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Dynamic Contests

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ABSTRACT

Dynamic Contests

by Kai A. Konrad *

Considering several main types of dynamic contests (the race, the tug-of-war, elimination contests and iterated incumbency fights) we identify a common pattern: the discouragement effect. This effect explains why the sum of rent-seeking efforts often falls considerably short of the prize that is at stake. It may cause violent conflict in early rounds, but may also lead to long periods of peaceful interaction.

ZUSAMMENFASSUNG

Dynamische Wettbewerbe

Unter Berücksichtigung verschiedener Haupttypen dynamischer Wettbewerbe (das Wettrennen, das Tauziehen, Ausscheidungskämpfe und wiederholte Kämpfe um Amtszeiten) identifizieren die Autoren ein gemeinsames Muster: den Entmutigungseffekt. Dieser Effekt erklärt, wieso die Rent-seeking-Bemühungen in Summe oft deutlich nicht an den auf dem Spiel stehenden Preis heranreichen. Der Effekt kann heftige Kämpfe in den ersten Runden des Wettbewerbs auslösen, aber auch zu langen Perioden friedlichen Zusammenspiels führen.

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1 Introduction

Conflict typically involves the choice of costly inputs (e.g, arming and the hiring of soldiers) by adversaries and the outcome of battles has an uncertain outcome. This structure – the choice of costly inputs that are combined adversarially with an uncertain outcome – is common to many other economic settings and is modeled as a contest. In a contest, decision makers expend effort and a contest success function maps the vector of efforts into win probabilities, or into the shares of the total prize they receive (Tullock 1980, Hillman and Riley 1989, Baye et al. 1996, Hirshleifer and Riley 1992). This game is considered as the core of a whole universe of strategic problems in very diverse contexts, including sports tournaments, labor market tournaments, internal tournaments inside organizations, R&D contests and patent races, contest games among animals, political competition such as electoral competition or military conflict. A closer look at many of these specific problems reveals a richer dynamic structure. In the course of most contests players have to make a whole series of decisions and effort choices. Some players may leave or drop out in the course of the contest, and others may enter. We generally entertain the idea that this dynamic process consists of a series of battle contests, each of which essentially follows the rules of a static contest. However, as these battle contests are imbedded into a larger, multi-stage game, a player’s behavior in a given battle contest will generally have manifold implications for the course of actions and battle outcomes in future battle contests. How current choices of effort and battle outcomes affect future options for a player also has profound implications for the behavior in earlier rounds of a dynamic contest. The future consequences of winning or losing a particular battle may actually induce little effort in a particular battle, as the future contest effort discounts what can be gained from winning; this effect has been called the *discouragement effect*. However, precisely this discouragement effect can also induce very high effort in early battles. The essay focuses on instances of both types.

To illustrate the dynamic structure of contests in politics, organisations, and practical life, consider some examples.

- Leaders of elected governments and political rulers such as kings or dictators extract rents from their incumbency. Emperors, kings or less dignified dictators may face competition from outside by other rulers, or from inside, often even from members of their own family or their supporters. If a ruler loses power, he or she may often be killed. Typically the new incumbent tries to take measures that prevent the former incumbent from re-entering into a competition for power. However, after some time a new challenger is likely to show up. In democratic regimes a structurally related type of repeated or dynamic interaction occurs: elections and electoral competition.
- Inside organizations, managers expend many kinds of effort to secure their own position or to achieve a promotion in an organizations. While the battle for promotion may be well depicted by a battle contest, a whole career

consists of multiple battles: incumbents need to defend their job against a sequence of contenders, and the process of moving up in the hierarchy is a process with battles among a group of contestants at each promotion stage. Rosen (1986) was the first to formally describe elimination tournaments of this type. Sometimes highly valued jobs are filled by way of organizing an elimination tournament. Jack Welch (2001), for instance, reports that he consciously designed the competition for his succession more than 6 years prior to the actual succession event. He first identified 23 possible candidates from inside GE, then narrowed down this list first to 8, later to 3 candidates. These three candidates knew they were competing against each other, and they also knew they would either be promoted to the CEO position, or would have to leave the firm when the succession took place.

- Firms expend considerable effort on research activities and on patent lawyers, trying to achieve valuable patents. R&D has actually been one of the first problems in which the theory of contests has been developed and proved useful as a description of the competition for patents (Nalebuff and Stiglitz 1983). Harris and Vickers (1985, 1987) highlighted the fact that patent races consist of a series of battle contests, and they described two benchmark cases for such multi-round contests: the tug-of-war in which the winner is the player who first accumulates a sufficient difference in the number of own battle victories and victories of the adversary.
- Sports contests most notably follow the rules of a contest, and it is the area in which the contest structure of the game is actually codified in the rules of the respective discipline. Dynamics, and the sequentiality of different battle contests, are rather important for understanding sports. A single tennis match actually consists of many single battles. Each win point is a little battle contest. The player who first makes four win points and has a lead of two win points in a game wins the game, or the game continues until one of the players has a two-point lead. But a series of such games must be won by a player to win a set, and the player who is first to win a given number of sets wins the match. Hence, the structure of a tennis match consists of a series of multi-battle contests. Note also that winning a match is typically only one of many steps needed for a player to win a tournament, as the winner is typically determined by an elimination tournament. Further, tournament victories are only battles in the larger picture: the competition for ATP rankings. Finally, efforts in the different battle contests in which a professional tennis player participates are not independent. The players typically start expending considerable effort when they are very young, and when their first major tournament is a possibility that may come up only many years later. Similar considerations can be made for many other sports disciplines.
- Firms expend large sums on advertising campaigns, trying to steal a share of the market from other firms in promotional competition. Marketing

researchers detected early on that promotional competition follows the rules of a contest.¹ Researchers studying this phenomenon realized early on that market share exerts some amount of inertia, much like a stock variable. For this reason, promotional effort can be seen as a flow variable affecting a stock variable.

