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Risk-return efficiency, financial distress risk, and bank financial strength ratings

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Risk-return Efficiency,
Financial Distress Risk, and
Bank Financial Strength Ratings

Changchun Hua and
Li-Gang Liu

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Abstract

This paper investigates whether there is any consistency between banks' financial strength ratings (bank rating) and their risk-return profiles. It is expected that banks with high ratings tend to earn high expected returns for the risks they assume and thereby have a low probability of experiencing financial distress. Bank ratings, a measure of a bank's intrinsic safety and soundness, should therefore be able to capture the bank's ability to manage financial distress while achieving risk-return efficiency. We first estimate the expected returns, risks, and financial distress risk proxy (the inverse z-score), then apply the stochastic frontier analysis (SFA) to obtain the risk-return efficiency score for each bank, and finally conduct ordered logit regressions of bank ratings on estimated risks, risk-return efficiency, and the inverse z-score by controlling for other variables related to each bank's operating environment. We find that banks with a higher efficiency score on average tend to obtain favorable ratings. It appears that rating agencies generally encourage banks to trade expected returns for reduced risks, suggesting that these ratings are generally consistent with banks' risk-return profiles.

JEL Classification: D21, D24, G21, G24, G28, G32

Contents

1.	Introduction.....	1
2.	Review of Literature.....	2
3.	Bank Rating and Risk-Return Profile: Empirical Issues and Methodology.....	3
	3.1 Empirical Issues.....	3
	3.2 Empirical Methodology.....	5
4.	Data.....	12
5.	Empirical Results and Analyses.....	16
	5.1 The AID System Estimation and Robustness of Estimates of Risks.....	16
	5.2 Bank Risk-Return Frontier and Efficiency.....	17
	5.3 Bank Ratings and Banks' Risk-Return Trade-offs	21
6.	Conclusions	25
	References	26
	Appendix I: The Optimization Problem for the Bank	29
	Appendix II: Restrictions of Parameters in the AID System	31

1. INTRODUCTION

A bank financial strength rating (bank rating) is a summary measure of a bank's intrinsic safety and soundness.¹ These ratings provide an indication as to how likely a bank will run into financial difficulty and request assistance from its owners, industry group, or the authorities (Estrella et al 2000). Banks with strong intrinsic safety and soundness generally take on risk prudently. They usually earn high expected returns for the risks they assume and thereby have a low probability of experiencing financial distress and a high probability of recovering from adverse exogenous circumstances (DeYoung, Hughes, and Moon 2001). It is expected that a bank rating that aims to capture bank intrinsic safety and soundness also reflects how well a bank manages its portfolios in order to achieve risk-return efficiency and avoid financial distress.

The objective of this paper is to investigate whether there is a consistency between banks' financial strength rating (bank rating) and their risk-return profiles. Specifically, we are interested in examining whether rating agencies assign higher bank ratings to those banks with a higher risk-return efficiency score. In addition, we are also interested in assessing whether the rating agencies treat banks with the same efficiency score similarly from a ratings consistency perspective. Such findings hope to reveal the importance the rating agencies attach to financial distress risk in their assessments of bank ratings.

Using a global banking database that consists of 1,049 banks from major economies of the world from 2000 to 2004 and building upon the latest development in estimating banks' risk-return efficiency and risk-adjusted production frontier, we find that banks with higher efficiency scores on average tend to obtain more favorable ratings and the rating agencies generally encourage banks to trade expected returns for reduced risks, indicating that bank ratings are generally consistent with risk-return profiles. But does Moody's view risk appetites differently from Fitch? Our empirical findings reveal that there indeed is a difference. It appears that the Fitch's Bank Individual Rating (FBIR) pays extra attention to financial distress risk, whereas the Moody's Bank Financial Strength Rating (MBFSR) does not.

The rest of the paper proceeds as follows: Section 2 provides a succinct review of related literature. Section 3 discusses the estimation framework in detail. Section 4 presents the key findings of the paper. Section 5 concludes.

¹ Two rating agencies, Moody's Investor Service (Moody's) and Fitch Ratings, Inc, have each published such ratings. In the case of the Moody's, it is called Moody's bank financial strengthen rating (MBFSR). In the case of the Fitch, it is called Fitch bank individual rating (FBIR).

2. REVIEW OF LITERATURE

The concept of how to best measure bank efficiency has evolved over time. Leibenstein (1966) introduced the term of X-efficiency. However, the X-efficiency theory is not underpinned in a framework of profit-maximization or cost-minimization for firms. Instead, it recognizes that individuals in firms have incomplete contracts, selective rationality, and effort discretions (Button and Weyman-Jones 1994). As a result, most of the literature follows the Farrell (1957) efficiency concept, which is based on a cost or profit optimization assumption.²

However, bank managers face a more complex optimization problem when selecting a loan portfolio. Because of risk and return trade offs, bank managers may have incentives to assume less risk in order to preserve the bank, and therefore, their jobs. In this case, bank managers may maximize their own utility (for example, job security) and trade high returns for low risk. Moreover, even though managers act in the interest of shareholders and maximize the value of shareholders' wealth, their risk preferences are still relevant in determining bank performance. As stated in Koch and MacDonald (2006), the value of shareholders' wealth is dependent on the underlying portfolio risk and return profile, which, in turn, is closely tied to management strategies pursued by managers and shaped by managers' risk preference. In general, if bank managers maximize expected profits, they rank production plans only by the first moment of the expected profit distribution; if they maximize their utility or maximize the value of shareholders' wealth, higher moments of the expected profit distribution will matter in the ranking (Hughes, Lang, Mester, and Moon 1999). This suggests the necessity and importance of applying models with more generality to estimate bank efficiency.³

Hughes and Moon (1995), based on Hughes, Lang, Mester, and Moon's (1995) framework, proposed a new measure on bank risk-return efficiency using the utility maximization assumption and the almost ideal demand (AID) system specification to obtain measures of predicted return on equity and risk, the standard error of the prediction, which in turn allow them to estimate a stochastic risk-return frontier and efficiency score. Under this framework,

2 In the literature on bank X-efficiency, parametric and non-parametric techniques have been developed to estimate cost or profit frontier and to compute the relative efficiency of decision-making units. The parametric methodology comprises Stochastic Frontier Analysis (SFA), Distribution-Free Analysis (DFA), and Thick Frontier Analysis (TFA), while the non-parametric methodology mainly includes Data Envelopment Analysis (DEA) and its special case—Free Disposal Hull (FDH) analysis (Coelli, Rao, O'Donnell, and Battese 2005). The two approaches primarily differ from each other in the assumptions imposed on the functional form of the efficient frontier, the existence of random error, and the probability distribution of the inefficiencies and random error (Bauer 1990; Berger and Humphrey 1997; Paradi, Vela, and Yang 2004).

3 In fact, some studies (e.g., Hughes and Mester 1994a, 1994b), have attempted to empirically investigate the risk preference of bank managers. They tested whether managers are risk neutral and maximize expected profits or they are risk averse and trade off profit for risk reduction; they found that bank managers in the US are generally risk-averse.

a bank's inefficiency at a certain level of risk is calculated as the difference in expected return between this bank and the best-practice banks on the efficient risk-return frontier. Because their framework links the structural model of production to an efficient risk-return frontier that is estimated using the predicted return and risk, the resulting measure of inefficiency does not assume non-neutrality toward risk and sub-optimal choices of quality-linked prices. Furthermore, this measurement may also be consistent with capital market pricing.⁴

Using the same framework, DeYoung, Hughes, and Moon (2001) investigated the consistency between supervisory capital, asset, management, equity, and liability (CAMEL) ratings and risk-return efficiency by examining the degree to which the CAMEL ratings reflect bank risk-return efficiency. The authors conducted an ordered logit regression of each bank's CAMEL rating in the United States in 1994 on the bank's corresponding expected return on the frontier, risk, risk-return inefficiency, and asset size. They found that bank supervisors are influenced not only by banks' risk-return choices, but also by how efficiently banks make this trade-off. This suggests that the CAMEL ratings are consistent with estimated risk-return efficiency.

