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Summary  
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Keywords: Concessions, Auctions, Award criteria, Service Rollout Time limits  

JEL Classification: L51, D44, D92  

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Auctioning Monopoly Franchises: Award Criteria and Service Launch Requirements

Cesare Dosi* and Michele Moretto†

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Abstract

We study the competition to acquire the exclusive right to operate an infrastructure service, by comparing two different specifications for the financial proposals - "lowest price to consumers" vs "highest concession fee", and two alternative contractual arrangements: a contract which imposes the obligation to immediately undertake the investment required to operate the concessioned service and a contract which simply assigns to the winning bidder the right to supply the market at a date of her choosing. By comparing the returns of these alternative award criteria and concessioning conditions, we show that concessioning without imposing rollout time limits may or may not provide a higher expected social value, depending on the bidding rule used to allocate the contract. In turn, the relative advantages of each award criterion are affected by the concessioning conditions.

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1 Introduction

"Over the past decades, governments have increasingly turned towards concessions as a way to raise funds and to improve services by applying private-sector expertise to investment, management and operation of infrastructure" (OECD, 2007, p.15).

The term "concession" has different meanings in different countries. Generally speaking, the term can be used to refer "to any arrangement in which a firm obtains the right to provide a particular service under conditions of significant market power" (World Bank and Inter-American Development Bank, 1998, p.10).

This broad economic definition, used throughout the paper, allows to include a wide range of legal arrangements concerning the nature and extent of the risk governments transfer to the franchisee, duration, exclusivity, service obligations, investment responsibilities, ownership of assets, and so on.

One important difference between contracts is whether the concedent authority imposes stringent rollout time limits for the concessioned service.¹ At one extreme, regulators can eliminate all scope for discretion by imposing the obligation to immediately undertake the investment required to provide the service. At the other, contracts can be designed so as to leave a large degree of autonomy to the franchisee, by simply assigning the right, as distinct from the obligation, to supply the market. These arrangements constitute a continuum, along which we find concessions which impose service obligations but give the franchisee a certain amount of time flexibility, and contracts which do not impose the obligation to supply the market, but the licensee has a limited amount of time to start using the licence after which it will be revoked.²

Whatever the terms and conditions specified in the contract, a concession can be granted essentially in three ways: direct negotiations, beauty contests, beauty contests, beauty contests.

¹Although service rollout time limits are often imposed, particularly when concessioning "socially significant services", such as the provision of potable water, governments also award contracts which do not impose stringent obligations. This, for example, has occurred in Europe when awarding 3G telecom (UMTS) licences. Many EU Member States did not specify rollout time limits, and even where specific service launch requirements were set, regulators agreed to change the date (Northstream, 2002).

²For example, the Minister for Communications, Information Technology, and the Arts instructed the Australian Communication Authority (ACA) to impose a use it or loose it condition on all licences for LPON (Low Power Open Narrowcasting) services that operate in the 87.5 - 88.0 MHz FM sub-band. Breaching this condition may result in cancellation of the apparatus licence by, the ACA, or refusal to renew the licence. As argued by ACA, the main reason to impose a use it or loose it condition is to avoid hoarding, i.e., to prevent firms from obtaining licences, then not operating them.
or competitive bidding mechanisms. Although concessions are still frequently awarded by a selection process based on a subjective evaluation of one or more quantitative or qualitative criteria, recent years have witnessed an increasing interest in using bidding processes\(^3\), which, particularly when the service is fairly standard (Bajari and Tadelis, 2001), provide a potentially effective mechanism for stimulating interest among a broader range of potential investors and selecting the best proposal.

When bidding for concessioning, governments must decide *inter alia* which specifications to include for the financial proposals.\(^4\) When the contract does not involve sale of existing assets, awarding authorities frequently base the bidding on lowest price charged to consumers, or on the highest fee paid by the concessionaire (or the lowest subsidy paid by the government).\(^5\)

The debate about the appropriate bidding rule is not new.\(^6\) However, the economics literature has not deeply explored the relation between award criteria and concessioning conditions, namely how the expected returns of alternative financial proposals are affected by service launch requirements, and vice versa, *i.e.* how the returns of alternative conditions are affected by the bidding rule used to allocate the contract.

