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“Why do I rob banks? Because that’s where the money is.” – Willie Sutton

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Abstract

We use a method similar to Google's PageRank procedure to rank banks in the Canadian Large Value Transfer System (LVTS). Along the way we obtain estimates of the payment processing speeds for the individual banks. These differences in processing speeds are essential for explaining why observed daily distributions of liquidity differ from the initial distributions, which are determined by the credit limits selected by banks.

JEL classification: C11, E50, G20

Bank classification: Payment, clearing, and settlement systems

Résumé

Les auteurs appliquent au système canadien de transfert de paiements de grande valeur une méthode similaire à l'algorithme PageRank de Google. Ils obtiennent ainsi une estimation de la vitesse de traitement des paiements dans chaque banque. L'analyse des différences entre banques à ce chapitre est essentielle si l'on veut comprendre pourquoi la distribution moyenne de la liquidité pour la journée diffère de la distribution observée en début de journée, qui est fonction des limites de crédit établies par les banques.

Classification JEL : C11, E50, G20

Classification de la Banque : Système de paiement, de compensation et de règlement

1 Introduction

Recently, economists have argued that the importance of banks within the financial system cannot be determined in isolation. In addition to its individual characteristics, the position of a bank within the banking network matters.¹ In this paper we examine the payments network defined by credit controls in the Canadian Large Value Transfer System (LVTS). We provide a ranking of LVTS participants with respect to predicted daily liquidity holdings, which we derive from the network structure. A bank is deemed to be “central” if, based on our network analysis, it is predicted to hold the most liquidity.²

We focus on the Tranche 2 component of the LVTS.³ In this component, participants set bilateral credit limits (BCLs) with each other that determine, via these limits and an associated multilateral constraint, the maximum amount of money any one participant can transfer to any other without offsetting funds. Because banks start off the day with zero outside balances, these credit limits define the initial liquidity holdings of banks.⁴ However, as payments are made and received throughout the day the initial liquidity holdings are shuffled around in ways that need not conform to the initial allocation. Banks with high credit limits granted to them may not be major holders of liquidity throughout

¹Allen and Gale (2000) analyze the role network structure plays in contagion of bank failures caused by preference shocks to depositors in a Diamond-Dybvig type model and find more complete networks are more resilient. Bech and Garratt (2007) explore how the network topology of the underlying payment flow among banks affects the resiliency of the interbank payment system.

²We are, of course, departing from the standard designation of a country or countries’ principal monetary authority as the central bank. The Bank of Canada is the central bank of Canada by that account. The proposed usage comes from the literature on social networks. In this literature, the highest ranked node in a network is referred to as the central node.

³See Arjani and McVanel (2006) for an overview of the Canadian LVTS.

⁴This is not the case in all payment systems. In Fedwire opening balances are with the exception of discount window borrowing and a few accounting entries equal to yesterday’s closing balance. In CHIPS each participant has a pre-established opening position requirement, which, once funded via Fedwire funds transfer to the CHIPS account, is used to settle payment orders throughout the day. The amount of the initial prefunding for each participant is calculated weekly by CHIPS based on the size and number of transactions by the participant. A participant cannot send or receive CHIPS payment orders until it transfers its opening position requirement to the CHIPS account.

the day if they make payments more quickly than they receive them. Likewise, banks that delay in making payments may tie up large amounts of liquidity even though they have a low initial allocation. Hence, knowledge of the initial distribution alone does not tell us how liquidity is allocated throughout the day, nor does it provide us with the desired ranking.

In order to predict the allocation of liquidity in the LVTs we apply a well known result from Markov chain theory, known as the Perron-Frobenius theorem. This theorem outlines conditions under which the transition probability matrix of a Markov chain has a stationary distribution.

In the present application, we define a transition probability matrix for the LVTs using the normalized BCL vectors for each bank. This approach is based on the premise that money flows out of a bank in the proportions given by the BCLs the bank has with the other banks. We also allow the possibility that banks will hold on to money. This is captured by a positive probability that money stays put. Assuming money flows through the banking system in a manner dictated by our proposed transition probability matrix, the values of its stationary vector represent the fraction of time a dollar spends at each location in the network. This stationary vector is our prediction for the distribution of daily liquidity. The bank with the highest value in the stationary vector is predicted to hold the most liquidity throughout the day and is thus the “central” bank.

