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## **Stock Prices and Monetary Policy. Re-examining the Issue in a New Keynesian Model with Endogenous Investment**

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**Abstract** In this paper, the authors present a New Keynesian quantitative model with endogenous investment and a stock-market sector to shed further light on two unsettled issues: whether central banks should include some financial indicator in their policy rules, and what indicator may be expected to generate better stabilization performance. For comparative purposes, the authors replicate the policy framework and assessment strategy of the well-known “no-inclusion” model of Bernanke–Gertler (1999, 2000) and assess performance of five policy rules. Two of these are “traditional” Taylor rules (i.e., do not incorporate financial indicators) that differ in the relative weight they put on output and inflation gaps. The other three are ‘financial’ Taylor rules. These involve the addition of one financial indicator in each case. Specifically, the deviation from trend of stock prices, of Tobin's  $q$  (the rate of change in stock prices relative to capital stock) and of investment. The authors obtain results that are at variance with Bernanke–Gertler, first, because the best performing rule of the traditional rules is output aggressive instead of inflation aggressive and, second, because the financial rule with Tobin's  $q$  outperforms the traditional inflation-aggressive one under all dimensions and cases. However, the authors cannot draw a univocal conclusion as regards the comparison between the financial rule with Tobin's  $q$  and the traditional but output aggressive rule.

**JEL** E5, E52

**Keywords** New Keynesian models; monetary policy; stock markets and bubbles

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## 1. Introduction

The financial crisis erupted in the US banking sector in 2007 has revived the issue of whether rule-based monetary policy aimed at price stability is also conducive to financial stability. Before briefly reviewing the state of the art, it is useful to dispel some semantic confusion.

Three different meanings of the question "should central banks react to asset price fluctuations?" are intertwined in the debate. The first meaning is "pricking" bubbles, the second is "targeting" asset prices, the third is "including some asset-market variable in the policy rule". Our interest here is limited to the third issue. The other two are less general, and there seems to be more agreement that the answer to the question should be 'no' (e.g. Roubini (2006)).

Assets may be real or financial. Those who are favourable or unfavourable to the inclusion of some asset-market variable in monetary policy rules do not generally draw a distinction among, say, real estate, houses or equities. To some extent these can be treated interchangeably. However, when a specific model is proposed, it is necessary to choose which kind of asset is considered, and this paper deals with financial assets, namely private company stocks.

Bernanke and Gertler (BG henceforth) (1999, 2001) epitomize the consensus view that asset-market variables should not be included in the policy rule (see also Bean (2003), Posen (2006), and Batini and Nelson (2000) in the context of a small open economy). Two key arguments are put forward by supporters of this view. The first is that (flexible) inflation targeting provides the central bank with sufficient tools to stabilize the economy with no direct reference to asset prices. The latter only matter insofar as they impinge upon the expected (consumer price) inflation path, and once account is taken of this, central banks need not care about asset prices directly (e.g. BG (2001, p.253)). The basic idea is simple: if asset prices are *too high* they also feed *too much* demand and inflation, hence an inflation-targeting central bank raises the interest rate thus stabilizing *both* the real economy *and* the asset market<sup>1</sup>. Then BG show that including the rate of change of stock prices in the policy rule does not improve, and may actually worsen, stabilization of output and inflation (measured in terms of standard deviations of the variables).

The second argument concerns central bank uncertainty under two dimensions. One is the hard-to-draw distinction between fundamental and non-fundamental changes in stock prices. The other is the life expectancy of

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<sup>1</sup> Note, in passing, that this conclusion would obviously apply to any variable which is correlated to (helps to predict) inflation and possibly output.

the non-fundamental bubble, which may be stochastic in nature. Acting upon wrong information may force the central bank to interfere with the allocative task of these markets with the undesirable effect of *destabilizing* both asset markets and the real economy (the informational problem was particularly stressed by the Fed's Chairman Greenspan on various occasions). Overall, this no-inclusion view, endorsed by Chairman Greenspan first, and by Chairman Bernanke later, can be dubbed the (pre-crisis) "Fed Consensus"

Cecchetti et al. (2000) open the way to the alternative views. Their key points are two. First, they use an optimized policy rule (derived from the standard quadratic loss function), whereas BG do not. Second, they employ the same BG model to show that controlling for *non-fundamental* movements in stock prices in the policy rule does improve stabilization. The same line of argument is also pursued by Filardo (2004), Disyatat (2005), Dupor (2005), Roubini (2006). Disyatat clarifies the reason why optimized policy rules, typically, contradict the no-inclusion claims. The reason is purely technical. In an optimization programme the central bank should take account of the functions determining the output and inflation gaps as constraints. If asset prices appear among these determinants (and nobody denies this) then they will also appear in the optimal policy rule as independent arguments, even though they are simple co-determinants of inflation and output as assumed by BG.

There is, however, more to it than this. The presumption that asset price fluctuations have no independent informational content for the central bank apart from predicting inflation is contradicted by a number of empirical studies on the macroeconomic consequences of asset-market boom-bust cycles (see Gerdesmeier et al. (2009) for a recent survey). These studies confirm that identifying a bubble or large non-fundamental swings of asset prices in real time is a difficult task. However, statistical results converge on the significant predictive power of some selected asset market indicators with respect to *medium run* (more than one year) adverse developments in asset markets and the real economy. The stress on the medium run is important because inflation targeting is typically theorized and practised on a short-term basis. On this basis, as shown in particular by Borio and Lowe (2002), inflation forecasts may fail to react to asset market imbalances, which typically accrue during, and at the same time create conditions for, prolonged periods of low interest rates, brisk economic activity and stable prices.<sup>2</sup> Therefore, these findings may explain why reacting directly to appropriate asset market indicators may improve the stabilization

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<sup>2</sup> Additional evidence is provided by Christiano et al. (2007), Allen and Gale (1999).

performance of monetary policy for both the real economy and the asset markets.

As regards the signal-uncertainty argument put forward by BG, it should not be overstated. If the central bank is assumed to have information with which it can compute the "natural rates" in the standard Taylor rule, then, by implication of general equilibrium, it is also able to disentangle the non-fundamental from the fundamental components of stock price changes. If, on the other hand, it is recognized that central banks operate under limited information about the determinants of the general-equilibrium path of the economy, this is a more general problem that requires reconsideration of monetary policy rules (e.g. Orphanides and Williams (2002), Tamborini (2009)). Bordo and Jeanne (2002), Filardo (2004), Dupor (2005) and Disyatat (2005) exemplify how uncertainty about the true nature of ongoing changes in stock prices in different contexts and set-ups still leads to optimized reaction functions inclusive of stock-market variables.

It may also be added that in the BG framework, monetary policy is impotent against bubbles because the latter are exogenous: that is, they are unaffected by changes in the interest rate. As a result, changing the interest rate in response to stock prices may harm the fundamental path of the economy while doing nothing against the bubble. This seems to be a radical simplification. If moving interest rates may be harmful to the fundamental path of the economy, it is because stock prices respond to the policy interest rate, and there is no reason why they should cease to respond if they are on a bubble, or why the sole bubble component should not respond. A number of major asset-price busts, not least the latest one in the US housing market, have been triggered by a shift to restrictive monetary policy (e.g. Allen and Gale (1999)).

Our contribution moves from BG and tries to take stock of the key critical elements emerging from the debate outlined above. First, we extend the basic New Keynesian model with sticky goods prices to include endogenous investment so as to focus more precisely on the endogenous link between stock prices and inflation and output gaps. This link is provided by the fact that stock prices affect investment, and investment is both a component of current aggregate demand and of the future capital stock. The latter in its turn determines potential output over time. As indicated by the empirical literature recalled above, during a stock market boom investment and capital accumulation are above trend, aggregate supply moves together with demand exerting downward pressure on prices. As a result, output and inflation gaps are negligible and the central bank is not alerted to raise the interest rate. This important element in the picture is often mentioned (e.g. BG (1999), Dornbusch (1999)), but has not been investigated extensively, an

exception being Dupor (2005). A recurrent argument is that the volatility of cyclical investment is quantitatively negligible (e.g. McCallum and Nelson (1999)). However, the data analyses mentioned above seem to show the opposite, since some have found that deviations of investment from trend are fairly good predictors of boom-bust cycles over the medium run (e.g. Borio and Lowe (2002), Gerdesmeier et al. (2009))<sup>3</sup>.

Second, we wish to portray stock price dynamics in a way that is both manageable and sufficiently rich, namely

- stock prices reflect the discounted value of expected payoff streams; these may be driven by fundamentals, non-fundamental components (bubbles) or both<sup>4</sup>
- the bubbles on expected payoffs follow stochastic processes with finite probability of collapse at each point of time
- stock prices are always arbitrated and sensitive to the policy interest rate (bubbles are endogenous)

Third, as to policy rules, for comparative purposes we replicate the policy framework adopted by BG, namely non-optimized, linear feed-back "Taylor rules" with different quantifications of the parameters. Our grid of policy rules ranges from "traditional" Taylor rules (only defined on output and inflation gaps) to "financial rules" augmented with different financial indicators, one of which is stock prices as in BG and the others are excess investment and Tobin's  $q$  (in practice, the rate of change of stock prices relative to the capital stock), all defined as deviation rates from steady state. The choice of these indicators complies with the claim that policy rules should contain no additional information than observable stock variables or variables that the central bank also consistently uses in the traditional Taylor rule. As regards the comparative assessment of the different rules, we also follow BG and look at the reduction of volatility of selected variables, in particular output and inflation.

The paper is organized in three parts. The first (section 2) presents the model. The second (section 3) presents and discusses the results of simulations of three types of stock price processes: one driven solely by a fundamental shock to productivity in an efficient stock market, one driven by a pure bubble, and one mixed. We show results that are at variance with

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<sup>3</sup> More direct econometric tests of investments' responsiveness to fundamental and non-fundamental movements in Tobin's  $q$  or stock prices yield mixed results: see e.g. Galeotti and Schiantarelli (1994), for positive responsiveness to non-fundamental movements, and Chirinko and Shaller (1996) for non responsiveness.

<sup>4</sup> Note, therefore, that our bubbles are not directly on prices as usually happens in these models. This difference is important in order to obtain bubbles that are endogenous and sustainable (this concept is defined in section 2).

BG. First, because among the traditional rules the best performing one is output aggressive instead of inflation aggressive. Second, because, whilst it is confirmed that the stock market indicator chosen by BG performs poorly, the financial rule with Tobin's  $q$  outperforms the traditional inflation aggressive one under all dimensions and cases, precisely because Tobin's  $q$  compensates for the missing inflation signal caused by the endogenous-investment, supply-side effects of the stock market boom. However, we cannot draw a univocal conclusion as regards the comparison between a traditional but output aggressive rule and the financial one with Tobin's  $q$ . The third part (section 4) provides a summary discussion of the latter issue and conclusions.