The aim of this paper is to fill this gap. In particular, we compare the two award criteria ("highest concession fee" vs. "lowest price") by looking at two alternative concessioning conditions which approximate actual practices: a

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\(^3\) For example, as far as spectrum licences are concerned, New Zealand legislated spectrum auctions in 1989, as did the United Kingdom in 1990. In the United States, where licences used to be assigned after comparative hearings held by the Federal Communications Commission (FCC), and then lotteries, Congress passed legislation giving the FCC the authority to auction licenses in 1993 (McMillan, 1994). More recently, in the E.U., where the allocation procedures of licences for 3G mobile communications were left to the discretion of Member States, while several States organised comparative bids, while others opted for an auction mechanism (Klemperer, 2003).

\(^4\) Specifications may include both technical and financial proposals. However, conceding authorities often adopt a two-stage process whereby technical proposals are evaluated before proceeding to the financial offers. The winning bidder is then selected on the basis of the best financial proposal from among those who passed the technical evaluation.

\(^5\) Conceding authorities may also base the auction on multiple financial criteria. However, using a single financial criterion tends to increase the transparency of the competitive process, although this can also be derived by using a pre-specified formula which combines multiple criteria and which, in the case of the price to consumers and the concession fee, may reflect the relative weight the government attaches to the consumer surplus and fiscal revenues.

\(^6\) For example, Alfred Marshall argued that "[...] the competition for the franchise shall turn on the price or the quality, or both, of the services or the goods, rather than on the annual sum paid for the lease". Quoted in Ekelund and Hebert (1981), p.471.
contract which imposes the obligation to immediately operate the concessioned service, and a contract which simply assigns the right to supply the market without imposing rollout time limits.\footnote{In our two-period model, the former concession arrangement may also be interpreted as an approximation of contracts which imposes the obligation to operate the service, but gives a certain amount of time flexibility. The second arrangement is an approximation of contracts which does not impose the obligation to supply the market, but gives the licensee a limited amount of time to start using the exclusive right of exercise, after which it will be cancelled.} We do this by developing a simple two-period model, by assuming that the concessioned service is fairly standard, the investment required to operate the service is irreversible and potential operators face a stochastic demand. Moreover, in order to avoid effects arising from assigning arbitrary weights to the consumer surplus and fiscal revenue, the comparative welfare analysis is carried out by assuming that from the concedent’s point of view, a euro in the pocket of consumers and a euro in the hand of public authorities are equally valuable.

Although the paper focuses on concessions, our analysis is related to the literature on procurement, namely to the branch of the literature which considers how to include quality other than sale price in the procurement process (Laffont and Tirole, 1987; Che, 1993). In particular, Che shows that the optimal buying mechanism distorts quality provided by the suppliers downwards relative to the first best levels. In other words, the buyer, acting as if she does not care about quality, may reduce the dispersion between suppliers and thus increase the level of procurement competition. Hence, if we interpret the construction time as an indicator of the procured project "quality" (Herbesman et al., 1995; Arditi et al., 1997), Che’s results suggest that the regulator may benefit from a reduced sale price in exchange for a project completion delay.

The paper contributes to concession literature in two ways. First, like Che, our findings suggest that concessioning without imposing the obligation to immediately supply the market (acting as if "quality" does not matter) may provide a higher expected value. Secondly, however, time flexibility \textit{per se} does not involve a higher social value. For instance, besides depending on the consumers’ maximum willingness-to-pay for the service ("the reservation price"), and the level of competition, the potential welfare gains arising by allowing the winning bidder to decide when to operate the service are affected by the bidding rule. \textit{Coeteris paribus}, when both the reservation price and competition are relatively low, allowing the franchisee to decide the date of service launch tends to provide a higher expected value, provided the concession is allocated according to the lowest price criterion. In contrast, the highest fee criterion may be more appropriate to allocate concessions which
do not impose rollout obligations when the number of bidders is relatively high. Finally, whatever the level of competition is, when the reservation price is relatively high, awarding authorities should impose rollout time limits and allocate the contract according to the lowest price criterion.

The rest of the paper is organized as follows. Section 2 outlines the model and describes the concession value, with and without time-flexibility. Section 3 looks at the bidding process, and Section 4 focuses on the expected welfare value of alternative award criteria and concessioning conditions. Section 5 concludes and the Appendix contains the proofs omitted in the text.

