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# A Model of Costly Capital Reallocation and Aggregate Productivity

by Shutao Cao

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## Abstract

The author studies the effects of capital reallocation (the flow of productive capital across firms and establishments mainly through changes in ownership) on aggregate labour productivity. Capital reallocation is an important activity in the United States: on average, its total value is 3–4 per cent of U.S. GDP. Firms with lower productivity are more likely to be reallocated to (i.e., bought by) more productive firms. Reallocated establishments experience an increase in productivity. The author develops a dynamic model of capital reallocation and compares its predictions with U.S. data. In the model, limited participation in acquisition markets by heterogeneous firms results in an increase in aggregate productivity. With reasonably chosen parameter values, policy experiments show that the increased reallocation of capital and labour contributed as much as a 17 per cent improvement in aggregate labour productivity in the mid-1980s. When a positive total-factor-productivity shock occurs, in steady state the increase in aggregate productivity arises entirely from this shock, and reallocation is unaffected.

JEL classification: E22, L16 Bank classification: Productivity; Economic models

## Résumé

L'auteur étudie les effets de la réaffectation du capital (flux du capital productif entre les entreprises et les établissements, surtout à la faveur d'un changement de propriété) sur la productivité globale du travail. Aux États-Unis, la réaffectation du capital a un rôle important puisque les montants concernés représentent en moyenne 3 à 4 % du PIB au total. Les firmes productives sont plus susceptibles d'acquérir les actifs de sociétés peu performantes, et les entreprises qui changent de propriétaire voient leur productivité augmenter. L'auteur construit un modèle dynamique pour quantifier la réaffectation du capital et compare les prédictions obtenues aux données américaines. Dans ce modèle, l'activité limitée de sociétés hétérogènes sur les marchés d'acquisition entraîne une hausse de la productivité globale. Des simulations réalisées à l'aide de valeurs paramétriques plausibles montrent que la réaffectation accrue du capital et de la main-d'œuvre est à l'origine d'une progression de 17 % de la productivité globale du travail au milieu des années 1980. Lorsqu'une hausse de la productivité totale des facteurs est observée en régime permanent, la croissance de la productivité globale s'explique entièrement par ce choc; la réaffectation du capital n'y est pour rien.

Classification JEL : E22, L16 Classification de la Banque : Modèles économiques; Productivité

#### 1. INTRODUCTION

It is well documented in the empirical literature that the reallocation of production inputs across firms and establishments is an important source of changes in aggregate productivity. As surveyed in Bartelsman and Doms (2000), there is a large amount of productivity dispersion. This dispersion is persistent over time, and much of the growth in aggregate productivity is attributable to resource reallocation.<sup>1</sup>

A change of ownership and therefore of assets across firms and establishments is an important component of resource reallocation. Data on publicly traded firms show that, between 1986 and 2004, an average of 4 per cent of firms exited each year from the data, of which 60 per cent were due to mergers and acquisitions. The share of capital of exited firms is 1.3 per cent of the total capital in an average year. Capital reallocation through changes in ownership has been increasing since the 1960s in the United States. In 1999, the total value of reallocation reached a record high: 15.4 per cent of the GDP. The ratio of annual reallocated capital over total capital stock was 8.7 per cent between 1971 and 2004.

How does a change in ownership of capital affect labour productivity and employment in an economy? This question is important for two reasons. First, studying the impacts of changes in ownership on productivity and employment helps explain the motives for changes in ownership. Second, and more importantly, government policies on changes in asset ownership can be evaluated in terms of productivity and welfare. If capital reallocation increases productivity overall, then policies that facilitate reallocation are productivity-improving.

This paper studies the effects that changes in ownership have on aggregate productivity in the United States, and it replicates the stylized facts of reallocation. Two types of reallocation are studied. One relates to entry and exit, and has been extensively investigated. For example, Davis, Haltiwanger, and Schuh (1996) find that about 20 per cent of job destruction and 15 per cent of job creation in manufacturing industries are due to entry and exit. The other type of reallocation is capital reallocation through changes in ownership. The model in this paper focuses more on capital reallocation by quantifying its contribution to changes in aggregate productivity.

In the industrial organization and finance literature, many studies analyze the motives of change in ownership and its relation to the stock market, competition, and firm organization. Only a few empirical models address the relation between changes in ownership and establishment productiv-

<sup>&</sup>lt;sup>1</sup>Studies that use plant-level data from the U.S. manufacturing sector show that about 25 per cent of increase in productivity during the 1980s resulted from a reallocation of resources from plants with low productivity to those with high productivity. The output share of establishments (which is related to reallocation) have positive effects on industry-level productivity. The growth rate of net industry productivity would be negative if there was no share effect between 1972 and 1977. In fact, at least 20 per cent of the growth in manufacturing productivity in the 1980s was due to resource reallocation. See Bartelsman and Dhrymes (1998) and Baily, Hulten, and Campbell (1992).

ity and employment. Lichtenberg and Siegel (1989) use data on large U.S. manufacturing plants from the Longitudinal Research Dataset (LRD) to study the effects of changes in ownership on productivity. About 21 per cent of the plants in their sample changed ownership at least once over a 10-year period. Lichtenberg and Siegel find that plants with lower total factor productivity (TFP) are more likely to be sold. They attribute the low productivity to bad random matches between plants and their parent firms (owners). Plants that change ownership experience a higher growth of productivity in the following years. The authors find that, for transferred plants, the average productivity residual increases by 23 per cent. In contrast, McGuckin and Nguyen (1995) use the LRD data for the food manufacturing industry and find that the labour productivity of plants with changes in ownership is about 20 per cent higher than the industry average at the time of the transaction, although, for large plants, the acquired plants tend to have lower productivity. Moreover, plants that experience changes in ownership gain productivity during the 5 to 9 years following these changes.

Maksimovic and Phillips (2001) extend these studies by looking into more broad ownership changes that include partial establishment sales in LRD from 1974 to 1992. They find that, on average, 3.89 per cent of establishments change ownership annually. Plants or whole firms that are transacted experience significant gains in productivity. The buyers tend to have higher TFP and tend to be larger than the sellers.<sup>2</sup>

More recently, using data on changes in asset ownership matched with the Longitudinal Business Data (LBD) from 1980 to 2005, Davis, Haltiwanger, Jarmin, Lerner, and Miranda (2008) investigate the impact of changes in ownership on employment growth. These authors find the acquired establishments experience an average -7 per cent of net employment growth in the year after transaction, and -11 per cent of net employment growth in the second year since transaction. The job destruction rates of these acquired establishments in the first and second year following transaction are respectively 18 per cent and 22 per cent, significantly higher than similar establishments that did not experience changes in ownership. However, the authors find that there is little difference in the post-transaction growth of the acquired establishments in the manufacturing sector

The above noted empirical papers provide empirical evidence for our model. Their findings are based on various microdata samples. But none of them investigates the impact of changes in capital ownership on aggregate productivity with a general-equilibrium model. The only macroeconomic models of capital reallocation are Jovanovic and Rousseau (2004) and Eisfeldt and Rampini (2006). Jovanovic and Rousseau (2004) use technology adoption to explain the waves of mergers in the United States. When a new technology is invented, firms can adopt it or choose to exit through

 $<sup>^{2}</sup>$ Siegel, Simons, and Lindstrom (2005) use the matched employer-employee data of the Swedish manufacturing plants, and find results similar to those obtained with the U.S. data.

acquisitions. Eisfeldt and Rampini (2006) study the cyclicality of capital reallocation. Neither paper quantifies the effects of reallocation on productivity.

This paper contributes to the research on reallocation by providing the first quantitative model of the contribution made by changes in ownership to aggregate productivity in a stationary economy. It also provides a framework for evaluating policies dealing with reallocation, such as an antitrust policy. A change of ownership of capital is modelled as a large investment by the buyers and a large disinvestment by the sellers. Firms are faced with heterogeneous shocks to their managerial ability. By managerial ability, we mean how effectively the manager organizes production and chooses technology. A firm meets its demand for capital by investing either in new capital markets or in acquisition markets. The optimal investment and capital reallocation is determined by the related costs and the shocks to managerial ability. The gain in productivity arises from labour saving as reallocated capital increases. The model is able to match the moments of investment and capital reallocation for the United States. With reasonably chosen parameter values, we find that increased capital reallocation accounts for 17 per cent of the increase in aggregate labour productivity in the mid-1980s. The model concludes that the economy-wide improvement in aggregate technology in the late 1990s has had a small impact on capital reallocation, and contributes more to the gain in productivity in that period by decreasing the demand for labour. Close to 40 per cent of the growth in aggregate labour productivity can be accounted for by changes in labour choice caused by the improvement in aggregate technology. Finally, the transition dynamics show that a reduced fixed cost of reallocation induces a large but temporary increase in aggregate labour productivity and a temporary decrease in aggregate capital in the economy.

Section 2 describes some stylized facts of capital reallocation and specifies the model. Section 3 defines the stationary equilibrium. Section 4 reports the results of two policy experiments and the decomposition of gains in productivity. Section 5 provides the results of transition dynamics. Section 6 offers some conclusions and identifies areas for future work.

#### 2. THE MODEL

The model is set up to be consistent with the following stylized facts of changes in capital ownership. First, establishments with lower productivity are more likely to be transferred than those with higher productivity. The probability of a change in ownership declines as plant size increases. Second, buyers tend to be larger and more productive than sellers. Third, establishments with changes in ownership experience significant growth in productivity, as found in McGuckin and Nguyen (1995), Lichtenberg and Siegel (1989), Lichtenberg (1992), and Maksimovic and Phillips (2001), among others. Finally, following the change in ownership, the acquiring firm's existing business does not show significant changes in productivity. The model economy is composed of one representative household and a continuum of firms. As stated earlier, firms are faced with heterogeneous shocks to their managerial ability. One homogeneous good is produced by firms and used for investment and consumption. The market is perfectly competitive. In each period, the firm makes decisions on investment in capital, on employment, and on capital purchased from other firms. The household in each period makes decisions about consumption, investment, and labour supply.

