the third period profit, we can plug it as given into Π_i^2(the total profit function of the second period).

Maximisation of incumbents is

$$\arg\max_{q_i^2} \Pi_i^2 = [\alpha - bQ_U^2]q_i^2 - c_i q_i^2 - m[\delta^2 q_i^2 - \frac{q_i^0}{Q^0}(E^2 - E_{NER}^2)] + d\pi_i^3 \tag{23}$$

Maximisation of new entrants is

$$\arg\max_{q_i^2} \Pi_i^2 = [\alpha - bQ_U^2]q_i^2 - c_i q_i^2 - m[\delta^2 q_i^2 - \delta\overline{q^2}] + d\pi_i^3 \tag{24}$$

First Order Condition of new entrants is identical to the First Order Condition of the incumbents and equals

$$\frac{d\Pi^2}{dq_i^2} = \alpha - bQ_U^2 - bq_i^2 - c_i - m\delta^2 + dm(E^3 - E_{NER}^3)p^2 \frac{Q_U^2 - q_i^2}{(Q_U^2)^2} = 0. \qquad (25)$$

Summing up N F.O.C, as the number of the companies in the market, we get

$$N\alpha - (N+1)bQ_U^2 - \sum c_i - Nm\delta^2 + d\frac{m(E^3 - E_{NER}^3)p^2}{(Q_U^2)'}(N-1) = 0.$$

Rearranging

$$Q_U^2 = \frac{\overline{Q^2}}{2} + \sqrt{\frac{(\overline{Q^2})^2}{4} + d\frac{m(N-1)(E^3 - E_{NER}^3)p^2}{(N+1)b}} > \overline{Q} \qquad (26)$$

A.1.2 First Period Total Output. Maximisation incumbents is

$$\arg\max_{q_i^1} \Pi_i^1 = [\alpha - bQ_U^1]q_i^1 - c_i q_i^1 - m[\delta^1 q_i^1 - \frac{q_i^0}{Q^0}(E^1 - E_{NER}^1)] + d\Pi_i^2 \qquad (27)$$

and new entrants is

$$\arg\max_{q_i^1} \Pi_i^1 = [\alpha - bQ_U^1]q_i^1 - c_i q_i^1 - m[\delta^1 q_i^1 - \delta\overline{q^1}] + d\Pi_i^2 \qquad (28)$$

Solving for the value of Q_U^1 we get

$$Q_U^1 = \frac{\overline{Q^1}}{2} + \sqrt{\frac{(\overline{Q^1})^2}{4} + d^2\frac{m(N-1)(E^3 - E_{NER}^3)p^1}{(N+1)b}} > \overline{Q} \qquad (29)$$

A.2. A.2. Choquet Expected value (CE). 1.Given that $\frac{q_i^1}{Q^1} > \frac{q_i^2}{Q^2}$, CE is

$$\begin{aligned} CE &= \begin{pmatrix} \min \\ p \in C(k) \end{pmatrix} \sum_{t=1}^{t=2} p \frac{q_i^t}{Q^t} = \begin{pmatrix} \min \\ p^1 \in C(k) \end{pmatrix} (p^1 \frac{q_i^1}{Q_A^1} + (1-p^1)\frac{q_i^2}{Q_A^2}) \\ &= \begin{pmatrix} \min \\ p^1 \in C(k) \end{pmatrix} p^1 (\frac{q_i^1}{Q^1} - \frac{q_i^2}{Q^2}) + \frac{q_i^2}{Q^2} \\ &= k^1 (\frac{q_i^1}{Q^1} - \frac{q_i^2}{Q^2}) + \frac{q_i^2}{Q^2} \end{aligned}$$

2. Given that $\frac{q_i^1}{Q^1} < \frac{q_i^2}{Q^2}$, CE is

$$\begin{aligned} CE &= \begin{pmatrix} \min \\ p \in C(k) \end{pmatrix} \sum_{t=1}^{t=2} p \frac{q_i^t}{Q^t} = \begin{pmatrix} \min \\ p^2 \in C(k) \end{pmatrix} ((1-p^2)\frac{q_i^1}{Q_A^1} + p^2 \frac{q_i^2}{Q_A^2}) \\ &= \begin{pmatrix} \min \\ p^2 \in C(k) \end{pmatrix} p^2 (\frac{q_i^2}{Q^2} - \frac{q_i^1}{Q^1}) + \frac{q_i^1}{Q^1} \\ &= k^2 (\frac{q_i^2}{Q^2} - \frac{q_i^1}{Q^1}) + \frac{q_i^1}{Q^1} \end{aligned}$$

A.3. Proof of Proposition 5.