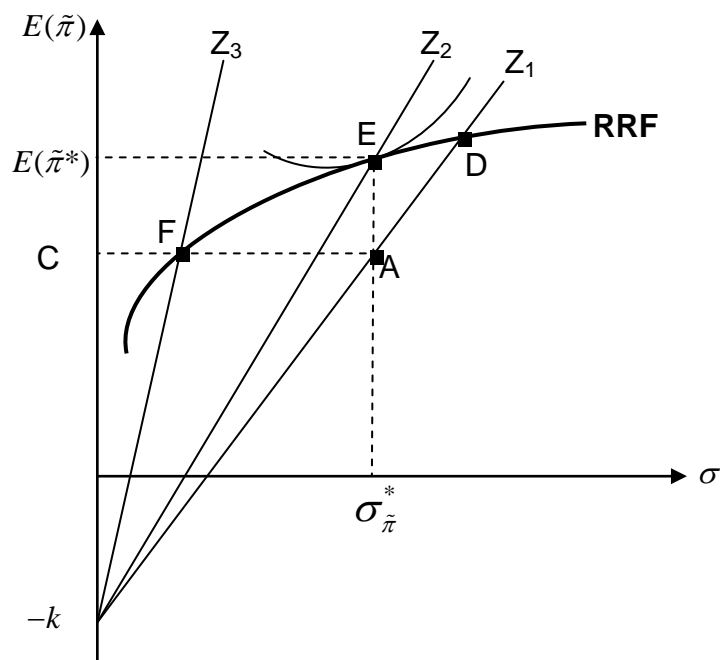
3. BANK RATING AND RISK-RETURN PROFILE: EMPIRICAL ISSUES AND METHODOLOGY

3.1 Empirical Issues

As shown in Figure 1, the risk-return frontier appears on the envelope *RRF* if no risk-free asset for banks is assumed. Not all banks, however, are able to operate their asset portfolios along the frontier. The bank at point *A*, for instance, is not taking risks efficiently, which could be due to management incapability or operating environment. The bank at point *A* has many options to improve its efficiency. For instance, it could increase its expected return to point *E* without changing risk profile. Meanwhile, the upper bound probability of financial distress also decreases as the *z*-score ray becomes steeper. If this bank is less conservative, it could move along the original *z*-score ray (i.e. DZ_1) to point *D*. This scenario would increase both its expected return and risk, but the upper bound probability of financial distress would remain unchanged. If this bank is quite conservative, it may choose to reduce both risk and financial distress probability to point *F* while maintaining the same expected return. For the bank to be on the frontier, it has to trade expected returns for reduced risks (e.g., from point *D* to *E*) and has a lower upper bound probability of financial distress.

⁴ Hughes and Moon (1995) showed that, for a sub-sample of publicly traded banks, their measures of expected profit and return risk explain 96% of the variation in the market value of the banks' equity, implying that their measure of inefficiency is also consistent with capital market pricing of banks.

Figure 1: Risk-Return Profile



Sources: Blair and Heggstad (1978); DeYoung, Hughes, and Moon (2001).

The risk-return profile in Figure 1 may also indicate the financial distress risk of a bank or the probability of a bank under financial distress. Financial distress generally means that a bank can not pay off, or has difficulty in paying off, its financial obligations, which can be equivalent to economic insolvency (i.e., the loss exceeds the equity capital). Thus, the financial distress risk can be defined as the likelihood of the event that a bank’s loss exceeds its total capital. Let $\tilde{\pi}$ be the random return with mean $E(\tilde{\pi})$ and the standard deviation $\sigma_{\tilde{\pi}}$.

Given the portfolio characteristics and the bank’s capital position, Chebyshev’s inequality may indicate the approximate probability of financial distress (Blair and Heggstad 1978). Chebyshev’s inequality suggests

$$P\{|\tilde{\pi} - E(\tilde{\pi})| \geq z\sigma_{\tilde{\pi}}\} \leq 1/z^2.$$

Denoting $E(\tilde{\pi}) - z\sigma_{\tilde{\pi}} = -k$, one can express the least upper bound probability of bank distress as follows

$$(1) \quad P\{\tilde{\pi} \leq -k\} \leq 1/z^2,$$

where $1/z = \frac{\sigma_{\tilde{\pi}}}{k + E(\tilde{\pi})} = \frac{\sigma_{\tilde{\pi}}/k}{1 + E(\tilde{\pi})/k}$ is the reciprocal of the slope of z-score rays (e.g.

DZ_1 , EZ_2 , and FZ_3). This suggests that the flatter the z-score ray, the higher the upper bound

probability that a bank would be under financial distress, while along the ray, different risk-return combinations have the same upper bound probability of financial distress (constant z-scores).

Banks with strong intrinsic safety and soundness should be able to manage risk efficiently, and therefore, they shall operate closely to or even on the risk-return frontier and avoid being exposed to costly financial distress. Bank ratings—MBFSR from Moody's and FBIR from Fitch—attempt to, as they are defined, reflect bank intrinsic safety and soundness or measure the likelihood that a bank will require assistance from third parties such as its owners or official institutions. Thus, the ratings also indicate how well a bank manages its portfolios—in other words, whether the bank achieves risk-return efficiency and is away from financial distress. To investigate whether these ratings are consistent with the risk-return profiles, this study examines whether rating agencies assign higher ratings to those banks on the frontier compared to those below the frontier. In addition, we are interested in finding out whether rating agencies such as Moody's and Fitch assign different ratings to those banks with the same efficiency score, taking financial distress risk into account.

3.2 Empirical Methodology

3.2.1 The AID System and the Most Preferred Demand Function System of Banks

To examine the above-mentioned empirical issues, it is necessary to estimate risk-return efficiency and financial distress probability. Here, we follow the framework of Hughes, Lang, Mester, and Moon (1995, 1996, and 1999). In these studies, the AID System is applied to specify the minimum expenditure necessary to attain a specific utility level at given prices for the bank managerial team. The AID System proposed by Deaton and Muellbauer (1980) is built on the theorems of Muellbauer (1975, 1976) about a specific class of preferences for aggregating consumers' demand and can be applied to obtain both theoretically consistent and empirically obtainable functional forms for the bank's most preferred demand functions for inputs and profits. The budget shares are assumed to have "price-independent generalized form" (PIGL) and market demands can be considered as the outcome of decisions by a rational representative consumer. As a member of PIGL family, the AID System has many desirable properties⁵, making it attractive in accommodating generalized managerial preferences. In this study, the AID expenditure function can be specified as follows:⁶

$$(2) \quad \ln E(\cdot) = \ln P + U \cdot \beta_0 (\prod_i y_i^{\beta_i}) (\prod_j w_j^{\nu_j}) p_{\pi}^{\mu} k^{\kappa},$$

where

⁵ For instance, it gives a first-order approximation to any demand system while it still precisely satisfies the axioms of choice. Furthermore, its functional form is consistent with household-budget data while it also aggregates perfectly over consumers without invoking parallel linear Engel curves.

⁶ The details of deriving this expenditure function can be found in Deaton and Muellbauer (1980).

$\ln E$	—	the expenditure of the bank management
$\ln P$	—	the price index
U	—	the generalized managerial utility for the bank
y_i	—	the output i
w_j	—	the price of input j
p_π	—	the price of after-tax profit (π) in terms of before-tax profit, equivalent to $1/(1-t)$ where t is tax rate
k	—	the equity capital
β, ν, μ, κ	—	parameters to be estimated

The translog form of the price index is shown as follows:

$$\begin{aligned}
 \ln P = & \alpha_0 + \alpha_p \ln \tilde{p} + \sum_i \delta_i \ln y_i + \sum_j \omega_j \ln w_j + \eta_\pi \ln p_\pi + \tau \ln r + \rho \ln k \\
 & + \frac{1}{2} \alpha_{pp} (\ln \tilde{p})^2 + \frac{1}{2} \sum_i \sum_j \delta_{ij} \ln y_i \ln y_j + \frac{1}{2} \sum_s \sum_t \omega_{st}^* \ln w_s \ln w_t \\
 & + \frac{1}{2} \eta_{\pi\pi} (\ln p_\pi)^2 + \frac{1}{2} \tau_{rr} (\ln r)^2 + \frac{1}{2} \rho_{kk} (\ln k)^2 \\
 & + \sum_j \theta_{pj} \ln \tilde{p} \ln y_j + \sum_s \phi_{ps} \ln \tilde{p} \ln w_s + \psi_{p\pi} \ln \tilde{p} \ln p_\pi + \psi_{pr} \ln \tilde{p} \ln r + \psi_{pk} \ln \tilde{p} \ln k \\
 & + \sum_j \sum_s \gamma_{js} \ln y_j \ln w_s + \sum_j \gamma_{j\pi} \ln y_j \ln p_\pi + \sum_j \gamma_{jr} \ln y_j \ln r + \sum_j \gamma_{jk} \ln y_j \ln k \\
 & + \frac{1}{2} \sum_s \omega_{s\pi}^* \ln w_s \ln p_\pi + \frac{1}{2} \sum_s \omega_{\pi s}^* \ln p_\pi \ln w_s + \sum_s \omega_{sr} \ln w_s \ln r + \sum_s \omega_{sk} \ln w_s \ln k \\
 & + \eta_{\pi r} \ln p_\pi \ln r + \eta_{\pi k} \ln p_\pi \ln k + \tau_{rk} \ln r \ln k \quad .
 \end{aligned}$$

Note that the average output price (\tilde{p}) instead of price for each output is used. This would help conserve degrees of freedom and avoid data unavailability problems in the estimation. The risk-free rate of return is r .

Most preferred input and profit share equations are obtained by applying Shephard's lemma to the expenditure function. These procedures yield the share equations for inputs and profit:⁷

⁷ The symmetry condition is applied to coefficients of input prices: $\omega_{si} = \omega_{is} = (\omega_{si}^* + \omega_{is}^*)/2$ and

$$\omega_{s\pi} = \omega_{\pi s} = (\omega_{s\pi}^* + \omega_{\pi s}^*)/2 .$$

$$(3) \quad \frac{\partial \ln E}{\partial \ln w_i} = \frac{w_i x_i}{\mathbf{p} \cdot \mathbf{y} + m} = \frac{\partial \ln P}{\partial \ln w_i} + v_i [\ln(\mathbf{p} \cdot \mathbf{y} + m) - \ln P]$$

$$= \omega_i + \sum_s \omega_{si} \ln w_s + \phi_{pi} \ln \tilde{p} + \sum_j \gamma_{ji} \ln y_j + \omega_{\pi i} \ln p_\pi$$

$$+ \omega_{ri} \ln r + \omega_{ki} \ln k + v_i [\ln(\mathbf{p} \cdot \mathbf{y} + m) - \ln P] ,$$

$$(4) \quad \frac{\partial \ln E}{\partial \ln p_\pi} = \frac{p_\pi \pi}{\mathbf{p} \cdot \mathbf{y} + m} = \frac{\partial \ln P}{\partial \ln p_\pi} + \mu [\ln(\mathbf{p} \cdot \mathbf{y} + m) - \ln P]$$

$$= \eta_\pi + \eta_{\pi\pi} \ln p_\pi + \psi_{p\pi} \ln \tilde{p} + \sum_j \gamma_{j\pi} \ln y_j + \sum_s \omega_{s\pi} \ln w_s$$

$$+ \eta_{\pi r} \ln r + \eta_{\pi k} \ln k + \mu [\ln(\mathbf{p} \cdot \mathbf{y} + m) - \ln P] ,$$

where \mathbf{P} and \mathbf{Y} represent the vectors of output price and output, respectively, and m represents other income.