Capital purchased from the household is called unbundled capital, and capital purchased from other firms is called bundled capital. Bundling is endogenously determined by the fixed reallocation cost and the relative price of the bundled capital. The fixed reallocation cost results in a limited participation in the bundled capital market. The reallocated capital experiences a gain in productivity in that it is used for production in combination with the buyer's superior managerial ability. In addition, less-productive firms experience gains in productivity because they downsize by selling more capital in the acquisition market.

#### $2.1.\ Firm$

At the firm level, our model shares features with two micro models, taking mergers and acquisitions as a method of investment. Erard and Schaller (2002) estimate a system of two equations of the Q regression. They find that both investment and acquisitions are correlated to the buyer's Q value. In a similar study, motivated by the facts that in nearly 70 per cent of mergers and acquisitions in the United States the buyer's Tobin's Q value is larger than the seller's Tobin's Qvalue, Jovanovic and Rousseau (2002) use the Q theory to explain why some firms buy others. The authors regress a bundled investment rate on a firm's Q values and find that the Q value and the bundled investment rate are significantly and positively correlated.

We abstract organizational issues by assuming that each firm is operated by a manager who maximizes the firm's profit.<sup>3</sup> We denote managerial ability using  $\varepsilon$ . Using the same amount of capital and labour, managers with higher  $\varepsilon$  produce more. A firm uses managerial ability, labour, l, and capital, k, to produce a good. The production function is  $f(k, l, \varepsilon) = z\varepsilon (k^{1-\alpha}l^{\alpha})^{\nu}$ , where

<sup>&</sup>lt;sup>3</sup>In reality, firms own plants. Here, we implicitly assume that all plants owned by the same firm receive the same management shock, which is firm-specific because all plants of a firm are under the same management. Griffith, Haskel, and Neely (2006) provide some evidence on productivity dispersion within a firm.

Under this assumption, if the firm owns n > 1 plants, and its production function is of the constant elasticity of substitution (CES) type, that is,  $y = z\varepsilon \left[\sum_{i=1}^{n} f(k_i, l_i)^{\gamma}\right]^{\frac{\nu}{\gamma}}$ , then the optimal sizes of all plants should be equal to each other, given that plants pay the same capital price and the same wage rate. It can be shown that the optimal number of plants, n, is undetermined. This is because the optimal capital is  $k_i^* = \left[\Delta n^{1-\frac{\nu}{\gamma}}\right]^{\frac{1}{\nu-1}}$ , with  $\Delta$  being a constant, including the shock. The optimal labour demand is  $l_i^* = \frac{r\alpha}{w(1-\alpha)}k^*$ . If we use the two-stage optimization, we can plug the optimal  $k_i^*$  and  $l_i^*$  into the firm's problem, and then it is straightforward to find that the optimal n is undetermined. This indicates that the firm can own many small plants or a few large plants to achieve the same level of profits.

 $\alpha \in (0,1)$  and  $\nu \in (0,1)$ . The decreasing returns to scale imply that the variable profit is positive in equilibrium, and that firm size is finite and bounded from above and zero.

The aggregate technology level is z, which is constant.<sup>4</sup> We use it for comparative statics in later sections.

With the above firm production technology, we ignore management as an agency problem by implicitly assuming that the manager maximizes the firm's profit. Agency problems can be serious in reallocation activities because, in some cases, the selling firm's managers can be offered a large amount of compensation in order to sell the business under their control.

ASSUMPTION 1 The shock to a firm's managerial ability follows an AR(1) process,  $\log \varepsilon_t = \rho_{\varepsilon} \log \varepsilon_{t-1} + \eta_t$ . The innovation term  $\eta_t$  is identically and independently distributed, with  $\eta_t \sim N(0, \sigma_n^2)$ .

The labour market is perfectly competitive. Adjusting labour does not incur any cost, so the firm's static labour choice is

$$l^* = \left(\frac{\alpha_2 z\varepsilon}{w}\right)^{\frac{1}{1-\alpha_2}} k^{\frac{\alpha_1}{1-\alpha_2}},$$

where  $\alpha_1 = (1 - \alpha)\nu$  and  $\alpha_2 = \alpha\nu$ . The firm's profit net of the labour cost is

$$R(\varepsilon, k) = \frac{1 - \alpha_2}{\alpha_2} w l^*.$$

The firm can buy (sell) capital from (to) the output market or the acquisition market. In the acquisition market, the firm buys capital from another firm. Let  $x_u$  denote the capital investment in the output market, and  $x_b$  denote the capital investment in the acquisition market. Following Jovanovic and Rousseau (2002), if the firm buys capital in the output market, the adjustment cost of capital  $h_1(x_u, k)$  is convex. If the firm buys capital from the acquisition market, it pays  $p_b$  per unit of capital. The adjustment cost for capital acquired from the acquisition market is  $h_2(x_b, k)$ , which is also convex. In addition, firms that participate in the acquisition market pay a fixed cost,  $\Phi k$ . We make the following assumption on convex adjustment costs:

Assumption 2 
$$\frac{\partial h_1(x_u,k)}{\partial x_u}|_{x_u=x} > \frac{\partial h_2(x_b,k)}{\partial x_b}|_{x_b=x}.$$

From a given investment level, the cost of an additional unit of investment in unbundled capital is larger than the cost of one additional unit of investment in bundled capital. This assumption

<sup>&</sup>lt;sup>4</sup>In a forthcoming paper, I extend the current model by allowing aggregate uncertainty, to study the cyclicality of reallocation and its effects on productivity dynamics.

implies that, without a fixed cost, the firm always buys or sells capital in the acquisition market.

Note that buying and selling the same amount of capital incurs the same level of adjustment cost. Adjusting employment is nevertheless costless. With these assumptions on the investment costs, if the demand for investment is small, the firm buys capital only from the output market, because in doing so the firm does not pay the fixed cost. As demand increases, the adjustment cost of investing in unbundled capital increases faster than the adjustment cost of investment in bundled capital. After some threshold is reached in the investment level, it is cheaper for the firm to buy a fraction of capital from the acquisition market. Therefore, we also refer to the acquisition market as the bundled capital market. The intuition is that assembling new (unbundled) capital, and training workers to use it, is more costly than if the firm buys bundled capital. However, because of the fixed cost of searching for bundled capital, the firm would rather buy unbundled capital when its demand for investment is low. As the demand for investment grows, the assembling costs and training costs are too large for the firm to invest only in unbundled capital; the firm will switch some investment to bundled capital, because with bundled capital the firm pays less for assembling capital and training workers. On the other hand, if the firm sells a large amount of capital, it is more costly to disassemble it for sale in the output market. It is then optimal to sell it in the bundled capital market.

Let  $v(\varepsilon, k)$  be the value of an incumbent firm in any period. The incumbent firm's recursive problem is

$$\begin{aligned} v(\varepsilon,k) &= \max_{\{x_u,x_b\}} R(\varepsilon,k) - c_f - x_u - p_b x_b - h_1(x_u,k) - h_2(x_b,k) - \Phi k \cdot \mathbf{1}_{\{x_b \neq 0\}} \\ &+ \frac{1}{1+r} (1 - \pi_d) E v(\varepsilon',k'), \end{aligned}$$

subject to  $k' = (1 - \delta)k + x_u + x_b$ . The fixed cost  $\Phi = 0$  if  $x_b = 0$ . The gross interest rate (1 + r) is endogenously determined in equilibrium. The exogenous exit probability is  $\pi_d$ .

#### Optimal investment

Let the investment rate be i = x/k, and the bundled capital investment rate be  $i_b = x_b/k$ . Given a total investment level, the firm's choice of splitting between  $x_b$  and  $x_u$  is determined by a static cost minimization. Let  $\tilde{i}$  be the investment rate, where the firm is indifferent between choosing  $i_b = 0$  and  $i_b \neq 0$ . Suppose that  $h_1(x_u, k) = \frac{\gamma_u}{2} \frac{x_u^2}{k}$  and  $h_2(x_b, k) = \frac{\gamma_b}{2} \frac{x_b^2}{k}$  with  $\gamma_b < \gamma_u$ . At  $\tilde{i}$ , the unit cost of investing in new capital only equals the unit cost of investing in both types of capital,

$$\tilde{i} + \frac{\gamma_u}{2}\tilde{i}^2 = \min_{i_b}\{\tilde{i} - i_b + p_b i_b + \frac{\gamma_u}{2}(\tilde{i} - i_b)^2 + \frac{\gamma_b}{2}i_b^2 + \Phi\}.$$

If the firm chooses both types of investment, the optimal choice of  $i_b$  is  $i_b^* = \frac{\gamma_u}{\gamma_u + \gamma_b}i + \frac{1-p_b}{\gamma_u + \gamma_b}$ . As *i* increases, both the left- and right-hand sides increase, but the right-hand side increases less as *i* rises, since  $\gamma_b < \gamma_u$ . When  $i < \tilde{i}$ , the left-hand side is smaller than the right-hand side. When  $i > \tilde{i}$ , the left-hand side becomes larger. The threshold investment level  $\tilde{i}$  can be obtained by solving the above equation after obtaining the optimal  $i_b$ . Because the fixed cost is proportional to the firm's capital stock, the threshold value  $\tilde{i}$  is determined only by the cost parameters; hence, it is independent of the firm's state variables. If the fixed cost is  $\Phi$ , instead of  $\Phi k$ , then the absolute value of  $\tilde{i}$  is decreasing in k.

PROPOSITION 1 When adjustment costs are quadratic and Assumption 2 holds, it is never optimal for the firm to choose  $x_u = 0$ .

In this set-up, the firm is allowed to buy (sell) capital in one market and sell (buy) in the other market. It can be shown that, when  $p_b < 1 - \gamma_b \sqrt{\frac{2\Phi}{\gamma_u + \gamma_b}}$ , the firm chooses  $i_b > 0$  and  $i_u < 0$ . When  $p_b > 1 + \gamma_b \sqrt{\frac{2\Phi}{\gamma_u + \gamma_b}}$ , the firm chooses  $i_b < 0$  and  $i_u > 0$ . See Appendix B for details.

Figure 1 illustrates the firm's trade-off between unbundled capital and bundled capital. Each curve shows the total expenditure of investment per unit of current capital stock (investment plus adjustment costs per unit of capital stock). To draw this figure, we choose  $\gamma_u = 0.758$ ,  $\gamma_b = 0.474$ ,  $\Phi = 0.059$ , and  $p_b = 1.10$ . Figure 2 shows an example of the optimal split between investments in unbundled and bundled capital.