## 2. The model

In this section we present our model for subsequent quantitative simulation. It is a standard New Keynesian model with sticky prices and endogenous investment, to which we add an explicit stock-market component such that investment depends on stock prices. In the BG approach, this dependence is *only* due to so-called financial frictions (firms face an external financial premium that is relaxed as a stock-price boom increases the net worth of firms so that they can increase investment). By contrast, we do not insert financial frictions (apart from stock price bubbles), and the impact of stock prices on investment directly takes place via Tobin's  $q$  under capital adjustment costs<sup>5</sup>. This latter transmission mechanism is also present in Dupor (2005), where, however, stock prices are not explicitly modelled, and a "bubble" is treated as a wedge directly driven into the Euler equation of the investing firms. Here we also provide an explicit treatment of the bubble/stock-price connection on the one hand, and of the stock-price/investment connection on the other, and explicit solutions for the stock-price and Tobin's  $q$  processes.

### 2.1. Producers, investment and the stock market

Producers are private stock companies that supply a homogeneous good that can be sold to retail traders that transform it for final consumption or can be transformed directly into additional capital. Production is

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<sup>5</sup> Empirical research on investments has also widely documented that, contrary to optimal investment theory, Tobin's  $q$  on its own is a poor explanatory variable of the observed variability of investment. Other variables, related to financial frictions such as those considered by BG, are deemed important. We are aware of this evidence, but we have preferred to keep the model simple and use the parameter of adjustment costs as a control device of the variability of investment.

characterized by a technology homogeneous of degree one in its arguments, with decreasing marginal returns and constant returns to scale. Let this be represented by the following Cobb-Douglas function

$$(1) \quad Y(I_t, K_t) = A_t K_t^\alpha L_t^{(1-\alpha)}$$

These firms operate in competitive markets for both inputs and outputs. As regards labour, this is freely available at the market real wage rate  $W_t$ . As regards capital, it is assumed that stockholders' dividends are always equal to profits, and that capital depreciates at the constant rate  $\delta$ . Moreover, we follow the general practice in this class of models and posit that, in order to obtain  $I_t$  units of additional capital from output, firms incur adjustment costs. Indeed, as will be seen later, this represents a key element in the model.

Adjustment cost functions, however, raise notorious problems concerning their theoretical as well as empirical foundations. For our purposes here, it is convenient to adopt one of the standard specifications in the New Keynesian literature; that is, the one proposed by Casares and McCallum (2006), with minor modifications:

$$(2) \quad \Gamma(I_t, K_t) = I_t \left( \gamma_2 \left( \frac{I_t}{\delta K_t} \right)^{\gamma_1} - 1 \right)$$

This function expresses adjustment costs in terms of *the rate of* investment with respect to capital depreciation, rather than the *absolute level* of investment.<sup>6</sup> The presence of the two parameters  $\gamma_1$  and  $\gamma_2$  provides useful flexibility for quantitative analysis. In particular, they make it possible to control for two different dimensions of the costs: their *scale* (dependent on  $\gamma_2$ ) and their *gradient* (dependent on  $\gamma_1$ ). Imposing  $\gamma_2 \geq 1$  ensures non-negative costs of capital maintenance ( $I_t = \delta K_t$ ), with  $\gamma_2 - 1$  measuring the unit cost of maintenance. Further, for  $\gamma_1 > 0$ , the function possesses the standard properties that  $\Gamma(0) = 0$ ,  $\Gamma_I > 0$  for  $I_t/\delta K_t \geq 1$ ,  $\Gamma_{II} > 0$ . That is, costs are increasing and convex as investment exceeds depreciation. For  $\gamma_1 = 1$ , the function is quadratic, a special case common in the investment literature.

Given investment, the capital stock evolves according to the following accumulation function<sup>7</sup>

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<sup>6</sup> Casares and MacCallum use the level of the capital stock instead of its depreciation.

<sup>7</sup> The accumulation function may take different forms, with (slight) differences in result. Here we have followed the "time-to-build" hypothesis, so that investment in  $t$  becomes operative capital in  $t+1$  (e.g. Casares and McCallum (2006)). Other models have that investment in  $t$  becomes operative in the same period (e.g. Chrinko and Schaller (1996)).



$$(3) \quad K_{t+1} = I_t + (1 - \delta)K_t$$

In each period  $t$ , firms demand labour  $L_t$  and additional capital  $I_t$  to maximize their expected real value, which is the discounted sum of present and future real profits  $Z_t$ . Since these firms are price-takers, we simply assume that they sell output to retailers at a price equal to the observed general price level (GPL)  $P_t$  (to be determined below) and hence deflate their nominal profits accordingly. Therefore, producers

$$(4) \quad \max : E_t \left[ Z_t + \sum_{j=1}^{\infty} Z_{t+j} R(j)^{-1} \right] \quad R(j) \equiv E_t \prod_{j=1}^{\infty} (1 + r_{t+j})$$

$$\text{s.t.} \quad (1 - \delta)K_t + I_t - K_{t+1} = 0$$

$$\text{given } Z_t = Y(L_t, K_t) - W_t L_t - (I_t + \Gamma(I_t, K_t))$$

where  $R(j)^{-1}$  is the real discount factor appropriate to stockholders (which will be introduced later). Hence the results for  $L_t$  and  $I_t$  satisfy

$$(5) \quad \left( \frac{K_t}{L_t} \right)^{\alpha} = W_t$$

$$(6) \quad I_t = \bar{Q}_t^{1/\gamma_1} \delta K_t$$

where  $\bar{Q}_t \equiv Q_t / \gamma_3$ ,  $\gamma_3 \equiv (1 + \gamma_1)\gamma_2$ , and  $Q_t$  is the shadow value of a marginal increment in capital (proof: see Appendix).

As a result,  $I_t$  is an increasing function of  $Q_t$ , with  $\gamma_3 > 1$  representing a "hurdle cost". In fact, firms wish to add net capital in addition to depreciation (positive net investment) only when  $Q_t > \gamma_3^8$ ; they invest less than capital depreciation (negative net investment) when  $Q_t < \gamma_3$ ; and when  $Q_t = \gamma_3$ , they only replace the depreciated capital (zero net investment). In fact, the capital stock, upon substituting (6) into (3), evolves according to

$$(7) \quad K_{t+1} = (1 + (\bar{Q}_t^{1/\gamma_1} - 1)\delta)K_t$$

As shown by Hayashi (1982), our assumed production function and the fact that firms are price takers imply that the "marginal"  $Q_t$  is also equal to the "average" Tobin's  $q$  (1969), the latter being defined as the future market value of the firm per unit of existing capital, i.e.

$$(8) \quad Q_t = V_t / K_t$$

where

$$V_t = E_t \left[ \sum_{j=1}^{\infty} Z_{t+j} R(j)^{-1} \right]$$

(see also Chirinko and Schaller (1996)).

It will be convenient to make use of this latter measure of Tobin's  $q$ . It implies that net investment is undertaken as long as  $V_t > \gamma_3 K_t$ . On this

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<sup>8</sup> The well-known basic condition for net investment without adjustment costs is  $Q_t > 1$ .

basis, we can also obtain the "fundamental value" of firms, that is, the value  $V_t^*$  associated with the optimized plan of firms

$$(9) \quad V_t^* = Q_t^* K_t$$

To develop this relationship, it is sufficient to recall that  $Q_t$  and the optimal path of capital are mutually related (namely in the first-order-condition with respect to the capital stock; see Chirinko and Schaller (1996)) so that solving this condition for  $Q_t$ , and substituting into (9) yields  $V_t^*$  (see Appendix)

Now we have the connection between the investment decision and the stock market. Let  $D_{t+j}$  be the dividend payoffs expected from firms for each period,  $j = 1 \dots \infty$ ; hence, the one-period market valuation of a stock held in  $t$  can be written as

$$(10) \quad V_t^m = E_t \left[ \frac{D_{t+1} + V_{t+1}^m}{1 + r_{t+1}} \right]$$

Forward iteration of the formula yields

$$(11) \quad V_t^m = E_t \left[ \sum_{j=1}^{\infty} D_{t+j} R(j)^{-1} \right] + E_t [V_{t+j}^m R(j)^{-1}]$$

Coincidence between expected dividends and profits, and the terminal condition that

$$\lim_{j \rightarrow \infty} E_t [V_{t+j}^m R(j)^{-1}] = 0$$

ensure that the stock market value at each point in time is equal to the fundamental value of firms given by (9), that is,

$$(12) \quad V_t^* = E_t \left[ \sum_{j=1}^{\infty} Z_{t+j}^* R(j)^{-1} \right]$$

where (\*) denotes optimized variables.

## 2.2. Fundamentals and bubbles

In order to provide the stock market with some life of its own, we allow for the insurgence of bubbles. Compare equations (11) and (12): bubbles are persistent deviations of stock market prices  $V_t^m$  from their fundamental value  $V_t^*$  due to the price expectational component represented by the last term in equation (11). Hence, in any period of time, it may happen that

$$V_t^m = V_t^* + B_t$$

As to the non-fundamental or bubble component  $B_t$ , for the sake of comparison we draw on BG treatment, with some modifications to make it consistent with our analytical framework.

In the first place, we assume that the bubble is due to a share  $\omega$  of "non-fundamentalists", that is, market participants who seek to price stocks

directly by extrapolating the total return of the stock, i.e. the numerator of (10), taken in the generic format

$$B_t = \Phi_{t,t+1} R(1)^{-1}$$

Consequently,

$$(13) \quad V_t^m = (1 - \omega)V_t^* + \omega\Phi_{t,t+1}R(1)^{-1}$$

The combination of equations (6), (8) and (13) yields the stock-market driven level of investment in each period.

It is worth stressing that in our model a possible bubble on  $V_t^m$  is in fact driven by an underlying bubble on  $\Phi_{t,t+1}$ . As far as the bubble process on  $\Phi_{t,t+1}$  is concerned, we consider a *stochastic* bubble like BG (1999): if the bubble exists at any time  $t$ , it has a probability  $p$  of existing, and  $(1 - p)$  of bursting, in  $t+1$ . The specification of this process is postponed to the analytical treatment of the model in section 3.

As will be appreciated later on, our formulation has various analytical advantages. First, it captures the widely reported fact that bubbles are ignited by extrapolations, often mouth-to-mouth, of future high returns to stocks (Akerlof and Shiller (2009)). Second, though we borrow from BG a non-rational bubble process, our treatment makes it possible to control for the bubble being profitable and hence *sustainable*.<sup>9</sup> Our basic and intuitive definition of sustainability is that, as long as  $\Phi_{t,t+1}$  is on the bubble, its discounted value, which determines  $B_t$ , should not fall. Indeed, the stock price equation (13) entails that the bubble on  $\Phi_{t,t+1}$  is continuously arbitrated away, so that investing one euro for one period yields no more and no less than the market return rate.<sup>10</sup> Third, and consequently, it is also intuitive at this stage that, unlike BG's, our bubble is *endogenous*: that is, its evolution is affected by that of the interest rate. This seems to us an appropriate feature since in most cases bubbles are sensitive to swings in interest rates, and conclusions about the role of monetary policy in the event of bubbles obviously depend on whether or not they are endogenous (Allen

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<sup>9</sup> The next expected value of the bubble at each point in time is systematically exceeded by its realized value as long as the bubble does not burst (see equation (28)). The well-known shortcoming of rational bubbles is that they are likely to grow forever at the same rate as the market interest rate. This is generally regarded as an unsatisfactory feature, and it is often relaxed as BG have done. The other side of the coin is that as long as a bubble grows, it must also be profitable, and this condition is provided by our formulation.

