Unsecured Debt, Consumer Bankruptcy, and Small Business

by Césaire A. Meh and Yaz Terajima
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Acknowledgements

We would like to thank participants at the 2005 Bank of Canada–University of Western Ontario joint workshop on “Housing and the Macroeconomy”, the 2005 NBER summer institute workshop “Aggregate Implications of Microeconomic Consumption Behavior”, Econometric Society World Congress 2005 in London, Society of Computational Economics 2005, the Federal Reserve Bank of Philadelphia, Université Laval, and University of Montreal. We also thank Jason Allen, Francisco Covas, Carlos De Resende, Shubhasis Dey, Walter Engert, Martin Gervais, Dirk Krueger, Wenli Li, Igor Livshits, Vincenzo Quadrini, and Victor Rios-Rull for comments.
Abstract

In this paper we develop a quantitative model of entrepreneurial activity (risk-taking) and consumer bankruptcy choices and use the model to study the effects of bankruptcy regulations on entrepreneurial activity, bankruptcy rate and welfare. We show that eliminating bankruptcy exemptions leads to a modest increase in the fraction of entrepreneurs, a large decrease in the overall bankruptcy rate and a significant welfare gain. In contrast, eliminating the whole consumer bankruptcy system leads to a large fall in the fraction of entrepreneurs and a substantial welfare loss. These two findings suggest that the consumer bankruptcy system is desirable but it must be well-designed with regard to bankruptcy asset exemptions. In particular, excessive bankruptcy exemptions can be counter-productive. Finally, we argue that entrepreneurial activity is important when studying different bankruptcy rules or regulations.

JEL classification: D31, E21, J23
Bank classification: Economic models; Financial stability; Financial system regulation and policies

Résumé

Les auteurs construisent un modèle quantitatif formalisant les choix des ménages quant à l’entrepreneuriat (prise de risque) et à la faillite. Ce modèle leur permet d’étudier l’incidence des règles de défaillance sur l’entrepreneuriat, le taux de faillite personnelle et le bien-être. Ils montrent que l’abandon des dérogations prévues aux mesures de liquidation donne lieu à une légère hausse de la proportion d’entrepreneurs, à une chute notable du taux global de faillite et à un accroissement sensible du bien-être. À l’inverse, l’abolition de toute possibilité de recours à la faillite personnelle réduit beaucoup la proportion d’entrepreneurs et le bien-être. Il ressort de ces deux résultats que la procédure de faillite est un dispositif législatif utile, qui doit toutefois être bien conçu en ce qui concerne les biens exemptés de saisie. En effet, des dérogations trop généreuses peuvent être contre-productives. En conclusion, les auteurs soutiennent que l’étude des lois et règlements relatifs à la faillite doit tenir compte du rôle important de l’entrepreneuriat.

Classification JEL : D31, E21, J23
Classification de la Banque : Modèles économiques; Stabilité financière; Réglementation et politiques relatives au système financier
1 Introduction

In this paper, we construct a quantitative model of entrepreneurship and consumer bankruptcy and use the model to address the following question: what are the effects of bankruptcy regulation on entrepreneurship, bankruptcy rate and welfare? This is motivated by the following observations:

- Entrepreneurial activity is widely regarded as essential for innovation, capital accumulation, and development.\(^1\)

- The consumer bankruptcy data show that a substantial fraction of debtors in bankruptcy under Chapter 7 of the U.S. bankruptcy code are entrepreneurs. Specifically, Sullivan, Warren, and Westbrook (2000) and Sullivan, Warren, and Westbrook (1989) show that about 20% of bankruptcy filings are accounted for by small businesses.

- A distinctive feature of bankrupt entrepreneurs is that they carry extremely large amounts of unsecured debt. In fact, they account for more than half of all unsecured debt listed among bankrupts.

- Entrepreneurs hold almost 3.5 times as much unsecured debt as non-entrepreneurs. Their debt-to-income ratio is higher than that of non-entrepreneurs.

Despite these observations, the interaction between consumer bankruptcy and entrepreneurship has received relatively little attention in quantitative macroeconomic models. In this paper, we propose a quantitative model that is consistent with these facts and then use it to examine the effects of tightening bankruptcy rules on entrepreneurial activity, bankruptcy rate and welfare. Specifically, we consider the elimination of bankruptcy asset exemptions (i.e., the level of household assets that a debtor declaring bankruptcy can keep) and the elimination of the whole consumer bankruptcy system.

We observe the presence of entrepreneurs in consumer bankruptcy because the U.S. personal bankruptcy system, although designed for consumers, also functions as a bankruptcy system for small businesses. When firms are noncorporate, debts of the firm are personal liabilities of the entrepreneur-owner. If the business fails, the owner of the firm can declare personal bankruptcy under Chapter 7, since both business and personal debts will be discharged.\(^2\) Small business owners declaring bankruptcy under Chapter 7 must give up all assets they own in excess of a pre-determined exemption level. However, all their future earnings are exempt from the obligation to repay pre-bankruptcy debt and they can start new businesses and take new jobs—this is known as the “fresh start” in bankruptcy.

The quantitative assessment of consumer bankruptcy laws on entrepreneurship and welfare entails the evaluation of two opposite effects. On the one hand, in incomplete markets, the personal

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\(^1\)Entrepreneurship is an important component of the aggregate economy. See Haltiwanger and Krizan (1999) and Bednarzik (2000) for the link between job growth and entrepreneurship, and Schumpeter (1934) and Banerjee and Newman (1993) for the entrepreneurship-economic growth nexus. See Cagetti and de Nardi (2006), Quadrini (2000), and Meh (2005) for entrepreneurship as an important factor in explaining wealth accumulation and its distribution.

\(^2\)Personal bankruptcy law is also important for many small corporations that fail. This is because creditors frequently demand personal guarantees from business owners, thus sidestepping the legal shield of an incorporated business.
bankruptcy system, by giving both the option to discharge debt and to have bankruptcy asset ex-
emptions, provides partial insurance against business failure and job loss. Because entrepreneurship
is risky, this partial insurance allows business owners to smooth consumption across state and as a
result provides risk averse agents with incentives to undertake entrepreneurial activity. On the other
hand, financial intermediaries charge a higher premium on borrowing to cover for default risks. To the
extent that financial constraints exist, such high interest rates may discourage entrepreneurship and
limit the ability of households to smooth consumption across time. There is indeed evidence suggest-
ing that entrepreneurs face financial constraints (Evans and Jovanovic (1989)) and that bankruptcy
makes it difficult for entrepreneurs to get funding in U.S. states with generous exemptions (Berkowitz
and White (2004)). We denote the first effect, the insurance effect and the second the credit supply
effect. Evaluating the effects of bankruptcy laws mainly involves assessing the trade-off between
the insurance and credit supply effects. These effects also exist with non-entrepreneurs, but with
entrepreneurs, there are direct implications for economic performance.

To conduct our quantitative analysis, we use an overlapping generation model of entrepreneurial
and bankruptcy choices that feature these two opposing effects. Every period, households can choose
to undertake risky entrepreneurial activity or to be workers. Workers face uninsurable idiosyncratic
earnings shocks while entrepreneurs face not only earnings risks but also uninsurable idiosyncratic
investment risks. Entrepreneurial activity requires a minimum scale. Households can save with
a risk-free asset and borrow from competitive financial intermediaries to smooth consumption, to
become entrepreneurs, and to increase the scale of their businesses. Borrowing entails a financial
intermediation cost per unit of funds borrowed. Every period, we allow households (workers and
entrepreneurs) to choose to default on their non-contingent unsecured debts. The default choice is
modelled as a decision to file for personal bankruptcy under U.S. Chapter 7 where unsecured debts are
fully discharged in exchange for assets in excess of pre-determined exemptions. Naturally, this default
option leads financial intermediaries to charge a default premium on loans to households. Following
Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) and Livshits, MacGee, and Tertilt (2007), we
assume that when making loans, financial intermediaries can observe a household’s current income,
occupation, level of business capital, savings, debt holdings and age. As a result, the equilibrium
price of loans to borrowers is a function of their current income, occupation, portfolio (savings and
business investment), age and level of debt.