In addition, it is also straightforward to include the first-order condition for equity capital in the system to account for its endogeneity. Since the conditional indirect utility function is the de facto Lagrangian function for the utility maximization problem, the first-order condition for equity capital can be obtained by computing $\frac{\partial V(\cdot)}{\partial k}$. The given level of utility can be substituted for by the indirect utility function in the expenditure function after solving the expenditure minimization problem, the dual problem of utility maximization (see Appendix I). Thus, inverting equation (2) yields

$$(5) \quad V(\cdot) = \frac{\ln(\mathbf{p} \cdot \mathbf{y} + m) - \ln P}{\beta_0 (\prod_i y_i^{\beta_i}) (\prod_j w_j^{\gamma_j}) p_\pi^\mu k^\kappa} .$$

Thus, the AID System's first-order condition for equity capital is given as follows:

$$\frac{\partial V(\cdot)}{\partial k} = \frac{\partial V(\cdot)}{\partial \ln k} \frac{\partial \ln k}{\partial k} = \frac{-1}{k [\beta_0 (\prod_i y_i^{\beta_i}) (\prod_j w_j^{\gamma_j}) p_\pi^\mu k^\kappa]} \left[\frac{\partial \ln P}{\partial \ln k} + \kappa [\ln(\mathbf{p} \cdot \mathbf{y} + m) - \ln P] \right] = 0 .$$

This yields equation (6):

$$(6) \quad \rho + \rho_{kk} \ln k + \psi_{pk} \ln \tilde{p} + \sum_j \gamma_{jk} \ln y_j + \sum_s \omega_{sk} \ln w_s + \tau_{\pi k} \ln r + \eta_{\pi k} \ln p_\pi + \kappa [\ln(\mathbf{p} \cdot \mathbf{y} + m) - \ln P] = 0 .$$

Parameters in the system of equations (3), (4), and (6) are restricted by symmetry, homogeneity, and adding-up conditions, which are listed in Appendix B. This system provides a robust framework to analyze many issues in the banking business, such as bank risk-return efficiency, financial distress, bank scale economies, among others. The nonlinear seemingly unrelated regression equations (Nonlinear SURE) model is applied to estimate the nonlinear AID system above. To allow for risk preferences to change over time, estimation of the system is done for each year. Also, as emphasized by DeYoung, Hughes, and Moon

(2001) and other studies, homoskedasticity on the error terms shall not be imposed so that the risk of the production plan can be measured.

3.2.2 Measuring Risk-return Efficiency and Financial Distress Risk

Before empirically estimating bank efficiency parameters, there are three important issues to be discussed: First, different measures of inputs and outputs may greatly affect efficiency scores and rankings, but unfortunately, there still seems to be no existing approach that is widely accepted. To solve this problem, researchers mainly undertake two approaches—the “production” approach and the “intermediation” approach—to define banking business activities (Berger and Humphrey 1997).⁸ The “production” approach views banking activities as the production of accounts and services (e.g., demand deposits, term and saving deposits, real estate loans, consumer loans, and business loans) to customers (both depositors and borrowers). This view is probably suitable for the case of local branches as these services are usually provided face-to-face to customers in local branch offices or delivered at customers’ premises. Hence, inputs in this approach are labor and physical capital while outputs are the number of deposit accounts and loan services. The alternative “intermediation” approach essentially reflects the asset transformation function of banks, namely, intermediating funds between savers and investors. This approach is mostly appropriate for banks’ main branches or themselves as they are responsible for the “transforming” activities instead of being directly in contact with customers. Inputs in this approach are primarily financial funds (deposits and other borrowed funds) while outputs are loans and outstanding investments. Neither approach, however, captures the whole picture of the modern banking activities. Furthermore, the controversy on whether deposits should count as inputs or outputs still remains.⁹

The second important concern on measuring bank efficiency, especially for international comparison, is how to control for environmental factors. Environment (e.g., market structure, credit culture, financial regulations, and macroeconomic stability) is also an important determinant for bank performance. However, it is mostly out of banks’ control. When one does not account for differences in environment and simply construct a common frontier for all banks across countries, the estimated efficiency score could be artificially high (low) for banks under a favorable (unfavorable) environment. To avoid this problem some studies restrict their study sample within a certain sub-region that has a similar economic environment. For instance, Berg, Forsund, Hjalmarsson, and Suominen (1993) evaluated

⁸ Freixas and Rochet (1997) and Paradi, Vela, and Yang (2004) each categorized the literature into three approaches. Besides the above-mentioned two approaches, the former also included a “modern” approach that attempts to incorporate risk management and information processing into banking activities, while the latter added the “profitability” approach that is designed to examine the process of how well a branch uses its inputs (expenses) to produce revenues.

⁹ This is probably due to the dual characteristics of deposits. Deposits are paid with interests and are the “raw material” of investible funds of banks. Also, banks compete for deposits as they are associated with liquidity, safekeeping, and payment services provided to depositors (Berger and Humphrey 1997).

bank efficiency only for Nordic countries. Some other studies (e.g., Lozano-Vivas, Pastor, and Hasan 2001) incorporate environmental factors in the basic data envelopment analysis (DEA) model by treating them as inputs or outputs. These solutions, however, are problematic and not satisfactory.¹⁰ The recent endeavor by Pastor (2002) and Avkiran (2007) to tackle this problem is to apply a three-stage DEA.¹¹ It seems that the three-stage DEA can isolate environmental effects and statistical noise, but it also involves tremendous calculations. Besides, it might also cause some other statistical problems by mixing parametric and nonparametric methods.

Third, incorporating risk in efficiency measurement is another challenge for standard efficiency studies. Managing risk is an intrinsic function of financial institutions. Therefore, it is natural to include bank risks in the frontier analysis. The fact that risk can arise from various sources (e.g., loans, interest rate movements, liquidity shortage, and contingent liabilities) complicates the analysis. The present common approach is to use a risk proxy, such as nonperforming loans (NPL) or loan loss provisions, as an input or a negative output of banks. As argued by many studies, the quality of loans can not represent all types of risks that banks face, although it may be a good proxy for credit risk. Furthermore, it may be inadequate to simply add risk measures to the profit maximization and cost minimization problems. For instance, one may need to account for the risk preference of managers in measuring bank efficiency. Some studies (e.g., Mester 1996) attempt to address it by treating financial capital as an input in the production process. As financial capital provides a cushion against losses, lower capital may indicate higher probability of default. Thus, a risk-averse manager may operate the bank with a level of financial capital that is different from the cost-minimizing one. In this sense, the level of financial capital may reflect risk preferences of bank managers, and therefore, counting financial capital as an input may control for risk preferences in measuring bank efficiency. While this is an insightful attempt, it is still an indirect approach to address the issue of risk and risk preferences of managers.

Based on the demand system equations (3), (4), and (6), we first obtain expected return and risk and then estimate bank risk-return efficiency using stochastic frontier analysis, accounting for environmental factors. In doing so, the estimation can avoid some of the issues listed above. The expected return on equity (ER) is measured as:

¹⁰ Restricting the study sample still does not account for environmental differences between countries. Treating environmental factors as inputs or outputs requires *a priori* knowledge of environmental influences, but it is often the case that we can not determine their effects.

¹¹ In the first stage, the basic DEA is applied to bank data to yield initial efficiency scores and slacks. In the second stage, SFA is adopted to regress estimated slacks against environmental variables and in this way, one can isolate managerial inefficiency from both environmental effects and statistical noise. Some studies use the tobit regression in the second stage but this can only isolate environmental effects. The last stage adjusts inputs or outputs with the three-part decomposed slacks obtained in the previous stage and conduct DEA to the adjusted dataset. The details of three-stage DEA can be found in Fried, Lovell, Schmidt, and Yaisawarng (2002).

$$(7) \quad ER = E(p_\pi \pi) / k = s_\pi(\hat{\beta}) \cdot [y \cdot + m] / k,$$

where $s_\pi(\hat{\beta})$ is the forecast profit share and $\hat{\beta}$ denotes the vector of estimated parameters in equation (4). Risk (RK) is measured by the degree of uncertainties in predicting return on equity, which is the standard error of return on equity. It is measured as:

$$(8) \quad RK = [\hat{\mathbf{X}}\hat{\beta}\hat{\mathbf{X}}'(\)^{-1}]^{1/2} y[(\cdot + m) / k],$$

where $\hat{\mathbf{X}} = \frac{\partial s_\pi(\hat{\beta})}{\partial \beta}$ and $\hat{\mathbf{X}}\hat{\beta}\hat{\mathbf{X}}'(\)^{-1}$ denotes the asymptotic variance of the fitted profit share, $s_\pi(\hat{\beta})$.

