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# The Optimal Pricing Strategy for an Insurer when Risk Preferences are Stochastically Distributed

by

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## Abstract

The present paper analyzes the demand for insurance when the insurer has incomplete information about types of potential customers. We assume that customers' risk preferences cannot be distinguished by the insurer. Therefore, the standard result in insurance economics that the insurer discriminates perfectly in prices cannot be applied. Instead, the present article examines the optimal pricing rule for an insurer faced with stochastic distribution of risk preferences. Within this general model framework, we show that an optimal strategy always exists. Both fixed and proportionate premium loadings (relative to expected loss) are considered.

**Keywords:** insurance demand, optimal insurance pricing, stochastically distributed risk preferences.

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## Introduction

In economic models of insurance demand, it is often assumed that the utility function of a (representative) individual, and thus his or her maximum premium he or she would be willing to pay for full insurance coverage, is known by the insurer. Under this assumption, the insurer maximizes its expected profit and offers full insurance to the individual who pays his or her reservation price in exchange. The reservation price is the expected loss of the policy plus the individual risk premium.<sup>1</sup>

However, when we extend this simple model of only two market participants (i.e. the insurer and the representative customer) to a market with many different potential customers looking for insurance, it means that the insurer must have perfect information about individual risk premiums (i.e., utility functions) in the population of potential insureds in order to set individually optimal prices. This assumption is very restrictive and may not hold in general.

Empirical findings suggest that individuals generally tend to make different decisions in identical risk situations, i.e. they value risk situations different, even if they are endowed with similar initial wealth. In other words, in a given population of potential customers, one will generally find different risk preferences. Individuals with higher risk aversion will have a higher willingness to pay for full insurance than individuals with less risk aversion. Therefore, in this paper we allow potential insureds to differ from each other by their risk attitudes.

Moreover, the insurer may not know individual risk attitudes, but will at most have some information about the distribution of risk preferences in the population of its potential customers. This more realistic case has not been studied in detail in the insurance literature yet. The present paper makes an attempt to fill this gap.

In a model with incomplete information, i.e. in a setting where the probability distribution of risk preferences is known to the insurer, but not individual risk attitudes, we derive the profit function of the insurer and determine the optimal pricing rule using a general model framework. We will begin by the simplest case of a fixed premium loading imposed by the insurer, then we introduce proportionate loadings, and finally we analyze the general insurance demand function including a combination of both premium loadings (i.e. a fixed loading fee and a proportionate loading factor on the expected value of the policy) together.

In general, an increase in premiums involves a decrease in demand. That is, there is a trade-off between the offered premium and the number of insureds. Since each potential insured has a maximum individual premium he or she is willing to pay for (full) insurance, in our model the number of insureds depends upon the insurance premium offered by the insurer. Due to this functional relationship between price setting of the insurer and number of insureds, we introduce a continuous model. The intention of the paper is therefore to develop a general insurance

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<sup>1</sup> This standard result can be found by using simple indifference curve analysis. See, for instance, Stiglitz (1977).

demand function and thereupon derive the optimal premium for an insurer, given incomplete information about the risk preferences in the population of potential insureds.<sup>2</sup>

An important result of this contribution is that very general continuity assumptions are indeed sufficient to ensure that an optimal pricing rule for the insurer always exists. The optimal pricing rule can be found in a general setting where the insurer knows the overall probability distribution of risk preferences in the customer pool, but cannot distinguish individual risk attitudes. We show how to find the expected-profit maximizing premium and its corresponding optimal number of insureds. Furthermore, we illustrate and confirm our findings in several examples where we include some explicit density or utility functions in the analysis. Finally, we derive some general results for the optimal pricing structure of the insurer by comparing our theoretical and numerical results.

This contribution may be embedded in the existing literature on insurance economics as follows. The paper is most related to Doherty (1975) and Schlesinger (1983). Doherty (1975) presents a basic model of insurance demand, building on earlier classics of Mossin (1968) and Smith (1968). He analyzes the optimal contract under different pricing structures of the insurer, that is under a fixed loading fee and a loading proportionate to the actuarial value of the policy. In the case of a fixed loading fee, full insurance or no insurance is optimally chosen. In contrast, in the case of a proportionate loading, partial insurance coverage (depending on the loading and the individual utility function) is optimal. These outcomes make the explicit assumption of a linear relationship between the insurance premium and the level of insurance coverage. We will differ from this traditional approach by analyzing the optimal premium for a given probability distribution of risk preferences in a considered customer pool. This approach seems reasonable, since an insurer will, in general, not be informed about the exact risk premium or exact willingness to pay for insurance of a customer.

Our paper also builds on Schlesinger (1983) who derives optimality conditions for insurance pricing in monopolistic and competitive insurance markets. However, we focus on the problem where risk preferences of potential insureds are not known to the insurer, and thereby offer a more general setting by allowing the resulting function of individually optimal proportion of insurance (coinsurance amount) to be non-differentiable and by explicitly taking into account marginal cost of the insurer. In this way, we take up an idea originally proposed by Schlesinger (1983) and generalize his interesting results by using weaker assumptions.

Further economic models of optimal insurance pricing related to the one presented here are few. One is by Kliger und Levikson (1998) who are concerned with the optimal premium and number of insureds that minimizes the expected loss due to insolvency of the insurer. The authors analyze a fixed premium loading in a discrete setting. Schlesinger (1987) considers the expected monopoly profit when the seller offers state-claims contracts to a risk-averse individual with state-dependent preferences. Landsberger und Meilijson (1994) examine the demand for insurance in a model with two types of risk-averse customers. Cleeton und Zellner (1993) use a comparative-static analysis in order to show how the degree of risk aversion of a consumer, the

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<sup>2</sup> We show a general connection between the classical economic model of monopoly pricing in standard economics and the basic model of demand for insurance. Such a transfer seems to make sense, since other economic models that rely on demand functions may be applied to insurance economics by drawing upon the present paper.

specification of the loss, and the price of insurance interact with income to affect the individual's net demand for insurance.

The paper is organized as follows. In the next section, we briefly comment on the motivation and economic background of our analysis. Here, we also point out the basic assumptions of our model. The following section presents the model with fixed premium loadings and full insurance. We illustrate the results of the model via selected numerical examples and graphs. Thereupon, we introduce proportionate premium loadings and partial insurance. Again, the results of the theoretical settings are illustrated by selected numerical examples and graphs. In the following section, we present and discuss the general case. The last section concludes.

## Motivation and Economic Background

As we have illustrated, a population of potential insureds (individuals and firms) is not, as frequently accepted, homogeneous, but rather heterogeneous, since the individuals or firms generally differ in both expected loss and risk preferences. However, in the first case actuarial classification criteria may be taken into account in order to divide insureds in relatively homogenous groups with almost equal expected loss. Then, the insurer may price each group separately.<sup>3</sup> But in the second case, when the insurer has limited information about individual risk preferences but knows individual expected loss, it is not possible to divide the insureds into homogenous groups. As a result, we will focus our analysis on a population of potential insureds with identical expected loss and different risk preferences. In such a setting, the optimization problem of the insurer is to find the optimal premium for this population. Taking these arguments into account, we will introduce a general model of insurance demand in the next section building on the following basic assumptions:

A risk neutral insurer is faced with a continuum of potential customers. The customers are identical in expected loss, but differ in their risk preferences, i.e. they differ in their risk premium  $r$ .<sup>4</sup> Each customer purchases one insurance contract at most. The different risk preferences in the considered customer pool may be interpreted in analogy to general demand theory as the individual willingnesses to pay for full insurance, i.e. more specifically, the net individual willingness to pay that exceeds the expected value of the policy. Those may be set up in ascending order to result in the probability distribution function of risk preferences.

