Firm Heterogeneity, Contract Enforcement, and the Industry Dynamics of Offshoring

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Summary
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Keywords: Offshoring, Heterogeneous Firms, Incomplete Contracts, Growth, Industry Dynamics

JEL Classification: D23, F23, L23, O31, O43

We are indebted to Elhanan Helpman and two anonymous referees for valuable comments that helped improve the paper considerably. We are also grateful to Rubén Segura-Cayuela for very helpful suggestions.

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Firm Heterogeneity, Contract Enforcement, and the Industry Dynamics of Offshoring∗

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May 13, 2009

Abstract

We develop an endogenous growth model to study the long run consequences of offshoring with firm heterogeneity and incomplete contracts. In so doing, we model offshoring as the geographical fragmentation of a firm’s production chain between a home upstream division and a foreign downstream one. On the positive side, we show that, when contracts are incomplete, the possibility of offshoring has favorable implications for economic growth. Yet, offshoring induced by a higher bargaining power of the upstream division can hamper growth: while there is always a positive correlation between upstream bargaining weight and offshoring activities, there is a non-monotonic relationship between these and growth. Whether offshoring with incomplete contracts also increases consumption depends on firm heterogeneity. On the normative side, we show that, whereas with complete contract efficiency is restored through a subsidy to R&D only, with incomplete contracts a production subsidy to offshored upstream divisions is needed too.

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1 Introduction

Offshoring, along with debates and literature related to it, has enjoyed an exponential growth in recent years. In particular, the controversy on the issue exploded in February 2004 when N. Gregory Mankiw rationalized offshoring through its long term positive consequences on the US economy. He argued that offshoring may release domestic resources that can be reallocated to the creation of new products, new technologies and thus new and better jobs to replace those lost to cheaper foreign countries.\(^1\) Trade economists have since rushed to support. Blinder (2006) calls offshoring the third industrial revolution, which can eventually be a sound occurrence for all workers, as the first and the second were regardless of initial skepticism. Baldwin (2006) calls the process "a second unbundling" that has occurred as a consequence of rapidly falling communication and coordination costs. Grossman and Rossi-Hansberg (2008) argue how traditional trade theory must give way for a paradigm more relevant to today’s world, namely trade in "tasks". They show the benefits of this phenomenon by pointing out its positive impact on real wages of all workers in the home country.\(^2\)

Finally, Rodriguez-Clare (2009) uses a dynamic model to show that the negative terms of trade effect of offshoring is outweighed by long-run gains as the origin country adjusts its research effort.

Our aim is to contribute to this debate by highlighting possible gains and losses from offshoring in an endogenous growth scenario with heterogeneous firms where the economic benefits of research and development (R&D) are not fully appropriable by innovators and some of the contracts supporting production abroad are incomplete.

We develop our argument by modeling an economy consisting of two countries, North and South, and two sectors, production and R&D. The North is the market for final products, which are horizontally differentiated. Varieties are supplied according to blueprints that are invented and patented by

\(^1\) Offshoring is frequently blamed by workers and trade unions for the slow pace of job growth in the United States and for the swelling wage differential between low and high skill workers (Feenstra and Hanson, 2001).

\(^2\) In their contribution, the positive effect of offshoring on wages is driven by a productivity effect as offshoring translates into a form of technological progress. These results are qualified in Kohler (2004) and in Baldwin and Robert-Nicoud (2007), where domestic workers only benefit if the labor cost savings of offshoring are associated with the labor intensive sector.
R&D labs. In the wake of Grossman and Helpman (1991), endogenous growth is introduced through a positive learning externality in R&D.

To enter the production market, entrepreneurs must first purchase a patent, and then engage in process innovation with uncertain outcome to find their capacity in using the patent. In particular, an entrepreneur’s capacity is determined by a random draw from some common productivity distribution as in Melitz (2003). Each entrepreneur organizes production along a vertical chain consisting of two stages, intermediate supply ("upstream") and final assembly ("downstream"), performed by two divisions within a vertically integrated firm. Vertical integration is due to the presence of tacit knowledge that cannot be transmitted outside firm boundaries. Both R&D and final assembly are assumed to take place in North only. Intermediates can be produced in North or South. This is assumed to be a potential site for the production of intermediates using a standardized traditional technology that offers productivity gains to entrepreneurs with bad draws, provided that they are willing to bear the additional trade costs associated with international shipments.\(^3\) We call "inshoring" an organizational structure in which both production stages take place in North and "offshoring" the alternative organizational structure in which intermediates are first produced in South and then assembled in North.

The two countries differ in terms of the quality of contract enforcement between divisions. Specifically, contracts are complete when both the upstream and the downstream divisions are located in North. They are incomplete when the upstream division is located in South due to the lack of credible institutions to perfectly enforce contracts.\(^4\) We model contractual incompleteness following recent contributions that study firms’ ownership and location choices in environments in which economic

\(^3\) Using Japanese firm level data from the period 1994-2000, Hijzen, Inui, and Todo (2007) give empirical evidence on how the scope for productivity improvements from offshoring depends negatively on the initial level of productivity of the firm. This in turn provides an effective channel for less productive firms to catch up and restore competitiveness.

\(^4\) Nunn (2007) for instance uses several proxies to measure contract incompleteness in the South: a weighed average of a number of variables that measure individuals’ perceptions of the effectiveness and predictability of the judiciary and the enforcement of contracts in 159 countries between 1997 and 1998 from Kaufmann et al. (2003); the measures of judicial quality and contract enforcement from Gwartney and Lawson (2003) and World Bank (2004).
interactions suffer from hold up problems. More precisely, we follow Grossman and Helpman (2002) in adopting the transaction cost approach à la Williamson (1975, 1985), whose key idea is that the quality of deliverables in a bilateral transaction is unobservable by third parties so that, after the deliverables have been produced, the stakeholders involved in the transaction have to bargain on some division of the surplus it would generate. However, by assuming that upstream-downstream transactions take place within the boundaries of firms, we abstract from the ownership decision and focus, instead, on the location decision. In other words, what generates contractual incompleteness is not the crossing of firms’ boundaries but rather the crossing of countries’ borders.

This setup generates new positive and normative insights on the dynamic and static aggregate effects of offshoring. On the positive side, we show that, when contracts are incomplete, the possibility of offshoring has favorable implications for economic growth. That does not happen when contracts are complete, in which case offshoring has no impact whatsoever on growth. The key parameter regulating the growth effect of offshoring with incomplete contracts is the bargaining power of upstream divisions. In particular, we show that, while a marginal increase in the bargaining power of upstream divisions always encourages more firms to offshore, it fosters growth only if such bargaining power is initially small enough. Otherwise, offshoring activities encouraged by a stronger upstream bargaining power slows down growth. Lastly, we show that whether offshoring with incomplete contracts also favors steady state consumption depends on firm heterogeneity. For example, when productivity draws are Pareto distributed, consumption increases when there are a lot of unproductive firms and very few productive ones.

On the normative side, we highlight that, just like in Grossman and Helpman (1991), with offshoring under complete contracts the endogenous growth rate of the economy is suboptimally low due the positive learning externality in R&D. In this case, efficiency is restored through a subsidy to R&D only. This is, instead, not enough under incomplete contracts as the hold up problem causes underproduction. Accordingly, with incomplete contracts, the R&D subsidy has to be complemented

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by a production subsidy to offshored upstream divisions.\textsuperscript{7}

To the best of our knowledge, our analysis represents the first attempt to study the long run consequences of offshoring with firm heterogeneity and incomplete contracts. A large branch of the international trade literature on firm organization has been devoted to the incomplete nature of contracts in arrangements between firms. On the dynamic side of this front, Naghavi and Ottaviano (2006, 2008, 2009) use a growth model à la Grossman and Helpman (1991) to study the potential tension that may arise between the static and dynamic implications of the fragmentation of production. They find that while outsourcing gives rise to complementary upstream and downstream innovation, incomplete contracts may prevent static gains of specialized production from carrying through in the long run. They also find that offshoring can slow growth by reducing the feedback from offshored plants to labs. Yet, in their model there is no firm heterogeneity so that, in equilibrium, firms either all outsource or they all vertically integrate.\textsuperscript{8} Grossman and Helpman (2004), Antràs and Helpman (2004) and Grossman, Helpman and Szeidl (2005) are among the first papers to study the organization of firms in the presence of heterogeneity in a static set up. Our contribution adds to this literature by studying the industry dynamics of firm organization, in particular the interactions between offshoring and growth.\textsuperscript{9} Our model also differs from previous work the organizational choices of heterogeneous firms as we do not apply the typical extra fixed cost that generally leads more productive firms to undertake a more costly form of organization. This helps us avert potential misleading assumptions as it is not clear how fixed costs can be ranked across organizational forms.\textsuperscript{10}

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 charac-

\textsuperscript{7} This stresses a novel reason to support FDI that supplements those already highlighted in the literature. See, e.g., Barba Navaretti and Venables (2004, Ch.10).
\textsuperscript{8} The same is true in the Ricardian growth model by Ottaviano (2009).
\textsuperscript{9} The only other growth models with heterogeneous firms to our knowledge are Baldwin and Robert-Nicoud (2008) and Segerstrom and Gustafson (2009), which explore the impact of trade liberalization on growth in the presence of heterogeneous firms. These papers, however, do not investigate the impact of contractual incompleteness.
\textsuperscript{10} For instance, Antràs and Helpman (2004) assume that fixed costs of vertical integration are larger while Grossman, Helpman and Szeidl (2005) suppose that outsourcing fixed costs are more substantial.
terizes the market equilibrium. Section 4 analyzes the interactions between offshoring, innovation and economic growth. Section 5 highlights the role of contractual incompleteness in determining the long run effects of offshoring. Section 6 concludes.

2 A Dynamic Model of Offshoring

The economy consists of two countries, North and South. We assume that all workers and consumers belong to the North but can be employed in South as expatriates to work in the offshored plants. Hence the South is simply a potential production site. This emphasizes the tacitness of knowledge by ruling out perfect substitutability of Southern labor for Northern labor, with the intention of abstracting from typical labor market debates on wages that have been widely studied empirically and are being widely studied theoretically parallel to the writing of this paper.\textsuperscript{11} This helps single out the additional impacts of offshoring on growth in the home country that have often been neglected in the literature. In addition, observed empirical evidence does not always approve of the phenomenon of one job shifted abroad being immediately one job released at home.\textsuperscript{12}

2.1 Overview and timing of events

Before getting into the details of the model, it is useful to provide a brief overview of the way it works. Consumers have CES preferences over a horizontally differentiated good $C$. The production of each variety of good $C$ requires a blueprint, an intermediate input and assembly. Blueprints are created by independent R&D labs that sell their blueprints to entrepreneurs. All labs operate in North. An entrepreneur discovers her ability to turn the acquired blueprint in a sellable product only after buying. Her ability is determined by a random productivity draw. Upon observing its productivity $\varphi$, the entrepreneur organizes her firm as a vertical value chain with an "upstream" division producing the intermediate input and a "downstream" division turning it into the final product. While final production takes place in North, the firm can either "inshore" intermediate production in North,

\textsuperscript{11}For recent theoretical analyses of offshoring as means of trade and its effects on real wages see Baldwin and Robert-Nicoud (2007), Grossman and Rossi-Hansberg (2008) and Rodriguez-Clare (2009).

\textsuperscript{12}See for instance Debande (2006) for the US and Japanese cases.
using its own technology, or "offshore" it to South using a cost-reducing standardized technology. In this way, offshoring offers a viable alternative to the least efficient firms only. This alternative comes with strings attached. On the one hand, shipping intermediates from South to North incurs trade costs. On the other hand, offshoring takes place under contractual incompleteness, which generates further costs due to ex post bargaining.

To summarize, in each period \( t \) the following sequence of events take place. First, independent labs engage in R&D to create new patented blueprints. Second, entrepreneurs enter by purchasing a blueprint, realize their productivity levels in terms of non-standardized production and choose the location of upstream divisions. Third, upstream divisions manufacture the inputs needed by their downstream counterparts. Fourth, once intermediate production is completed, the upstream and downstream divisions of producers that have offshored bargain over the share of total revenues from final sales and inputs are handed over by the former to the latter. Lastly, final assembly takes place and final products are sold to households.

### 2.2 Demand side

There are \( L \) infinitely-lived households with identical preferences defined over the consumption of a horizontally differentiated good \( C \). The utility function is assumed to be instantaneously Cobb-Douglas and intertemporally CES with unit elasticity of intertemporal substitution:

\[
U = \int_0^\infty e^{-\rho t} \ln C(t) dt, \tag{1}
\]

where \( \rho > 0 \) is the rate of time preference and

\[
C(t) = \left[ \int_0^{n(t)} c(i, t)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}
\]

is a CES quantity index in which \( c(i, t) \) is the consumption of variety \( i \), \( n(t) \) is the number of available varieties of good \( C \), and \( \sigma \) is the own and cross demand elasticity of any variety, and thus an inverse measure of the degree of product differentiation between varieties. Households have perfect foresight and they can borrow and lend freely in a perfect capital market at instantaneous interest rate \( R(t) \).

Using multi-stage budgeting to solve their utility maximization problem, households first allocate
their income flow between savings and expenditures. This yields a time path of total expenditures $E(t)$ that obeys the Euler equation of a standard Ramsey problem:

$$\frac{\dot{E}(t)}{E(t)} = R(t) - \rho$$

(2)

where we have used the fact that the intertemporal elasticity of substitution equals unity. By definition, $E(t) = P(t)C(t)$ where $P(t)$ is the exact price index associated with the quantity index $C(t)$:

$$P(t) \equiv \left[ \int_0^{n(t)} p(i, t)^{1-\sigma} \, di \right]^{1/(1-\sigma)}.$$  

(3)

Households then allocate their expenditures across all varieties, which yields the instantaneous demand function

$$c(i, t) = A(t)p(i, t)^{-\sigma} \quad i \in [0, n(t)]$$

(4)

for each variety. In (4) $p(i, t)$ is the price of variety $i$ and

$$A(t) = \frac{E(t)}{P(t)^{1-\sigma}}$$

(5)

is aggregate demand. Throughout the rest of the paper, we leave the time dependence of variables implicit when this does not generate confusion.