2 The concession value

Consider a natural monopoly industry facing demand uncertainty which is beyond the supplier’s control. The infrastructure service under consideration is fairly standard, i.e. the supplier is unable to exercise any degree of discretion with respect to the type of investment to be undertaken and product quality.\(^8\)

The service can be operated only by acquiring an exclusive right of exercise auctioned by a public authority (hereafter "the government"). Depending on the bidding rule, the franchise will be awarded to the applicant proposing the lowest cost to the consumers ("the price"), or offering the highest up-front payment to the government ("the fee"). In both formats, if there are more bidders that submit the same price (fee), then a random drawing determines the winner.

Before focussing on the two award criteria, we derive the private value of the contract, by taking the price as given and by ignoring the fee. We make the following assumptions.

**Assumption 1** To operate the service the concessionaire must afford a capital cost \(I\). The instantaneous investment is sunk, it can neither be changed, nor temporarily stopped, nor shut down. Operating and maintenance costs are comparatively small and set to zero.

**Assumption 2** The price \((p)\) offered by the winning bidder is constant over the franchise term.

\(^8\)An example is provided by toll roads. Demand for a highway is largely beyond the franchise holder, traffic forecasts are notoriously imprecise, and it is difficult to make accurate traffic predictions especially in the long term (Engel, Fisher, and Galetovic, 2001). Moreover, the service is fairly standard, and there is a limited scope for creativity on the part of an operator.
Assumption 3  At any time \( t \geq 0 \) there is a mass \( y_t \) of identical consumers. Each consumer has an inelastic demand for one unit of the service up to some reservation price \( p^{\text{max}} \).

Assumption 4  The timing of the demand is as follows. Current demand \((t = 0)\) is \( y_0 \). At \( t = 1 \) it may either rise to \((1+u)y_0\) with probability \( q \), or decrease to \((1-d)y_0\) with probability \( 1-q \) (where \( u > 0 \), \( 0 < d < 1 \) and \( q \in (0,1) \)):

\[
\begin{align*}
y^+ &= (1+u)y_0 \quad \text{with probability } q \\
y^- &= (1-d)y_0 \quad \text{with probability } 1-q
\end{align*}
\]

We first derive the concession value when the franchisee faces the obligation to operate the service immediately. By indicating with \( \rho > 0 \) the constant discount rate, we get the following Lemma.

Lemma 1  The concession expected Net Present Value at \( t = 0 \) is:

\[
NPV^0 = pK_0 - I \equiv (p - \hat{p})K_0
\]

(1)

where:

\[
\hat{p} \equiv \frac{I}{K_0}, \quad \text{and} \quad K_0 \equiv \left[ 1 + q \frac{1+u}{1+\rho} + (1-q) \frac{1-d}{1+\rho} \right] y_0
\]

Proof. Straightforward

Consider now the case where the franchisee does not face the obligation to supply the market. In this case, the condition \( NPV^0 > 0 \) is no longer sufficient for immediately undertaking the required investment, since it does not account for the franchisee’s ability to react to unfavorable market conditions, i.e. demand falling short of expectations. Since in our setting a period is sufficient to obtain information on investment profitability, the decision to wait is economically significant only if operating the service becomes profitable under the upward realization of the demand level \((y_0^+)\). From now on we restrict the analysis to this case only, i.e. we assume that \( py_0 \frac{1-d}{1+\rho} < \frac{I}{1+\rho} < py_0 \frac{1+u}{1+\rho} \) (Dixit and Pindyck, 1994, p. 40-41).

Lemma 2  The concession expected Net Present Value at \( t = 1 \) as of today is:

\[
NPV^1 = p(K_0 - K_1) - \frac{q}{1+\rho}I \equiv (p - \hat{p})K_0 + (\hat{p} - p)K_1
\]

(2)

where:

\[
\hat{p} = \frac{1+\rho - q}{1+\rho} \frac{I}{K_1}, \quad \text{and} \quad K_1 = \left[ 1 + (1-q) \frac{1-d}{1+\rho} \right] y_0
\]
Proof. See Appendix A ■

By putting (1) and (2) together, we get the concession value, which accounts for how much the option to delay the service operation is worth.