Let us suppose a realistic setting in which the insurer is not perfectly informed about the risk preferences in the customer pool, that is the insurer cannot differentiate between different risk preferences, but knows the probability density  $f(r)$  in the customer pool. In this case, the insurer cannot skim all the consumer rent by maximizing its profit, but may maximize its expected profit given the known probability distribution of risk preferences in the considered pool of potential insurance buyers.

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<sup>3</sup> See Ramsay (2005), p. 38. For more on risk classification, see for example Finger (1996).

<sup>4</sup> The risk premium, as we use it here, was originally introduced by Pratt (1964). For any given strictly increasing and concave utility function  $U(\cdot)$  the risk premium is defined by  $EU(X) = U(W_0 - E(X) - r)$ . The risk premium  $r$  tells us the maximum amount an individual is willing to pay to securely receive the expected value of a given lottery instead of having the lottery itself.

We assume that the insurer cannot distinguish between individual customers. Therefore, a complete discrimination in prices is not possible. The insurer may set a premium following the structure  $P = E(X) + p$ , where the fixed premium loading  $p$  is the same for all potential customers. If the insurer knew the risk premium of each customer, it would achieve a complete discrimination in prices by setting the premium exactly equal to the amount the individual potential customer is prepared to pay, i.e. the individual gross reservation price. This outcome results when the insurer knows the utility function of each potential insured. As mentioned above, this will not probably be the case since a complete discrimination in prices will generally involve high transaction cost. Therefore, it seems realistic to assume that the insurer has incomplete information about individual risk preferences, or alternatively that a complete discrimination in prices may indeed seem "too expensive" for an insurer to be reasonably undertaken.

Clearly, the insurer may also set its premium according to the structure  $P = qE(X)$ , where  $q$  indicates a proportionate loading factor. Here, the potential customer has to pay a proportionate loading factor upon the expected value of the policy, so that the individual utility function determines the coinsurance rate he or she will actually choose. Therefore, the utility function determines both the risk premium and the individually chosen coinsurance rate. The loading factor is again the same for all potential customers. The distribution of individual utility functions and therefore the pool of potential customers is again given by the density function  $f(r)$ .

Finally, the insurer may set a premium following the general structure  $P = qE(X) + p$ , where we allow for both fixed loading fee and proportionate loading factor simultaneously. In this case, the premium structure incorporates the potential to extract expected profit from both fixed loading fee and proportionate loading factor, respectively. Therefore, this general pricing strategy seems to be a powerful instrument. In the following, we will address each possibility separately.

As a result, we will show that given the insurer knows the distribution of risk preferences in the customer pool, a general insurance demand function can always be derived. The insurer cannot distinguish individual risk preferences, but it can nevertheless determine an optimal price. Therefore, we analyze a situation similar to the classical monopoly model. Interestingly, the model might also be of concern in all situations where an insurer exerts some market power, i.e. in all cases where price setting behavior on the part of the insurer is important.

### **The Model: Full Insurance**

The probability density of the customers' risk preferences is given by  $f(r)$  over the closed interval  $r \in [0, \bar{r}]$ . The customers differ in their risk aversion (i.e.,  $r > 0$ ). Hence, at one extreme, we have a maximum risk aversion  $\bar{r}$  in the customer pool, and at the other extreme, we find the least risk-averse customer is indeed risk neutral ( $r = 0$ ). With these assumptions, we include all potential insureds in the analysis, since risk loving customers (those with  $r < 0$ ) would not ask for insurance.

All potential insureds dispose of initial wealth  $W_0$  and face a potential loss  $L < W_0$  that occurs with probability  $p$ . Thus, the size of loss  $X$  is a random variable with expected value  $E(X)$  that may be written as  $pL$ .<sup>5</sup> In order to ensure non-negative expected profits of the insurer, the insurance premium must not be lower than the expected value of the policy. Furthermore, the premium should not exceed the reservation price of the most risk-averse customer (the maximum risk aversion  $\bar{r}$ ), since then there would be no demand for insurance. As a consequence, the profit maximizing premium of the insurer  $P$  will lie in the interval  $E(X) \leq P \leq E(X) + \bar{r}$ .

To keep the analysis simple, let us normalize the number of potential insureds to one. Then, let  $x(p) \in [0,1]$  be the fraction of customers purchasing insurance given the net insurance premium is  $p$ . This so called *economic premium* presents the net price of insurance (adjusted to the expected value of the policy).<sup>6</sup> Hence, the gross insurance premium  $P$  is determined by the expression  $p = P - E(X)$ .

Insurance demand is given by

$$x(p) = \int_p^{\bar{r}} f(r) dr = 1 - F(p), \quad (1)$$

where  $F(\cdot)$  is the probability distribution according to the density  $f(\cdot)$ . When  $p$  tends to zero, all risk-averse customers will purchase insurance coverage.

In order to further analyze the function in (1), we first consider the points of axis interception:

$$x(0) = 1 - F(0) = 1 \quad (2)$$

$$0 = x(p) = 1 - F(p) \Leftrightarrow F(p) = 1 \Leftrightarrow p = \bar{r}. \quad (3)$$

Furthermore, the function is injective (one-to-one), since we have  $x'(p) = -f(p) < 0$  with  $p \in [0, \bar{r}]$ . Therefore, the inverse function always exists, and is given by

$$p(x) = F^{-1}(1 - x) \quad \text{with } x \in [0,1]. \quad (4)$$

This function is strictly decreasing due to

$$p'(x) = -\frac{1}{f(1-x)} < 0 \quad \text{for } x \in [0,1]. \quad (5)$$

Let us assume that the insurer has the following general cost function  $C(x) = cx + k$ , where - without loss of generality - we may suppose fixed cost of zero. Taking into account the average (and marginal) cost of the insurer  $c < \bar{r}$ , we may calculate the expected profit of the insurer as follows:

$$G(p) = p \cdot x(p) - c \cdot x(p) \quad \text{with } p \in [0, \bar{r}]. \quad (6)$$

The First Order Condition of an interior maximum of the profit function is given by

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<sup>5</sup> The random variable  $X$  denotes the size of potential loss. It takes the value  $L$  with probability  $p$  and the value 0 with probability  $1-p$ . Thus we have  $E(X) = pL$ .

<sup>6</sup> See also Kliger and Levikson (1998), p. 245.

$$G'(p) = \frac{dx}{dp} \cdot p + x(p) - c \cdot \frac{dx}{dp} = 0, \quad (7)$$

where the profit maximizing premium of the insurer is implicitly defined by

$$p_{opt} = \frac{1 - F(p_{opt})}{f(p_{opt})} + c. \quad (8)$$

Therefore, the optimal (i.e., the profit maximizing) gross premium of the insurer is given by

$$P^* = E(X) + p_{opt}. \quad (9)$$

Since  $x(p)$  is a continuous function due to (1), the expected profit of the insurer is a continuous function in  $p$ , as well. An optimal premium maximizing (6) always exists, since we maximize over the compact interval  $[0, \bar{r}]$ , and continuous functions over a compact interval always have a maximum.<sup>7</sup> Thus, we have shown that - for any given continuous probability distribution of risk preferences of potential insureds - an optimal premium for the insurer always exists. This optimal premium does not necessarily need to be unique. In summary, if the insurer is only informed about the distribution of risk preferences in the customer pool, but cannot distinguish individual customers, the insurer can nevertheless always determine an optimal price by taking into account the resulting insurance demand function.