An attractive feature of our application of Markov chain theory is that it allows us to estimate an important, yet unobservable characteristic of banks, namely, their relative waiting times for using funds. The Bank of Canada observes when payments are processed by banks, but does not know when the underlying payment requests arrive at the banks. We are able to estimate these wait times using a Bayesian framework. We find that processing speed plays a significant factor in explaining the liquidity holdings of banks throughout the

day and causes our ranking of banks to be different from the one suggested by the initial distribution of liquidity. In particular, the bank which is central based on initial liquidity holdings is not central in terms of liquidity flows over the day.

Once we have estimates for the wait times we are able to see how well the daily stationary distributions match the daily observed distributions of liquidity. We find that they match closely. This validates our approach and suggests that Markov analysis could be a useful tool for examining the impact of changes in credit policies (for example a change in the system wide percentage) by the central bank on the distribution of liquidity in the LVTS and for examining the effects of changes in the credit policies of individual banks.

Our approach has much in common with Google's PageRank procedure, which was developed as a way of ranking web pages for use in a search engine by Sergey Brin and Larry Page.⁵ In the Google PageRank system, the ranking of a web page is given by the weighted sum of the rankings of every other web page, where the weights on a given page are small if that page points to a lot of places. The vector of weights associated with any one page sum to one (by construction), and hence the matrix of weights is a transition probability matrix that governs the flow of information through the world wide web. Google's PageRank ranking is the stationary vector of this matrix (after some modifications which are necessary for convergence). In PageRank the main diagonal elements of the transition probability matrix are all zeros. In contrast, we allow these elements, which represent the probabilities that banks delay in processing payment requests, to be positive.

The potential usefulness of Markov theory for examining money flows was proposed by Borgatti (2005). He suggests that the money exchange process

⁵The PageRank method has also been adapted by the founders of Eigenfactor.org to rank journals. See Bergstrom (2007)

(between individuals) can be modelled as a random walk through a network, where money moves from one person to any other person with equal probability. Under Borgatti’s scenario, the underlying transition probability matrix is symmetric. Hence, as he points out, “the limiting probabilities for the nodes are proportional to degree.” The transition probability matrix defined by the BCLs and the patience parameters of banks is not symmetric and hence, this proportionality does not hold in our application.

Others have looked at network topologies of banking systems defined by observed payment flows. Boss, Elsinger, Summer, and Thurner (2004) used Austrian data on liabilities and Soramäki, Bech, Arnold, Glass, and Beyeler (2006) used U.S. data on payment flows and volumes to characterize the topology of interbank networks. These works show that payment flow networks share structural features (degree distributions, clustering etc.) that are common to other real world networks and, in the latter case, discuss how certain events (9/11) impact this topology. In terms of methodology our work is completely different from these works. We prespecify a network based on fixed parameters of the payment system and use this network to predict flows. The other papers provide a characterization of actual flows in terms of a network.

2 Data

The data set used in the study consists of all Tranche 2 transactions in the LVTS from October 1st 2005 to October 31st 2006. This data set consists of 272 days in which the LVTS was running.

The participants in the sample consist of members of the LVTS and the Bank of Canada. For the purposes of this study we exclude the Bank of Canada since it does not send any significant payments in Tranche 2.⁶⁷

⁶We discuss implications of this in section 3.

⁷While we remove the Bank of Canada payments we do not remove the BCLs that the

	BCLs	abs diff
min	0.0	0.0
25 percentile	50.0	0.0
median	200.0	0.0
mean	417.3	59.5
75 percentile	698.6	16.3
max	2464.7	1201.1
std. dev.	495.8	182.5

Table 1: Daily cyclical limits in millions of Canadian dollars

2.1 Credit controls

The analysis uses data on daily cyclical bilateral credit limits set by the fourteen banks over the sample period. Sample statistics for the daily cyclical limits are presented in Table 1. BCLs granted by banks vary by a large amount (at least an order of magnitude). The BCLs are fairly symmetric since the minimum through the 50th percentile of absolute differences of the BCLs between pairs of banks are zero and even the 75 percentile of the cyclical is only 16 million compared to the average cyclical BCL of 699 million. While it is not evident from table 1, BCLs vary across pairs of banks by a large amount (at least an order of magnitude) in some instances.