<sup>10</sup> Our formulation can also be viewed as consistent with "weak efficiency" of the stock market (Fama (1970)). As is well-known, weak efficiency means that prices exhaust profitable trade by incorporating all available *public information*. This is represented by  $\Phi_{t,t+1}$ , and the point is that it does not contain (only) fundamental information.

and Gale (1999)). Finally, (13) is a flexible specification that can easily accommodate three important processes that can be treated comparatively:

- a *pure fundamental process*, where a shock to  $V_t^*$  occurs with no bubble,  $\omega = 0$
- a *pure bubble process*, with no shock to  $V_t^*$  and an exogenous start-up of the bubble on  $\Phi_{t,t+1}$
- a *mixed process*, where both  $V_t^*$  and  $\Phi_{t,t+1}$  are shocked simultaneously by the same amount.

The mixed process is the most interesting one for several reasons. The emerging story is one where the non-fundamentalists in the stock market react to news about increased productivity by anticipating higher stock returns (recall that these consist of next dividends plus stock price). Since these initially increase for fundamental reasons, the bubble grows as a self-sustained bet on growing returns. All in all, our "non-fundamentalists" are only mildly so, because they look at the discounted value of their expected payoff and, at least initially, they extrapolate on the basis of truly fundamental information.<sup>11</sup> According to BG (2001), mixed processes are also important because they pose the policy dilemma that raising the interest rate may unduly damage the real economy (as will be seen,  $V_t^*$  in fact depends on the interest rate as well).

### 2.3. Retail traders

Retailers buy the homogenous product at the current GPL  $P_t$ , and transform it into  $n$  differentiated products  $C_{nt}$  destined to consumption that can be sold at the individual price  $P_{nt}$ . Each retailer is specialized in one single brand, and there is a continuum of brands (retailers) of mass 1.

Differentiation is a self-employment activity that takes place by means of a common linear technology that delivers 1 unit of differentiated product per unit of homogeneous product,  $Y_{nt} = C_{nt}$ , and requires  $b$  units of labour to process one unit of homogeneous product, i.e.  $L_{nt} = bC_{nt}$ . Retailers are risk-neutral and value their own work effort at the market real wage  $W_t$ . In each period they wish to maximize their expected real surplus, which is the discounted sum of each period's real surplus given by the difference between the real value of sales and of resources employed

$$(14) \quad \begin{aligned} Z_{nt} &= C_{nt}(P_{nt}/P_t) - (W_t L_{nt} + Y_{nt}) \\ &= C_{nt}(P_{nt}/P_t - 1 - bW_t) \end{aligned}$$

In setting the optimal price, each retailer can exploit a standard Dixit-Stiglitz brand demand function of consumers:

$$(15) \quad C_{nt} = C_t(P_{nt}/P_t)^{-\theta} \quad \theta > 1$$

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<sup>11</sup> The same idea was first developed by Froot and Obstfeld (1991).

where

$$C_t = \left[ \int_0^1 C_{nt}^{(\theta-1)/\theta} dn \right]^{\theta/(\theta-1)}$$

and

$$P_t = \left[ \int_0^1 P_{nt}^{1-\theta} dn \right]^{1/(1-\theta)}$$

Upon substituting (15) into (14), each brand optimal price results

$$(16) \quad P_{nt}^* = \mu MC_t$$

that is, a mark-up  $\mu = \theta(\theta-1)^{-1} > 1$  above the nominal marginal costs  $MC_t = P_t(1 + bW_t)$ .<sup>12</sup> Note that this price rule is the same for all retailers (symmetric pricing), so that the GPL is also equal to (16), and we shall drop the subscript  $n$ .

We now introduce price stickiness by means of a standard Calvo-pricing scheme, where for each retailer in each period there exists a probability  $v$  that the price is kept unchanged and a probability  $(1 - v)$  that it can be reoptimized (if necessary). The standard result is that all retailers that can reoptimize in period  $t$  will set the (symmetric) price

$$(17) \quad P'_t = (1 - v\beta)P_t^* + v\beta E_t P'_{t+1}$$

where  $\beta < 1$  is a time discount factor. At the same time, all retailers that cannot reoptimize will set  $P''_t = P_{t-1}$ . Consequently, the GPL is

$$(18) \quad P_t = (1 - v)P'_t + vP_{t-1}$$

This implies that  $E_t P_{t+1} = (1 - v)E_t P'_{t+1} + vP_t$ , and therefore, upon collecting  $E_t P'_{t+1}$  and substituting into (17), expression (18) becomes

$$(19) \quad P_t = \frac{1}{1 + \beta v^2} [v\beta E_t P_{t+1} + (1 - v)(1 - v\beta)P_t^* + vP_{t-1}]$$

This is the basis for the typical New Keynesian forward-looking price equation. The parameter  $v$  is also a measure of the degree of stickiness of prices, and is easily seen that for  $v = 0$  (full price flexibility),  $P_t = P_t^*$  would obtain.

## 2.4. Households

For the purposes of this model, we focus on the role of households as corporate stockholders, which is not present in the standard NK model, while we leave other features in the background. We characterize households as being endowed with a fixed amount of labour force (normalized to 1,  $\bar{L}=1$ ), which is supplied inelastically, and a number of corporate stocks  $S_t$ . Hence in each  $t$  households earn real labour income  $W_t$

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<sup>12</sup> Due to linear technology, marginal costs coincide with constant unit costs.

and returns to stocks. As explained before, stocks carried from the previous period  $S_{t-1}$  entitle households to earn dividends  $D_t$  and net proceeds from sales and purchases at the market value  $V^m_t$ . Rearranging according to expression (10), these stock market operations amount to  $(S_{t-1}V^m_{t-1}(1 + r_t) - S_tV^m_t)$ , in real terms.

Households can also hold a liquid asset issued by the central bank  $M_t$ . Liquid means that it is freely tradable and has a fixed exchange price equal to 1 in nominal terms. This asset, if held for one period  $t$ , pays a nominal interest rate  $\imath_t$  at the beginning of the next period. With two assets in place, we have a financial setup that corresponds to the one discussed by Woodford (2003, sec. 3.2). He explains how, in a "cashless economy", the central bank can nonetheless gain control over the nominal interest rate by being the monopolistic supplier of the "special asset" described above. The mechanism is one of instantaneous arbitrage whereby the rates of return (prices) of all other assets are set consistently with the nominal rate set by the central bank, even though the "special asset" does not even come into circulation (see pp. 35-37). An advantage of this approach is that one need not impose transaction balances on agents (which is typically done by circuitous analytical means). A disadvantage is that actual economies are not cashless economies, and monetary policy models without actual money in circulation remain rather abstract exercises. For our purposes, however, we think that the advantage overcomes the disadvantage, and hence we proceed with Woodford's cashless economy.<sup>13</sup>

Now define,  $\underline{M}_{t-1} \equiv M_{t-1}/P_{t-1}$ , the real stock of the liquid asset at the beginning of period  $t$ ;  $\pi_t \equiv P_t/P_{t-1} - 1$ , the inflation rate of the GPL in period  $t$ ;  $(1 + r_t) = (1 + \imath_{t-1})/(1 + \pi_t)$ , the real interest rate accruing in period  $t$ . Consequently, in each  $t$ , households choose  $C_t$ ,  $S_t$  and  $M_t$  to maximize

$$\mathbb{E}_t \left[ C_t + \sum_{j=1}^{\infty} \beta^j U(C_{t+j}) \right]$$

subject to the real budget constraint

$$W_t + S_{t-1}V^m_{t-1}R(0) + \underline{M}_{t-1}R(0) = C_t + \underline{M}_t + S_tV^m_t$$

To obtain a tractable solution, we now posit the standard CES utility function that is generally employed in this class of models, namely

$$(20) \quad U(C_t) = \frac{1}{1-\sigma} C_t^{1-\sigma} \quad \sigma > 0$$

The solution for  $C_t$  is

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<sup>13</sup> The model could easily be extended to include transaction balances by means of usual tools such as "money in the utility function" or a "cash in advance constraint". As said above, this extension would unduly complicate the model without adding important insights, since tracking transaction balances is unessential in the problem of interest here.

$$(21) \quad C_t = \beta^{-1/\sigma} E_t C_{t+1} R(1)^{-1/\sigma}$$

with  $R(1) \equiv E_t[(1 + \imath_t)/(1 + \pi_{t+1})]$ .

The result is therefore the Euler equation of the standard New Keynesian model. The absence of the stock market value from the consumption equation may appear at odds with the popular idea that wealth-effects on consumption provide a major transmission mechanism from the stock market to the real economy. BG (1999) include this effect in their model, but they find it relatively small. Our result is not surprising since it is consistent with the stock-pricing model in paragraph 2.2. In fact, owing to arbitrage, no matter whether a bubble is present or not, holding stocks from one period to the next should yield the one-period real rate of return  $r_t$  for whatever *level* of stock prices.<sup>14</sup> To obtain stock-market wealth effects a non-arbitraged bubble is necessary. Here we do not wish to engage in speculations about these processes that may always be questionable. As a consequence, our model only captures investment as the key variable in the transmission mechanism of stock market fluctuations. This limitation may be significant in regard to the quantitative dimension of the phenomena of interest, but, on the other hand, the investment channel has been much less explored than the consumption channel.

## 2.5. Central bank

In our economy the central bank is the institution that supplies the liquid asset and determines its nominal interest rate. As in BG, we do not examine the case where the central bank seeks to optimize some welfare function. We simply posit that the central bank adopts a linear reaction function of the Taylor type, and we shall be concerned with assessing to what extent a standard Taylor rule enables the central bank to stabilize the economy, or whether modifications that take stock-market variables into account may improve its stabilization policy.

As to the latter, we shall consider two main stock-market variables that can be found in the relevant literature. The first is simply the current stock-price index  $V_t^m$ . The second is Tobin's  $q$ ,  $Q_t = V_t^m/K_t$ . Note that, therefore, we do not require the central bank to disentangle fundamental from non-fundamental movements in  $V_t^m$  or  $Q_t$ . The capital stock  $K_t$  is a statistic that the central bank should know in order to compute potential output as well. To these we add a third indirect proxy drawn from the empirical literature on leading indicators of boom-bust cycles, namely above-trend growth of investment (e.g. Gerdemeister et al. (2009)).

