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The Impact of Intermediary Remuneration in Differentiated Insurance Markets

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Abstract
This article deals with the impact of intermediaries on insurance market transparency and performance. In a market exhibiting product differentiation and coexistence of perfectly and imperfectly informed consumers, competition among insurers leads to non-existence of a pure-strategy market equilibrium. Consumers may become informed about product suitability by consulting an intermediary. We explicitly model two intermediary remuneration systems: commissions and fees. We find that social welfare under fees is first-best efficient but fees lead to lower expected profits of insurers and non-existence of a pure-strategy market equilibrium. Commissions, in contrast, cause 'overinformation' of consumers relative to minimal social cost, but yield a full-information equilibrium in pure strategies associated with higher expected profits of insurers. This might explain why intermediaries are generally compensated by insurers.

Keywords: product differentiation, intermediation, insurance oligopoly

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1 Introduction

Despite many potential sources of information about insurance products available today – like the internet, financial magazines, informative advertising or simply word-of-mouth information – a fraction of consumers looking for insurance coverage is often not well-informed about the insurance product or provider which best suits their individual preferences. A reason for this phenomenon might be that information is sometimes not easy or too costly to obtain or simply that some consumers do not trust in information publicly available. This paper considers a duopolistic insurance market with product differentiation where some consumers are uninformed about product ‘fit’.

Common economic models of insurance markets often consider either a monopolistic insurer or a perfectly competitive insurance market. These contrarian analyses are due to two implicit assumptions. The first assumption is that insurance is a homogeneous good. The second is that any insurer can potentially serve the whole market demand. Given these assumptions, if an insurer offers a marginally smaller price than its competitors, it gains (and serves) the whole market demand. Therefore, if at least two insurers compete in premiums, Bertrand competition yields zero profits and the premium to the insureds is actuarially fair. This is the so called Bertrand Paradox.$^1$$^2$

The Bertrand Paradox seems to be one reason to explain that while monopolistic and competitive insurance markets are subject to vast research in insurance economics, very few theoretical work examines the intermediate case of oligopolistic insurers sustaining positive profits in equilibrium. Polborn [1998] considers an insurance oligopoly with two risk-averse insurers engaged in Bertrand competition. The insurers face a trade-off between profit and risk, so that in equilibrium premiums tend to exceed marginal cost. Schlesinger and Schulenburg [1991] study oligopolistic competition of insurers in a product differentiation framework exhibiting search and switching costs. They show that introducing search and switching costs of insureds provides some market power to incumbent insurers and reduces market shares to new entrants.

1 The Bertrand Paradox is named after Bertrand [1883] who argued that if duopolistic firms compete via price, the noncooperative equilibrium price would fall to marginal cost since the firms would keep undercutting each other. See Bertrand [1883].

2 Sonnenholzner and Wambach [2003] argue that Bertrand competition seems indeed to be a plausible mechanism in insurance markets since competition takes place via premiums due to the fact that insurers cannot produce coverage in advance.
In contrast to the small theoretical literature on insurance oligopolies, empirical evidence suggests that insurance markets are rather oligopolistic than perfectly competitive. It is widely known that insurance providers tend to make positive profits. It seems thus interesting to develop a theoretical model of an oligopolistic insurance market where insurers actually make profits in equilibrium. We contribute to fill this gap and suggest a solution to the Bertrand Paradox.

Most insurance models interpret an 'insurance contract' as a pair of only two parameters, the insurance premium paid by the insured regardless of state and the indemnification payment paid by the insurer in case of loss. As pointed out by Schlesinger and Schulenburg [1991], in reality such an insurance contract is rather a long description of contingencies in which the contract pays out and in which it does not. One should make a reasonable distinction between the "insurance contract" and the "insurance product". The insurance product represents a service that may differ from other services. The actual "fit" of the product to the needs of a consumer therefore also differs. The service may include claims settlement and risk management services, the availability of local agents or method-of-payment options. Warranties and embedded options in life insurance are also a means by which insurance products may be differentiated. We may consider life insurance policies with the same premium but different maturities or embedded options. Product characteristics differ between insurers, even though insurance contracts seem to be identical. For Germany, for instance, empirical evidence of perceived product heterogeneity in insurance markets is found by Schlesinger and Schulenburg [1993].

Taking these arguments into account, insurance is a rather sophisticated and multidimensional product about which characteristics consumers are often poorly informed. Although consumers might be well-informed about existence and prices of insurance products (and maybe even some attributes differentiating them), information about which product is actually best suited for them is not easy to obtain. Consumers may observe prices and product characteristics of

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3 See, for instance, Nissan and Caveny [2001] and Murat et al. [2002]. Murat et al. [2002] also find that their evidence is consistent with a trade-off between competition and industry stability. See Murat et al. [2002], p. 477. Interestingly, our findings suggest that more intense competition (due to existence of imperfectly informed consumers in the market) makes the market less stable.

4 According to the Insurance Information Institute, for instance, property-casualty insurers in the US earned net income after taxes of about 44 billion dollars in 2005 and 64 billion dollars in 2006.

existing insurance products but they may not easily determine their best matching product due to lack of information. Our framework highlights this lack of information and aims at explaining the existence of intermediaries in insurance markets. An intermediary plays a matchmaking role of 'market maker' in recommending the appropriate insurance product, i.e. in matching buyers with appropriate insurance providers. Thus 'uninformed' consumers can be informed about their best matching insurance product by consulting a broker. Therefore, in our model the role of brokers is to provide information about optimal product match. This information, however, comes at a cost. We analyze two remuneration systems: commissions and fees. We show that fees lead to higher social welfare than commissions but imply lower expected profits of insurance providers and non-existence of a pure-strategy market equilibrium. Commissions, in contrast, cause 'overinformation' of consumers relative to minimal social cost, but yield a full-information market equilibrium in pure strategies.

