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Abstract

What are the dynamic consequences of comprehensive integration shocks? The answer to this question appears all but trivial. We set up a dynamic macroeconomic model of a small open economy where both capital and labor are mobile and there are increasing returns to scale at the aggregate level. The model features multiple equilibria as well as (local and global) indeterminacy. Hence, expectations matter for resulting equilibrium dynamics. Despite its simplicity, the model creates a rich set of plausible implications. Our analysis contributes to a better understanding of the interaction between expectations and fundamentals in models with indeterminacy. The model is applied to replicate two striking empirical characteristics of macroeconomic development in East Germany since 1991.

JEL-Code: E600, H200, O400.

Keywords: increasing returns to scale, capital mobility, migration, multiple equilibria, indeterminacy, history vs. expectations.

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1 Introduction

There are numerous examples in economic history of economies that have experienced a comprehensive integration into a larger economic entity or into the global economy. Examples comprise the first era of globalization (about 1820 until 1913) with nearly perfect capital mobility and mass migration among Atlantic economies (O’Rourke and Williamson, 1999), the unification of South and North Italy in 1862 (Boltho et al., 1997), the German Reunification in 1991 (Sinn, 2002) and the enlargement of the European Union in 2004 to include many of the East European economies (Mora et al., 2004). What are the dynamic consequences with regard to income and welfare of such comprehensive integration shocks for specific regions or economies? The answer to this question appears all but trivial.

We argue that a careful analysis should account for increasing returns to scale (IRS) and factor mobility. Employing a simple one-sector model of a small open economy we find that the success of economic development in response to factor market integration (FMI) is determined by history and expectations. Our analysis contributes to a better understanding of the interaction between expectations and fundamentals in models with indeterminacy. It is shown that (i) the relative importance of expectations as determinant of economic success depends on total factor productivity (TFP) and (ii) admissible expectations depend on fundamentals (initial conditions and TFP) in a systematic fashion. This insight has far reaching economic implications. For instance, it is shown that bad institutions may hinder
favorable economic development because sufficiently optimistic expectations cannot be fundamentally warranted under this side condition.

In search for an answer to our research question, international trade theory seems to be the natural starting point. Provided that a set of neoclassical conditions (including constant returns to scale) holds, trade liberalization induces, first, (incomplete) specialization in goods production according to the principle of comparative advantages and, second, international equalization of factor rewards. Additional FMI would add no further implications. However, the consequences of goods trade and FMI differ drastically if one believes in IRS. Goods trade then induces complete specialization in goods production, whereas factor price equalization does not apply anymore. FMI, on the other hand, may lead to declining areas or set the stage for prospering regions by attracting capital and labor. Interestingly, Paul Romer has recently proposed an innovative development strategy, labeled "charter cities", which rests on IRS, sound institutions and complete factor mobility (Romer, 2010).

There is indeed strong evidence, both direct and indirect, in favor of (aggregate) IRS. For instance, Schmitt-Grohé (1997) reviews the empirical literature on IRS. She reports that the degree of IRS (at the level of industries) ranges from 1.03 to 1.4 (Schmitt-Grohé, 1997, Table 4). Graham and Temple (2006) argue that models with IRS and multiple equilibria (ME) can explain a substantial part of the international income disparity. Depending on the strength of the positive externality in the manufacturing sector, they attribute between 18 percent and 50 percent to
the presence of ME. Consequently, a careful discussion of the question raised above should take IRS and factor mobility into account. Moreover, IRS in endogenous input factors, as is well known, may give rise to ME. An important question, then, is how the process of equilibrium selection works. Most theoretical models imply that initial conditions are crucial (see, for instance, Galor, 1996; Deissenberg et al., 2001). There are, however, also models showing that expectations may play an important role in the process of equilibrium selection (Krugman, 1991; Galor, 1992; Graham and Temple, 2006).

There is also substantial, although at this stage inconclusive, evidence on the importance of expectations as driving force for economic growth. Sengupta and Okamura (1995) estimate an error-correction model to identify the determinants of economic growth in Japan between 1965 and 1990. They conclude that "future expectations may play a stronger role than the historical endowments" (Sengupta and Okamura, 1995, p. 494). Sauer et al. (2003) use time-series data for selected industries (eight emerging countries in East Asia and Eastern Europe) to test for the big push industrialization hypothesis (Murphy et al., 1989). Their results support the basic idea that optimistic expectations play an important role in the process of economic development. Harris and Ioannides (2000) investigate the relative importance of history versus expectations in US urban development by testing whether asset prices (housing or land) are Granger causal for population. The authors are more reluctant and conclude that "the results confirm the notion that expectations

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1See also the literature discussed in Graham and Temple (2006, Section 2).
at best help history along in determining whether a city dominates or languishes in the periphery" (Harris and Ioannides, 2000, p. 9). Alesina et al. (1996) analyze the relationship between political instability and per capita GDP growth in a sample of 113 countries for the period 1950 through 1982. Economic growth in countries and time periods with a high probability of government collapse is found to be significantly lower than otherwise. They conclude that political instability induces expectations about unfavorable (uncertain) future economic policy which unfolds a detrimental impact on growth.

The list of theoretical papers dealing with the dynamic consequences of FMI under IRS appears limited. Dellas and de Vries (1995) employ an OLG model with learning externalities in final output production. The model features ME. The consequences of drastic (precipitous) and gradual (piecemeal) FMI are investigated. The authors identify conditions, in terms of initial endowments, under which a drastic FMI leads to permanently lower, whereas a gradual integration leads to a permanently higher level of world per capita income. Faini (1996) employs a two-regions, small open economy model with labor mobility to investigate the link between convergence, growth and factor mobility. The tradeable goods sector exhibits constant returns to scale, while the non-tradeable goods sector exhibits IRS. It is shown that neither long-run convergence nor long-run divergence is compelling. Both results are possible and the outcome depends on the underlying set of parameters. Reichlin and Rustichini (1998) employ a two-countries, one-sector model with labor heterogeneity, IRS, and labor mobility to explain persistent migration flows and the lack
of cross-country convergence. The main result is that a labor exporting country may either experience an increase or a decline in per capita GDP depending on the composition of migration on early stages. These papers are clearly instructive and show that the consequences of FMI under IRS are all but trivial. None of these contributions takes, however, the role of expectations in the process of equilibrium selection into account.

We develop a simple one-sector model of a small open economy to study the dynamic consequences of FMI under IRS. FMI means that both input factors get access to an outside option, i.e. capital can be invested abroad and workers may start to emigrate. Agents (capital owners and workers) form rational expectations about future factor rewards. The model features ME as well as (local and global) indeterminacy. There is a multiplicity of expectations which give rise to self-fulfilling prophecies. A number of non-trivial implications are obtained: (i) The success of economic development in response to FMI is determined by history and expectations. (ii) The relative importance of expectations as determinant of economic success depends on TFP. The economic reason is simply that the interdependence of decisions (across groups) to engage in the domestic market sector increases with TFP. (iii) The model implies that there is a multiplicity of admissible initial expectations. Admissibility means that initial expectations must be fundamentally warranted. Initial expectations do hence interact with fundamentals (initial conditions and TFP) in a systematic fashion. This implies that, for instance, bad institutions (red tape and corruption), which have a detrimental impact on TFP, may block favorable economic
development because sufficiently optimistic expectations cannot be fundamentally warranted under this side condition.

The paper is structured as follows. In Section 2 the model is developed and its main properties are described. Section 3 is devoted to the interaction between expectations and fundamentals. In Section 4 the model is applied to replicate three stylized facts of macroeconomic development in East Germany since 1991. Section 5 summarizes and concludes.

2 The model

Consider a dynamic one-sector model of a perfectly competitive, small open economy. To enable a discussion of the dynamics of the regional income distribution, we view this economy as being composed of a number of \( n \geq 1 \) regions. If \( n = 1 \) there is just one single region which comprises the entire, small open economy. The alternative case, i.e. \( n > 1 \), allows for a regional disaggregation.\(^2\) For simplicity regions do not interact.\(^3\) FMI means that both input factors get access to an outside option, i.e. capital can be invested abroad and workers may emigrate. Moving input factors out of or into the domestic market sector is associated with mobility

\(^2\)This aspect will become relevant when applying the model to better understand the stylized facts of macroeconomic development in East Germany (Section 4).