Many important aspects of dynamic contests are disregarded here. One important element of dynamics refers to the possibility of the sequencing of contest effort, and possibly repeated effort choices, with efforts made over time accumulated and turned into the relevant overall effort of each player, with the win probability being a function of this overall effort. Although these are relevant issues, we do not focus on these branches of the literature.²

2 Types of dynamic contest interaction

Dynamic contests are defined as games which consist of a sequence of component contests or battles. Each of these component contests or battles can typically be seen as a simple contest. To be more concrete, a simple contest is a game with two or more players $i = 1, \dots, n$. Each player chooses a non-negative effort e_i from a set of possible effort choices. Player i 's payoff is given as

$$\pi_i(e_1, \dots, e_n) = p_i(e_i, \dots, e_n)v_i - e_i. \quad (1)$$

Here, v_i is player i 's (possibly idiosyncratic) valuation of the prize that is awarded to the winner of the contest, and a mapping $p_i(e_i, \dots, e_n)$ that constitutes a probability distribution describes players' probability of winning for different combinations of effort. This mapping is called the *contest success function*. A prominent version of this function is the symmetric lottery function introduced by Tullock (1980), by which $p_i(e_1, \dots, e_n) = e_i/(e_1 + \dots + e_n)$ if at least one contest effort is strictly positive, and $p_i(0, 0, \dots, 0) = 1/n$. Another prominent version is the contest success function of the all-pay auction without noise, by which the player with the highest effort wins with a probability of one (see, e.g., Hillman and Riley 1989 and Baye, Kovenock and de Vries 1996). The payoff of player i is then equal to the product of his win probability and his valuation of winning, minus his cost of effort. For the analysis here, the cost of effort is identified with the amount of effort itself.

¹The lottery contest success function is known as the fundamental theorem of market shares in the context of marketing (see, e.g., Kotler and Bliemel (2001, 277). Some of the most early theoretical analyses of contests, such as Friedman (1958), are motivated by the idea of promotional competition and the relationship between marketing efforts and market shares.

²For games with sequential effort choice see, e.g., Baik and Shogren (1992), Leiniger (1991, 1993), Pérez-Castrillo and Verdier (1992), Linster (1993). Baik and Shogren (1992) and Leiniger (1993) also explain that sequential choice of effort can emerge endogenously in equilibrium, if players can move early or late. Konrad and Leiniger (2007) show how these results generalize for all-pay auctions with multiple players. Romano and Yildirim (2005) also consider repeated effort making, with total contest effort accumulating from these effort contributions.

We distinguish between different dynamic contest structures and sort them into (1) the *racing contest* and the *tug-of-war*, alluding to dynamic structures in which the set of contestants stays constant over time, but in which the prize is awarded as a function of the outcomes of a whole sequence of battle outcomes, (2) the *elimination contest*, characterized by the feature that battles take place at several stages, and the set of participants slowly narrows down over a series of stages, and (3) *iterated incumbency fights*, characterized by the fact that, at each point of time, there is one or a group of incumbents who can be attacked by new entrants who, if victorious, take over the incumbency role.³

2.1 Racing and the tug-of-war

There is a well-known difference between winning a battle and winning the war. Battles are important components of war, and the outcome of a war can be seen as a function of the outcomes of its battles and possibly of further determinants. The pattern according to which an outcome is determined by the outcomes of a whole set of component contests of the nature of the simple contests just described is not limited to warfare, and the function by which the set of battle outcomes maps into the final outcome of the overall competition are often more precisely defined as in the military context. The rules of tennis, for instance, explain how winning a game is a function of the points accumulated by the two players, winning a set is a function of the games won by each player, and the winner of the match is determined by the number of sets each player wins. As the game of tennis suggests, the functions that map battle outcomes into the outcome of the overall contest can be fairly complex. Tennis also incorporates and combines two types of overall contests which we will briefly discuss in isolation.

Consider first a *race*. For simplicity consider a race between two players. Both players accumulate battle victories, and the player who first accumulates a given number of battle victories wins the overall contest. These component battles are similar to the simple contest; both contestants expend efforts, and the winner is determined as a deterministic or stochastic function of these efforts. Staying with the example of tennis, for the male Wimbledon tennis final, for instance, the structure of the race can be depicted as in Figure 1. Players start at state (3,3): each player needs to win 3 sets prior to the other player in order to win the match. They play the first set. They both expend effort, and one of them wins. If player A wins, we move to state (2,3), meaning that player A wins if he wins two further sets prior to player B winning three sets. At this state, if player A also wins the next set, the process moves on to state (1,3), otherwise it moves to state (2,2). Hence, if A wins, A is only one set away from

³Battle contests are often also part of a larger, dynamic structure. E.g., a patent race among firms may be followed by a stage in which firms trade the patents received, with production and a market game following this trading stage, as has been pointed out by Shapiro (2001) and studied by Clark and Konrad (2008). However, we remove this type of dynamics from the picture and concentrate on dynamic contests, i.e., a multi-stage game in which the different stage games can be seen as battle contests.

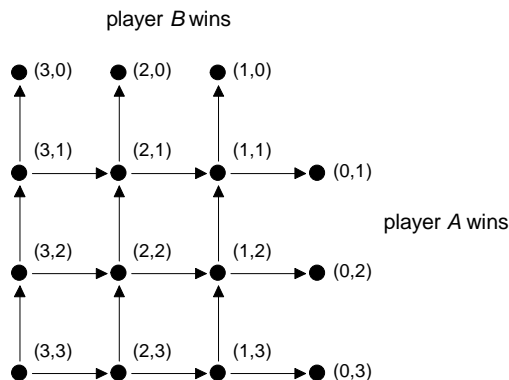


Figure 1: A symmetric race starting at $(3,3)$. The player wins who first wins 3 battles.

winning the match, whereas B wins the match only if B wins the next three sets in a row. In contrast, if B wins the second set and the process moves to $(2,2)$, both contestants return to a state of symmetry in which both are equally distant from final victory, etc.