The capital adjustment cost and fixed participation cost are employed to summarize the frictions in the capital market and the acquisition market. Alternative modelling of frictions could be the firm's financial constraint, information asymmetry, and adverse selection in the bundled capital market. Adverse selection due to asymmetric information seems to be relevant in the acquisition market. Empirical evidence shows that a significant portion of capital reallocation fails to improve productivity. As Kaplan (2000) points out, the extent to which buyers understand the target before acquisition is an important factor affecting the performance of acquired capital. Acquisition failure can occur because the buyers do not have sufficient information about the target. House and Leahy (2004) develop an sS model of the used car market where the agent's adjustment costs of purchasing and selling a car arise from an adverse selection problem. Eisfeldt (2004) considers a model similar to House and Leahy (2004), but focuses on the interaction between liquidity and adverse selection in an equity market. We can interpret the fixed cost as it arises from the adverse selection problem or financial frictions. Our specification of investment costs summarizes all possible frictions, so that our model is general enough to allow for frictions other than from information and financial markets. Meanwhile, we do not need to explicitly model these forms of friction, which can be very



FIGURE 1: Comparing Costs of Investment



FIGURE 2: Optimal split between bundled and unbundled capital

complicated.

#### Entry

A continuum of potential entrants exists, who decide whether to enter the industry. At the beginning of period t, the entrant makes the start-up investment decision before it draws an idiosyncratic productivity shock (by hiring a manager) from the distribution  $\Omega(\varepsilon)$ . It is assumed that  $\Omega(\varepsilon)$  is an independent and identical distribution across entrants and over time. The entrant pays a cost  $(\psi k + c_e)$  and decides upon the start-up capital level by solving the following problem<sup>5</sup>:

(1) 
$$v^0(k_0) = \max_k \int_{\varepsilon \in \mathcal{E}} \frac{1}{1+r} v(\varepsilon, k) d\Omega(\varepsilon) - \psi k - c_e.$$

The free-entry condition implies that  $v^0(k_0) \leq 0$ . When  $v^0(k_0) < 0$ , there is zero entry. In the rest of this paper, only the case with positive entry,  $v^0(k_0) = 0$ , is discussed.

The entrants' productivity shocks are drawn from a log-uniform distribution, with the same support as the invariant distribution of the incumbents' productivity shock. The incumbents' invariant distribution of productivity shocks is log-normal. Hence the entrants are, on average, more productive than the incumbents, but their productivity shocks are more dispersed. In computing the model, we discretize the AR(1) process of  $\varepsilon$  so that the log-uniform distribution has a bounded support.

#### 2.2. Firm distribution

Let the firm's policy function be  $k' = g(\varepsilon, k) \in \mathcal{K}$ . The firm distribution over  $(\varepsilon, k)$  can be summarized by the probability measure  $\mu$  defined on S, where S is the  $\sigma$  field generated by the open subsets of product space  $(\mathcal{E}, \mathcal{K})$ . Let M be the mass of entering capital. The evolution of the firm distribution is

(2) 
$$\mu'(\varepsilon',k') = \int_{\mathbb{S}} (1-\pi_d)\pi(\varepsilon',\varepsilon) \mathbf{1}_{\{k'=g(\varepsilon,k)\}} d\mu(\varepsilon,k) + \mathbf{1}_{\{k_0\in\mathcal{K}\}} M\Omega(\varepsilon'),$$

where  $\pi(\varepsilon', \varepsilon)$  is the transition matrix of  $\varepsilon$ . The term  $1_{\{k_0 \in \mathcal{K}\}}$  is a vector of zeros, except that where  $k = k_0$  it is one, because all entrants enter with the same amount of capital. On the right-hand side, the first term is the conditional distribution of firms that stay in the industry. The second term is the distribution of entrants on S. Let  $P(\varepsilon', k'|\varepsilon, k) = \pi(\varepsilon', \varepsilon) 1_{\{k'=g(\varepsilon,k)\}}$ . It is the transition matrix of the state  $(\varepsilon, k)$ . Then,  $P(\varepsilon', k'|\varepsilon, k)\mu(\varepsilon, k) = \int_{\mathbb{S}} P(\varepsilon', k'|\varepsilon, k)d\mu(\varepsilon, k)$ . The evolution of the firm distribution can be written as

$$\mu'(\varepsilon',k') = (1 - \pi_d) P(\varepsilon',k'|\varepsilon,k) \mu(\varepsilon,k) + \mathbb{1}_{\{k_0 \in \mathcal{K}\}} M\Omega(\varepsilon').$$

If the invariant distribution exists, then we have

(3) 
$$\mu(\varepsilon, k) = [I_0 - (1 - \pi_d)P]^{-1} \cdot [M1_{\{k_0 \in \mathcal{K}\}}\Omega(\varepsilon)],$$

<sup>&</sup>lt;sup>5</sup>For simplicity, we have assumed that the entrant buys capital from the unbundled capital market. Otherwise, the relative price of bundled capital,  $p_b$ , would be present in the costs of entry.

where  $I_0$  is the identity matrix. The timing of this process is as follows: at the beginning of period t, the firm distribution over  $(\varepsilon, k)$  is  $\mu_t$ . Of this distribution, a proportion,  $\pi_d$ , of the firms exit before commencing production. The entrants enter at the beginning of period t with the entry measure  $1_{\{k_0 \in \mathcal{K}\}}M$ . The period t entrants then draw their productivity shock from  $\Omega(\varepsilon)$ . Therefore, the measure of entrants  $1_{\{k_0 \in \mathcal{K}\}}M\Omega(\varepsilon')$  is not part of the distribution  $\mu_t$ . With the new entrants and those that exit by shutting down, the industry reaches the end of period t with a new firm distribution.

#### 2.3. Household

The economy has one representative household with preference

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) - \xi L_t],$$

where  $c_t$  is consumption and  $L_t$  is the fraction of individuals employed. This preference is used in Hansen (1985) and Rogerson (1988). Since there is no aggregate uncertainty, the household's optimization is deterministic. The household owns all firms. In each period, the household chooses the optimal consumption, the labour supply, and investment in firm shares. Let  $w(\mu)$  be the wage rate relative to the output price, and let  $dQ(\varepsilon, k)$  be the household's portfolio of the one-period shares of firms with  $\varepsilon$  and k. Also, let  $\rho(\varepsilon, k, \mu)$  be the share price of all firms with  $\varepsilon$  and k. The household buys the firm portfolios at the beginning of each period. The household's recursive optimization problem is

$$W(Q,\mu) = \max_{c,L,Q'} \left[ u(c,L) + \beta W(Q',\mu') \right],$$

subject to

$$c + \int_{\mathbb{S}} \rho(\varepsilon',k') dQ'(\varepsilon',k') \leq w(\mu)L + \int_{\mathbb{S}} v(\varepsilon,k) dQ(\varepsilon,k)$$

On the left-hand side of the budget constraint,  $\rho(\varepsilon', k', \mu)$  is the price of the firm that enters the next period with  $\varepsilon'$  and k'. In a stationary equilibrium, the number of firms with  $\varepsilon'$  is certain and given by the invariant distribution of  $\varepsilon$ . In this sense, the household's portfolio  $dQ(\varepsilon, k)$  is risk free.

Let the shadow price of the firm's output be p; it is the Lagrange multiplier in the household's optimization problem. The first-order conditions are  $u_1(c,L) = p$ ,  $w = -\frac{u_2(c,L)}{u_1(c,L)}$ , and  $\rho(\varepsilon',k') = \beta \frac{p'}{p} v(\varepsilon',k')$ . The last condition is the standard asset-pricing equation. The price of the firm equals the discounted firm value. It is discounted because the household buys the shares for the next

period in the current period.

#### 3. EQUILIBRIUM ANALYSIS

Our focus is on the stationary equilibrium with positive entry and exit. First, the market clearing conditions are analyzed. Then, the definition of equilibrium is given.

#### 3.1. Output market

Let the household preference be  $u(c, L) = \log c - \xi L$ . From the household's problem, the aggregate consumption is  $C = \frac{1}{p}$ . Not surprisingly for the quasi-linear preference, the optimal aggregate consumption is independent of the non-labour revenue. In addition, the wage rate is  $w = \frac{\xi}{p}$ .

In the output market, the net aggregate output that the household can use for consumption in each period is computed as the weighted output level of all firms. In the stationary equilibrium, the continuing firm has the following real revenue:

$$\Pi(\varepsilon,k) = z\varepsilon(k^{1-\alpha}l^{\alpha})^{\nu} - c_f - x_u(\varepsilon,k) - p_b x_b(\varepsilon,k) - h_1(x_u(\varepsilon,k),k) - h_2(x_b(\varepsilon,k),k).$$

This is the net output before paying the labour cost, so it equals the total output minus all the investment costs and the fixed production cost. If the firm exits, the sell-off value is zero. The labour income is cancelled out in the budget constraint condition. The net aggregate output is the output of all the firms net of investment and related costs by incumbent firms and entrants:

$$C^{s}(\mu) = (1 - \pi_{d}) \int_{\mathbb{S}} \Pi(\varepsilon, k) \mathbf{1}_{\{g(\varepsilon, k) \in \mathcal{K}\}} d\mu(\varepsilon, k) + \int \Pi(\varepsilon, k) \mathbf{1}_{\{k_{0} \in \mathcal{K}\}} M d\Omega(\varepsilon') - M(\psi k_{0} + c_{e}).$$

The first term on the right-hand side is the aggregate output of continuing firms, and the last term is the total cost of entry. Note that, in equilibrium, the aggregate net investment in bundled capital is zero. The acquisition market affects the consumption only because it is costly to participate in the acquisition market.