\(^3\)This simplifying assumption is uncritical with regard to the major implications. For a, say, declining region it does not matter whether capital and workers move to another region or move abroad.
costs. Agents form rational expectations about future factor rewards. Regions are identical except for the initial amount of capital and labor allocated to the domestic market sector and the degree of initial optimism or pessimism. The model features IRS at the aggregate level due to external economies.

2.1 Production technology and factor prices

The output technology of a typical firm $j \in [0, 1]$ in region $i \in \{1, ..., n\}$ reads

$$Y_{j,i} = AK_j^a K_i^\alpha L_j^a L_i^b, \quad (1)$$

where $A$ denotes a constant technology parameter (capturing TFP), $K_{j,i}$ capital employed by firm $j$ in region $i$, $K_i$ the overall stock of capital in region $i$, $L_{j,i}$ the amount of labor employed by firm $j$ in region $i$, $L_i$ the overall amount of labor employed in region $i$, and $0 < \alpha, \beta, a, b < 1$ constant parameters, which satisfy $\beta + b < 1, \alpha + a < 1$. There are constant returns to scale at the level of the individual firm, i.e. $\alpha + \beta = 1$. Aggregating over firms gives the aggregate output technology of region $i$ to read $Y_i = AK_i^{\alpha+a} L_i^{\beta+b}$. At the aggregate level there are IRS, i.e. it is assumed that $\alpha + \beta + a + b > 1$. The major implications, described below, do not depend on the specific mechanism which gives rise to aggregate IRS. Marshallian externalities represent one widely used mechanism (e.g., Krugman, 1991; Graham and Temple, 2006). Competitive factor prices are given by

$$r_i = \alpha AK_i^{\alpha+a-1} L_i^{\beta+b}, \quad (2)$$

$$w_i = \beta AK_i^{\alpha+a} L_i^{\beta+b-1}, \quad (3)$$
where denotes \( r_i \) the rate of return to capital and \( w_i \) the competitive wage rate in region \( i \), respectively.

Equation (2) implies \( \frac{\partial r_i}{\partial L_i} > 0 \) and \( \frac{\partial^2 r_i}{\partial L_i \partial A} > 0 \). Hence, the positive impact of additional workers employed in the domestic market sector on the incentive for capital owners to engage in the domestic market economy, reflected by \( r_i \), increases with the TFP parameter \( A \). Similarly, equation (3) implies \( \frac{\partial w_i}{\partial K_i} > 0 \) and \( \frac{\partial^2 w_i}{\partial K_i \partial A} > 0 \). Within the context of the underlying model, as will become clear below, this implies that the interdependence of decisions (across groups) to engage in the domestic market sector increases with TFP. As a consequence, a higher level of TFP gives expectations a greater role in the process of equilibrium selection. In addition, a high TFP level may facilitate sufficiently optimistic expectations such that an economy experiences a favorable economic development despite comparably unfavorable initial conditions.

Under (aggregate) IRS, factor rewards and per capita income are crucially determined by the amount of capital and labor allocated to the domestic market sector. We therefore turn to the allocation decisions of capital owners and workers at next. Despite considering both outward and inward capital flows as well as outward and inward migration, the terminology is based on the perspective of domestic expatriates. The domestic region / economy is often denoted as "the source", whereas the rest of the world is denoted as "the destination".
2.2 Capital owners

Every region is populated by a continuum of length one of identical capital owners.\(^4\) Every capitalist is endowed with \(\bar{K}_i^S\) units of capital. Capital can be employed in the region’s domestic market sector (source) earning a rate of return \(r_i\). Alternatively, capital can be invested abroad (destination) to earn the fixed rate of return \(\bar{r} > 0\).

The representative capital owner in the source maximizes the present value of an infinite income stream, i.e. solves the following problem

\[
\max_{\{v^S_{K_i}\}} \int_0^\infty \left[ r_i K_i^S + \bar{r}(\bar{K}_i^S - K_i^S) - \frac{1}{2\gamma_K} (v^S_{K_i})^2 \right] e^{-\rho t} dt
\]

s.t. \(\dot{K}_i^S = v^S_{K_i}\)

\(K_i^S(0) = \bar{K}_i^S, 0 \leq K_i^S \leq \bar{K}_i^S,\)

where \(\rho > 0\) denotes the time preference rate and \(\dot{K}_i^S := dK_i^S/dt\). Moving capital from the region’s domestic market sector to the outside option, or vice versa, causes (symmetric and convex) capital adjustment costs which reduce current income, as captured by the term \(\frac{1}{2\gamma_K} (v^S_{K_i})^2\), where \(v^S_{K_i}\) denotes a control variable. More precisely, moving one additional unit of capital from, say, the source to the destination causes relocation costs of \(\frac{\dot{K}_i^S}{\gamma_K}\), i.e. marginal adjustment costs increase with the flow of capital being transferred.\(^5\) The parameter \(\gamma_K > 0\) is an inverse mea-

\(^4\)To simplify, we assume that capital owners don’t move. The location of capital owners is not important since we are interested in the determinants of GDP, not in the determinants of GNP.

\(^5\)There are indeed convex capital reallocation costs at the individual level. This is consistent with the following interpretation. If the individual capital owner transfers financial funds, no
sure for the importance of adjustment costs. We assume that the typical capital owner has all his capital \( \bar{K}^S_i \) allocated to the domestic market economy initially, i.e. \( K_i^S(0) = \bar{K}^S_i \). Thus he can either keep all his capital inside the region’s domestic market sector or start investing abroad. An increase in the region’s stock of capital requires capital inflows from abroad. To simplify the analysis, we assume that foreign capital investments are irreversible. This implies that capital once invested in, say, the source cannot be invested again in the destination.

Let \( H^S := H^S(K_i^S, v_{K_i}^S) \) denote the associated (current value) Hamiltonian function. The first-order conditions may then be expressed as follows

\[
\frac{\partial H^S}{\partial v_{K_i}^S} = -\frac{1}{\gamma_K} v_{K_i}^S + \lambda_{K_i} = 0 \quad \Rightarrow \quad v_{K_i}^S = \gamma_K \lambda_{K_i} \tag{5}
\]

\[
\dot{\lambda}_{K_i} = \rho \lambda_{K_i} - \frac{\partial H^S}{\partial K_i^S} = \rho \lambda_{K_i} - (r_i - \bar{r}) . \tag{6}
\]

Equation (5) says that, in equilibrium, marginal moving costs \( v_{K_i}^S \) must equal the shadow price \( \lambda_{K_i} \). Equation (6) indicates that \( \lambda_{K_i}(0) = \int_0^\infty (r_i - \bar{r}) e^{-\rho t} dt \), i.e. \( \lambda_{K_i}(0) \) gives the difference between earnings in the domestic market sector and in the outside option in present value terms.\(^6\) Since the competitive rate of return \( r_i \) depends on the amount of capital and labor employed in the domestic market sector, \( \lambda_{K_i}(0) \) captures expectations about future economic development.

\(^6\)This solution for \( \lambda_{K_i}(0) \) requires that the boundary condition \( \lambda_{K_i}(T) = 0 \) (the so-called "soft landing condition") holds, where \( T \) denotes the point in time where the economy hits a boundary (see Section 3.1).
The problem of the typical capital owner in the destination (rest of the world) may be expressed as follows

$$\max_{\{v_{K_i}^D\}} \int_0^\infty \left[ \tilde{r} \left( \dot{K}_i^D - \sum_{i=1}^n K_i^D \right) + \sum_{i=1}^n r_i \left( \frac{K_i^D}{v_{K_i}^D} \right)_{t=0} - \sum_{i=1}^n \frac{1}{2\gamma_K} (v_{K_i}^D)^2 \right] e^{-\rho t} dt \tag{7}$$

s.t. $\dot{K}_i^D = v_{K_i}^D \quad \forall \ i \in \{1, ..., n\}$

$$K_i^D(0) = 0 \quad \forall \ i \in \{1, ..., n\}, 0 \leq K_i^D \leq \bar{K}_i^D.$$ 

where $\bar{K}_i^D > 0$ denotes the capital endowment, $K_i^D \geq 0$ the amount of capital invested in region $i$, $\tilde{r}$ the rate of return to capital in the destination, $r_i$ the rate of return in region $i$, and $v_{K_i}^D$ choice variables, respectively. We assume $K_i^D(0) = 0 \forall \ i \in \{1, ..., n\}$ implying that the initial capital endowment is invested completely in the destination. Similarly to the case considered above, the first-order conditions $\dot{\lambda}_{K_i} = \rho \lambda_{K_i} - (r_i - \tilde{r})$, together with appropriate border conditions, imply that the shadow value placed on capital invested in region $i$ is given by $\lambda_{K_i}(0) = \int_0^\infty (r_i - \tilde{r}) e^{-\rho t} dt$.