We will present some simple selected examples of the probability density function for the risk preferences in the pool of potential insureds in the next section.

## Selected Examples

### The Uniform Distribution

In the following, we will analyze the uniform distribution as a simple illustrative example of our model. When risk preferences are uniformly distributed, we have the probability density to read

$$f(r) = \begin{cases} 1/\bar{r} & r \in [0, \bar{r}] \\ 0 & otherwise \end{cases} \quad (10)$$

and the probability distribution function is given by

$$F(r) = \begin{cases} 0 & r \leq 0 \\ r/\bar{r} & 0 < r < \bar{r} \\ 1 & r \geq \bar{r} \end{cases}. \quad (11)$$

In this case, the function of insurance demand yields

$$x(p) = 1 - F(p) = 1 - \frac{p}{\bar{r}} \quad \text{with } p \in [0, \bar{r}]. \quad (12)$$

From (6), we find the following profit maximizing price and quantity

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<sup>7</sup> For a general proof, the reader is referred to the "Extreme Value Theorem". See, for instance, Dugundji (1970), p. 227.



$$p_{opt} = \frac{\bar{r} + c}{2} \quad x^*(p_{opt}) = \frac{l}{2} - \frac{c}{2\bar{r}}. \quad (13)$$

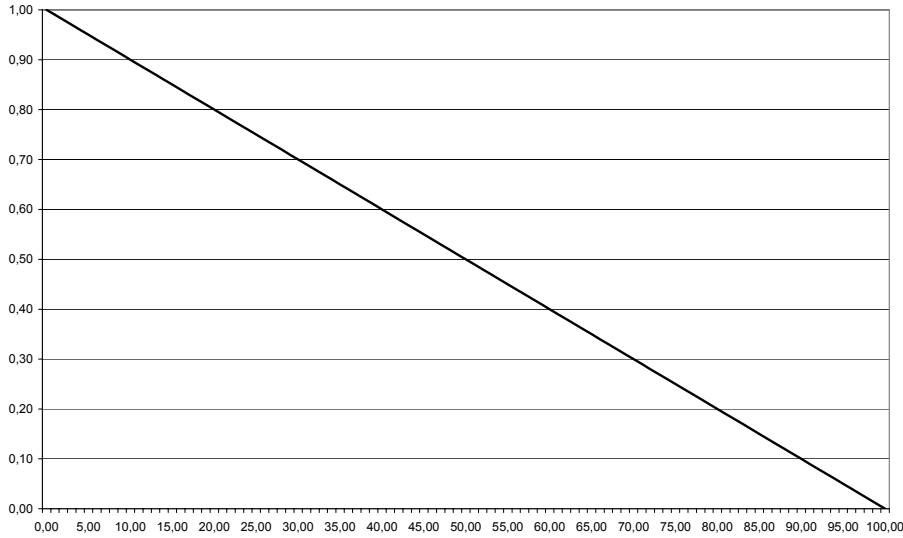
Then, we may calculate the profit maximizing gross premium to be charged by the insurer as

$$P_{opt} = E(X) + \frac{\bar{r} + c}{2}. \quad (14)$$

The resulting function of insurance demand may be depicted graphically as follows:

**Figure 1**

Function of insurance demand with a uniform distribution of risk preferences



### The Normal Distribution

As a second example, let us consider normally distributed risk preferences in the population. Together with

$$f(r) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{[r-\mu]^2}{2\sigma^2}\right) \quad \text{and} \quad F(r) = \int_{-\infty}^r \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{[\rho-\mu]^2}{2\sigma^2}\right) d\rho, \quad (15)$$

we find the density and distribution functions for the potential insureds, respectively:

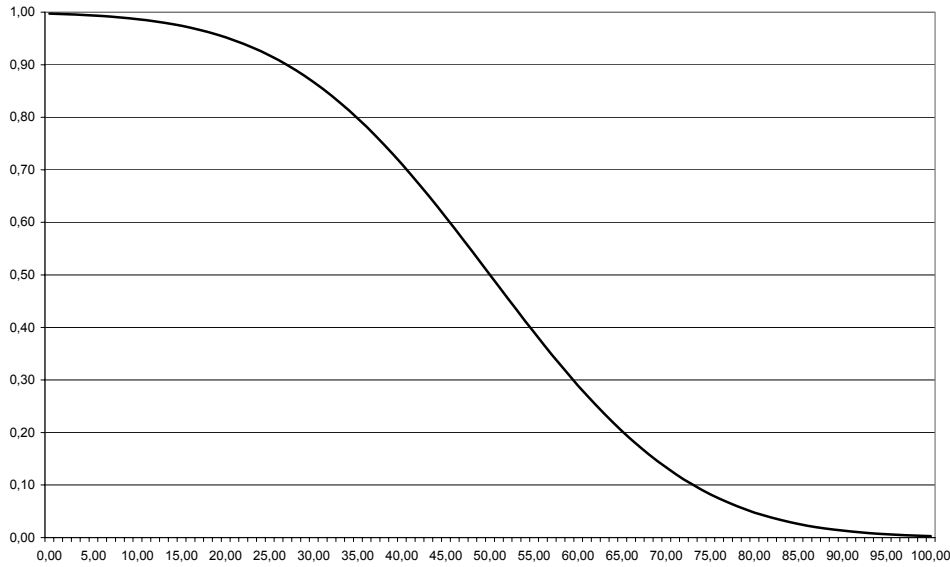
$$\tilde{f}(r) = \begin{cases} \frac{l}{F(\bar{r}) - F(0)} f(r) & r \in [0, \bar{r}] \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad (16)$$

$$\tilde{F}(r) = \begin{cases} 0 & r \leq 0 \\ \frac{1}{F(\bar{r}) - F(0)} [F(r) - F(0)] & 0 < r < \bar{r} \\ 1 & r \geq \bar{r} \end{cases}. \quad (17)$$

The resulting function of insurance demand is depicted graphically in Figure 2.

## Figure 2

Function of insurance demand with a normal distribution of risk preferences



## Coinsurance

In this section, we will concentrate on a setting where the insurer determines its price according to a premium structure that includes a loading factor proportional to the expected value of the policy. Within this framework, we do not restrict our analysis to full insurance any more, but include partial insurance coverage as the optimal insurance decisions of potential insureds. If the insurer offers a proportionate premium loading, the expected-utility maximizing customers will not necessarily want to fully insure (or even not insure at all), but instead will choose partial insurance coverage in optimum.<sup>8</sup>

If the insurer calculates its premium on the basis of a proportionate loading factor  $q$  upon the actuarial value of the policy, the premium structure becomes  $P = \alpha \cdot qE(X)$ , where  $\alpha$  indicates the proportion of insurance. In the following, we will assume that the premium loading is a net loading, that means the loading is cost-adjusted. The policyholder chooses the proportion of insurance  $\alpha$  which maximizes his or her expected utility of final wealth, i.e.

$$\max_{\alpha} EU(W) = (1 - p) \cdot U(W_0 - \alpha qpL) + p \cdot U(W_0 - \alpha qpL - (1 - \alpha)L), \quad (18)$$

where  $W_0$  denotes initial wealth. The First Order Condition for an interior maximum is given by

$$-\frac{1 - p}{p} \cdot \frac{U'(W_1)}{U'(W_2)} = -\frac{1 - qp}{qp}, \quad (19)$$

where  $W_1 = W_0 - \alpha qpL$  and  $W_2 = W_0 - \alpha qpL - (1 - \alpha)L$  represent final wealth of the insured in the no loss state and the loss state, respectively.