A.3.1 Second period Total Output. Maximisation incumbents is

$$\arg\max_{q_i^2} \Pi_i^2 = [\alpha - bQ_A^2]q_i^2 - c_i q_i^2 - m[\delta^2 q_i^2 - \frac{q_i^0}{Q^{0\prime}}(E^2 - E_{NER}^2)] + d\pi_i^3 \quad (30)$$

First Order Condition of incumbents which satisfy the condition of $\frac{q_i^1}{Q^1} < \frac{q_i^2}{Q^2}$ is

$$\frac{d\Pi^2}{dq_i^2} = \alpha - b(Q_A^2) - bq_i^2 - c_i - m\delta^2 + dm(E^3 - E_{NER}^3)k^2 \frac{Q_A^2 - q_i^2}{(Q_A^2)^2} = 0 \quad (31)$$

First Order Condition of incumbents which satisfy the condition of $\frac{q_i^1}{Q^1} > \frac{q_i^2}{Q^2}$ is

$$\frac{d\Pi^2}{dq_i^2} = \alpha - bQ_A^2 - bq_i^2 - c_i - m\delta^2 + dm(E^3 - E_{NER}^3)(1-k^1)\frac{Q_A^2 - q_i^2}{Q_A^{2\,2}} = 0 \quad (32)$$

Maximisation of new entrants is

$$\arg\max_{q_i^2} \Pi_i^2 = [\alpha - bQ_A^2]q_i^2 - c_i q_i^2 - m[\delta^2 q_i^2 - \delta^2 \overline{q^2}] + d\pi_i^3 \quad (33)$$

First Order Condition of new entrants is identical to First Order Condition of incumbents which satisfy the condition of $\frac{q_i^1}{Q^1} < \frac{q_i^2}{Q^2}$. It is important to notice that for the new entrants which enters in period $t = 2$ it is always the case that their relative production in period $t = 1$ is smaller than in period $t = 2$ ($\frac{q_i^1}{Q^1} < \frac{q_i^2}{Q^2}$). As they do not produce in period $t = 1$, and they are assigned with the benchmarking ratio. Therefore it is reasonable to assume that the entrants increase their share in total output.

Summing up N FOC we get

$$N\alpha - (N+1)bQ_A^2 - \sum c_i - Nm\delta^2 + d\frac{m(E^3 - E_{NER}^3)}{(Q_A^2)^2}\{\sum^{N-G}(Q_A^2 - q_i^2)k^2 + \sum^{G}(Q_A^2 - q_i^2)(1-k^1)\} = 0$$

Given that $\sum k^t < 1$ we get

$$N\alpha - (N+1)bQ_A^2 - \sum c_i - Nm\delta^2 + d\frac{m(E^3 - E_{NER}^3)(N-1)k^2}{Q_A^2} < 0, \tag{34}$$

and

$$N\alpha - (N+1)bQ_A^2 - \sum c_i - Nm\delta^2 + d\frac{m(E^3 - E_{NER}^3)(N-1)(1-k^1)}{Q_A^2} > 0 \tag{35}$$

Solving the eq. (34) and eq. (35) we get a range for Q_A^2:

$$Q_A^2 > \frac{\overline{Q^2}}{2} + \sqrt{\frac{(\overline{Q^2})^2}{4} + d\frac{m(N-1)(E^3 - E_{NER}^3)k^2}{2(N+1)b}} \tag{36}$$

$$Q_A^2 < \frac{\overline{Q^2}}{2} + \sqrt{\frac{(\overline{Q^2})^2}{4} + d\frac{m(N-1)(E^3 - E_{NER}^3)(1-k^1)}{2(N+1)b}} \tag{37}$$

A.3.2 First period Total Output. Maximisation of incumbents is

$$\arg\max_{q_i^1} \Pi_i^1 = [\alpha - bQ_A^1]q_i^1 - c_i q_i^1 - m[\delta^1 q_i^1 - \frac{q_i^{-1}}{Q^{-1}}(E^1 - E_{NER}^1)] + d\Pi_i^2 \tag{38}$$