Physical capital in region $i$, denoted as $K_i$, increases due to capital inflows $\dot{K}_i^D \geq 0$ and decreases due to capital outflows $\dot{K}_i^S \leq 0$, i.e. $\dot{K}_i = \dot{K}_i^D + \dot{K}_i^S$. Physical capital in region $i$ therefore changes according to (for details see the appendix)

$$\dot{K}_i = \gamma_K \lambda_{K_i} \leq 0 \quad \text{for} \quad \lambda_{K_i} \leq 0. \tag{8}$$

### 2.3 Workers

There are $L_i(0) = L_{i,0}$ identical workers initially in region $i$, who live forever. Every worker supplies one unit of time per period, independent of the wage rate, to the
labor market. Domestic workers have the possibility to emigrate. Migration decisions are modelled according to Braun (1993). The benefit of emigration at \( t = 0 \) is reflected by

\[
\lambda_{L_i}^D(0) = \int_0^\infty (\bar{w} - w_i) e^{-\rho t} dt, \quad (9)
\]

where \( \rho > 0 \) denotes the time preference rate, \( w_i \) is the domestic competitive wage rate, and \( \bar{w} \) is the wage rate that can be earned in the destination. The benefit of emigrating at \( t = 0 \) from the source to the destination, denoted as \( \lambda_{L_i}^D(0) \), is given by the difference between earnings in the outside option (destination) and in the domestic market sector (source) in present value terms. Since the domestic competitive wage rate \( w_i \) depends on the amount of capital and labor employed in the domestic market sector, \( \lambda_{L_i}^D(0) \) captures expectations about the future economic development.

The moving costs per migrant are given by

\[
m_{c_i} = \frac{1}{\gamma_L} \dot{L}_{i}^{SD}, \quad (10)
\]

where \( \dot{L}_{i}^{SD} \geq 0 \) denotes the flow of migrants from the source to the destination per period of time. Equation (10) shows that (individual) migration costs increase with the number of migrants, which may be due to congestion externalities (cf. Braun, 1993, p. 24). The parameter \( \gamma_L > 0 \) is an inverse measure, given \( \dot{L}_{i}^{SD} \), for the importance of the moving costs. The individual moving decision is a binary

---

7The Braun (1993) model of migration and growth represents a dynamic, representative-agent model (see also Barro and Sala-i-Martin, 2004, Chapter 9.1.3).

8The key property is that the cost of moving for the marginal mover rises with the number of
decision. Workers leave the economy if the benefit of emigration, reflected by \( \lambda_{Li}^D \), exceeds the migration costs, as given by \( mc_i \). In a competitive equilibrium the benefit of emigration must equal the costs of migration, i.e. \( \lambda_{Li}^D = \frac{1}{\gamma_L} \dot{L}_i^{SD} \). Moreover, if the benefit of emigration is zero, \( \lambda_{Li}^D = 0 \), no one will want to emigrate, i.e. \( \dot{L}_i^{SD} = 0 \). Similarly, provided that \( \lambda_{Li}^D < 0 \), one gets \( \dot{L}_i^{SD} = 0 \). Migration from the source to the destination (outward migration) may hence be described as follows

\[
\dot{L}_i^{SD} = \begin{cases} 
\gamma_L \lambda_{Li}^D & \text{for } \lambda_{Li}^D > 0 \\
0 & \text{for } \lambda_{Li}^D \leq 0
\end{cases}
\]  

(11)

Consider at next the typical worker in the destination (i.e. the rest of the world). Let \( \lambda_{Li}^S (0) = \int_0^\infty (w_i - \bar{w}) e^{-\rho t} dt \) denote the value of being in the source (domestic region) rather than being in the destination (rest of the world). Notice also that \( \lambda_{Li}^D = -\lambda_{Li}^S \). The moving costs per migrant are again given by \( mc_i = \frac{1}{\gamma_L} \dot{L}_i^{DS} \), where \( \dot{L}_i^{DS} \geq 0 \) denotes the flow of migrants from the rest of the world to the domestic region per period of time. Applying an analogous reasoning as before yields

\[
\dot{L}_i^{DS} = \begin{cases} 
\gamma_L \lambda_{Li}^S & \text{for } \lambda_{Li}^S > 0 \\
0 & \text{for } \lambda_{Li}^S \leq 0
\end{cases}
\]  

(12)

The rate of change of workers in the domestic region is the difference between inward migration and outward migration, i.e. \( \dot{L}_i = \dot{L}_i^{DS} - \dot{L}_i^{SD} \). The number of movers. This relation would also hold if there were heterogeneity with respect to moving costs. The persons with lower costs would move sooner, and the cost of moving would therefore rise at the margin with the number of movers (cf. Barro and Sala-i-Martin, 2004, p. 401).
workers in region $i$ therefore changes according to (for details see the appendix)

$$
\dot{L}_i = \gamma_L \lambda^S_{L_i} \leq 0 \text{ for } \lambda^S_{L_i} \leq 0.
$$

(13)

A clarification may be warranted. Consider a set of domestic regions with $\lambda^S_{L_i} > 0$. Does the model imply that the region with the highest $\lambda^S_{L_i}$ attracts workers from the rest of the world first and then other regions develop at next? Assuming that there are enough workers outside the economy for a simultaneous development of all regions with $\lambda^S_{L_i} > 0$, the answer is no.

As stated above, the focus is on the source. Therefore, we consider variables that relate to the source only. Moreover, to simplify the notation we set $\lambda^S_{L_i} = \lambda_{L_i}$. This notation then implies that $\lambda_{L_i} < 0$ captures pessimism in the sense that the difference between $w_i$ and $\bar{w}_i$, in present value terms, is negative.

### 2.4 Dynamic system and steady states

Noting (6), (8), (13) and differentiating $\lambda_L(0) = \int_0^\infty (w - \bar{w}) e^{-\mu t} dt$ with respect to $t$, the dynamic system which governs the evolution of the economy within the interior of the state space may be expressed as follows

$$
\dot{K} = \gamma_K \lambda_K
$$

(14)

$$
\dot{L} = \gamma_L \lambda_L
$$

(15)

$$
\dot{\lambda}_K = \rho \lambda_K - (r - \bar{r})
$$

(16)

---

9To simplify notation, the region index is suppressed whenever this does not lead to confusion.
\[ \dot{L}_L = \rho \lambda_L - (w - \bar{w}) \]  
(17)

\[ K(0) = K_0, \quad L(0) = L_0, \]

where \( r \) and \( w \) are given by (2) and (3). Provided that \( \lambda_K(0) \) and \( \lambda_L(0) \) are specified, the above system describes a unique trajectory in four-dimensional \((K, L, \lambda_K, \lambda_L)\)-space. However, \( \lambda_K(0) \) and \( \lambda_L(0) \) are not uniquely determined. There is rather a multiplicity of shadow price combinations \([\lambda_K(0), \lambda_L(0)]\) which are admissible as self-fulfilling prophecies. Moreover, any set of admissible expectations \([\lambda_K(0), \lambda_L(0)]\) is restricted to be fundamentally warranted, as will be explained below.