This race has some interesting properties. The properties are most pronounced for battle contests that follow the rules of a symmetric all-pay auction without noise (see Konrad and Kovenock 2009a for a detailed exposition, allowing also for intermediate battle prizes and for asymmetry). However, the qualitative properties are more general.⁴ To develop these insights, suppose that the final payoff from winning the overall race as in Figure 1 is normalized to unity and is the same for both players. We can solve the race depicted in Figure 1 recursively. The continuation values at (final) states $(j, 0)$ are 0 for player A and 1 for player B, respectively. Once the process reached these states, the contest is over. By design, the winner is handed out the prize, without further contest action being taken at this stage. The reverse payoffs emerge at final states $(0, j)$. At $(1, 1)$, both players engage in a fully symmetric all-pay auction without noise. Hence, the subgame at $(1, 1)$ is analogous to a symmetric static contest for a prize of size 1. As is known from Hillman and Riley (1989), the equilibrium contest effort for the all-pay auction without noise fully dissipates the prize in expectation at this point. Hence, the players' continuation values at state $(1, 1)$ are both zero. Turn now to a state like $(1, 2)$. Here, A's benefit from winning the battle contest and reaching $(0, 1)$ is equal to the prize of value 1 which A receives from reaching this state. If, instead, A loses at $(1, 2)$, the process reaches $(1, 1)$. The continuation value for A reaching $(1, 1)$ is zero, as

⁴ Alcalde and Dahm (2010) show that all-pay contests with little noise have full-dissipation equilibria with payoffs just as in the all-pay auction without noise. Accordingly, the insights on the discouragement effect can be generalized for all-pay contests with little noise. Qualitatively similar, but weaker results can also be derived for contests with strong random elements, such as the Tullock (1980) lottery contest.

just been discussed. Accordingly, A values winning the battle at $(1, 2)$ by the difference $1 - 0 = 1$. Consider now B 's situation at $(1, 2)$. Player B receives zero if the process should move on to $(0, 2)$, i.e., to A 's final victory. But player B 's benefit from winning and moving to $(1, 1)$ is also zero, as B has a continuation value of zero there, too. Accordingly, B is indifferent about whether B wins or loses at $(1, 2)$. The player B would not expend positive effort at $(1, 2)$. Intuitively, B could fight at this point, and may win, bringing the process back to $(1, 1)$, in which both players again have the same chance of winning. However, B anticipates that such a battle victory at $(2, 1)$ is not worth anything for B , as the continuation values at $(1, 1)$ and at $(0, 2)$ are both zero for B . Hence, fighting does not pay for B . Hence, the continuation value of arriving at $(1, 2)$ is equal to 1 for player A , and zero for player B . (A similar argument can be made for state $(2, 1)$, with A and B changing roles.) Turning now to $(2, 2)$ the continuation values at $(2, 1)$ and $(1, 2)$ show that both players attribute a value of winning at $(2, 2)$ which is equal to the full value of the prize. Accordingly, A and B expend considerable effort at $(2, 2)$, – so much effort that, in expectation, they dissipate the whole prize, just as if $(2, 2)$ were the final state $(1, 1)$. The contest outcome at $(2, 2)$ is decisive, provided that A and B play according to subgame perfect equilibrium in the continuation game.

This consideration illustrates a principle that holds more generally, also for a race that is not fully symmetric: There is a range of states (for symmetric contests, this range is a straight line through $(1, 1)$, $(2, 2)$...) in which players contest fiercely. If the process has moved along a trajectory sufficiently far away from this range of states into a region in which one player has accumulated a sufficient lead, then competition slacks off. The race reveals an important property of dynamic contests that has been called the *discouragement effect*. Players in a dynamic contest anticipate that winning or losing a given component contest typically brings them into a new state with a new contest. They will expend contest effort only if they anticipate that the improvement of their continuation value from winning a single battle at some point is worth the effort. This discouragement effect, and the property that competition tends to slack off once one contestant becomes sufficiently disadvantaged is a more general phenomenon.

The discouragement effect is strongest if the battle contests follow the rules of an all-pay auction without noise or with very little noise. The intensity of competition need not fall to zero, for instance, if the battle contest follows different rules, or if players win an additional prize from winning single battles (e.g., a tennis player may prefer losing the Wimbledon final 2:3 rather than 0:3). The strong outcome by which effort drops to zero once players move away from the initial state towards asymmetric states is due to the absence of such considerations and to the assumed contest success function. Analyses by Klumpp and Polborn (2006) in the context of US Primaries and by Harris and Vickers (1985) in the context of R&D reveal similar discouragement effects for other rules of the battle contest. Qualitatively similar discouragement effects emerge, for instance, if each battle follows the rules of the Tullock lottery contest, by which the win probability of a player i for a given battle is given (for positive efforts) by the ratio of this player's effort and the sum of the two players' efforts.

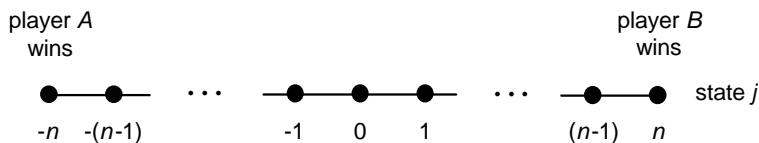


Figure 2: The tug-of-war

To see this, consider state $(1, 2)$. If player A wins at this state, A finally wins and receives a prize equal to 1. If player B wins at $(1, 2)$, the process moves to $(1, 1)$. At this state the continuation game is equivalent to the standard symmetric Tullock lottery contest for a symmetric prize of 1. By the standard results on the Tullock lottery contest (see, e.g., Tullock 1980), the equilibrium continuation values of both players at $(1, 1)$ are equal to $1/4$. Hence, for A the prize of winning the battle at $(1, 2)$ is the difference between winning (i.e., receiving the prize of 1), and losing (i.e., entering into state $(1, 1)$ with a continuation value of $1/4$). Player A 's valuation of winning the battle at $(1, 2)$ is therefore equal to $(1 - (1/4)) = 3/4$. For player B , the value of winning at $(1, 2)$ is only $1/4$. The player B receives zero and the game ends if B loses at this state, and if B wins at $(1, 2)$ then B enters into $(1, 1)$, with a continuation value of $1/4$. Accordingly, the difference in continuation values for B is $1/4$ only. This consideration makes the battle contest at $(1, 2)$ asymmetric in prizes. And according to standard results on asymmetry in contests, the larger prize which player A has from winning at $(1, 2)$ induces this player to expend more effort and makes A win with a higher probability than B at this state.