When a manager chooses an ex ante riskier production plan or loan profile, the manager has less confidence in predicting profit compared to an ex ante lower risk production plan. Thus, this measure can indicate the risk inherent in the bank's production plan: The higher standard error of ER suggests larger risk in the production plan. This risk measure depends on, among others, bank size, asset composition, average return on assets, costs of financial and nonfinancial inputs, the marginal tax rate on profits, the risk premium. It can, therefore, comprehensively indicate the risk inherent in the production plan.

Having computed expected returns and risks, we then estimate the following risk-return frontier

$$(9) \quad ER_{it} = \Gamma_0 + \Gamma_1 RK_{it} + \Gamma_2 RK_{it}^2 + \mathbf{Z}\zeta + v_{it} - u_{it},$$

where $v_{it} \sim IID N(0, \sigma_v^2)$,

$$u_{it} \geq 0 \sim IID N^+(0, \sigma_u^2).$$

The composite error term contains inefficiency term (u) and statistical noise (v). \mathbf{Z} is a vector of environmental factors, for instance, banking sector structure, bank regulations and macroeconomic fluctuations. These factors not only indirectly influence the expected profit through manager's decisions over production plans but also directly affect the expected profit due to systematic differences or heterogeneity across banks. A bank's inefficiency is measured by the conditional expectation of u on $\varepsilon (= v - u)$,

$$(10) \quad E(u|\varepsilon) = \left(\frac{\sigma_u \sigma_v}{\sigma} \right) \left[\frac{\phi\left(\frac{\varepsilon\lambda}{\sigma}\right)}{\Phi\left(-\frac{\varepsilon\lambda}{\sigma}\right)} - \frac{\varepsilon\lambda}{\sigma} \right],$$



where $\lambda = \sigma_u^2 / (\sigma_u \sigma_v)$ and $\sigma = \sqrt{\sigma_u^2 + \sigma_v^2}$.

As for financial distress, its economic meaning is different from the bankruptcy, as emphasized in the literature (e.g., Dietrich, 1984). Financial distress can be equivalent to the economic insolvency (the loss exceeds the equity capital), while the bankruptcy is a legal term rather than an economic term. Econometric models—either probit/logit model or quadratic discriminant analysis—using bankruptcy data could under-predict or over-predict financial distress. Some firms may choose bankruptcy voluntarily while their economic conditions are not yet insolvent in the case of strategic bankruptcy. Estimation on the sample including these cases may over-predict financial distress. On the contrary, some firms are economically insolvent but they are not bankrupt due to financial support from the third parties (e.g. shareholders, industry group, and governmental institutions). In this case, econometric models may under-predict financial distress. To avoid these problems, we simply use estimated returns and risks from the system equations above to construct the indicator of financial distress probability (FD):

$$(11) \quad FD = \frac{1}{z} = \frac{RK}{1 + ER}.$$

The drawback of this measure, however, is that it only indicates the upper bound financial distress risk while the actual one could be different. However, given the data limitation, this may be the best approximation indicator to address the economic insolvency risk of banks.

3.2.3 Bank Risk-Return Efficiency, Financial Distress and Bank Ratings

Given the nature of the ranking data, the ordered response model would be proper to link risk-return efficiency and financial distress to bank ratings. Let BR represent the rating score assigned by Moody's or by Fitch. The ordered logit model can be shown as follows,

$$(12) \quad \begin{aligned} BR_{it}^* &= \beta \mathbf{X}_{it} + e_{it}, \\ BR_{it} &= \begin{cases} 0 & \text{if } y_{it}^* \leq \mu_0 = 0 \\ 1 & \text{if } \mu_0 < y_{it}^* \leq \mu_1 \\ 2 & \text{if } \mu_1 < y_{it}^* \leq \mu_2 \\ \vdots & \\ n & \text{if } y_{it}^* > \mu_{n-1} \end{cases}, \end{aligned}$$

where the error term in the latent model (e_{it}) is assumed to follow the logistic distribution.

BR^* is a continuous and unobserved latent variable, which can be interpreted as the hypothetical financial strength. The ordinal variable, BR , is coded on the $n+1$ -point scale from 0 to n , where 0 represents the poorest condition and n superior financial strength. \mathbf{X} is a

vector of independent variables, including frontier expected return, risk level, efficiency, financial distress risk and bank size, in addition to year and region dummies.

4. DATA

The sample used in this paper consists of 1,094 banks from the BankScope database that obtained ratings from either Moody's or Fitch during the period 2000–2004. The three-year average value is used for financial and economic variables in estimations.¹² MBFSR and FBIR are mapped into eight numerical scores, with zero denoting the lowest rating, E, and eight representing the highest rating, A.¹³ Note that Moody's appends '+' and '-' to each category (from E up to A) to distinguish intermediate categories while Fitch only uses the slash to express intermediate categories. To make the rating system comparable, we rescaled Moody's intermediate categories according to Fitch's methodology. As shown in Table 1, banks rated by Fitch generally obtained higher ratings, compared to those rated by Moody's. In the Fitch rated sample, 3.2% were assigned A, while only 0.8% of Moody's rating sample were rated A.

¹² The usage of three-year average values in the study is consistent with the rating methods of both rating agencies as they mainly use three-year financial data to issue ratings.

¹³ If MBFSR and FBIR are updated in a year, the latest ratings are used.



Table 1: Transformation of Letter to Numeric Variable for Ratings

Moody's		Fitch		Interpretation ^b	Mapping
Rating	Frequency ^a	Rating	Frequency ^a		
A	26 (0.79%)	A	101 (3.2%)	<i>Superior intrinsic financial strength</i> : highly valuable and defensible business franchises, outstanding financial fundamentals, and a very predictable and stable operating environment.	8
A-,B+	161 (4.9%)	A/B	304 (9.62%)		7
B	328 (9.99%)	B	895 (28.32%)	<i>Strong intrinsic financial strength</i> : valuable and defensible business franchises, good financial fundamentals, a predictable and stable operating environment, and no major concerns.	6
B-,C+	590 (17.97%)	B/C	685 (21.68%)		5
C	472 (14.38%)	C	425 (13.45%)	<i>Adequate intrinsic financial strength</i> : limited but still valuable business franchises, possessing one or more problems with financial fundamentals or the operating environment.	4
C-,D+	705 (21.47%)	C/D	280 (8.86%)		3
D	349 (10.63%)	D	299 (9.46%)	<i>Modest intrinsic financial strength</i> (potentially requiring some outside supports): a weak business franchise, deficient financial fundamentals in one or more respects, or an unpredictable and unstable operating environment.	2
D-,E+	480 (14.62%)	D/E	127 (4.02%)		1
E	172 (5.24%)	E	44 (1.39%)	<i>Very modest intrinsic financial strength</i> (very serious problems, either requiring or being likely to require external supports): a weak and limited business franchise, materially deficient financial fundamentals in one or more respects, or a highly unpredictable or unstable operating environment.	0

Note: ^a The frequency is calculated based on 3,283 observations for Moody's and 3,160 observations for Fitch.

^b The Interpretations of ratings are from websites of Moody's and Fitch.

Source: Authors and Moody's and Fitch databases.

In this paper, we primarily follow the intermediate approach to define bank outputs and inputs. Outputs comprise interbank loans (y_1), customer loans (y_2), securities investments (y_3), and off-balance sheet activities (y_4). Off-balance sheet items, mainly consisting of non-traditional activities (e.g. loan commitments, securitizations, and derivatives), are considered as an output of banks since omitting them in the estimation of bank efficiency could result in a misspecification of bank output and lead to incorrect conclusions.¹⁴ The average price or yield of outputs (\tilde{p}) is defined as the ratio of operating income (the sum of interest income, securities investment and trading income, and fees) to total outputs. Bank inputs are physical assets (x_1), employees (x_2), and borrowed financial funds (x_3), and their prices are w_1 (depreciation and other operating expenses over fixed assets), w_2 (personnel expenses per employee), and w_3 (total interest expenses over total borrowed financial funds), respectively.

The price of after-tax profit (p_π) is defined as $1/(1-t)$, where the mean marginal tax rate on pre-tax profit (t) is measured by running a simple regression of taxes paid by banks on pre-tax profits for each country and each year.

The nonoperating income (m) is adjusted to take into account the standard error of the measured tax rate. The share of each input (SW_i) is calculated as the ratio of each input's expenses to total revenue ($\tilde{p}y + m$). The sum of shares for inputs and profit is 100%. Table 2 presents the summary statistics of bank production variables from 2000 to 2004.

¹⁴ For more detailed discussions on this topic, please refer to studies such as, among others, Rogers (1998), Stiroh (2000), Clark and Siems (2002).