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<sup>8</sup> See also Doherty (1975), pp. 451-52.

The degree of individually optimal proportion of insurance  $\alpha^*(U_r, q)$  results implicitly from the above equation. For any insurer-determined proportionate loading factor  $q > 1$ , an individual  $I_r$  with utility  $U_r$  (i.e., risk premium  $r$ ) will choose an individually optimal proportion of insurance  $\alpha^*(U_r, q)$ . The sales of the insurer are determined by the integral over the density-weighted aggregate insurance demand. Given the assumptions in this section, the insured chooses his or her individually optimal proportion of insurance  $\alpha^*(U_r, q)$ <sup>9</sup>. The aggregate sales of the insurer are then given by<sup>10</sup>

$$x(q) = \int_0^{\bar{r}} \alpha^*(U_r, q) f(r) dr . \quad (20)$$

Hence, we have shown that the sales of the insurer and thus the general insurance demand function may be depicted as a classical expected value of random variables  $1_{[r \geq p]}$  and  $\alpha^*(U_r, q)$ , respectively.<sup>11</sup>

In order to ensure the existence of the integral in (20) and in the following, we need two general conditions for the function  $\alpha_r(q) = \alpha^*(U_r, q)$  to hold:

$$\text{for all } q \in [1, \bar{q}] , \alpha_r(q): r \mapsto \alpha_r(q) \text{ is measurable, and} \quad (21)$$

$$\text{for all } r \in [0, \bar{r}] , \alpha_r(\cdot): q \mapsto \alpha_r(q) \text{ is continuous on the interval } q \in [1, \bar{q}]. \quad (22)$$

These assumptions will be retraced in the following section, where we present some illustrative examples. Furthermore, we may state the result that the above conditions are basically fulfilled in the case where the functions

$$g_r(\alpha) = \frac{U'_r(W_0 - \alpha pqL)}{U'_r(W_0 - \alpha pqL - (1 - \alpha)L)} \quad (23)$$

are uniformly continuous in  $r \in [0, \bar{r}]$ , i.e.

$$\|g_{r_n} - g_r\|_\infty = \sup_{0 \leq \alpha \leq 1} |g_{r_n}(\alpha) - g_r(\alpha)| \rightarrow 0 \text{ for all } r_n \rightarrow r \in [0, \bar{r}].^{12} \quad (24)$$

This assumption of uniform continuity generally holds for all important functions of the HARA (Hyperbolic Absolute Risk Aversion) class, i.e. exponential, quadratic and isoelastic utility

<sup>9</sup> The dependence on the risk premium  $r$  is modeled in such a way that the parameter of the utility function  $t(r)$  is calculated in dependence on the risk premium. Using the first derivative of the utility function, we obtain the individually optimal proportion of insurance.

<sup>10</sup> Note that equation (1) in the last section may be written as

$$x(p) = \int_0^{\bar{r}} 1_{[r \geq p]} f(r) dr ,$$

and thus we can easily see that an individual will choose full insurance coverage or no coverage at all (depending on his or her individual risk premium). The above insurance demand function may then be interpreted as follows. An individual will demand full insurance if his or her individual risk premium is at least as high as the fixed premium loading of the insurer. Otherwise, no insurance will be purchased.

<sup>11</sup> If the proportionate premium loading of the insurer  $q$  approaches one (i.e., the premium becomes actuarially fair), the individually optimal proportion of insurance  $\alpha^*(U_r, q)$  will also approach one. That is, all potential insureds indeed ask for insurance and choose full insurance in optimum.

<sup>12</sup> See the Appendix (1.) for proof of this result.

functions. The same holds for root functions. The uniform continuity can be shown by applying the Theorem of Dini.<sup>13</sup>

For a given individual  $I_r$ , the profit of the insurer amounts to

$$G_r(q) = \alpha^*(U_r, q)(q-1)E(X) \quad (25)$$

The aggregate profit of the insurer, given all risk-averse policyholders, is then given by<sup>14</sup>

$$G(q) = \int_0^{\bar{r}} G_r(q) f(r) dr = (q-1)E(X) \cdot \int_0^{\bar{r}} \alpha^*(U_r, q) f(r) dr . \quad (26)$$

Making the reasonable assumption that  $q$  lies in a compact interval  $[1, \bar{q}]$ , our problem is to maximize a continuous function over a compact interval. Therefore, a profit maximizing  $q$  always exists.<sup>15,16</sup>

In summary, our main theoretical result may be seen as follows. We introduce a very general assumption, i.e. the uniform continuity of  $g_r(\alpha)$ , which implies conditions (i) and (ii) for the function  $\alpha^*(U_r, q)$ . Subsequently, we obtain that the profit function of the insurer  $G(q)$  is continuous on the compact interval  $[1, \bar{q}]$  implying a general maximum. In words, the insurer can always find an insurance premium for all potential insureds that maximizes its expected profit. In addition, we offer a theoretical overview showing that our introductory general assumption does indeed hold for commonly used utility functions.

## Selected Examples

In the following, we will demonstrate the general results of the last section, and at the same time analyze these results graphically using some concrete probability density function. As an illustrative example, let us consider the family of utility functions given by

$$U_t(x) = \sqrt[t]{x} = x^{\frac{1}{t}} \text{ with } t \in [1, t(\bar{r})]. \quad (27)$$

In the above, the parameter  $t$  indicates the curvature (i.e., degree of concavity) of the utility function, so that the respective risk premium of a considered individual may be determined by

$$r(t) = px_1 + (1-p)x_2 - U_t^{-1}\{pU_t(x_1) + (1-p)U_t(x_2)\} \text{ with} \quad (28)$$

$$x_1 = W_0 - L \quad , \quad x_2 = W_0 \quad \text{and} \quad U_t^{-1}(y) = y^t .$$

Therefore, we have

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<sup>13</sup> The Theorem of Dini states that a sequence of monotone and continuous functions which tends in pointwise convergence to a continuous function is uniformly convergent. The assumptions of the Theorem of Dini follow directly from the continuity and monotonicity assumptions of the utility parameter function  $t(r)$ .

<sup>14</sup> A similar representation of the aggregate profit of the insurer using a different approach is shown in Schlesinger (1983), p. 75-78. However, note that our findings are based on an explicit development of the insurance demand function. This profit function results directly from the insurance demand function.