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<sup>14</sup> On the other hand, this outcome implies that the rate of change of consumption is uncorrelated with the rate of return to stocks; hence, according to the CCAPM, stocks pay no risk premium in spite of households being risk averse.

### 3. Log-linearization and simulations

We now treat the model in order to obtain its version in log-linearized deviations from steady-state general equilibrium, and then move to simulations. Since the procedure is standardized we skip the details and move to the final equations (for the investment and stock price equations, see Appendix).

#### 3.1 The baseline model

The baseline model consists of the structural equations of aggregate demand, aggregate supply, stock market price index and the monetary policy rule. The latter is specified as a Taylor rule where the target level of output is the "technical" general-equilibrium level of output  $Y_t^*$ , given technology and production factors (that is, the central bank does not take account of the efficiency loss due to monopolistic competition). Consequently, associated with zero output gap is zero inflation target.

Using small-case letters to denote log-deviation rates from corresponding s-s variables ( $x_t \equiv \ln(X_t/X^{ss})$ , except for  $\imath$  and  $\pi$ ), we obtain:

##### A) Aggregate demand

$$y_t = y_c c_t + y_i i_t$$

where  $C^{ss}/Y^{ss} \equiv y_c$ ,  $I^{ss}/Y^{ss} \equiv y_i$

$$c_t = E_t c_{t+1} - \sigma^{-1} \hat{r}_{t+1}$$

where  $\hat{r}_{t+1} \equiv (\imath_t - E_t \pi_{t+1} - r^{ss})$

$$i_t = \gamma_1^{-1} q_t + k_t$$

$$q_t = v^m_t - k_t$$

##### B) Aggregate supply

$$y_t^* = a_t + \alpha k_t$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$$

where  $\kappa \equiv (1 - v)(1 - v\beta)v^{-1}$

$$k_{t+1} = \delta i_t + (1 - \delta)k_t$$

$$x_t = y_t - y_t^*$$

##### C) Stock price index

$$v^m_t = (1 - \omega)v^*_t + \omega(\varphi_{t,t+1} - \hat{r}_{t+1})$$

$$v^*_t = [E_t v^*_{t+1} + \psi E_t a_{t+1} - (1 + \psi(1 - \alpha)k_{t+1})] R^{ss-1} - \hat{r}_{t+1} + k_t$$

where  $\psi \equiv r^{ss} + \delta(1 + \gamma_1)^{-1}$

##### D) Monetary policy

$$\imath_t = r^{ss} + \phi_\pi E_t \pi_{t+1} + \phi_x x_t + z_t$$

where  $z_t = \{0, \phi_v v^m_t, \phi_q q_t, \phi_i i_t\}$



By means of simple substitutions we may work with the following semi-reduced form

$$(22) \quad x_t = x_1 E_t c_{t+1} - x_2 \hat{r}_{t+1} - x_3 k_t + x_4 [(1 - \omega) v_t^* + \omega \phi_{t,t+1}] - a_t$$

where  $x_1 \equiv y_c$ ,  $x_2 \equiv y_c \sigma^{-1} + \omega y_i \gamma_1^{-1}$ ,  $x_3 \equiv \alpha - y_i (1 - \gamma_1^{-1})$ ,  $x_4 \equiv y_i \gamma_1^{-1}$

$$(23) \quad \pi_t = \beta E_t \pi_{t+1} + \kappa x_t$$

$$(24) \quad v_t^* = v_1 E_t v_{t+1}^* + v_2 E_t a_{t+1} - v_3 k_{t+1} - \hat{r}_{t+1} + k_t$$

where  $v_1 \equiv R^{ss-1}$ ,  $v_2 \equiv \psi R^{ss-1}$ ,  $v_3 \equiv (1 + \psi(1 - \alpha)) R^{ss-1}$

$$(25) \quad k_{t+1} = k_1 [(1 - \omega) v_t^* + \omega \phi_{t,t+1}] - k_2 \hat{r}_{t+1} + k_3 k_t$$

where  $k_1 \equiv \delta \gamma_1^{-1}$ ,  $k_2 \equiv \omega \delta \gamma_1^{-1}$ ,  $k_3 = 1 - \delta \gamma_1^{-1}$

$$(26) \quad \imath_t = r^{ss} + \phi_\pi E_t \pi_{t+1} + \phi_x x_t + z_t$$

The first equation is the output gap, the second is the Phillips curve, the third is the fundamental stock-market value, the fourth is the capital stock and the fifth is the Taylor rule (TR), where the index variable  $z_t$  can take one of the four values indicated depending on whether the central bank ignores stock-market indicators ( $z_t = 0$ ) or whether it considers one of them.

To prepare the ground for simulation analysis, let us single out the channels through which stock prices influence the economy. There are *direct* and *indirect* channels. *Direct* channels include

- the *output gap* (via investment and aggregate demand, through coefficient  $x_4 > 0$ )
- *capital accumulation* (via investment, coefficient  $k_1 > 0$ ).

*Indirect* channels include

- the *output gap* (via capital accumulation and potential output); unless capital adjustment costs are particularly heavy (e.g.  $\gamma_1 \gg 2$ ) then  $x_3 > 0$  which means that a stock-price spike spurs higher investment, capital accumulation and future potential output which compresses the output gap; notably, this term is absent from the main New-Keynesian models with bubbles, which only contemplate a positive demand effect like  $x_4$ <sup>15</sup>
- the *inflation rate* (via the output gap), if the previous effect operates, then a stock-price bubble also dampens inflation.

In parallel, let us look at the monetary policy channels. These are identified by the variable  $\hat{r}_{t+1}$ , which measures deviations of the market real interest rate from the s-s "natural rate", engineered by the central bank raising the nominal rate above expected inflation. *Direct* channels include

- the *output gap* (via consumption, investment, and hence aggregate demand, coefficient  $x_2 > 0$ ); note that investment is affected indirectly by

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<sup>15</sup> See the reduced-form models employed by Bean (2003), Filardo (2004), Disyatat (2006).

way of the negative effect of the interest rate on stock prices, and hence on Tobin's  $q$

- *stock prices* (via both the fundamental and non-fundamental component)
- *capital accumulation* (via investment)

*Indirect* channels include

- the *inflation rate* (via output gap).

### 3.2. Exogenous shocks to total factor productivity and stock prices

In order to obtain well focused and controllable results, we concentrate on only two drivers of our economy, namely shocks to total factor productivity  $a_t$ , and to non-fundamental expected stock payoffs  $\phi_{t,t+1}$ .

The former are assumed to follow a first-order autoregressive process

$$(27) \quad a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \quad \rho_a < 1, \mathbb{E}_t \varepsilon_{a,t} = 0, \text{cov}(\varepsilon_{a,t}, \varepsilon_{a,t-1}) = 0$$

The latter are assumed to follow the bubble process posited by BG (1999), such that

$$(28) \quad \phi_{t,t+1} = \begin{cases} \frac{\mu}{p} \phi_{t-1,t}; & p \\ 0; & (1-p) \end{cases}$$

where  $\mu > p$  is the momentum of the bubble, and  $p$  is the probability of not bursting. Recall that here  $\phi_{t,t+1}$  indicates the rate of deviation of expected stock payoffs from fundamental valuation. As a result, this rate of deviation grows in each period at the constant rate  $\mu p^{-1} - 1 > 0$ <sup>16</sup>.

Note that substituting (28) into the equation for  $v_t^m$  conditional on the bubble not being burst yields

$$v_t^m = (1 - \omega)v_t^* + \omega(\mu p^{-1} \phi_{t-1,t} - \hat{r}_{t+1})$$

Consequently, for the bubble on payoffs to be reflected on stock prices two conditions should occur

- $\omega > 0$ , i.e. there must be non-fundamentalists in the market
- $\mu p^{-1} \phi_{t-1,t} > \hat{r}_{t+1}$ ; since the first term is the rate of overvaluation of stock payoffs, and the second is the change in the interest rate relative to the s-s rate, this condition means that as long as the bubble is growing its discounted value cannot fall

- we rule out negative bubbles, and impose the further condition that

$$\phi_{t,t+1} = 0 \text{ if } \mu p^{-1} \phi_{t-1,t} < \hat{r}_{t+1}$$

- we also rule out that burst bubbles can restart again.

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<sup>16</sup> If, say,  $\phi_{t-1,t}$  indicates an overvaluation of 10%,  $\mu = 0.6$  and  $p = 0.5$ , then  $\phi_{t,t+1} = 12\%$ , so that overvaluation has increased by 20%.

### 3.2. Simulations

We now present the results of some selected simulations of the model. Since our aim was to compare the performances of alternative monetary policy rules in a framework as "representative" as possible, for the model's basic parameters (see table 1) we have adopted a grid in the range of calibration results provided by some well-known quarterly models of the US economy.<sup>17</sup>

TABLE 1 BASIC PARAMETERS					
$y_c$	consumption share of GDP	0.75	$v$	price stickiness	0.855
$y_i$	investment share of GDP	0.25	$\delta$	capital depreciation rate	0.025
$\alpha$	capital share of GDP	0.35	$\rho_a$	autocorrelation tech. shocks	0.95
$\beta$	time discount rate	0.995	$\sigma_a$	st. dev. tech. shocks	0.01
$\sigma$	consumption utility	5			
STOCK MARKET AND INVESTMENT PARAMETERS					
$\omega$	share of non-fundamentalists	0 - 1	$\gamma_1$	invest. adj. costs (exponent)	0.1
$\mu$	momentum of bubbles	0.99	$\gamma_2$	invest. adj. costs (intercept)	1.05
$p$	prob. of collapse of bubbles	0.5			

Among the basic parameters,  $\beta$  implies the steady-state real interest rate  $r^{ss} = 2\%$  on a year basis; the other utility parameter  $\sigma$  yields an intertemporal elasticity of consumption  $\sigma^{-1} = 0.2$ ; with respect to price stickiness, the 85.5% probability that a retailer keeps price unchanged implies  $\kappa = 0.015$  in the Phillips curve<sup>18</sup>.

As to the stock market parameters, those regulating the evolution of bubbles have been drawn from BG (1999); hence bubbles double in each period and last for 5 periods. The share of non-fundamentalists which affect stock market valuations may range from 0 (stock prices equal fundamental values) to 1. The measure of the adjustment costs of capital is instead one of

<sup>17</sup> See Casares and McCallum for models with endogenous investment, and the survey in Woodford (2003) ch. 5, sec. 2.

<sup>18</sup> This may appear a small value, but it is in line with Woodford (2003). We have also tried with a higher value (0.025), but the results do not change substantially.

the most controversial issues in the investment literature. Having two parameters, the model allows for control of both the scale dimension of the costs ( $\gamma_2$ ) and their gradient ( $\gamma_1$ ), recalling that high (low)  $\gamma_1$  entails low (high) responsiveness of investment to stock prices via Tobin's  $q$ . Following Casares and McCallum (2006),  $\gamma_2$  has been set so that the unit adjustment cost in s-s is 5% of total investment, or about 1% of GDP. As to  $\gamma_1$ , the value given in table 1 has been the result of different trials that are discussed in the Appendix A5.