Table 2: Summary Statistics of Bank Production Variables (2000–2004)

Variable		Description	Mean	Std. Dev.	Minimum	Maximum
y_1	¹⁾	Inter-bank loans	25600.00	56100.00	6.40	517000.00
y_2	¹⁾	Customer loans	5894.98	19400.00	0.03	299000.00
y_3	¹⁾	Securities	13300.00	39100.00	0.0001	578000.00
Investments						
y_4	¹⁾	Off-balance sheet items	23300.00	245000.00	0.025	9120000.00
\tilde{p}	²⁾	Avg. price of outputs	8.12	5.04	0.03	61.08
x_1	¹⁾	Physical assets	571.29	1344.07	0.10	16600.00
x_2	³⁾	Employees	12192.96	29348.24	10	323937
x_3	¹⁾	Borrowed funds	42300.00	96400.00	26.01	893000.00
w_1	²⁾	Price of physical assets	348.05	2704.27	0.15	87985.05
w_2	¹⁾	Wage rate	0.077	0.74	0.0001	25.41
w_3	²⁾	Price of borrowed funds	4.36	3.13	0.05	34.22
π	¹⁾	Profit after-tax	381.06	1042.23	-2229.59	16900.00
p_π	²⁾	Price of after-tax profit	1.37	0.19	0.38	3.14
t	²⁾	Mean tax rate	25.20	11.99	-162.85	68.18
$\tilde{p}y + m$	¹⁾	Total revenue	3319.29	7619.38	5.08	98800.00
$SW1$	²⁾	Input share of Physical assets	0.19	0.10	0.001	0.78
$SW2$	²⁾	Input share of labor	0.18	0.08	0.001	0.52
$SW3$	²⁾	Input share of borrowed funds	0.47	0.20	0.001	1.69
$SW\pi$	²⁾	Share of pre-tax profit	0.17	0.15	-1.46	0.82
k	¹⁾	Equity capital	4545.94	11800.00	3.40	195000.00
A	¹⁾	Total assets	50800.00	121000.00	32.59	1280000.00
r	²⁾	Risk-free rate	5.35	7.52	0.05	68.57

Notes: N=3366; ¹⁾ in millions of US dollar; ²⁾ in percentage; ³⁾ the number of people. Std. Dev means standard deviation.

Source: Authors.

5. EMPIRICAL RESULTS AND ANALYSES

5.1 The AID System Estimation and Robustness of Estimates of Risks

The nonlinear SURE model is applied to estimate the nonlinear AID system during 2000–2004.¹⁵ It is worth noting that homoskedasticity on the error terms is not imposed so that the risk of the production plan can be measured. Therefore, the statistical significance is not shown in the table.

To check the robustness of estimated risks, we also conducted a robustness test by regressing some traditional risk proxies on risk measures obtained from the AID system, i.e. *RK* and *FD*. As the main financial risk for banks is credit risk, the robustness test selects several financial ratios to indicate credit risk. These financial ratios include loan-asset ratio, NPL ratio, the standard deviation of NPL over three years and the ratio of loan-loss provision to total assets. The loan-asset ratio attempts to capture the total exposure of a bank to credit risk, while the NPL ratio shows the past credit risk and the loan-loss provision implies the managers' view on future credit risk. The standard deviation of NPL over three years indicates the fluctuation of credit risk. The banking literature suggests that the higher these ratios, the riskier a bank would be. The liquidity risk is indicated by the ratio of liquid assets to total assets. Holding other factors constant, the higher the liquidity ratio, the less probability a bank would run into liquidity shortage. The deposit mix represents the diversification of a bank's deposit-taking. The operating risk is mainly indicated by three ratios, including gross operating income and the ratios of operating and other operating expenses to total assets. Higher income or lower expenses can indicate that a bank can manage its operating risk well. The equity-asset ratio is included to examine overall insolvency risk. Many studies have suggested that the less equity a bank holds, the more moral hazard a bank may have. In addition, facing financial shocks, well-capitalized banks would have more funds to buffer their losses, compared to weakly-capitalized banks. Thus, banks with high equity-asset ratio tend to have less risk, holding other factors constant.

The fixed effects estimations of these two regressions are shown in Table 3. The estimations show that traditional risk proxies are, in general, statistically significant with correct signs in both regressions. However, the overall *R*-square is not high in either model. It seems that the two risk measures obtained from the AID system do capture the major bank risks indicated by these financial ratios, while the low *R*-square might suggest that these risk measures may contain much richer information than these financial ratios.

¹⁵ The coefficients of the nonlinear AID system can be obtained from the authors upon request.

Table 3: Relationship Between Traditional Risk Proxies and Risk Estimates of the AID System

Financial Ratios	Prediction Risk (RK)	Financial Distress (FD)	Descriptive Statistics
	Coeff. (std. err.)	Coeff. (std. err.)	Mean (Std. Dev.)
(1) Total loans / total assets	0.005** (0.002)	0.001 (0.001)	56.146 (18.234)
(2) Nonperforming loans / total assets	0.014*** (0.005)	0.013*** (0.003)	4.917 (6.901)
(3) Standard deviation of NPL ratio in 3 years	-0.002 (0.006)	0.001 (0.003)	1.171 (2.467)
(4) Loan-loss provision / total assets	-0.013 (0.02)	0.028*** (0.01)	0.673 (1.33)
(5) Liquid assets / total assets	-0.004* (0.002)	-0.001*** (0.001)	18.018 (15.387)
(6) Deposit mix	-0.902*** (0.195)	-0.303*** (0.102)	0.555 (0.267)
(7) Gross operating income	-0.937*** (0.133)	-0.468*** (0.069)	14.987 (0.494)
(8) Operating expenses / total assets	0.039*** (0.009)	0.013*** (0.005)	3.693 (3.457)
(9) Other operating expenses / total assets	0.003** (0.001)	0.003*** (0.001)	29.538 (31.598)
(10) Equity / total assets	-0.054*** (0.007)	-0.029*** (0.003)	8.641 (6.94)
(11) Size	-0.348*** (0.046)	-0.122*** (0.024)	16.174 (1.755)

Note: Fixed effects estimates are presented in the table. Coeff. means coefficient; std.err. means standard error; and Std. Dev. Means standard deviation.

*, **, *** denote the significance level of 10%, 5% and 1%, respectively.

Source: Authors.

5.2 Bank Risk-Return Frontier and Efficiency

We applied the stochastic frontier analysis (SFA) to the pooled cross-sectional sample. To examine the robustness of the risk-return frontier, two specifications were estimated. The first specification includes the estimated risk and its square, in addition to year dummies, while the second specification also contains environmental factors (GDP growth rate, domestic

credit to private sector over GDP, and banking sector structural variables and regulatory factors).¹⁶

GDP growth rate (*gGDP*) and domestic credit expansion (*gCredit*) indicates the economic and credit cyclical fluctuations, respectively. Herfindahl index (*BHI*) indicates the competition degree in one banking sector, while government ownership (*BSOBR*) and foreign ownership (*BFOBR*) indicate ownership structure in the banking sector.¹⁷ A binary variable (*BDepress*) is included to indicate whether a banking sector experiences serious depression or crisis.¹⁸ Entry restriction (*EntryREG*), activity restriction (*ActREG*), and capital adequacy regulation (*CAPREG*), and disclosure requirements (*PMI*) vary across countries, indicating different regulatory environment.¹⁹ Supervisory power (*SuperPower*) and Supervisory agency independence (*SuperIndep*) are included to indicate how strong the supervisory authority can correct bank activities.²⁰ The deposit insurance scheme (*DIS*, *MHI*, and *DISPower*) is also included.²¹

¹⁶ The environmental variables included in risk-return frontier estimation are mainly obtained from International Financial Statistics (IFS) of the International Monetary Fund, Demircuc-Kunt, Karacaovali, and Laeven (2005), Caprio and Klingebiel (2003), and regulatory surveys conducted by Barth, Caprio and Levine in the World Bank.

¹⁷ Herfindahl index here is calculated using bank assets data, namely, $BHI = \frac{\sum_{i=1}^n (\text{asset}_i)^2}{(\sum_{i=1}^n \text{asset}_i)^2}$, with smaller values indicating that a banking sector is less concentrated and more competitive. *BSOBR* and *BFOBR* are measured by their shares in the banking sector assets. These variables are aggregated using the data from BankScope.

¹⁸ Banking depression dummy variable takes one if there is a serious banking depression and zero otherwise, and it is defined in greater detail in Caprio and Klingebiel (2003).

¹⁹ The entry restriction (*EntryREG*) ranges in value from 0 to 8, with higher values representing more restrictiveness. Bank activity restrictions (*ActREG*) are regulations on the ability of banks to engage in securities underwriting, brokering and dealing, insurance underwriting and selling, as well as real estate investment, development and management. This variable ranges in value from 1 to 4, with higher values indicating greater restrictiveness. Capital stringency regulation (*CAPREG*) refers to regulations on minimum capital ratios, definitions of capital, risk-based guidance, among others. This variable ranges in value from 0 to 7, with higher values indicating greater stringency. Disclosure requirements (*PMI*) specify the auditor, non-performing loans disclosure, off-balance-sheet items disclosure, among others. It ranges in value from 0 to 7 as well. These variables are obtained from the surveys conducted by Barth, Caprio, and Levine.

²⁰ Supervisory power (*SuperPower*) takes values 0 to 16, the sum of 16 binary indicators. Supervisory agency independence (*SuperIndep*) takes values 1, 2, and 3, with higher values indicating higher dependence degree of the supervisory agency. These variables are obtained from the surveys conducted by Barth, Caprio and Levine.