**Proposition 1** For any given \( p \), the concession value at \( t = 0 \) is:

\[
V(p) = \max \left[ NPV^0, NPV^1 \right] \\
\equiv (p - \bar{p})K_0 + \max [ (\hat{p} - p)K_1, 0 ]
\]

Proof. Straightforward from Lemma 1 and 2. ■

The second term on the r.h.s. (3) represents the option value embedded in a contract which does not impose the obligation to immediately afford sunk capital costs. Since \( K_0 - K_1 > 0 \), by defining \( \bar{p} \equiv \phi \hat{p} + (1 - \phi)\hat{p} \), where \( \phi \equiv \frac{K_0}{K_0 - K_1} > 1 \) and \( (1 - \phi) \equiv -\frac{K_1}{K_0 - K_1} < 0 \), equation (3) can be rewritten as follows:

\[
V(p) = \max [ (p - \bar{p})K_0, (p - \bar{p})(K_0 - K_1) ].
\]

Finally, for the rest of the paper we assume that \( \hat{p} \equiv \frac{1 + \rho - q}{1 + \rho} \frac{K_0}{K_1} > 1 \) and \( \bar{p} \equiv \frac{q}{1 + \rho} \frac{K_0}{K_0 - K_1} < 1 \), which ensure that \( 0 < \bar{p} < \hat{p} < \hat{p} \), and imply the following optimal decision rule (See Figure 1):

\[
\begin{cases} 
\text{if } p > \hat{p} \text{ it is optimal to invest at } t = 0 \\
\text{if } \bar{p} < p < \hat{p} \text{ it is optimal to invest at } t = 1 \\
\text{if } p < \bar{p} \text{ it is never optimal to invest .}
\end{cases}
\]

\footnote{It is easy to see that \( \bar{p} \equiv \frac{1 + \rho - q}{1 + \rho} \frac{K_0}{K_1} > 0 \)}
3 Award criteria and rollout obligations

A firm can operate the service only after submitting a successful bid. We consider the following alternative first-price-sealed-auction formats:10

- The concession is awarded to the bidder offering the Lowest Price (hereafter LP award rule, or LP).
- The concession is awarded to the bidder offering the Highest Fee (HF).

---

10We do not derive the optimal procedure to award a monopoly franchise (see Riordan and Sappington, 1987; Laffont and Tirole, 1987; McAfee and McMillan, 1987), and we only consider a first-price auction, in order to examine the welfare properties of alternative specifications for the financial proposals.
These bidding rules will be compared by considering the following alternative concessioning conditions:

- The service must be operated at \( t = 0 \), i.e. the winning bidder is not allowed to delay the investment (Case I).
- The concession does not impose roll-out requirements, i.e. the winning bidder acquires the right to supply the market at a date of her choosing (Case II).

We conclude the model set-up by adding the following assumptions.

**Assumption 5** There are \( N > 1 \) competing firms.

**Assumption 6** Each bidder \( i \) (\( i = 1, 2, \ldots, N \)) observes \( y_0 \) and the multiplicative parameters \( (u, d) \), knows the distribution \( (q, 1-q) \) and the realization of the investment cost \( I_i \), and only knows that \( I_j, j \neq i \) are independent random variables, with the same absolutely continuous distribution \( G \), with positive density \( g \) over the interval \( I = [I^l \geq 0, I^u] \subseteq \mathbb{R}^+ \). For the sake of simplicity, we assume that capital costs are uniformly distributed on \( I_i \), with \( I^l = 0 \).\(^{11}\)

**Assumption 7** \( p_{max} \geq \tilde{p}^u \equiv \frac{I^u}{R^0} \), i.e. the reservation price is such that it would ensure a non-negative concession value to the less efficient bidder, even when the contract imposes the obligation to immediately operate the service.

**Assumption 8** Bidders are not subject to any liquidity or budget constraints, so that each firm \( i \) has sufficient resources to pay the up-front fee after winning the auction.

Finally, rollout obligations are verifiable and enforceable, the price (or alternatively the up-front fee) offered by the winning bidder cannot be renegotiated, and all aspects of the bidding situation are known to the government except for investment costs \( I_i \) (\( i = 1, 2, \ldots, N \)) known only by each firm itself.