<sup>15</sup> Generalizing the optimality conditions for the insurer (which are originally derived in Schlesinger (1983), p. 75-78), we do not need to assume differentiability in  $q$  here, but need only continuity. Under these general and weaker assumptions, we demonstrate the existence of an optimal premium maximizing the insurer's profit.

$$r(t) = px_1 + (1-p)x_2 - \left\{ p \cdot {}^t\sqrt{x_1} + (1-p) \cdot {}^t\sqrt{x_2} \right\}^t. \quad (29)$$

The function  $r(t)$  is a composition of continuous functions and thus it is also continuous. Due to<sup>17</sup>

$$\begin{aligned} r'(t) = & - \underbrace{\left\{ p \cdot {}^t\sqrt{x_1} + (1-p) \cdot {}^t\sqrt{x_2} \right\}}_{<0} \cdot \underbrace{\ln \left\{ p \cdot {}^t\sqrt{x_1} + (1-p) \cdot {}^t\sqrt{x_2} \right\}}_{>0} \\ & \cdot \underbrace{\left\{ -\frac{1}{t^2} p \cdot \ln x_1 \cdot {}^t\sqrt{x_1} - \frac{1}{t^2} (1-p) \cdot \ln x_2 \cdot {}^t\sqrt{x_2} \right\}}_{<0} \geq 0 \end{aligned} \quad (30)$$

the inverse function  $t(r)$  basically exists for any  $r \in [0, \bar{r}]$ . As a consequence,  $t(r)$  is continuous.<sup>18</sup>

In the following, we will determine the individually optimal coverage ratio  $\alpha^*(U_r, q)$  for a given proportionate premium loading  $q$ . To continue, we rewrite the First Order Condition

$$-\frac{1-p}{p} \cdot \frac{U'(W_1)}{U'(W_2)} = -\frac{1-qp}{qp}$$

to read

$$\frac{(W_0 - \alpha qpL)^{\frac{1-t}{t}}}{(W_0 - \alpha qpL - (1-\alpha)L)^{\frac{1-t}{t}}} = \frac{(1-qp)p}{qp(1-p)} \quad \text{or} \quad (31)$$

$$\Leftrightarrow \frac{(W_0 - \alpha qpL)}{(W_0 - \alpha qpL - (1-\alpha)L)} = \left\{ \frac{(1-qp)p}{qp(1-p)} \right\}^{\frac{t}{1-t}}. \quad (32)$$

Solving for  $\alpha$  results in

$$\alpha^* = \alpha^*(U_r, q) = \frac{-W_0 \left\{ \frac{(1-qp)p}{qp(1-p)} \right\}^{\frac{t(r)}{1-t(r)}} + W_0 + L \left\{ \frac{(1-qp)p}{qp(1-p)} \right\}^{\frac{t(r)}{1-t(r)}}}{qpL - qpL \left\{ \frac{(1-qp)p}{qp(1-p)} \right\}^{\frac{t(r)}{1-t(r)}} + L \left\{ \frac{(1-qp)p}{qp(1-p)} \right\}^{\frac{t(r)}{1-t(r)}}} \quad \text{for all } t(r) > 1. \quad (33)$$

The ratio of insurance coverage chosen by the insured must lie between zero and one. Therefore, we define

<sup>16</sup> A formal proof can be found in the Appendix (2.).

<sup>17</sup> Note that  $p \cdot {}^t\sqrt{x_1} + (1-p) \cdot {}^t\sqrt{x_2} > p + (1-p) = 1$  for  $t > 1$  and  $x_1, x_2 > 1$ .

<sup>18</sup> The continuity follows from the theorem concerning the continuity of the inverse function, which states that the inverse function of a continuous function is continuous, as well.

$$\alpha^*(U_r, q) = \max \left[ \frac{-W_0 \left\{ \frac{(1-qp)p}{qp(1-p)} \right\}^{\frac{t(r)}{1-t(r)}} + W_0 + L \left\{ \frac{(1-qp)p}{qp(1-p)} \right\}^{\frac{t(r)}{1-t(r)}}}{qpL - qpL \left\{ \frac{(1-qp)p}{qp(1-p)} \right\}^{\frac{t(r)}{1-t(r)}} + L \left\{ \frac{(1-qp)p}{qp(1-p)} \right\}^{\frac{t(r)}{1-t(r)}}}, 0 \right] \quad 0 < r \leq \bar{r}, 1 \leq q \leq \bar{q}, \quad (34)$$

$$\alpha^*(U_r, q) = 0 \text{ for } r = 0, 1 < q \leq \bar{q} \text{ and } \alpha^*(U_r, q) = 1 \text{ for } r = 0, q = 1.$$

It can easily be seen that given an actuarially fair premium ( $q = 1$ ), the coverage ratio chosen is one, i.e., the policyholder chooses full insurance.<sup>19</sup> Continuity is ensured since for all  $0 < r \leq \bar{r}, 1 \leq q \leq \bar{q}$  we have  $\alpha^*(U_r, q)$  which is a composition of continuous functions, as well.

For any  $r = 0, 1 < q \leq \bar{q}$  assume  $(r_n, q_n)$  to be a sequence with  $(r_n, q_n) \rightarrow (0, q)$ . Due to

$$\frac{-W_0 \left\{ \frac{(1-q_n p)p}{q_n p(1-p)} \right\}^{\frac{t(r)}{1-t(r)}} + W_0 + L \left\{ \frac{(1-q_n p)p}{q_n p(1-p)} \right\}^{\frac{t(r)}{1-t(r)}}}{qpL - qpL \left\{ \frac{(1-q_n p)p}{q_n p(1-p)} \right\}^{\frac{t(r)}{1-t(r)}} + L \left\{ \frac{(1-q_n p)p}{q_n p(1-p)} \right\}^{\frac{t(r)}{1-t(r)}}} \rightarrow \frac{-W_0 + L}{-qpL + L} < 0,^{20} \quad (35)$$

$\alpha^*(U_{r_n}, q_n) = 0$  results for  $n \in N$  to be sufficiently large. Therefore,  $\alpha^*$  is continuous in  $(0, q)$ .

Only if  $r = 0$  and  $q = 1$ , we find that  $\alpha^*$  is not continuous.<sup>21</sup> Hence, we have shown that our continuity assumptions are indeed fulfilled for the family of root functions (except for one point which is negligible). The general argumentation is given in section 5.

A concrete graph of the function of  $\alpha^*$  is depicted in Figure 3. As can easily be seen, the individually optimal proportion of insurance  $\alpha^*$  is equal to one – for all possible risk preferences – given an actuarially fair premium (i.e.  $q=1$ ). If risk preferences tend to risk neutrality, the function is plunging down with increasing premium loading. When risk aversion increases, this phenomenon becomes more moderate, since the risk-averse insured tends to accept a higher proportionate loading. This rationale can also be found in Figure 3 by taking  $q$  as fixed, so that the function of  $r$  increases monotonically. That is, with increasing risk aversion the individually optimal proportion of insurance increases as well, and the insured indeed demands more insurance coverage.

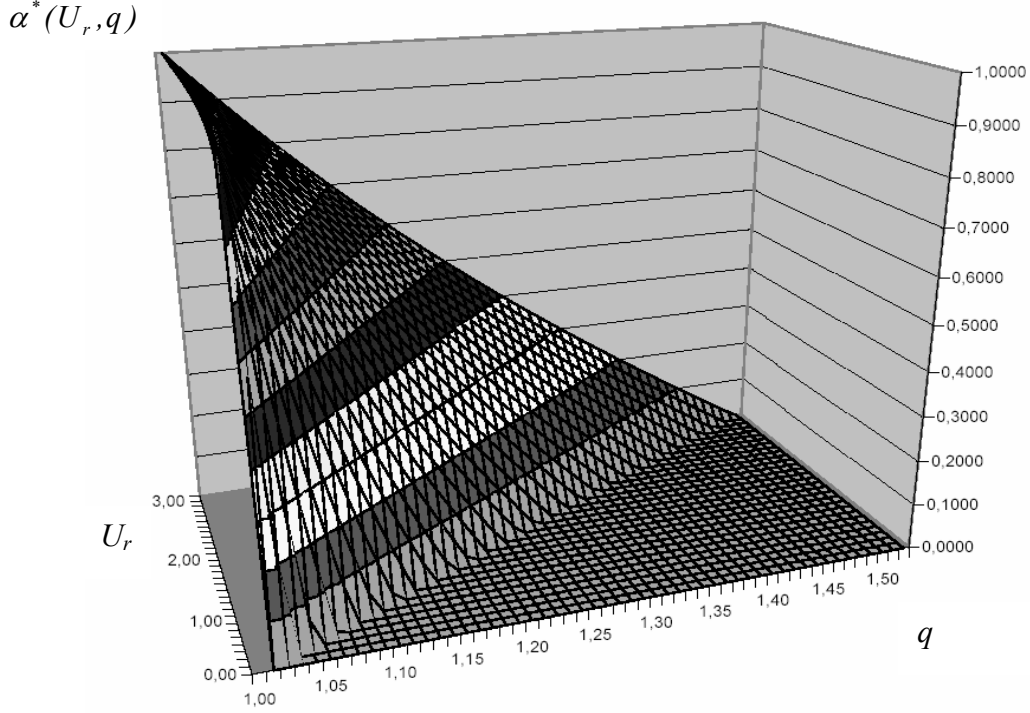
<sup>19</sup> For  $q=1$  the expression in brackets is one and so the numerator and the denominator are equal.