²¹ Deposit insurance (*DIS*) takes value 0 to 2 indicating how well the deposit is protected. Deposit insurance scheme design (*MHI*) is to indicate the moral hazard caused by the scheme. It is the first principal component of nine factors (e.g., insurance coverage, management, premium, fund, source of fund, etc.), and it explains around 84.5% of the total variation in these nine variables. Deposit insurance agency's power (*DISPower*)

As shown in Table 4, the coefficients of RK and RK^2 are consistently significant and are similar in terms of size in both specifications. The positive slope of estimated risk suggests that banks are not pure profit maximizers, but rather are trading returns for reduced risks. This result is consistent with those in DeYoung, Hughes, and Moon (2001), Koetter (2004, 2006), among others. Furthermore, SFA results also suggest that domestic financial deepening, market competition, and the authorities' power (including regulatory authorities and deposit insurers) to make prompt corrective action on banks would shift the risk-return frontier outward while entry restrictions, capital regulation, bank disclosure requirements, and banking sector depression would shift the frontier inward. Having accounted for market competition effects, it seems that foreign bank participation does not really move the risk-return frontier outward. The risk-return frontier for sampled banks is shown in Figure 2, which has taken into account of environmental factors.

reflects whether the agency has the authority to intervene banks, whether the agency can take legal actions against banks, and whether the legal actions have been taken. This variable takes values from 1 to 4, with higher values indicating stronger power. These values are obtained from Demirguc-Kunt, Karacaovali, and Laeven (2005).

Table 4: Risk-return Stochastic Frontier Analysis Results (2000-2004)

Dependent variable: *Expected Return*

	(1) Risk-Return Frontier without Environmental Factors		(2) Risk-Return Frontier with Environmental Factors	
	Coeff.	(Std.Err)	Coeff.	(Std.Err.)
Constant	0.266 ^{***}	(0.007)	0.443 ^{***}	(0.044)
Risk	0.996 ^{***}	(0.004)	0.991 ^{***}	(0.004)
Risk-square	-0.011 ^{***}	(0.000)	-0.011 ^{***}	(0.000)
gGDP			-0.001	(0.001)
Credit to private sector/GDP			0.0001 ^{***}	(0.000)
BHI			-0.102 ^{**}	(0.049)
BSOBR			0	(0.000)
BFOBR			-0.0002 ^{**}	(0.000)
BDEPRESS			-0.036 ^{***}	(0.009)
EntryREG			-0.008 ^{**}	(0.004)
ActREG			0.005	(0.006)
CapREG			-0.006 ^{**}	(0.002)
SuperPower			0.004 ^{***}	(0.001)
SuperIndep			-0.002	(0.005)
PMI			-0.018 ^{***}	(0.004)
DIS			-0.057 ^{***}	(0.008)
DISPOWER			0.011 ^{***}	(0.004)
MHI			0.002	(0.002)
Y00	0.061 ^{***}	(0.009)	0.055 ^{***}	(0.010)
Y01	-0.212 ^{***}	(0.008)	-0.221 ^{***}	(0.009)
Y02	0.002	(0.009)	0	(0.009)
Y03	-0.005	(0.009)	-0.005	(0.010)
$\lambda = \sigma_u^2 / (\sigma_u^2 + \sigma_v^2)$	2.354 ^{***}	(0.063)	2.233 ^{***}	(0.066)
$\sigma = \sqrt{\sigma_u^2 + \sigma_v^2}$	0.222 ^{***}	(0.001)	0.211 ^{***}	(0.001)
LogL	1,687.56		1,851.42	
N	3,366		3,366	

Source: Authors.

Figure 2: The Best-practice Stochastic Risk-return Frontier

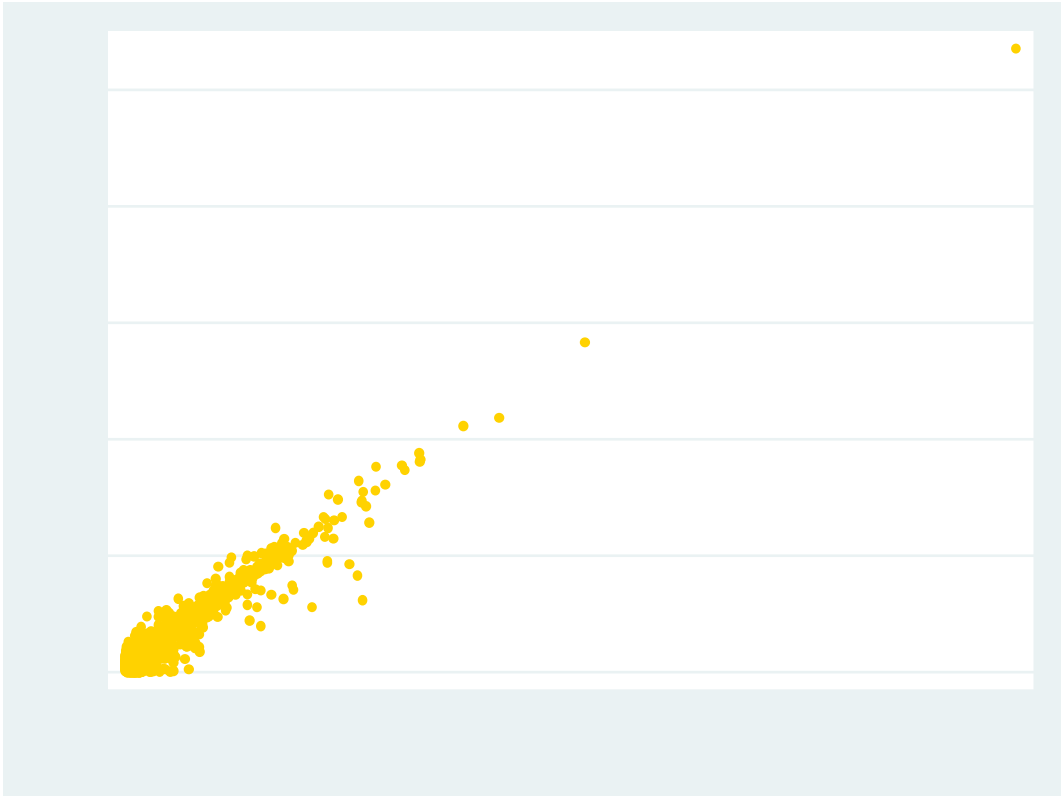


Table 5: Summary Statistics on Estimates of Risk-return Profile

<i>Variable</i>	2000	2001	2002	2003	2004	Total
	N=570	N=628	N=678	N=729	N=761	N=3,366
<i>Expected Return</i>	1.22 (1.27)	0.02 (0.14)	0.39 (0.3)	0.34 (0.27)	0.32 (0.28)	0.43 (0.69)
<i>Risk</i>	1.1 (1.79)	0.16 (0.18)	0.27 (0.35)	0.22 (0.28)	0.2 (0.27)	0.36 (0.85)
<i>Efficiency</i>	0.86 (0.07)	0.85 (0.11)	0.87 (0.07)	0.87 (0.06)	0.87 (0.05)	0.86 (0.08)
<i>Financial Distress Risk</i>	0.43 (0.14)	0.17 (0.32)	0.17 (0.15)	0.14 (0.13)	0.13 (0.11)	0.2 (0.21)

Note: Standard deviation is provided in parenthesis.

Source: Authors.

Figure 3: Bank Ratings in the Risk-return Profile (2000–2004)