The paper is organized as follows. The next section presents a short literature review. Section 3 introduces our model in a Hotelling framework. As a point of reference, we first study a differentiated insurance market without intermediation in section 4 and show that no equilibrium exists in pure strategies. Section 5 introduces intermediation. We show that intermediation may lead to a pure strategy full-information equilibrium under commissions, but not under fees. We also analyze social welfare under both remuneration systems. Conclusions are summarized in the final section.

2 Related Literature

Previous research on efficiency of intermediation addresses several functions intermediaries might generally fulfill in markets exhibiting imperfect information. For instance, Biglaiser [1993] presents a basic bargaining framework where middlemen might reduce inefficiencies that arise from adverse selection problems. Brokers as search agents whose function is to match trading partners are studied by Rubinstein and Wolinsky [1987], Posey and Yavas [1995], Cosimano [1996], Posey and Tennyson [1998] as well as Seog [1999]. Rubinstein and Wolinsky [1987] present a general framework where time-consuming negotiations between buyers, sellers and middlemen take place. Cosimano [1996] introduces a monopolist intermediary who lowers the probability of an unsuccessful trade among

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6 See Cummins and Doherty [2006], p. 393.
buyers and sellers. More specifically, Posey and Yavas [1995] and Posey and Tennyson [1998] refer to insurance markets. They offer search models where insurers and consumers may engage in costly search for matching partners. Both insurers and consumers can be of either high or low search cost type. Insurers may transfer the search task to brokers. Brokers then fulfill a search function by looking for potential customers at a commission fee to be paid by the insurer. Seog [1999] examines an insurance market where consumers are poorly informed about the price distribution and focuses on dynamic aspects of price search by intermediaries in order to find a long-run equilibrium where dependent and independent brokers might coexist. Cummins and Doherty [2006] argue that in an insurance market where insurers cannot observe loss probabilities but do know overall average loss probability, brokers might prevent a market failure due to adverse selection by informing insurers about loss probabilities of their customers.

In insurance markets, the brokers’ most important function can be seen in a matchmaking function. Gravelle [1994] offers a model for a competitive insurance market with intermediation. He assumes that the brokers’ function lies in determining the best-matching insurance product for consumers. Insurers offer only one type of insurance product and engage in Bertrand competition via brokers. Consumers are heterogeneous in their preferences for the insurance product and uninformed about whether the product actually offers a good match with their individual preferences. They know the overall distribution of mismatch in society and the expected mismatch but are unable to determine their individual degree of mismatch from buying the insurance product. A broker can inform a consumer about his individual degree of mismatch. In competitive equilibrium marginal cost pricing and thus zero profits of insurers result. This is true under both the commission system and the fee-for-advice system. Hence, both systems appear equivalent from a profitability viewpoint of insurers. Gravelle also finds that neither fees nor commissions might achieve an even second-best efficient equilibrium solution in his framework. Unlike Gravelle, we look at an oligopolistic insurance market with differentiated products

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7 Although there may be other functions of intermediaries in insurance markets, such as providing risk management consulting, loss mitigation or assistance with claims settlement, the matchmaking role actually seems to be the most important.

where insurance providers may interact strategically.

Focht et al. [2006] build upon a product differentiation framework by Schultz [2004]. There are some uninformed consumers who can neither observe prices nor product varieties in the market. The authors show that a pure-strategy equilibrium might exist given that all consumers have rational and identical expectations about prices and product characteristics. Without intermediation, equilibrium profits of insurance providers are higher compared to profits given intermediation. Furthermore, equilibrium profits under commissions and fees are equivalent. Hence, from the insurers’ viewpoint, intermediation is associated with lower profits (but higher social welfare). We differ from this view. In our framework prices are observable by all consumers in the insurance market. The rationale underlying this assumption is that unobservability of prices seems not very common, since price information is actually quite easy to obtain (for instance via the internet) while information about individual product match is less easily determined. We will see that this different and perhaps more realistic information structure leads to completely opposite results.

3 The Model

Our model follows the well-known product differentiation approach by Hotelling [1929] who formulated the following model of location and price choice in duopoly: Consumers are uniformly distributed on the unit line. A single good is produced at zero cost by two firms, each of which selects a location in the unit line and a price. Consumers have travel cost proportional to the distance to firms, and buy one unit of the good from the firm for which price plus travel cost is lowest. The model is a two-stage game between the two firms. In the first stage each firm (simultaneously) selects a location on the unit line, in the second stage, having observed the locations selected, each firm (simultaneously) offers a price. The Hotelling model has been discussed in different contexts by many authors. Our setting follows d’Aspremont et al. [1979].

Consider an insurance market with a continuum of consumers. The number of consumers in the market is normalized to one. Consumers are uniformly distributed on a line of unit length. There are two insurers in the market,

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9 The Hotelling model is a model of horizontal product differentiation. For a detailed discussion of horizontal product differentiation models, see Martin [2002], pp. 84-105.