A steady state is determined by \( \dot{K} = \dot{L} = \dot{\lambda}_K = \dot{\lambda}_L = 0 \). We first turn to the interior steady state. From \( \dot{K} = \gamma_K \lambda_K \) and \( \dot{L} = \gamma_L \lambda_L \) one recognizes that \( \dot{K} = \dot{L} = 0 \) requires \( \lambda_K = \lambda_L = 0 \). From (16), (17), \( \lambda_K = \lambda_L = 0 \) (implying \( \dot{\lambda}_K = \dot{\lambda}_L = 0 \)) one gets \( r = \bar{r} \) and \( w = \bar{w} \). These two equations in \( K \) and \( L \) characterize the interior steady state in \((K, L)\)-plane.\(^{10}\)

Noting (2) and (3) and solving for \( L \) gives

\[ L = \left( \frac{\bar{r}}{\alpha A} \right)^{\frac{1}{\beta + b}} K^{\frac{1-\alpha-a}{\beta+b}} \]  
(18)

\[ L = \left( \frac{\bar{w}}{\beta A} \right)^{\frac{1}{\beta+b-1}} K^{\frac{\alpha+a}{1-\beta-b}}. \]  
(19)

Since \( \beta + b < 1, \alpha + a < 1, \) and \( \alpha + \beta + a + b > 1 \), it follows that, first, the RHS of (18) is an increasing and concave function of \( K \) due to \( 0 < \frac{1-\alpha-a}{\beta+b} < 1 \) and, second, the RHS of (19) is an increasing and convex function of \( K \) due to \( \frac{\alpha+a}{1-\beta-b} > 1 \). Hence,\(^{10}\)

\(^{10}\)Notice, however, that \( r = \bar{r} \) and \( w = \bar{w} \) is necessary but not sufficient for \( \dot{K} = 0 \) and \( \dot{L} = 0 \); sufficient for \( \dot{K} = 0 \) (\( \dot{L} = 0 \)) is \( \lambda_K = 0 \) (\( \lambda_L = 0 \)).
there is a unique interior solution \((K^*, L^*)\), as illustrated by point A in Figure 1.

There are also two boundary steady states. The lower (inferior) steady state is \((K = 0, L = 0)\), point C in Figure 1. The upper (superior) steady state reads \((K = K^{**}, L = L)\), point B in Figure 1.

As regards the dynamics at the border of the state space, two aspects need to be clarified. First, the economy remains at the boundary once it touches the border of the state space (see the appendix for details). Second, in a world of IRS we need to ensure that factor inflows sooner or later come to a halt. It is assumed that the maximum number of workers that can move into a specific region cannot exceed \(\bar{L}\). This may be interpreted as capturing the importance of a third (fixed) factor, land, in the background. The maximum amount of capital, \(K^{**}\), is then endogenously
determined by \( r(K^*, \bar{L}) = \bar{r} \). To illustrate, assume that the economy hits, say, the upper \( L \)-boundary at \( t = T \), i.e. \( L(T) = \bar{L} \), with \( 0 < K(T) < K^* \). The dynamics of the economy are then governed by (14), (16), and \( L = \bar{L} \). Capital inflows take place until \( r = \bar{r} \). This movement is sluggish because of convex adjustment costs (for details see the appendix).

\[ \text{3 Expectations and fundamentals} \]

\[ \text{3.1 Admissible expectations and equilibrium dynamics} \]

The dynamic system (15) to (16), evaluated at the interior steady state, exhibits three eigenvalues with positive real part. The interior steady state (point A in Figure 1) is hence locally unstable. Moreover, there is indeterminacy in the sense of a multiplicity of admissible initial shadow prices \([\lambda_K(0), \lambda_L(0)]\). Admissible expectations and equilibrium trajectories must fulfill the following criteria (see the appendix for details)

1. Equilibrium trajectories must approach the border of the state space tangential, i.e. must satisfy one of the "soft-landing conditions" \( \dot{K}(T) = \lambda_K(T) = 0 \) or \( \dot{L}(T) = \lambda_L(T) = 0 \). Once the economy hits the border it does not return into the interior of the state space. Instead it moves along the boundary to one of the border equilibria (B or C in Figure 1).

2. Given initial conditions \( K(0) = K_0 \) and \( L(0) = L_0 \), there is a multiplicity
of initial shadow prices \([\lambda_K(0), \lambda_L(0)]\) which are admissible as self-fulfilling prophecies. Hence, there is an infinite number of equilibrium trajectories, indexed by initial shadow prices.

3. The set of admissible shadow prices must, first, equal the present value of expected earning differentials and, second, must give rise to a trajectory that satisfies the soft-landing condition.

### 3.2 Interaction between expectations and fundamentals

Any region is characterized by a predetermined history in the sense of an initial amount of capital and labor allocated to the domestic market sector. If the region under study starts inside a specific \((K, L)\)-set, expectations (initial shadow prices) determine whether the region moves towards the superior or inferior steady state, i.e. the model exhibits global indeterminacy. Within this area of global indeterminacy - or overlap, a term coined by Krugman (1991) - knowledge about initial state variables is not sufficient to determine the final outcome. In contrast, if the economy starts with sufficiently unfavorable initial conditions (i.e. southwest of the overlap) it converges to the inferior steady state. Similarly, if it starts with sufficiently favorable initial conditions (i.e. northeast of the overlap) it converges to the superior steady state.

To visualize the area of global indeterminacy, we have discretized the state space, i.e. we have defined a grid of points in \((K, L)\)-space. Then we have checked whether,
for a specific \((K, L)\)-combination, there is at least one admissible set of expectations \([\lambda_K(0), \lambda_L(0)]\) which gives rise to a trajectory leading to the superior steady state and, for the same \((K, L)\)-combination, there is at least one set of admissible expectations \([\lambda_K(0), \lambda_L(0)]\) which gives rise to a trajectory leading to the inferior steady state. If this condition is satisfied, the \((K, L)\)-combination under consideration is an element of the overlap. If this is not the case, the specific \((K, L)\)-combination lies outside the overlap.\(^{11}\)

Figure 2: Three exemplary regions (D, E, F) and a multiplicity of equilibrium trajectories

\(^{11}\)A description of the underlying numerical procedure is available from the authors upon request.
Figure 2 illustrates the basic logic of the model. For expositional convenience, the size of the factor box was chosen such that the interior steady state is centered in $L$-dimension and $K$-dimension and, in addition, is normalized to one. The area of global indeterminacy is represented by the set of grey rectangles. Consider a region starting with an initial endowment given by the coordinates of point D. Provided that agents are sufficiently optimistic, both capital owners and workers increasingly engage in the region’s domestic market sector. The region prospers and converges to the superior equilibrium at B. In contrast, if agents are pessimistic both capital and labor leave the region’s domestic market sector. The region declines and approaches the inferior equilibrium at C.\(^{12}\) The region with initial endowment D, therefore, exhibits global indeterminacy.

Local indeterminacy means that there is a multiplicity of equilibrium trajectories leading to, say, the superior equilibrium. Local indeterminacy may also be of substantial economic interest. Consider again a region with an initial endowment given by D and assume that expectations are such that the region prospers, i.e. approaches the superior equilibrium at point B. The multiplicity of trajectories indicates that the model can explain both a pattern of immigration and capital inflows (one of the upper trajectories) or a pattern of emigration followed by immigration (i.e. migration reversals) and capital inflows (one of the lower trajectories). The trajectory which describes a migration reversal is based on $\lambda_L(0) < 0$ and $\lambda_K(0) > 0$.

\(^{12}\)Equilibrium trajectories could, of course, also hit the (lower or upper) $K$-boundary for interior $L$-values. This pattern is, however, rarely observed for plausible calibrations.
such that labor emigrates initially, \( \dot{L} < 0 \), but capital is being attracted, \( \dot{K} > 0 \).

Both \( \dot{L} < 0 \) and \( \dot{K} > 0 \) push the wage rate up such that \( \lambda_L(t) \) turns positive at some \( t = t^* \) implying that labor starts immigrating, i.e. \( \dot{L} > 0 \), for \( t > t^* \).

Next, consider a region with initial endowment given by E (i.e. northeast of the overlap). It will unambiguously prosper and converge to the superior equilibrium at B. Similarly, a region with initial endowment given by F (i.e. southwest of the area of global indeterminacy) must converge towards the inferior equilibrium at C.