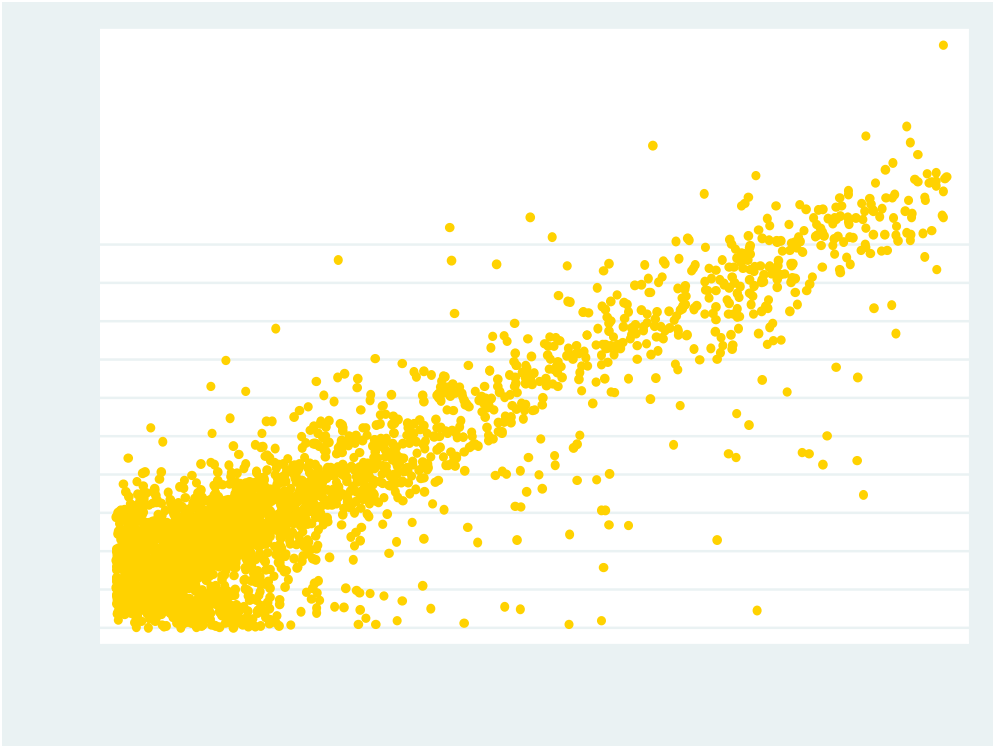
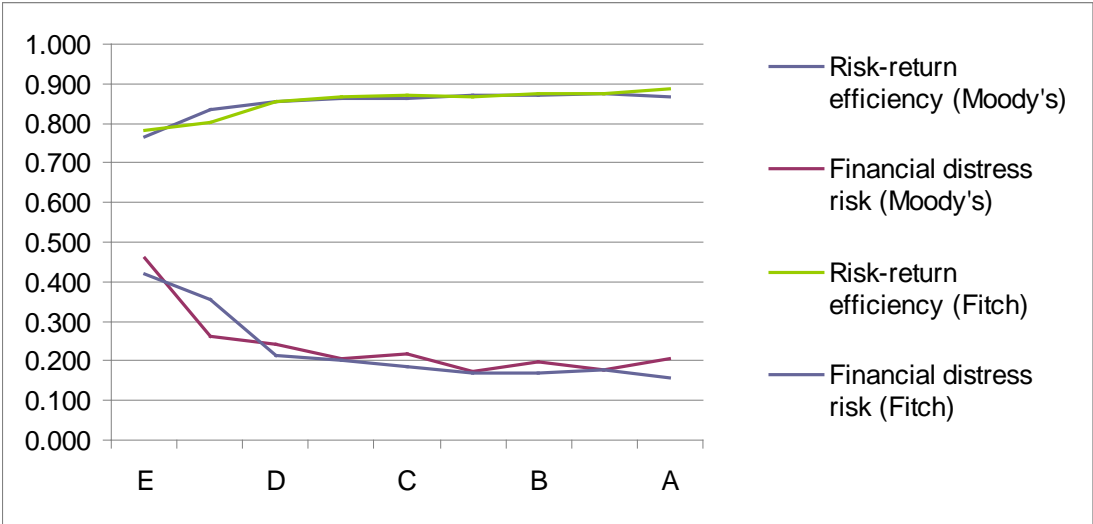


Figure 4: Risk-return Efficiency and Financial Distress Risk by Bank Ratings (2000–2004)



Source: Authors.

The ordered logit estimates for both Moody’s and Fitch are presented in Table 6. The explanatory variables include the estimated frontier expected return (corresponding to point E in Figure 1), the estimated risk, the estimated upper bound financial distress risk, the estimated risk-return efficiency, size (logarithm of total assets), and dummies for time (Y00-Y03) and market maturity (OECD). The sign and significance level of most independent variables (e.g. the frontier expected return, the estimated risk, efficiency, size, and dummy variables for mature markets) are the same for both ratings. This shows that both rating agencies assign higher ratings to banks with higher risk-return efficiency and encourage banks to trade expected returns for reduced risks. In other words, both rating models are consistent with the estimated risk-return efficiency ranks. However, one noticeable difference between these two bank rating models is the significance level of the upper bound financial distress risk. After controlling for other factors, Moody’s attaches no importance on the upper bound financial distress risk while Fitch does in the rating process. That is, while both rating agencies encourage banks to trade expected returns for reduced risks, Fitch probably encourages banks to take less risks than Moody’s does.

Table 6: Ordered Logit Model Regressions (Random Effects Estimates, 2000–2004)

	MBFSR		FBIR	
	Coeff.	(Std.Err.)	Coeff.	(Std.Err.)
Constant	-15.792 ^{***}	(0.835)	-12.414 ^{***}	(0.854)
Frontier Expected Return	2.794 ^{***}	(0.912)	7.463 ^{***}	(0.869)
Risk	-3.536 ^{***}	(0.878)	-8.399 ^{***}	(0.846)
Financial Distress Risk	-0.224	(0.244)	-0.436 [*]	(0.26)
Efficiency	5.318 ^{***}	(0.69)	7.477 ^{***}	(0.739)
Size	0.896 ^{***}	(0.028)	0.553 ^{***}	(0.024)
OECD	2.330 ^{***}	(0.101)	2.327 ^{***}	(0.108)
Y00	0.709 ^{***}	(0.168)	0.914 ^{***}	(0.171)
Y01	1.042 ^{***}	(0.234)	1.933 ^{***}	(0.222)
Y02	0.195 [*]	(0.116)	0.094	(0.113)
Y03	0.103	(0.113)	0.012	(0.111)
Log-L	-3,414.94		-3,754.70	
N	2,237		2,429	

Notes: *, **, *** denote the significance level of 10%, 5% and 1%, respectively.

Source: Authors.

Table 7 presents the marginal effects of several estimated factors (the frontier expected return, the estimated risk, the upper bound financial distress risk, and risk-return efficiency) on the probability of each rating score. Again, it seems that these factors have very similar marginal effects on each rating score of both ratings. Careful examination, however, may show a small difference in the marginal effects of the upper bound financial distress risk on low rating scores. As seen in the table, financial distress risk shows no significance in affecting Moody's lowest two rating scores while it significantly affects the lowest two rating scores of Fitch's. This may imply that Fitch pays extra attention to the risk-taking behavior of those banks with low financial strength.

Table 7: Marginal Effects on the Probability of Each Grade (2000–2004)

	<i>Frontier Expected Return</i>	<i>Risk</i>	<i>Financial Distress Risk</i>	<i>Efficiency</i>
MBFSR				
<i>E</i>	-0.007 ^{***}	0.009 ^{***}	0.001	-0.014 ^{***}
<i>E+,D-</i>	-0.074 ^{***}	0.093 ^{***}	0.006	-0.140 ^{***}
<i>D</i>	-0.122	0.154	0.01	-0.231
<i>D+,C-</i>	-0.435 [*]	0.551	0.035	-0.829
<i>C</i>	-0.013	0.016	0.001	-0.024
<i>C+,B-</i>	0.41	-0.519 ^{***}	-0.033 ^{***}	0.780 ^{**}
<i>B</i>	0.185 [*]	-0.234 ^{**}	-0.015 ^{**}	0.353 ^{**}
<i>B+,A-</i>	0.05	-0.063	-0.004	0.094
<i>A</i>	0.006 ^{**}	-0.007 ^{**}	-0.001	0.011 ^{**}
FBIR				
<i>E</i>	-0.008 ^{***}	0.009 ^{***}	0.001 [*]	-0.008 ^{***}
<i>E/D</i>	-0.039 ^{***}	0.044 ^{***}	0.002 [*]	-0.039 ^{***}
<i>D</i>	-0.271	0.305	0.016	-0.271
<i>D/C</i>	-0.439	0.494	0.026	-0.44
<i>C</i>	-0.793	0.892	0.046	-0.794
<i>C/B</i>	-0.186	0.209	0.011	-0.186
<i>B</i>	1.231 ^{***}	-1.385 ^{***}	-0.072 ^{***}	1.233 ^{***}
<i>B/A</i>	0.39	-0.439	-0.023	0.391
<i>A</i>	0.113 ^{***}	-0.128 ^{***}	-0.007	0.114 ^{***}

Notes: *, **, *** denote the significance level of 10%, 5% and 1%, respectively.

Source: Authors.

6. CONCLUSIONS

This study empirically investigated the consistency between bank ratings and risk-return profile. We applied an AID system to measure the bank risk-return efficiency and the upper bound financial distress risk. Applying the ordered logit method, we found that both the Moody's and the Fitch generally assign higher ratings to banks with higher risk-return efficiency and encourage them to trade expected returns for reduced risks. Thus, it seems that both rating models do take the risk-return profiles of banks into consideration. However, the difference between these two rating agencies is that they seem to attach different importance of the banks' distress risk in their rating assignments. It seems that Fitch is more risk averse since it pays extra attention to banks' financial distress risk, thus assigning higher ratings to those banks with the characteristics of low risks and low returns. This might be the reason why we tend to observe that the Fitch assigns higher ratings to banks than does Moody's.

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APPENDIX I: THE OPTIMIZATION PROBLEM FOR THE BANK

In the banking industry, debt leverage is usually large, and therefore, risk management is important for banks so as to avoid financial risk and financial fragility. Because of the serious consequence of financial fragility, bank managers may trade expected returns for reduced risks in their production plan so that they can preserve the bank. This results in financial stability as well as maintaining bank managers' jobs. Thus, ignoring bank risks and managers' risk preference in the efficiency analysis may give a wrong evaluation of bank performance and probably undervalues managers' efforts to maintaining financial stability.