Our quantitative model successfully replicates the above regularities on entrepreneurship and
bankruptcy. In particular, the model accounts for the extraordinarily high level of debt of en-
trepreneurs in bankruptcy, the high bankruptcy rate of entrepreneurs relative to workers, and the
high debt-to-income ratio of entrepreneurs relative to workers. The model also generates the hump-
shape profile of the bankruptcy rate over the life-cycle. Given the success of our model in replicating
the data, we use it to conduct two policy experiments that are meant to capture the tightening
of bankruptcy rules as recently approved in the U.S. The first policy experiment consists of the
elimination of the bankruptcy exemption. In such a case the only benefit associated with declaring
bankruptcy is the full discharge of debt (the fresh start). The second policy reform is the elimination
of the entire bankruptcy system which implies that the default option is removed. We find that elim-

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inating the bankruptcy exemption leads to an increase in the number of entrepreneurs, a decrease in
the bankruptcy rate, and most importantly a significant welfare gain. The welfare gain comes from
the increase in entrepreneurial activity and the ability to better smooth consumption over time with
the decrease in interest rate. The credit supply effect of the bankruptcy exemption thus dominates
the insurance effect.

In contrast, eliminating the entire consumer bankruptcy system leads to a large decrease in
the number of entrepreneurs and a significant welfare loss. The insurance effect of the consumer
bankruptcy system is more valuable than the credit supply effect and thus promotes entrepreneurship
and smooths consumption across states. The fact that the bankruptcy exemption is welfare-reducing
while the whole bankruptcy system is welfare-improving provides a very clear message: the consumer
bankruptcy system is essential for its insurance role but must be well designed in the sense that
bankruptcy exemptions have to be low.

Another key finding of this paper is that entrepreneurial activity plays a central role in consumer
bankruptcy law analysis. When entrepreneurship is not explicitly modelled, the welfare implications
of bankruptcy regulations are small and may even have different directions relative to an economy
in which entrepreneurial activity is accounted for. For example, in the model without entrepreneurs,
eliminating bankruptcy asset exemptions leads to a small welfare gain (0.024% of lifetime consump-
tion) while the welfare gain is large in the model with entrepreneurs (1.78%). With respect to the
elimination of the entire consumer bankruptcy system, the direction of the welfare effects varies
depending on whether or not entrepreneurs are included in the economy. In contrast to the case
when entrepreneurs are considered, eliminating the entire consumer bankruptcy system in the model
without entrepreneurs increases welfare, although the welfare gain is relatively small. This result is
consistent with Athreya (2002) and also with Livshits, MacGee, and Tertilt (2006) when there are
no expense shocks and only earnings shocks are present.

A number of recent papers have focused on studying the magnitude of the economic effects of per-
sonal bankruptcy reform in quantitative dynamic equilibrium models (e.g., Athreya (2001), Athreya
(2004), Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007), Li and Sarte (2006), Livshits, MacGee,
and Tertilt (2006)) These papers have, however, ignored entrepreneurship. As we have shown in this
paper, accounting for entrepreneurial activity is crucial for studying the determinants of bankruptcy
and for understanding the economic implications of bankruptcy policy. Chatterjee, Corbae, Nak-
ajima, and Ríos-Rull (2007) make theoretical and quantitative contributions in a macroeconomic
model with a bankruptcy option in which prices of unsecured loans depend on loan size and house-
hold characteristics. They show that means-testing under Chapter 7 leads to large welfare gains.
Livshits, MacGee, and Tertilt (2006) in a bankruptcy model with wage garnishment, illustrate the
importance of expense shocks and life-cycle effects when comparing bankruptcy rules. Li and Sarte
(2006) consider aggregate production and the choice between Chapter 7 and Chapter 13 and find
that eliminating consumer bankruptcy and lowering the bankruptcy exemption increase welfare. In
their model, however, all households borrow at the same interest rate regardless of their type or the
amount borrowed. In contrast to Li and Sarte (2006), Athreya (2002), who also assumes a single
interest for all borrowers, finds that eliminating the bankruptcy system improves welfare. None of
these models account for entrepreneurial activity and the effects of the consumer bankruptcy system
on income through risk-taking.

Our paper is also related to an independent work of Akyol and Athreya (2007), who also study bankruptcy with occupational choices. In their model, however, only entrepreneurs are allowed to file for bankruptcy under Chapter 7 and workers are not. This is not only inconsistent with the data, but it also substantially exaggerates the effects of the bankruptcy asset exemption on entrepreneurship and welfare. Another important feature of their framework that also contributes to exaggerate the effects of the bankruptcy exemption is that in their economy, only poor households with little employment prospects move to entrepreneurship. This feature makes self-employment a disguised form of unemployment and more sensitive to bankruptcy. In our work, on the other hand, individuals with good employment prospects could decide to enter entrepreneurship if they have good business ideas.

Our paper is related to several empirical studies of bankruptcy. Fan and White (2003) in their probit analysis show that in states with high homestead exemptions (exemption of on housing equity for homeowners), the probability of becoming self-employed is high. This result may, however, be due to state-specific effects such as tax laws, banking regulations, and geographical location. One of their results that may support the case for state-specific effects is that renters in these states have also a high probability of becoming self-employed since renters do not have equity. Moreover, given the reduced form approach of their analysis, they are unable to conduct a policy experiment. Using cross-country analysis, Armour and Cumming (2005) argue that the tighter is the personal bankruptcy law the lower is the self-employment rate across countries. For instance, they find that a ten-year reduction in the time to discharge unsecured debt increases self-employment rates by approximately 1.5% in the countries considered. This result is consistent with our finding, though much like Fan and White, their analysis suffers from country-specific effects.

The plan of the paper is as follows. In the next section we briefly describe the bankruptcy law in the U.S., its link to entrepreneurship, and data unsecured debt and debt-income ratio in the population. Section 3 presents the model economy. Section 4 parameterizes the model and Section 5 discusses the benchmark results. Section 6 conducts the quantitative analysis using the parameterized version of the model. Section 7 considers the importance of entrepreneurship for bankruptcy analysis. Section 8 concludes.

2 Consumer bankruptcy law and entrepreneurship

Bankruptcy law In many countries, households can declare bankruptcy on their unsecured debts, such as credit card debts. In the U.S., for example, households can choose either to file under Chapter 7, “total liquidation” or Chapter 13, “payout plan.” Chapter 7 fully discharges the unsecured debts, but requires household debtors to give up all assets above an exemption level. Future earnings of debtors are exempt from the obligation to repay. This is known as the “fresh start” in bankruptcy. Filing under Chapter 7 precludes the debtor from declaring bankruptcy under Chapter 7 for six

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4 Congressional Budget Office (2000) provides an excellent review of the empirical literature.
5 The equivalent of Chapter 7 and Chapter 13 in Canada are the “Straight Bankruptcy” and the “Consumer Proposal” respectively.
years. Most bankruptcy filings (about 70% of total) are done under Chapter 7.

Under Chapter 13, debtors do not receive any immediate discharge of their unsecured debts. Instead they must propose a plan to use some of their future earnings to repay part of or all their unsecured debt over 3 to 5 years while keeping all their assets. If they respect this rescheduling plan the remaining unsecured debts are fully discharged. This procedure is generally less favorable to households (such as business owners) than Chapter 7 because often they do not have any nonexempt assets. This may explain why most bankruptcy filings are done under Chapter 7. Since Chapter 7 is the most preferred bankruptcy procedure by households, we therefore model incentives to file for personal bankruptcy as a decision under Chapter 7.

Although the bankruptcy law is uniform across the U.S., bankruptcy exemptions are set by the states and vary widely. A bankruptcy exemption is the level of household assets that a debtor declaring bankruptcy can keep. Most states have several types of exemptions: for equity in owner-occupied housing (the homestead exemption), for equity in cars, for cash, and for various types of goods. In most states, the homestead is the largest, and other exemptions are small.

Causes of bankruptcy Sullivan, Warren, and Westbrook (2000) study a survey of bankrupt households in 1991 where they asked these debtors the reasons for their bankruptcy. They find that 67.5% of bankrupt debtors identify job loss (particularly, unemployment or personal business failure) as the main cause of bankruptcy. Hence, anything, such as business failure, that causes household income to fall exposes them to the risk for bankruptcy filings. The next two immediate reported causes of personal bankruptcy are family (divorce, unexpected children) and medical problems. Other sources are for example, credit problem and housing difficulties.