3 Initial versus average liquidity holdings

Let W_t denote the array of Tranche 2 debit caps (or BCLs) in place at time t , where element w_{ijt} denotes the BCL bank j has granted to bank i on date t . The initial distribution of liquidity is determined by the bilateral debit caps that are in place when the day begins. By taking the row sum of the matrix W_t , we obtain the sum of bilateral credit limits granted to bank i . However, a bank's initial payments cannot exceed this amount times the system wide

Bank of Canada grants to other banks in Tranche 2. As this would have an impact on the T2NDCs between member banks.

percentage, 24% during the sample period.⁸ Using the notation from Arjani and McVanel (2006), let

$$T2NDC_{it} = .24 * \sum_j w_{ijt}, \quad (1)$$

denote the Tranche 2 multilateral debit cap of bank i on date t . Since we are summing over the BCLs that each bank $j \neq i$ has granted to bank i , this is the conventional measure of the status (a la Katz (1953)) of bank i . The BCL bank j grants to i defines i 's ability to send payments to j . Hence, in terms of the weighted, directed network associated with W_t , w_{ijt} is the weight on the directed link from i to j . Hence, $T2NDC_{it}/.24$ is also the (weighted) outdegree centrality of bank i on date t .

The multilateral debit caps specified in (1) represent the amount of liquidity available to each bank for making payments at the start of the day. Thus, the initial distribution of liquidity on date t is $d_t = (d_{1t}, \dots, d_{nt})$, where

$$d_{it} = \frac{T2NDC_{it}}{\sum_{j=1}^n T2NDC_{jt}}, \quad i = 1, \dots, n.$$

During the day the liquidity holdings of bank i change as payments are sent and received. The average amount of liquidity that bank i holds on date t , denoted Y_{it} , is the time weighted sum of their balance in Tranche 2 over the day on date t and the maximum cyclical T2NDC on date t . To compute this we divide the day into K_i (not necessarily equal) time intervals, where K_i is the number of transactions that occurred that day for bank i . Then

$$Y_{it} = \sum_{k_i=0}^{K_i} p_{it}^k \delta_t^{k_i, k_i+1} + T2NDC_{it} \quad (2)$$

where $\delta_t^{k_i, k_i+1}$ is the length of time between transaction k_i and $k_i + 1$ on date t

⁸The system wide percentage is currently 30% and was changed on May 1st 2008.

and $p_{it}^{k_i}$ is i 's aggregate balance of incoming and outgoing payments on date t following transaction k_i .

In a closed system the aggregate payment balances at any point must sum to zero across all participants. Therefore the total potential liquidity in the system is the sum of the T2NDCs. In practice this is not quite true since the Bank of Canada is also a participant in the LVTS and acts as a drain of liquidity in Tranche 2. Specifically, the Bank of Canada receives payments on behalf of various other systems (e.g. Continuous Linked Settlement (CLS) Bank pay-ins). Therefore, in practice the summation of net payments across participants sums to a negative number; since the Bank of Canada primarily uses Tranche 1 for outgoing payments. To account for this drain, we use as our definition of liquidity in the system at any one time the summation, across all banks, of (2). Thus, the average share of total liquidity that i has on date t is equal to

$$y_{it} = \frac{Y_{it}}{\sum_{i=1}^{14} Y_{it}}. \quad (3)$$

The vector $y_t = (y_{1t}, \dots, y_{nt})$ is our date t measure of the observed average liquidity holdings for the n banks.

A comparison of the initial liquidity holdings, d_t , to the average liquidity holdings, y_t , over the 272 days of the sample period is shown in Figure 1. Each point in the figure represents a matching initial and average value (the former is measured on the horizontal axis and the latter is measured on the vertical axis) for a given bank on a given day. Hence, there are $272 \times 14 = 3808$ points on the graph. If the two liquidity distributions matched exactly, all the points would lie on the 45 degree line.

The worst match between the average liquidity holdings and the initial holdings occurs for points on the far right of Figure 1. This vertical clustering below the 45 degree line reflects the fact that for some banks the value in the