The discouragement effect has two sides. Disadvantaged contestants (e.g., contestant B at $(1, 2)$) are discouraged from fighting, as they anticipate that they won't win much even if they win the battle, because winning brings them back to a state at which their payoff is low, due to high dissipation that takes place at such a state. The other side of this effect can be illustrated considering state $(2, 2)$. Here, each player would have to win several (perhaps many) single battles before receiving the final prize. However, the future interaction is not as much discouraging for the players at this state, due to the discouragement effect that reduces their efforts in more asymmetric states. For this reason, they expend more resources at $(2, 2)$, as the battle victory at this state brings them very close to the final overall victory. It is the discouragement effect that emerges in asymmetric states that makes the competition at $(2, 2)$ so fierce.

The importance of the discouragement effect for particularly high or particularly low effort in the early rounds of a dynamic contest can also be studied in a related dynamic structure of contest that has been called a tug-of-war. In this type of interaction two players A and B contest against each other in a sequence of battles. The tug-of-war starts in some initial state which is denoted as state 0 in Figure 2. Players A and B expend efforts in this state, and a contest success function determines whether A or B wins. If player A wins, the process moves from state 0 to state -1 . If B wins, the process moves to state

+1 in the next period. In the new state a new battle takes place that follows the same rules and either moves to -2 in the period thereafter, or back to 0, depending on whether A or B wins at -1 . The process continues along similar lines, and possibly for an infinite number of periods, as the process can potentially move back and forth. The process ends, however, once the state $-n$ or the state $+n$ is reached. At these final states the process comes to an end and the winner (A in $-n$, and B in $+n$) receives a winner prize u and the loser receives a loser prize v , with $u > v$. Payoffs seen from period $t = 0$ essentially consist of the discounted prize (if the player wins) minus the discounted amounts of effort expended. For instance, if the sequences of players' efforts are a_0, a_1, \dots, a_t for player A and b_0, b_1, \dots, b_t for player B , and if player A is eventually the receiver of the winner prize in period t , the payoffs are

$$\pi_A = \delta^t u - \sum_{\tau=0}^t \delta^\tau a_\tau \text{ and } \pi_B = \delta^t v - \sum_{\tau=0}^t \delta^\tau b_\tau, \quad (2)$$

where $\delta \in [0, 1)$ is the players' discount factor.

Konrad and Kovenock (2005) solved a more general version of this game for the case in which the battle at each state follows the rules of an all-pay auction without noise, i.e., makes the player with the higher effort at this state win the battle and for $u > v = 0$. They find that, in the Markov perfect equilibrium, players expend considerable effort only at the state 0. Once one of the players is advantaged (e.g., player A for states $-i < 0$), the other player stops fighting and the process moves straight to the final victory of the advantaged player. The intuitive reason for this outcome is, again, the discouragement effect. To illustrate this, suppose the process moved from 0 to -1 . At this point, if B loses, the process moves further to A 's final victory at $-n$. Player B could also try to turn things around and win the battle at -1 . However, B does not win anything from this. If B brings the process back to state 0, then the situation is symmetric for both players again. Here, due to the nature of the all-pay auction without noise, both players expend efforts that are, in expectation, equal to what they gain from moving the process in their preferred direction. Accordingly, B 's benefit from returning to 0 is zero, because all rents that accrue are dissipated at 0 in the fierce contest between A and B at 0. This fierce contest discourages B from trying to 'turn things around'. Much like in the race at symmetric states (j, j) , the battle at state 0 is not the final battle, but using Markov perfect equilibrium play in the continuation games, state 0 is the decisive state.

An even more puzzling result that is also based on the discouragement effect is derived by McMillan (2000). Essentially he considers a variant of the tug-of-war similar to the above, but with $u > 0$, $v < 0$, and $u + v < 0$. An implication of the analysis by McAfee (2000) for the conceptually simplified framework here is the possibility of eternal peace. Consider what happens given these values of final victory and defeat if the process comes close to a final state, say, state $-(n - 1)$. Players A and B have a substantial interest in winning the battle at this state. The material interest of player B to avoid this defeat exceeds the material interest of player A who is close to final victory. By the nature of

the all-pay contest without noise, in such a contest the player with the smaller valuation has a payoff of zero and the player with the higher valuation has an equilibrium payoff that is equal to the difference between the two players' valuations. Players A and B will expend considerable resources at this point, and by $u < (-v)$, the expected payoff of A is zero. As A does not win anything from reaching $-(n-1)$ and B clearly loses something from reaching $-(n-1)$ and having to fight against final defeat, both players have an interest in staying in the interior range and not coming close to the terminal states. Accordingly, the tug-of-war may permanently stay close to the middle state 0 in a situation in which both players expend zero effort.⁵ Again, the mechanism is driven by the discouragement effect. The players know that it is useless to push the process towards the terminal state they prefer. They would benefit from reaching their respective final win state; but just before reaching this victorious state, the adversary's incentives to fight against his defeat are so strong that the eventual payoff of a player who would like to push the process towards his victorious terminal state is zero.