10 Our setting follows d’Aspremont et al. [1979].
i ∈ \{1, 2\}, offering each some variant of the insurance product. Insurers are located at the extreme points of the unit line: Provider 1 is located at 0 and provider 2 at 1. The position of a consumer on the Hotelling line represents his individual preferences for the insurance product offered. A consumer located at \(x \in (0, 1)\) cannot have a perfectly matching product, so there is some disutility (called transport cost) involved in purchasing this product.

We make the following basic assumptions concerning the distribution of information in the insurance market: All consumers can observe prices of all existing insurance products, but some consumers cannot observe product varieties in order to determine which insurance product actually offers the best match for their type. We will refer to these consumers as 'uninformed' because, technically, they do not know their position on the Hotelling line. In particular, we assume that only a fraction \(\delta \in (0, 1)\) of consumers is able to determine the best matching insurance product to their type. The uninformed consumers, however, following the 'principle of insufficient reason', have rational expectations of being in the middle of the market and thus tend to purchase insurance at the cheapest provider. Both types of consumers are uniformly distributed on locations. This is common knowledge.

We assume that all consumers are risk-averse. Consumers differ in expected loss and in their preferences for insurance products. Thus, individual suitability

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11 Our results generalize to the case of \(N\) firms in an insurance market. Maintaining the symmetric variety pattern of the Hotelling duopoly market, the \(N\) firm case can be analyzed in a circular street market as in Salop [1979] where product varieties are symmetric. See Polo [1991], p. 712.

12 This simplification of 'maximal differentiation' represents no restriction to our analysis as is shown in the appendix. The assumption only serves to simplify our exposition.

13 Alternatively, we might assume that all consumers observe prices and varieties, but some consumers are unable to process this information in order to find their best matching insurance product.

14 Since those consumers do have information about insurance premiums, they are in fact "partially informed" consumers. However, for simplicity we refer to them as "uninformed" since they differ from the other group in that they do not know their position on the Hotelling line and thus cannot determine optimal product match.

15 The 'principle of insufficient reason' was first expressed by Jacob Bernoulli. It states that if an agent is ignorant of the ways an event might occur (and therefore has no reason to believe that one way will occur rather than another), the event will occur equally likely. Keynes referred to the principle as the 'principle of indifference', formulating it as "if there is no known reason for predicating of our subject one rather than another of several alternatives, then relatively to such knowledge the assertions of each of these alternatives have an equal probability." See Keynes [1921], pp. 52-53.
of insurance products to consumers also differs. Insurers offer premiums $P_i = EX + p_i$, where $EX$ is expected loss of the policy and $p_i$ represents the loading. Since premiums are marginally fair, i.e. include a fixed loading fee, full insurance is optimal for consumers as long as the cost of insurance net of expected loss (i.e. the sum of premium loading and individual disutility of mismatch) does not exceed their individual risk premium $r$. The amount paid by policyholders above actuarial cost is the price of insurance. Hence, we refer to $p_i$ as the price for an insurance product of provider $i$. We assume there is no moral hazard problem, i.e. expected loss is verifiable and contractible by the insurer.

We might introduce a zero stage into the game where insurers first simultaneously compete in locations (i.e. product characteristics) on the Hotelling-line before competing in prices. We do not explicitly consider this stage. Locations of insurers are not very interesting from our point of view since we are mainly interested in gaining insight into the performance of broker remuneration systems. Note, however, that for $\delta = 1$, it is easy to show that insurance providers choose their locations 0 and 1, respectively.

4 Insurance Market without Intermediation

Given some positive transport cost $t > 0$, a consumer located at $x \in [0, 1]$ faces disutility $tx^2 (t(1 - x)^2)$ from purchasing an insurance product from insurer 1 (2). Consumers are sufficiently risk-averse so that the market is completely

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16 Expected loss plus risk premium may be interpreted as the individual willingness to pay for full risk reduction of a consumer. See the seminal articles by Pratt [1964], Arrow [1963], Mossin [1968], Smith 1968] or Doherty [1975].

17 D’Aspremont et al. [1979] have shown that if all consumers are informed about product characteristics and firms are restricted to locate in $[0, 1]$, firms will locate in 0 and 1, respectively. In the same way, we may derive optimal product characteristics for insurers in our framework. Since our focus is on an equilibrium in prices (second stage of the original Hotelling game), we suppose locations 0 and 1 in the following. This is without any loss of generality as we show in the appendix.

18 The heterogeneity parameter $t$ represents the marginal disutility of mismatch and thus measures the intensity of product differentiation. This disutility is often referred to as "transportation cost". In our setting, however, we refer to it as disutility to emphasize the utility loss from purchasing a product which does not perfectly match a consumer’s type.