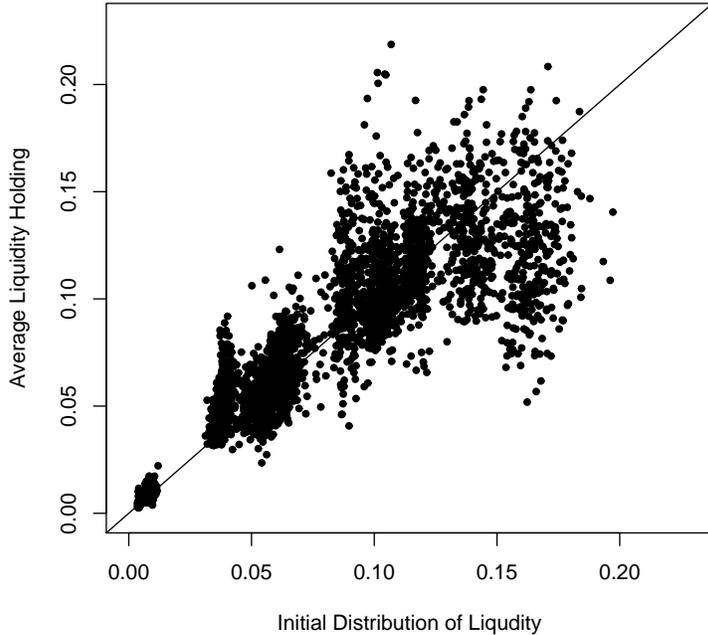


Figure 1: Initial versus average liquidity holdings.

initial distribution is almost always greater than the average liquidity holdings over the day. This occurs because, as we shall see in section 5, these banks, in particular bank 11, are speedy payment processors.

4 Markov Analysis

We begin with the weighted adjacency matrix W_t defined from the BCLs in Section 3 and normalize the components so that the rows sum to one. That is, we define the stochastic matrix $W_t^N = [w_{ijt}^N]$, where

$$w_{ijt}^N = \frac{w_{ijt}}{\sum_j w_{ijt}}. \quad (4)$$

Row i of W_t^N is a probability distribution over the destinations of a dollar that leaves bank i that is defined using the vector of BCLs granted to bank i from all the other banks on date t . Conditional on the fact that a dollar leaves bank

i , its movement is described by the matrix W_t^N . However, we need to make an important modification to address the fact that banks sometimes delay in processing payment requests.

Delay is accounted for by (i) specifying delay probabilities θ_i for each bank i and (ii) re-scaling the off-diagonal elements of W_t^N to make these the appropriate conditional probabilities. Specifically, we create a new stochastic matrix $B_t = [b_{ijt}]$, where

$$b_{iit} = \theta_i, \quad i = 1, \dots, n, \quad \text{and} \quad b_{ijt} = (1 - \theta_i)w_{ijt}^N \quad \text{for } i \neq j. \quad (5)$$

The delay parameters θ_i can be interpreted as the probability that bank i sends a payment to itself. These are allowed to differ across banks, but not across time. That is, the θ_i 's are taken to be primitives of the payment process (like preference parameters) and are assumed to be constant over the period of analysis.⁹

By the Perron-Frobenius theorem (see, for example, Seneta (1981, chapter 1)) we know that the power method applied to the matrix B_t converges to a unique, positive stationary vector from any starting point so long as B_t is stochastic, irreducible and aperiodic. These conditions are met by construction and because of the high degree of connectedness of banks in the LVTS.¹⁰ Given a vector of delay parameters $\theta = (\theta_1, \dots, \theta_n)$, the desired stationary vector, which we denote by $x_t(\theta)$, is the leading (left) eigenvector of B_t :

$$x_t^T(\theta) = x_t^T(\theta)B_t. \quad (6)$$

⁹We discuss the implications of allowing the θ_i 's to vary over time in Section 6.

¹⁰In the case of Google, many pages exist which do not link to other pages and hence the transition probability matrix constructed from the world wide web using links is only substochastic. Moreover, this hyperlink matrix, as it is called in Langville and Meyer (2006), is neither irreducible nor aperiodic. Hence, modifications of the initial hyperlink matrix are required to derive the Google rankings.

Where do the θ_i 's come from? Unfortunately data is available on when payment requests are processed, but not on when they were first received by the bank. Hence, we do not have data on the delay tendencies of each bank. Consequently we estimate the delay parameters using our assumption that on average the distribution of liquidity in the system throughout the day achieves the stationary distribution that corresponds to the transition probability matrix B_t .

5 Estimation of the delay parameters

We want to choose the vector θ so that over the sample period the eigenvectors defined by (6) are as close as possible to the observed distributions of average liquidity.