<sup>20</sup> Note that we always assume  $q \leq \bar{q} \leq 1/p$ .

<sup>21</sup> This only discontinuity point is not critical to the analysis: Economically spoken, for the profit function of the insurer in case  $r = 0$  and  $q = 1$  the insurer actually makes zero profits, as well in case  $r = 0$  and  $q > 1$  the insured does not ask for insurance coverage. In both cases, the insurer makes zero expected profits. Mathematically, this problem is solved by showing the continuity of the profit function for  $q = 1$  directly by using an upward estimation of the integral with a sequence tending to zero and for  $q > 1$ , as will be shown in the Appendix (9.2).

**Figure 3**

Function of insurance coverage ratio for  $W=100$ ,  $L=5$  and  $p=0,25$ .



In the following, we will derive the profit of the insurer in an explicit way. According to (26), we have

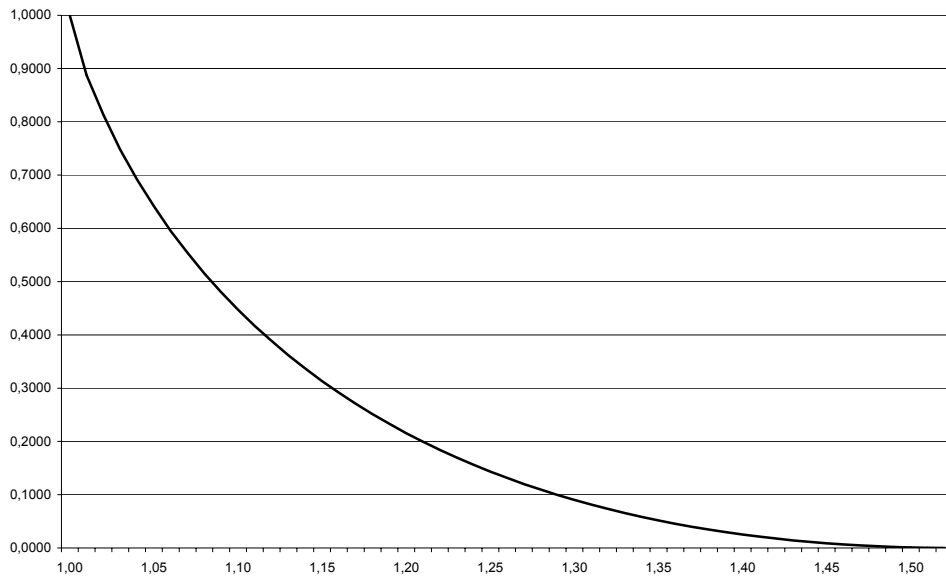
$$G(q) = \int_0^{\bar{r}} \max \left[ \frac{-W_0 \left\{ \frac{(1-qp)p}{qp(1-p)} \right\}^{\frac{t}{1-t(r)}} + W_0 + L \left\{ \frac{(1-qp)p}{qp(1-p)} \right\}^{\frac{t}{1-t(r)}}}{qpL - qpL \left\{ \frac{(1-qp)p}{qp(1-p)} \right\}^{\frac{t}{1-t(r)}} + L \left\{ \frac{(1-qp)p}{qp(1-p)} \right\}^{\frac{t}{1-t(r)}}}, 0 \right] (q-1)EXf(r)dr. \quad (36)$$

This expression can be solved numerically.<sup>22</sup> Hence, in the case of a uniform distribution of risk preferences  $f(r)$ , we basically obtain the main result of an insurance demand function as well as a resulting function of expected profit of the insurer. These functions are illustrated in figures 4 and 5, respectively. The profit maximum is attained at  $q = 1,14$ .

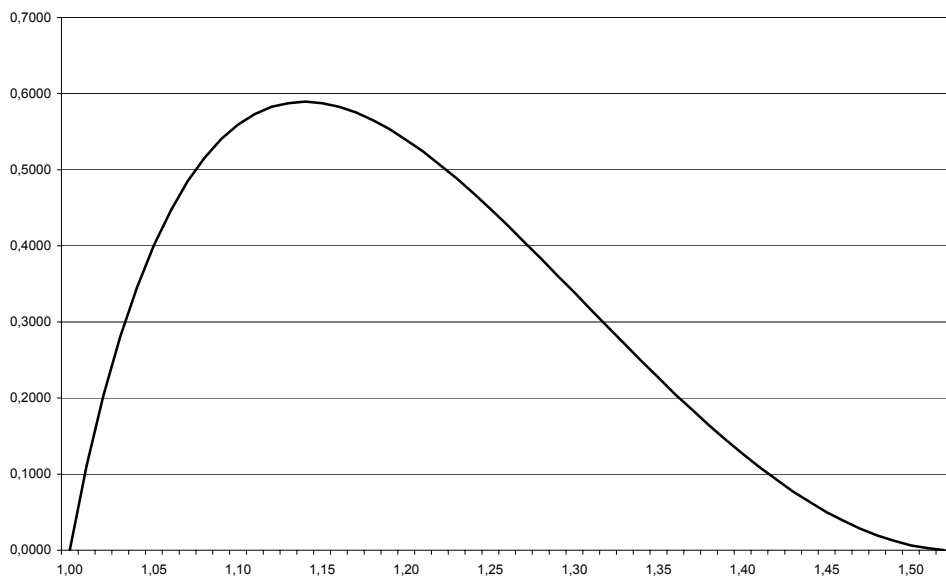
<sup>22</sup> Due to continuity of the integrand in  $r$ , the integral can be approximated using a decomposition of  $[0, \bar{r}]$  and respective intermediate vectors.

**Figure 4**

Function of insurance demand with a proportionate premium loading for  $W=100$ ,  
 $L=50$ ,  $p=0,25$

**Figure 5**

Function of expected profit of the insurer with a proportionate premium loading  
for  $W=100$ ,  $L=50$ ,  $p=0,25$





## The General Case

Finally, we may consider the general case of a combination of fixed and proportionate premium loading. In this case, the insurer demands a premium of  $P = \alpha \cdot q E(X) + p$ . The fixed loading fee presents some kind of entry cost into the insurance market. Generally, a customer will decide on the basis of his or her individual risk premium whether he or she enters into the insurance market or not. Once this entry cost has been paid, the insured chooses his or her individual level of insurance coverage. The function of expected profit of the insurer is then given by

$$G(p, q) = \int_0^{\bar{r}} \alpha^*(U_r, q)(q-1)E(X)f(r)dr + \{1 - F(r_0(p, q))\} \cdot p \quad \text{with} \quad (37)$$

$$r_0(p, q) = \inf \{ r \in [0, \bar{r}] \mid \alpha^*(U_r, q) \cdot I_{[r \geq p]} > 0 \}. \quad (38)$$

Therefore,  $G(p, q)$  is a composition of continuous functions, and thus is itself continuous on the compact set  $[0, \bar{r}] \times [1, \bar{q}]$ . As a consequence, the function has a maximum.