First Order Condition of incumbents which satisfy the condition of $\frac{q_i^1}{Q^1} < \frac{q_i^2}{Q^2}$:

$$\frac{d\Pi^1}{dq_i^1} = \alpha - bQ_A^1 - bq_i^1 - c_i - m\delta^1 + d^2 m(E^3 - E_{NER}^3)(1-k^2)\frac{Q_A^1 - q_i^1}{(Q_A^1)^2} = 0 \tag{39}$$

First Order Condition of incumbents which satisfy the condition of $\frac{q_i^1}{Q^1} > \frac{q_i^2}{Q^2}$:

$$\frac{d\Pi^1}{dq_i^1} = \alpha - bQ_A^1 - bq_i^1 - c_i - m\delta^1 + d^2 m(E^3 - E_{NER}^3)k^1\frac{Q_A^1 - q_i^1}{(Q_A^1)^2} = 0 \tag{40}$$

Maximisation of new entrants is:

$$\arg\max_{q_i^1} \Pi_i^1 = [\alpha - bQ_A^1]q_i^1 - c_i q_i^1 - m[\delta^1 q_i^1 - \overline{\delta^1 q^1}] + d\Pi_i^2 \tag{41}$$

First Order Condition of new entrants is identical to First Order Condition of incumbents satisfying the condition of $\frac{q_i^1}{Q^1} < \frac{q_i^2}{Q^2}$(above, Appendix A.3.1).

Summing up N F.O.C of the firms in the market:

$$N\alpha - (N+1)bQ_A^2 - \sum c_i - Nm\delta^1 + d^2\frac{m(E^3 - E_{NER}^3)}{(Q_A^1)^2}\{\sum^{N-G}(Q_A^1 - q_i^1)(1-k^2) + \sum^{G}(Q_A^1 - q_i^1)k^1\} = 0 \tag{42}$$

Given that $\sum k^t < 1$ we get

$$N\alpha - (N+1)bQ_A^1 - \sum c_i - Nm\delta^1 + d^2 \frac{m(E^3 - E^3_{NER})(N-1)k^1}{Q_A^1} < 0 \tag{43}$$

and

$$N\alpha - (N+1)bQ_A^1 - \sum c_i - Nm\delta^1 + d^2 \frac{m(E^3 - E^3_{NER})(N-1)(1-k^2)}{Q_A^1} > 0 \tag{44}$$

Solving the eq. (43) and eq. (44) we get a range for Q_A^1:

$$Q_A^1 > \frac{\overline{Q^1}}{2} + \sqrt{\frac{(\overline{Q^1})^2}{4} + d^2 \frac{m(N-1)(E^3 - E^3_{NER})k^1}{2(N+1)b}} \tag{45}$$

$$Q_A^1 < \frac{\overline{Q^1}}{2} + \sqrt{\frac{(\overline{Q^1})^2}{4} + d^2 \frac{m(N-1)(E^3 - E^3_{NER})(1-k^2)}{2(N+1)b}} \tag{46}$$

A.4. Proof of Proposition 6. Substituting spread allocation with an additional parameter θ to eq.(16) has no effect on the maximisation in period $t = 3$, as the parameter θ cancels out. Therefore, an increase of uncertainty has no effect on the total output in the sector. However, in the case of ambiguity, increase of ambiguity has a different effect on total output.

An increase of parameter ε^2 has the following effect on the total output:

- In case we increase ε^2, in period $t = 1$ the lower bound remains tha same

$$Q_A^1 > \frac{\overline{Q^1}}{2} + \sqrt{\frac{(\overline{Q^1})^2}{4} + d^2 \frac{m(N-1)(E^3 - E^3_{NER})\left(\min_{p^1 \in C(k)}\right)p^1}{2(N+1)b}}$$

whereas upper bound increases

$$Q_A^1 < \frac{\overline{Q^1}}{2} + \sqrt{\frac{(\overline{Q^1})^2}{4} + d^2 \frac{m(N-1)(E^3 - E^3_{NER})\left(\min_{p^2 \in C(k)}\right)(1-p^2)}{2(N+1)b}}$$

Therefore, the total effect on Q_A^1 is positive as Q_A^1 increases.

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