5.1 Bayesian Estimation Procedure

Our model of the observable distribution of liquidity is

$$y_{it} = x_{it}(\theta) + \epsilon_{it}, \tag{7}$$

where θ is the vector of unknown diagonal parameters of B_t , y_{it} is the average amount of liquidity held by bank i on date t , $x_{it}(\theta)$ is the stationary amount of liquidity held by i on date t according to (6), and ϵ_{it} is the forecast error, which has a mean zero symmetric distribution.

In this preliminary exploration we are interested in explaining mean levels of liquidity as opposed to the forecast errors. Therefore, we assume a simple distribution of errors that is independent across observations.¹¹ The process of

¹¹A plausible next step would be to include the correlations between the errors on a given data t induced by the fact that the y_{it} variables have to sum to one for each bank. Given the difficulty in estimating the mean parameters, estimating these covariance parameters, as well as accounting for heteroskedasticity, is left for a later exercise.

finding the unobservable θ s can be done either via a GMM estimation or via a Bayesian framework; the latter is described below.

The family of distributions used for the forecast error is the normal family with precision τ .¹² In this case the likelihood for an observation is

$$L(y_{it}|\theta, B_t, \tau) = N(y_{it}|x_{it}(\theta), \tau).$$

Assuming independence of the errors, a likelihood for the whole sample is

$$L(\{y_{it}\}_{t=1}^T|\theta, \{B_t\}_{t=1}^T, \tau) = \prod_{t=1}^T \prod_{i=1}^n L(y_{it}|\theta, B_t, \tau).$$

We assume a flat uniform prior on the θ s and a diffuse Gamma prior on the precision with a shape parameter of 1/2 and a scale parameter of 2. The former distribution embodies our lack of information about the θ s and the latter distribution embodies our lack of information of the error term, and also exploits the conjugacy of the normal-gamma likelihood. The natural question is how diffuse are the priors used. A natural accounting for this would be to use a Jefferys prior. In our case this is impossible since this requires taking the Hessian of the functioning mapping the θ s to an eigenvalue. That being the case we think a uniform prior on the interval $[0, 1]$ is suitable uninformative for our purposes.

The MCMC algorithm used to calculate the above model is a Metropolis-in-Gibbs. The first block is a draw of τ (conditional on the current realization of the θ s) from its posterior distribution of Gamma with the scale parameter of $1/2+nT$ where nT is the total number observations, and a shape parameter of $1 + \text{SSE}$ where SSE is the sum of squared errors (i.e the sum of squared differences between the cash distribution and the stationary distribution). The second block is a random walk Metropolis-Hastings step to draw a realization

¹²The precision is just the inverse of the variance.

of the θ s conditional on the current realization of τ . The proposal density is a multivariate normal distribution with mean of the current θ s and a covariance matrix tuned so that the acceptance probability is approximately 25%-30%. The drawing procedure consists of simultaneously drawing the mean of the θ s, which is denoted $\bar{\theta}$, and then drawing deviations of this mean, which are denoted $\theta_{\epsilon,i}$. An individual θ is then defined as

$$\theta_i = \bar{\theta} + \theta_{\epsilon,i},$$

$i = 1, \dots, n$. This allows good movement along the likelihood surface as described by Gelman and Hill (2007).

6 Empirical results

The algorithm was started at θ_i equal to 0.5 for all banks except for bank eleven which was set at (roughly) 0.3. After this, the MCMC algorithm was run for 530,000 iterations and a posterior sample was collected.¹³ The first 30,000 iterations were discarded as a burn-in phase. Total computing time was roughly 84 hours.

The posterior sample averages and the 95% Highest Probability Densities (HPDs) are presented in Table 2. Precise estimates of θ have a fairly large amount of uncertainty to them. This is due to an identification problem in how the θ 's are defined. If all θ s are identical (say zero), then the stationary distribution that comes from this set of θ 's will be the same as that from any other identical vector of θ s. This holds for the case when all θ 's are identical and not equal to one. Another issue is that the surface of the likelihood is very flat in certain directions (e.g. the direction of the unit vector) and falls off rapidly in other directions. Because of this the sampler can only move slowly

¹³The identification problems discussed below necessitated the large amount of iterations.