Entrepreneurs in bankruptcy Sullivan, Warren, and Westbrook (1989) show that a substantial fraction of bankrupts are entrepreneurs. Specifically, they find that 20% were either entrepreneurs at the time of filing or had been entrepreneurs before they filed for bankruptcy. Given that the fraction of entrepreneurs in the population is lower than the fraction of entrepreneurs in bankruptcy, entrepreneurs are over-represented in bankruptcy so that they are about twice more likely to be bankrupt than workers. A key feature of entrepreneurs in bankruptcy is that they typically have extremely high debt levels, with particularly high levels of unsecured debt although they represent only 20% of all bankrupt debtors. Entrepreneurs account for more than half of the total unsecured debt. This is illustrated in Table 1 which presents the balance sheets of entrepreneurs and workers (non-entrepreneurs) in bankruptcy. The table shows that entrepreneurs have not only higher assets than workers, but also higher debts (secured and unsecured debts) than workers. For example, the mean of unsecured debts for entrepreneurs is almost $40,000 for entrepreneurs and only just less than $10,000 for workers.

Entrepreneurs are more heavily burdened by debts. This is shown in Table 2 which presents debt-income ratios of workers and entrepreneurs. The average debt-income ratio of entrepreneurs is 7.1 which implies that a typical entrepreneur debtor in bankruptcy would owe debts greater than

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6 Multiple responses were permitted during the survey.
Table 1: Distribution of Debts and Assets for Entrepreneurs and Workers in Bankruptcy

<table>
<thead>
<tr>
<th></th>
<th>Total Debt</th>
<th>Unsecured Debt</th>
<th>Secured Debts</th>
<th>Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Entrepreneurs</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>$85,824</td>
<td>$38,087</td>
<td>$47,897</td>
<td>$55,434</td>
</tr>
<tr>
<td>25th percentile</td>
<td>24,931</td>
<td>7,966</td>
<td>6,362</td>
<td>8,985</td>
</tr>
<tr>
<td>Median</td>
<td>53,193</td>
<td>19,039</td>
<td>25,976</td>
<td>40,592</td>
</tr>
<tr>
<td>75th percentile</td>
<td>109,899</td>
<td>46,305</td>
<td>59,000</td>
<td>71,727</td>
</tr>
<tr>
<td><strong>Workers</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>$26,753</td>
<td>$9,827</td>
<td>$16,850</td>
<td>$22,835</td>
</tr>
<tr>
<td>25th percentile</td>
<td>8,715</td>
<td>3,475</td>
<td>2,068</td>
<td>2,196</td>
</tr>
<tr>
<td>Median</td>
<td>16,953</td>
<td>6,063</td>
<td>8,010</td>
<td>10,595</td>
</tr>
<tr>
<td>75th percentile</td>
<td>35,750</td>
<td>10,577</td>
<td>25,275</td>
<td>37,942</td>
</tr>
</tbody>
</table>


seven years’ income. By contrast, a typical worker would owe debts equal to less than three years’ income—i.e., debt-income ratio equal to 2.5.

Table 2: Distribution Debt-Income Ratios for Entrepreneurs and Workers in Bankruptcy

<table>
<thead>
<tr>
<th></th>
<th>Entrepreneurs</th>
<th>Workers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>7.1</td>
<td>2.5</td>
</tr>
<tr>
<td>25th percentile</td>
<td>1.4</td>
<td>0.7</td>
</tr>
<tr>
<td>Median</td>
<td>2.7</td>
<td>1.3</td>
</tr>
<tr>
<td>75th percentile</td>
<td>7.5</td>
<td>2.3</td>
</tr>
</tbody>
</table>


**Unsecured debts by occupation** So far we have reported information on entrepreneurs and workers in the bankruptcy sample. It is also important to have know the amount of unsecured debt and debt income ratio in the economy in a non-bankruptcy data set. To do so, we use the Survey of Consumer Finance of 2001 to present average amount of unsecured debt and the debt-income-ratio of entrepreneurs and workers. The results are reported in Table 3. There are a few things to note from the table. First, entrepreneurs have higher debt-income ratio than workers. Put differently, entrepreneurs owe on average more than seven years of income in unsecured debts while workers owe just five years of income. Second, entrepreneurs hold more that 30% of total unsecured debt and their unsecured debt is almost three and half times the unsecured debts of workers.
Table 3: Unsecured debts and Debt-Income Ratios for Entrepreneurs and Workers in the SCF

<table>
<thead>
<tr>
<th></th>
<th>Entrepreneurs</th>
<th>Workers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unsecured debt to income ratio</td>
<td>7.14</td>
<td>5.14</td>
</tr>
<tr>
<td>Unsecured debt</td>
<td>$10,009</td>
<td>$3,034</td>
</tr>
<tr>
<td>Fraction of Unsecured debt</td>
<td>31.03%</td>
<td>69.97%</td>
</tr>
</tbody>
</table>


There is also indirect evidence (not from the SCF) that suggests that entrepreneurs use credit cards to finance their businesses. In 1998 a study from Arthur Anderson reported in Sullivan, Warren, and Westbrook (2000) argue that 47% of small businesses use credit cards to finance investment.

The effects of bankruptcy law on entrepreneurship Entrepreneurs benefit from higher personal bankruptcy exemptions because exemptions provide partial wealth insurance, but at the cost of a reduction in credit availability (credit rationing and higher interest rate) as the exemption level rises. Using the family-level panel data, Survey of Income and Program Participation (SIPP) and the variation of bankruptcy exemptions across states, Fan and White (2003) investigate whether households are more likely to become entrepreneurs if they live in states with higher homestead exemptions. They find that households who are homeowners are about 35% more likely to be business owners if they live in states with unlimited exemption—hence, the wealth insurance effect of homestead exemptions encourages business ownership. Moreover, Berkowitz and White (2004), using the 1993 National Survey of Small Business Finance (NSSBF), find that small firms are more likely to be denied credit if they are located in states with unlimited homestead exemptions than low homestead exemptions, and that even when they receive the loan, the loans are on average smaller and lending rates are higher. Armour and Cumming (2005) provide cross-country evidence and show that the tighter is the personal bankruptcy law the lower is the self-employment rate across countries.

3 Model economy

The model is similar to Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) and Livshits, MacGee, and Tertilt (2006) and the main difference is that entrepreneurial activity is explicitly modelled. Specifically, the key characteristics are as follows: first, the model features a bankruptcy choice as well as an entrepreneurial choice. Second, there are two sources of idiosyncratic uncertainty: uninsurable idiosyncratic investment risks and earnings risks. There is no aggregate uncertainty.

3.1 Preferences

The model economy is populated by overlapping generations of individuals who live for $J$ periods. The population grows at a constant rate $n$. Each individual of age $j$ maximizes her expected
discounted lifetime utility,
\[
E_0 \sum_{j=1}^{J} \beta^{j-1} u \left( \frac{c_j}{s_j} \right), \quad u \left( \frac{c}{s} \right) = \frac{(\frac{c}{s})^{1-\sigma} - 1}{1-\sigma},
\]
where \( \beta > 0 \) is the intertemporal discount factor, \( \sigma > 1 \) is the coefficient of relative risk aversion, \( c_j \) is the consumption good at age \( j \), and \( s_j > 0 \) is an exogenous parameter that captures the evolution of family sizes over the life-cycle. The introduction of the family size over the life-cycle is important for explaining the hump-shaped life-cycle profile of consumption (Attanasio and Rios-Rull (1999)).

### 3.2 Endowments and stochastic processes

In each period, age-\( j \) agents are endowed with earnings \( y(\varepsilon, j) = \bar{y}_j \varepsilon \), where \( \bar{y}_j \) is the deterministic average life-cycle profile of earnings and \( \varepsilon \in \mathcal{E} = \{\varepsilon_1, ..., \varepsilon_{N_\varepsilon}\} \) is an idiosyncratic earnings shock. The shock \( \varepsilon \) is observed at the beginning of the period and follows a first-order Markov process with a transition probability \( Q_\varepsilon (\varepsilon', \varepsilon) \).\(^7\) The shock received by age-1 agents are drawn from the stationary distribution \( Q_\varepsilon^* (\varepsilon) \).

Every period, each age-\( j \) household can run a risky technology that returns, \( G(k, \eta) \), in the next period with capital input \( k \). The variable \( \eta \) is an idiosyncratic business shock that is unknown when \( k \) is chosen. The shock, \( \eta \in \{\eta_1, ..., \eta\} \), follows a first-order Markov process with a transition probability, \( Q_\eta (\eta', \eta) \). Age-1 agents draw \( \eta \) from the invariant distribution \( Q_\eta^* (\eta) \) of \( Q_\eta \). We assume that there is a minimum size of business \( k \). We call households that decide to run the risky technology with \( k \geq k \) entrepreneurs and the remaining of the population workers. The gross return, \( G \), is given by the following equation:
\[
G(\eta, k) = \eta k^\nu + (1-\delta)k.
\]
The first component is the output produced and \( \nu < 1 \) is the return-to-scale parameter. The second component is the non-depreciated capital, where \( \delta \) is the depreciation rate.