### 2.3 Supply side

There are two factors of production in the economy. Labor is inelastically supplied by households and each household supplies one unit of labor so that we can use $L$ to refer both to the number of households and the total endowment of labor. Labor is freely mobile between countries and it is chosen as numeraire. The other factor is knowledge capital in the form of blueprints for the production of differentiated varieties. Blueprints are protected by infinitely lived patents and depreciate at a constant rate $\delta$.

There are two sectors, innovation (R&D) and production. Perfectly competitive labs invent blueprints for the production of the differentiated varieties. The production of each variety requires a single blueprint and consists of an upstream and a downstream stage. Entrepreneurs enter by buying
the rights to use the blueprints and split their activities between an upstream division supplying intermediates and a downstream division assembling them. Assembly takes place only in North whereas intermediate inputs can be produced also in South using an older standardized traditional technology ("offshoring"). Southern production takes place through a standardized traditional technology, which allows one unit of labor to produce $\varphi_f > 0$ units of intermediates.

Shipping the intermediate inputs back to the North for assembly incurs iceberg trade costs: $\tau > 1$ units must be shipped for one unit to reach destination. Trade costs can be embedded into the productivity parameter of the South without loss of generality. Hence, throughout the rest of the paper, we will use $\varphi_o = \varphi_f / \tau$ to denote the standard southern technology inclusive of trade costs. Northern production can rely on new advanced technologies that are generated by process innovation. This is a risky endeavor as long as its outcome is uncertain and the property rights on patents have to be bought in advance before experimenting new production processes. Specifically, after buying the rights to use the blueprints from labs, producers randomly draw their productivity level $\varphi$ from a continuous cumulative distribution $G(\varphi)$ with support $[0, \infty)$ so that offshoring offers productivity gains to producers with bad draws $\varphi < \varphi_o$. Final assembly in turn needs one unit of the intermediate component for each unit of the final good no matter where intermediates originate from. Intermediates are variety-specific: once produced for a certain assembly line, they have no alternative use.

Offshoring is associated with contractual costs that arise from weak legal institutions in the South. Specifically, only high quality variety-specific intermediates can be processed whereas low quality ones are useless even though supplied at zero cost. Contracts between the upstream and the downstream divisions are complete when both are located in North, but incomplete when the upstream division is offshored to South. In this case the quality of intermediates can not be assessed by third parties. That generates a hold up problem: after the upstream division has supplied its specific input, it has to reach an agreement with the downstream division on how to share the joint surplus (revenues) from final sales. The agreement is reached through Nash bargaining and we denote the bargaining weight of the upstream division by $\omega$. 

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Finally, we introduce endogenous growth by assuming that R&D faces a learning curve so that the marginal R&D cost of blueprints decreases with the number of blueprints that have been successfully introduced in the past. Specifically, the invention of a new blueprint requires \( k/n \) units of labor where \( k > 0 \) is a parameter and \( n \) is the total number of blueprints that have already been patented. Given the chosen functional form, some initial stock of implemented blueprints \( n_0 > 0 \) is needed to have finite costs of innovation at all times. We assume that this stock belongs to North.

### 3 Market Equilibrium

#### 3.1 Production

At time \( t \) the instantaneous equilibrium is found by solving the model backwards from final production to R&D. Varieties can be sold to final customers by two types of producers: "inshorers" have both divisions in North whereas "offshorers" have their upstream divisions in South and their downstream ones in North. Under inshoring, as contracts are complete, the upstream and downstream divisions of the same firm first maximize the firm’s profit and then share it according to their bargaining weights. This implies that the upstream division of a producer with labor productivity \( \varphi \) selects intermediate output \( x(\varphi) \) to maximize operating profit \( \pi_u(\varphi) = r_u(\varphi)/\sigma = p_u(\varphi)y_u(\varphi)/\sigma \) where \( r_u(\varphi), p_u(\varphi) \) and \( y_u(\varphi) \) are final revenues, final price and final output (itself equal to intermediate production). Given the demand curve (4), profit maximization yields markup pricing

\[
p_u(\varphi) = \frac{\sigma}{\sigma - 1} \frac{1}{\varphi}
\]

with associated output \( x_u(\varphi) = y_u(\varphi) = Ap_u(\varphi)^{-\sigma} \) and operating profit \( \pi_u(\varphi) = r_u(\varphi)/\sigma = Ap_u(\varphi)^{1-\sigma}/\sigma. \) A share \( \omega \) of \( \pi_u(\varphi) \) goes to the upstream division and the rest to the downstream one.

---

\(^{13}\)The assumed shape of the learning curve serves analytical solvability and the comparison with Grossman and Helpman (1991). In equilibrium it yields a ‘size effect’, meaning that larger countries grow faster. To avoid this prediction that runs against the empirical evidence, one could assume that the intensity of the learning spillover is lower, i.e. \( k/n^\xi \) with \( 0 < \xi < 1 \) (Jones, 1995). This would turn our setup into a quasi-endogenous growth model in the wake of Segerstrom (1998).
Under offshoring, the producer uses the standardized technology with upstream labor productivity $\varphi_o$ and gets the joint surplus of its divisions under incomplete contracts. This surplus is given by the revenues from final sales and is divided between divisions through ex post Nash bargaining. Absent any outside option, revenues are therefore split according to the bargaining weights of the two parties with a share $(1-\omega)$ going to the downstream division and the remaining share $\omega$ going to the upstream one. The upstream division decides how much input $x_o$ to produce anticipating that bargaining outcome. Hence, it maximizes

$$\pi_u = \omega p_o y_o - x_o/\varphi_o$$

where $p_o$ and $y_o$ are final price and final output (itself equal to intermediate production). Given the demand curve (4), this yields markup pricing for final sales

$$p_o = \frac{\sigma}{\sigma - 1} \frac{1}{\omega \varphi_o}$$

with associated output $x_o = y_o = A p_o^{-\sigma}$ and revenues $r_o = p_o y_o = A p_o^{1-\sigma}$. A share $\pi_d = (1-\omega)r_o$ goes to the downstream division while the complementary share goes to the upstream one. Accordingly, after subtracting labor costs, the upstream division is left with $\pi_u = \omega r_o/\sigma$: larger upstream bargaining weight and stronger product differentiation shift a larger share of a given joint surplus $r_o$ from downstream to upstream divisions. Hence, the overall operating profit of the offshorer is

$$\pi_o = \pi_d + \pi_u = [1 + (\sigma - 1)(1-\omega)] r_o/\sigma.$$ 

Since the downstream division does not contribute anything before the bargaining stage, the joint surplus $r_o$ (and the joint profit $\pi_o$ as well) is at its maximum when $\omega$ goes to one. In other words, when $\omega$ goes to one, the incomplete contract outcome converges to the complete contract one.

As producers can freely choose between inshoring and offshoring, the operating profits they earn are equal to $\pi(\varphi) \equiv \max[\pi_u(\varphi), \pi_o]$. The fact that $\pi_u(\varphi)$ is an increasing function of productivity $\varphi$ implies that there exists a unique threshold productivity level ("cutoff") $\varphi^*$ above which producers

14 The upstream division does not face an incentive constraint as the optimal output is always positive.
15 For the upstream division the adverse incentive due to ex post bargaining under incomplete contracts has exactly the same impact as an iceberg trade cost that melts a fraction $(1-\omega)$ of intermediate output shipped from South to North, and therefore does not generate revenues for that division. The fact that here the fraction $(1-\omega)$ of revenues is recovered by the downstream division explains why the overall operating profit of the offshorer is larger than that of the simple iceberg case.

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prefer to inshore. This cutoff solves $\pi_v(\varphi^*) = \pi_o$ and is therefore equal to

$$
\varphi^* = (\omega \varphi_o) \left[ 1 + (1 - \omega)(\sigma - 1) \right]^{-1}
$$

(6)

The cutoff is decreasing in $\sigma$ because weaker product differentiation shifts surplus from upstream to downstream divisions exacerbating intermediate underproduction and thus promoting inshoring.

For symmetric reasons, the cutoff is increasing in the upstream bargaining weight $\omega$. It is also increasing in $\varphi_o$ as offshoring is fostered by any improvement in the productivity of the standardized technology $\varphi_o$ or any fall in trade cost $\tau$.

We can therefore highlight:

**Proposition 1** A marginal increase in the bargaining weight of upstream divisions encourages more firms to offshore.

Since $1/(\omega \varphi_o)$ is the amount of labor embedded in unit revenues, it will turn out to be useful to denote by $\widehat{\varphi}_o \equiv \omega \varphi_o$ the "delivered" productivity of offshored labor. We will call this simply "offshored productivity" and we will contrast it with producer-specific "inshored productivity" $\varphi$. Note that (6) shows that a marginal producer drawing exactly $\varphi^*$ has higher inshored than offshored productivity ($\varphi^* > \widehat{\varphi}_o$) so that the range ($\widehat{\varphi}_o, \varphi^*$) identifies producers whose decisions to offshore reduce aggregate productivity. Moreover, due to $\partial \varphi^*/\partial \omega > 0$, the cutoff $\varphi^*$ achieves its maximum value $\varphi_o$ at $\omega$ equal to 1, so any other value of $\omega$ implies $\varphi^* < \varphi_o$. Hence, incomplete contracts generate two adverse effects of offshoring on aggregate productivity. First, firms drawing values of $\varphi$ between $\widehat{\varphi}_o$ and $\varphi^*$ offshore while they have higher inshored productivity. Second, firms drawing values between $\varphi^*$ and $\varphi_o$ do not offshore whereas in the absence of contractual frictions doing so would increase their productivity. In Figure 1 we call these adverse effects *penalization 1* and *penalization 2* respectively. Indeed, when $\omega$ is equal to 1, (6) implies $\varphi^* = \widehat{\varphi}_o = \varphi_o$ so that all firms with a productivity level $\varphi < \varphi_o$ offshore and their decision to do so improves aggregate productivity.

To summarize, producers’ organizational choices give the following cutoff results for prices and
overall profits:

\[
p(\varphi) = \begin{cases} \frac{1}{\frac{\sigma}{\varphi} + \frac{1}{\rho}} & \text{and } \pi(\varphi) = \begin{cases} \frac{1+(\sigma-1)\lambda}{\sigma}Ap(\varphi)^{1-\sigma} & \text{for } \varphi \in [0, \varphi^*) \\ \frac{1}{\sigma}Ap(\varphi)^{1-\sigma} & \text{for } \varphi \in [\varphi^*, \infty) \end{cases} \end{cases}
\]

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\] (7)

### 3.2 Innovation

At the innovation stage, labs invent new blueprints at a marginal cost that depends on acquired experience \(k/n\) and their output determines the law of motion of \(n\). In particular, we have

\[\dot{n} = \frac{nL^I}{k} - \delta n,\] (8)

where \(\dot{n} \equiv dn/dt\), \(L^I\) is labor employed in inventing new blueprints, \(n/k\) is its productivity and \(\delta\) is the rate of depreciation.

Due to learning, as innovation cumulates, it becomes increasingly cheaper to introduce new blueprints and, being priced at marginal cost, their value falls through time. Specifically, if we call \(J\) the asset value of a patented blueprint, marginal cost pricing gives \(J = k/n\), which implies \(\dot{J}/J = -\dot{n}/n\).

Labs pay their researchers by borrowing at the interest rate \(R\) and know that the resulting patents will generate instantaneous dividends equal to the expected profits of the corresponding producers \(\pi\). Arbitrage in the capital market then requires the dividends \(\pi\) and capital gains \(\dot{J}\) to match interest payments \(RJ\) and depreciation \(\delta J\) so that:

\[R + \delta = \frac{\pi n}{k} - \frac{\dot{n}}{n}\] (9)

where the equality is granted by the definition of \(J\).

### 3.3 Aggregation

In characterizing the aggregate behavior of our heterogeneous economy, we follow Melitz (2003) and define average (output-weighted) productivity as:

\[\bar{\varphi} \equiv \left\{ G(\varphi^*)\varphi_o^{\sigma-1} + \left[1 - G(\varphi^*)\right]\varphi_0^{\sigma-1} \right\}^{\frac{1}{\sigma-1}}\] (10)
The Figure 1: The Productivity Distribution and an Increase in $\varphi$

Figure 1: Industry partition

where, as already mentioned, $\bar{\varphi}_o \equiv \omega \varphi_o$ is the common productivity of offshorers and

$$\bar{\varphi}_v = \left[ \frac{1}{1 - G(\varphi^*)} \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} dG(\varphi) \right]^{\frac{1}{1-\sigma}}$$

is the average (output weighted) productivity of inshorers. Since $\varphi^* > \bar{\varphi}_o$, we have $\bar{\varphi}_v > \varphi^* > \bar{\varphi}_o$, $\bar{\varphi}_v > \bar{\varphi}_o$, and $d\bar{\varphi}_v/d\varphi^* > 0$. Figure 1 shows a ranking of the productivity levels.$^{16}$

We also define $E_o$ as the share of expenditures going to offshorers, and $P_o$ and $C_o$ as the corresponding exact price and quantity indices such that $P_o C_o = E_o$. Analogously, we define $E_v$ as the share of expenditures going to inshorers, and $P_v$ and $C_v$ as the corresponding exact price and quantity indices such that $P_v C_v = E_v$ and $E_o + E_v = E$. Then we have:

$$\bar{p}_v = \frac{\sigma}{\sigma - 1} \frac{1}{\bar{\varphi}_v}, P_v = \left\{ n [1 - G(\varphi^*)] \right\}^{\frac{1}{1-\sigma}} \bar{p}_v, C_v = \frac{E_v}{P_v}, E_v = \left( \frac{P_v}{P} \right)^{1-\sigma} E \quad (11)$$

where $\bar{p}_v$ is the average price of inshorers. Analogously, we can write

$$p_o = \frac{\sigma}{\sigma - 1} \frac{1}{\bar{\varphi}_o}, P_o = \left\{ n G(\varphi^*) \right\}^{\frac{1}{1-\sigma}} p_o, C_o = \frac{E_o}{P_o}, E_o = \left( \frac{P_o}{P} \right)^{1-\sigma} E$$

and

$$\bar{p} = \frac{\sigma}{\sigma - 1} \frac{1}{\bar{\varphi}}, P = \left\{ n G(\varphi^*) \bar{p}_o^{1-\sigma} + n [1 - G(\varphi^*)] \bar{p}_o^{1-\sigma} \right\}^{\frac{1}{1-\sigma}}, C = \frac{E}{P}$$

$^{16}$While $\bar{\varphi}$ is larger than $\varphi^*$ in Figure 1, this is not always necessarily the case.
3.4 Financial market clearing

Since producers discover their productivity only after acquiring the right to use a patented blueprint, the dividends they are willing to pay to labs equal their expected operating profits

$$\pi = G(\varphi^*) \pi_o + [1 - G(\varphi^*)] \bar{\pi}_v$$

(12)

where, since $r_o = E_o/[nG(\varphi^*)]$, the operating profit of a typical offshorer $\pi_o$ can be rewritten in terms of aggregate variables as

$$\pi_o = \frac{[1 + (\sigma - 1)(1 - \omega)] E_o}{\sigma n G(\varphi^*)}$$

and, by definition, the average operating profit of inshorers equals

$$\bar{\pi}_v = \frac{E_v}{\sigma n [1 - G(\varphi^*)]}$$

By (11), expected operating profits (12) simplify to

$$\pi = \frac{E}{n} \left[ \frac{1}{\sigma} + \frac{\sigma - 1}{\sigma} \Omega \right]$$

(13)

where $\Omega \equiv (1 - \omega) E_o / E$ is the share of aggregate expenditures accruing to the downstream divisions of offshorers. Expression (13) shows that expected profits are an increasing function of $\Omega$. When a higher share of expenditures in the economy goes to offshorers, expected profits are larger in the industry. To see this, note that keeping fixed total expenditure $E$ and the proportion of offshoring firms $G(\varphi^*)$, shifting a unit of expenditures from inshorers $E_v$ to offshorers $E_o$ increases average profits because the downstream offshored divisions earn a share $1 - \omega$ of revenues $r_o$ while the inshored ones only earn a share $1 - \omega$ of profits $\pi_v = r_v / \sigma$.17 This also explains why the positive impact of $\Omega$ on $\pi$ is larger when $\sigma$ is larger. Larger downstream bargaining power $(1 - \omega)$ also increases expected profits at the time of entry.