#### 3.2. Acquisition market

Appendix B shows that there exists a threshold value,  $\tilde{i}$ , at which the firm is indifferent in choosing  $x_b = 0$ . Given the price  $p_b$ , and if the firm participates in the acquisition market, we can use the firm's optimal split-decision conditions to find that the firm  $(\varepsilon, k)$  buys bundled capital if its total investment rate  $i(\varepsilon, k) > \tilde{i_2}$ , and it sells bundled capital if  $i(\varepsilon, k) < \tilde{i_1}$ , where  $\tilde{i_1} = \frac{1}{\gamma_n}(p_b - 1 - \sqrt{2\Phi(\gamma_n + \gamma_a)})$  and  $\tilde{i_2} = \frac{1}{\gamma_n}(p_b - 1 + \sqrt{2\Phi(\gamma_n + \gamma_a)})$ . As the price,  $p_b$ , increases, the firm tends to buy less or sell more bundled capital.

The measure of capital being sold is

$$\Lambda(p_b) = (1 - \pi_d) \int_{\mathbb{S}} \mathbb{1}_{\{x_b(\varepsilon, k) < 0\}} x_b(\varepsilon, k) d\mu(\varepsilon, k).$$

 $\Lambda(p_b)$  is negative and a decreasing function of price  $p_b$ . The measure of capital being purchased is

$$\Psi(p_b) = (1 - \pi_d) \int_{\mathbb{S}} \mathbb{1}_{\{x_b(\varepsilon, k) > 0\}} x_b(\varepsilon, k) d\mu(\varepsilon, k),$$

which is positive and decreasing in  $p_b$ . In equilibrium, the two measures sum to zero.

In this model, we assume that the production technology is not embodied in capital. If technology were embodied in capital, the price of bundled capital would be determined differently. In that case, the older the capital, the more outdated the technology, hence the less attractive it is in the market. Thus, firms with older capital tend to sell more capital in the unbundled capital market than firms with relatively newer capital. This may drive up the price of bundled capital.<sup>6</sup>

#### 3.3. Recursive equilibrium

We focus on the stationary recursive equilibrium, which is defined as a set of functions,

$$\left(w, p, r, p_b, \rho, v, L^d, K', W, C, L^s, M, Q, \mu\right)$$

such that the household and firms maximize their expected values, and the markets for reallocation, assets, labour, and output clear<sup>7</sup>:

- (i) Given prices, v solves the firm's Bellman equation, and l(ε, k, μ) and g(ε, k, μ) are the firm's policy functions for labour and capital, for all (ε, k) ∈ S. The aggregate future capital is K'(μ) = ∫<sub>μ∈S</sub> g(ε, k, μ)dμ.
- (ii) Given prices, W satisfies the household's problem.  $(C, L^s)$  are the associated policy functions.
- (iii) The acquisition market clears:  $p_b$  solves  $\Psi(p_b) + \Lambda(p_b) = 0$ .
- (iv) The asset market clears:  $Q(\varepsilon, k) = \mu(\varepsilon, k)$  for all  $(\varepsilon, k) \in \mathbb{S}$ .
- (v) The output market clears:  $C = C^{s}(\mu)$ .
- (vi) The firm distribution is given by Equation (3).
- (vii) The free-entry condition is satisfied; i.e,  $v^0(k_0) = 0$ .

<sup>&</sup>lt;sup>6</sup>I thank Jon Willis for pointing this out.

<sup>&</sup>lt;sup>7</sup>For the existence of the equilibrium and the invariant distribution  $\mu(\varepsilon, k)$ , see Stokey, Lucas, and Prescott (1989) and Hopenhayn and Prescott (1992). These authors prove the existence of stationary equilibrium without the aggregate uncertainty. In our case, the proof is left to be done, although the equilibrium seems to exist after the heuristic check.

As noted earlier, the household's first-order necessary conditions are  $-\frac{u_2(c,L)}{u_1(c,L)} = w(\mu)$  and  $p = u_1(c,L)$ . In equilibrium, we have  $\frac{1}{1+r} = \frac{\beta u_1(c',L')}{u_1(c,L)}$  or  $1 = \frac{p}{p'} = \beta(1+r)$ , so the interest rate is  $\frac{1}{\beta} - 1$ . The interpretation of these necessary conditions is standard: the household chooses the optimal consumption to equalize the market values of marginal utility between two periods. Within the period, the household's marginal rate of substitution of consumption and leisure equals the wage rate.

#### 3.4. Model solution

To solve for an equilibrium, note that the price of output satisfies  $p(\mu) = \frac{1}{C}$  and  $w(\mu) = \xi C$ . From the equilibrium condition with respect to output, we know that  $\beta(1+r) = \frac{p}{p'}$ . The equilibrium can be computed by solving the Bellman equation, which combines the firm's dynamic problem and the household's first-order conditions. Define  $V = p \cdot v$ . After plugging the household's optimal decision rules and condition  $\beta(1+r) = 1$  into the firm's problem, the reformulated recursive problem is obtained as follows:

(4)  

$$V(\varepsilon, k, \mu) = \max_{\{x_u, x_b\}} p(\mu) [R(\varepsilon, k, \mu) - c_f - x_u - p_b(\mu) x_b - h_1(x_u, k) - h_2(x_b, k) - \Phi_k \cdot 1_{\{x_b \neq 0\}}] + \beta (1 - \pi_d) EV(\varepsilon', k', \mu').$$

For given prices, each firm with  $(\varepsilon, k)$  makes the decisions on labour as shown in Section 2. Let the convex adjustment cost be  $h_1(x_u, k) = \frac{\gamma_u}{2} \frac{x_u^2}{k}$  and  $h_2(x_b, k) = \frac{\gamma_b}{2} \frac{x_b^2}{k}$  with  $0 < \gamma_b < \gamma_u$ . The optimal investment decision is

$$k' = g(\varepsilon, k, \mu), \quad \text{and} \quad x_b = \begin{cases} \left(\frac{\gamma_u}{\gamma_u + \gamma_b} \cdot i + \frac{1 - p_b}{\gamma_u + \gamma_b}\right) k, & \text{if } |i| > \tilde{i}; \\ 0, & \text{otherwise.} \end{cases}$$

The total investment in the unbundled capital is

$$X_u = (1 - \pi_d) \int_{\mathbb{S}} (x_u(\varepsilon, k)) \, d\mu(\varepsilon, k)$$

In principle, it can be optimal for firms to buy unbundled capital while selling bundled capital in equilibrium. This is not arbitrage, since the firm that does so is unable to buy an infinitely large amount of new capital and sell it in the acquisition market, because of the cost structure in both markets. In equilibrium, no firm sells capital in one market and buys capital in the other.

The computation of equilibrium will be based on the following two propositions. The proofs are shown in Appendix B.

**PROPOSITION 2** With the assumptions about production function and utility, the firm value func-

tion  $v(\varepsilon, k)$  is non-decreasing in the output price p.

An increase in p will cause the variable profit  $R(\varepsilon, k)$  to increase, because the latter is decreasing in wage w. Meanwhile, consumption decreases as a response to the increase in p. Hence, the investment in unbundled capital will increase, meaning that firms tend to buy more (or sell less) of the unbundled capital. From the Euler equation for the unbundled investment, the expected value function will increase as the investment increases. Therefore, the higher the price, the larger the value function.

PROPOSITION 3 With the assumptions about production function and utility, in equilibrium, the firm value function  $v(\varepsilon, k)$  is non-decreasing in the entry mass, M.

This proposition is obtained by the equilibrium output condition and the previous proposition. As M increases, the aggregate net output also increases. For the output market to be clearing, p must decrease and consumption increase. By the previous proposition,  $v(\varepsilon, k)$  also increases.

#### 4. REALLOCATION AND PRODUCTIVITY

This section quantifies how exogenous policy shocks in the economy affect reallocation and aggregate productivity measures. First, we describe the baseline computation, and then the steady-state results. The policy experiments try to link the model to the two recent waves of mergers in the United States, using the calibrated model to investigate the role of reallocation in changes in aggregate productivity.

The aggregate labour productivity is defined as  $Y_l = \frac{\int y_l(\varepsilon,k) d\mu(\varepsilon,k)}{\int d\mu(\varepsilon,k)}$ , where

(5) 
$$y_l = (\ln y - \ln l) = \ln z + \ln \varepsilon + (1 - \alpha)\nu \ln k + (\alpha \nu - 1) \ln l.$$

After plugging the optimal labour choice into this definition, we obtain the firm's labour productivity as a function of state variables:

$$y_l = \ln w - \ln \alpha - \ln \nu$$

Then, the change in the aggregate labour productivity between two steady-state equilibria is  $\Delta Y_l = y_l^1 - y_l^0$ . Another definition of aggregate labour productivity is  $Y'_l = \ln Y - \ln L$ , where Y is the aggregate output and L is the aggregate employment. It follows, then, that

$$Y'_l = \ln\left(\frac{\int y(\varepsilon, k)d\mu(\varepsilon, k)}{\int l(\varepsilon, k)d\mu(\varepsilon, k)}\right)$$

Because the output-labour ratios at the firm level are equal, this measure of aggregate productivity  $Y'_l$  reduces to  $Y_l$ .

Gains in productivity from reallocation are determined by the equilibrium wage rate and the firm distribution. Any barrier that decreases the wage rate and the firm's total measure necessarily causes a productivity loss. Firm distribution is important because it also affects the equilibrium wage rate.

#### 4.1. Productivity decomposition

Baily, Hulten, and Campbell (1992) decompose the aggregate TFP growth into incumbents, entrants, and exiting firms. In this model, the firm TFP measure is  $\varepsilon$ , which is assumed to be an AR(1) process. Its distribution is assumed to be exogenous. But with entry and exit, the equilibrium distribution of  $\varepsilon$  will change, depending on the level of entry and exit. The entrants are more productive, on average, than the incumbents. Thus, following entry, the distribution will change. The incumbents' productivity shock does not increase, but the share of incumbents changes. Therefore, all firms contribute to the aggregate TFP growth. The more interesting case is labour productivity, which can be decomposed similarly.