Bank	θ_i	Lower 95% HPD	Upper 95% HPD
1	0.3126	0.2396	0.4538
2	0.2285	0.0178	0.4632
3	0.3305	0.2580	0.4682
4	0.3251	0.2344	0.4677
5	0.4220	0.0357	0.7454
6	0.3815	0.0809	0.6019
7	0.1992	0.0921	0.3671
8	0.3348	0.2611	0.4721
9	0.4131	0.3400	0.5359
10	0.4154	0.3504	0.5369
11	0.0778	0.0021	0.2649
12	0.3591	0.2867	0.4923
13	0.4158	0.3222	0.5438
14	0.4962	0.4287	0.6015

Table 2: Posterior Averages

around the surface of the likelihood.¹⁴

The most striking feature of the data presented in Table 2 is the degree of heterogeneity among the estimates. Looking at the most extreme case we see that bank 14 is on average over 6 times more likely to delay in making a payment than bank 11. To date there are no theories that explain why some banks would process payments more quickly than others. And, we do not attempt to explain the variation in the θ_i 's here. However, we do note that there does seem to exist a negative relationship between delay tendencies and initial liquidity holdings. Classical ordinary least squares regression of the average initial distribution of liquidity for the fourteen banks on the θ_i s provides estimates of .4056 for the intercept (standard deviation equals 0.0472) and -0.9669 for the slope (standard deviation equals 0.5442). This suggests that banks with higher liquidity holdings delay less, however this is not quite significant at the 10% confidence level (The p-value of the slope of the trend line is .1009).

¹⁴This is a problem of the likelihood not the method. In a classical exercise, like GMM, the optimizer would get stuck at non-optimal points since as the optimizer gets close to (for example) the unit vector it will stop moving (or slow down in its movements) due to the flatness.

The broad range in the estimates of the θ_i 's are produced by the Bayesian estimation procedure in order to resolve differences between the initial distributions of liquidity and the average daily liquidity. We already saw a snapshot of these differences in Figure 1. These differences are also reflected in the rankings of the banks according to initial versus average liquidity holdings. Bank 11 had the highest initial liquidity of all banks on 265 days of the sample period (272 days), but it had the highest average liquidity on only 5 days. In contrast, bank 1 had the highest initial liquidity of all banks on only 4 days, but it had the highest average liquidity on 260 days. From Table 2 we see that bank 11 has a delay parameter of only .0778 compared to .3126 for bank 1. Hence, despite its relatively lower level of initial liquidity bank 1 is over 4 times more likely to hold onto liquidity sent to it than bank 11, and hence bank 1 holds more liquidity over the course of the day. Returning to the Sutton epigraph, suppose that on some random day at some random time that Willie could steal the liquidity from one of the fourteen banks in the LVTS.¹⁵ Which bank should Willie rob? An important message of this paper is that it is not the “central” bank in the sense of Katz (1953) (i.e., the one with the highest initial liquidity). Rather, it is necessary to factor in processing speeds which, until now, were unknown.

Figure 2 shows a boxplot of the average stationary distribution of liquidity over the sample period, computed using our transition probability matrices B_t specified in (5) and our posterior averages for the delay parameters specified in Table 2. Each individual box covers the middle half (25% to 75% percentiles) of a bank's liquidity holdings according to the 272 stationary distributions we computed over the sample period. The line in the middle of the box represents the median value. The whiskers from a given box extend to the most extreme non-outlying observation (i.e. an observation less than 1.5 times the

¹⁵This is, of course, a purely hypothetical question since liquidity in the system is in the form of electronic balances rather than cash and Willie Sutton died in 1980.

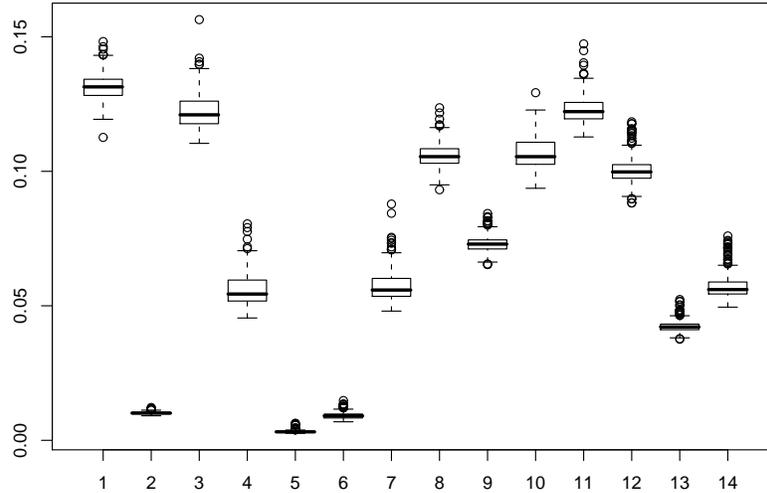


Figure 2: Boxplot of the average stationary distribution of liquidity for banks in the LVTS.

length of the given box). Observations beyond the whiskers are individually plotted.¹⁶ Our centrality predictions coincide with our declarations of the highest ranked banks according to observed (average) liquidity holdings. Bank 1 has the highest predicted liquidity according to the stationary distribution and is thus central on 260 of 272 days, bank 3 is the central bank on 7 days and bank 11 is central on the remaining 5 days.