To account for, and assess, different monetary policies, we considered different parametrizations of the TR (26) according to the following table.

TABLE 2 TAYLOR-RULE PARAMETERS					
	$y_t$	$E_t\pi_{t+1}$	$v^m_t$	$q_t$	$i_t$
"inflation aggressive"	1	2	0	0	0
"output aggressive"	2	1	0	0	0
"stock prices"	1	1	1	0	0
"Tobin's $q$ "	1	1	0	1	0
"investment"	1	1	0	0	1

The first two lines are variants of the baseline TR with no stock-market indicators (let us call them "traditional TRs"). The second three lines include the three different stock-market indicators available in the model (let us call them "financial TRs"). In view of comparison across rules, we have adopted a simple on-off scheme of the various indicators (see also BG (2001))<sup>19</sup>. In the financial rules we have set unit output and inflation parameters in order to assess to what extent the inclusion of the relevant financial variable substitutes for "aggressivity" in the traditional rules. To ease reading and comparison, each parametrization has an identification label<sup>20</sup>. As explained in section 2, in this setup we have studied three different processes of stock prices.

<sup>19</sup> The inflation parameter  $\phi_\pi = 1$  lies just on the boundary of monetary policy effectiveness according to the so-called Taylor Principle.

<sup>20</sup> A larger array of parameter values and combinations has been tried. However, the computational cost increases sharply due to the large dimensionality of the grid, while the comparability of results falls. It should be considered that, as is typical with the DSGE methodology, dynamic paths are generally monotonically stable. Hence, one may expect (and we have indeed found) that increasing or decreasing the value of the parameters has small quantitative, monotonic effects on the paths that do not modify the qualitative ordering of the results that is presented in the paper.

### A) A pure fundamental process when the stock market is efficient

( $\varepsilon_a = 1\%$ ,  $\omega = 0$ )

The economy is hit by a positive 1% technology shock. The stock market is efficient in the sense that it always reflects fundamental values ( $\omega = 0$ ). Room for policy action is still warranted as temporary misalignments between aggregate demand and supply following shocks may give rise to undesired non-zero gaps in output and inflation. Figure 1 reports the impulse-response graphs for the variables  $[y_t, y_t^*, x_t, \pi_t, \iota_t - r^{ss}, c_t, r_t, k_t, v_t^*, v_t^m, q_t]$ .

[Figure 1]

On impact, the positive technology shock has the effect of raising the prices of stocks (which reflect all available information in this context) because firms have become more profitable. These in turn receive a positive signal from Tobin's  $q$  and trigger new investment. The ensuing transmission mechanism from the stock market to the real economy, and vice versa, may be quantitatively quite different depending on the sensitivity of investment to Tobin's  $q$  ( $\gamma_1^{-1}$ ). In order to gauge the role of this variable, figure 1 reports the tracks with our calibration  $\gamma_1 = 0.1$  (high sensitivity) and with  $\gamma = 1$  (low sensitivity). With  $\gamma_1 = 0.1$ , capital increases faster, the rise in potential output is larger and more persistent. However, investment rises enough to keep aggregate demand in line with the production capacity of the system: output and inflation gaps are smaller and virtually negligible. As a consequence, the policy response on the interest rate, too, is minimal. Thus, the increase in stock prices is smoothed<sup>21</sup>.

### B) A pure bubble process ( $\varepsilon_a = 0$ , $\omega = 1$ ).

A pure bubble process is an exogenous growth of stock prices which is not driven by fundamental valuation. There is no technology shock, and the weight of non-fundamentalists is  $\omega = 1$ . Bubbles are modelled as explained above in equation (28): they begin with a positive exogenous shock of 1 percentage point to non-fundamental expected stock payoffs,  $\phi$ . The equation is parameterized so that  $\phi$  doubles in each period as long as the bubble persists. After 5 periods, the bubble crashes and non-fundamental agents' expectations immediately align with the fundamentals.

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<sup>21</sup> Note that with  $\gamma_1 = 1.0$ , potential output rises more than aggregate demand, and a (slight) deflationary pressure arises. As a consequence, the central bank engineers *a cut in the interest rate* in order to realign aggregate demand. Since monetary policy is expansionary, *stock market revaluation overshoots and remains sustained*. This scenario does not seem consistent with common evidence of periods of stock-market booms, which are typically associated with buoyant economic activity, inflationary pressures and non accommodative monetary policy. Hence we are confirmed that our choice of low  $\gamma$  is more appropriate for our model.

Here we discuss the different monetary policy options according to the data summarized in table 4. Recall that the chain goes from the impact of the bubble on stock prices (increase) to Tobin's  $q$  (increase), investment and capital (increase), potential output (increase). Consumption only reacts to anticipated changes in the real interest rate. All these first round effects are similar to those of our benchmark case A, but now they are undesirable since they are triggered by a non-fundamental bubble.<sup>22</sup>

As a preliminary observation, no single rule outperforms all the others under all dimensions. If, to begin with, we focus on the two key variables typically associated with central banks' objective functions, output ( $x$ ) and inflation ( $\pi$ ) gaps, we see that

- among traditional TRs, "output aggressive" outperforms the other
- among financial TRs, "Tobin's  $q$ " outperforms the others
- "Tobin's  $q$ " performs better than "output aggressive" in terms of output stabilization, but the reverse is true in terms of inflation stabilization.

TABLE 4 TAYLOR RULES AND VOLATILITY OF SELECTED VARIABLES St. deviations (%) <sup>a</sup>					
Variables	$x$	$\pi$	$i$	$c$	$v^m$
Taylor rules					
"inflation aggressive"	2.06	0.24	17.10	3.15	1.68
"output aggressive"	1.23	0.15	15.71	3.75	1.54
"stock prices"	8.01	3.18	9.23	11.17	0.86
"Tobin's $q$ "	0.87	0.23	15.10	3.90	1.48
"investment"	0.96	3.10	20.01	6.29	1.95
<sup>a</sup> Standard deviations are computed on a mid-term basis of 20 quarters					

The first result dispels the traditional wisdom that, should the central bank stick to traditional TRs, the preferable parametrization is an "inflation aggressive" <sup>23</sup> one (BG (1999), Smets and Wouters (2005)). The reason why the "output aggressive" parametrization is to be preferred is closely related to the transmission mechanism of the stock-price bubble via endogenous investment. The bubble-induced increase in investment and capital generates a comovement in aggregate demand and supply that checks inflationary pressures. This pattern seems to capture fairly well the stylized

<sup>22</sup> Though it is irrelevant in this scenario, note that the fundamental value of stocks is also affected and *falls*. This is due to two main reasons: excess accumulation of capital, and high real interest rate

<sup>23</sup> A parameter greater than 2 seems unrealistic, and in any case it would worsens the trade-off with output stabilization.

facts reported by the cited works by Borio and Lowe (2002) and others, and perhaps the allegedly excessive laxity of the US Federal Reserve between 2003 and 2007. The increase in the interest rate gap under "output aggressive" is constantly greater than under "inflation aggressive", and at peak the difference is 1/3 of a point. Consequently, the output and inflation gaps are constantly lower (the peak output gap is reduced by about 40%).

It is worth noting that this result raises a non trivial consistency problem. The proverbial conservative banker would ex ante commit to the "inflation aggressive" rule so that "output aggressive" may not be an option when a bubble grows. Otherwise the central banker should be ready to switch from one rule to the other conditionally on being in ordinary times or on a bubble, which not only is unorthodox but implies the central bank's ability to detect the bubble.

The results concerning the financial TRs are half in agreement with the BG central tenet, namely that including stock prices in the TR may be destabilizing: indeed, the "stock prices" rule yields the absolute worst results for both output and inflation. It is interesting to note that this rule suffers from the opposite fault of the traditional ones: it is too volatile and monetary policy is forced to over-react. In particular, the impulse response data (not reported) reveal that the "stock prices" rule generates the most dramatic departure from the pattern of the other rules. This, as also explained by BG (1999), is mostly due to the abnormal fall in consumption in anticipation of a sharp increase in the real interest rate. As a result, the response paths of output, output gap and inflation gap are reversed into sharp negative values, forcing the central bank to eventually switch to an accommodative policy.

But we are also half in disagreement with BG, since their conclusion is not true for other possible financial indicators. In line with the above-mentioned evidence indicating that deviations of investment from trend are good predictors of financial instability, this indicator in the TR fares better than "stock prices", at least in terms of output stabilization. However, the most interesting case is "Tobin's  $q$ ", which not only is the best performing financial TR, but it also definitely outperforms the traditional "inflation aggressive" for all variables except consumption. The comparison between the impulse responses of "Tobin's  $q$ " and "inflation aggressive" can be seen in figure 2. The advantage of the former is similar to the one pointed out for the "output aggressive" rule, namely that it is more responsive to the developments in the real economy than the "inflation aggressive" alone.

[Figure 2]

As can be seen in the impulse response tracks, the real interest rate rises more (2/3 of a point at peak), while investment, capital, output, potential

output and the output gap all rise less (the peak output gap is 20% smaller). Of course, the stabilization benefits to be gained shrink if the sensitivity of Tobin's  $q$  to stock market bubbles, and that of investment to Tobin's  $q$ , are low. But this is tantamount to saying that stock market bubbles do not represent a major macroeconomic problem. If they do,  $q$ -augmenting the TR seems the appropriate move.

Hence we are left with the comparison between the traditional "output aggressive" and the financial "Tobin's  $q$ " rules. The figures in table 4 reveal a trade-off between the two. The conservative central banker might prefer the outcome of the traditional "output aggressive" since it stabilizes inflation better, while "Tobin's  $q$ " stabilizes output better. However, as also stressed by Dupor (2005), the welfare problem raised by stock-price bubbles with endogenous investment is that their distortionary effect is to be seen in the output boom. Therefore, stabilization should be stronger on that side. In this respect, "Tobin's  $q$ " seems preferable.

### **C) A mixed process of a shock to the fundamental value of stocks and a bubble ( $\epsilon_a > 0$ , $\omega = 0.5$ )**

In our last experiment, we have studied how our model reacts to a stock-market bubble which shows up in connection with rising productivity. It is often argued that the central bank can hardly distinguish between bubbles and movements in stock prices which are driven by fundamentals, and hence that it should not react to stock price swings in order to avoid destabilizing effects, or "throwing the baby out with the bathwater". Indeed, none of our policy rules is based on unobservable financial variables; hence it is interesting to see how they perform in the presence of a mixed process of stock prices.