For convenience, we use the discrete firm distribution. Let  $\mu_{i0}$  be the share of firms with the same state  $(\varepsilon, k)_i$  at the steady state 0. The share of firms can be decomposed into entrants, incumbents, and exiting firms, with their shares being  $\theta_{i0}^n = \mu_{i0}^n / \sum_i \mu_{i0}$ ,  $\theta_{i0}^s = \mu_{i0}^s / \sum_i \mu_{i0}$ , and  $\theta_{i0}^x = \mu_{i0}^x / \sum_i \mu_{i0}$ , respectively. Then the steady-state labour productivity is decomposed as

$$Y_{l}^{0} = \left(\sum_{i} \theta_{i0}^{n} y_{l}^{i0} - \sum_{i} \theta_{i0}^{x} y_{l}^{i0} + \sum_{i} \theta_{i0}^{s} y_{l}^{i0}\right).$$

The aggregate labour productivity change between steady-state 0 and steady-state 1 is

(6) 
$$\Delta Y_l = Y_l^1 - Y_l^0 = \sum_i (\theta_{i1}^n y_l^{i1} - \theta_{i0}^n y_l^{i0}) - \sum_i (\theta_{i1}^x y_l^{i1} - \theta_{i0}^x y_l^{i0}) + \sum_i (\theta_{i1}^s y_l^{i1} - \theta_{i0}^s y_l^{i0}).$$

On the right-hand side, the first term is the contribution of entrants to the change in labour productivity. The share of entrants potentially changes if the barriers to reallocation change (e.g., fixed cost). Note that the firm-level labour productivity  $y_l^i$  does not necessarily change, depending on whether the shock is to the production technology or to the reallocation cost. The second term is the contribution of exiting firms, and the last term is the contribution of firms staying in the industry.

#### 4.2. Parameters

The household discount factor is  $\beta = 0.9615$ , to reflect an annual interest rate of 4 per cent. The decreasing return to scale is set as  $\nu = 0.896$ . The Cobb-Douglas technology parameter is  $\alpha = 0.7143$ , which implies that the labour share in production is close to 0.64, as in Prescott (1986). Given this parameter, the capital output ratio is about 2.353. The capital depreciation rate is  $\delta = 0.07$ , as in Cooper and Haltiwanger (2006). The share of leisure in utility is  $\xi = 0.94$ , so that around 80 per cent of the population works in the steady state.

The first-order autocorrelation process of the firm-specific shock is from Cooper and Haltiwanger (2006),  $\rho_{\varepsilon} = 0.885$ ,  $\sigma_{\eta} = 0.30$ , where the AR(1) equation is  $\varepsilon_t = \rho_{\varepsilon}\varepsilon_{t-1} + \eta_t$ . These values are estimated by Cooper and Haltiwanger (2006) using the U.S. plant data. Their sample comprises mainly the large plants, most of which are owned by the publicly held firms. The aggregate technology is normalized to 1, and it changes in the second policy experiment.

The fixed cost,  $c_f$ , is set as zero. The exit probability is set as  $\pi_d = 0.015$ , so that the capital exit rate is 1.5 per cent.<sup>8</sup> The entry cost parameter is  $\psi = 1.0705$ , such that the entry rate and exit rate of capital are equal. The fixed entry cost is set as  $c_e = 6.40$ , with which the firm distribution has an approximately total measure of 1.

We estimate the parameters  $\gamma_u$ ,  $\gamma_b$ , and  $\Phi$  with the simulated method of moments where the model does not have entry and exit. The estimation method is borrowed from Cooper and Haltiwanger (2006), but our estimation is based on the general-equilibrium model with prices being endogenous. The estimation procedure consists of an inner loop and an outer loop. In the inner loop, equilibrium prices are solved given cost parameters, and the outer loop searches for parameter values to match moments. The moments we match in estimation are the average annual investment rate, the average annual reallocation rate, the proportion of firms with positive spikes, the proportion of firms with inaction, and the proportion of firms that participate in the acquisition market. The data moments are calculated from Compustat. Appendix A provides more details on the data moments.

The estimated parameter values are  $\gamma_u = 0.758$ ,  $\gamma_b = 0.474$ , and  $\Phi = 0.0585$ . The estimate of  $\gamma_u$  is within the reasonable range of estimates in Cooper and Haltiwanger (2006) and Hall (2004). The estimate of  $\gamma_b$  is close to the estimate implied in the Q theory of mergers by Jovanovic and Rousseau (2002), who estimate that the coefficient value for Q is 1.916, which implies that the quadratic adjustment cost in the bundled capital market is 0.52. In contrast, Erard and Schaller (2002) obtain a very small estimated value of 0.003, but they do not allow for the fixed cost.

Table 1 summarizes all parameter values we use for calibration.

 $<sup>^{8}</sup>$ In Compustat, from 1986 to 2004, on average, 4 per cent of firms exited from the data set each year. Of these firms, close to 60 per cent exited due to mergers and acquisition.

Table 1: Parameter Values Used for Calibration

$\beta$	ξ	$\alpha$	ν	δ	$ ho_{arepsilon}$	$\sigma_\eta$	$\gamma_u$	$\gamma_b$	$\Phi$
0.96	0.94	0.7143	0.896	0.07	0.885	0.30	0.758	0.474	0.059
$\pi_d$	$c_f$	$\psi$	$c_e$						
0.015	0.0	1.07	6.40						

#### 4.3. Baseline results

The equilibrium price p is obtained from the free-entry condition, and the output market clearing condition gives the entry mass, M. The acquisition market clearing condition gives the equilibrium  $p_b$ . The following algorithm is implemented<sup>9</sup>:

- (i) Fix a value of M.
- (ii) Given p and  $p_b$ , solve the reformulated Bellman equation. Corner solutions for bundled capital investment are taken care of by allowing firms to choose  $x_b = 0$ .
- (iii) Find the optimal capital at entry.
- (iv) Compute the stationary distribution.
- (v) Compute the aggregate output and the aggregate bundled capital investment.
- (vi) Check the acquisition market clearing. If it is not cleared, change  $p_b$  and go back to step (ii).
- (vii) Check the free-entry condition. If it does not hold, change p and go back to step (ii).
- (viii) Check the output market clearing. If it is not cleared, choose a new value of M and go back to step (vi), until it is cleared.

The computed moments are matched with the U.S. firm-level data. The investment and reallocation moments in the data are computed for the manufacturing industries in Compustat. Appendix A summarizes the data set and the procedures for computing the data moments. Table 2 summarizes our baseline results. The investment rate is the ratio of the average of firm-level values of unbundled capital investment to capital. The reallocation rate is the ratio of the average of firm-level values of bundled capital investment to capital. The positive spike rate is the average annual proportion of firms with an unbundled investment of greater than 20 per cent. Inaction is the average annual proportion of firms with an unbundled investment of less than 1 per cent. The participation rate is the average annual proportion of firms that have a non-zero bundled capital investment. The model moments are computed analogous to the data moments.

<sup>&</sup>lt;sup>9</sup>An alternative method to compute the equilibrium is by simulation. Given an initial firm distribution, we can simulate a panel of a large number of firms. In each simulation period, we need to solve for equilibrium prices to determine each firm's investment decisions. The invariant firm distribution can be obtained when the distributions from two consecutive periods are very close to each other.



FIGURE 3: Optimal Splits between Bundled and Unbundled Capital

	Inv. rate	$\operatorname{Spike}^+$	Spike <sup>-</sup>	Inaction	Cap. sale/Cap.	Acqui./Cap.	Participation
Data	0.18	0.23	-	0.031	0.014	0.087	0.24
Model	0.16	0.30	0.08	0.027	0.057	0.10	0.20

Table 2: Investment and Reallocation

In equilibrium, no firm buys capital in one market while selling capital in the other market. The baseline results show that our model can match the long-run data moments well. We also compute the aggregate productivity measures (Table 3). The output per unit of capital is computed as the sum of weighted outputs divided by the sum of weighted capital. The output per unit of labour and the output-to-investment ratio are computed in the same way. But the cost per unit of output is calculated as the total cost in the economy divided by the total output.

Figure 3 shows optimal splits between two types of capital.

Table 3: Output and Prices in the Model

	Output/cap.	Output/labour	Inv./output	Inv. cost/output	Price	$p_b$	Entry mass
Model	0.31	3.35	0.10	0.063	0.438	1.064	0.014

The ratio moments are the averages over firm distribution. It is thus not surprising that labour productivity is the same for all firms, proportional to the equilibrium wage rate. The ratio of investment cost to output is the total investment and reallocation costs divided by the total output. The total cost accounts for 6 per cent of the total output, which is consistent with the estimates based on plant-level data reported by Cooper and Haltiwanger (2006).

In the late 1980s, the United States experienced a wave of mergers and acquisitions. The annual total value of all acquisitions reached 4.84 per cent of GDP in 1988, from 3.11 per cent in 1984, as documented in Holmstrom and Kaplan (2001). In the mid- and late 1980s, a large proportion of acquisitions were leveraged buyouts, with investment groups and managers using borrowing to buy back firm shares. Nearly half of the U.S. corporations received a takeover offer in this period. After this wave of acquisitions, some firms went private. Meanwhile, from 1985 to 1989, aggregate labour productivity, aggregate capital productivity, and 4-factor TFP grew by 12.3 per cent, 6.0 per cent, and 2.0 per cent, respectively. These productivity measures are weighted averages of 4-digit standard industrial classification (SIC) industries. We use the real shipment-value share of each industry as their weight.<sup>10</sup>

During this period, the average investment rate declined and the reallocation rate increased (Table 4).

Year	Inv./cap.	Acq./cap.	Output/labour	Output/cap.	TFP
1985	0.21	0.074	213.2	2.56	0.98
1986	0.21	0.146	222.5	2.55	0.97
1987	0.19	0.108	232.7	2.75	1.00
1988	0.18	0.098	238.7	2.78	1.01
1989	0.17	0.097	239.5	2.72	1.00

Table 4: Investment, Reallocation, and Productivity, 1985-89

Note: Investment and reallocation rates are computed from Compustat. These are the unweighted mean over firm-level values. Productivity measures are computed using the NBER-CES Manufacturing Industry database. The productivity measures are the weighted average of 458 4-digit SIC industries.

Source: Author's calculation from Compustat and NBER-CES Manufacturing Industry Database.