2.2 Elimination contests

In many types of dynamic contests the set of contestants changes over time. Rosen (1986) was the first to analyze a type of contest in which the number of contestants slowly melts down in a series of battle-rounds, because some contestants drop out of the competition over time. Such elimination contests are frequently observed and are most explicit in sports competition, with a series of qualification rounds followed by intermediate rounds, eventually ending in a final round. They occur, however, in many applications. Rosen (1986) focused on labor market contests. Rather than aiming at a high effort, such contests are often motivated by the problem of selecting top performers. The successorship tournament designed by Jack Welch (2001) that was discussed here in the introduction is an example. Similar structures are also important in Biology and have been considered as 'knock-out conflicts', e.g., by Broom, Cannings and Vickers (2000, 2001). Other examples are beauty contests (between individuals in genuine beauty contest, or firms competing for government contracts). All these structures have in common that the set of contestants is narrowed down slowly in several rounds.

One of the most frequent formats has players being teamed up first pairwise, with the losers leaving the competition, and the winners forming a new set of players, and being matched again pairwise etc, up to the quarter-finals, semi-finals and eventually the final. Many structures of elimination tournaments can be envisioned (and actually exist in reality). The characterizing element of elimination tournaments is that the outcome of the previous rounds determines whether a player is eliminated from future rounds of the tournament, or whether the player can take part in a further round, typically competing with a reduced set of contestants. A player who wins or belongs to the group of winners in a

⁵This simplified example is worked out in more detail in Konrad (2009a, 173-177).

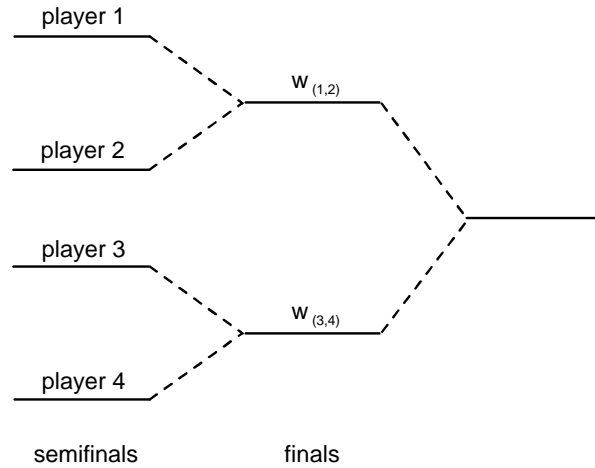


Figure 3: The two-by-two elimination tournament

given round may be awarded a winner prize (that need not be positive, and is larger or smaller than what a loser in a given period receives), but, in addition a winner wins something in comparison to dropping out of the contest which is different from a prize in a static contest: he wins the right to take part in a further contest. A player who is admitted may calculate the expected payoff in case of being admitted, and this expected payoff constitutes a major part of the incentive that makes winning the previous round desirable. However, as the contest in the next round will also require effort and has an uncertain outcome, the monetary benefit of being admitted to a further round is typically much smaller than the prize that is to be awarded to the winner of this next round.

The most prominent result in Rosen (1986) addresses the difference between the incentives that apply in the final round of the contest and the incentives in previous rounds. He studies this problem for a general contest structure as in Figure 3, asking what prize structure is required to induce the same effort choice by all players in all rounds of the contest in the case of full information and perfect symmetry between all players. He finds that, in order to incentivize finalists to make the same effort as players in the semi-finals or earlier, the prize in the final must exceed the period prize of winners in an earlier round. Intuitively, contestants in earlier rounds compete for the period prize, plus for the right to continue in the competition, which adds something to their valuation of winning in a given period. Accordingly, the prize awarded in a given period other than the final exceeds the actual prize that is paid in the respective period. In contrast, in the final all that is at stake is the difference between the winner prize and the loser prize that is paid out at the end of the final. To make the stakes in the final as big as the stakes in the semi-final or earlier on, the prize paid in the final should exceed the prize in earlier rounds.

To illustrate this result for the most simple case more formally, consider a game with four players, 1,2,3 and 4. In a first stage player 1 plays against player 2 and player 3 plays against player 4. Each player chooses a non-negative effort, with the vector of efforts denoted as (x_1, x_2, x_3, x_4) . A contest success function determines the probability by which player i wins against the player j he is teamed up with. We denote this probability by p_i and assume that $p_i(x_i, x_j) = x_i/(x_i + x_j)$ for $x_i + x_j > 0$, and $p_i(x_i, x_j) = 1/2$ for $x_i = x_j = 0$ (as in the lottery contest). These "semifinals" in stage 1 determine a winner from each team, say $w_{(1,2)}$ and $w_{(3,4)}$. The two winners are then admitted to the "final", which is again a contest which follows the same rules, with the non-negative effort choices of the participants denoted as $y_{w_{(1,2)}}$ and $y_{w_{(3,4)}}$, with the same Tullock lottery contest success function determining the winner of the final.

A winner in the semifinal receives a monetary prize equal to v_2 and the loser in the semifinal receives a loser prize that is equal to zero. Further, a winner in the final receives a final winner prize equal to v_1 , whereas the loser in the final receives a monetary reward of zero in this stage. Assuming that the players' efforts are equal to their monetary cost of making this effort, if players are risk neutral, then their payoffs are equal to $-x_i$ for a player who loses in the semi-final, $v_2 - x_i - y_i$ for a player who wins in the semi-final but loses in the final, and $v_2 + v_1 - x_i - y_i$ for a player who successively wins the semi-final and the final. Solving this game for the efforts in the unique subgame perfect equilibrium yields efforts $y_{(1,2)} = y_{(3,4)} = y^* = \frac{1}{4}v_1$ in the final. Therefore, the expected payoff of a player from participating in the final is equal to $\frac{1}{2}v_1 - \frac{1}{4}v_1 = \frac{1}{4}v_1$. Going back one stage from the final to the semi-final, the total value which is attributed to a player for winning a semi-final is equal to $v_2 + \frac{1}{4}v_1$. This is the total prize which is at stake for each semi-finalist. The equilibrium efforts for each player in the semi-final in the subgame perfect equilibrium are $x^* = \frac{1}{4}(v_2 + \frac{1}{4}v_1)$.