Once substituted into (9), expression (13) allows us to restate the Euler condition (2) as:

$$\frac{\dot{E}}{E} = \frac{E}{\sigma k} \left[ 1 + (\sigma - 1) \Omega \right] - \frac{n}{n} - \delta - \rho$$

(14)

17Inshores have a higher sensitivity to the elasticity of substitution as with offshoring all variable costs of producing intermediates are passed over to upstream suppliers. See Grossman and Helpman (2002, p. 102) for more detail.
3.5 Labor market clearing

Aggregate labor endowment $L$ is absorbed by innovation ($L_I$) as well as by inshored and offshored upstream production. Inshorers' and offshorers' employment levels amount to $L_v = E_v/(\bar{p}_v\hat{\varphi}_v)$ and $L_o = E_o/(p_o\varphi_o)$ respectively. Accordingly, given (11), total employment in upstream production simplifies to

$$L_v + L_o = \frac{\sigma - 1}{\sigma} E(1 - \Omega)$$

which, together with (8), allows us to rewrite the labor market clearing condition $L = L_I + L_v + L_o$ as

$$L = k \left( \frac{\dot{n}}{n} + \delta \right) + \frac{\sigma - 1}{\sigma} E(1 - \Omega) \quad (15)$$

Employment in production is a decreasing function of the share $\Omega$ of aggregate expenditures accruing to the downstream divisions of offshorers. This is the dual of the previously discussed result that expected profits increase with $\Omega$ as long as larger expected profits induce a reallocation of labor from production to R&D.

4 Offshoring and Growth

The market clearing conditions (14) and (15) define a dynamic system in two unknowns: the growth rate of the stock of patents ($\dot{n}/n$) and the expenditures level ($E$). A unique balanced growth path exists along which these variables are constant and is achieved without any transition dynamics.18 Calling the corresponding growth rate and expenditures level by $g_s$ and $E_s$ respectively, then imposing $\dot{n}/n = g_s$, $E = E_s$ and $\dot{E} = 0$ in (14) and (15) allows us to find:

$$g_s = \frac{L}{k} \left( \frac{1}{\sigma} + \frac{\sigma - 1}{\sigma} \Omega \right) - \frac{\sigma - 1}{\sigma} (1 - \Omega) \rho - \delta, \quad E_s = L + \rho k \quad (16)$$

While expenditures $E_s$ do not depend on $\Omega$, the growth rate $g_s$ is instead an increasing function of $\Omega$. The reason is that, by definition, a rise in $\Omega$ shifts expenditures from inshorers to offshorers. This shift, as discussed above, generates larger expected profits and smaller employment in production.

The resulting reallocation of labor from production to R&D promotes innovation and growth.

18 See Grossman and Helpman (1991, ch.3) for details.
Since $E_s$ does not depend on $\Omega$, the bargaining weight $\omega$ does not affect expenditures. It affects, however, the growth rate through various channels funneled through the impact of $\Omega$ on $g_s$. To disentangle these channels, we use (10) and (11) to rewrite the share $\Omega$ of expenditures accruing to the downstream divisions of offshorers as

$$\Omega = (1 - \omega) \frac{E_o}{E} = (1 - \omega) \left( \frac{P_o}{T} \right)^{1-\sigma} = (1 - \omega) G(\varphi^*) \left( \frac{\varphi_o}{\bar{\varphi}} \right)^{\sigma-1}$$

where $\varphi_o \equiv \omega \varphi_o$ is offshored productivity and $\bar{\varphi}$ is the average (offshored and inshored) productivity as defined in (10). Since $dg_s/d\Omega > 0$, the sign of the impact of $\omega$ on $g_s$ depends on the sign of $d\Omega/d\omega$. This can be decomposed as:

$$\frac{d\Omega}{d\omega} = -s_o + (1 - \omega) \frac{ds_o}{d\omega}$$  \hspace{1cm} (17)$$

where $s_o \equiv E_o/E$.

Consider a marginal increase in $\omega$. The first term on the right hand side is the direct effect of larger $\omega$. It is negative as it identifies the corresponding fall in the share of expenditures accruing to the downstream divisions of offshorers holding the overall share of expenditures accruing to offshorers constant. It captures a pure surplus reallocation between divisions as a higher upstream bargaining weight transfers surplus from downstream to offshored upstream divisions.

The second term on the right hand side of (17) is the indirect effect. It identifies the change in the overall share of expenditures accruing to offshorers. This adjustment takes place along two margins: the relative number of offshorers as determined by $G(\varphi^*)$ ("extensive margin") and their relative size with respect to the average producer $r_o/\bar{r} = (\varphi_o/\bar{\varphi})^{\sigma-1}$ ("intensive margin"):

$$\frac{ds_o}{d\omega} = \frac{dG(\varphi^*)}{d\omega} (r_o/\bar{r}) + G(\varphi^*) \frac{d(r_o/\bar{r})}{d\omega},$$

where $\bar{r} = A\bar{\varphi}^{1-\sigma}$. The impact of larger $\omega$ is positive on both margins. Since a larger bargaining weight of upstream divisions alleviates their underproduction of intermediates, as $\omega$ rises not only more producers decide to offshore, but also offshorers become larger. Along the extensive margin, by (6) we have $d\varphi^*/d\omega > 0$ and thus $dG(\varphi^*)/d\omega > 0$. Along the intensive margin, using $\tilde{\varphi}_o \equiv \omega \varphi_o$
and (10), we have
\[
\frac{r_\omega}{\bar{r}} = \left( \frac{\bar{\varphi}_o}{\bar{\varphi}} \right)^{\sigma-1} \frac{\bar{\varphi}_o^{\sigma-1}}{G(\varphi^*)\bar{\varphi}_o^{\sigma-1} + [1 - G(\varphi^*)] \bar{\varphi}_o^{\sigma-1}} = \frac{(\omega \varphi_\omega)^{\sigma-1}}{G(\varphi^*) (\omega \varphi_\omega)^{\sigma-1} + \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} dG(\varphi)}
\] (18)
which, given \(d\varphi^*/d\omega > 0\) and \(dG(\varphi^*)/d\varphi^* > 0\), is an increasing function of \(\omega\) (see Appendix 1 for a proof that \(\partial (r_\omega/\bar{r})/\partial \omega > 0\)). Hence, we can conclude that \(ds_\omega/d\omega > 0\), which, given (17), implies that improved contract enforcement in the South has ambiguous effects on \(\Omega\) and, therefore, on \(g_s\).

To shed light on such ambiguity, we can manipulate (17) to show that a higher bargaining weight of upstream divisions promotes growth when the elasticity of the offshorers’ market share \(s_o\) to \(\omega\) is larger than the ratio between their upstream and downstream divisions’ bargaining weights, i.e.
\[
\frac{d \ln s_o}{d \ln \omega} > \frac{\omega}{1 - \omega}
\] (19)
If the reverse is true, a higher bargaining weight of upstream divisions hampers growth. Figure 1 shows the effect of an increase in \(\omega\) on the productivity distribution of firms graphically. Larger \(\omega\) directly raises \(\bar{\varphi}_o\), while it reduces \([1 - G(\varphi^*)] \bar{\varphi}_o\) through a change in \(\varphi^*\). This makes the change in average productivity \(\bar{\varphi}\) ambiguous in \(\omega\), yet inferior to the rise in \(\bar{\varphi}_o\). Also, the productivity range along which offshoring raises aggregate productivity increases if the change in the intensive margin is larger than that in the extensive margin \((\partial \bar{\varphi}_o/\partial \omega > \partial \varphi^*/\partial \omega)\) so that the two values converge, and falls if the opposite holds so that they diverge. Finally, note that the values achieved by \(d \ln s_o/d \ln \omega\) at \(\omega = 0\) and at \(\omega = 1\) are both strictly positive and finite provided that the elasticity of the extensive margin \(d \ln G(\varphi^*)/d \ln \varphi^*\) is also positive and finite, as in the case of all the commonly used families of cumulative density functions (see Appendix 2 for a proof). Then, since \(\omega/(1 - \omega)\) equals zero at \(\omega = 0\) and goes to infinity when \(\omega\) goes to one, there must exist a threshold value of \(\omega\) below which the inequality (19) holds and above which it is violated. Hence, there exists a unique threshold value for the bargaining weight of upstream divisions \(\omega\) such that \(d \Omega/d\omega > (<)0\) and, thus, \(dg_s/d\omega > (<)0\) if and only if \(\omega\) falls below (above) that value.

Hence, we can state:

**Proposition 2** A marginal increase in the bargaining weight of upstream divisions fosters growth if it is initially small and hampers growth if it is initially large enough.

18
To sum up, Proposition 1 tells us that a higher bargaining weight of upstream divisions unambiguously promotes offshoring. Proposition 2 tells us that this rise of offshoring stimulates growth as long as that bargaining weight is not too large. The reason is that a higher bargaining weight $\omega$ increases the share of expenditures accruing to offshorers through two positive indirect effects on their relative size ("intensive margin") and number ("extensive margin"). On the other hand, a higher $\omega$ has a negative direct effect on the fraction of offshorers' revenues appropriated by their downstream divisions. While the indirect effects boost offshorers' profitability and thus growth by reallocating labor from production to R&D, the direct effect works in the opposite direction. This effect comes to dominate when $\omega$ is large enough, generating a non-linear relation between the bargaining weight of upstream divisions and the growth rate.

5 Contracts and Welfare

The aim of this section is twofold. On the one hand, we want to clarify how our results depend on the quality of contract enforcement. In so doing, we first characterize the steady state outcome when offshoring takes place under complete contracts. We then use this characterization to unveil the different performance of the economy when the transition from no offshoring to offshoring happens under incomplete rather than complete contracts. This will also highlight the role of firm heterogeneity. On the other hand, we want to highlight how incomplete contracts lead to different policy implications with respect to complete contracts.

5.1 Complete contracts

To better understand the role of contractual incompleteness for our results, it is useful to characterize the steady state outcome of the model in two scenarios: one in which offshoring is inhibited (so that the hold up problem is not an issue) and the other in which offshoring takes place under complete contracts.

In both cases at the aggregate level the model with heterogeneous firms is homomorphic to a model with homogeneous firms à la Grossman and Helpman (1991) in which all firms are identical to
the average heterogeneous firm. Accordingly, average productivity (and ultimately the productivity cutoff) provides a sufficient statistic to describe the aggregate behavior of the economy. When offshoring is inhibited, average productivity is

$$\tilde{\varphi}_h = \left[ \int_0^\infty \varphi^{\sigma-1} dG(\varphi) \right]^{\frac{1}{1-\sigma}}$$

due to the fact that, by definition, no entrepreneurs relies on the foreign standardized technology. On the other hand, when offshoring is allowed for and contracts are complete, average productivity is equal to

$$\tilde{\varphi}_c = \left\{ G(\varphi_o^c) \varphi_o^c + [1 - G(\varphi_o^c)] \tilde{\varphi}_{vc}^{\sigma-1} \right\}^{\frac{1}{1-\sigma}}$$

where from (6) we have \( \varphi_o^c = \varphi_o > \varphi^* > \tilde{\varphi}_o \) as the productivity cutoff under contractual completeness, and \( \tilde{\varphi}_{vc} \) is the corresponding average productivity of inshorers

$$\tilde{\varphi}_{vc} = \left[ \int_{\tilde{\varphi}_1}^{\infty} \varphi^{\sigma-1} dG(\varphi) \right]^{\frac{1}{1-\sigma}}$$

In both case, as in Grossman and Helpman (1991), efficiency in production is immaterial for steady state growth and expenditures because it conveys equal incentives to production and innovation. As a result, steady state growth and expenditures are the same in the two complete contract scenarios:

$$g_h = g_c = \frac{1}{\sigma} \frac{L}{k} - \frac{\sigma - 1}{\sigma} \rho - \delta$$

and

$$E_h = E_c = E_s = L + \rho k$$

In addition, steady state expenditures are also the same as those with incomplete contracts.

Average productivity affects, instead, the aggregate quantities consumed in the various scenarios as these quantities are given by expenditures divided by the corresponding average prices

$$D_h = \frac{\sigma-1}{\sigma} \tilde{\varphi}_h (L + \rho k), \quad D_c = \frac{\sigma-1}{\sigma} \tilde{\varphi}_c (L + \rho k), \quad D_s = \frac{\sigma-1}{\sigma} \tilde{\varphi}_s (L + \rho k)$$

where, for notational symmetry, we use \( \tilde{\varphi}_s \) to relabel the average productivity (10) that prevails with incomplete offshoring contracts.