The reallocation rate rose from 7.4 per cent in 1985 to an annual average of 12 per cent in 1986-88, representing an increase of 58 per cent. Many firms became private and were delisted from Compustat in this period; thus, the reallocation rate increase is very likely underestimated.<sup>11</sup>

Capital reallocation intensified in the late 1980s, mainly because the stock market price was low relative to the cost of building new capacity. Expanding through takeovers of the capital of other firms appeared to be less costly than building up new plants. Jensen (1993) takes the

<sup>&</sup>lt;sup>10</sup>With equally weighted measures, the growth rates are, respectively, 16.2 per cent, 9.3 per cent, and 3.5 per cent.

<sup>&</sup>lt;sup>11</sup>We plot the firm size distributions in 1985 and 1989, and find that the size distribution in capital shifted to the left-hand side between 1985 and 1989.

view that acquisitions in the 1980s were a reaction of capital market participation to corporate mismanagement of conglomerates. In addition, the antitrust law was less strictly enforced than before. In 1982 and 1984, the U.S. government introduced merger guidelines that clearly defined the industry concentration index. It is believed that the clearer definitions of the merger guidelines induced more mergers.

It seems that fixed cost decreases help explain this wave of acquisitions in the United States. From 1985 to 1989, 4-factor TFP increased by only 2 per cent while output per worker increased by 12 per cent. The model can generate gains in productivity arising from capital reallocation that are consistent with the facts during this period. We take the year 1985 as the base year with low reallocation, and take the period 1986-88 as the years with high reallocation. The aim is to change the fixed costs such that the model can generate a similar magnitude of reallocation increase. The two steady-state equilibria are compared to study the gains in productivity. In order to examine the effects of reallocation, we do not change the aggregate technology, but only let the fixed cost decrease from  $\Phi = 0.068$  to  $\Phi = 0.025$ .

When the fixed reallocation cost decreases, the value of the firm increases. This induces more firms to entry, since the entry cost does not change. The experiment accommodates this increased entry by allowing a higher exit rate, so that the new steady-state equilibrium holds. This requires that the capital cost of entrants change from 1.071 to 1.0735 (Table 5).

Table 5: Effects of Fixed Reallocation Cost

	Inv. rate	Acqui./cap.	Output/cap.	Output/labour	Inv. cost/output	Price	$p_b$
$\Phi = 0.068, \psi = 1.071$	0.16	0.097	0.315	3.34	0.061	0.44	1.07
$\Phi = 0.025, \psi = 1.073$	0.15	0.137	0.304	3.41	0.063	0.43	1.04

Figure 4 shows the distribution change in the two stationary equilibria. As reallocation increases, the model economy becomes larger.

When the fixed cost drops, the reallocation rate rises by 41 per cent and the participation rate rises from 17 per cent to 37 per cent. The gain in aggregate (or average) labour productivity is positive but small relative to that in the data. The output per unit of labour increases by 2.1 per cent and the output per unit of capital drops by 3.5 per cent. Comparing this result with the data, we see that increased capital reallocation boosts aggregate productivity. The 2.1 per cent increase accounts for 17 per cent of the aggregate labour productivity increase. Figure 5 shows the changes in the firm's policy functions. The solid lines are splits in investment of two types of capital when the fixed cost is high, and the dashed lines are the investment splits when the fixed cost is low. When the fixed cost drops, the interval of choosing not to participate in the acquisition market shrinks; hence, investment in the acquisition market increases and investment in unbundled capital



FIGURE 4: Cumulative Distributions

decreases. Because capital is less costly, the firms tend to increase the capital size and reduce the labour demand. Indeed, the firm's average labour demand decreases by 2 per cent. Therefore, the capital-labour ratio increases. With the decreasing returns to scale in production technology, the output-capital ratio will decrease and the output-labour ratio will increase if capital is larger and labour demand is less.



FIGURE 5: Changes in Investment as Fixed Cost Drops

The labour productivity in logarithm form can be decomposed into contributions from each production factor, as in Equation (5). The change in labour productivity is 0.021, of which labour and capital contribute 0.039 and -0.018, respectively. The change in aggregate labour productivity in logarithm form can also be decomposed into contributions from entry, exit, and reallocation,

following Equation(6). We find that  $\Delta Y_l = 0.021$ .<sup>12</sup> The net entry and exit effects cancel each other out. The gain in productivity is due to capital reallocation across firms. The contribution from the entry and exit components arises from the higher average productivity shocks of entrants, but this contribution is very small.

As a consequence of the cost reduction, household aggregate consumption increases by 2.1 per cent, exactly the same magnitude as the wage increase. This is due to the quasi-linearity of the utility function, in which the labour supply is perfectly elastic.

#### 4.5. Aggregate technology and reallocation

In the late 1990s, the United States experienced extraordinarily high economic growth, accompanied by a wave of investment in information technology. In 1999, the total asset value being reallocated reached a record high of 15 per cent of the GDP. Unlike the wave of mergers in the late 1980s, in the 1990s wave, 58 per cent of the transactions were financed with stocks (Andrade, Mitchell, and Stafford (2001)).

Year	Inv./cap.	Acq./cap.	Output/hour	Output/cap.	TFP
1995	0.20	0.10	96.5	100.6	99.2
1996	0.22	0.14	100.0	100.0	100.0
1997	0.21	0.18	103.8	101.4	103.1
1998	0.19	0.25	108.9	101.7	105.7
1999	0.16	0.17	114.0	101.7	108.7
2000	0.21	0.13	118.3	101.0	111.3

Table 6: Investment, Reallocation, and Productivity, 1995-2000

Note: productivity measures are from the Bureau of Labour Statistics (BLS). All productivity measures are indexed with 100 in 1996. TFP is the five-factor total productivity.

Source: Author's calculation from Compustat and the BLS productivity tables

Table 6 shows that, from 1996 to 2000, TFP increased by 11.3 per cent, capital productivity increased by 1 per cent, and labour productivity increased by 18.3 per cent.

We are interested in whether the change in aggregate technology causes the reallocation increase, and by how much the gain in labour productivity is accounted for by the increase in reallocation. As

<sup>12</sup>This value is obtained from the following equation:  

$$\Delta Y_l = \left[\frac{0.0149}{0.974} \cdot \log(3.41) - \frac{0.0144}{0.960} \cdot \log(3.34)\right] - \left[\frac{0.0149}{0.974} \cdot \log(3.41) - \frac{0.0144}{0.960} \cdot \log(3.34)\right] + \left[\frac{0.959}{0.974} \cdot \log(3.41) - \frac{0.945}{0.960} \cdot \log(3.34)\right].$$

shown in section 1, the threshold investment rate at which the firm is indifferent between  $x_b = 0$  and  $x_b \neq 0$  is independent of the state variables. Since the whole economy is more productive, one would expect that the wage rate goes up and the output price goes down, and this is true in the model. However, the demand for investment and labour does not increase in steady states responding to the increased z. If all firms in the model economy adopt the new aggregate technology at the same time, the model predicts that the optimal splits between bundled capital and unbundled capital are unaffected and the investment level does not change. Further, the whole economy upsizes, but the shape of firm distribution does not change, since the labour choice is unaffected by the change in aggregate technology in steady states. The change in aggregate technology affects only the wage rate, not reallocation, as shown by the firm's optimal labour choice. Hence, the labour productivity gain is entirely attributed to technology change, not reallocation.

The above observation indicates that, for the change in aggregate technology to induce increased reallocation, it should be the case that firms respond differently to the change. For example, only some firms are able to adopt the new technology, while other firms continue to use the old technology. Jovanovic and Rousseau (2004) study the role of technology adoption on reallocation, with only some of the firms being able to adopt the new technology. When the new technology is created, the firm can adopt it by reorganizing its production internally, or it can choose to exit through acquisitions.

Let all firms first have the same aggregate technology  $z_1 = 0.9476$ . When a new technology,  $z_2 = 1.0553$ , emerges, some firms are able to adopt  $z_2$ , but others cannot. Assume that the transition between the aggregate technologies is a two-state Markov chain, as follows:

$$\pi_z = \begin{bmatrix} 0.40 & 0.60 \\ 0.20 & 0.80 \end{bmatrix}$$

In steady state, 75 per cent of firms ultimately adopt the technology  $z_2$ . The aggregate technology increases by 8.5 per cent. Thus, two steady-state equilibria are compared: one with all the firms having technology  $z_1$ , and the other after some firms have moved from  $z_1$  to  $z_2$  according to the transition matrix  $\pi_z$ .

The results are shown in Table 7. When the economy converges to the new steady state, where 25 per cent of the firms are left to use the old technology,  $z_1$ , the aggregate investment in new capital does not change, while the acquisition rate increases by 1 per cent. Consequently, the 8.5 per cent change in aggregate technology causes a 14.6 per cent increase in labour productivity. Figure 6 shows the change in capital distribution between the two steady states. Clearly, the new economy has more, large firms.

	Inv. rate	Acqui./cap.	Output/cap.	Output/labour
$z_1$ only	0.16	0.105	0.3136	3.08
$z_1$ and $z_2$	0.16	0.106	0.3129	3.53
	Cost/output	Price	$p_b$	
$z_1$ only	0.063	0.477	1.064	
$z_1$ and $z_2$	0.063	0.416	1.064	

Table 7: Effects of a Change in Aggregate Technology



FIGURE 6: Cumulative Distribution of Log(k)

The change in labour productivity is driven by both the change in aggregate technology and the related investment by firms. We next decompose the change in aggregate labour productivity into two components: one arising from the change in aggregate technology, and the other arising from the related changes in capital and employment. Let the joint distribution of firms be  $\mu_z(z, \varepsilon, k)$ . The aggregate labour productivity is defined similarly by:

$$y_l = \frac{1}{\int d\mu_z(z,\varepsilon,k)} \int [\log z + \log \varepsilon + \nu(1-\alpha)\log k + (\nu\alpha - 1)\log l] d\mu_z(z,\varepsilon,k).$$

The aggregate labour productivity in logarithm increases from 1.1247 to 1.2619, an increase of 12.2 per cent. The contribution of labour increases from 1.33 to 1.39. Thus, close to 37 per cent of the change in productivity is accounted for by changes in employment. The capital component decreases by 0.002, from -0.153 to -0.155. Thus, the capital component contributes -1.6 per cent to the increase in aggregate labour productivity.