Rosen's (1986) primary question for this example could be rephrased: what is the relationship between the payout v_2 for winners in the semi-final and the payout v_1 for the winner in the final that yields the same effort choices in both rounds, i.e., leads to $x^* = y^*$? The answer is $v_2 = \frac{3}{4}v_1$. In order to induce the same effort both in the semi-final and in the final, the prize in the final needs only be 3/4 of the size of the payout in the semi-final.

The importance of this result is not the specific payout structure that is required to sustain constant effort across the different rounds in an elimination tournament, as the answer to this specific question may be of limited practical relevance. The important aspect is that the expected payoff from admittance to the next round in the elimination contest typically has non-negative value and is therefore part of the prize of winning an intermediate round in such a contest. Note that, assuming symmetry among the players, a player who enters into the final wins the prize awarded to the finalist with a probability of one half in the equilibrium. Nevertheless, the valuation of being admitted to the final is typically much less than $v_1/2$. If the final follows the rules of a Tullock lottery contest with two players, the prize of admittance is equal to 1/4 of the

winner prize in the final. The valuation of being admitted to the final is smaller than the expected prize money received due to the fact that finalists expend effort trying to win, and the cost of this effort diminishes the value of being admitted to the final. The fact that the final prize is contested for and players will generally expend effort trying to win it in the final makes participation much less desirable. This is, again, the discouragement effect.

Rosen (1986) studied a number of issues that emerge in elimination contests. These include aspects of asymmetry between players and aspects of incomplete information: if players can differ in their cost of expending effort or in their valuations of winning prizes, then this provides them with different incentives to expend contest effort, leading to asymmetric win probabilities. Accordingly, observed differences in players' effort choices, or the outcomes of parallel stage contests in elimination contests are potentially informative about future adversaries. In turn, this raises issues of signaling.

Many other dimensions have been explored. Gradstein and Konrad (1999) discuss that the number of rounds as well as the size of the subbattles in which players are grouped in the rounds prior to the final and in the final are essentially a matter of choice from the perspective of contest designers.⁶ They focus on contest designers who aim at maximizing the total expected amount of effort expended by all contestants summed up across all rounds. They find that a single-stage Tullock contest between all n contestants or an elimination contest with many elimination rounds and pairwise elimination contests at each stage can be optimal, depending on the nature of the contest success function, and Fu and Lu (2009) develop this question further, allowing for a larger set of designs of the elimination contest.

A variant of elimination contests is a multi-round game, in which groups of players compete against each other in one or several early rounds in what will be called the "inter-group contest", followed by one or several contests among the players that constituted the winning group or groups in the early rounds, in what will be called the "intra-group contest".⁷ Examples for this type of elimination contests can be found in many areas. In the context of political competition, for instance, it is common that groups of politicians form a team and contest against other teams or parties. Once one party is victorious in this competition, the members of the victorious group may start fighting about how to allocate the governance rent among themselves. The two triumvirates in Ancient Rome are perhaps the most prominent examples. A similar phenomenon can be observed in the context of military conflict. Several countries may form an alliance against an enemy (which may be a single country or another alliance); but once the enemy is defeated, the members of the alliance may start fighting among each other.

⁶ Amegashie (1999) considers a very similar question for a less general structure.

⁷ This structure has been formally analysed by Katz and Tokatlidu (1996), using the Tullock contest success function as the allocation rule both in the inter-group contest and in the intra-group contest. Wärneryd (1998) applied this structure to the context of rent-seeking in a federation, and Müller and Wärneryd (2001) explored the role of a related structure in the context of corporate governance.

This type of elimination contest has interesting strategic aspects. The members of a group anticipate that, should their group win a prize, the group members will expend effort trying to increase their own share in the prize. Due to this intra-group effort the group as a whole values winning the prize by the value of the prize minus the sum of intra-group efforts. This is the prize-reduction effect of intra-group contest within the victorious group. When considering their individual contributions to the group's fighting effort in the contest against another group, group members anticipate that, what the group fights for is only the winner prize net of intra-group effort. They anticipate the price reduction effect. Moreover, contributions to group effort increase the whole group's probability of winning the contest. They are contributions to a group-specific public good. Accordingly, group members may be inclined to free-ride on other members' effort.

This free-rider effect jointly with the prize-reduction effect tend to reduce the effort that is expended in the inter-group effort. The sum of inter-group efforts and intra-group efforts may then fall short of the total effort that would be expended by the set of all players in a situation in which all players compete with each other in a single grand contest among all individuals. This reduction in total contest effort cannot be taken for granted, however. Konrad (2004) considers an inter-group contest followed by an intra-group contest, showing that the intra-group composition is essential for whether the dynamic, nested contest or the grand simultaneous contest among all individuals induces higher total effort. He shows that groups which consist of group members who are heterogeneous as regards their fighting ability (the difference between the strongest and the second strongest fighter in the group matters in particular) tend to perform better in the inter-group contest than homogenous groups, even if the members of the homogenous group are strong. Intuitively, homogenous groups dissipate a large share of the prize in the intra-group contest, should this group win the inter-group contest. Heterogenous groups dissipate a smaller share of the prize, should the group win the inter-group contest. Accordingly, winning the prize is more valuable for heterogenous groups, which tends to make them expend more effort, which gives some members in the heterogenous groups stronger fighting incentives. For suitable contest success functions the total effort expended can be higher in the dynamic nested contest, and the prize may end up in the hands of individuals who, compared to the set of all players, need not be strong fighters, or need not value winning the prize particularly highly.

The discussion of the nested dynamic contest with an inter-group contest followed by an intra-group contest suffers from a free-rider problem in the inter-group contest, and from the disincentives for group effort coming from the prize-reduction effect. Would players then have an interest in forming such groups (alliances), or would they be better-off fighting on their own? Esteban and Sákovics (2003) analyzed whether a voluntary formation of such alliances can be favorable for the members of the alliance, compared to a decentralized contest in which everyone fights on his own. They find that alliance formation reduces the equilibrium payoffs of the players who form the alliance, due to the free-rider problem and the prize-reduction effect of possible intra-alliance fighting.