19 We do not treat transportation cost as linear in distance, as we interpret it as a proxy for disutility from purchasing an insurance product other than the individually most preferred one. Thus, it seems plausible that unit disutility rises more than proportionately
covered. A consumer purchases insurance from provider 1 if his net utility is higher, i.e.\(^{20}\)

\[ r - (p_1 + tx^2) \geq r - (p_2 + t(1 - x)^2) \iff x \leq \frac{p_2 - p_1 + t}{2t}, \quad (1) \]

where \( p_1 + tx^2 \) (\( p_2 + t(1 - x)^2 \)) represent the total cost of purchase for a \( x \)-type consumer when purchasing from provider 1(2). A consumer is indifferent between purchasing from provider 1 or 2 if he is located at\(^{21}\)

\[ \hat{x} = \frac{p_2 - p_1 + t}{2t}. \quad (2) \]

We assume that only a fraction \( \delta \) of consumers is aware of the "fit" of insurance products to their type. Uninformed consumers, represented by \((1 - \delta)\), minimize expected cost of purchase by choosing the cheapest provider (since both product varieties are equally likely to be associated with each price). Hence, for uninformed consumers, we have classic Bertrand competition. The fraction \( \delta \) of informed consumers might be interpreted as a measure of market transparency: the higher \( \delta \), the more transparent is the insurance market.\(^{22}\) The market demand of insurer 1 is therefore given by

\[ D_1(p_1, p_2, \delta) = \delta\left(\frac{p_2 - p_1 + t}{2t}\right) + (1 - \delta) \begin{cases} 1 & \text{if } p_1 < p_2 \\ 1/2 & \text{if } p_1 = p_2 \\ 0 & \text{if } p_1 > p_2 \end{cases} \quad (3) \]

with distance. Given linear transportation costs, the so called "principle of maximum differentiation" (i.e. firms locate as far from each other as possible in equilibrium of a two-stage game) is replaced by the "principle of minimum differentiation". See, for instance, Martin [2002], p. 99. However, one should not expect minimum differentiation as advocated by Hotelling. This is because the price subgame in Hotelling’s model fails to have a pure strategy equilibrium if firms are located “too close” to each other (but not at the same point). Correcting the non-existence problem in Hotelling’s original model, d’Aspremont et al. [1979] verify that with quadratic transportation costs a price equilibrium exists for all possible locations.

\(^{20}\) Note that the individual risk premium \( r \) can differ among consumers. We only assume that all consumers are sufficiently risk-averse in order to ask for insurance coverage: \( \inf r > \sup \left( p_1 + tx^2, p_2 + t(1 - x)^2 \right) \). See also footnote 25.

\(^{21}\) Throughout the analysis, we assume that \( \hat{x} \) is positive, i.e. \( t \) is sufficiently high to ensure non-negative demand.

\(^{22}\) See Schultz [2004], p. 175.
and demand for insurer 2 is then
\[ D_2(p_1, p_2, \delta) = \delta(1 - \frac{p_2 - p_1 + t}{2t}) + (1 - \delta) \begin{cases} 
1 & \text{if } p_2 < p_1 \\
1/2 & \text{if } p_1 = p_2 \\
0 & \text{if } p_2 > p_1 
\end{cases} \] (4)

Given the market shares of providers 1 and 2, expected profits are

\[ \Pi_1(p_1, p_2, \delta) = p_1 \cdot D_1(p_1, p_2, \delta) \] (5)

\[ \Pi_2(p_1, p_2, \delta) = p_2 \cdot D_2(p_1, p_2, \delta). \] (6)

Intuition suggests that the price game has no (Bertrand-Nash-) equilibrium in pure strategies. This seems intuitively clear since the presence of uninformed consumers provides the insurers with an incentive to slightly undercut the rival in order to capture all uninformed consumers without losing informed ones. Undercutting seems worthwhile as long as the premium is not "too low". Then, it may be profitable for an insurer to become a 'niche player' and set a high premium supplying informed consumers only, which, again, makes undercutting worthwhile for the rival insurer, and so on. Polo [1991] shows that the game under imperfect information has indeed no pure-strategy equilibrium in prices.\(^{23}\)

To illustrate this for our framework, note first that the usual Bertrand result cannot be an equilibrium. One of the providers might increase its price by some amount \( \varepsilon > 0 \) in order to increase its profit: In particular, assume provider 1 increases its price \( p_1 \) from \( p_1 = p_2 = 0 \) to \( \tilde{p}_1 = p_2 + \varepsilon = \varepsilon \). Of course, at \( p_1 = p_2 \) profits are zero. At \( \tilde{p}_1 = \varepsilon \), however, due to (3) we obtain

\[ \check{\Pi}_1(\tilde{p}_1, p_2, \delta) = \tilde{p}_1 \cdot \delta \frac{p_2 - \tilde{p}_1 + t}{2t} = \frac{\varepsilon \delta (t - \varepsilon)}{2t} > 0 \quad \text{for } t > \varepsilon > 0. \] (7)

Thus one of the two insurers could always increase its profit by increasing its price, meaning that fair insurance \( p_1 = p_2 = 0 \) could not be an equilibrium.

It seems intuitively clear that symmetric pricing \( p_1 = p_2 >> 0 \) cannot be an equilibrium since a provider may slightly undercut its rival and gain all uninformed consumers without losing informed ones. We may illustrate equilibrium

\(^{23}\) See Polo [1991], p. 708.
behavior graphically in order to see that there is no other (asymmetric) equilibrium. We have

\[
\frac{\partial \Pi_1}{\partial p_1}_{p_1 \neq p_2} = 1_{[0, p_2]}(p_1) \cdot \left[ \frac{\delta}{2t} (p_2 - 2p_1 + t) + (1 - \delta) \right] + 1_{(p_2, \infty)}(p_1) \cdot \left[ \frac{\delta}{2t} (p_2 - 2p_1 + t) \right]
\]