\(^{11}\)None of the results depend on the assumption that \( G(I) \) is a uniform distribution as long as \( I + \frac{G(I)}{g(I)} \) is a monotone increasing function (Krishna, 2002, p.69).
3.1 Case I

When the government imposes the obligation to operate the service immediately, LP implies that the firms’ pricing problem reduces to a Bertrand game where each bidder picks up the lowest price $p$ that maximizes her expected net present value as defined in (1):

$$\max_{p_i} NPV^0(p_i; \hat{p}_i) \Pr \left[ \min_{j \neq i} p_j \geq p_i \right]$$

(5)

The equilibrium strategy is summarized in the following Proposition.

Proposition 2 When the concessionaire must immediately operate the service, the LP award rule involves the following unique symmetric equilibrium strategy rule:

$$p_i = p(\hat{p}_i) \equiv (1 - \frac{1}{N})\hat{p}_i + \frac{1}{N}\tilde{p}^u \leq \tilde{p}^u$$

(6)

Proof. See Appendix B.

Going back to the definition of $NPV^0$, since by assumption 7 the threshold levels $\hat{p}_i$ are distributed uniformly within the support $\hat{P} = [0, \tilde{p}^u]$, equation (6) implies that $NPV^0_i$ are also uniformly distributed over the interval $[0, NPV^0_u]$, with interim profits positive for all types except the less efficient firm, which never wins and whose $NPV^0_i$ is equal to zero even if it does win. By substituting (6) back into $NPV^0_i$, (1) can be rewritten as a function of the proposed price:

$$NPV^0_i \equiv \frac{1}{N-1}(\tilde{p}^u - p(\hat{p}_i))K_0$$

(7)

In other words, the bidder offering the lowest price is the one with the highest $NPV^0$.

Under HF, firms will bid on the basis of the monopoly price. For instance, since the franchisee will be able to fix the price up to the reservation price, conditionally on $p^{\text{max}}$, we obtain the proposed fee ($R$) by maximizing:

$$\max_{R_i^0} \left[ NPV^0(p^{\text{max}}; \hat{p}_i) - R_i^0 \right] \Pr \left[ \max_{j \neq i} R_j^0 \leq R_i^0 \right]$$

(8)

The equilibrium strategy is summarized in the following Proposition.

Proposition 3 When the concessionaire must immediately operate the service, the HF award rule involves the following unique symmetric equilibrium strategy rule:

$$R_i^0 = \frac{N-1}{N}NPV^0_i \equiv \frac{N-1}{N}(p^{\text{max}} - \hat{p}_i)K_0$$

(9)
\textbf{Proof.} See Appendix C \hfill ■

By substituting (9) back into (8), the net benefit \([NPV^0(p_{\text{max}}; \tilde{p}_i) - R_i^0]\) can be rewritten as a function of the threshold levels \(\tilde{p}_i\):

\[NPV^0(p_{\text{max}}; \tilde{p}_i) - R_i^0 = \frac{1}{N}(p_{\text{max}} - \tilde{p}_i)K_0\] (10)

Since \(\tilde{p}_i\) are distributed uniformly within the support \(\tilde{P} = [0, \tilde{p}^u]\), the bidder reporting the highest fee is the one with the highest net benefit.

\subsection{Case II}

When the contract does not impose the obligation to immediately supply the market, under LP the bidders’ pricing problem can still be described as a Bertrand game, where the concession value to be maximized is now given by \(V(p_i) = \max [NPV^0, NPV^1]\). In other words, each agent selects two prices contingent to the investment time, and reports the one that maximizes her probability of winning the auction, i.e. \(p_i^{\text{option}} = \min[p(\tilde{p}_i), p(\bar{p}_i)]\).

The equilibrium strategy is summarized in the following Proposition.

\textbf{Proposition 4} When the concessionaire does not face the obligation to immediately operate the service, the LP award rule involves the following unique symmetric equilibrium strategy rule:

\[p_i^{\text{option}} = p(\bar{p}_i) = (1 - \frac{1}{N})\tilde{p}_i + \frac{1}{N}\tilde{p}^u \leq \tilde{p}^u\] (11)

\textbf{Proof.} See Appendix D \hfill ■

By direct inspection of (6) and (11), we can show that:

\[p(\tilde{p}_i) \leq p(\bar{p}_i), \quad \text{for all } i.\] (12)

Disequality (12) implies that competing by maximizing \(NPV^1_i\) is a dominant strategy when the price plays a key role in winning the auction. In other words, under LP firms always exploit time flexibility. By doing so, bidders are able to submit a price \((p(\bar{p}_i))\) lower than the one they would have offered if the government had asked immediate rollout of the service.