Figure 3: Frequency distribution (unit area histograms) of admissible initial expectations of workers, \( \lambda_L(0) \), corresponding to the three exemplary regions (D, E, F) shown in Figure 2
Figure 3 displays the frequency distribution of admissible initial expectations of workers $\lambda_L(0)$.$^{13}$ The upper graph shows the admissible $\lambda_L(0)$ for region E. Notice that endowment E (together with the underlying TFP parameter) guarantees optimism in the sense of $\lambda_L(0) > 0$. Moreover, there is obviously a multiplicity of equilibrium expectations, illustrating local indeterminacy. The middle graph shows the frequency distribution of admissible $\lambda_L(0)$ for region D. This distribution comprises two separate parts, which corresponds to the global indeterminacy implication. For $\lambda_L(0)$ taken from the left part of the distribution (i.e. comparably unfavorable expectations), the economy declines and converges to the inferior equilibrium at C, while for $\lambda_L(0)$ taken from the right part (i.e. comparably favorable expectations), the economy prospers and moves towards the superior equilibrium at B. Finally, the lower graph shows the admissible initial shadow prices for region F. Notice that $\lambda_L(0) < 0$ throughout, i.e. endowment F (together with the underlying TFP parameter) dooms region F to be pessimistic.

Figure 4 shows that the size of the overlap increases with the TFP parameter $A$. The small overlap applies for $A = 0.05$, whereas the large overlap applies for $A = 0.1$. This makes good economic sense since an increase in TFP (due to $\frac{\partial r_i}{\partial L_i} > 0$, $\frac{\partial^2 r_i}{\partial L_i \partial A} > 0$ and $\frac{\partial w_i}{\partial K_i} > 0$, $\frac{\partial^2 w_i}{\partial K_i \partial A} > 0$) strengthens the interdependence among the agents’ decisions to engage in the domestic market sector across groups. As will be explained below, this mechanism implies that sound institutions, industrial traditions, and a strict supply-side policy give expectations a greater role in the

$^{13}$A similar graph can be shown for the admissible initial expectations of capital owners $\lambda_K(0)$. 

22
process of equilibrium selection.

Figure 4: Overlap in response to an increase in TFP (small overlap applies for $A = 0.05$; large overlap applies for $A = 0.1$)

Is a large overlap good or bad? The answer is that it is neither good nor bad. A large overlap may imply that, even under unfavorable initial conditions, the economy is capable, due to strong optimism, of moving towards the superior steady state. In contrast, even under favorable initial conditions there is the risk that, due to a high degree of pessimism, the inferior steady state is ultimately realized. In this sense, the economy becomes more vulnerable against bad moods. Hence, an adequate, although fairly general, proposition states that the relative importance of
expectations vis-à-vis history increases with the size of the overlap.

Figure 5 shows the distribution of admissible initial expectations $\lambda_L(0)$ across all regions in the factor endowment box for alternative TFP-levels. Due to the multiplicity of admissible expectations a specific region contributes to distinct classes of initial expectations. The graph demonstrates that an increase in TFP, holding initial endowments constant, shifts probability mass to the right. This observation demonstrates that a higher level of TFP enables a higher degree of optimism to be fundamentally warranted. Moreover, one recognizes that expectations are not completely exogenous. Initial expectations can be seen as being drawn from a distribution which is determined by fundamentals, i.e. initial endowments and the level of TFP.

Figure 5: Frequency distribution (unit area histograms) of $\lambda_L(0)$ across all regions for $A = 0.05$ (upper diagram) and for $A = 0.1$ (lower diagram)
In summary, an increase in TFP is ambiguous since it gives expectations a greater role in the process of equilibrium selection, as demonstrated by Figure 4. This may imply that, due to sufficiently unfavorable expectations, a region converges to the inferior equilibrium despite comparably favorable fundamentals. On the other hand, an increase in TFP is a good thing in that it increases, on average, the admissible (i.e. fundamentally warranted) initial expectations, as demonstrated by Figure 5. A more optimistic view becomes possible, implying that the chance for a prosperous economic development increases.

3.3 Deep determinants of TFP

Since the interaction between expectations and TFP is of outstanding importance in the process of equilibrium selection, as explained above, we sketch three prominent approaches which point to the deep determinants of TFP.

Assume that final output is produced according to \( Y = L^{1-\alpha} \sum_{i=1}^{N} x_i^\alpha \), where \( x_i \) denotes the amount of intermediate good \( i \in \{1, ..., N\} \) and \( 0 < \alpha < 1, \ N > 0 \) (Ethier, 1982; Romer, 1990). Employing the symmetry property \( x_i = x \) for all \( i \) and using \( x = \frac{K}{N} \), where \( K \) denotes aggregate capital, one gets \( Y = N^{1-\alpha} L^{1-\alpha} K^\alpha \). Assume that it costs \( F > 0 \) units of final output to set up a new \( x \)-firm and that there is free entry into the \( x \)-sector. The number of \( x \)-firms in equilibrium may then be stated as \( N = N(F) \) with \( N'(F) < 0 \). TFP, given by \( N^{1-\alpha} \), is inversely related to \( F \), which may reflect the consequence of bad institutions, i.e. red tape or corruption. Our model therefore implies that bad institutions may block favorable economic
development because sufficiently optimistic expectations cannot be fundamentally warranted under this side condition.

Suppose next that the \( Y \)-technology is given by \( Y = K^\alpha (BL)^{1-\alpha} \) with labor efficiency (or average human capital) \( B \) depending on cumulated output per capita according to \( \dot{B} = \kappa y - \delta_B B \), where \( y = k^\alpha B^{1-\alpha} \) is output per capita and \( 0 < \kappa < 1 \) (Arrow, 1962; Romer, 1986). The steady state level of \( B \) (determined by \( \dot{B} = 0 \)) then reads \( B^* = \left( \frac{\kappa}{\delta_B} \right)^{\frac{1}{\alpha}} k \). Accordingly, *industrial traditions* in the sense of a high level of cumulated output, i.e. a high level of \( B \), increases TFP and strengthens the relative importance of expectations in the process of equilibrium selection. In addition, it enables a high degree of optimism to be fundamentally warranted.

Finally, assume that \( Y = G^\beta K^\alpha L^{1-\alpha} \), where \( G \) denotes *productive government expenditures* (Barro, 1990), financed according to \( G = q \tau Y \), where \( 0 < q < 1 \) denotes the share of tax receipts devoted to productive government expenditures and \( 0 < \tau < 1 \) the tax rate, respectively. The reduced form technology then is \( Y = (q \tau)^{\frac{\beta}{\alpha-\beta}} K^{\frac{\alpha}{\alpha-\beta}} L^{\frac{1-\alpha}{\alpha-\beta}} \). TFP, as given by \( (q \tau)^{\frac{\beta}{\alpha-\beta}} \), now depends positively on \( q \) and \( \tau \). Competitive factor rewards net of taxes are given by \( r = (1-\tau)\alpha (q \tau)^{\frac{\beta}{1-\beta}} K^{\frac{\alpha-1+\beta}{1-\beta}} L^{\frac{1-\alpha}{1-\beta}} \) and \( w = (1-\tau)(1-\alpha)(q \tau)^{\frac{\beta}{\alpha-\beta}} K^{\frac{\alpha}{\alpha-\beta}} L^{\frac{\beta-\alpha}{\alpha-\beta}} \). An increase in \( q \) enhances TFP with the implications sketched above. An increase in \( \tau \) unfolds, however, a non-monotonic impact on \( w \) and \( r \) and, hence, on the relative importance of expectations.
4 The case of East Germany

The German reunification provides a drastic example of an economy that was subject to a comprehensive integration shock. In 1991 a whole economy with hitherto highly restricted capital and labor mobility, namely East Germany, became integrated into a nearly frictionless world capital and labor market. We ask now whether the model at hand is instructive when trying to better understand macroeconomic development in East Germany in response to a comprehensive integration shock.  

4.1 Three striking empirical observations

Economic development in East Germany since the German reunification in 1991 exhibits three striking empirical characteristics.

Massive factor movements. There has been substantial labor outflow and massive capital inflow. Between 1991 and 2009 about 60,000 people (0.4 percent of the population) emigrated from East Germany per annum (Statistisches Bundesamt, 2005, 2009). There are important regional differences. Some regions shrank substantially and, at the same time, there are regions which attracted people to a substantial extent. At the same time, there has been a substantial inflow of (private) capital.