### 3.3 Financial intermediation

There is a competitive intermediation sector that processes all borrowing and lending. The return on savings \( s \geq 0 \) is given by the risk-free rate \( r^* \). Household can borrow unsecured debts \( (d \geq 0) \), that is non-collateralized debts, such as credit cards loans. When making loans to households, financial intermediaries incur transaction cost \( \phi \) per unit of loan. Individuals can declare bankruptcy on unsecured debts. Following Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) and Livshits, MacGee, and Tertilt (2006), we assume that financial intermediaries charge individual-specific prices on unsecured debts to capture the type-specific risk of bankruptcy. The underlying assumption is that, when making unsecured loans to a household, intermediaries observe the level of total unsecured loans, the business investments, the current earnings shock, the current business shock, and the age of the borrower. As a result, the price of the unsecured debt charged to a household with a

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\(^7\)All variables denoted by prime (’) refer to next period variables.
portfolio $\mathcal{P} = (s, k, d)$, shocks, $x = (\varepsilon, \eta)$, and age, $j$, is given by $q_d(s, k, d, \varepsilon, \eta, j)$ which in turn leads to an endogenous borrowing limit. The corresponding gross interest rate of this unsecured loan is $R_d(s, k, d, \varepsilon, \eta, j) = 1/q_d(s, k, d, \varepsilon, \eta, j)$. Because of perfect competition financial intermediaries make zero profit on unsecured loans made to each type of households. This implies there is no cross subsidization of interest rates across different types of borrowers. Let us define $\Lambda (x, \mathcal{P}', j)$ to be the probability that an age-$j$ household with next period portfolio, $\mathcal{P}' = (s', k', d')$, and current shocks, $x$, will file for bankruptcy in the next period. The zero profit condition on unsecured loans to each type of household borrower implies the following expression:

$$q_d(\mathcal{P}', x, j) = [1 - \Lambda (\mathcal{P}', x, j)] \overline{q_d} + \Lambda (\mathcal{P}', x, j) E \left[ \max\left\{ s' + G(\eta', k'), 0 \right\} \middle| b = 1 \right] \overline{q_d}$$

where $\overline{q_d} = \frac{1}{1+r^*+\phi}$ is the price of unsecured loans when the probability of default is zero, $\chi$ is the government specified bankruptcy exemption, and $\Lambda$ is the probability of default (which is defined in the equilibrium). The variable $b \in \{0, 1\}$ represents the decision of whether to declare bankruptcy or not. Note that the definition of the price of $q_d$ takes into account how much financial intermediaries can collect when individuals declare bankruptcy. This is explained below in section 3.4.

### 3.4 Bankruptcy provisions

Households can declare bankruptcy on their unsecured debts. Bankruptcy in the model will be similar to U.S. Chapter 7 fresh start provision.\(^8\) Filing for bankruptcy leads to a full discharge of unsecured debt $d$ of the household borrower. The household must, however, give up any assets in excess of a specified bankruptcy exemption level, $\chi$, to the creditor. More specifically, the amount of asset that a household with a business of size, $k$, a business shock, $\eta$, and savings, $s$, can keep in bankruptcy is given by the following expression:

$$\min \{G(\eta, k) + s, \chi\},$$

where the first term represents business income plus savings.

In addition to losing non-exempted assets a bankrupt household is also subject to the following punishment. The bankrupt households lose a fraction $\lambda$ of their current labor income. The parameter $\lambda$ summarizes all non-exemption costs associated with bankruptcy filing: (i) the stigma cost, (ii) the pecuniary costs of a bad credit rating, and (iii) the exclusion from the credit market.

We end this section by assuming that a household cannot die with debts or assets. Therefore at age $J$ we have the following: $d = 0$, $s = 0$, and $k = 0$.

\(^8\)In the U.S. households can also file under Chapter 13. Contrary to Chapter 7, filing under Chapter 13 does not discharge all unsecured debts, instead the debts are rescheduled. In the U.S., however, around 70% of all bankruptcy filings occur under Chapter 7.
3.5 Timing of events

It is convenient to define \( a \equiv s + G(\eta, k) - d + y(\varepsilon, j) \) as the resources at hand after the bankruptcy decision. The timing of events within a period is as follows. At the beginning of the period, households observe the earnings and business shocks, \( x = (\varepsilon, \eta) \). Given shocks, \( x \), and the beginning-of-period composition of the household’s portfolio, \( P = (s, k, d) \), the agent decides whether to declare bankruptcy or not. If the household declares bankruptcy, the non-exempted assets are seized by the creditor, a fraction, \( \lambda \), of current labor income is lost, and all unsecured debts are fully discharged. Bankrupt households and non-bankrupt households differ mainly in their debt or asset position. Households choose their current consumption, savings, borrowing, and business investments decisions. This is summarized as follows.

<table>
<thead>
<tr>
<th>Pre-bankruptcy state:</th>
<th>Post-bankruptcy state:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( j, \varepsilon, \eta, s, d, k )</td>
<td>( j, \varepsilon, \eta, a )</td>
</tr>
<tr>
<td>( t )</td>
<td>( t+1 )</td>
</tr>
<tr>
<td>Bankruptcy decision:</td>
<td>Consumption-portfolio decisions:</td>
</tr>
<tr>
<td>( b )</td>
<td>( c, s', d', k' )</td>
</tr>
</tbody>
</table>

3.6 The individual’s decision problem

We define \( V_j(x, a) \) to be the post-bankruptcy decision value function of an age-\( j \) household with current shocks, \( x = (\varepsilon, \eta) \), and debt, \( d \). Similarly, define \( W_j(x, P, a) \) as the pre-bankruptcy decision value function of an age-\( j \) household whose beginning-of-period shocks are \( x \) and beginning-of-period portfolio is \( P \). We assume that the value function at age \( J + 1 \) is \( V_{J+1}(\cdot) = 0 \).

The pre-bankruptcy decision value function of an age-\( j \) individual is given by

\[
W_j(x, \Omega; \chi) = \max_{b \in \{0,1\}} V_j(x, a; \chi) \tag{5}
\]

where

\[
a = \begin{cases} 
  s + G(\eta, k) - d + y(\varepsilon, j) & \text{if } b = 0 \\
  \min \{s + G(\eta, k), \chi\} + (1 - \lambda)y(\varepsilon, j) & \text{if } b = 1 
\end{cases} \tag{6}
\]
If the household chooses not to declare bankruptcy \((b = 0)\), then his or her post-bankruptcy decision asset level is given by saving, plus business income, minus debt and plus labor income. If the household chooses to declare bankruptcy \((b = 1)\) his or her asset level is the exempted assets plus labor income net of bankruptcy costs.

The post-bankruptcy decision value function is defined as follows:

\[
V_j(\varepsilon, \eta, a) = \max_{c, s', k', d'} \left( \frac{c}{n_j} \right) + \beta E \left[ W_{j+1}(\varepsilon', \eta', s', k', d') | \varepsilon, \eta \right]
\]  

subject to

\[
c + \frac{s'}{1 + r^*} + k' - q_d(s', k', d', \varepsilon, \eta, j) d' = a.
\]  

\[
c, d', s' \geq 0, \quad \text{and} \quad k' \geq k \text{ or } k' = 0.
\]

Equation (8) is the budget constraint. On the right hand side of the budget constraint are assets (or resources at hand), \(a\). On the left hand side of the budget constraint, \(c\) is the current consumption, \(s'\) is savings with the price \(q^* = 1/(1 + r^*)\), \(d'\) is unsecured loans with the price function \(q_d(x, \mathcal{P}', j)\) defined in equation (3), and \(k'\) is business investments for the next period. If \(k' = 0\), the household chooses to become a worker in the next period and if \(k' \geq k\) the household becomes an entrepreneur. Equation (9) presents the lower limits on the choice variables.

### 3.7 Equilibrium

We are now ready to define the competitive equilibrium.