By substituting (11) back in \(NPV^1_i\), (2) can be rewritten as a function of the proposed price:

\[NPV^1_i \equiv \frac{1}{N-1}(\tilde{p}^u - p(\bar{p}_i))(K_0 - K_1)\] (13)
Again, the bidder with the lowest price is the one with the highest $NPV^1$.

Similarly, under HF, each agent selects two fees contingent to the investment time and reports the one that maximizes her probability of winning the auction, i.e. $R^\text{option}_i = \max \{R^0_i, R^1_i\}$. The equilibrium strategy is summarized in the following Proposition.

**Proposition 5** When the concessionaire does not face the obligation to immediately operate the service, the HF award criterion involves the following unique symmetric equilibrium strategy rule:

$$
\begin{align*}
R^\text{option}_i &= R^0_i \quad \text{if} \quad p^\text{max} \geq \hat{p}_i \\
R^\text{option}_i &= R^1_i \quad \text{if} \quad p^\text{max} < \hat{p}_i
\end{align*}
$$

(14)

where $R^0_i$ is given by (9), and $R^1_i = \frac{N-1}{N} [(p^\text{max} - \bar{p}_i)(K_0 - K_1)]$.

**Proof.** See Appendix D

As in Case I, by substituting (14) back into (8), it is easy to show that the bidder reporting the highest fee is the one with the highest net benefit.

According to Proposition 5, and by direct inspection of $R^0_i$ and $R^1_i$ we can conclude that the unique symmetric equilibrium strategy rule for all bidders is:

$$
\begin{align*}
R^\text{option}_i &= R^0_i \quad \text{for all } i, \text{ if } p^\text{max} \geq \hat{p}^u \\
R^\text{option}_i &= \max \{R^0_i, R^1_i\} \quad \text{for all } i, \text{ if } \hat{p}^u < p^\text{max} < \hat{p}^u
\end{align*}
$$

To put Proposition 4 and 5 into words, whilst auctioning a contract which does not impose rollout obligations always induces all bidders to lower the price charged to consumers ($p^\text{option}_i < p_i$), it does not necessarily provide a higher fiscal revenue. For instance, if the reservation price (the investment cost, $I_i$) is sufficiently high (low), i.e. if $p^\text{max} \geq \hat{p}^u$, all bidders will offer the same fee they would have reported when bidding for a concession which imposes the obligation to immediately operate the service ($R^\text{option}_i = R^0_i$).

Generally speaking, the higher the reservation price, the lower tends to be the additional fiscal revenue arising from allowing the franchisee to optimally decide the date of service launch.

### 4 Welfare analysis

#### 4.1 Case I

Let us first consider the case where the concedent authority imposes rollout time limits. In order to compare the expected social value ($W$) arising from
the two bidding rules, we assume that from the government’s point of view, a euro in the pocket of consumers and a euro of fiscal revenue are equally valuable.

By indicating with $S^0$ the consumer surplus when the concessionaire operates the service at $t = 0$, i.e. immediately after winning the auction:

$$S^0 = \sum_{t=0}^{1} \frac{1}{(1 + \rho)^t} \int_{p_{(\hat{b})}}^{p_{\text{max}}} E_0(y_\hat{t}) dp$$

we get the following Lemma.

**Lemma 3** When the concessionaire must immediately operate the service,

i) the LP award rule provides the following expected welfare value:

$$W_{LP} = E[S^0] = (p_{\text{max}} - \frac{2}{N+1} \tilde{p}^u)K_0$$

ii) the HF award rule provides the following expected welfare value:

$$W_{HF} = E[R^0] = \frac{N - 1}{N+1}p_{\text{max}}K_0 + \frac{N - 1}{N(N + 1)}(p_{\text{max}} - \tilde{p}^u)K_0$$

**Proof.** See Appendix F

Defining $\Delta W_{LP/HF}^0 = E[S^0] - E[R^0]$, by assumption 8 and Lemma 3, we get the following Proposition.