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15 East-West migration is much more important compared to movements within East Germany. The ratio of East-West migration and movements within East Germany fluctuates between 70
For instance, Burda (2006, p. 368) reports that capital inflows between 1991 to 2004 amounted to 80 to 90 billion EUR, or about 20 percent of GDP, each year.

*Limited East-West convergence.* Real GDP per capita (per employee) in East Germany relative to GDP per capita (per employee) in West Germany stood at 39 percent (41 percent) in 1991. It then increased at an impressive pace up to 63 percent (69 percent) in 1996. Subsequently, the process of East-West convergence slowed down substantially. Relative GDP per capita (per employee) reached 69 percent (77 percent) in 2010. This leveling points to limited convergence of GDP per capita (cf. also Uhlig, 2006, p. 383).

*Regional divergence and emergence of twin peaks.* Vollmer et al. (2011) decompose the (entire) German distribution of annual GDP per employee into two components using a two-component normal mixture model for the years 1992 until 2006. This allows them to identify those counties ("Kreise") that moved from the first to the second component. Subsequently, a standard cross-sectional growth regression is extended to include a mover dummy, which is highly significant. Vollmer et al. (2011, p. 11) conclude that "...there are two distinct convergence clubs for GDP per employee in the East of Germany.". There is also impressive descriptive evidence for regional divergence and the emergence of a twin-peak structure in the regional distribution of real GDP per capita (available from the authors). The kernel density for the regional (at the level of counties) distribution of real GDP per capita shows an unimodal income distribution in 1996 and a bimodal distribution in 2006.

percent and 85 percent (Statistisches Bundesamt, 1993-2011).
Both limited East-West convergence and regional divergence are in stark contrast to the neoclassical model. This is reinforced by noting, first, that real GDP per capita in East Germany relative to West Germany stood at about 120 percent before WWII (Boltho et al., 1997, p. 257) and, second, the degree of factor mobility appears to be quite high. East Germany has unrestricted access to the international capital market and major migration costs associated with cultural and lingual differences do not apply (Hunt, 2006). Moreover, there are substantial productive government expenditures, funded by the central government, which aim at a "harmonization of living conditions" (as prescribed by the German constitution) by uniformly distributed public infrastructure investment.

4.2 Sketch of the underlying procedure

The model is evaluated in the following manner. First, we consider a "large number" of regions $i \in \{1, \ldots, n\}$. Every region starts with a specific combination $[K_i(0), L_i(0)]$ which is restricted to fall inside the upper left of the state plane (i.e. above the $r = \bar{r}$-isocline and to the left of the $w = \bar{w}$-isocline). This assumption implies $r_i > \bar{r}$ and $w_i < \bar{w}$. This choice is part of the calibration strategy since we choose $[K_i(0), L_i(0)]$ such that the model generates (aggregate) capital inflows and (aggregate) labor outflows. Second, we assume that, in every region, there are $0 < L < \bar{L}$ inhabitants who do not condition their migration decision on the wage differential, i.e. they stay in
the source irrespective of $\lambda_{L_i}$. Third, we focus on interior dynamics throughout. This implies that we hold the coordinates $[K_i(t), L_i(t)]$ fix once a region has hit the boundary of the state space. Accounting additionally for dynamics at the border would not change the qualitative implications. Fourth, initial shadow prices $\lambda_{L_i}(0)$ for every $i \in \{1, ..., n\}$ are drawn from a (non-parametric) probability distribution, which is region specific, as shown in Figure 3. The values of $\lambda_{K_i}(0)$ are then determined by the soft-landing condition, i.e. $\lambda_{L_i}(T) = 0$ or $\lambda_{K_i}(T) = 0$, where $T$ denotes the point in time when the economy hits a boundary. Once, $K_i(0), L_i(0), \lambda_{K_i}(0)$ and $\lambda_{L_i}(0)$ are specified, one can trace out $K_i(t), L_i(t)$ for all $t \in [0, T]$ and $i \in \{1, ..., n\}$. Finally, we calculate the time path of average per capita income $y(t) := \frac{\sum Y_i(t)}{\sum L_i(t)}$.

4.3 Calibration

Initial conditions $[K_i(0), L_i(0)]$ are selected, as mentioned above, such that $r_i > \bar{r}$ and $w_i < \bar{w}$. This is in line with the observation of (aggregate) capital inflows and (aggregate) labor outflows. The technology parameters are chosen as follows: $\alpha = 0.3$, $\beta = 0.7$, $a = 0.075$, $b = 0.075$. The implied degree of IRS of 1.15 lies inside the empirically plausible range (Schmitt-Grohé, 1997; Graham and Temple, 2006). The rate of return on capital is set to $\bar{r} = 0.04$ and the wage rate in the outside option is normalized to one, i.e. $\bar{w} = 1$. The time preference rate is set to $\rho = 0.01$.

\footnote{The calibration below assumes a value for $L_0$ and initial conditions $L_i(0)$ such that, on average, about 4 percent of the domestic population are assumed not to move.}
and the TFP parameter reads $A = 1.1$.\(^{17}\) Moreover, we have set $\bar{L} = 1.5L^*$, where $L^*$ is the interior steady state value. $K^{**}$ is then endogenously determined. This implies that a successful region, in the steady state, exhibits a GDP per employee which is 5 percent higher than (average) GDP per employee in West Germany.

The mobility cost parameter $\gamma_L = 0.025$ is determined as follows. Equation (15) says that $\gamma_L = \frac{\hat{L}}{\lambda_L}$. To determine $\hat{L}$ we notice that about 2.4 percent of the East German labor force left East Germany in 1991 (Statistisches Bundesamt, 2005). Hence, the appropriately scaled $\hat{L}$ reads $\hat{L} = -0.024 \sum_i L_i(0)$ Given $\hat{L}$, the initial conditions $[K_i(0), L_i(0)]$, the above mentioned parameters, and an initial guess for $\gamma_L$ and $\gamma_K$, we can calculate a frequency distribution across all regions; in analogy to the frequency distributions shown in Figure 5. This frequency distribution gives us $E[\lambda_{L_i}(0)]$. Now we adjust $\gamma_L$, and repeat the simulation iteratively, until the implied $E[\lambda_{L_i}(0)]$ matches $\gamma_L = \frac{\hat{L}}{E(\lambda_L)}$. Capital mobility costs are set equal to labor mobility costs, i.e. $\gamma_K = \gamma_L$.\(^{18}\)

\(^{17}\)The value $A = 1.1$ does not mean that the overlap comprises nearly the entire state plane, as one could conjecture with regard to the $A$-values employed in Section 3. The reason is twofold. First, the overlap seems to converge to a limiting set (a subset of the state plane) as $A$ increases and, second, other parameters, like $\gamma_L$ and $\gamma_K$, also have an impact.

\(^{18}\)One might object that capital is more mobile than labor, i.e. $\gamma_K > \gamma_L$. However, this is not clear if we think of real capital instead of financial capital. Also, we have experimented with other constellations, i.e. $\gamma_K > \gamma_L$ and $\gamma_K < \gamma_L$. The qualitative results do not change.
4.4 Limited East-West convergence and twin peaks

We now demonstrate that the model can replicate limited East-West convergence as well as the emergence of a twin-peak structure in the regional income distribution. Notice at first that, within our model, \( \frac{Y}{L} \) may be interpreted either as GDP per capita or as GDP per employee, since the model abstracts from unemployment.