Comparing the two complete contract scenarios, we see that the fact that less efficient firms offshore for \( \varphi < \varphi_o \) implies \( \tilde{\varphi}_c > \tilde{\varphi}_h \) and thus \( D_c > D_h \). Hence,
Proposition 3 Making offshoring possible under contractual completeness generates a static welfare gain due to more production but no dynamic welfare effect through a change in the growth rate.

The reason for this is that offshoring operates as "neutral" technological progress that evenly augments both the profitability of R&D and the profitability of production. As a result, when firms offshore there is no reallocation of labor between R&D and production. All that happens is that more efficient labor generates more output.

5.2 Incomplete contracts

We can now compare the steady state outcomes with or without incomplete contracts. As for growth rates, expressions (16) and (21) show that offshoring with incomplete contracts leads to faster growth with respect to both no offshoring and offshoring under complete contracts, the more so the larger is $\Omega$. In particular, we have

$$g_s - g_h = \frac{\sigma - 1}{\sigma} \Omega \left( \frac{L}{k} + \rho \right) > 0$$

Hence:

Proposition 4 Making offshoring possible under contractual incompleteness generates dynamic welfare gains due to faster growth.

The reason for this is that offshoring operates as "biased" technological progress that augments the profitability of R&D more than the profitability of production. As a result, when firms offshore there is a reallocation of labor from production to R&D.

Turning to consumption, we have shown in Section 3.1 that incomplete contracts generate two adverse effects of offshoring on aggregate productivity. First, firms drawing values of $\varphi$ between $\varphi_o$ and $\varphi^*$ offshore while they have higher inshored productivity. This does not happen under complete contracts for which we have $\tilde{\varphi}_o = \varphi^*$. Second, firms drawing values between $\varphi^*$ and $\varphi_o$ do not offshore whereas in the absence of contractual frictions doing so would increase their productivity. These adverse effects imply $\tilde{\varphi}_k < \tilde{\varphi}_c$ and, therefore, $D_s < D_c$ so that with offshoring output is always higher under complete than incomplete contracts.
Differently, when it comes to comparing offshoring under contractual incompleteness and no offshoring only the first adverse effect is relevant as firms with productivity levels between \( \varphi^* \) and \( \varphi_o \) do not offshore in either case. The ranking in terms of output is therefore ambiguous. In particular, we have \( \tilde{\varphi}_s > \tilde{\varphi}_h \) if and only if

\[
\int_0^{\tilde{\varphi}_o} \left( \varphi_o^{\alpha-1} - \varphi^{\alpha-1} \right) dG(\varphi) > \int_{\tilde{\varphi}_o}^{\varphi^*} \left( \varphi^{\alpha-1} - \varphi_o^{\alpha-1} \right) dG(\varphi)
\]

that is if the cumulated productivity gain of offshores that are less productive under onshoring (0 < \( \varphi < \tilde{\varphi}_o \)) is larger than the cumulated productivity loss for offshorers that are more productive under onshoring (\( \tilde{\varphi}_o < \varphi < \varphi^* \)).

These results allow us to state the following:

**Proposition 5** Making offshoring possible under contractual incompleteness generates static welfare gains if the cumulated productivity gains of offshores that would be less productive under onshoring are larger than the cumulated productivity losses of offshorers that would be more productive under onshoring.

Whether this is indeed the case or not clearly depends on the way \( G(\varphi) \) distributes the productivity draws \( \varphi \) between the two intervals (0, \( \tilde{\varphi}_o \)) and (\( \tilde{\varphi}_o, \varphi^* \)). This highlights the crucial role of heterogeneity. For example, if \( \varphi \) is Pareto distributed with \( G(\varphi) = 1 - (\varphi_m)^k \varphi^{-k} \), then \( \tilde{\varphi}_s > \tilde{\varphi}_h \) whenever \( k < (\sigma - 1)/[2 + (1 - \omega)(\sigma - 1)] \). In other words, with incomplete contracts offshoring also generates static gains when there are a lot of unproductive firms and few productive ones.

### 5.3 Optimal policy

We are now ready to investigate how incomplete contracts lead to different policy implications with respect to complete contracts when the objective is to implement an efficient outcome maximizing (1) subject to the aggregate resource constraint without any hold up problem. The efficient outcome is readily characterized by remembering again that, with complete contracts, at the aggregate level the model with heterogeneous firms is homomorphic to a model with homogeneous firms in which
all firms are identical to the average heterogeneous firm. We can, therefore, invoke the results in Grossman and Helpman (1991) to assert that the efficient steady state is characterized by

$$g_w = \frac{L}{\eta} - (\sigma - 1) \rho - \delta, \quad E_w = E_h = E_c = E_s$$  \hspace{1cm} (23)$$

with quantity consumed $D_w = D_c$.

The comparison between $g_s$ and $g_w$ shows that, as in Grossman and Helpman (1991), due to the positive learning externality in R&D, at the market outcome with complete contracts our economy grows too slowly. In this case, optimal intervention then requires an R&D subsidy that equalizes $g_h = g_c$ to $g_w$. Specifically, if we call $\phi_w^R$ the optimal fraction of R&D expenditures paid by the government, such fraction satisfies

$$\phi_w^R = \frac{g_w + \delta}{g_w + \delta + \rho}$$

With incomplete contracts the R&D subsidy alone is not enough due to the underproduction of the intermediate input and, therefore, of the final output. Optimal intervention here requires the government to subsidize also offshorers’ upstream production. If we call $\phi_w^P$ the optimal fraction of offshorers’ upstream production costs paid by the government, then we have

$$\phi_w^P = 1 - \omega$$

Hence, both R&D and production subsidies are needed to implement the first best under incomplete contracts.

To summarize, we can write:

**Proposition 6** With complete contracts welfare is maximized through a subsidy to innovation only. With incomplete contracts welfare maximization also requires a subsidy to offshored production.

### 6 Conclusion

We have used an endogenous growth model of North-South offshoring with heterogeneous firms to study its dynamic and static effects on the economy when contracts are incomplete in the South.
In so doing, we have modelled offshoring as the geographical fragmentation of a firm’s production chain between a home upstream division and a foreign downstream one.

On the positive side, we have shown that, when contracts are incomplete, the possibility of offshoring may have favorable implications for economic growth. The key parameter regulating the growth effect of offshoring is the bargaining power of the upstream division through a non-linear relation. While a larger upstream bargaining weight unambiguously promotes offshoring, it (hence increased offshoring) only stimulates growth up to a critical level. Under complete contracts, offshoring has no impact whatsoever on growth. The reason for this is that under complete contracts, offshoring evenly augments the profitability of R&D and production, whereas with incomplete contracts the gains in profitability are biased towards R&D.

Whether offshoring with incomplete contracts also increases consumption depends on firm heterogeneity. For example, when productivity draws are Pareto distributed, consumption increases when there are a lot of unproductive firms and very few productive ones.

On the normative side, we have show that, whereas with complete contracts efficiency can be restored through a subsidy to R&D only, with incomplete contracts a production subsidy to offshored upstream divisions is needed too.

Contrary to the existing literature, our study uses the industry dynamics of firm organization to reveal the possibility of adverse long term effects of offshoring for the North. In addition, it emphasizes the role of firm heterogeneity for the social gains from offshoring. This raises the question whether analyses on the consequences of offshoring based on real wages can fully absorb the mechanisms through which it influences the economy performance. Our analysis has its limitation and leaves much work for future research on the issue.

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7 Appendix

7.1 Stronger contract enforcement in the South increases the relative size of offshorers

The cutoff $\varphi^*$ is an increasing function of $\omega$:

$$
\frac{d\varphi^*}{d\omega} = \frac{d}{d\omega} \left( \omega \varphi^* \right) \left( 1 + (1 - \omega)(\sigma - 1) \varphi^* \right) = (1 - \omega) \sigma \varphi^* \left( 1 + (1 - \omega)(\sigma - 1) \frac{\varphi^*}{\omega} \right) > 0
$$
Since $\varphi^*$ is an increasing function of $\omega$, the average productivity of inshorers $\bar{\varphi}_o$ is a decreasing function of $\omega$:

$$\frac{d\bar{\varphi}_o}{d\omega} = -\frac{1}{\sigma - 1} \left[ \frac{\int_\varphi^\infty \varphi^{\sigma-1} dG(\varphi)}{1 - G(\varphi^*)} \right]^{\frac{1}{\sigma - 1}} (\varphi^*)^{\sigma - 1} G'(\varphi^*) \frac{d\varphi^*}{d\omega} < 0$$

where

$$\frac{d}{d\varphi^*} \left[ \int_\varphi^\infty \varphi^{\sigma-1} dG(\varphi) \right] = -\frac{d}{d\varphi^*} \left[ \int_\varphi^\infty \varphi^{\sigma-1} G'(\varphi) d\varphi \right] = - (\varphi^*)^{\sigma - 1} G'(\varphi^*)$$

is granted by the fundamental theorem of calculus. Moreover, since $\varphi^*$ is an increasing function of $\omega$, the relative size of offshorers with respect to the average producer $r_o/\bar{r}$ is an increasing function of $\omega$:

$$\frac{d (r_o/\bar{r})}{d\omega} = \frac{d}{d\omega} \left[ \frac{(\omega \varphi_o)^{\sigma - 1}}{G(\varphi^*) (\omega \varphi_o)^{\sigma - 1} + \int_\varphi^\infty \varphi^{\sigma-1} dG(\varphi)} \right] = \frac{num}{[G(\varphi^*) (\omega \varphi_o)^{\sigma - 1} + \int_\varphi^\infty \varphi^{\sigma-1} dG(\varphi)]^2}$$

$$num = (\omega \varphi_o)^{\sigma - 1} \left\{ \frac{\sigma - 1}{\omega} \left[ \int_\varphi^\infty \varphi^{\sigma-1} dG(\varphi) \right] - \left[ (\omega \varphi_o)^{\sigma - 1} - (\varphi^*)^{\sigma - 1} \right] G'(\varphi^*) \frac{d\varphi^*}{d\omega} \right\}$$

With

$$\varphi^* = (\omega \varphi_o) \left[ 1 + (1 - \omega)(\sigma - 1) \right]^{\frac{1}{\sigma - 1}}$$

we have

$$num = (\omega \varphi_o)^{\sigma - 1} (\sigma - 1) \left\{ \frac{1}{\omega} \left[ \int_\varphi^\infty \varphi^{\sigma-1} dG(\varphi) \right] + (1 - \omega) (\omega \varphi_o)^{\sigma - 1} G'(\varphi^*) \frac{d\varphi^*}{d\omega} \right\}$$

which is positive since $d\varphi^*/d\omega > 0$.

Hence:

$$\frac{d (r_o/\bar{r})}{d\omega} = \frac{num}{[G(\varphi^*) (\omega \varphi_o)^{\sigma - 1} + \int_\varphi^\infty \varphi^{\sigma-1} dG(\varphi)]^2} > 0$$

### 7.2 Properties of the elasticity of offshorers’ market share to the quality of the contractual environment

Recall the definition

$$s_o \equiv \frac{E_o}{E} = G(\varphi^*) \left( \frac{\tilde{\varphi}_o}{\bar{\varphi}} \right)^{\sigma - 1} = G(\varphi^*) \frac{r_o}{\bar{r}} = \frac{G(\varphi^*) \tilde{\varphi}_o^{\sigma - 1}}{G(\varphi^*) \tilde{\varphi}_o^{\sigma - 1} + [1 - G(\varphi^*)] \tilde{\varphi}_o^{\sigma - 1}} = \frac{G(\varphi^*) (\omega \varphi_o)^{\sigma - 1}}{G(\varphi^*) (\omega \varphi_o)^{\sigma - 1} + \int_\varphi^\infty \varphi^{\sigma-1} dG(\varphi)}$$
Then the elasticity of offshorers’ market share \( s_o \) to the quality of the contractual environment \( \omega \) evaluates to

\[
\frac{d\ln s_o}{d\ln \omega} = \frac{d\ln G(\varphi^*)}{d\ln \omega} + \frac{d\ln (r_o/\bar{r})}{d\ln \omega}
\]

with

\[
\frac{d\ln (r_o/\bar{r})}{d\ln \omega} = (\sigma - 1) \left[ \int_{\varphi_o}^{\infty} \varphi^{\sigma-1} dG(\varphi) + (1 - \omega) (\omega \varphi_o)^{\sigma-1} \frac{d\ln G(\varphi^*)}{d\ln \omega} G(\varphi^*) \right] / G(\varphi^*) (\omega \varphi_o)^{\sigma-1} + \int_{\varphi_o}^{\infty} \varphi^{\sigma-1} dG(\varphi)
\]

as derived in the previous section. Accordingly

\[
\frac{d\ln s_o}{d\ln \omega} = \left[ 1 + (\sigma - 1)(1 - \omega)s_o \right] \frac{d\ln G(\varphi^*)}{d\ln \omega} + (\sigma - 1) (1 - s_o)
\]

\[
= \left[ 1 + (\sigma - 1)(1 - \omega)s_o \right] \frac{d\ln G(\varphi^*)}{d\ln \varphi^*} \frac{d\ln \varphi^*}{d\ln \omega} + (\sigma - 1) (1 - s_o)
\]

where \( d\ln G(\varphi^*)/d\ln \varphi^* \) is the elasticity of the extensive margin, i.e., the percentage change in the fraction of offshorers when the cutoff changes by one per cent.