**Proposition 6** When the concessionaire must immediately operate the service,

$$\Delta W_{LP/HF}^0 = 0, \text{ if } p_{\text{max}} = \tilde{p}^u$$

and

$$\Delta W_{LP/HF}^0 = \frac{1}{N}(p_{\text{max}} - \tilde{p}^u)K_0 > 0 \text{ if } p_{\text{max}} > \tilde{p}^u$$

**Proof.** Straightforward from Lemma 3

To put the Proposition into words, the two bidding rules provide the same expected value when the reservation price ($p_{\text{max}}$) is equal to the price which would make the concession net present value for the less efficient bidder equal to zero ($\tilde{p}^u$). Otherwise, LP will always provide the highest value. Moreover, comparative statics shows that the higher (the lower) the reservation price (the level of competition) is, the higher are the additional benefits arising from LP.\(^\text{12}\)

For instance:

$$\frac{\partial \Delta W_{LP/HF}^0}{\partial p_{\text{max}}} > 0 \text{ and } \frac{\partial \Delta W_{LP/HF}^0}{\partial N} < 0$$

\(^\text{12}\)An example is provided by water services. Because of their essential nature, and since
4.2 Case II

Let us now consider the case where the regulator does not impose time limits for the provision of the concessioned service. We still assume that the government does not attach different weights to the consumer surplus and the fiscal revenue.

Since under LP all bidders will exploit time flexibility, by indicating the consumer surplus with $S^1$, evaluated at $t = 1$, and only for $y^+_t$:

$$S^1 = q \left\{ \frac{1}{(1 + \rho)} \int_{p_i(\tilde{p}_i)}^{p_{\text{max}}} y^+_t \, dp \right\}$$

we get the following Lemma.

**Lemma 4** When the concessionaire does not face the obligation to immediately operate the service,

i) the LP award rule provides the following expected welfare value

$$W_{LP} = E[S^1] \equiv \left( p_{\text{max}} - \frac{2}{N+1} \tilde{p}_u \right) (K_0 - K_1)$$

ii) the HF award criterion provides the following expected welfare value:

$$W_{HF} = E[R^{\text{option}}] = \begin{cases} E[R^0] & \text{if } p_{\text{max}} \geq \tilde{p}_u \\
E[R^0] - \frac{N-1}{N(N+1)} (p_{\text{max}} - \tilde{p}_u) K_1 & \text{if } \tilde{p}_u < p_{\text{max}} < \tilde{p}_u \end{cases}$$

where $E[R^0] \equiv \frac{N-1}{N+1} p_{\text{max}} K_0 + \frac{N-1}{N(N+1)} (p_{\text{max}} - \tilde{p}_u) K_0$

**Proof.** See Appendix F □

By defining $\Delta W_{LP/HF}^1 = E[S^1] - E[R^{\text{option}}]$, according to Lemma 4 and substituting the expressions for $E[S^1]$ and $E[R^0]$, we get the following Proposition.

the concessionaire does not face competition from other service modes, the reservation price tends to be relatively high. Moreover, because of the highly specialized nature of this industry, competition to acquire a concession is relatively low. For example, in Italy, between 1998 and 2005 in regions where new water services were assigned by competitive bidding, the average number of bidders was 1.2 (Dipartimento per le Politiche di Sviluppo - UVAL, 2006). Generally, concedent authorities adopted the lowest price award rule, but some authority referred to the highest fee. Since, in Italy, water concessions involve stringent investment plans and service launch obligations, the choice of latter award criterion does not appear consistent with our results.

13 In other words, by assuming that the interest rate ($\rho$) reflects the government’s time preferences, the social cost (the "quality loss") stemming from allowing the franchisee to delay the operation of the service is captured by discounting the expected consumer surplus.
Proposition 7  When the concessionaire does not face the obligation to immediately operate the service

\[ \Delta W_{1}^{\text{LP/HF}} = \Delta W_{0}^{\text{LP/HF}} - \frac{N - 1}{N + 1} \hat{p} u K_1 \]

\[ - \begin{cases} (p_{\text{max}} - \hat{p} u) K_1 & \text{if } p_{\text{max}} > \hat{p} u \\ (1 + \frac{N - 1}{N(N+1)}) (p_{\text{max}} - \hat{p} u) K_1 & \text{if } \tilde{p} u < p_{\text{max}} < \hat{p} u \end{cases} \]

Proof. Straightforward from Lemma 4 ■

Unlike the previous case, when the government does not impose rollout time limits, we cannot say that one or the other award rule is better, insofar the sign of \( \Delta W_{1}^{\text{LP/HF}} \) strongly depends on the level of competition \( (N) \). In particular, it is easy to show that whatever is the the value of \( p_{\text{max}} \):

\[ \frac{\partial \Delta W_{1}^{\text{LP/HF}}}{\partial N} < 0 \]

i.e. as the number of bidders increase, it is more profitable (or less disadvantageous) to use HF rather than LP.