When it comes to a comparison between model implications and empirical data, it is clearly appropriate to interpret \( \frac{Y}{L} \) as GDP per employee. With regard to limited East-West convergence of GDP per employee we focus on the following metric. The competitive wage rate in West Germany, which constitutes the outside option for East German workers, is given by \( \bar{w} = \beta \frac{Y^W}{L^W} \). This implies that GDP per employee in West Germany may be written as \( \frac{Y^W}{L^W} = \frac{\bar{w}}{\beta} \). East German GDP per employee, averaged across different regions, reads \( \frac{Y^E}{L^E} = \frac{\sum_i Y^E_i}{\sum_i L^E_i} \). Hence, limited East-West convergence requires that

\[
\lim_{t \to \infty} \frac{Y^E/L^E}{Y^W/L^W} = \lim_{t \to \infty} \frac{\sum_i Y^E_i/\sum_i L^E_i}{\bar{w}/\beta} < 1.
\]

Figure 6 shows the time path \( \frac{Y^E/L^E}{Y^W/L^W} \) resulting from model simulation by applying the procedure and calibration described above (solid line). The empirical series

<table>
<thead>
<tr>
<th>Technological elasticities</th>
<th>( \alpha = 0.3; \beta = 0.7; a = 0.075; b = 0.075 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preference and outside option</td>
<td>( \rho = 0.01; \bar{r} = 0.04; \bar{w} = 1 )</td>
</tr>
<tr>
<td>Expectations and mobility costs (implied)</td>
<td>( E[\lambda_{L_i}(0)] \simeq -10; \gamma_L = \gamma_K = 0.025 )</td>
</tr>
</tbody>
</table>

Table 1: Set of parameters
for \( \frac{Y^E/L^E}{Y^W/L^W} \) is represented by the dashed line. One can recognize that relative GDP per employee starts at \( \frac{Y^E/L^E}{Y^W/L^W} \approx 0.41 \) and converges to \( \frac{Y^E/L^E}{Y^W/L^W} \approx 0.83 \). This illustrates that the model can replicate limited convergence. About 5 percent of regions converge to the superior, while 95 percent of regions converge to the inferior steady state. The hump-shaped pattern of \( \frac{Y^E/L^E}{Y^W/L^W} \) between \( t = 0 \) and \( t = 5 \), displayed in Figure 6 (solid line), results from the fact that those regions which converge to the inferior steady state may temporarily experience an increase in GDP per employee since labor may leave individual regions more rapidly than capital flows out of this same region, implying that \( \frac{Y^E_L}{L^E} \) increases temporarily.

Figure 6: Limited East-West convergence of real GDP per employee (dashed line: empirical series; solid line: theoretical series)
We finally turn to regional divergence and the emergence of twin peaks in the regional income distribution. Figure 7 (left panel) demonstrates that the model, given the initial set of \((K^E_i, L^E_i)\)-coordinates according to the calibration strategy outlined above, does indeed imply an unimodal regional income distribution, which is in line with empirical observations. Figure 7 (right panel) shows that the regional income distribution after 20 years is clearly bimodal. Thus, the model can replicate regional divergence (the maximum deviation has increased) and the emergence of a twin-peak structure in East Germany.

Figure 7: Regional income distribution (kernel densities) at \(t = 0\) (left graph) and at \(t = 20\) (right graph)

In summary, the model under study can replicate limited East-West convergence as well as the emergence of twin peaks in the regional income distribution. The model is also in line with regional heterogeneity in labor movements as well as
migration reversals (cf. also Figure 2), which can indeed be observed for a number of East German counties (Statistisches Bundesamt, 1993-2011). Hence, we believe that the model points to an underlying economic structure and associated mechanisms that appear helpful when trying to better understand East German macroeconomic development since 1991.

5 Summary and conclusion

Does a comprehensive integration shock set the stage for ongoing prosperity or does it doom to gradual economic decline? This question has been discussed by employing a model of a small open economy under increasing returns to scale. Due to multiple steady states as well as local and global indeterminacy, the underlying model creates a rich set of plausible implications. It has been shown that the outcome depends on a number of side conditions, like initial state variables, total factor productivity, and expectations about the future domestic factor rewards.

Our analysis contributes also to a better understanding of the interaction between expectations and fundamentals in models with indeterminacy. It has been shown that (i) the relative importance of expectations as determinant of economic success depends on total factor productivity (TFP) and (ii) expectations depend on fundamentals (initial conditions and TFP) in a systematic fashion. This insight has far reaching economic implications. For instance, bad institutions may block favorable economic development because sufficiently optimistic expectations cannot
be fundamentally warranted under this side condition.

We believe that our analysis is not only important with regard to factor market integration. The main implications should also apply to cases where the factors of production become endogenous. This may be due to institutional or technological changes. For instance, instead of using the metaphor "agents get the opportunity to move across borders" one may think of an economy that starts developing a capital market or an educational system.

Finally, the model has been applied to replicate two striking empirical characteristics of macroeconomic development in East Germany since 1991. It has been demonstrated that it can replicate limited East-West convergence and the emergence of a twin-peak structure in the regional income distribution, which represents a puzzle for the standard neoclassical model. In addition, the model is in line with regional heterogeneity in labor movements as well as migration reversals. We believe that the model points to an underlying economic structure and associated mechanisms that are helpful when trying to better understand macroeconomic development in East Germany since 1991.
6 Appendix

6.1 Equations (8) and (13)

Capital - equ. (8). The (current-value) Hamiltonian function associated with problem (7) and the first-order conditions read

\[
H^D := \bar{r} \left( \bar{K}^D - \sum_{i=1}^{n} \bar{K}^P_i \right) + \sum_{i=1}^{n} r_i \bar{K}^D_i \left|_{t=0} \right. - \sum_{i=1}^{n} \frac{1}{2\gamma_K} (v^D_{K_i})^2 + \sum_{i=1}^{n} \lambda_K v^D_{K_i}
\]

\[
\frac{\partial H^D}{\partial v^D_{K_i}} = -\frac{1}{\gamma_K} v^D_{K_i} + \lambda_K = 0 \quad \Rightarrow \quad v^D_{K_i} = \gamma_K \lambda_K_i \quad \forall \ i \in \{1, ..., n\} \tag{20}
\]

\[
\dot{\lambda}_K_i = \rho \lambda_K_i - \frac{\partial H^D}{\partial \bar{K}^D_i} = \rho \lambda_K_i - (r_i - \bar{r}) \quad \forall \ i \in \{1, ..., n\}. \tag{21}
\]

Physical capital in region \(i\), denoted as \(K_i\), increases due to capital inflows \(\dot{K}^D_i \geq 0\) and decreases due to capital outflows \(\dot{K}^S_i \leq 0\), i.e.

\[
\dot{K}_i = \underbrace{\dot{K}^D_i}_{\geq 0 \ (\text{capital inflows})} + \underbrace{\dot{K}^S_i}_{\leq 0 \ (\text{capital outflows})}. \tag{21}
\]

From the first-order condition \(\frac{\partial H^S}{\partial \bar{K}^S_i} = 0\) associated with problem (4) and the first-order conditions \(\dot{K}^D_i = \gamma_K \lambda_K_i \quad \forall \ i \in \{1, ..., n\}\), implied by (20), it follows that\(^{19}\)

\[
\dot{K}^S_i = \begin{cases} 
\gamma_K \lambda_K_i & \text{for } \lambda_K_i < 0 \\
0 & \text{for } \lambda_K_i \geq 0 
\end{cases} \tag{22}
\]

\[
\dot{K}^D_i = \begin{cases} 
\gamma_K \lambda_K_i & \text{for } \lambda_K_i > 0 \\
0 & \text{for } \lambda_K_i \leq 0 
\end{cases} \tag{23}
\]

We now draw the following case distinction

\(^{19}\) We also assume that there is enough capital outside the economy for a simultaneous development of all regions heading towards the superior equilibrium.
1. Assume $\lambda_{K_i} < 0$. Noting (21), (22) and (23) this yields

$$\dot{K}_i = \dot{K}_i^D + \dot{K}_i^S = \gamma_K \lambda_{K_i} < 0.$$  

2. Assume $\lambda_{K_i} = 0$. Noting (21), (22) and (23) this yields

$$\dot{K} = \dot{K}_i^D + \dot{K}_i^S = 0.$$  

3. Assume $\lambda_{K_i} > 0$. Noting (21), (22) and (23) this yields

$$\dot{K}_i = \dot{K}_i^D + \dot{K}_i^S = \gamma_K \lambda_{K_i} > 0.$$

In summary, the equation of motion for $K_i$ reads

$$\dot{K}_i = \gamma_K \lambda_{K_i} \leq 0 \text{ for } \lambda_{K_i} \leq 0. \quad (24)$$

This is equ. (8) in the main text.