Given

\[
\varphi^* = (\omega \varphi_o) \left[ 1 + (1 - \omega)(\sigma - 1) \right]^{1/\sigma}
\]

we have

\[
\frac{d\ln \varphi^*}{d\ln \omega} = \frac{(1 - \omega) \sigma}{1 + (1 - \omega)(\sigma - 1)}
\]

Therefore

\[
\frac{d\ln s_o}{d\ln \omega} = \frac{(1 - \omega) \sigma}{1 + (1 - \omega)(\sigma - 1)} \frac{d\ln G(\varphi^*)}{d\ln \varphi^*} + (\sigma - 1) \left[ \frac{\sigma(1 - \omega)^2}{1 + (1 - \omega)(\sigma - 1)} s_o \frac{d\ln G(\varphi^*)}{d\ln \varphi^*} + (1 - s_o) \right]
\]

In the limit, this implies that at \( \omega = 0 \) the elasticity of offshorers’ market share to the quality of the contractual environment equals

\[
\frac{d\ln s_o}{d\ln \omega} = \frac{d\ln G(\varphi^*)}{d\ln \varphi^*} + (\sigma - 1) > 0
\]

since \( \varphi^* = 0 \) (all firms offshore), whereas at \( \omega = 1 \) it equals

\[
\frac{d\ln s_o}{d\ln \omega} = (\sigma - 1) (1 - s_o) = (\sigma - 1) \frac{\int_{\varphi_o}^{\infty} \varphi^{\sigma-1} dG(\varphi)}{G(\varphi_o) (\varphi_o)^{\sigma-1} + \int_{\varphi_o}^{\infty} \varphi^{\sigma-1} dG(\varphi)}
\]

since \( \varphi^* = \varphi_o \). Hence, \( d\ln s_o/d\ln \omega \) is strictly positive and finite at both \( \omega = 0 \) and \( \omega = 1 \).
Firm Heterogeneity, Contract Enforcement, and the Industry Dynamics of Offshoring

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May 13, 2009

Abstract

We develop an endogenous growth model to study the long run consequences of offshoring with firm heterogeneity and incomplete contracts. In so doing, we model offshoring as the geographical fragmentation of a firm’s production chain between a home upstream division and a foreign downstream one. On the positive side, we show that, when contracts are incomplete, the possibility of offshoring has favorable implications for economic growth. Yet, offshoring induced by a higher bargaining power of the upstream division can hamper growth: while there is always a positive correlation between upstream bargaining weight and offshoring activities, there is a non-monotonic relationship between these and growth. Whether offshoring with incomplete contracts also increases consumption depends on firm heterogeneity. On the normative side, we show that, whereas with complete contract efficiency is restored through a subsidy to R&D only, with incomplete contracts a production subsidy to offshored upstream divisions is needed too.

Keywords: offshoring, heterogeneous firms, incomplete contracts, growth, industry dynamics.


*We are indebted to Elhanan Helpman and two anonymous referees for valuable comments that helped improve the paper considerably. We are also grateful to Rubén Segura-Cayuela for very helpful suggestions.

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1 Introduction

Offshoring, along with debates and literature related to it, has enjoyed an exponential growth in recent years. In particular, the controversy on the issue exploded in February 2004 when N. Gregory Mankiw rationalized offshoring through its long term positive consequences on the US economy. He argued that offshoring may release domestic resources that can be reallocated to the creation of new products, new technologies and thus new and better jobs to replace those lost to cheaper foreign countries.\footnote{Offshoring is frequently blamed by workers and trade unions for the slow pace of job growth in the United States and for the swelling wage differential between low and high skill workers (Feenstra and Hanson, 2001).} Trade economists have since rushed to support. Blinder (2006) calls offshoring the third industrial revolution, which can eventually be a sound occurrence for all workers, as the first and the second were regardless of initial skepticism. Baldwin (2006) calls the process "a second unbundling" that has occurred as a consequence of rapidly falling communication and coordination costs. Grossman and Rossi-Hansberg (2008) argue how traditional trade theory must give way for a paradigm more relevant to today’s world, namely trade in "tasks". They show the benefits of this phenomenon by pointing out its positive impact on real wages of all workers in the home country.\footnote{In their contribution, the positive effect of offshoring on wages is driven by a productivity effect as offshoring translates into a form of technological progress. These results are qualified in Kohler (2004) and in Baldwin and Robert-Nicoud (2007), where domestic workers only benefit if the labor cost savings of offshoring are associated with the labor intensive sector.}

Finally, Rodriguez-Clare (2009) uses a dynamic model to show that the negative terms of trade effect of offshoring is outweighed by long-run gains as the origin country adjusts its research effort.

Our aim is to contribute to this debate by highlighting possible gains and losses from offshoring in an endogenous growth scenario with heterogeneous firms where the economic benefits of research and development (R&D) are not fully appropriable by innovators and some of the contracts supporting production abroad are incomplete.

We develop our argument by modeling an economy consisting of two countries, North and South, and two sectors, production and R&D. The North is the market for final products, which are horizontally differentiated. Varieties are supplied according to blueprints that are invented and patented by
R&D labs. In the wake of Grossman and Helpman (1991), endogenous growth is introduced through a positive learning externality in R&D.

To enter the production market, entrepreneurs must first purchase a patent, and then engage in process innovation with uncertain outcome to find their capacity in using the patent. In particular, an entrepreneur’s capacity is determined by a random draw from some common productivity distribution as in Melitz (2003). Each entrepreneur organizes production along a vertical chain consisting of two stages, intermediate supply ("upstream") and final assembly ("downstream"), performed by two divisions within a vertically integrated firm. Vertical integration is due to the presence of tacit knowledge that cannot be transmitted outside firm boundaries. Both R&D and final assembly are assumed to take place in North only. Intermediates can be produced in North or South. This is assumed to be a potential site for the production of intermediates using a standardized traditional technology that offers productivity gains to entrepreneurs with bad draws, provided that they are willing to bear the additional trade costs associated with international shipments.3 We call "inshoring" an organizational structure in which both production stages take place in North and "offshoring" the alternative organizational structure in which intermediates are first produced in South and then assembled in North.

The two countries differ in terms of the quality of contract enforcement between divisions. Specifically, contracts are complete when both the upstream and the downstream divisions are located in North. They are incomplete when the upstream division is located in South due to the lack of credible institutions to perfectly enforce contracts.4 We model contractual incompleteness following recent contributions that study firms’ ownership and location choices in environments in which economic

3Using Japanese firm level data from the period 1994-2000, Hijzen, Inui, and Todo (2007) give empirical evidence on how the scope for productivity improvements from offshoring depends negatively on the initial level of productivity of the firm. This in turn provides an effective channel for less productive firms to catch up and restore competitiveness.

4Nunn (2007) for instance uses several proxies to measure contract incompleteness in the South: a weighed average of a number of variables that measure individuals’ perceptions of the effectiveness and predictability of the judiciary and the enforcement of contracts in 159 countries between 1997 and 1998 from Kaufmann et al. (2003); the measures of judicial quality and contract enforcement from Gwartney and Lawson (2003) and World Bank (2004).
interactions suffer from hold up problems. More precisely, we follow Grossman and Helpman (2002) in adopting the transaction cost approach à la Williamson (1975, 1985), whose key idea is that the quality of deliverables in a bilateral transaction is unobservable by third parties so that, after the deliverables have been produced, the stakeholders involved in the transaction have to bargain on some division of the surplus it would generate. However, by assuming that upstream-downstream transactions take place within the boundaries of firms, we abstract from the ownership decision and focus, instead, on the location decision. In other words, what generates contractual incompleteness is not the crossing of firms’ boundaries but rather the crossing of countries’ borders.

This setup generates new positive and normative insights on the dynamic and static aggregate effects of offshoring. On the positive side, we show that, when contracts are incomplete, the possibility of offshoring has favorable implications for economic growth. That does not happen when contracts are complete, in which case offshoring has no impact whatsoever on growth. The key parameter regulating the growth effect of offshoring with incomplete contracts is the bargaining power of upstream divisions. In particular, we show that, while a marginal increase in the bargaining power of upstream divisions always encourages more firms to offshore, it fosters growth only if such bargaining power is initially small enough. Otherwise, offshoring activities encouraged by a stronger upstream bargaining power slows down growth. Lastly, we show that whether offshoring with incomplete contracts also favors steady state consumption depends on firm heterogeneity. For example, when productivity draws are Pareto distributed, consumption increases when there are a lot of unproductive firms and very few productive ones.

On the normative side, we highlight that, just like in Grossman and Helpman (1991), with offshoring under complete contracts the endogenous growth rate of the economy is suboptimally low due the positive learning externality in R&D. In this case, efficiency is restored through a subsidy to R&D only. This is, instead, not enough under incomplete contracts as the hold up problem causes underproduction. Accordingly, with incomplete contracts, the R&D subsidy has to be complemented

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by a production subsidy to offshored upstream divisions.⁷

To the best of our knowledge, our analysis represents the first attempt to study the long run consequences of offshoring with firm heterogeneity and incomplete contracts. A large branch of the international trade literature on firm organization has been devoted to the incomplete nature of contracts in arrangements between firms. On the dynamic side of this front, Naghavi and Ottaviano (2006, 2008, 2009) use a growth model à la Grossman and Helpman (1991) to study the potential tension that may arise between the static and dynamic implications of the fragmentation of production. They find that while outsourcing gives rise to complementary upstream and downstream innovation, incomplete contracts may prevent static gains of specialized production from carrying through in the long run. They also find that offshoring can slow growth by reducing the feedback from offshored plants to labs. Yet, in their model there is no firm heterogeneity so that, in equilibrium, firms either all outsource or they all vertically integrate.⁸ Grossman and Helpman (2004), Antràs and Helpman (2004) and Grossman, Helpman and Szeidl (2005) are among the first papers to study the organization of firms in the presence of heterogeneity in a static set up. Our contribution adds to this literature by studying the industry dynamics of firm organization, in particular the interactions between offshoring and growth.⁹ Our model also differs from previous work the organizational choices of heterogeneous firms as we do not apply the typical extra fixed cost that generally leads more productive firms to undertake a more costly form of organization. This helps us avert potential misleading assumptions as it is not clear how fixed costs can be ranked across organizational forms.¹⁰

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 charac-

⁷ This stresses a novel reason to support FDI that supplements those already highlighted in the literature. See, e.g., Barba Navaretti and Venables (2004, Ch.10).

⁸ The same is true in the Ricardian growth model by Ottaviano (2009).

⁹ The only other growth models with heterogeneous firms to our knowledge are Baldwin and Robert-Nicoud (2008) and Segerstrom and Gustafsson (2009), which explore the impact of trade liberalization on growth in the presence of heterogeneous firms. These papers, however, do not investigate the impact of contractual incompleteness.

¹⁰ For instance, Antràs and Helpman (2004) assume that fixed costs of vertical integration are larger while Grossman, Helpman and Szeidl (2005) suppose that outsourcing fixed costs are more substantial.
terizes the market equilibrium. Section 4 analyzes the interactions between offshoring, innovation and economic growth. Section 5 highlights the role of contractual incompleteness in determining the long run effects of offshoring. Section 6 concludes.

2 A Dynamic Model of Offshoring

The economy consists of two countries, North and South. We assume that all workers and consumers belong to the North but can be employed in South as expatriates to work in the offshored plants. Hence the South is simply a potential production site. This emphasizes the tacitness of knowledge by ruling out perfect substitutability of Southern labor for Northern labor, with the intention of abstracting from typical labor market debates on wages that have been widely studied empirically and are being widely studied theoretically parallel to the writing of this paper. This helps single out the additional impacts of offshoring on growth in the home country that have often been neglected in the literature. In addition, observed empirical evidence does not always approve of the phenomenon of one job shifted abroad being immediately one job released at home.

2.1 Overview and timing of events

Before getting into the details of the model, it is useful to provide a brief overview of the way it works. Consumers have CES preferences over a horizontally differentiated good $C$. The production of each variety of good $C$ requires a blueprint, an intermediate input and assembly. Blueprints are created by independent R&D labs that sell their blueprints to entrepreneurs. All labs operate in North. An entrepreneur discovers her ability to turn the acquired blueprint in a sellable product only after buying. Her ability is determined by a random productivity draw. Upon observing its productivity $\phi$, the entrepreneur organizes her firm as a vertical value chain with an "upstream" division producing the intermediate input and a "downstream" division turning it into the final product. While final production takes place in North, the firm can either "inshore" intermediate production in North,
using its own technology, or "offshore" it to South using a cost-reducing standardized technology. In this way, offshoring offers a viable alternative to the least efficient firms only. This alternative comes with strings attached. On the one hand, shipping intermediates from South to North incurs trade costs. On the other hand, offshoring takes place under contractual incompleteness, which generates further costs due to ex post bargaining.

To summarize, in each period \( t \) the following sequence of events take place. First, independent labs engage in R&D to create new patented blueprints. Second, entrepreneurs enter by purchasing a blueprint, realize their productivity levels in terms of non-standardized production and choose the location of upstream divisions. Third, upstream divisions manufacture the inputs needed by their downstream counterparts. Fourth, once intermediate production is completed, the upstream and downstream divisions of producers that have offshored bargain over the share of total revenues from final sales and inputs are handed over by the former to the latter. Lastly, final assembly takes place and final products are sold to households.

### 2.2 Demand side

There are \( L \) infinitely-lived households with identical preferences defined over the consumption of a horizontally differentiated good \( C \). The utility function is assumed to be instantaneously Cobb-Douglas and intertemporally CES with unit elasticity of intertemporal substitution:

\[
U = \int_0^{\infty} e^{-\rho t} \ln C(t) dt, \tag{1}
\]

where \( \rho > 0 \) is the rate of time preference and

\[
C(t) = \left[ \int_0^{n(t)} c(i, t^{\sigma-1}) \right]^{\frac{1}{\sigma-1}}
\]

is a CES quantity index in which \( c(i, t) \) is the consumption of variety \( i \), \( n(t) \) is the number of available varieties of good \( C \), and \( \sigma \) is the own and cross demand elasticity of any variety, and thus an inverse measure of the degree of product differentiation between varieties. Households have perfect foresight and they can borrow and lend freely in a perfect capital market at instantaneous interest rate \( R(t) \).

Using multi-stage budgeting to solve their utility maximization problem, households first allocate
their income flow between savings and expenditures. This yields a time path of total expenditures $E(t)$ that obeys the Euler equation of a standard Ramsey problem:

$$\frac{\dot{E}(t)}{E(t)} = R(t) - \rho$$

(2)

where we have used the fact that the intertemporal elasticity of substitution equals unity. By definition, $E(t) = P(t)C(t)$ where $P(t)$ is the exact price index associated with the quantity index $C(t)$:

$$P(t) \equiv \left[ \int_0^{n(t)} p(i, t)^{1-\sigma} di \right]^{1/(1-\sigma)} .$$

(3)

Households then allocate their expenditures across all varieties, which yields the instantaneous demand function

$$c(i, t) = A(t)p(i, t)^{-\sigma} \quad i \in [0, n(t)]$$

(4)

for each variety. In (4) $p(i, t)$ is the price of variety $i$ and

$$A(t) = \frac{E(t)}{P(t)^{1-\sigma}}$$

(5)

is aggregate demand. Throughout the rest of the paper, we leave the time dependence of variables implicit when this does not generate confusion.