**Definition 1** Given the risk-free bond price, \(q^*\) and the bankruptcy exemption, \(\chi\), a recursive competitive equilibrium is a set of value functions, \(V(\cdot)\) and \(W(\cdot)\), and a set of policy functions, \(s(\cdot), d(\cdot), k(\cdot), c(\cdot),\) and \(b(\cdot)\), an unsecured debt price function, \(q_d(\cdot)\), and a probability of declaring bankruptcy, \(\Lambda(x, \mathcal{P}', j)\) such that

1. \(V(\cdot)\) and \(W(\cdot)\) solve the household problem, and \(s(\cdot), d(\cdot), k(\cdot), c(\cdot),\) and \(b(\cdot)\) are the associated decision rules;

2. The unsecured debt price function, \(q_d(\cdot)\) is obtained by the zero profit conditions for financial intermediaries in equation (3);

3. The default probability is defined by

\[
\Lambda(x, \mathcal{P}', j) = \sum_{x'} b(x', \mathcal{P}', j + 1) Q_\varepsilon(\varepsilon', \varepsilon) Q_\eta(\eta', \eta).
\]
4 Benchmark Parametrization

To obtain numerical solutions and conduct policy analysis, we need to choose particular values for the parameters of the model economy. The parameters to be calibrated are related to the households preferences, the stochastic process for labor efficiency, the stochastic process of business income, technology in the entrepreneurial sector, technology in the intermediation sector, and the bankruptcy system.

4.1 Demographics

We choose the model period to be 5 years. Agents start their life at age 20 and die at age 80 with 12 periods ($J = 12$). The annual population growth rate is 0.01 ($n = 1.01^{0.01} - 1$). The life-cycle profile of family size $s_j$, is calculated by multiplying the reciprocal of the average family size of each age group from 2001 SCF, $j$, by the family equivalence scale given by Nelson (1993).

4.2 Preferences

The preference parameters include the coefficient of relative risk aversion, $\sigma$, and the discount factor, $\beta$. The relative risk aversion parameter, $\sigma$, is set 2.0. This number is in the range of estimates suggested by Auerbach and Kotlikoff (1987) and Prescott (1986). The annual discount factor is set 0.96 so that $\beta = 0.96^{0.01}$.

4.3 Labor ability

The average age-profile of earnings $\bar{y}_j$ is taken from Hansen (1993) and adjusted for social security. The persistent idiosyncratic shock, $\varepsilon$, is assumed to follow a four-state Markov process with transition probability $Q_{\varepsilon}$. To calibrate $Q_{\varepsilon}$, it is assumed that the logarithm of the household’s labor earnings follows a first-order autoregressive process:

$$\ln (\varepsilon_{t+1}) = \rho \varepsilon \ln (\varepsilon_t) + \xi_{t+1}, \quad \xi_{t+1} \sim N(0, \sigma^2_\xi).$$ (11)

Following Storesletten, Telmer, and Yaron (2004) and Livshits, MacGee, and Tertilt (2006) we set annual the value of $\rho = 0.99$ and $\sigma^2_\xi = 0.016$. To convert these values into the five year period we assume that: $\rho = 0.99^5$ and $\sigma^2_\xi = (1 + \rho + \rho^2 + \rho^3 + \rho^4) \times 0.016$.

Given ($\rho, \sigma_\xi$), the procedure described in Tauchen and Hussey (1991) is used to approximate the above autoregression by a four-state Markov chain.

4.4 Intermediation technology

Two parameters need to be set in this section: the exogenous risk-free rate, $r^*$, and the transaction cost parameter, $\phi$. The annual risk-free rate is set equal to 4%, which is the average return on capital reported by McGrattan and Prescott (2000). The risk-free rate for a five year period is thus $r^* = (1.04)^5 - 1 = 21.67\%$. The transaction costs are calibrated to match the observed interest rate spread between secured and unsecured debt. Following Athreya (2004), the annual transaction cost
on unsecured borrowing are 6.4%. The five-year interest rate on unsecured borrowing is $(1.04)^5 - 1 = 64\%$. This implies that the transaction cost for the five-year period is the difference between the five-year borrowing rate and the five-year risk free rate: $\phi = 64\% - 21.67\% = 42.33\%$.

### 4.5 Business technology

There are three parameters to be set in this section: the returns-to-scale parameter, $\nu$, the minimum business size $k$, and the depreciation rate, $\delta$. Following Cagetti and de Nardi (2006), we set $\nu$ to 0.7. The minimum size is set to match the fraction of entrepreneurs in the data. Using the 2001 Survey of Consumer Finance the fraction of population that is business owner is 12%. This number ranges between 8% and 18% (see Gentry and Hubbard (2000) and Quadrini (1999)). Finally, the annual depreciation rate is set to $\delta = 0.062$.

### 4.6 Bankruptcy law

The bankruptcy law in this economy is characterized by two parameters: the bankruptcy exemption, $\chi$, and the bankruptcy cost of filing, $\lambda$. In the U.S., while bankruptcy is a matter of federal law and the procedure is uniform across the country, Congress gave the states the right to set their own bankruptcy exemption levels, and these vary widely. Most states have several type of exemptions which can be broadly classified into two groups: the homestead exemption and the non-homestead exemption. The homestead exemption is the exemption on the equity in owner-occupied principal residence. The non-homestead exemption is: for equity in cars, for cash, and for various types of goods (furniture, clothing, cooking utensils, farm implements, family bibles, and tools for trade, etc). In most states, the homestead exemption is the largest, and the non-homestead exemption is small. In the U.S. the average national homestead bankruptcy is $25,000 and while the non-homestead exemption is $6,797 in 1994. In the model, we will consider bankruptcy exemption as the sum of both homestead and non-homestead exemptions. In the U.S., the average annual earnings is about $58,000 in 1994. The annual mean total bankruptcy exemption to mean earnings ratio is 0.79. Normalizing mean earnings to unity implies that $\chi = 0.158 = 0.79 \div 5$.

The parameter, $\lambda$, is calibrated to match the overall bankruptcy rate. Fay, Hurst, and White (2002) report that the national bankruptcy rate varies between 0.33 percent and 0.88 percent over the period 1984-1995. The targeted annual bankruptcy rate in the model is chosen to 0.88. The bankruptcy rate for the five year period is obtained by multiplying the annual bankruptcy rate by five.

The bankruptcy rate of entrepreneurs is obtained by using (i) the fact that 20% of bankrupts are entrepreneurs (Sullivan, Warren, and Westbrook (2000)), (ii) the fraction of entrepreneurs in the economy (12%) and (iii) the overall bankruptcy rate. The bankruptcy rate of entrepreneurs is given by 0.20 times the overall bankruptcy rate and divided by the measure of entrepreneurs.
4.7 Stochastic process of business shocks

We assume that the transition probability function, \( Q_\eta \), of the business shock is a four-state discrete approximation, à la Tauchen and Hussey (1991), to an AR(1):

\[
\ln \eta_t = (1 - \rho_\eta) \mu_\eta + \rho_\eta \ln \eta_{t-1} + \zeta_t, \quad \text{with} \quad \zeta_t \sim N(0, \sigma^2_\zeta). \tag{12}
\]

\( Q^*_\eta \) is the invariant distribution of the transition matrix \( Q_\eta \). We have four parameters (\( \lambda, k, \mu_\eta, \sigma_\zeta \)) that are set endogenously to match the following five targets in equilibrium: (1) the fraction of entrepreneurs in the population, 12 percent, where entrepreneurs are defined as business owners who have active management in the 2001 Survey of Consumer Finances. (2) The fraction of bankrupt households that are entrepreneurs. Sullivan, Warren, and Westbrook (1989) report that 20 percent of bankrupt households are entrepreneurs. (3) The overall five-year bankruptcy rate of 4.4 percent. (4) The entrepreneurs’ income to workers’ income ratio is 2.72 in the 2001 SCF. The annual values for the targets are summarized in Table 4.

Table 4: Moments Targeted in the Benchmark in Annual Values

<table>
<thead>
<tr>
<th>Moment</th>
<th>Values</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of Entrepreneurs</td>
<td>12%</td>
<td>SCF (2001)</td>
</tr>
<tr>
<td>Fraction of bankrupts that are</td>
<td>20%</td>
<td>Sullivan, Warren, and Westbrook (1989)</td>
</tr>
<tr>
<td>entrepreneurs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall annual bankruptcy rate</td>
<td>0.88%</td>
<td>Athreya (2004)</td>
</tr>
<tr>
<td>Entrepreneurs’ income to workers’</td>
<td>2.72</td>
<td>SCF (2001)</td>
</tr>
<tr>
<td>income ratio</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The fixed parameters and the endogenously set parameters (\( \lambda, k, \mu_\eta, \sigma_\zeta, \rho_\eta \)) are listed in Table 5.