We have assumed that the system is hit by a positive 1% technical shock following the auto-regressive process (27) as in the efficient stock-market case A. Now the stock market is populated by a 50% share of non-fundamentalists and 50% of fundamentalists ( $\omega = 0.5$ ). Initially, both types of agents read the available information correctly ( $\phi = 1\%$ ). But soon the non-fundamentalists become over-optimistic: after the second period, they start inflating a bubble according to the process (28). The boom period ends after 5 quarters, and the bubble bursts.

Table 5 reports the performance of the various TRs in terms of volatility of selected variables. Volatilities are reported both in absolute (percent) terms and relative to the stock-market efficiency case A. We think that this latter relative measure may be more informative in regard of the issue of not killing the desirable component of adjustments driven by the fundamentals.



TABLE 5  
TAYLOR RULES AND VOLATILITY OF OUTPUT,  
INFLATION AND STOCK PRICES

Taylor rules	St. deviations (%) <sup>a</sup>			Relative to case A (ratios)		
	$x$	$\pi$	$v^m$	$x$	$\pi$	$v^m$
"inflation aggressive"	0.070	0.026	0.271	13.490	4.040	1.311
"output aggressive"	0.050	0.019	0.264	9.611	3.028	1.279
"stock prices"	0.183	0.590	0.223	32.274	92.267	1.079
"Tobin's $q$ "	0.062	0.011	0.218	11.964	1.712	1.057
"investment"	0.585	0.575	0.084	112.541	89.901	0.410

<sup>a</sup>Standard deviations are computed on a mid-term basis of 20 quarters

As can be seen from the table, under all TRs the economy displays substantial excess volatility with respect to case A as a consequence of the bubble component, though less markedly so in the case of stock prices. If stock-price stability were pursued for its own sake (in absolute terms or relative to case A), all rules would be almost equivalent, whereas the rules fare quite differently in terms of excess volatility of output and inflation. For direct comparison, under the same "inflation aggressive" rule embedded in case A, output volatility is now magnified about 13 times and inflation volatility 4 times. Looking for better rules, we can see analogies and differences with respect to the pure bubble case B.

Again, the "inflation aggressive" rule performs poorly with respect to both "output aggressive" and "Tobin's  $q$ ", and these two rules stand out as the best performing ones; yet they still present a trade-off. Interestingly, however, the trade-off is reversed with respect to case B. Now "Tobin's  $q$ " outperforms "output aggressive" for inflation stability, while the ordering is reversed for output stability. Thus, if the argument of preserving the benefits of the fundamental process underlying the bubble concerns the *real* benefits, then more weight may be attached to minimizing excess volatility of output with respect to case A, and hence the preferred rule should be "output aggressive". If instead the usual argument prevails in favour of minimizing excess volatility of inflation, then "Tobin's  $q$ " should be preferred. We also present the impulse response graphs of these two rules *vis-à-vis* case A (see figure 3). Inspection of the graphs provides useful insights into the pros and cons of the two selected rules.

[Figure 3]

If we first look at the real variables, we clearly see that, in the course of the bubble process, "Tobin's  $q$ " is more restrictive. Investment rises less, consumption falls more; output and potential output rise less, and the output gap becomes negative. As regards this latter variable, "output aggressive" exactly yields the specular result, with case A being in the middle of the two. Hence, "Tobin's  $q$ " over-kills the real effects of the stock-market boom, whereas "output aggressive" rule keeps the baby together with some bathwater. On the other hand, "Tobin's  $q$ " is more effective in controlling inflation and curbing "irrational exuberance" in stock prices.

## 4. Conclusions

In the years to come, it is likely that central banks will be required to include financial stability among their macroeconomic responsibilities more directly and explicitly than they used to do before the 2008-09 crisis. Here we have provided a New Keynesian quantitative model that may shed further light on two unsettled specific issues: whether central banks should include some financial indicator in their policy rules, and which indicator may be expected to generate better stabilization performance. The model has been designed to take stock of pre-crisis debates and to overcome various limitations that emerged from those debates as well as from empirical reconstructions and stylized facts of boom-bust cycles. In this regard, the noteworthy features of the model are endogenous investments and capital accumulation (and hence evolution of aggregate supply) in response to stock price booms, the possibility of mixed processes of fundamental stock price movements and non-fundamental, finite-life bubbles, an endogenous component of bubbles (in the sense that the bubble dynamics is affected by changes in the interest rate), the availability of observable stock-market indicators. In the light of the simulations presented here, we cannot reach a clear-cut, univocal conclusion, but we can single out two competing rules, "output aggressive" and "Tobin's  $q$ ", that are at variance with the Fed consensus.

Our distinction from the Fed consensus lies in that 1) the best performing traditional rule (i.e. with no financial indicators) is "output aggressive" instead of "inflation aggressive"; 2) while we confirm that financial indicators such as the pure stock price index (or deviations of investment from trend) are too volatile and induce destabilizing effects of monetary policy, this is not the case with "Tobin's  $q$ ". One of the key reasons for these results is that, when capital accumulation supply-side effects are taken into account, stock-price bubbles unfold without producing sufficiently strong goods-price inflation signals. Traditional policy rules fail to react to the

required extent, even if greater weight is attached to inflation gaps. Therefore, better stabilization is obtained by shifting weight to output or by including a financial indicator such as Tobin's  $q$ , which signals the undesirable spikes in investment and capital accumulation fairly well, allowing the central bank to take a more restrictive stance.

We cannot give a univocal ranking between "output aggressive" and "Tobin's  $q$ " rules because of dependence on the type of stock-price process (pure bubble or mixed) and on the central bank's own ranking between output and inflation stabilization. "Tobin's  $q$ " yields better output stabilization, and "output aggressive" better inflation stabilization, in the case of pure bubbles. When bubbles are mixed up with fundamental stock-price movements, the ranking is reversed. If we introduce the specific welfare ordering criterion suggested by Dupor (2005), according to which in the presence of bubbles (whether pure or mixed) it is the excess output boom that should be minimized, then the central bank may wish to switch from "Tobin's  $q$ " in the case of a pure bubble to "output aggressive" in the case of a mixed process. The additional problem is that, in ordinary times, an orthodox central bank would also like to re-switch to being "inflation aggressive".

This switching across rules, which would imply a significant shift of policy stance with respect to the ordinary mandate, is clearly problematic, at least for orthodox central banks which are committed to price stability as a priority. Here we have not provided a formal analysis of this choice problem because it would inevitably involve arbitrary assumptions concerning the policy maker's or society's preferences, and hence we leave it to further analysis. In this perspective, we only wish to suggest that if the central bank wishes to abide with one single publicly known policy rule unconditionally, and the chosen rule contains no financial indicator, switching to an "output aggressive" policy stance during a stock market boom, with possibly vibrant economic activity and low inflation, would be uneasy to communicate and politically difficult to implement. Hence, prudentially, it may be advisable that the central bank *ex ante* includes a financial indicator into its systematic rule in consideration of the recommendations that central banks should communicate their framework for policy choices as clearly as possible, and that such a framework should remain reliably stable across different contingencies. Our analysis shows that Tobin's  $q$  is a serious candidate.

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Figure 1. Impulse-response graphs. Stock-market efficiency