An alternative approach would be to assume that the θ 's vary by day; since it could be argued that θ captures both processing speed and other unobserved factors.¹⁷ One way to implement this would be to find the θ vectors that fit the distribution exactly each day and look at the resulting time series of the θ 's. Preliminary work on this shows that the averages of these time series are comparable to the above posterior averages and that no discernible pattern of dependence across days is apparent.

¹⁶ Assuming normality 1.5 times the interquartile range is roughly 2 standard deviations.

¹⁷We thank Thor Koepl for pointing this out.

6.1 Comparison of the stationary distribution to the observed distribution of liquidity.

Figure 3 shows the daily stationary distributions (using the posterior means for the θ vector) and the observed average liquidity distributions over the 272 days of the sample period.¹⁸ Each point in the figure represents a matching stationary distribution value and observed value (the former is measured on the horizontal axis and the latter is measured on the vertical axis) for a given bank on a given day. Different colors represent different banks. As in Figure 1, there are 3808 points on the graph and if the two distributions matched exactly all the points would lie on the 45 degree line.

Compared to Figure 1, which involves the initial distribution of liquidity, there is improved clustering around the 45 degree line. In particular, the cluster of points associated with the fastest processor, bank 11, (magenta) is centered closely on the 45 degree line. In Figure 1 bank 11 was one of the several banks which contributed to the vertical clustering below the forty-five degree line. This was due to the fact that in Figure 1 the speed with which bank 11 (among others) processes payments was not taken into account.

7 Conclusion

In this paper we have developed an empirical measure of which banks in the Canadian LVTS payment system are likely to be holding the most liquidity at any given time. This measure is based on the implicit network structure defined by the BCLs that LVTS members grant each other.

Our measure of predicted daily liquidity holdings is based on the idea that credit limits are a good indicator of likely liquidity flows. This idea is borne

¹⁸An animated presentation of the data is available at <http://www.econ.ucsb.edu/~garratt/daily.m1v>.

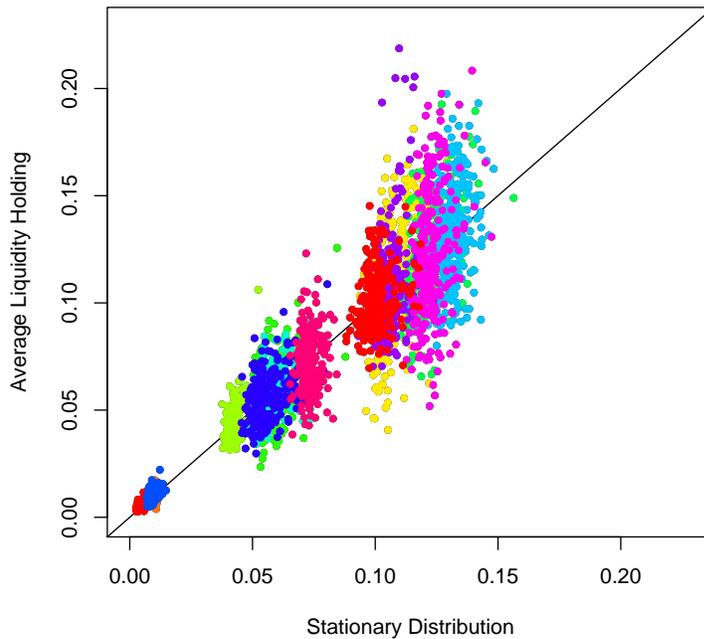


Figure 3: Observed liquidity and the stationary distribution at the posterior average values of θ .

out by comparing predicted liquidity with the realized average liquidity. One crucial parameter that we estimate is an unobserved processing speed parameter. We show that when processing speed is taken into account our measure of predicted liquidity is a good predictor of daily average liquidity holdings. Ignoring differences in processing speed leads to poorer predictions of average liquidity.

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