Table 5: Calibrated Parameters of the Benchmark Economy

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fixed parameters</strong></td>
<td></td>
</tr>
<tr>
<td>( \sigma ) Relative risk aversion</td>
<td>2</td>
</tr>
<tr>
<td>( \beta ) Discount factor</td>
<td>0.815</td>
</tr>
<tr>
<td>( J ) Lifetime</td>
<td>12</td>
</tr>
<tr>
<td>( \nu ) Degree of return to scale</td>
<td>0.700</td>
</tr>
<tr>
<td>( \rho_\varepsilon ) Coefficient of autocorrelation of labor income</td>
<td>0.951</td>
</tr>
<tr>
<td>( \sigma_\zeta^2 ) Annual variance of labor income</td>
<td>0.007</td>
</tr>
<tr>
<td>( r^* ) Risk-free rate (%)</td>
<td>21.67</td>
</tr>
<tr>
<td>( \phi ) Annual transaction costs on unsecured debt</td>
<td>0.064</td>
</tr>
<tr>
<td>( \chi ) Bankruptcy exemption</td>
<td>0.158</td>
</tr>
<tr>
<td><strong>Endogenously calibrated parameters</strong></td>
<td></td>
</tr>
<tr>
<td>( \mu_\eta ) Average business shock</td>
<td>0.224</td>
</tr>
<tr>
<td>( \sigma_\zeta ) Standard deviation of business shock</td>
<td>0.657</td>
</tr>
<tr>
<td>( \rho_\eta ) Coefficient of autocorrelation of business shocks</td>
<td>—</td>
</tr>
<tr>
<td>( k ) Minimum business size</td>
<td>1.273</td>
</tr>
<tr>
<td>( \lambda ) Bankruptcy punishment</td>
<td>0.466</td>
</tr>
</tbody>
</table>
5 Benchmark results and properties of model

In this section, we present the calibration results and the properties of the model. Table 6 reports the target statistics and the implied results from the model. The table shows that, the model matches relatively well the targets. For example, in the data the overall annual bankruptcy rate and the fraction of entrepreneurs are 0.88% and 12% respectively; the model obtains 0.89% and 12% respectively for these variables.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Targets</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of Entrepreneurs (%)</td>
<td>12.00</td>
<td>12.00</td>
</tr>
<tr>
<td>Overall annual bankruptcy rate (%)</td>
<td>0.88</td>
<td>0.89</td>
</tr>
<tr>
<td>Entrepreneur annual bankruptcy rate (%)</td>
<td>1.467</td>
<td>1.49</td>
</tr>
<tr>
<td>Entrepreneurs’ income to workers’ income</td>
<td>2.72</td>
<td>2.60</td>
</tr>
</tbody>
</table>

Our model economy provides attractive implications by occupational status. Table 7 presents debt and bankruptcy statistics for entrepreneurs and workers. With the exception of the bankruptcy rate of entrepreneurs, the statistics in this table are not targeted, they are implied by the model. Therefore, they provide an additional way to gauge the success of our model in terms of being consistent with the data. Several key results emerge from the table. First, it is apparent from the table that entrepreneurs account for most of the debts in bankruptcy. This is consistent with the stylized facts of Sullivan, Warren, and Westbrook (2000) that we reported in Table 1. Entrepreneurs account for about 60% of total debt in default. Put it differently, the amount of debt defaulted of entrepreneurs is more than six times the amount of debt discharged of workers. This is explained by the fact that entrepreneurship is risky and business owners borrow a large amount of funds for entrepreneurial activity. Second, entrepreneurs account for more than 30% of total debts in the economy. Third, the average interest rate among workers, which is weighted by the number of workers, is lower than that among entrepreneurs. This is justified by the fact that the default premium for entrepreneurs is on average higher than that for worker since entrepreneurs face greater default risks.

Properties of loan interest rates Figure 5 shows the interest rate schedule of entrepreneurs and workers as a function of the debt (d) to asset (k + s) ratio. There is not a unique interest rate for an individual household, it is a function of the amount of debt, portfolio, and household’s type. For illustration, we report the interest rate schedule for low η, middle-aged households and medium earnings shock ε and fixed asset at (k + s = 1.5). On the figure, blue and green lines represent entrepreneurs (k > 0) and workers (k = 0) respectively. There are a few points to note. First, the loan interest rate schedule is increasing with debt to asset ratio. The higher is the debt level, the more vulnerable are households to fluctuations in income which in turn increases the default rate.
Table 7: Bankruptcy, Debt and Loan Interest Rates for Entrepreneurs and Workers in the Benchmark Economy

<table>
<thead>
<tr>
<th></th>
<th>Entrepreneurs</th>
<th>Workers</th>
<th>Ratio Entrep. to Workers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual bankruptcy rate (%)</td>
<td>1.490</td>
<td>0.726</td>
<td>2.1</td>
</tr>
<tr>
<td>Average debt</td>
<td>1.51</td>
<td>0.47</td>
<td>3.2</td>
</tr>
<tr>
<td>Average amount of debt defaulted</td>
<td>0.13</td>
<td>0.0199</td>
<td>6.5</td>
</tr>
<tr>
<td>Average annual borrowing interest rate (%)</td>
<td>11.12</td>
<td>10.07</td>
<td>1.1</td>
</tr>
<tr>
<td>Fraction of total debt held</td>
<td>30.46%</td>
<td>69.54%</td>
<td></td>
</tr>
<tr>
<td>Fraction of total debt defaulted</td>
<td>64.64%</td>
<td>35.36%</td>
<td></td>
</tr>
</tbody>
</table>

Second, for higher debt to asset ratios, the loan interest rate schedule for entrepreneurs is above that for workers. This is because business and earnings shocks are persistent and the default rate among entrepreneurs is much higher than that among workers.

Figure 2 reports the interest rate schedule for different levels of business productivity $\eta$. As can be seen from the figure, the interest rate schedule is lower when business productivity is higher. Given the persistent nature of the business productivity shock, the probability of default decreases with business productivity.

Our benchmark economy features several interesting life-cycle results. We define young, middle-aged, and old as households of age less than 35, between 36 and 65, and above 65 respectively. As can be seen from panel A of Table 8, the model does relatively a good job of matching the hump-shape of bankruptcy rate over the life-cycle.\(^9\)

The second panel also shows that the number of entrepreneurs replicates the age-profile of entrepreneurship. This is due to the fact the presence of financial constraints and the cost of external financing makes wealth holding an important element for starting a business. Young agents have not time to accumulate enough wealth while middle-age households and old have accumulated assets to become entrepreneurs.

Table 8: Bankruptcy and Entrepreneurship Rates over the Life-Cycle

<table>
<thead>
<tr>
<th></th>
<th>Baseline (%)</th>
<th>Data (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bankruptcy rate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Young</td>
<td>1.52</td>
<td>2.90</td>
</tr>
<tr>
<td>Middle</td>
<td>6.32</td>
<td>3.07</td>
</tr>
<tr>
<td>Old</td>
<td>5.53</td>
<td>0.42</td>
</tr>
<tr>
<td>Fraction of entrepreneurs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Young</td>
<td>6.17</td>
<td>7.05</td>
</tr>
<tr>
<td>Middle</td>
<td>19.06</td>
<td>17.07</td>
</tr>
<tr>
<td>Old</td>
<td>10.65</td>
<td>11.24</td>
</tr>
</tbody>
</table>

\(^9\)The bankruptcy rates over the life-cycle in data are from Table A4 in Sullivan, Warren, and Westbrook (2000).
To summarize this section, in our framework:

- Entrepreneurial households are more likely to file for personal bankruptcy than non-entrepreneurs (workers). This pattern holds not only for the overall economy but also within age groups.

- Although, entrepreneurs represent 20% of all bankrupts households they account for 60% of all debts in bankruptcy.

- Entrepreneurs account for a significant fraction of all consumer debt. More specifically, they hold 30% of total debts.

6 Policy experiments

In this section we use our model to study the economic consequences of changes in bankruptcy regulation that seek to tighten bankruptcy rules under Chapter 7. We consider two bankruptcy regulations:

1. The reduction of bankruptcy asset exemptions. Lowering asset exemptions is an element of the bankruptcy reform that has been approved by President Bush (Bankruptcy Abuse Prevention and Consumer Protection Act of 2005).
2. The full elimination of the personal bankruptcy system. This policy eliminates the partial insurance coming from full discharge of unsecured debt as well as the insurance coming from the exemptions.