(8)

and

\[
\frac{\partial \Pi_2}{\partial p_2}_{p_1 \neq p_2} = 1_{[0, p_1]}(p_2) \cdot \left[ \frac{\delta}{2t} (p_1 - 2p_2 + t) + (1 - \delta) \right] + 1_{(p_1, \infty)}(p_2) \cdot \left[ \frac{\delta}{2t} (p_1 - 2p_2 + t) \right]
\]

(9)

The price reaction functions of the two insurers are then given by

\[
p_1^R(p_2)_{p_1 \neq p_2} = 1_{[0, p_2]}(p_1) \cdot \left[ \frac{1}{2} p_2 + t \left( \frac{1}{\delta} - \frac{1}{2} \right) \right] + 1_{(p_2, \infty)}(p_1) \cdot \left[ \frac{1}{2} p_2 + \frac{t}{2} \right]
\]

(10)

and

\[
p_2^R(p_1)_{p_1 \neq p_2} = 1_{[0, p_1]}(p_2) \cdot \left[ \frac{1}{2} p_1 + t \left( \frac{1}{\delta} - \frac{1}{2} \right) \right] + 1_{(p_1, \infty)}(p_2) \cdot \left[ \frac{1}{2} p_1 + \frac{t}{2} \right]
\]

(11)

These reaction functions are represented graphically in Figure 1. As can easily be seen from the figure, the reaction functions don’t intersect. There is no equilibrium in pure strategies.

If some consumers are uninformed about their best product match, discontinuities in the reaction functions imply that the game will have no pure-strategy equilibrium. Thus any game structure leaving some consumers uninformed about their optimal product match cannot lead to a market equilibrium in pure

\textsuperscript{24} Here $1_{[0, p_2]}(p_1)$ etc. represent the indicator functions meaning $1_A(y)$ which for every $y \in A$ has value 1 and for $y \notin A$ has value 0.
strategies. Building upon a result by Dasgupta and Maskin [1986], it can be shown that there is a symmetric equilibrium in mixed strategies.\textsuperscript{25}

Real-world decisions of insurance providers concerning their prices seem, however, not randomly made. A stable pure-strategy equilibrium seems a more convincing concept in order to offer a plausible hypothesis on empirical market behavior of firms in oligopolistic interaction. The unsatisfactory instability of prices within a certain range (given mixed strategies) could be overcome when intermediation is added to our analysis. We suggest this solution in the next section.

\textsuperscript{25} Mixed strategy equilibria in Hotelling’s original model are examined by Dasgupta and Maskin [1986] and Osborne and Pitchik [1987]. Osborne and Pitchik [1987] show that the range of possible outcomes with mixed strategy equilibria may be large. Dasgupta and Maskin [1986] demonstrate that under full information there may be infinite equilibria in mixed strategies. Introducing imperfect information, Polo analyzes the second stage of Hotelling’s game. He considers a slightly different setting than we do. For our framework and results, it is sufficient to assume that the market is completely covered which means that we assume $t_j^{-} := 0$ and $t_j^{+} := 1$ in Polo’s framework. See Polo [1991], pp. 703, 708-711. This assumption is standard and avoids the possibility of an issue discussed by Wang and Yang [1999] which may be interpreted as follows. When risk aversion of consumers reaches a sufficiently low level, less than maximum differentiation might result in the pure-strategy equilibrium since insurers might have an incentive to move towards the middle of the market in order to capture consumers in the central area. Since our focus is on the second stage of the original game, this problem is not interesting from our view and we thus avoid it.
As we have shown in the previous section, the existence of uninformed and thus price-sensitive consumers plays an important role since those consumers provoke a failure of market equilibrium. In this situation, it seems interesting and reasonable to introduce an honest intermediary who improves the matching of uninformed consumers and insurers in the market. We assume that an intermediary incurs some variable cost $k > 0$ of performing a risk analysis and reveals the position of a consumer on the Hotelling-line perfectly. Thus a broker transforms a previously uninformed consumer into an informed consumer.

Generally, there are two remuneration systems how the broker may be paid: the broker might be paid directly by consumers (we refer to this system as ”fee” or ”fee-for-advice” system) or the broker might be paid for his service by the insurer (we refer to this system as ”commission” remuneration system). We first study the latter and then compare our results with the former system. The sequence of play we consider is depicted in Figure 2.

### 5.1 Commissions

Under the commission remuneration system, the broker is paid by insurance providers. Since information from a broker is costless for consumers at stage three of the game, all uninformed consumers prefer to become informed by the broker. The insurer has to pay the brokers’ fee $m$ for each risk analysis. Given the broker market is competitive, marginal cost pricing leads to the brokers’ fee

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26 A risk analysis is usually expensive since it requires expertise not only in finance, but also in actuarial science, law and engineering. See Cummins and Doherty [2006], p. 392.
This leads us to

**Proposition 1** Under commissions, there is a full-information market equilibrium in pure strategies. Equilibrium profits of insurance providers are \( t/2 \).