Capital reallocation is affected by the change in aggregate technology, but the effect is small, because the threshold investment rate at which the firm is indifferent between unbundled capital and bundled capital does not depend upon the productivity shocks. Moreover, aggregate technology affects the labour choice directly without relying on the reallocation of capital. In fact, aggregate technology is a substitute for labour. Labour productivity rises because labour demand decreases and aggregate technology improves.

#### 5. TRANSITION DYNAMICS WITHOUT ENTRY/EXIT

To this point, we have focused on the changes in productivity in steady states. This section examines the transition dynamics when the fixed cost is dropped permanently.

The transition dynamics are computed without entry and exit because it is not clear whether the entry and exit rate of capital should be equal during the transition. To simplify the computation, the entry and exit are shut down. The transition between steady states is computed to examine the short-run effects of a reduction in fixed costs and a change in aggregate technology. The parameter change is permanent and unanticipated by the firms. The transitional paths are shown in Appendix C.

When the fixed reallocation cost drops, two effects are created on the capital markets. First, it is relatively less costly for firms to enter the acquisition market. More firms participate in the acquisition market and invest less in the unbundled market. On the other hand, total investment is less costly. As a result, the prices decrease and the investment in unbundled capital decreases. The net effect on capital markets is that the reduced fixed cost temporarily reduces the demand for capital, and therefore the aggregate capital decreases temporarily. When the price goes down, the wage rate increases, so firms should hire less labour. But, in fact, the aggregate labour increases temporarily before it goes down. This indicates that various firms respond differently. Those firms that increase their capital (for example, firms that previously sold off capital) have a higher probability mass. Because the capital of these firms increases, they hire more labour.

#### 6. CONCLUSION AND FUTURE RESEARCH

Given the fact that some firms are more productive than others and some managers have better managerial ability than others, it would be most efficient to have all capital and labour organized for production in the most productive firms. In reality, both the production technology (returns to scale) and reallocation frictions prevent capital and labour from flowing from the less productive firms to the more productive firms. These frictions are not only informational and technical, but they can also be related to policy and institutions. The model in this paper is a step toward quantifying the benefits of reducing these frictions. Our analysis demonstrates that removing reallocation frictions boosts the productivity of aggregate labour. As much as 20 per cent of the growth in productivity can be accounted for by increased reallocation. The implication is that government policy should facilitate the reallocation of capital and labour. The results of this paper also raise the issue of whether the enforcement of antitrust policies should take into consideration the gains in productivity that result from mergers.

Our model assumes the existence of persistent productivity shocks and a perfectly competitive labour market. In equilibrium, all firms have the same labour productivity, which equals the equilibrium wage rate. However, empirical evidence shows that labour productivity at the firm and plant levels is not equal among establishments. This begs an extension to the current model to explain the dispersion of labour productivity across firms, and to investigate the contribution of worker reallocation to growth of productivity. This can be done by either assuming that firms have different abilities to use the same workers, or introducing heterogeneous workers who have a different productivity or quality. At the aggregate level, a gain in productivity arises either from the increased share of firms using workers more efficiently or from increased worker quality.

Further detailed study of the transition dynamics is needed to understand how the capital market and the labour market react to reallocation frictions.

Finally, the agency problems related to the incentives of managers and to information asymmetry in the acquisition market could be modelled explicitly. In particular, the importance of informational friction should be quantified.

#### APPENDIX A: DATA

The U.S. data on manufacturing are from Compustat. We choose periods between 1971 and 2004 for computing moments because the reallocation variable in Compustat is available only during that period. Adding the mineral industries and construction industries only changes moment values very slightly. All dollar values are deflated using CPI (with 100 in 1996). Firms with capital value less than 0.5 million dollars are removed. Capital is the level at the beginning of a year (data item 7). Investment is the expenditures on plant, property, and equipment (data item 128). Capital sale is the dollar value of sold plant, property, and equipment (data item 107). Acquisition is the total expense of acquiring the plant, property, and equipment from other firms (data item 129). Employment is the total number of employees (data item 29).

#### Moments for Manufacturing Industries

Table A.1 gives the investment and reallocation moments. In this table, micro moments are computed as the averages of firm-level values. Investment rate is the average of firm investment rates. Spike is the proportion of firm-year observations with an investment rate larger than 20 per cent. Inaction is the proportion of firm/year observations with an investment rate less than 1 per cent. Participation is the proportion of firm-year observations that have non-zero acquired capital value.

	Inv. rate	Spike	Inaction	Cap. sale/cap.	Acqui./cap.	Participation
Micro	0.181	0.23	0.031	0.014	0.087	0.24
Macro	0.114	0.23	0.031	0.012	0.075	0.24

Table A.1: Investment and Reallocation Moments

Macro moments are computed as the averages of annual moments. Investment rate is calculated as the average annual investment rate, which is the yearly total investment of all firms divided by the yearly total capital stock of all firms.

Figure A.2 and Figure A.3 show the changes in the distribution of firm size, measured with capital and employment. The moments of these distributions are given in Table A.2.

Table A.2: Firm Size Distributio	n
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	Mean	Std dev	Skewness	Kurtosis
Capital	$1.88 \ (0.356)$	$2.22 \ (0.160)$	$0.49\ (0.068)$	2.80(0.110)
Employment	2.74(0.292)	1.92(0.180)	$0.25 \ (0.053)$	2.72(0.123)



Figure A.2: The U.S. Firm Distribution in Capital

Capital is the logarithm of demeaned data item 7. First we divide the firm's capital by the yearly industry average capital, and then we take the logarithm. Employment is computed in the same way. All moments are calculated as the averages of yearly values during 1971-2004. Values in brackets are standard deviations of moments.



Figure A.3: The U.S. Firm Distribution in Employment

#### Moments for Mineral, Construction, and Manufacturing

The moments in Table A.3 are computed by choosing all firms in the mineral, construction, and manufacturing industries. Moments of firm size distribution in these industries are given in Table A.4.

	Inv. rate	Spike	Inaction	Cap.sale/cap.	Acqui./cap.	Participation
Micro	0.180	0.25	0.036	0.016	0.088	0.22
Macro	0.116	0.25	0.036	0.012	0.078	0.22

Table A.3: Moments: Investment and Reallocation Rates

Table A.4: Firm Size Distribution

	Mean	Std dev	Skewness	Kurtosis
Capital	$1.95\ (0.336)$	$2.21 \ (0.160)$	$0.45 \ (0.066)$	$2.75 \ (0.137)$
Employment	$2.64 \ (0.325)$	2.04(0.221)	$0.052\ (0.130)$	$2.90 \ (0.192)$

#### A.1. Productivity

We use the NBER-CES industry data to compute productivity moments for the years between 1971 and 1996. The data are on the 4-digit SIC industry level. The aggregate productivity measures are mean productivity measures weighted by share of industry shipment in total shipment. The standard deviation of productivity measures is also weighted by shipment values.

The productivity measures between 1997 and 2001 are computed from the BLS productivity website at <a href="http://www.bls.gov/>.">http://www.bls.gov/>.</a>

#### APPENDIX B: PROOFS

#### B.1. When does the firm choose $x_b = 0$ ?

If the firm chooses to invest in both types of capital, the optimal investment rate in bundled capital is

(B-1) 
$$i_b = \frac{1}{\gamma_u + \gamma_b} (\gamma_u i + 1 - p_b).$$

#### B.2. When is $x_b = 0$ optimal?

The firm chooses  $x_b = 0$  if

$$i + \frac{\gamma_u}{2}i^2 \le \min\{i - i_b + p_b i_b + \frac{\gamma_u}{2}(i - i_b)^2 + \frac{\gamma_b}{2}i_b^2 + \Phi, p_b i + \frac{\gamma_b}{2}i^2 + \Phi\}.$$

Let  $\tilde{i}$  be the total investment rate at which the firm is indifferent between  $x_b = 0$  and  $(x_u \neq 0)$ 

 $(0, x_b \neq 0)$ , then  $\tilde{i} = \frac{1}{\gamma_u} (p_b - 1 \pm \sqrt{2\Phi(\gamma_u + \gamma_b)})$ . That is, when

$$(B-2) \quad i \in \left(\frac{1}{\gamma_u}(p_b - 1 - \sqrt{2\Phi(\gamma_u + \gamma_b)}), \frac{1}{\gamma_u}(p_b - 1 + \sqrt{2\Phi(\gamma_u + \gamma_b)})\right),$$

the firm chooses  $x_b = 0$ , investing in unbundled capital only. Next, we check the conditions under which the firm does not choose  $x_u = 0$  over  $x_b = 0$ . We compare the costs of choosing  $x_u = 0$  and  $x_b = 0$ .

If the firm chooses  $x_b = 0$ , then we have  $p_b i + \frac{\gamma_b}{2} i^2 + \Phi > i + \frac{\gamma_u}{2} i^2$ . Then, if

(B-3) 
$$i \in \left(\frac{1}{\gamma_u - \gamma_b}(p_b - 1 - \sqrt{(p_b - 1)^2 + 2\Phi(\gamma_u - \gamma_b)}), \frac{1}{\gamma_u - \gamma_b}(p_b - 1 + \sqrt{(p_b - 1)^2 + 2\Phi(\gamma_u - \gamma_b)})\right)$$

the firm would choose  $x_b = 0$  over  $x_u = 0$ .

Therefore, when the investment rate is in the intersection of the above two sets (B-2) and (B-3), the firm will choose  $x_b = 0$ , investing in unbundled capital only. From inequality (B-4) (below), we know that the cost of  $x_u = 0$  is always larger than the cost of  $x_u \neq 0, x_b \neq 0$ . Therefore, the condition (B-3) is redundant.

#### B.3. Proof of Proposition 1

For the proposition to hold, the investment rate must satisfy

$$p_b i + \frac{\gamma_b}{2} i^2 + \Phi < \min\{i - i_b + p_b i_b + \frac{\gamma_u}{2} (i - i_b)^2 + \frac{\gamma_b}{2} i_b^2 + \Phi, i + \frac{\gamma_u}{2} i^2\}.$$

First, we need to find the range of investment rate at which the firm chooses  $x_u = 0$  over investing in both types of capital. The difference between the two cost functions of  $x_u = 0$  and  $(x_u \neq 0, x_b \neq 0)$  is

$$= p_b i + \frac{\gamma_b}{2} i^2 - [i - i_b + p_b i_b + \frac{\gamma_u}{2} (i - i_b)^2 + \frac{\gamma_b}{2} i_b^2]$$
  
$$= \frac{1}{\gamma_c + \gamma_c} (\gamma_b i - 1 + p_b)^2 \ge 0,$$

where in the second line Equation (B-1) is used. This positive difference of investment costs between the two cases implies that, for any investment rate, the firm never chooses  $x_u = 0$  over  $(x_u \neq 0, x_b \neq 0)$ .