6.1 The elimination of bankruptcy exemptions

In this section we present the effects of our first policy, a tightening of bankruptcy rules under Chapter 7 that eliminates the asset exemption ($\chi = 0$). With the exemption eliminated, the only benefit that remains from filing for bankruptcy is the insurance that comes from the discharge of debt.

Before discussing, we present qualitatively the different effects arising from reducing bankruptcy exemptions. Reducing exemptions leads to the following effects. First, a lower exemption implies greater confiscation of assets, and at the margin less benefit from filing for bankruptcy. This effect tends to decrease bankruptcy filings. Second, a lower exemption reduces the partial insurance provided by the presence of exemption. This tends to discourage entrepreneurial activity. Third, because more assets are confiscated in bankruptcy, a lower exemption reduces incentives to save and to accumulate capital. Finally, because the creditor can confiscate more assets in the event of default, and because the incentive to default decreases, the interest rate charged to the individual falls. This tends to (i) increase unsecured borrowing and (ii) encourage entrepreneurial activity to the extent that entrepreneurs’ financial constraints are relaxed.
6.1.1 Aggregate effects and welfare implications

Table 9 reports the implications of eliminating the bankruptcy exemption on entrepreneurship, the overall bankruptcy rate, the amount of debt, the average borrowing interest rate, and welfare.

**Bankruptcy, entrepreneurship and debts** It is apparent from the table that the elimination of the bankruptcy exemption generates the expected results regarding bankruptcy filings. Indeed, the bankruptcy rate is much lower after the reform and falls by about 33%.

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>No Exemptions</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of entrepreneurs (%)</td>
<td>12.00</td>
<td>12.68</td>
<td>5.7</td>
</tr>
<tr>
<td>Bankruptcy rate (%)</td>
<td>0.892</td>
<td>0.602</td>
<td>-32.5</td>
</tr>
<tr>
<td>Average annual borrowing interest rate (%)</td>
<td>10.65</td>
<td>10.55</td>
<td>-2.4</td>
</tr>
<tr>
<td>Average unsecured debt</td>
<td>0.68</td>
<td>0.91</td>
<td>34.3</td>
</tr>
<tr>
<td>Amount of debt defaulted</td>
<td>0.044</td>
<td>0.031</td>
<td>-29.4</td>
</tr>
<tr>
<td>Total business capital per entrepreneurs</td>
<td>1.34</td>
<td>1.37</td>
<td>2.2</td>
</tr>
<tr>
<td>Welfare (equivalence consumption variation %)</td>
<td></td>
<td>1.78</td>
<td></td>
</tr>
</tbody>
</table>

The table also shows that the number of entrepreneurs increases after eliminating the bankruptcy exemption. To understand this result, recall that the bankruptcy exemption has two opposite effects on entrepreneurship. On one hand, bankruptcy provides partial insurance against business failure and thus encourages entrepreneurial activity. On the other hand, a direct consequence of this partial insurance is an increase in the default rate. To compensate for this high default risk, the financial intermediary charges a higher premium. This high borrowing rate tightens borrowing constraints of entrepreneurs and tends to discourage entrepreneurship. The change in entrepreneurial activity is the net effect of these two opposing forces: the partial insurance and credit supply effects. As the table shows, the average borrowing rate drops substantially after the reform (since the default rate falls) and the number of entrepreneurs and entrepreneurial investments increase. This suggests that the increase in entrepreneurial activity comes from the credit supply effect.

Because of the lower borrowing rate, the amount of debt increases significantly. In fact, it increases by more than 34%. Despite a large increase in the amount of debt held, the average amount of debt defaulted decreases substantially. The decrease in the amount of debt discharge falls by almost 30%.

**Welfare implications** Our measure of welfare is the ex ante expected lifetime utility of a newborn agent with zero asset in our economy. To describe welfare, we will report the equivalent consumption variation (ECV), which is the percentage increase in lifetime consumption required to compensate
for the difference in welfare between the two policy regimes. A positive number means that the new policy regime is a better regime under the welfare criteria.

The second panel of Table 9 shows that the elimination of the bankruptcy exemption leads to a significant welfare gain of 1.78% of lifetime consumption. The welfare gain stems from the decrease in the borrowing rate which not only allows for better consumption smoothing over the life-cycle but also increases entrepreneurial activity by relaxing their financial constraints. Another factor that contributes to the improvement in welfare is that the removal of bankruptcy asset exemption eliminates relatively bad projects with high business risk that were undertaken because of the partial insurance coming from the exemption. This improvement in welfare suggests that tightening bankruptcy rule through exemption reductions is desirable. This result is consistent with Li and Sarte (2006).

6.1.2 Aggregate effects by occupation

Table 10 presents statistics for entrepreneurs and non-entrepreneurs. The table shows that taking away the bankruptcy exemption reduces bankruptcy rates among both entrepreneurs and workers, though more so among entrepreneurs. More specifically, the bankruptcy rate among entrepreneurs decreases by almost 40%, while the rate among workers falls by about 30%. The decrease in the bankruptcy rate is larger among entrepreneurs because of the following effects: (i) with less insurance the quality of start-up businesses is higher for a given wealth holdings of entrepreneurs and thus relatively less risky on average, and (ii) a higher fraction of individuals with relatively high wealth, who will thus need to borrow less, enter entrepreneurship since there is less insurance in case of business failure. Note that, despite the fact that the default rate of entrepreneurs decreases more after the reform, it is still higher than the default rate of workers. This is not surprising since entrepreneurs are on average subject to greater risks and carry more debts than workers. With the default rate decreasing more among entrepreneurs than among workers, the average interest rates faced by entrepreneurs decrease by more than those facing workers. In fact, the average interest rate of workers decreases only modestly. The table also shows that the amount of debt defaulted falls for both entrepreneurs and workers, though less dramatically for workers. Moreover, the amount of debt carried by entrepreneurs rises less than the one of workers. This comes from the fact that entrepreneurs rely relatively less on borrowing.

6.1.3 Life-cycle effects

We present default rates, the fraction of entrepreneurs, and the amount of debt over the life-cycle after the policy change. The first panel of Table 11 shows that bankruptcy rates retain their hump shaped profile, though with a decrease in bankruptcy filing for each age group. The decrease in bankruptcy rates is largest for the eldest cohort. This is because, without exemptions, all the accumulated savings of the old will be confiscated by the financial intermediary if they default.

\[^{10}\text{Given that the quality of business increases after the reform, this means that the optimal size of business increases and as a result entrepreneurs will prefer a lower interest borrowing rate.}\]
Table 10: The Effects of Eliminating Bankruptcy Exemptions for Entrepreneurs and Workers

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>No Exemptions</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bankruptcy rate (%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entrepreneur</td>
<td>1.49</td>
<td>0.862</td>
<td>-38.83%</td>
</tr>
<tr>
<td>Worker</td>
<td>0.726</td>
<td>0.484</td>
<td>33.33%</td>
</tr>
<tr>
<td>Average debt</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entrepreneur</td>
<td>1.51</td>
<td>1.58</td>
<td>4.12%</td>
</tr>
<tr>
<td>Worker</td>
<td>0.47</td>
<td>0.73</td>
<td>55.41%</td>
</tr>
<tr>
<td>Average amount of debt defaulted</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entrepreneur</td>
<td>0.13</td>
<td>0.08</td>
<td>-38.99%</td>
</tr>
<tr>
<td>Worker</td>
<td>0.0199</td>
<td>0.0165</td>
<td>-17.06%</td>
</tr>
<tr>
<td>Average annual borrowing interest rate (%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entrepreneur</td>
<td>11.12</td>
<td>10.65</td>
<td>-0.47</td>
</tr>
<tr>
<td>Worker</td>
<td>10.07</td>
<td>10.06</td>
<td>-0.01</td>
</tr>
</tbody>
</table>

The second panel of Table 11 shows that the number of entrepreneurs in each age group increases. The number of young entrepreneurs increases by 10% while the number of middle-aged and old entrepreneurs increases by 6% and 2% respectively. There is a larger increase in the number of young entrepreneurs because young entrepreneurs were initially constrained. The reduction of borrowing interest rates generated by lower default risk allows them to borrow to become entrepreneurs.

Table 12 presents the amount of debt and average interest rate by age cohort after the reform. As expected, the amount of debt increases for each age cohort. The highest increase in debt is for the youngest cohort, where the amount of debt after the reform more than doubles. This high increase comes from the fact that the young were initially constrained and can now borrow at low interest rates. These increases in the amount of debt are consistent with the decreases in the borrowing interest rates for each age cohort.