**Proof:** Profits of insurers are

\[
\Pi_1^c = (p_1 - (1 - \delta)k) \cdot \left( \frac{p_2 - p_1 + t}{2t} \right) \tag{12}
\]

and

\[
\Pi_2^c = (p_2 - (1 - \delta)k) \cdot \left( 1 - \frac{p_2 - p_1 + t}{2t} \right). \tag{13}
\]

First-order conditions for a profit maximum are

\[
\frac{\partial \Pi_1^c}{\partial p_1} = \frac{p_2 - 2p_1 + t + (1 - \delta)k}{2t} = 0 \tag{14}
\]

and

\[
\frac{\partial \Pi_2^c}{\partial p_2} = \frac{p_1 - 2p_2 + t + (1 - \delta)k}{2t} = 0. \tag{15}
\]

Hence, price reaction functions are well-behaved and given by

\[
p_1^R(p_2) = \frac{1}{2}p_2 + \frac{t}{2} + \frac{1 - \delta}{2}k \tag{16}
\]

and

\[
p_2^R(p_1) = \frac{1}{2}p_1 + \frac{t}{2} + \frac{1 - \delta}{2}k. \tag{17}
\]

The symmetric pure-strategy price equilibrium is then

\[
p_1^* = p_2^* = t + (1 - \delta)k \tag{18}
\]

\[\text{In the US, for example, there is indeed intense competition in the intermediary market, especially for small and medium-sized risks. Overall competitiveness tends to vary by market segment. See Cummins and Doherty [2006], p. 369-370. An alternative interpretation of marginal cost pricing might be that the participation constraint of the broker must hold and implies that he participates if he is indifferent between offering his service and not offering. Therefore, the providers would like to pay him } m = k. \text{ Without loss of generality, we might suppose that when the broker is indifferent between offering his service and not offering it, he still offers his service.}\]
and equilibrium profits are
\[ \Pi_1^c = \Pi_2^c = \frac{t}{2}. \] (19)

The equilibrium is illustrated in Figure 3 by the intersection point of price reaction functions. As compared to the standard Bertrand-equilibrium premiums \( p_1^B = p_2^B = 0 \), it is easy to see that equilibrium premiums are higher than fair. By informing all uninformed consumers about product suitability via insurance intermediaries, the commission system leads to a symmetric pure-strategy full-information equilibrium. Insurers make positive profits and share market demand equally. Intermediation via the commission system is desirable for both uninformed consumers and insurers because uninformed consumers are optimally matched to product varieties and insurers earn higher profits than without intermediation.\(^{28}\) Interestingly, equilibrium profits in (19) would be the same in a Hotelling-market with full information and no intermediation which suggests that costs of commissions are entirely allocated to policyholders via insurance premiums. The key point of the commission system is therefore that in equilibrium uninformed consumers are subsidized by informed ones.

\(^{28}\) This follows easily from the proof of proposition 2 in the next section.
5.2 Fees

While under the commission system, all uninformed consumers automatically ask for information from an intermediary since it is costless for them, under the fee system, consumers must pay the broker for his service before purchasing insurance from one of the two providers. It seems realistic to assume that consumers differ in their individual willingness to pay for the brokers’ information since they generally differ in expected loss (loss size and/or probability of loss) and thus being mismatched with the wrong provider matters differently. In particular, a mismatch matters more for "high risk" consumer types than for "low risk" types. To describe consumer heterogeneity, we introduce a utility parameter \( \theta \in [0, \overline{\theta}] \) with appropriate density function \( f(\theta) \) and distribution function \( F(\theta) \).

The broker market is again competitive and marginal cost pricing leads to the brokers’ fee \( m = k \). We conclude that uninformed consumers decide to ask for information and become informed if the individual value of the information from the broker exceeds the cost of providing it, i.e.\(^{30}\)

\[
\theta - k - \left( \frac{1}{4} \right)^2 t \geq - \left( \frac{1}{2} \right)^2 t \iff \frac{3}{16} t + \theta \geq k. \tag{20}
\]

Making the reasonable assumption that \( \overline{\theta} > \theta > \frac{3}{16} t \), consumers with \( \theta \geq \hat{\theta} := k - \frac{3}{16} t \) ask for product suitability analysis from the broker and consumers with \( \theta < \hat{\theta} \) do not. Since \( \hat{\theta} > 0 \) but \( \theta \in [0, \overline{\theta}] \) there always exist some consumers who do not buy from the broker. Some consumers remain uninformed about their best product match and no pure-strategy insurance market equilibrium can result under the fee system.

Hence, only a part of previously uninformed consumers \( 1 - \delta \) decide to become informed by a broker. This leads to a new proportion of informed consumers under the fee system to read

\[
\overline{\delta} = \delta + (1 - \delta)(1 - F(\hat{\theta})). \tag{21}
\]

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\(^{29}\) While transport cost \( t \) represents disutility of mismatch resulting from not having an ideal insurance product, \( \theta \) measures the impact of this mismatch. Uninformed consumers are indifferent between insurance providers. However, they value the impact of mismatch differently due to their different underlying risks. A higher risk makes a claim more likely which results in stronger manifestation of mismatch.

\(^{30}\) For an uninformed consumer, the mean distance to an insurance provider is 1/2. When he is informed, his location is between 0 and 1/2 or 1/2 and 1. Thus the mean distance for an informed consumer is 1/4.
Proposition 2 Under a fee system, some consumers remain uninformed and no pure-strategy equilibrium exists. The equilibrium is then in mixed strategies and insurers’ expected profits are lower than under commissions.