**Workers - equ. (13).** The number of workers in the domestic region, denoted as $L_i$, changes according to

$$\dot{L}_i = \dot{L}_i^{DS} - \dot{L}_i^{SD}. \quad (25)$$

We now draw the following case distinction

1. Assume $\lambda_{L_i}^S < 0$ (i.e. $\lambda_{L_i}^D > 0$). Noting (11), (12), (13), this gives

$$\dot{L}_i = \dot{L}_i^{DS} - \dot{L}_i^{SD} = \gamma_L \lambda_{L_i}^S < 0.$$  

2. Assume $\lambda_{L_i}^S = 0$ (i.e. $\lambda_{L_i}^D = 0$). Noting (11), (12), (13), this gives

$$\dot{L}_i = \dot{L}_i^{DS} - \dot{L}_i^{SD} = 0.$$  

3. Assume $\lambda_{L_i}^S > 0$ (i.e. $\lambda_{L_i}^D < 0$). Noting (11), (12), (13), this gives

$$\dot{L}_i = \dot{L}_i^{DS} - \dot{L}_i^{SD} = \gamma_L \lambda_{L_i}^S > 0.$$
In summary, the equation of motion for $L_i$ may be expressed as follows

$$\dot{L}_i = \gamma_L \lambda^S_{L_i} \leq 0 \quad \text{for} \quad \lambda^S_{L_i} \leq 0.$$  \hspace{1cm} (26)

This is equ. (13) in the main text.

### 6.2 Notes on stability

We proceed in two steps. First, we assume that $\alpha = \beta = a = b = 0.5$. The dynamic system then becomes linear such that eigenvalues can be determined analytically. Second, we set $\alpha = 0.3$ and $\beta = 0.7$, as in the baseline set of parameters in the main text, and determine the eigenvalues for alternative values of $a$ and $b$ numerically.

Assuming $\alpha = \beta = a = b = 0.5$ the Jacobian matrix of system (15) to (16) is

$$J = \begin{pmatrix}
0 & 0 & \gamma_L & 0 \\
0 & 0 & 0 & \gamma_K \\
0 & -0.5A & \rho & 0 \\
-0.5A & 0 & 0 & \rho
\end{pmatrix}.$$  

It can be readily shown that the eigenvalues of the Jacobian matrix are given by

$$r_{1,2} = \frac{1}{2} \left[ \rho \pm \sqrt{\rho^2 - 2A\sqrt{\gamma_L \gamma_K}} \right]$$

$$r_{3,4} = \frac{1}{2} \left[ \rho \pm \sqrt{\rho^2 + 2A\sqrt{\gamma_L \gamma_K}} \right].$$

Several remarks are at order:

1. Eigenvalues $r_{3,4}$ are real. Moreover, $r_3 > 0$ and $r_4 < 0$. This requires $\rho < \sqrt{\rho^2 + 2A\sqrt{\gamma_L \gamma_K}}$, which boils down to $2A\sqrt{\gamma_L \gamma_K} > 0$ being always true.
2. As regards \( r_{1,2} \), we need a case distinction: Provided that \( \rho^2 < 2A \sqrt{\gamma L \gamma K} \) eigenvalues \( r_{1,2} \) are conjugate complex with positive real parts \( \frac{1}{2} \rho > 0 \).

3. If, on the other hand, \( \rho^2 > 2A \sqrt{\gamma L \gamma K} \) eigenvalues \( r_{1,2} \) are real. Eigenvalue \( r_1 > 0 \). Eigenvalue \( r_2 > 0 \) iff \( \rho > \sqrt{\rho^2 - 2A \sqrt{\gamma L \gamma K}} \). This condition boils down to \( 2A \sqrt{\gamma L \gamma K} > 0 \), which is always true.

Figure A1 demonstrates that the eigenvalue configuration, given \( \alpha = 0.3 \) and \( \beta = 0.7 \), is robust with respect to changes in \( a \) and \( b \).

![Figure 8: Eigenvalues (real parts) in response to changes in \( a \) and \( b \). Notice that the horizontal line represents the two (identical) real parts of a pair of conjugate complex eigenvalues](image)

To summarize, there are always three eigenvalues with positive real part and one eigenvalue with negative real part. Since there are two jump variables and three
unstable roots, the interior steady state is unstable. There is a three-dimensional unstable manifold leading away from the interior steady state. Since there are two (predetermined) state variables and two jump variables, there is indeterminacy in the sense of a multiplicity of admissible initial shadow prices \([\lambda_L(0), \lambda_K(0)]\).

6.3 Notes on equilibrium dynamics

Reasoning of Fukao and Benabou (1993). Fukao and Benabou (1993, Proposition 2) have shown that, within the one-factor Krugman (1991) model, equilibrium trajectories must satisfy two conditions: (i) the shadow price of the factor reaching the boundary must approach zero and (ii) once the boundary has been touched, equilibrium implies that the economy remains at the boundary forever. The reasoning relies on an arbitrage condition, which must hold in equilibrium, and applies also to the model under study: Assume that the economy hits, say, the lower \(L\)-boundary at \(t = T\) (i.e. \(L(T) = 0\) with \(K > 0\)) with \(\lambda_L(T) < 0\). In this case, each individual worker has an incentive to deviate from the trajectory under consideration since he can realize the gain, reflected by \(\lambda_L(T) < 0\), an instant in time later and thereby avoid all relocation costs (the individual is of measure zero) by moving one instant in time later. Hence, any equilibrium trajectory must hit the \(L\)-boundary such that \(\lambda_L(T) = 0\).

A similar reasoning applies to the case when the economy is located at the boundary and remains there forever. Assume the economy is located at the lower \(L\)-boundary (i.e. \(L = 0\) and \(K > 0\)). In this case \(w > \bar{w}\) applies. It would indeed be
optimal for workers to return into the domestic market sector. This will, however, never happen. Each individual worker has an incentive to realize the gain, reflected by \( w > \bar{w} \), an instant in time later by moving alone and thereby avoid relocation costs. Hence, the fact that the economy does not return into the interior of the state region is essentially due to a coordination failure in market equilibrium.

The arbitrage argument used here relies on one crucial assumption, namely that the individual agent is of measure zero. This guarantees that the deviation of any individual from a given trajectory does not change competitive factor rewards and hence leaves \( \lambda_K \) and \( \lambda_L \) unchanged. Moreover, this assumption guarantees that relocation costs are zero if one agent moves in isolation.\(^{20}\) Therefore, this reasoning extends to the two-factor case under consideration with atomistic agents implying that equilibrium trajectories must approach the border of the state region tangential, i.e. satisfying either \( \dot{L}(T) = \lambda_L(T) = 0 \) or \( \dot{K}(T) = \lambda_K(T) = 0 \) and, in addition, remains at the border of the state space once the economy hits the boundary.

**Boundary dynamics.** Assume that the economy hits, say, the \( L \)-border at \( t = T \), i.e. \( L(T) = 0 \) or \( L(T) = \bar{L} \), with \( 0 < K(T) < \bar{K} \). The dynamics are then governed by (14) and (16) (noting that \( L(T) = 0 \) or \( L(T) = \bar{L} \)). The shadow price \( \lambda_K \) at \( t = T \) jumps in order to satisfy the transversality condition. Next assume that the economy hits the \( K \)-border at \( t = T \), i.e. \( K(T) = 0 \) or \( K(T) = \bar{K} \), with \( 0 < L(T) < \bar{L} \). The dynamics are then governed by (15) and (17) (noting

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\(^{20}\)Notice that "relocation costs" are essentially congestion costs, i.e. marginal moving costs are zero at the origin.
that \( K(T) = 0 \) or \( K(T) = \bar{K} \). The shadow price \( \lambda_L \) at \( t = T \) jumps in order to satisfy the transversality condition. A non-formal sketch of equilibrium dynamics at the border of the state space is as follows. Assume that the economy touches the \( K \)-axis at \( t = T \), i.e. \( L(T) = 0 \). The rate of return then is \( r(T) = 0 \) and, hence, capital leaves the domestic market sector (in finite time). This movement is sluggish because of convex adjustment costs. An equivalent reasoning applies for \( K(T) = 0 \) and \( w(T) = 0 \). Now assume that the economy touches the upper border of the state region, i.e. \( L(T) = \bar{L} \) for some \( t = T \). Assume further that \( r(\bar{L}, \bar{K}) > \bar{r} \). Capital allocated to the domestic market sector is increased until \( r = \bar{r} \). An equivalent reasoning holds true if the economy hits the right border of the state region \( K = \bar{K} \).

References