6.2 The elimination of the personal bankruptcy system

This section analyzes the effects of eliminating bankruptcy provisions entirely. This means that there is no debt discharge nor any exemption and therefore the insurance against bad luck brought by personal bankruptcy system is no longer present. In this new economy, households will borrow up to the point they can pay back their debt with certainty.

Table 13 presents the implications of complete elimination of the personal bankruptcy system. Because borrowers will pay back their debt with probability one, the financial intermediary does not charge any risk premium and therefore the average borrowing interest falls and it is just equal to \( r^* + \phi = 10.4\% \). In contrast to the case where the bankruptcy exemption is eliminated, the table shows that eliminating the entire consumer bankruptcy system considerably decreases the fraction of
entrepreneurs. The fraction of entrepreneurs falls from 12% to 9.16%—that is, by nearly 24%. This decrease in entrepreneurial activity comes mainly from the fact that the insurance offered by both the debt discharge and the exemption no longer exist. With entrepreneurship being risky, business owners value this insurance even if such insurance increases the cost of borrowing.

Another key result is that, taking away the whole bankruptcy system leads to a sizeable welfare loss. Put differently, in this incomplete-market economy, the presence of the bankruptcy system is desirable. The welfare loss of eliminating bankruptcy system is about 4.94% of life-time consumption. This large welfare loss is due to the fact that (i) the benefits of smoothing consumption across states dominate the benefits of the intertemporal consumption smoothing and (ii) the large decrease in entrepreneurial activity. On the whole, the system’s insurance effect thus dominates the credit supply effect.

Table 14 describes the effects of removing the bankruptcy system on entrepreneurship and debt holdings over the life-cycle. As can be seen from the table, the fraction of entrepreneurs decreases for all three age groups. As in the the first policy experiment, the change in the fraction of entrepreneurs is largest for young individuals. Because these young agents have low wealth and low age profile of labour efficiency, they are more sensitive to policy changes.
### Table 13: The Effects of Eliminating Bankruptcy System

|                          | Baseline | No Bankruptcy | Change (%)
|--------------------------|----------|---------------|------------
| Fraction of entrepreneurs (%) | 12.00    | 9.16          | -23.7      |
| Bankruptcy rate (%)      | 0.89     | 0.00          | -100.0     |
| Average annual borrowing interest rate (%) | 10.65    | 10.40         |            |
| Average unsecured debt   | 0.68     | 0.90          | 32.4       |
| Amount of debt defaulted | 0.044    | 0.00          | -100.0     |

Welfare (equivalence consumption variation %) -4.94

### Table 14: The Effects of Eliminating Bankruptcy System over the Life-Cycle

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>No Bankruptcy</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entrepreneurship rate (%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Young</td>
<td>6.17</td>
<td>2.955</td>
<td>-52.1</td>
</tr>
<tr>
<td>Middle</td>
<td>19.06</td>
<td>16.18</td>
<td>-15.1</td>
</tr>
<tr>
<td>Old</td>
<td>10.65</td>
<td>9.276</td>
<td>-12.9</td>
</tr>
<tr>
<td>Average unsecured debt</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Young</td>
<td>0.50</td>
<td>1.15</td>
<td>128.8</td>
</tr>
<tr>
<td>Middle</td>
<td>0.81</td>
<td>0.95</td>
<td>17.1</td>
</tr>
<tr>
<td>Old</td>
<td>0.85</td>
<td>0.88</td>
<td>2.9</td>
</tr>
</tbody>
</table>

### 7 Importance of entrepreneurship

A key point in this paper is that entrepreneurship plays an important role in understanding bankruptcy options. To make this point, we will compare our two bankruptcy reforms in two different economies: one economy with entrepreneurs and another without entrepreneurs. We will contrast the effects of a given bankruptcy reform in the two economies in terms of the change in bankruptcy rates and the welfare implications. We take away the occupational choice so that everybody is a worker. Table 15 summarizes the findings of these experiments.

Before discussing the implications of the policy reforms, it is important to note that the bankruptcy rate is lower in the baseline economy without entrepreneurs than in the economy with entrepreneurs. This is not surprising since in the economy with no entrepreneurs, the only source of uncertainty responsible for default is earnings risk. In the economy with entrepreneurs, on the other hand, there is an additional source of uncertainty: investment risk. In such an economy, with entrepreneurs facing greater risks and taking on more debt to finance investments, the probability of default is higher. This can be seen from Table 15 where the overall annual bankruptcy rate in the economy with workers only is 0.776% while it is 0.88% in the model with entrepreneurs.

We now return to the policy experiments. It is apparent from the last two columns of Table 15
that, entrepreneurship plays an important role in understanding bankruptcy regulations. The second column shows that the elimination of the bankruptcy exemption still leads to an improvement welfare in the model without entrepreneurs, but the improvement in welfare is relatively small. The reduction in interest rates that comes from a lower default premium facilitates intertemporal smoothing of consumption over the life-cycle. Finally, the last column illustrates that eliminating the entire bankruptcy system leads to a welfare gain in the economy without entrepreneurs though it leads to a welfare loss in the economy with entrepreneurs. However, this welfare gain is also relatively small. This finding is in line with Athreya (2002). With no more investment risks in the economy without entrepreneurs, the need to smooth consumption across states is less desirable.

Overall, the presence of entrepreneurship increases the welfare effects of bankruptcy regulation. Specifically, accounting explicitly for entrepreneurship makes the bankruptcy exemption less desirable and the entire bankruptcy system more attractive. This suggests that accounting for entrepreneurship is essential in consumer bankruptcy analysis.

Table 15: The Effects of Eliminating Bankruptcy Exemptions and the Bankruptcy System on Bankruptcy Rate and Welfare in Economies with and without Entrepreneurs (in Percentage)

<table>
<thead>
<tr>
<th>Without Entrepreneurs</th>
<th>Baseline</th>
<th>No Exemption</th>
<th>No Bankruptcy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bankruptcy rate</td>
<td>0.776</td>
<td>0.576</td>
<td>0.00</td>
</tr>
<tr>
<td>Welfare (consumption equivalence)</td>
<td>0.024</td>
<td>0.081</td>
<td></td>
</tr>
<tr>
<td>With Entrepreneurs (baseline)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bankruptcy rate</td>
<td>0.892</td>
<td>0.602</td>
<td>0.00</td>
</tr>
<tr>
<td>Welfare (consumption equivalence)</td>
<td>1.78</td>
<td>-4.96</td>
<td></td>
</tr>
</tbody>
</table>

8 Conclusion

In this paper, we developed a quantitative model of consumer bankruptcy and entrepreneurship and used it to assess the importance of the whole bankruptcy procedure and bankruptcy asset exemptions. A key feature of the model is the presence of two types of uninsurable idiosyncratic uncertainty: earnings risks and investment risks where entrepreneurs face both types of risks while workers face only earnings risks. Because entrepreneurs are subject to an additional risk, their default probability is higher on average than that of workers. Another key feature of the model is that interest rates on consumer loans depend on observable individual characteristics (such as income, labor productivity and business productivity) and the amount of consumer debt owed by the household.

We showed that eliminating bankruptcy asset exemptions leads to a modest increase in entrepreneurship, a large decrease in the overall bankruptcy rate, and an increase in the amount of
debt. More importantly, this policy reform generates a significant welfare gain. In contrast, eliminating the entire bankruptcy system leads a sizeable welfare loss and a large decrease in the number of entrepreneurs. The results of these two policy experiments provide a very important message: the consumer bankruptcy system is desirable but it must be well designed in a way that carefully limits asset exemptions.

A second key message from this paper is that entrepreneurial activity is important for quantitatively assessing the macroeconomic and welfare implications of bankruptcy regulations. For example, in the model without entrepreneurs, the elimination of the entire consumer bankruptcy system generates a welfare gain (although small) though it leads to a large welfare loss when entrepreneurial activity is explicitly modeled.

Our model can be extended in several important directions. An interesting extension is to take into account durable goods (such as housing and cars). With housing, we would be able to study different types of exemptions in particular homestead exemption which is the largest exemption in the data. Another interesting extension is to allow the coexistence of secured and unsecured debt as a high fraction of consumer debts is secured (e.g., mortgages).
References


——— (2004): “Shame as it ever was: stigma and personal bankruptcy,” Economic Quarterly Review of the Federal Reserve Bank of Richmond, spring, 1–19.


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