In general, if bank managers maximize expected profits, they rank production plans only by the first moment of the expected profit distribution. However, if they maximize a managerial utility function, higher moments of the expected profit distribution will matter in the ranking (Hughes, Lang, Mester, and Moon, 1999). More importantly, the standard approach of maximizing expected profits or minimizing expected costs is a special case of the new approach of maximizing *a generalized managerial utility function* when managers are risk-neutral.²² If bank managers trade profit for other objectives (e.g. reduced risks), then their utility function will include other arguments (such as risk) in addition to profit. However, it is difficult to measure all types of risk and include them in the function. Hughes, Lang, Mester, and Moon (1995) defined *the generalized managerial utility function* over production plans, input mix, and profits conditional on endogenous prices. Such definition enables managers to rank production plans and profit according to their risk preferences and their expectations about the probability distribution of profit. The merit of this framework is that the derived cost and profit functions allow for higher moments of the conditional distribution of profits and, therefore, allow for non-neutral risk preference of managers.²³

Banks are assumed to produce a portfolio of loans and securities with labor, physical capital, and financial funds (deposits, equity capital, and other borrowed funds) given production technology and environmental constraints. The portfolio production plan is designated by $(\mathbf{y}, \mathbf{x}, k)$. The transformation function, $T(\mathbf{y}, \mathbf{x}, k) \leq 0$, defines the feasible set of portfolio production plans, given production technology and environmental constraints. As shown in Hughes, Lang, Mester, and Moon (1995), the generalized managerial utility function is defined as $U(\pi, \mathbf{y}, \mathbf{x}, \mathbf{p}, r, k)$.²⁴ Note that the managerial utility function is defined differently from the individual utility function which is usually defined over consumption and leisure. The managerial utility function is defined to capture the risk preferences of a group of bank managers and shows how satisfied these managers in a bank is in organizing the resources of the bank and selecting certain profit targets, subject to the production constraint that the input mix must produce the given output vector. The input mix is also included in the managerial utility function mainly because in organizing the production plan, risk-averse bank managers may fund their loans from more costly but less volatile sources, such as core deposits. In other words, the risk characteristics of the input mix should also be taken into account when managers are not risk neutral (Hughes, Lang, Mester, and Moon, 1995).

Managers choose their most preferred production plan or their most preferred subjective conditional probability distribution of profit, and hence, managers maximize the generalized

²² Indeed, the utility maximization approach has been widely employed in the literature on modeling bank's behavior (Santomero 1984).

²³ Non-neutral risk preference could be risk-loving or risk-averse.

²⁴ In Hughes, Lang, Mester, and Moon (1995) and their companion studies, NPLs are also used to indicate the output quality, but it is dropped here. The reason is twofold: first, as expressed in Hughes, Lang, Mester, and Moon (1995), this variable only indicates ex post the output quality and can not show the exact quality of assets that are being held by banks; second, the data for this variable are generally either unavailable or not comparable, especially in the context of international comparison of banking efficiency.

utility function with respect to profit and input, subject to the profit identity and the transformation functions. The utility-maximizing problem (UMP) can be shown as follows:

$$\begin{aligned} & \max_{\pi, \mathbf{x}} U(\pi, \mathbf{x}, \mathbf{y}, k, \mathbf{p}, r) \\ & s.t. \quad \mathbf{p} \cdot \mathbf{y} + m - \mathbf{w} \cdot \mathbf{x} - p_{\pi} \pi = 0, \\ & \quad T(\mathbf{y}, \mathbf{x}, k) \leq 0 \end{aligned}$$

where \mathbf{W} is a vector of input prices. Note that the UMP is conditioned on the output vector, \mathbf{Y} , to facilitate the computation of economies of scale and on equity capital, k , to allow the profit demand function to be normalized by equity capital to obtain the rate of return on equity (Hughes, Lang, Mester, and Moon, 2000).²⁵ The solution to this utility maximization problem gives the managers' most preferred production plan.

The dual problem of the UMP is the expenditure minimization problem (EMP) which tries to minimize the managerial expenditure on profits and inputs for a specific level of utility (U^0) at given prices, output quantities, output quality, and financial capital.

The EMP can be expressed as follows:

$$\begin{aligned} & \min_{\pi, \mathbf{x}} \quad \mathbf{w} \cdot \mathbf{x} + p_{\pi} \pi \\ & s.t. \quad U^0 - U(\pi, \mathbf{x}, \mathbf{y}, k, \mathbf{p}, r) = 0. \\ & \quad T(\mathbf{y}, \mathbf{x}, k) \leq 0 \end{aligned}$$

Note that the EMP is not intended to minimize profit; instead, it attempts to minimize the managerial expenditure for after-tax profit. The difference between $p_{\pi} \pi$ and π is the tax on profit. Thus, the EMP for $p_{\pi} \pi$ mainly means that, given the tax rate on profit (i.e., given the price of after-tax profit), bank managers choose certain target of after-tax profit to minimize their efforts (the managerial expenditure) on obtaining the target. In this sense, $p_{\pi} \pi$ also reflects managerial expenditure to obtain such level's profit.

Solving UMP yields the manager's most preferred production plan, $\mathbf{x}^*(\mathbf{y}, \mathbf{v}, m, k)$, and the managers' most preferred profit function, $\pi^*(\mathbf{y}, \mathbf{v}, m, k)$, where $\mathbf{v} = (\mathbf{w}, \mathbf{p}, r, p_{\pi})$ denotes the price and tax environment of a bank. As it reflects bank managers' risk-preference, the resulting profit function does not necessarily achieve the maximum profits since profits may be used to trade for less risk. Furthermore, as it is conditional on the risk-preference, the most preferred profit demand function can be used to estimate the risk-return frontier. The solution of EMP gives Hickian or constant-utility demand functions, $\mathbf{x}^u(\mathbf{y}, \mathbf{v}, k, U^0)$, $\pi^u(\mathbf{y}, \mathbf{v}, k, U^0)$, and the minimum expenditure function, $E(\mathbf{y}, \mathbf{v}, k, U^0)$. Inverting the expenditure function, one can obtain the indirect utility function, $V(\mathbf{y}, \mathbf{v}, m, k)$. Substituting U^0 with V , one can obtain the following three equations:

$$\begin{aligned} \mathbf{x}^u(\mathbf{y}, \mathbf{v}, k, V(\mathbf{y}, \mathbf{v}, m, k)) &= \mathbf{x}^*(\mathbf{y}, \mathbf{v}, m, k), \\ \pi^u(\mathbf{y}, \mathbf{v}, k, V(\mathbf{y}, \mathbf{v}, m, k)) &= \pi^*(\mathbf{y}, \mathbf{v}, m, k), \\ E(\mathbf{y}, \mathbf{v}, k, V(\mathbf{y}, \mathbf{v}, m, k)) &= \mathbf{p} \cdot \mathbf{y} + m. \end{aligned}$$

²⁵ Profit demand function mainly describes managers' demand for profit or managers' profit target.

APPENDIX II: RESTRICTIONS OF PARAMETERS IN THE AID SYSTEM

The partial differentiation of the expenditure function gives the following symmetry conditions:

$$(S1) \delta_{ij} = \delta_{ji} \quad \forall i, j ,$$

$$(S2) \omega_{s\pi} = \omega_{\pi s} \quad \forall s , \text{ and}$$

$$(S3) \omega_{si} = \omega_{is} \quad \forall s, i .$$

The first symmetry condition needs to be held since the constituent coefficients can not be identified separately, while it is not necessary to hold the last two.

The expenditure function is homogenous of degree one, and therefore, inputs and profit share equations are homogeneous of degree zero in (w, \tilde{p}, r, p_π) , which yields the following homogeneity conditions:

$$(H1) \sum v_j + \mu = 0 ,$$

$$(H2) \alpha_p + \sum \omega_j + \eta_\pi + \tau = 1 ,$$

$$(H3) \alpha_{pp} + \sum_t \phi_{pt} + \psi_{pr} + \psi_{p\pi} = 0 ,$$

$$(H4) \phi_{pt} + \sum_s \omega_{st} + \omega_{tr} + \omega_{t\pi} = 0 \quad \forall t ,$$

$$(H5) \tau_{rr} + \psi_{pr} + \sum_s \omega_{sr} + \eta_{\pi r} = 0 ,$$

$$(H6) \theta_{pj} + \sum_t \gamma_{jt} + \gamma_{j\pi} + \gamma_{jr} = 0 \quad \forall j ,$$

$$(H7) \eta_{\pi\pi} + \psi_{p\pi} + \sum_s \omega_{s\pi} + \eta_{\pi r} = 0 ,$$

$$(H8) \psi_{pk} + \sum_s \omega_{sk} + \tau_{rk} + \eta_{\pi k} = 0 , \text{ and}$$

$$(H9) \frac{1}{2} \alpha_{pp} + \frac{1}{2} \sum_s \sum_t \omega_{st} + \sum_t \phi_{pt} + \frac{1}{2} \tau_{rr} + \frac{1}{2} \eta_{\pi\pi} + \psi_{p\pi} + \psi_{pr} + \sum_s \omega_{sr} + \sum_s \omega_{s\pi} + \eta_{\pi r} = 0 .$$

The sum of input and profit shares equal one, requiring the following adding-up conditions:

$$(A1) \sum_i \omega_i + \eta_\pi = 1 ,$$

$$(A2) \sum_i \omega_{si} + \omega_{s\pi} = 0 \quad \forall s ,$$

$$(A3) \sum_i \phi_{pi} + \psi_{p\pi} = 0 ,$$

$$(A4) \sum_i \gamma_{ji} + \gamma_{j\pi} = 0 \quad \forall j ,$$

$$(A5) \sum_i \omega_{\pi i} + \eta_{\pi\pi} = 0 ,$$

$$(A6) \sum_i \omega_{ir} + \eta_{\pi r} = 0 ,$$

$$(A7) \sum_i \omega_{ik} + \eta_{\pi k} = 0, \text{ and}$$

$$(A8) \sum v_j + \mu = 0.$$