Proof: The first part has already been shown. The equilibrium under fees is in mixed strategies due to $\tilde{\delta} \in (0, 1)$. Polo [1991] shows that there is a symmetric equilibrium in mixed strategies. Equilibrium prices are defined over a finite interval which lies below the full information price and shifts down as the mass of uninformed consumers increases (i.e. $\delta$ decreases).\(^{31}\) Given the symmetry of the problem, equilibrium expected profits will be lower than under commissions.\(\square\)

5.3 Social Welfare

Let us now look at social welfare (or social cost) under both remuneration systems. In order to compare both systems, social cost may be represented by the sum of disutilities of mismatch over all consumers and the information cost of the brokers. In a welfare optimum social cost is minimized. From a social welfare viewpoint, the fee system seems superior since it induces only those consumers to buy the brokers’ information for whom it is indeed worthwhile to do so. Hence, the fee system leads to the social optimum: Social costs are minimized. Under the commission system, in contrast, costs of becoming informed are zero (at stage three of the game) so that all uninformed consumers ask for information, even those for whom the value of information is smaller than the cost of providing it. Hence, uninformed consumers are subsidized by informed consumers. The commission system misses the social optimum. We summarize this in

Proposition 3 Social welfare under fees is first-best efficient. Social welfare under commissions leads to ‘overinformation’ of consumers compared to the social optimum.

Proof: Under a commission system, all uninformed consumers ask for a risk analysis. Social cost is then\(^{32}\)

\(^{31}\) See Polo [1991], proposition 4, p. 708 and proposition 5, p. 711. Note that full information prices in Hotelling’s model are smaller than in (18).

\(^{32}\) Overall disutility of informed consumers is $-(1/4)^2 t$. Overall disutility of uninformed consumers is $-(1/2)^2 t$. 

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\[ S^c = \left( \frac{1}{4} \right)^2 t + (1 - \delta)k, \]  
(22)

where the proportion of previously uninformed consumers is given by \( 1 - \delta \). Under a fee-for-advice system, we may write social cost to read\(^{33}\)

\[ S^f = \tilde{\delta} \left( \frac{1}{4} \right)^2 t + (1 - \tilde{\delta}) \left( \frac{1}{2} \right)^2 t + (\tilde{\delta} - \delta)k. \]  
(23)

Comparing (23) to (22), simple calculation reveals

\[ S^c - S^f = (1 - \tilde{\delta})(k - \frac{3}{16}t) > 0, \]  
(24)

i.e. social cost under commissions is higher than under fees. This implies that social welfare is indeed higher under fees than under commissions. \( \square \)

Summarizing our results, we may conclude that the fee system is socially optimal but has no equilibrium in pure strategies while the commission system leads to ‘overinformation’ of consumers relative to the social optimum, but is preferable from the view of insurance providers due to the fact that it leads to a full-information market equilibrium in pure strategies and higher expected profits.

6 Concluding Remarks

The introduction of product differentiation and intermediation into insurance demand broadens the range of possible market equilibria compared to the standard Bertrand models. In an insurance market exhibiting product differentiation and coexistence of perfectly and imperfectly informed consumers, competition among insurers may lead to discontinuities in the reaction functions so that a (subgame perfect) pure-strategy equilibrium fails to exist. The introduction of intermediation, however, may result in a symmetric pure-strategy equilibrium, positive profits of insurers and full information of consumers in the insurance market. The rationale for the latter is that increasing market transparency via intermediation increases product differentiation (since previously uninformed

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\(^{33}\) Actually, it is not clear where insurance providers will locate in \([0, 1]\) since there is no pure-strategy equilibrium in the price game. For comparison, however, we assume the same locations.
consumers become informed ones), and therefore leads to less tight competition. This mechanism ensures positive profits of insurers and full information of consumers in equilibrium. While higher transparency on the consumer side is usually thought to promote competition, our framework where consumers become informed about product match via intermediation suggests that increasing consumers' information makes the market less competitive and thus leads to positive profits. Insurance premiums are thus higher than actuarially fair in competitive equilibrium.

Interestingly, our results are opposite to those obtained by Schultz [2004] and Focht et al. [2006]. While these authors find that, given prices and product characteristics are unobservable by some consumers, increasing market transparency on the consumer side implies less product differentiation, lower prices and profits, we show that – when prices are observable by consumers – increasing market transparency (for example via intermediation) increases product differentiation (since previously uninformed consumers become informed ones), and therefore leads to less intense competition associated with higher prices and profits. Uninformed consumers are price sensitive since the price is the only criterion which actually matters to them. In the models by Schultz [2004] and Focht et al. [2006], consumers act stochastically since they do not have any information at all. This suggests that basic assumptions about the information structure of the game seem to play an important role.

Our results also add to those obtained by Cummins and Doherty [2006] who argue that in a perfectly competitive insurance market where insurers cannot observe loss probabilities, brokers might prevent a market failure by informing insurers about loss probabilities of their customers. We show that (in an oligopolistic insurance market) even if loss probabilities are perfectly observable, a market failure might arise without intermediation due to lack of information about product match. The market failure might be prevented by introducing information intermediaries.