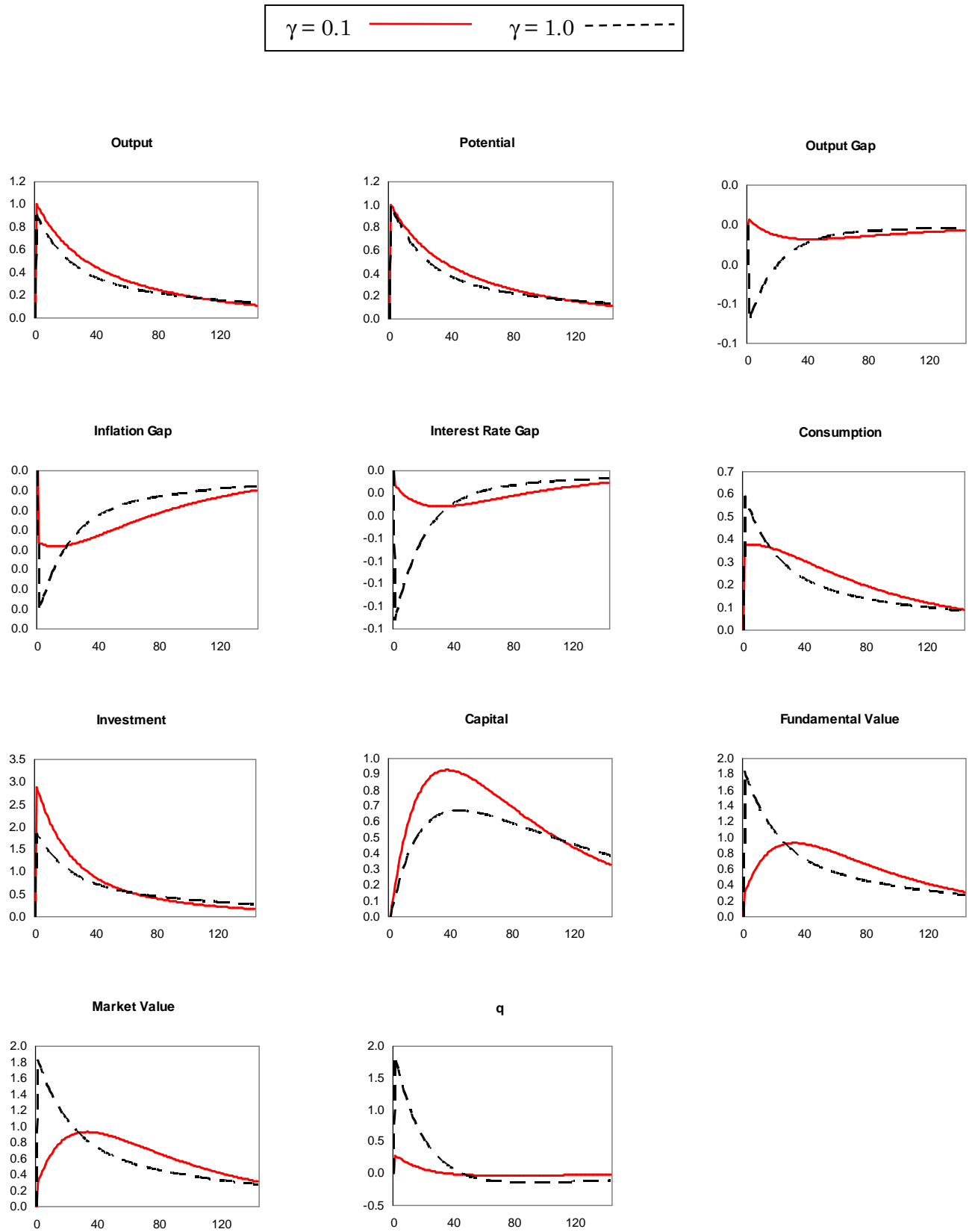


Figure 2. Impulse-response graphs. Pure bubble

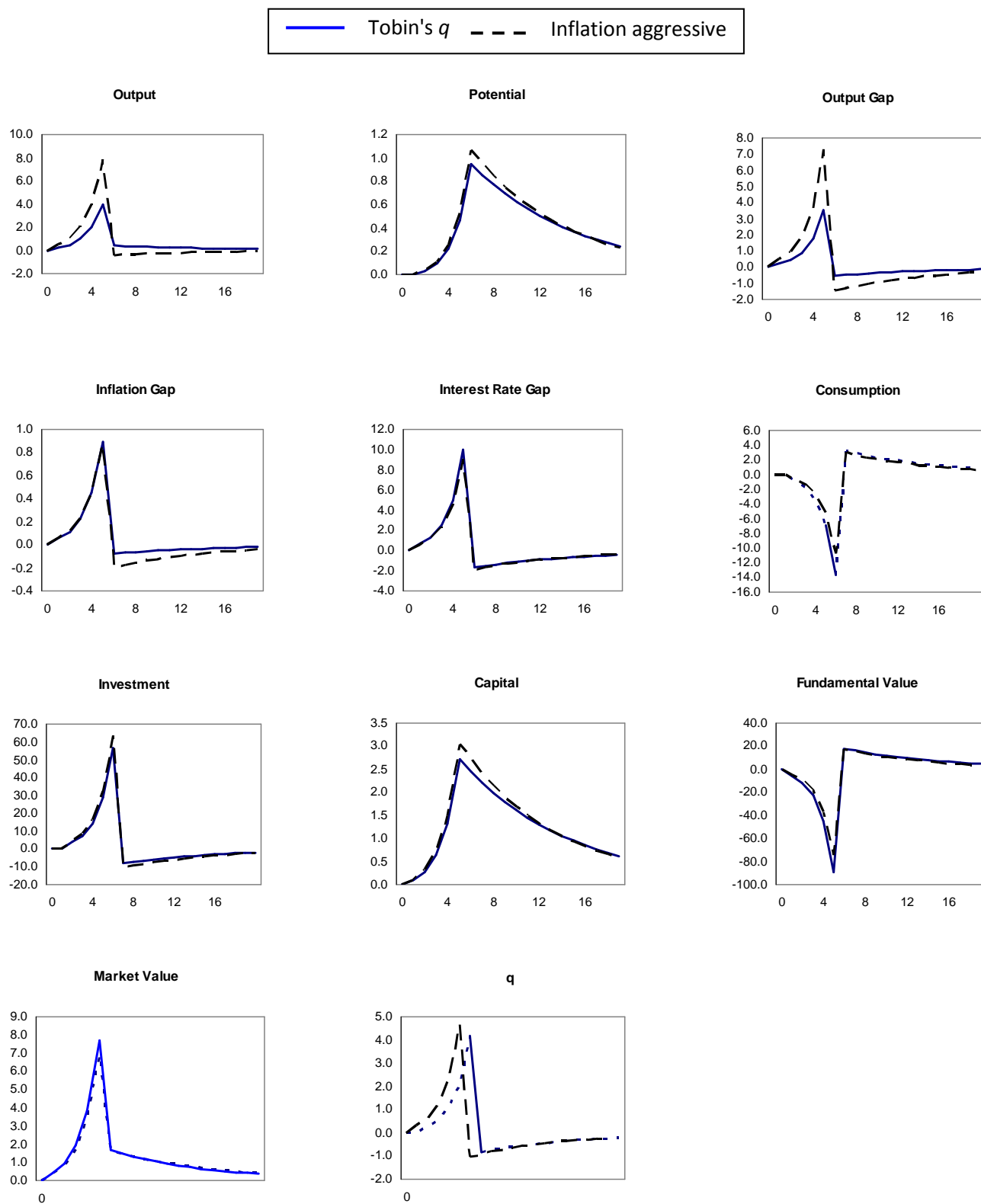
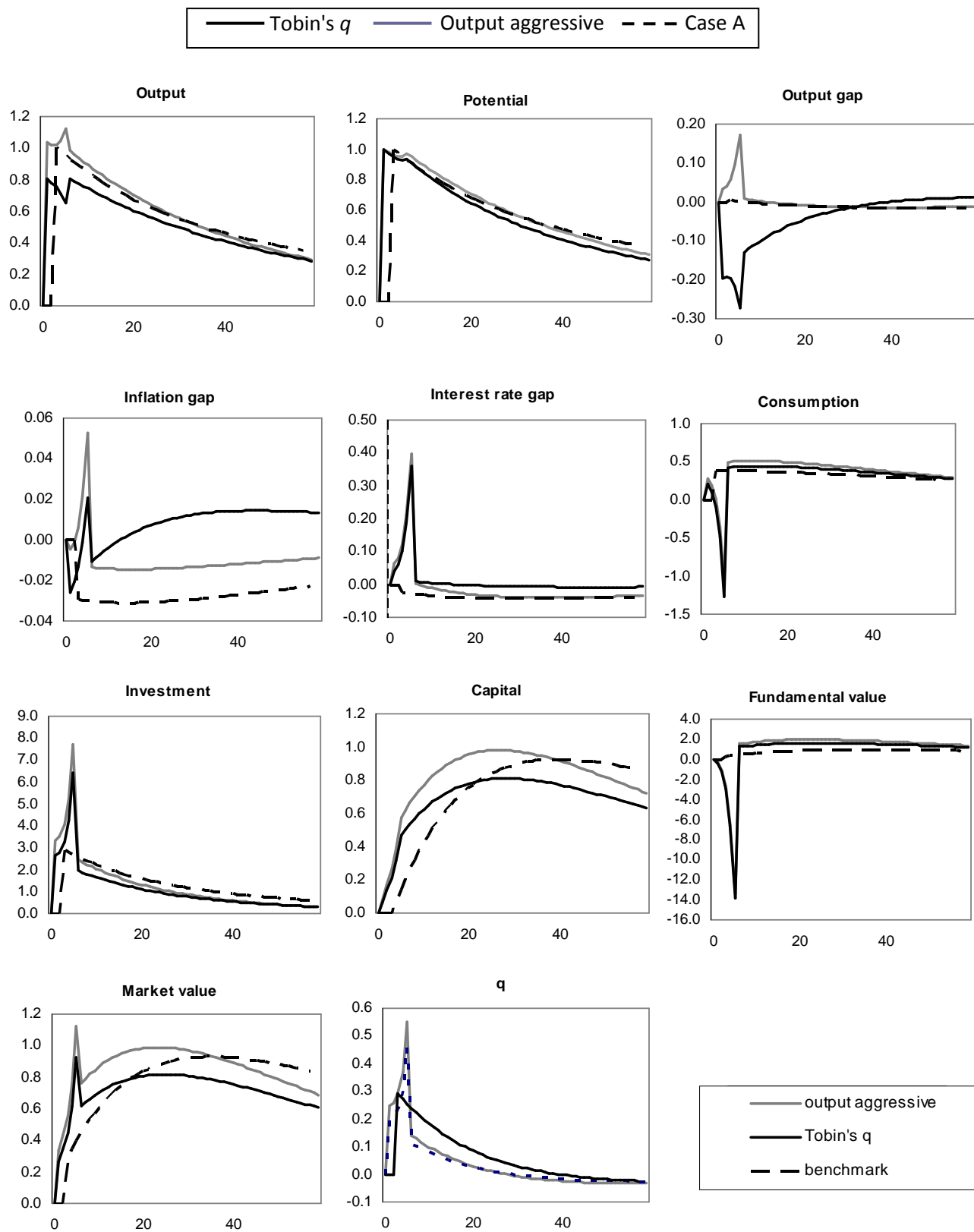


Figure 3. Impulse-response graphs. Mixed process





## Appendix

### A1. Capital stock and investment

Producers maximize

$$(A1) \quad \mathbb{E}_t \left[ Z_t + \sum_{j=1}^{\infty} Z_{t+j} R(j)^{-1} \right] \quad R(j) = \prod_{j=1}^{\infty} (1 + r_{t+j})$$

s.t.  $(1 - \delta)K_t + I_t - K_{t+1} = 0$

given

$$\begin{aligned} Z_t &= Y(L_t, K_t) - W_t L_t - (I_t + \Gamma(I_t, K_t)) \\ Y(L_t, K_t) &= A_t K_t^\alpha L_t^{(1-\alpha)} \\ \Gamma(I_t, K_t) &= I_t \left( \gamma_2 \left( \frac{I_t}{\delta K_t} \right)^{\gamma_1} - 1 \right) \end{aligned}$$

Given  $K_t$ , which is predetermined by previous investments and cannot be changed in  $t$ , the choice variables as of  $t$  are  $L_t$ ,  $I_t$ ,  $K_{t+1}$ . Indicating with  $\mathcal{L}$  the Lagrangean function and with  $\lambda_t$  its multipliers for each period  $t$ , the f.o.c. are

$$\begin{aligned} \partial \mathcal{L} / \partial L_t &= Y_{L,t} - W_t = 0 \\ \partial \mathcal{L} / \partial I_t &= -(1 + \Gamma_{I,t}) + \lambda_t = 0 \\ \partial \mathcal{L} / \partial K_{t+1} &= -\lambda_t + \mathbb{E}_t [Y_{K,t+1} - \Gamma_{K,t+1} + (1 - \delta)\lambda_{t+1}] R(1)^{-1} = 0 \\ \partial \mathcal{L} / \partial \lambda_t &= (1 - \delta)K_t + I_t - K_{t+1} = 0 \end{aligned}$$

In each period  $t$ ,  $\lambda_t$  measures the variation in the firm's value due to a marginal increment in the capital stock. Hence, this is the marginal variation in the firm's value after investment  $I_t$ . By Hayashi theorem,

$$\lambda_t \equiv Q_t = V_t / K_t$$

$$\text{where } V_t = \mathbb{E}_t \left[ \sum_{j=1}^{\infty} Z_{t+j} R(j)^{-1} \right]$$

(see also Chirinko and Schaller (2006)).

The f.o.c. for  $I_t$  states that the optimal  $I_t$  is such that its marginal cost equates  $Q_t$ . Therefore,

$$(1 + \gamma_1) \gamma_2 \left( \frac{I_t}{\delta K_t} \right)^{\gamma_1} = Q_t$$

$$(A2) \quad I_t = \bar{Q}_t^{1/\gamma} \delta K_t$$

where  $\bar{Q}_t \equiv Q_t / \gamma_3$ ,  $\gamma_3 \equiv (1 + \gamma_1) \gamma_2$ ,

Hereafter we shall make use of the log-linearization method whereby any variable  $X_t$  in level is expressed as log-deviation from its s-s value,  $X_t = X^{ss} \exp(\ln(X_t / X^{ss}))$ . Then, the log-deviation is denoted  $\ln(X_t / X^{ss}) \equiv x_t$  and the variable in level is approximated by  $X_t = X^{ss}(1 + x_t)$ .