2.3 Supply side

There are two factors of production in the economy. Labor is inelastically supplied by households and each household supplies one unit of labor so that we can use $L$ to refer both to the number of households and the total endowment of labor. Labor is freely mobile between countries and it is chosen as numeraire. The other factor is knowledge capital in the form of blueprints for the production of differentiated varieties. Blueprints are protected by infinitely lived patents and depreciate at a constant rate $\delta$.

There are two sectors, innovation (R&D) and production. Perfectly competitive labs invent blueprints for the production of the differentiated varieties. The production of each variety requires a single blueprint and consists of an upstream and a downstream stage. Entrepreneurs enter by buying
the rights to use the blueprints and split their activities between an upstream division supplying intermediates and a downstream division assembling them. Assembly takes place only in North whereas intermediate inputs can be produced also in South using an older standardized traditional technology ("offshoring"). Southern production takes place through a standardized traditional technology, which allows one unit of labor to produce $\varphi_f > 0$ units of intermediates.

Shipping the intermediate inputs back to the North for assembly incurs iceberg trade costs: $\tau > 1$ units must be shipped for one unit to reach destination. Trade costs can be embedded into the productivity parameter of the South without loss of generality. Hence, throughout the rest of the paper, we will use $\varphi_o = \varphi_f / \tau$ to denote the standard southern technology inclusive of trade costs. Northern production can rely on new advanced technologies that are generated by process innovation. This is a risky endeavor as long as its outcome is uncertain and the property rights on patents have to be bought in advance before experimenting new production processes. Specifically, after buying the rights to use the blueprints from labs, producers randomly draw their productivity level $\varphi$ from a continuous cumulative distribution $G(\varphi)$ with support $[0, \infty)$ so that offshoring offers productivity gains to producers with bad draws $\varphi < \varphi_o$. Final assembly in turn needs one unit of the intermediate component for each unit of the final good no matter where intermediates originate from. Intermediates are variety-specific: once produced for a certain assembly line, they have no alternative use.

Offshoring is associated with contractual costs that arise from weak legal institutions in the South. Specifically, only high quality variety-specific intermediates can be processed whereas low quality ones are useless even though supplied at zero cost. Contracts between the upstream and the downstream divisions are complete when both are located in North, but incomplete when the upstream division is offshored to South. In this case the quality of intermediates can not be assessed by third parties. That generates a hold up problem: after the upstream division has supplied its specific input, it has to reach an agreement with the downstream division on how to share the joint surplus (revenues) from final sales. The agreement is reached through Nash bargaining and we denote the bargaining weight of the upstream division by $\omega$. 

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Finally, we introduce endogenous growth by assuming that R&D faces a learning curve so that the marginal R&D cost of blueprints decreases with the number of blueprints that have been successfully introduced in the past. Specifically, the invention of a new blueprint requires $k/n$ units of labor where $k > 0$ is a parameter and $n$ is the total number of blueprints that have already been patented.\footnote{The assumed shape of the learning curve serves analytical solvability and the comparison with Grossman and Helpman (1991). In equilibrium it yields a 'size effect', meaning that larger countries grow faster. To avoid this prediction that runs against the empirical evidence, one could assume that the intensity of the learning spillover is lower, i.e. $k/n^{\xi}$ with $0 < \xi < 1$ (Jones, 1995). This would turn our setup into a quasi-endogenous growth model in the wake of Segerstrom (1998).} Given the chosen functional form, some initial stock of implemented blueprints $n_0 > 0$ is needed to have finite costs of innovation at all times. We assume that this stock belongs to North.

3 Market Equilibrium

3.1 Production

At time $t$ the instantaneous equilibrium is found by solving the model backwards from final production to R&D. Varieties can be sold to final customers by two types of producers: "inshorers" have both divisions in North whereas "offshorers" have their upstream divisions in South and their downstream ones in North. Under inshoring, as contracts are complete, the upstream and downstream divisions of the same firm first maximize the firm’s profit and then share it according to their bargaining weights. This implies that the upstream division of a producer with labor productivity $\varphi$ selects intermediate output $x(\varphi)$ to maximize operating profit $\pi_u(\varphi) = r_u(\varphi)/\sigma = p_u(\varphi)y_u(\varphi)/\sigma$ where $r_u(\varphi)$, $p_u(\varphi)$ and $y_u(\varphi)$ are final revenues, final price and final output (itself equal to intermediate production). Given the demand curve (4), profit maximization yields markup pricing

$$p_u(\varphi) = \frac{\sigma}{\sigma - 1} \frac{1}{\varphi}$$

with associated output $x_u(\varphi) = y_u(\varphi) = Ap_u(\varphi)^{-\sigma}$ and operating profit $\pi_u(\varphi) = r_u(\varphi)/\sigma = Ap_u(\varphi)^{1-\sigma}/\sigma$. A share $\omega$ of $\pi_u(\varphi)$ goes to the upstream division and the rest to the downstream one.
Under offshoring, the producer uses the standardized technology with upstream labor productivity $\varphi_o$ and gets the joint surplus of its divisions under incomplete contracts. This surplus is given by the revenues from final sales and is divided between divisions through ex post Nash bargaining. Absent any outside option, revenues are therefore split according to the bargaining weights of the two parties with a share $(1 - \omega)$ going to the downstream division and the remaining share $\omega$ going to the upstream one. The upstream division decides how much input $x_o$ to produce anticipating that bargaining outcome. Hence, it maximizes $\pi_u = \omega p_o y_o - x_o/\varphi_o$ where $p_o$ and $y_o$ are final price and final output (itself equal to intermediate production). Given the demand curve (4), this yields markup pricing for final sales

$$p_o = \frac{\sigma}{\sigma - 1} \frac{1}{\omega \varphi_o}$$

with associated output $x_o = y_o = A p_o^{-\sigma}$ and revenues $r_o = p_o y_o = A p_o^{1-\sigma}$. A share $\pi_d = (1 - \omega) r_o$ goes to the downstream division while the complementary share goes to the upstream one. Accordingly, after subtracting labor costs, the upstream division is left with $\pi_u = \omega r_o/\sigma$: larger upstream bargaining weight and stronger product differentiation shift a larger share of a given joint surplus $r_o$ from downstream to upstream divisions. Hence, the overall operating profit of the offshorer is $\pi_o = \pi_d + \pi_u = [1 + (\sigma - 1)(1 - \omega)] r_o/\sigma$. Since the downstream division does not contribute anything before the bargaining stage, the joint surplus $r_o$ (and the joint profit $\pi_o$ as well) is at its maximum when $\omega$ goes to one. In other words, when $\omega$ goes to one, the incomplete contract outcome converges to the complete contract one.

As producers can freely choose between inshoring and offshoring, the operating profits they earn are equal to $\pi(\varphi) \equiv \max[\pi_u(\varphi), \pi_o]$. The fact that $\pi_u(\varphi)$ is an increasing function of productivity $\varphi$ implies that there exists a unique threshold productivity level ("cutoff") $\varphi^*$ above which producers

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14 The upstream division does not face an incentive constraint as the optimal output is always positive.

15 For the upstream division the adverse incentive due to ex post bargaining under incomplete contracts has exactly the same impact as an iceberg trade cost that melts a fraction $(1 - \omega)$ of intermediate output shipped from South to North, and therefore does not generate revenues for that division. The fact that here the fraction $(1 - \omega)$ of revenues is recovered by the downstream division explains why the overall operating profit of the offshorer is larger than that of the simple iceberg case.
prefer to inshore. This cutoff solves \( \pi_v(\varphi^*) = \pi_o \) and is therefore equal to

\[
\varphi^* = (\omega \varphi_o) \left[ 1 + (1 - \omega)(\sigma - 1) \right]
\] (6)

The cutoff is decreasing in \( \sigma \) because weaker product differentiation shifts surplus from upstream to downstream divisions exacerbating intermediate underproduction and thus promoting inshoring. For symmetric reasons, the cutoff is increasing in the upstream bargaining weight \( \omega \). It is also increasing in \( \varphi_o \) as offshoring is fostered by any improvement in the productivity of the standardized technology \( \varphi_o \) or any fall in trade cost \( \tau \).

We can therefore highlight:

**Proposition 1** A marginal increase in the bargaining weight of upstream divisions encourages more firms to offshore.

Since \( 1/(\omega \varphi_o) \) is the amount of labor embedded in unit revenues, it will turn out to be useful to denote by \( \tilde{\varphi}_o \equiv \omega \varphi_o \) the "delivered" productivity of offshored labor. We will call this simply "offshored productivity" and we will contrast it with producer-specific "inshored productivity" \( \varphi \). Note that (6) shows that a marginal producer drawing exactly \( \varphi^* \) has higher inshored than offshored productivity \( (\varphi^* > \tilde{\varphi}_o) \) so that the range \((\tilde{\varphi}_o, \varphi^*)\) identifies producers whose decisions to offshore reduce aggregate productivity. Moreover, due to \( \partial \varphi^*/\partial \omega > 0 \), the cutoff \( \varphi^* \) achieves its maximum value \( \varphi_o \) at \( \omega \) equal to 1, so any other value of \( \omega \) implies \( \varphi^* < \varphi_o \). Hence, incomplete contracts generate two adverse effects of offshoring on aggregate productivity. First, firms drawing values of \( \varphi \) between \( \tilde{\varphi}_o \) and \( \varphi^* \) offshore while they have higher inshored productivity. Second, firms drawing values between \( \varphi^* \) and \( \varphi_o \) do not offshore whereas in the absence of contractual frictions doing so would increase their productivity. In Figure 1 we call these adverse effects **penalization 1** and **penalization 2** respectively. Indeed, when \( \omega \) is equal to 1, (6) implies \( \varphi^* = \tilde{\varphi}_o = \varphi_o \) so that all firms with a productivity level \( \varphi < \varphi_o \) offshore and their decision to do so improves aggregate productivity.

To summarize, producers’ organizational choices give the following cutoff results for prices and
overall profits:

\[
p(\varphi) = \begin{cases} 
\frac{1}{\varphi^{\frac{1}{\sigma - 1}}} & \text{and } \pi(\varphi) = \begin{cases} 
\frac{1+(\sigma-1)(1-\omega)}{\sigma}A p(\varphi)^{1-\sigma} \\
\frac{1}{\sigma}A p(\varphi)^{1-\sigma}
\end{cases} 
\end{cases}
\text{ for } \varphi \in \left[0, \varphi^*\right) \cup \left[\varphi^*, +\infty\right)
\]

3.2 Innovation

At the innovation stage, labs invent new blueprints at a marginal cost that depends on acquired experience \(k/n\) and their output determines the law of motion of \(n\). In particular, we have

\[
\dot{n} = \frac{nL^I}{k} - \delta n,
\]

where \(\dot{n} \equiv dn/dt\), \(L^I\) is labor employed in inventing new blueprints, \(n/k\) is its productivity and \(\delta\) is the rate of depreciation.

Due to learning, as innovation cumulates, it becomes increasingly cheaper to introduce new blueprints and, being priced at marginal cost, their value falls through time. Specifically, if we call \(J\) the asset value of a patented blueprint, marginal cost pricing gives \(J = k/n\), which implies \(\dot{J}/J = -\dot{n}/n\).

Labs pay their researchers by borrowing at the interest rate \(R\) and know that the resulting patents will generate instantaneous dividends equal to the expected profits of the corresponding producers \(\pi\). Arbitrage in the capital market then requires the dividends \(\pi\) and capital gains \(\dot{J}\) to match interest payments \(RJ\) and depreciation \(\delta J\) so that:

\[
R + \delta = \frac{\pi n}{k} - \frac{\dot{n}}{n}
\]

where the equality is granted by the definition of \(J\).

3.3 Aggregation

In characterizing the aggregate behavior of our heterogeneous economy, we follow Melitz (2003) and define average (output-weighted) productivity as:

\[
\bar{\varphi} \equiv \left\{G(\varphi^* )\bar{\varphi}_o^{-1} + \left[1 - G(\varphi^* )\right]\bar{\varphi}_o^{-1}\right\}^{\frac{1}{\sigma - 1}}
\]
where, as already mentioned, $\bar{\varphi}_o \equiv \omega \varphi_o$ is the common productivity of offshorers and

$$
\bar{\varphi}_v = \left[ \frac{1}{1 - G(\varphi^*)} \int_{\varphi^*}^{\infty} \varphi^{\sigma - 1} dG(\varphi) \right]^{\frac{1}{\sigma - 1}}
$$

is the average (output weighted) productivity of inshorers. Since $\varphi^* > \bar{\varphi}_o$, we have $\bar{\varphi}_v > \varphi^* > \bar{\varphi}_o$, $\bar{\varphi}_v > \bar{\varphi}$, and $d\bar{\varphi}_v/d\varphi^* > 0$. Figure 1 shows a ranking of the productivity levels.\(^{16}\)

Figure 1: Industry partition

We also define $E_o$ as the share of expenditures going to offshorers, and $P_o$ and $C_o$ as the corresponding exact price and quantity indices such that $P_o C_o = E_o$. Analogously, we define $E_v$ as the share of expenditures going to inshorers, and $P_v$ and $C_v$ as the corresponding exact price and quantity indices such that $P_v C_v = E_v$ and $E_o + E_v = E$. Then we have:

$$
\tilde{p}_v = \frac{\varphi^*}{\sigma - 1 \bar{\varphi}_v}, P_v = \left\{n [1 - G(\varphi^*)] \right\}^{\frac{1}{\sigma - 1}} \tilde{p}_v, C_v = E_v \frac{P_v}{P_o}, E_v = \left( \frac{P_v}{P_o} \right)^{1 - \sigma} E \quad (11)
$$

where $\tilde{p}_v$ is the average price of inshorers. Analogously, we can write

$$
p_o = \frac{\bar{\varphi}_o}{\sigma - 1 \bar{\varphi}_o}, P_o = \left\{n G(\varphi^*) \right\}^{\frac{1}{\sigma - 1}} p_o, C_o = E_o \frac{P_o}{P_o}, E_o = \left( \frac{P_o}{P} \right)^{1 - \sigma} E
$$

and

$$
\tilde{p} = \frac{\bar{\varphi}}{\sigma - 1 \bar{\varphi}}, P = \left\{n G(\varphi^*) \bar{p}^1 - \sigma + n [1 - G(\varphi^*)] \tilde{p}^1 - \sigma \right\}^{\frac{1}{\sigma - 1}}, C = \frac{E}{P}
$$

\(^{16}\)While $\bar{\varphi}$ is larger than $\varphi^*$ in Figure 1, this is not always necessarily the case.
3.4 Financial market clearing

Since producers discover their productivity only after acquiring the right to use a patented blueprint, the dividends they are willing to pay to labs equal their expected operating profits

$$\pi = G(\phi^*) \pi_o + [1 - G(\phi^*)] \tilde{\pi}_v$$

(12)

where, since $$r_o = E_o / [nG(\phi^*)]$$, the operating profit of a typical offshorer $$\pi_o$$ can be rewritten in terms of aggregate variables as

$$\pi_o = \frac{[1 + (\sigma - 1) (1 - \omega)] E_o}{\sigma n G(\phi^*)}$$

and, by definition, the average operating profit of inshorers equals

$$\tilde{\pi}_v = \frac{E_v}{\sigma n[1 - G(\phi^*)]}$$

By (11), expected operating profits (12) simplify to

$$\pi = \frac{E}{n} \left[ \frac{1}{\sigma} + \frac{\sigma - 1}{\sigma} \Omega \right]$$

(13)

where $$\Omega \equiv (1 - \omega) E_o / E$$ is the share of aggregate expenditures accruing to the downstream divisions of offshorers. Expression (13) shows that expected profits are an increasing function of $$\Omega$$. When a higher share of expenditures in the economy goes to offshorers, expected profits are larger in the industry. To see this, note that keeping fixed total expenditure $$E$$ and the proportion of offshoring firms $$G(\phi^*)$$, shifting a unit of expenditures from inshorers $$E_v$$ to offshorers $$E_o$$ increases average profits because the downstream offshored divisions earn a share $$1 - \omega$$ of revenues $$r_o$$ while the inshored ones only earn a share $$1 - \omega$$ of profits $$\pi_v = r_v / \sigma$$. This also explains why the positive impact of $$\Omega$$ on $$\pi$$ is larger when $$\sigma$$ is larger. Larger downstream bargaining power $$(1 - \omega)$$ also increases expected profits at the time of entry.