We explicitly model commissions versus fees. We find that fees minimize social cost but yield lower profits for insurers and failure of an equilibrium in pure strategies. Commissions imply that more consumers are informed than would be socially optimal. In equilibrium uninformed consumers are subsidized by informed ones. While the fee system appears to be unstable, the commission system seems preferable from the viewpoint of insurance providers since it yields both a stable full-information market equilibrium and higher expected profits. This might explain why intermediaries are generally compensated by insurance
providers in practice. The superiority of the commission system over the fee system from the insurers’ viewpoint has not been shown yet. Existing models predict equivalence of both systems and thus cannot explain the prevalence of commissions in practice.\textsuperscript{34} A further interesting result is that commissions are passed on in the premiums, a phenomenon which can be empirically confirmed as well.\textsuperscript{35}

Finally, our theoretical model abstracts from some aspects which might also be important in insurance markets with intermediation. For instance, we abstain from the brokers’ advice quality and their incentives to inform consumers perfectly. While the fee system seems to offer brokers a high incentive to inform consumers truthfully about their ‘best matching product’, the commission system might offer lower incentives.\textsuperscript{36} However, even when we assume these incentive problems away, the commission system misses the social optimum. We show that in a ‘perfect world’ where all intermediaries act non-strategically and honestly reveal the best product for their customers, the commission system – which is \textit{inferior} from a social welfare viewpoint – will be established in the industry due to its capacity of being associated with higher expected profits for insurance providers in market equilibrium.

\textsuperscript{34} See Gravelle [1994] and Focht et al. [2006].
\textsuperscript{35} See Cummins and Doherty [2006], p. 381-383.
\textsuperscript{36} Sass and Gisser [1989], for instance, argue that turning a broker who acts under the commission system into an exclusive agent (like one acting under the fee system for one particular insurer) will induce the broker to use a higher effort level for this particular insurer than before. This is because the broker has no possibility to sell rival products any more which lowers his opportunity cost.
Appendix

Throughout our analysis, we have supposed 'locations' 0 and 1 of insurers, respectively. Our concern here is to verify these equilibrium locations resulting from the game when there is a 'stage zero', i.e. when providers can at an initial zero-stage freely choose locations on the Hotelling line. Product characteristics may be represented by a location $q_i$ of provider $i$ on the unit line. Without loss of generality, we assume that provider 1 is to the left of provider 2: $0 \leq q_1 \leq q_2 \leq 1$. Of course, at $q_1 = q_2$, we have no product differentiation: Products are identical and Bertrand competition yields the well-known Bertrand result. In our relevant case where $\delta = 1$, the structure of the game can be reduced as follows: In the first stage, firms choose their product characteristics, and then, in the second stage, firms compete in prices. D’Aspremont et al. [1979] have shown that in case of a two-stage game where firms choose a location at an initial stage, and then compete in prices at the second stage, there will be 'maximum differentiation', i.e. the firms select locations as far apart from each other as possible. We show that maximum differentiation also holds in our framework under the commission system. Given some positive transportation cost $t > 0$, a consumer located at $x \in [0, 1]$ faces disutility $t(q_i - x)^2$ from purchasing a product from firm $i$. By choosing different locations, however, firms can ensure themselves positive profits. A consumer purchases from provider 1 if his net utility is higher, so

$$r - p_1 - t(x - q_1)^2 \geq r - p_2 - t(q_2 - x)^2$$

which, together with $q_2 - q_1 := \Delta q$ and $(q_1 + q_2)/2 := \bar{q} \in [0, 1]$, is equivalent to

$$x \leq \frac{p_2 - p_1}{2t\Delta q} + \bar{q}.$$  \hspace{1cm} (26)

A consumer is indifferent between purchasing from provider 1 or 2 at

$$\hat{x} = \frac{p_2 - p_1}{2t\Delta q} + \bar{q}.$$ \hspace{1cm} (27)

Market shares for providers 1 and 2 are then $\hat{x}$ and $1 - \hat{x}$, respectively. Profits of insurers are

$$\Pi_1 = (p_1 - (1 - \delta)k) \cdot \left(\frac{p_2 - p_1}{t\Delta q} + \bar{q}\right)$$ \hspace{1cm} (28)

and

$$\Pi_2 = (p_2 - (1 - \delta)k) \cdot (1 - \frac{p_2 - p_1}{t\Delta q} - \bar{q}).$$ \hspace{1cm} (29)
Price reaction functions are then given by

\[ p_R^1(p_2) = \frac{1}{2}p_2 + \frac{(1 - \delta)k}{2} + \bar{q} \cdot t \Delta q \]  
(30)

and

\[ p_R^2(p_1) = \frac{1}{2}p_2 + \frac{(1 - \delta)k}{2} + (1 - \bar{q}) \cdot t \Delta q. \]  
(31)

Equilibrium prices are

\[ p^*_1(q_1, q_2) = (1 - \delta)k + t \Delta q \cdot \frac{2}{3}(1 + \bar{q}) \]  
(32)

and

\[ p^*_2(q_1, q_2) = (1 - \delta)k + t \Delta q \cdot \frac{2}{3}(2 - \bar{q}). \]  
(33)

We can now establish reduced profit functions by substituting equilibrium prices in (28) and (29). We then have

\[ \Pi_{red.}^1(q_1, q_2) = \frac{1}{18}t(2 + q_1 + q_2)^2(q_2 - q_1) \]  
(34)

and

\[ \Pi_{red.}^2(q_1, q_2) = \frac{1}{18}t(4 - q_1 + q_2)^2(q_2 - q_1). \]  
(35)

It is now easy to see that insurance providers choose so called 'maximum differentiation'. Due to

\[ \frac{\partial \Pi_{red.}^1}{\partial q_1} < 0 \]  
(36)

and

\[ \frac{\partial \Pi_{red.}^2}{\partial q_2} > 0, \]  
(37)

providers will optimally select locations as far apart from each other as possible. Therefore, when providers are restricted to locate in \([0, 1]\), they will locate in \(q^*_1 = 0\) and \(q^*_2 = 1\), respectively.
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