Next, from the above algebra we know that the firm will choose  $x_u = 0$  over  $x_b = 0$  if investment rate *i* is located in the complement of (B-3). This implies that the firm must also choose ( $x_u \neq 0, x_b \neq 0$ ) over  $x_b = 0$ , because the set (B-3) is larger than set (B-2). But inequality (B-4) implies that the firm always chooses ( $x_u \neq 0, x_b \neq 0$ ) over  $x_u = 0$ .

(B-4)

Therefore, the firm will never choose  $x_u = 0$ .

The above proof is also valid if the fixed cost is  $\Phi$ , instead  $\Phi k$ . However, these two forms of fixed cost induce different cut-off values,  $\tilde{i}$ , at which the firm is indifferent between  $x_b = 0$  and not.

#### B.4. When does the firm choose to buy capital in one market and sell it in another market?

From the above, we know that the optimal  $i_b = \frac{1}{\gamma_u + \gamma_b} (\gamma_u i + 1 - p_b)$ . Therefore, if  $i_b > 0$ , then  $i > \frac{1-p_b}{\gamma_b}$ . The two inequalities hold together only if  $p_b < 1$ . Meanwhile, the investment rate must be located in the complement of (B-2). It can be shown that it is possible that  $i_b > 0$  and  $i_u < 0$  only when  $i > \frac{1}{\gamma_u} (p_b - 1 + \sqrt{2\Phi(\gamma_u + \gamma_b)})$ . For i to not be located in an empty set, we require that  $\frac{1-p_b}{\gamma_b} > \frac{1}{\gamma_u} (p_b - 1 + \sqrt{2\Phi(\gamma_u + \gamma_b)})$ . Hence, the proposition holds, and i must be larger than  $\frac{1}{\gamma_u} (p_b - 1 + \sqrt{2\Phi(\gamma_u + \gamma_b)})$ .

In a similar way, we can show that, if  $p_b > 1 + \gamma_b \sqrt{\frac{2\Phi}{\gamma_u + \gamma_b}}$ , the firm chooses  $i_b < 0$  and  $i_u > 0$ . At that time, the investment rate must be lower than  $\frac{1}{\gamma_u}(p_b - 1 - \sqrt{2\Phi(\gamma_u + \gamma_b)})$ .

#### B.5. Proof of Proposition 2 and Proposition 3

**Proposition 2.** With the assumptions about production function and utility, the firm value function  $v(\varepsilon, k)$  is non-decreasing in the output price p.

First, note that  $\frac{\partial R(\varepsilon,k,\mu)}{\partial w} < 0$ , hence  $\frac{\partial R(\varepsilon,k,\mu)}{\partial p} > 0$ . The derivative of the value function with respect to p is

$$\frac{\partial v(\varepsilon,k)}{\partial p} = \frac{\partial R(\varepsilon,k,\mu)}{\partial p} + \frac{\partial x_u}{\partial p} \left[ -1 - \frac{\partial h_1(x_u,k)}{\partial x_u} + \frac{1}{1+r} E \frac{\partial v(\varepsilon',k')}{\partial x_u} \right] \\
+ \frac{\partial x_b}{\partial p} \left[ -p_b - \frac{\partial h_2(x_b,k)}{\partial x_b} + \frac{1}{1+r} E \frac{\partial v(\varepsilon',k')}{\partial x_b} \right].$$

The terms in square brackets are Euler equations, respectively, for two types of investment, and they are zero for the optimal investment. If the firm does not participate in the acquisition market, the last term is zero. Therefore, the value function is non-decreasing in p.

**Proposition 3.** With the assumptions about production function and utility, in equilibrium, the firm value function  $v(\varepsilon, k)$  is non-decreasing in the entry mass, M.

This proposition trivially follows the previous proposition and the market clearing condition for the output.

#### APPENDIX C: TRANSITION DYNAMICS

The transition path is computed as follows, starting with the stationary equilibrium for  $\phi = 0.065$ . Assume that the steady states are in period 0 and T + 1. The unanticipated shock occurs in period 1.

- (i) Fix T. Compute the initial steady-state equilibrium  $\{\mu^*, p^*, p_b^*\}$  and the final steady-state equilibrium  $\{\mu^{**}, p^{**}, p_b^{**}\}$ .
- (ii) Guess a sequence of prices  $\{p_{b,t}\}_{t=1}^T$  and  $\{p_t\}_{t=1}^T$ .
- (iii) Use backward induction to obtain the policy functions for each period  $\{g(\varepsilon_t, k_t, p_t, b_{b,t}\}_{t=1}^T$ .
- (iv) Given the period 0 firm distribution, use the policy function to obtain the series of firm distribution  $\{\mu_t\}_{t=1}^T$ .
- (v) In each period, check the output clearing condition and the acquisition market clearing condition. If the excess demands are positive, increase the prices; if negative, decrease the prices. Go back to step (ii).
- (vi) Stop when the output clearing condition and the acquisition market clearing condition are satisfied.
- We set T = 25. Increasing T does not improve the results.



#### REFERENCES

- ANDRADE, G., M. MITCHELL, AND E. STAFFORD (2001): "New Evidence and Perspectives on Mergers," Journal of Economic Perspectives.
- BAILY, M. N., C. HULTEN, AND D. CAMPBELL (1992): "Productivity Dynamics in Manufacturing Plants," Brookings Papers on Economic Activity. Microeconomics, 1992, 187–267.
- BARTELSMAN, E. J., AND P. J. DHRYMES (1998): "Productivity dynamics: U.S. manufacturing plants, 1972-1986," Journal of Productivity Analysis, 9(1), 5–34.
- BARTELSMAN, E. J., AND M. DOMS (2000): "Understanding Productivity: Lessons from Longitudinal Microdata," Journal of Economic Literature, 38(3), 569–594.
- COOPER, R. W., AND J. C. HALTIWANGER (2006): "On the Nature of Capital Adjustment Costs," *Review of Economic Studies*.
- DAVIS, S., J. HALTIWANGER, R. JARMIN, J. LERNER, AND J. MIRANDA (2008): "Private Equity and Employment," Working Paper 08-07, Center for Economic Studies, U.S. Census Bureau.
- DAVIS, S., J. HALTIWANGER, AND S. SCHUH (1996): Job Creation and Destruction. MIT Press, Cambridge, MA.
- EISFELDT, A. L. (2004): "Endogenous Liquidity in Asset Markets," Journal of Finance.
- EISFELDT, A. L., AND A. A. RAMPINI (2006): "Capital reallocation and liquidity," *Journal of Monetary Economics*, 53(3), 369–399.
- ERARD, B., AND H. SCHALLER (2002): "Acquisitions and Investment," Economica, 69(275), 391-413.
- GRIFFITH, R., J. HASKEL, AND A. NEELY (2006): "Why is Productivity so Dispersed?," Oxford Review of Economic Policy, 22(4), 513–525.
- HALL, R. E. (2004): "Measuring Factor Adjustment Costs," The Quarterly Journal of Economics.
- HANSEN, G. D. (1985): "Indivisible labor and the business cycle," Journal of Monetary Economics, 16(3), 309–327.
- HOLMSTROM, B., AND S. N. KAPLAN (2001): "Corporate Governance and Merger Activity in the United States: Making Sense of the 1980s and 1990s," *Journal of Economic Perspectives*, 15(2), 121–144.
- HOPENHAYN, H. A., AND E. C. PRESCOTT (1992): "Stochastic Monotonicity and Stationary Distributions for Dynamic Economies," *Econometrica*, 60(6), 1387–406.
- HOUSE, C. L., AND J. V. LEAHY (2004): "An sS Model with Adverse Selection," Journal of Political Economy.
- JENSEN, M. C. (1993): "The Modern Industrial Revolution, Exit, and the Failure of Internal Control Systems," Journal of Finance.
- JOVANOVIC, B., AND P. L. ROUSSEAU (2002): "The Q-Theory of Mergers," American Economic Review, 92(2), 198–204.
- (2004): "Mergers as Reallocation," Working paper, New York University.
- KAPLAN, S. N. (2000): "Introduction," in *Mergers and Productivity*, ed. by Mergers, and Productivity. The University of Chicago Press.
- LICHTENBERG, F. R. (1992): Corporation Takeovers and Productivity. MIT Press, Cambridge, MA.
- LICHTENBERG, F. R., AND D. SIEGEL (1989): "The Effect of Control Changes on the Productivity of U.S. Manufacturing Plants," Journal of Applied Corporate Finance, pp. 60–67.
- MAKSIMOVIC, V., AND G. PHILLIPS (2001): "The Market for Corporate Assets: Who Engages in Mergers and Asset Sales and Are There Efficiency Gains?," *Journal of Finance*, 56(6), 2019–2065.
- MCGUCKIN, R. H., AND S. V. NGUYEN (1995): "On Productivity and Plant Ownership Change: New Evidence from the Longitudinal Research Database," *RAND Journal of Economics*, 26(2), 257–276.
- PRESCOTT, E. C. (1986): "Theory ahead of business cycle measurement," Discussion paper.
- ROGERSON, R. (1988): "Indivisible labor, lotteries and equilibrium," Journal of Monetary Economics, 21(1), 3-16.
- SIEGEL, D. S., K. L. SIMONS, AND T. LINDSTROM (2005): "Ownership Change, Productivity, and Human Capital: New Evidence from Matched Employer-Employee Data in Swedish Manufacturing," Rensselaer Working Papers in Economics 0502, Rensselaer Polytechnic Institute, Department of Economics.
- STOKEY, N. L., R. E. LUCAS, AND E. C. PRESCOTT (1989): *Recursive Methods in Economic Dynamics*. Harvard University Press, Cambridge, MA.