In the general case of a combination of both potential premium loadings, the resulting profit function of the insurer (37) according to our example above may be depicted as in Figure 6. The maximum is obtained at  $p = 1,50$  and  $q = 1,14$ . This result is plausible since  $q$  takes the same value as in the case with only a proportionate loading. Furthermore, we have  $r_0(p, q) = p$  for  $q = 1,14$  in a neighborhood of  $p = 1,50$ . Outside this neighborhood, we have  $r_0(p, q) \geq p$ . Due to the assumption of a uniform distribution, the profit maximum is achieved at  $p = \bar{r}/2 = 1,50$  as shown above.

In general, given that in the proportionate loading scheme for a profit-maximizing  $q$  exists a neighborhood of  $p = \bar{r}/2$  with  $r_0(p, q) = p$ , the profit maximum is achieved at  $(\bar{r}/2, q)$ . If this neighborhood does not exist, we may vary the loadings  $p$  and  $q$  as follows: Let us begin by a profit maximizing  $q$  in the proportionate loading scheme and choose any  $q' < q$ . Then, define the reduction in profit following (26) according to

$$\Delta_1 = (q' - 1)E(X) \cdot \int_0^{\bar{r}} \alpha^*(U_r, q')f(r)dr - (q - 1)E(X) \cdot \int_0^{\bar{r}} \alpha^*(U_r, q)f(r)dr. \quad (39)$$

We have  $\Delta_1 \leq 0$ . Due to  $r_0(p, q') \leq r_0(p, q) \forall p$  and consequently  $\{1 - F(r_0(p, q'))\} \cdot p \geq \{1 - F(r_0(p, q))\} \cdot p \forall p$ , the resulting difference in profit for the fixed loading scheme is given by

$$\Delta_2 = \max_p \{1 - F(r_0(p, q'))\} \cdot p - \max_p \{1 - F(r_0(p, q))\} \cdot p, \quad (40)$$

and therefore always positive. Economically, we may ask the question: Can the reduction in profit given a proportionate loading scheme be compensated by the increase in profit given a fixed loading scheme? We should repeat this procedure as long as we have found a  $q'$  maximizing  $\Delta_1 + \Delta_2$ .<sup>23</sup> For  $q' > q$ , due to  $r_0(p, q') \geq r_0(p, q) \forall p$  as well as

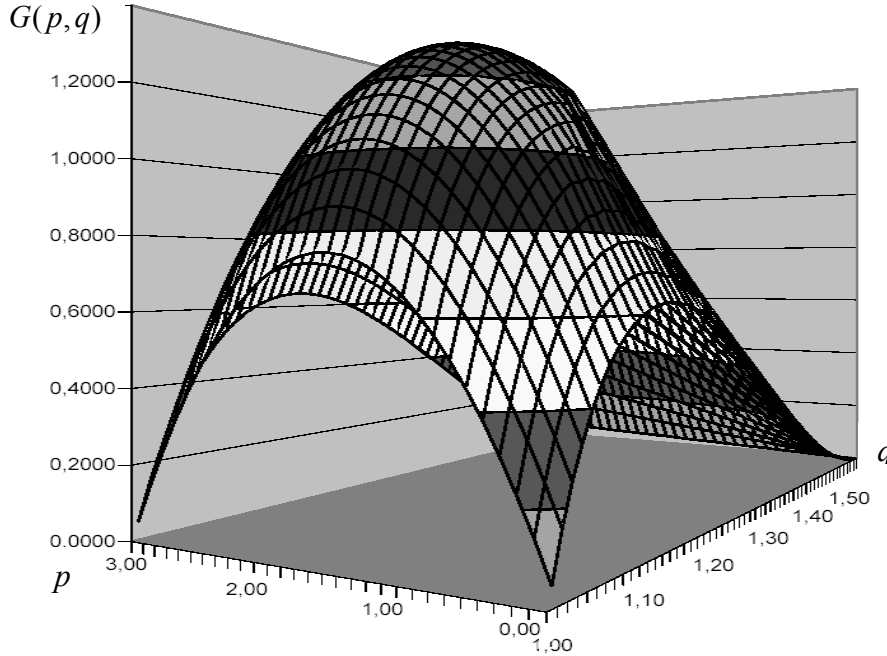
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<sup>23</sup> In case that the function of optimal proportion of insurance may be differentiable, the necessary first order conditions can be found in Schlesinger (1983), p. 77.

$\{1 - F(r_0(p, q'))\} \cdot p \leq \{1 - F(r_0(p, q))\} \cdot p \quad \forall p \quad \Delta_2$  is negative. Hence, we cannot achieve any improvement for those  $q'$ .

### Figure 6

Function of expected profit with a fixed and proportionate premium loading for  $W=100, L=50, p=0,25$



Finally, we may consider how the expected profit of the insurer changes with a variation of the premium loading, i.e. price structure of the insurer. Generally, the expected profit of the insurer will change when the premium changes from a fixed premium loading to a proportionate one or even to a combination of both. In the case of a combination of loadings, the expected profit of the insurer is always at least as high as in each single case, that is, when each pricing rule is applied separately.<sup>24</sup> As a result, the general premium structure incorporates the potential to extract expected profit from both fixed loading fee and proportionate loading factor, respectively. Therefore, this general pricing strategy seems to be a powerful instrument.

The improvement of expected profit is depicted for different density functions in Figure 7. The table illustrates the percentage increase in profit when the insurer switches from a simple fixed or proportionate loading scheme to a combination of both loadings, respectively.

<sup>24</sup> This follows directly from  $\{1 - F(r_0(p, q))\} \cdot p \geq 0$  and  $\max G(p, q) \geq \max G(p, 1) = \max(1 - F(p)) \cdot p$ .

**Figure 7**Increase in expected profit when switching to a combined loading scheme<sup>25</sup>

<i>Distribution of risk preferences in the population of potential insureds</i>	<i>Percentage increase in profit starting by fixed premium loading</i>	<i>Percentage increase in profit starting by proportionate premium loading</i>
Uniform distribution	78,69 %	127,08 %
Triangular distribution	83,12 %	120,30 %
Normal distribution ( $\alpha=1,5$ , $\beta=1$ )	78,66 %	127,13 %
Normal distribution ( $\alpha=0,5$ , $\beta=1$ )	83,14 %	120,28 %

When we compare the columns in the table, we find that the resulting percentage increase in expected profit of the insurer is higher in case of an extension of the pricing strategy from a solely proportionate loading strategy to the general combined premium loading scheme. In contrast, the extension to a general combined premium loading scheme given solely a fixed loading strategy involves less additional expected profit for the insurer. Therefore, an insurer using a pricing strategy with a proportionate premium loading only might be worse off compared to an insurer using a pricing strategy with a fixed loading fee only. Note that this result is not due to our exemplified class of utility functions above (see (27)), but is valid for other utility functions in the HARA class, as well. Therefore, we may state that for commonly used density and utility functions the numerical results above seem to be robust.