Once substituted into (9), expression (13) allows us to restate the Euler condition (2) as:

$$\frac{E_v}{E} = \frac{E}{\sigma E} [1 + (\sigma - 1) \Omega] - \frac{\dot{n}}{\bar{n}} - \delta - \rho$$

(14)

17 Inshores have a higher sensitivity to the elasticity of substitution as with offshoring all variable costs of producing intermediates are passed over to upstream suppliers. See Grossman and Helpman (2002, p. 102) for more detail.
3.5 Labor market clearing

Aggregate labor endowment $L$ is absorbed by innovation ($L_I$) as well as by inshored and offshored upstream production. Inshorers’ and offshorers’ employment levels amount to $L_v = E_v/(p_v \bar{\varphi}_v)$ and $L_o = E_o/(p_o \varphi_o)$ respectively. Accordingly, given (11), total employment in upstream production simplifies to

$$L_v + L_o = \frac{\sigma - 1}{\sigma} E(1 - \Omega)$$

which, together with (8), allows us to rewrite the labor market clearing condition $L = L_I + L_v + L_o$ as

$$L = k \left( \frac{\dot{n}}{n} + \delta \right) + \frac{\sigma - 1}{\sigma} E(1 - \Omega)$$

Employment in production is a decreasing function of the share $\Omega$ of aggregate expenditures accruing to the downstream divisions of offshorers. This is the dual of the previously discussed result that expected profits increase with $\Omega$ as long as larger expected profits induce a reallocation of labor from production to R&D.

4 Offshoring and Growth

The market clearing conditions (14) and (15) define a dynamic system in two unknowns: the growth rate of the stock of patents ($\dot{n}/n$) and the expenditures level ($E$). A unique balanced growth path exists along which these variables are constant and is achieved without any transition dynamics.\(^{18}\) Calling the corresponding growth rate and expenditures level by $g_s$ and $E_s$ respectively, then imposing $\dot{n}/n = g_s$, $E = E_s$ and $\dot{E} = 0$ in (14) and (15) allows us to find:

$$g_s = L \left( \frac{1}{\sigma} + \frac{\sigma - 1}{\sigma} \Omega \right) - \frac{\sigma - 1}{\sigma} (1 - \Omega) \rho - \delta, \quad E_s = L + \rho k$$

While expenditures $E_s$ do not depend on $\Omega$, the growth rate $g_s$ is instead an increasing function of $\Omega$. The reason is that, by definition, a rise in $\Omega$ shifts expenditures from inshorers to offshorers. This shift, as discussed above, generates larger expected profits and smaller employment in production.

The resulting reallocation of labor from production to R&D promotes innovation and growth.

\(^{18}\)See Grossman and Helpman (1991, ch.3) for details.
Since $E_s$ does not depend on $\Omega$, the bargaining weight $\omega$ does not affect expenditures. It affects, however, the growth rate through various channels funneled through the impact of $\Omega$ on $g_s$. To disentangle these channels, we use (10) and (11) to rewrite the share $\Omega$ of expenditures accruing to the downstream divisions of offshorers as

$$\Omega = (1 - \omega) \frac{E_o}{E} = (1 - \omega) \frac{P_o}{P} \left( \frac{\bar{\varphi}_o}{\bar{\varphi}} \right)^{1-\sigma} = (1 - \omega) \frac{G(\varphi^*)}{G(\bar{\varphi})} \left( \frac{\bar{\varphi}_o}{\bar{\varphi}} \right)^{\sigma-1}$$

where $\bar{\varphi}_o \equiv \omega \bar{\varphi}_o$ is offshored productivity and $\bar{\varphi}$ is the average (offshored and inshored) productivity as defined in (10). Since $dg_s/d\Omega > 0$, the sign of the impact of $\omega$ on $g_s$ depends on the sign of $d\Omega/d\omega$. This can be decomposed as:

$$\frac{d\Omega}{d\omega} = -s_o + (1 - \omega) \frac{ds_o}{d\omega}$$

(17)

where $s_o \equiv E_o/E$.

Consider a marginal increase in $\omega$. The first term on the right hand side is the direct effect of larger $\omega$. It is negative as it identifies the corresponding fall in the share of expenditures accruing to the downstream divisions of offshorers holding the overall share of expenditures accruing to offshorers constant. It captures a pure surplus reallocation between divisions as a higher upstream bargaining weight transfers surplus from downstream to offshored upstream divisions.

The second term on the right hand side of (17) is the indirect effect. It identifies the change in the overall share of expenditures accruing to offshorers. This adjustment takes place along two margins: the relative number of offshorers as determined by $G(\varphi^*)$ ("extensive margin") and their relative size with respect to the average producer $r_o/\bar{r} = (\bar{\varphi}_o/\bar{\varphi})^{\sigma-1}$ ("intensive margin"):

$$\frac{ds_o}{d\omega} = \frac{dG(\varphi^*)}{d\omega} \left( \frac{r_o/\bar{r}}{G(\varphi^*)} \right) \frac{d(r_o/\bar{r})}{d\omega},$$

where $\bar{r} = A \bar{\varphi}_o^{1-\sigma}$. The impact of larger $\omega$ is positive on both margins. Since a larger bargaining weight of upstream divisions alleviates their underproduction of intermediates, as $\omega$ rises not only more producers decide to offshore, but also offshorers become larger. Along the extensive margin, by (6) we have $d\varphi^*/d\omega > 0$ and thus $dG(\varphi^*)/d\omega > 0$. Along the intensive margin, using $\bar{\varphi}_o \equiv \omega \bar{\varphi}_o$
and (10), we have

\[
\frac{r_\omega}{\bar{r}} = \left( \frac{\varphi_\omega}{\varphi} \right)^{\sigma-1} \frac{\varphi_\omega^{\sigma-1}}{G(\varphi^*) \varphi_\omega^{\sigma-1} + [1 - G(\varphi^*)] \varphi^{\sigma-1}} = \frac{(\omega \varphi_\omega)^{\sigma-1}}{G(\varphi^*) \varphi_\omega^{\sigma-1} + \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} dG(\varphi)} \tag{18}
\]

which, given \(d\varphi^*/d\omega > 0\) and \(dG(\varphi^*)/d\varphi^* > 0\), is an increasing function of \(\omega\) (see Appendix 1 for a proof that \(\vartheta(r_\omega/\bar{r})/\vartheta \omega > 0\)). Hence, we can conclude that \(ds_\omega/d\omega > 0\), which, given (17), implies that improved contract enforcement in the South has ambiguous effects on \(\Omega\) and, therefore, on \(g_\omega\).

To shed light on such ambiguity, we can manipulate (17) to show that a higher bargaining weight of upstream divisions promotes growth when the elasticity of the offshorers’ market share \(s_\omega\) to \(\varphi\) is larger than the ratio between their upstream and downstream divisions’ bargaining weights, i.e.

\[
\frac{d \ln s_\omega}{d \ln \omega} > \frac{\omega}{1 - \omega} \tag{19}
\]

If the reverse is true, a higher bargaining weight of upstream divisions hampers growth. Figure 1 shows the effect of an increase in \(\omega\) on the productivity distribution of firms graphically. Larger \(\omega\) directly raises \(\varphi_\omega\), while it reduces \([1 - G(\varphi^*)] \varphi_\omega\) through a change in \(\varphi^*\). This makes the change in average productivity \(\bar{\varphi}\) ambiguous in \(\omega\), yet inferior to the rise in \(\varphi_\omega\). Also, the productivity range along which offshoring raises aggregate productivity increases if the change in the intensive margin is larger than that in the extensive margin \((\vartheta \varphi_\omega/\vartheta \omega > \vartheta \varphi^*/\vartheta \omega)\) so that the two values converge, and falls if the opposite holds so that they diverge. Finally, note that the values achieved by \(d \ln s_\omega/d \ln \omega\) at \(\omega = 0\) and at \(\omega = 1\) are both strictly positive and finite provided that the elasticity of the extensive margin \(d \ln G(\varphi^*)/d \ln \varphi^*\) is also positive and finite, as in the case of all the commonly used families of cumulative density functions (see Appendix 2 for a proof). Then, since \(\omega/(1 - \omega)\) equals zero at \(\omega = 0\) and goes to infinity when \(\omega\) goes to one, there must exist a threshold value of \(\omega\) below which the inequality (19) holds and above which it is violated. Hence, there exists a unique threshold value for the bargaining weight of upstream divisions \(\omega\) such that \(d\Omega/d\omega > (>)0\) if and only if \(\omega\) falls below (above) that value.

Hence, we can state:

**Proposition 2** A marginal increase in the bargaining weight of upstream divisions fosters growth if it is initially small and hampers growth if it is initially large enough.
To sum up, Proposition 1 tells us that a higher bargaining weight of upstream divisions unambiguously promotes offshoring. Proposition 2 tells us that this rise of offshoring stimulates growth as long as that bargaining weight is not too large. The reason is that a higher bargaining weight \( \omega \) increases the share of expenditures accruing to offshorers through two positive indirect effects on their relative size ("intensive margin") and number ("extensive margin"). On the other hand, a higher \( \omega \) has a negative direct effect on the fraction of offshorers' revenues appropriated by their downstream divisions. While the indirect effects boost offshorers' profitability and thus growth by reallocating labor from production to R&D, the direct effect works in the opposite direction. This effect comes to dominate when \( \omega \) is large enough, generating a non-linear relation between the bargaining weight of upstream divisions and the growth rate.

5 Contracts and Welfare

The aim of this section is twofold. On the one hand, we want to clarify how our results depend on the quality of contract enforcement. In so doing, we first characterize the steady state outcome when offshoring takes place under complete contracts. We then use this characterization to unveil the different performance of the economy when the transition from no offshoring to offshoring happens under incomplete rather than complete contracts. This will also highlight the role of firm heterogeneity. On the other hand, we want to highlight how incomplete contracts lead to different policy implications with respect to complete contracts.

5.1 Complete contracts

To better understand the role of contractual incompleteness for our results, it is useful to characterize the steady state outcome of the model in two scenarios: one in which offshoring is inhibited (so that the hold up problem is not an issue) and the other in which offshoring takes place under complete contracts.

In both cases at the aggregate level the model with heterogeneous firms is homomorphic to a model with homogeneous firms à la Grossman and Helpman (1991) in which all firms are identical to
the average heterogeneous firm. Accordingly, average productivity (and ultimately the productivity cutoff) provides a sufficient statistic to describe the aggregate behavior of the economy. When offshoring is inhibited, average productivity is
\[
\tilde{\varphi}_h = \left[ \int_0^\infty \varphi^{\sigma-1} dG(\varphi) \right]^{\frac{1}{\sigma-1}}
\]
due to the fact that, by definition, no entrepreneurs relies on the foreign standardized technology.

On the other hand, when offshoring is allowed for and contracts are complete, average productivity is equal to
\[
\tilde{\varphi}_c = \{ G(\varphi^*_c)\varphi^*_{\sigma-1} + [1 - G(\varphi^*_c)] \tilde{\varphi}^{\sigma-1}_{vc} \}^{\frac{1}{\sigma-1}}
\]
where from (6) we have \( \varphi^*_e = \varphi_o > \varphi^* > \tilde{\varphi}_o \) as the productivity cutoff under contractual completeness, and \( \tilde{\varphi}_{vc} \) is the corresponding average productivity of inshorers
\[
\tilde{\varphi}_{vc} = \left[ \frac{1}{1 - G(\varphi^*_c)} \int_{\varphi^*_c}^\infty \varphi^{\sigma-1} dG(\varphi) \right]^{\frac{1}{\sigma-1}}
\]

In both case, as in Grossman and Helpman (1991), efficiency in production is immaterial for steady state growth and expenditures because it conveys equal incentives to production and innovation. As a result, steady state growth and expenditures are the same in the two complete contract scenarios:
\[
g_h = g_c = \frac{1}{\sigma k} \frac{\sigma - 1}{\sigma} \rho - \delta \quad \text{and} \quad E_h = E_c = E_s = L + \rho k \quad (21)
\]

In addition, steady state expenditures are also the same as those with incomplete contracts.

Average productivity affects, instead, the aggregate quantities consumed in the various scenarios as these quantities are given by expenditures divided by the corresponding average prices
\[
D_h = \frac{\sigma - 1}{\sigma} \tilde{\varphi}_h (L + \rho k), \quad D_c = \frac{\sigma - 1}{\sigma} \tilde{\varphi}_c (L + \rho k), \quad D_s = \frac{\sigma - 1}{\sigma} \tilde{\varphi}_s (L + \rho k) \quad (22)
\]
where, for notational symmetry, we use \( \tilde{\varphi}_s \) to relabel the average productivity (10) that prevails with incomplete offshoring contracts.