As alliance formation occurs voluntarily and is a frequent phenomenon, this establishes a puzzle. A number of effects have been discovered that may solve the puzzle. These include increasing returns (Skaperdas 1998), the technological benefits of possible resource transfers between members of an alliance (Kovenock and Roberson 2008) and budget constraints (Konrad and Kovenock 2009b).

2.3 Repeated incumbency fights

Iterating incumbency fights constitute a further class of dynamic contests. Games of this type typically start with a player who is the "incumbent". The incumbent receives a flow of income from incumbency. From time to time the incumbent is challenged by a rival player. If this happens, the incumbent and the rival compete with each other in a contest. Both expend amounts of effort. One of them wins, the other loses. The winner henceforth becomes the incumbent, the loser typically gets a lower payoff. The new incumbent benefits from incumbency some time, until a new challenger emerges. This challenger typically comes from a larger pool of players. Former incumbents who lost a competition may, but need not belong to this pool.

To give a few examples for iterated incumbency fights, consider sports, politics and business.

- Kings or dictators extract leadership rents from their empire while being in their leadership role. Emperors, kings or less dignified dictators may face competition from outside by other rulers, or from inside, often even from members of their own family or their supporters. If a ruler loses power, he or she may often be killed. Typically the new incumbent tries to take measures that prevent the former incumbent from re-entering into a competition for power. However, after some time a new challenger is likely to show up.
- Political parties allocate valuable offices to the inner circle while being in power. However, after some time political parties face general elections. If a party loses, it typically loses allocation rights for government office; hence, the party leaders lose some of their rents. Parties typically stay active, however, and a party that lost power is likely to be the main rival of the new incumbent in the next general elections.
- Company leaders receive nice compensation packages while being in charge. As incumbents they face several threats from rivals. They may be ousted by rivals from inside, or their firm may face a take-over threat. Company leaders may survive several of these struggles, but it is likely that they will lose power eventually. The consequences are typically less drastic for company leaders than for defeated dictators. It also holds in this context that former incumbents typically do not return as challengers, but new rivals emerge and challenge the new incumbent.
- Champions in sports typically earn money from promotion contracts and TV commercials for business companies. They must also defend their title

in a fight or a tournament from time to time. If they lose, they typically never come back and a new incumbent earns the incumbency rent for some time.

The examples reveal the general structure of an iterated incumbency fight. Iterated incumbency fights can differ along a number of dimensions that can also be extracted from the examples. First, they differ in the nature of the fight for incumbency. Incumbent and rival may have similar or asymmetric means of fighting, cost of fighting, resource endowments etc. The cost of effort and the contest technology may, more generally, be time invariant. These technological aspects may alternatively follow a deterministic or a stochastic pattern over time. A pioneering analysis of iterated incumbency fights is by Stephan und Ursprung (1998), but much is left to be done in this area. Stephan and Ursprung (1998) consider a framework with an infinite sequence of periods. A simplified version of their analysis shows the importance of repetition. For this purpose, suppose that the following stage game is played in each of an infinite sequence of periods. There is an incumbent player and a rival. The incumbent and the rival choose non-negative contest efforts x_t and y_t , respectively, which cause a cost which is equal to the effort and cannot be recovered, regardless of whether a player wins or loses the contest. A contest success function maps these efforts into a probability $p(x_t, y_t)$ by which the incumbent wins, and there is a corresponding win probability $(1 - p(x_t, y_t))$ for the rival. The player who wins receives a period prize that is normalized to 1 here. The loser in the period contest disappears from the picture with a default payoff of zero in all future periods. The winner becomes the incumbent in period $t + 1$, and the rival of the incumbent is randomly drawn from an infinitely large set of possible rivals, all being identical as regards their payoffs and contest technologies.

Solving for Markov perfect equilibrium in stationary strategies, we denote u^* the equilibrium value of being the incumbent at the beginning of a period. This gives the objective function of the incumbent as

$$u_t(x_t, y_t) = p(x_t, y_t)(1 + \delta u^*) - x_t. \quad (3)$$

The incumbent receives $(1 + \delta u^*)$ if he wins the period fight, which happens with probability $p(x_t, y_t)$. The incumbent player receives zero (for all future periods) if he loses the incumbency fight in period t , which happens with a probability of $(1 - p(x_t, y_t))$. Here $\delta \in (0, 1)$ is the discount factor that makes the accumulated present value of the future benefits of being an incumbent at the beginning of period $t + 1$ comparable with the period payoff of 1 in period t . Also, the incumbent has a non-recoverable cost of effort equal to x_t .

The objective function of the rival in period t is

$$w_t(x_t, y_t) = (1 - p(x_t, y_t))(1 + \delta u^*) - y_t. \quad (4)$$

This objective function has a very similar interpretation. The rival wins with the complementary probability $(1 - p(x_t, y_t))$, and if he wins he receives the winner prize in period t , and becomes the incumbent from period $t + 1$ on.

Accordingly, he has the same discounted present value from being in this role from period $t + 1$ on. The discount factor used by the rival is assumed to be same as for the incumbent. Also, if the rival loses, he receives the default payoff of zero from there on.