**Conclusion**

Empirical findings suggest that individuals generally tend to make different decisions in identical risk situations. In other words, in a given population of potential customers, one may generally find different risk preferences. Individuals with higher risk aversion will have a higher willingness to pay for full insurance than individuals with less risk aversion. Therefore, in this paper we allow potential insureds to differ from each other by their risk attitudes. Moreover, the insurer may not know individual risk attitudes, but will at most have some information about the distribution of risk preferences in the population of customers. This more realistic case has not been studied in detail in the insurance literature yet.

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<sup>25</sup> Note that, when the insurer switches from a fixed premium loading strategy to a combination of loadings, models with asymmetric underlying density functions and a weight on less risk-averse insureds seem to benefit more from a combination of loadings than models with symmetric density functions. Vice versa, when the insurer switches from a proportionate premium loading strategy to a combination, models with symmetric underlying density functions seem to benefit more.

The present paper generalizes the standard model of insurance demand. It presents a general approach to derive the optimal price for an insurer that cannot distinguish between individual customers. In a model with incomplete information, i.e. in a setting where the probability distribution of risk preferences is known to the insurer, but not individual risk attitudes, we derive the resulting profit function of the insurer and find the optimal pricing strategy. We show that if the probability distribution of risk preferences is known to the insurer, an optimal price always exists. We further demonstrate how to find the expected-profit maximizing premium and its corresponding optimal number of insureds.

It is of interest to compare the different pricing strategies of the insurer and to discuss potential gains in expected profit when the insurer extends its premium structure to the general case involving both a proportionate and a fixed premium loading. Interestingly, in the case of commonly used density and utility functions, we find that given a pricing strategy with a proportionate premium loading only, an insurer might be worse off compared to an insurer using a pricing strategy with fixed premium loadings.

Since the combined premium structure incorporates the potential to extract expected profit from both fixed loading fee and proportionate loading factor, respectively, this general pricing strategy is superior when compared to the simple premium strategies, and therefore represents a powerful pricing instrument. Hence, an insurer should always prefer the general pricing strategy to the simple ones and calculate optimal premium parameters as we have shown above.

## Appendix

### 1. ad (i)

We will show that  $\alpha_r(q)$  is continuous and thus measurable. Therefore, let  $r \in [0, \bar{r}]$  be fix and  $r_n \in [0, \bar{r}]$  with  $r_n \rightarrow r$ . For a given  $\delta > 0$  we define

$$\varepsilon = \min(g_r(\alpha) - g_r(\alpha - \delta), g_r(\alpha + \delta) - g_r(\alpha)) \quad (\text{A1})$$

with  $\alpha = \alpha_r(q)$ .

For each  $\varepsilon > 0$  a  $n_0 \in \mathbb{N}$  exists, so that for all  $n \geq n_0$  we have

$$\|g_{r_n} - g_r\|_\infty = \sup_{0 \leq \alpha \leq 1} |g_{r_n}(\alpha) - g_r(\alpha)| < \varepsilon \quad (\text{A2})$$

and therefore

$$g_{r_n}(\alpha - \delta) = g_{r_n}(\alpha - \delta) - g_r(\alpha - \delta) + g_r(\alpha - \delta) < \varepsilon + g_r(\alpha - \delta) \leq g_r(\alpha) = \frac{p}{1-p} \cdot \frac{1-pq}{pq}. \quad (\text{A3})$$

Likewise, we have

$$\begin{aligned} g_{r_n}(\alpha + \delta) &= g_{r_n}(\alpha + \delta) - g_r(\alpha + \delta) + g_r(\alpha + \delta) \geq g_r(\alpha + \delta) - |g_r(\alpha + \delta) - g_{r_n}(\alpha + \delta)| \\ &> g_r(\alpha + \delta) - \varepsilon \geq g_r(\alpha) = \frac{p}{1-p} \cdot \frac{1-pq}{pq}. \end{aligned} \quad (\text{A4})$$

Since the utility function is assumed to be strictly concave and continuously differentiable,  $g_r(\cdot)$  is strictly monotonically increasing and continuous.

Due to this monotonicity and the defined characteristics, it follows that  $g_{r_n}(\alpha_{r_n}(q)) = \frac{p}{1-p} \cdot \frac{1-pq}{pq}$  and  $\alpha_{r_n}(q) \in ]\alpha - \delta, \alpha + \delta[$ . Since  $\delta > 0$  can arbitrarily be chosen, it follows that we have  $\alpha_{r_n}(q) \rightarrow \alpha = \alpha_r(q)$ .  $\square$

### ad (ii)

We will show the proof by contradiction. Therefore, we assume that for a sequence  $q_n \rightarrow q$ ,  $\alpha_r(q_n)$  does not converge to  $\alpha_r(q)$ . Therefore, a partial sequence  $q_{n_k}$  and a  $\varepsilon > 0$  with  $\alpha_r(q_{n_k}) > \alpha_r(q) + \varepsilon$  or  $\alpha_r(q_{n_k}) < \alpha_r(q) - \varepsilon$  exist, and due to monotonicity of  $g_r(\cdot)$ , we have

$$\frac{p}{1-p} \cdot \frac{1-pq_{n_k}}{pq_{n_k}} = g_r(\alpha_r(q_{n_k})) > g_r(\alpha_r(q) + \varepsilon) > g_r(\alpha_r(q)) = \frac{p}{1-p} \cdot \frac{1-pq}{pq} \quad (\text{A5})$$

or

$$\frac{p}{1-p} \cdot \frac{1-pq_{n_k}}{pq_{n_k}} = g_r(\alpha_r(q_{n_k})) < g_r(\alpha_r(q) - \varepsilon) < g_r(\alpha_r(q)) = \frac{p}{1-p} \cdot \frac{1-pq}{pq}. \quad (\text{A6})$$

Apparently, this is a contradiction to  $q_{n_k} \rightarrow q$ .  $\square$

## 2.

In the following, we shall show the continuity of  $G(q) = \int_0^{\bar{r}} G_r(q)f(r)dr$  in  $q$ . For that purpose, assume  $q_n \rightarrow q$  to be any sequence with  $q_n, q \in [1, \bar{q}]$ . Due to measurability of  $\alpha^*(U_r, q)$  in  $r$ , the functions  $G_r(q_n)f(r)$  and  $G_r(q)f(r)$  are measurable in  $r$ .

Resulting from continuity of  $\alpha^*(U_r, q)$  in  $q$ , we get

$$G_r(q_n)f(r) = \alpha^*(U_r, q_n)(q_n - 1)E(X)f(r) \rightarrow \alpha^*(U_r, q)(q - 1)E(X)f(r) = G_r(q)f(r). \quad (\text{A7})$$

In addition, we have

$$|G_r(q_n)f(r)| = |\alpha^*(U_r, q_n)(q_n - 1)E(X)f(r)| \stackrel{r \in [0, \bar{r}]}{\leq} 1 \bar{q} E(X) \max_{r \in [0, \bar{r}]} f(r) =: g \quad (\text{A8})$$

for an integrable function  $g$ . Following the Dominated Convergence Theorem<sup>26</sup>, we may interchange integration and limes so that

$$\int G_r(q)f(r)dr = \lim_{n \rightarrow \infty} \int G_r(q_n)f(r)dr. \quad (\text{A9})$$

As a consequence, we obtain  $G(q_n) \rightarrow G(q)$ . This proves the continuity of  $G$  in  $q$ .  $\square$

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<sup>26</sup> For the Dominated Convergence Theorem, see Browder (1996), p. 230.

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