Comparing the two complete contract scenarios, we see that the fact that less efficient firms offshore for \( \varphi < \varphi_o \) implies \( \tilde{\varphi}_c > \tilde{\varphi}_h \) and thus \( D_c > D_h \). Hence,
Proposition 3 Making offshoring possible under contractual completeness generates a static welfare gain due to more production but no dynamic welfare effect through a change in the growth rate.

The reason for this is that offshoring operates as "neutral" technological progress that evenly augments both the profitability of R&D and the profitability of production. As a result, when firms offshore there is no reallocation of labor between R&D and production. All that happens is that more efficient labor generates more output.

5.2 Incomplete contracts

We can now compare the steady state outcomes with or without incomplete contracts. As for growth rates, expressions (16) and (21) show that offshoring with incomplete contracts leads to faster growth with respect to both no offshoring and offshoring under complete contracts, the more so the larger is $\Omega$. In particular, we have

$$g_s - g_n = \sigma \frac{1}{\sigma} \Omega \left( \frac{L}{k + \rho} \right) > 0$$

Hence:

Proposition 4 Making offshoring possible under contractual incompleteness generates dynamic welfare gains due to faster growth.

The reason for this is that offshoring operates as "biased" technological progress that augments the profitability of R&D more than the profitability of production. As a result, when firms offshore there is a reallocation of labor from production to R&D.

Turning to consumption, we have shown in Section 3.1 that incomplete contracts generate two adverse effects of offshoring on aggregate productivity. First, firms drawing values of $\varphi$ between $\varphi_o$ and $\varphi^*$ offshore while they have higher inshored productivity. This does not happen under complete contracts for which we have $\varphi_o = \varphi^*$. Second, firms drawing values between $\varphi^*$ and $\varphi_o$ do not offshore whereas in the absence of contractual frictions doing so would increase their productivity. These adverse effects imply $\tilde{\varphi}_s < \tilde{\varphi}_c$ and, therefore, $D_s < D_c$ so that with offshoring output is always higher under complete than incomplete contracts.
Differently, when it comes to comparing offshoring under contractual incompleteness and no offshoring only the first adverse effect is relevant as firms with productivity levels between \( \varphi^* \) and \( \varphi_o \) do not offshore in either case. The ranking in terms of output is therefore ambiguous. In particular, we have \( \tilde{\varphi}_a > \tilde{\varphi}_h \) if and only if

\[
\int_{0}^{\tilde{\varphi}_o} \left( \tilde{\varphi}_o^{\sigma-1} - \varphi^{\sigma-1} \right) dG(\varphi) > \int_{\tilde{\varphi}_o}^{\varphi^*} \left( \varphi^{\sigma-1} - \tilde{\varphi}_o^{\sigma-1} \right) dG(\varphi)
\]

that is if the cumulated productivity gain of offshores that are less productive under inshoring (\( 0 < \varphi < \tilde{\varphi}_o \)) is larger than the cumulated productivity loss for offshorers that are more productive under inshoring (\( \tilde{\varphi}_o < \varphi < \varphi^* \)).

These results allow us to state the following:

**Proposition 5** Making offshoring possible under contractual incompleteness generates static welfare gains if the cumulated productivity gains of offshores that would be less productive under inshoring are larger than the cumulated productivity losses of offshorers that would be more productive under inshoring.

Whether this is indeed the case or not clearly depends on the way \( G(\varphi) \) distributes the productivity draws \( \varphi \) between the two intervals \((0, \tilde{\varphi}_o)\) and \((\tilde{\varphi}_o, \varphi^*)\). This highlights the crucial role of heterogeneity. For example, if \( \varphi \) is Pareto distributed with \( G(\varphi) = 1 - (\varphi_M)^{k} \varphi^{-k} \), then \( \tilde{\varphi}_a > \tilde{\varphi}_h \) whenever \( k < (\sigma - 1)/(2 + (1 - \omega)(\sigma - 1)) \). In other words, with incomplete contracts offshoring also generates static gains when there are a lot of unproductive firms and few productive ones.

### 5.3 Optimal policy

We are now ready to investigate how incomplete contracts lead to different policy implications with respect to complete contracts when the objective is to implement an efficient outcome maximizing (1) subject to the aggregate resource constraint without any hold up problem. The efficient outcome is readily characterized by remembering again that, with complete contracts, at the aggregate level the model with heterogeneous firms is homomorphic to a model with homogeneous firms in which
all firms are identical to the average heterogeneous firm. We can, therefore, invoke the results in Grossman and Helpman (1991) to assert that the efficient steady state is characterized by

$$g_w = \frac{L}{\eta} - (\sigma - 1) \rho - \delta, \ E_w = E_h = E_c = E_s$$

(23)

with quantity consumed $D_w = D_c$.

The comparison between $g_s$ and $g_w$ shows that, as in Grossman and Helpman (1991), due to the positive learning externality in R&D, at the market outcome with complete contracts our economy grows too slowly. In this case, optimal intervention then requires an R&D subsidy that equalizes $g_h = g_c$ to $g_w$. Specifically, if we call $\phi^*_w$ the optimal fraction of R&D expenditures paid by the government, such fraction satisfies

$$\phi^*_w = \frac{g_w + \delta}{g_w + \delta + \rho}$$

With incomplete contracts the R&D subsidy alone is not enough due to the underproduction of the intermediate input and, therefore, of the final output. Optimal intervention here requires the government to subsidize also offshorers’ upstream production. If we call $\phi^p_w$ the optimal fraction of offshorers’ upstream production costs paid by the government, then we have

$$\phi^p_w = 1 - \omega$$

Hence, both R&D and production subsidies are needed to implement the first best under incomplete contracts.

To summarize, we can write:

**Proposition 6** With complete contracts welfare is maximized through a subsidy to innovation only.

With incomplete contracts welfare maximization also requires a subsidy to offshored production.

## 6 Conclusion

We have used an endogenous growth model of North-South offshoring with heterogeneous firms to study its dynamic and static effects on the economy when contracts are incomplete in the South.
In so doing, we have modelled offshoring as the geographical fragmentation of a firm’s production chain between a home upstream division and a foreign downstream one.

On the positive side, we have shown that, when contracts are incomplete, the possibility of offshoring may have favorable implications for economic growth. The key parameter regulating the growth effect of offshoring is the bargaining power of the upstream division through a non-linear relation. While a larger upstream bargaining weight unambiguously promotes offshoring, it (hence increased offshoring) only stimulates growth up to a critical level. Under complete contracts, offshoring has no impact whatsoever on growth. The reason for this is that under complete contracts, offshoring evenly augments the profitability of R&D and production, whereas with incomplete contracts the gains in profitability are biased towards R&D.

Whether offshoring with incomplete contracts also increases consumption depends on firm heterogeneity. For example, when productivity draws are Pareto distributed, consumption increases when there are a lot of unproductive firms and very few productive ones.

On the normative side, we have show that, whereas with complete contracts efficiency can be restored through a subsidy to R&D only, with incomplete contracts a production subsidy to offshored upstream divisions is needed too.

Contrary to the existing literature, our study uses the industry dynamics of firm organization to reveal the possibility of adverse long term effects of offshoring for the North. In addition, it emphasizes the role of firm heterogeneity for the social gains from offshoring. This raises the question whether analyses on the consequences of offshoring based on real wages can fully absorb the mechanisms through which it influences the economy performance. Our analysis has its limitation and leaves much work for future research on the issue.

References


7 Appendix

7.1 Stronger contract enforcement in the South increases the relative size of offshorers

The cutoff $\varphi^*$ is an increasing function of $\omega$:

$$\frac{d\varphi^*}{d\omega} = \frac{d}{d\omega} \left( \frac{(\omega \varphi_0) (1 + (1 - \omega)(\sigma - 1))}{\varphi_0} \right) = (1 - \omega) \sigma \varphi_0 (1 + (1 - \omega)(\sigma - 1)) \frac{\varphi^*}{\varphi_0} > 0$$
Since $\varphi^*$ is an increasing function of $\omega$, the average productivity of inshorers $\bar{\varphi}_o$ is a decreasing function of $\omega$:

$$
\frac{d\bar{\varphi}_o}{d\omega} = -\frac{1}{\sigma - 1} \left[ \frac{1}{1 - G(\varphi^*)} \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} dG(\varphi) \right]^{\frac{1}{\sigma-1}} \left( \varphi^* \right)^{\sigma - 1} G'(\varphi^*) \frac{d\varphi^*}{d\omega} < 0
$$

where

$$
\frac{d}{d\varphi^*} \left[ \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} dG(\varphi) \right] = -\frac{d}{d\varphi^*} \left[ \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} G'(\varphi) d\varphi \right] = -\left( \varphi^* \right)^{\sigma - 1} G'(\varphi^*)
$$

is granted by the fundamental theorem of calculus. Moreover, since $\varphi^*$ is an increasing function of $\omega$, the relative size of offshorers with respect to the average producer $r_o/\bar{r}$ is an increasing function of $\omega$:

$$
\frac{d (r_o/\bar{r})}{d\omega} = \frac{d}{d\varphi^*} \left[ \left( \omega \varphi_o \right)^{\sigma-1} + \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} dG(\varphi) \right] / \omega \left( \omega \varphi_o \right)^{\sigma-1} + \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} dG(\varphi) = \text{num}
$$

$$
\text{num} = (\omega \varphi_o)^{\sigma-1} \left\{ \frac{\sigma - 1}{\omega} \left[ \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} dG(\varphi) \right] - \left[ (\omega \varphi_o)^{\sigma-1} - \left( \varphi^* \right)^{\sigma - 1} \right] G'(\varphi^*) \frac{d\varphi^*}{d\omega} \right\}
$$

With

$$
\varphi^* = (\omega \varphi_o) \left[ 1 + (1 - \omega)(\sigma - 1) \right]^{\frac{1}{\sigma - 1}}
$$

we have

$$
\text{num} = (\omega \varphi_o)^{\sigma-1} (\sigma - 1) \left\{ \frac{1}{\omega} \left[ \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} dG(\varphi) \right] + (1 - \omega) (\omega \varphi_o)^{\sigma-1} G'(\varphi^*) \frac{d\varphi^*}{d\omega} \right\}
$$

which is positive since $d\varphi^*/d\omega > 0$.

Hence:

$$
\frac{d (r_o/\bar{r})}{d\omega} = \frac{\text{num}}{\left[ (\omega \varphi_o)^{\sigma-1} + \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} dG(\varphi) \right]^2} > 0
$$

### 7.2 Properties of the elasticity of offshorers’ market share to the quality of the contractual environment

Recall the definition

$$
s_o \equiv \frac{E_o}{E} = G(\varphi^* \left( \frac{\bar{\varphi}_o}{\varphi} \right)^{\sigma-1} = \frac{G(\varphi^*)}{G(\varphi^*) \bar{\varphi}_o^{\sigma-1} + \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} dG(\varphi)} = \frac{G(\varphi^*) (\omega \varphi_o)^{\sigma-1} + \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} dG(\varphi)}{G(\varphi^*) (\omega \varphi_o)^{\sigma-1} + \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} dG(\varphi)}
$$
Then the elasticity of offshorers’ market share $s_o$ to the quality of the contractual environment $\omega$ evaluates to

$$\frac{d \ln s_o}{d \ln \omega} = \frac{d \ln G(\varphi^*)}{d \ln \omega} + \frac{d \ln (r_o/\bar{r})}{d \ln \omega}$$

with

$$\frac{d \ln (r_o/\bar{r})}{d \ln \omega} = (\sigma - 1) \left[ \int_{\varphi_o}^{\infty} \varphi^{\sigma-1} dG(\varphi) + (1 - \omega) (\omega \varphi_o)^{\sigma-1} \frac{d \ln G(\varphi^*)}{d \ln \omega} G(\varphi^*) \right]$$

$$\frac{d \ln \omega}{d \ln \varphi^*} \left[ \int_{\varphi_o}^{\infty} \varphi^{\sigma-1} dG(\varphi) \right]$$

as derived in the previous section. Accordingly

$$\frac{d \ln s_o}{d \ln \varphi^*} = \left[ 1 + (\sigma - 1) (1 - \omega) s_o \right] \frac{d \ln G(\varphi^*)}{d \ln \omega} + (\sigma - 1) (1 - s_o)$$

$$\frac{d \ln \varphi^*}{d \ln \omega} = \frac{(1 - \omega) \sigma}{1 + (1 - \omega)(\sigma - 1)}$$

Therefore

$$\frac{d \ln s_o}{d \ln \omega} = \frac{(1 - \omega) \sigma}{1 + (1 - \omega)(\sigma - 1)} + (\sigma - 1) \left[ \frac{\sigma (1 - \omega)^2}{1 + (1 - \omega)(\sigma - 1)} s_o \frac{d \ln G(\varphi^*)}{d \ln \varphi^*} + (1 - s_o) \right]$$

In the limit, this implies that at $\omega = 0$ the elasticity of offshorers’ market share to the quality of the contractual environment equals

$$\frac{d \ln s_o}{d \ln \omega} = \frac{d \ln G(\varphi^*)}{d \ln \varphi^*} + (\sigma - 1) > 0$$

since $\varphi^* = 0$ (all firms offshore), whereas at $\omega = 1$ it equals

$$\frac{d \ln s_o}{d \ln \omega} = (\sigma - 1) (1 - s_o) = (\sigma - 1) \left[ \int_{\varphi_o}^{\infty} \varphi^{\sigma-1} dG(\varphi) 
\frac{G(\varphi_o)}{(\varphi_o)^{\sigma-1} + \int_{\varphi_o}^{\infty} \varphi^{\sigma-1} dG(\varphi)} \right]$$

since $\varphi^* = \varphi_o$. Hence, $d \ln s_o/d \ln \omega$ is strictly positive and finite at both $\omega = 0$ and $\omega = 1$. 

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(lxxxv) This paper has been presented at the 14th Coalition Theory Network Workshop held in Maastricht, The Netherlands, on 23-24 January 2009 and organised by the Maastricht University CTN group (Department of Economics, http://www.feem-web.it/ctn/12d_maa.php).