Consequently,

$$I^{ss}(1 + i_t) = \bar{Q}^{ss(1/\gamma)} \delta K^{ss}(1 + \gamma^1 q_t + k_t)$$

Since  $I^{ss} = \bar{Q}^{ss(1/\gamma)} \delta K^{ss}$ , and  $\bar{Q}_t / \bar{Q}^{ss} = Q_t / Q^{ss}$ , then

$$(A3) \quad i_t = \gamma^1 q_t + k_t$$

## A2. Stock prices

The market value of a stock at any time  $t$  consists of the fundamental component  $V_t^*$  and possibly the bubble component  $B_t = \Phi_{t,t+1} R(1)^{-1}$

$$(A4) \quad V_t^m = (1 - \omega) V_t^* + \omega \Phi_{t,t+1} R(1)^{-1}$$

where  $\omega$  captures the weight of "non fundamentalists" in the market.

To begin with, we express the stock-market valuation (A4) in terms of log-deviations from s-s, i.e.:

$$V^{ss}(1 + v_t^m) = (1 - \omega) V^{ss}(1 + v_t^*) + \omega \Phi^{ss} R^{ss-1} (1 + \phi_{t,t+1} - \hat{r}_{t+1})$$

Since  $\Phi^{ss} R^{ss-1} = V^{ss}$ ,

$$(A5) \quad v_t^m = (1 - \omega) v_t^* + \omega (\phi_{t,t+1} - \hat{r}_{t+1})$$

As to the fundamental value  $V_t^*$ , we follow Chirinko and Schaller's (2006) method. Let us re-write the f.o.c. condition for  $K_{t+1}$  for a maximum of firms' value (labour is constant and standardized to 1):

$$(A6) \quad -Q_t + E_t[Y_{K,t+1} - \Gamma_{K,t+1} + (1 - \delta)Q_{t+1}] R(1)^{-1} = 0$$

Therefore, the fundamental value of  $Q_t$  associated with the optimal capital stock that satisfies (A6) is

$$(A7) \quad Q_t^* = E_t[Y_{K,t+1} - \Gamma_{K,t+1} + (1 - \delta)Q_{t+1}] R(1)^{-1}$$

where

$$Y_{K,t+1} = \alpha A_{t+1} K_{t+1}^{(\alpha-1)}$$

$$\Gamma_{K,t+1} = -\delta \gamma_1 \gamma_2 \left( \frac{I_{t+1}}{\delta K_{t+1}} \right)^{1+\gamma_1} = -\delta \gamma_1 \gamma_2 \bar{Q}_{t+1}^{(1+\gamma)/\gamma}$$

Variables in terms of log-deviations from s-s can be expressed as follows

$$Q^{ss} R^{ss} (1 + q_t + \hat{r}_{t+1}) = E_t[Y_K^{ss} (1 + a_{t+1} - (1 - \alpha)k_{t+1}) + \\ - \Gamma_K^{ss} (1 + (1 + \gamma_1) \gamma_1^{-1} q_{t+1}) + (1 - \delta) Q^{ss} (1 + q_{t+1})]$$

As to the s-s variables, we know that

$$Q^{ss} = \gamma_3$$

$$\Gamma_K^{ss} = -\delta \gamma_1 \gamma_2$$

$$Q^{ss} (R^{ss} - (1 - \delta)) = Y_K^{ss} - \Gamma_K^{ss}$$

Therefore,

$$q_t^* = [E_t a_{t+1} + \psi E_t [a_{t+1} - (1 - \alpha)k_{t+1}]] R^{ss-1} - \hat{r}_{t+1}$$

where  $\psi \equiv R^{ss} + \delta(1 + \gamma_1)^{-1}$

The fundamental stock-market value is defined as

$$V_t^* = Q_t^* K_t$$

Log-deviations from s-s are

$$v_t^* = q_t^* + k_t$$

Deriving  $E_t q_{t+1}$  from the previous expression one period forward, and factoring the other variables,

$$(A8) \quad v_t^* = [E_t v_{t+1}^* + \psi E_t a_{t+1} - (1 + \psi(1 - \alpha)k_{t+1})]R^{ss-1} - \hat{r}_{t+1} + k_t$$

### A3. The Phillips curve

Let us start from the GPL equation

$$(A9) \quad P_t = \frac{1}{1 + \beta v^2} [v\beta E_t P_{t+1} + (1 - v)(1 - v\beta)P_t^* + vP_{t-1}]$$

In terms of log-deviations we obtain

$$P^{ss}(1 + \hat{p}_t) = \frac{1}{1 + \beta v^2} [v\beta P^{ss}(1 + E_t \hat{p}_{t+1}) + (1 - v)(1 - v\beta)P^{ss}(1 + \hat{p}^*) + v P^{ss}(1 + \hat{p}_{t-1})]$$

Now let us consider the following relationships

$$\pi_t = \hat{p}_t - \hat{p}_{t-1}$$

$$P^{ss}(1 + \hat{p}^*) = \mu MC^{ss}(1 + mc_t + \hat{p}_t), \text{ from (16)}$$

$$\hat{p}^* = mc_t + \hat{p}_t$$

where  $mc_t$  is the log-deviation rate of the real marginal cost. Using these, we obtain,

$$(A10) \quad \pi_t = \beta E_t \pi_{t+1} + (1 - v)(1 - v\beta)v^{-1}mc_t$$

As to the variable  $mc_t$ , we know that the real marginal cost is  $1 + bW_t$ . Recall that  $W_t$  is the retailers' reservation value of self-employment, which is established on the competitive labour market in the production sector. Given that labour is in fixed supply  $L_t = 1$ , it must be that  $W_t = (1 - \alpha)Y_t$ , which is the labour's income share. Consequently, the log-deviation rate of  $1 + b(1 - \alpha)Y_t$  is just the same as  $Y_t$ , i.e.  $mc_t = x_t$ . Hence we can write

$$(A11) \quad \pi_t = \beta E_t \pi_{t+1} + \kappa x_t$$

where  $\kappa \equiv (1 - v)(1 - v\beta)v^{-1}$

### A4. Consumption

Households maximize

$$(A12) \quad E_t \left[ C_t + \sum_{j=1}^{\infty} \beta^j U(C_{t+j}) \right]$$

$$\text{s.t. } W_t + S_{t-1} V_{t-1}^m R(0) + \underline{M}_{t-1} R(0) - C_t - \underline{M}_t - S_t V_t^m = 0$$

given

$$U(C_t) = \frac{1}{1 - \sigma} C_t^{1 - \sigma} \quad \sigma > 0$$

Note that the budget constraint implies that each period a) opens with the predetermined asset stocks  $\underline{M}_{t-1}$  and  $S_{t-1}$ , and b) returns to assets are

calculated on the predetermined stock-values. Consequently, the choice variables in any  $t$  are  $C_t$ ,  $M_t$ ,  $S_t$ , and should satisfy the respective f.o.c.

$$\partial \mathcal{L} / \partial C_t = U_{c,t} - \lambda_t = 0$$

$$\partial \mathcal{L} / \partial M_t = -\lambda_t + \beta E_t \lambda_{t+1} = 0$$

$$\partial \mathcal{L} / \partial S_t = -\lambda_t V_t^m + \beta V_t^m E_t \lambda_{t+1} R(1) = 0$$

From the f.o.c. for  $S_t$  and  $C_t$ , we obtain  $\lambda_t = \beta E_t \lambda_{t+1} R(1)$ ,  $\beta E_t \lambda_{t+1} = \beta E_t U_{c,t+1}$ , and therefore

$$\begin{aligned} U_{c,t} &= \beta E_t U_{c,t+1} R(1) \\ (A13) \quad C_t &= \beta^{-1/\sigma} E_t C_{t+1} R(1)^{-1/\sigma} \end{aligned}$$

Note also that, as in Woodford (2003),  $M_t$  and  $S_t$  are perfect substitutes so that the interest-rate policy of the central bank works through instantaneous arbitrage between the two assets.

Log-deviations from s-s are

$$C^{ss}(1 + c_t) = \beta^{-1/\sigma} C^{ss} R^{ss-1/\sigma} (1 + E_t c_{t+1} - \sigma^{-1} \hat{r}_{t+1})$$

Since  $\lambda_t = \beta E_t \lambda_{t+1} R(1)$  implies  $R^{ss} = \beta^{-1}$ , then

$$(A14) \quad c_t = E_t c_{t+1} - \sigma^{-1} \hat{r}_{t+1}$$

## A5. Calibration of the investment function

The quantification of investment adjustment costs is notoriously controversial. Casares and McCallum (2006) have introduced the idea that these adjustment costs should be large (typically a cost function with power no less than 2) in order to avoid extremely high volatility not comparable with the data.<sup>24</sup> However, these empirical assessments are, inevitably, dependent on the specific model employed, and on complementary conditions such as the monetary policy stance. In order to have a quantitative check, we have replicated the Casares-McCallum procedure, that is, a comparison among the standard deviations of output, consumption and investment obtained in our model under three parametrizations of  $\gamma_1$  (0.1, 0.5, 1) with those in the real US quarterly data (see table 6). In order to bypass problems of scale of shocks (standard deviations are scale sensitive) we present the data on relative volatility to GDP. In consideration of the use of long-run data, we have adopted the benchmark specification of the model, which is generally regarded as representative of the prevailing operating conditions of the economy, that is the efficient stock market regime A with the traditional "inflation aggressive" TR.

A couple of preliminary observations regarding real data are in order. The first is that the three main components of private investments have clearly different volatility, with the residential component being significantly

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<sup>24</sup> In their model, the adjustment-cost parameter equivalent to our  $\gamma_1$  is calibrated to 3.14.

higher than the non-residential one. The second is that the boom-bust cycle 2003-2009 has left its imprint in the larger volatility of all investment components.

Clearly, our simulations confirm that the volatility of investment increases as  $\gamma_1$  decreases; and yet, the model does not seem to suffer from excess volatility for low  $\gamma_1$ .

TABLE 6  
RELATIVE VOLATILITY OF INVESTMENT  
St. deviations (%)

	US economy <sup>a</sup> (1980:1-2009:7)	US economy <sup>a</sup> (2003:1-2009:7)	Model economy $\gamma_1=0.1$ $\gamma_1=0.5$ $\gamma_1=1$		
Priv.investment/GDP					
-total	4.81	5.55			
-non-residential	3.71	4.77	2.93	2.15	1.54
-residential	5.64	6.67			

<sup>a</sup>Detrended with Hodrick-Prescott filter 1600. Source: U.S. Department of Commerce: Bureau of Economic Analysis: Quarterly data, seasonally adjusted..

Since our focus is on investment dynamics, and we only have firms' investment in our model economy, the more sensible benchmark for the model should be the volatility of non-residential investment relative to GDP. Hence we have concluded that, for both qualitative and quantitative considerations, the most sensible parametrization for our purposes is  $\gamma_1 = 0.1$ .

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