Suppose for the sake of illustration that $p(x_t, y_t)$ is described by Tullock's simple symmetric lottery contest, with $p(x_t, y_t) = x_t/(x_t + y_t)$ if at least one of the players' efforts is strictly positive, and $p(0, 0) = 1/2$. First-order conditions for players' effort choices then yield

$$\frac{y_t}{(x_t + y_t)^2}(1 + \delta u^*) = \frac{x_t}{(x_t + y_t)^2}(1 + \delta u^*) = 1.$$

This yields the symmetric solution $x_t^* = y_t^* = (1 + \delta u^*)/4$. As the win probability in the equilibrium is equal to $1/2$ for both players, we can calculate the equilibrium payoffs using the stationarity of $u_t(x_t^*, y_t^*) = u^*$ and find $u^* = \frac{1}{4-\delta}$. The equilibrium efforts are also equal to $\frac{1}{4-\delta}$. If the future is important (δ close to 1), then it is more valuable to be an incumbent, but not by much. Most of the future incumbency rent is dissipated in the future incumbency fights. If the future is completely irrelevant ($\delta = 0$), then the problem degenerates to the problem in the static Tullock contest, with $u^* = 1/4 = x^* = y^*$.

The aspect of iterated incumbency fights that makes them different from static contests is the fact that all or a subset of the players interacting today may also fight in the future. Apart from aspects of information, information revelation or signaling, even in a context of complete information, the future of fighting and its rules strongly affect incumbent's and rivals' fighting effort today. Even if the incumbency fight between an incumbent and a rival in a given period is perfectly symmetric as regards their cost of expending contest effort, possible budget constraints, or the contest success function, what an incumbent wins from defending his leadership may differ from what a rival wins from replacing the old incumbent leader: due to the implications of winning or losing, the induced prize that is at stake in the given period may differ for an incumbent and for a rival, and the future fighting may compound even small asymmetries in the static framework.

One possible illustration of these aspects is the analysis in Konrad (2009b). He considers the owner of an asset in a world with incomplete property rights. In each period the asset owner is approached by a bandit who tries to appropriate the asset for himself. They fight, and if the bandit wins the bandit has a given benefit from using the asset, whereas the asset owner is left with nothing from there on. The key aspect of this analysis is to compare two regimes that differ in the nature of the bandit. In one regime the asset owner (as long as he has not lost the asset in a previous period) is approached by a different bandit each period. In the other regime it is always the same bandit who repeatedly tries to appropriate the asset until the bandit is eventually successful. It turns out that the option to come back and try again makes rivals less aggressive in each single incumbency fight: a bandit who loses in this period but has another opportunity to appropriate in the next period is better-off than a bandit who missed his only chance to appropriate the good. The formal comparison is less straightforward,

because the changed win probability for the incumbent asset owner and the changes in his equilibrium fighting efforts will generally affect also the present value of his incumbency, which, in turn, changes the bandit's ambitions. Konrad (2009b) considers a borderline case of contest success function in which the player who expends more effort wins with probability one (sometimes called the all-pay auction without noise). However, the quantitative results do not depend on this choice.

A different type of repeated incumbency contests is *Cold War*, variants of which has been studied, e.g., by Polborn (2006) and Bester and Konrad (2005) and Konrad and Skaperdas (2007). The nature of this game is as follows. There is an incumbent and a set of rivals. The incumbent receives some incumbency rent in each of a series of periods. The rival (or a rival from a group of rivals, or the group of rivals) may challenge the incumbent in a contest. If the attack or the coup is successful, the incumbent is replaced; the player who replaces the former leader may be the rival from the group of former supporters who attacked the former leader, or a member of this group. The rules that govern the determination of successorship are important and constitute one of the differences in these analyses. In all the analyses of this structure the interaction between the leader/incumbent and his possible challenger(s) may be peaceful for a long time. The option to attack in the future makes attacking today less attractive. Attackers may wait for the most advantageous period of attack. Accordingly, the incumbent may enjoy an incumbency rent for quite some time, until a period emerges in which the incumbent is weak, making a successful attack sufficiently inexpensive (as in Bester and Konrad 2005) or an attack may never occur (as in Konrad and Skaperdas 2007). As explained in Konrad and Skaperdas (2007), the incumbent may consider paying the rivals for delaying their attack, and the interaction inside the group of rivals may also play a considerable role. In particular, if the rivals risk losing a flow of transfers from the incumbent in case of a challenge, and if the group of rivals enter into heavy fighting for who among them replaces the former incumbent, the payment to supporters that is needed to prevent rivals from an attack can be very small.

Note that, on a more abstract level, these incumbency games have in common with other dynamic contests that the option of future contest, or the threat of an emerging contest governs current action. In the case of cold war, the future consequences of a current attack may lead to delay or even to the absence of violent conflict.

3 General conclusions

In many practical situations the outcome of a contest does not simply allocate a prize that is given, but brings players into a situation which is again characterized by a contest. Winning or losing a battle affects the role of a player in this future situation. What drives a player's contest effort in a given period, hence, is the difference in the continuation values from winning or losing a given component contest.

What are the general conclusions from the specific structures we considered? First, future contest invokes a discouragement effect: given that a large share of a possible prize may be dissipated away in future contests, this makes competition much less desirable. Tullock (1980) raised the puzzle why, empirically, rent-seeking effort in lobbying contests is so small compared to the large stakes. The dynamics of contests yield a natural explanation: if the owner of a valuable asset expends effort today and successfully defends it against other players who would like to appropriate the asset for themselves, the owner finds himself in a situation in which he has to protect the same asset again. The actual prize of winning is, hence, more similar to the period flow of benefits from the asset than the uncontested asset value itself. The discouragement effect may cause contests to be much less resource wasteful if incumbent owners have to defend their property again and again. In elimination tournaments with a single final prize but no intermediate prizes, the discouragement effect can explain why contestants would expend little effort in the early rounds of an elimination contest. In the race and the tug-of-war the discouragement effect may also have counterintuitive indirect effects. It may make contests particularly fierce in the early states of symmetry, with competition slacking off when the dynamics of the contest reach states at which one player gains a considerable advantage. For large negative loser prizes the anticipation of fierce competition when entering into a final showdown of a tug-of-war contestants may avoid entering into such a showdown phase. This may explain why rivals who would gain from a final defeat of their adversary may stay peaceful for a long time, or even forever. Peaceful equilibrium can also emerge in the case of cold war for similar reasons.

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