As for the reservation price, it is easy to prove that:

\[ \frac{\partial \Delta W_{1}^{\text{LP/HF}}}{\partial p_{\text{max}}} = \begin{cases} \frac{1}{N} K_0 - K_1 & \text{if } p_{\text{max}} > \hat{p} u \\ \frac{1}{N} K_0 - K_1 - \frac{N - 1}{N(N+1)} K_1 & \text{if } \tilde{p} u < p_{\text{max}} < \hat{p} u \end{cases} \]

While for relatively low values of \( N \), the higher the reservation price is, the higher tend to be the potential benefits of LP with respect to HF (i.e. \( \frac{\partial \Delta W_{1}^{\text{LP/HF}}}{\partial p_{\text{max}}} > 0 \)), the reverse applies when \( N \) is sufficiently high (\( \frac{\partial \Delta W_{1}^{\text{LP/HF}}}{\partial p_{\text{max}}} < 0 \)). In other words, unlike a contract which imposes service launch requirements, when the winning bidder is allowed to decide the date of service launch, if the competitive pressure is sufficiently high, the higher \( p_{\text{max}} \) is, the higher tend to be the relative benefits arising by granting the contract according to the HF award rule.

4.3 Time-flexibility vs. no time-flexibility

We conclude the analysis by comparing the two concessioning conditions. To do so, we evaluate the difference between the expected welfare value from the two award criteria, with and without time-flexibility.

Proposition 8  i) Under the LP award rule, the differential expected welfare value arising from allowing the winning bidder to decide the date of service
launch is:

\[ E[S^1] - E[S^0] = -(p_{\text{max}} - \frac{2}{N+1} \hat{p}^u)K_1 \equiv \left\{ \begin{array}{ll}
< 0 & \text{if } p_{\text{max}} \geq \hat{p}^u \\
? & \text{if } \hat{p}^u < p_{\text{max}} < \hat{p}^u
\end{array} \right. \]

ii) Under the HF award rule, the differential expected welfare value arising from allowing the winning bidder to decide the date of service launch is:

\[ E[R^\text{option}] - E[R^0] = \left\{ \begin{array}{ll}
0 & \text{if } p_{\text{max}} \geq \hat{p}^u \\
\frac{N-1}{N(N+1)}(p_{\text{max}} - \hat{p}^u)K_1 > 0 & \text{if } \hat{p}^u < p_{\text{max}} < \hat{p}^u
\end{array} \right. \]

Proposition 8 confirms the intuition that imposing rollout time limits may or may not provide higher expected value, depending on the financial criterion used to allocate the contract. Specifically, whatever the discount rate (\( \rho > 0 \)), when the contract is allocated according to LP, imposing time limits tends to provide a higher social value. This is always true for \( p_{\text{max}} \geq \frac{2}{N+1} \hat{p}^u \), and holds over the entire range as \( N \) increases a bit. On the contrary, under HF, auctioning a contract which does not impose rollout obligations provides higher fiscal revenue, although the additional revenue arising from allowing the winning bidder to decide the date of service launch tends to vanish as the reservation price increases.

5 Final remarks

Concession arrangements and award procedures can take different forms and entail various legal and economic issues. In this paper we have focused on competitive bidding mechanisms, by comparing two award criteria, "the lowest price to consumers" (LP) and "the highest concession fee" (HF), and two alternative concessioning conditions, namely a contract which imposes service launch requirements and a contract which simply assigns the right, as distinct from the obligation, to supply the market.

We have shown that the expected returns arising from these bidding rules may be significantly affected by the concessioning conditions, and vice versa. In other words, when deciding whether or not to impose service launch requirements, governments should take into account which specifications will be included in the financial proposals. The main findings and policy implications can be summarised as follows.

If the government focuses on rollout speed, i.e. when concedent authorities impose the obligation to immediately operate the concessioned service, and do not assign different weights to the consumer surplus and fiscal revenues, the
concession should be awarded by adopting the LP bidding rule: the lower (the higher) the level of competition $N$ (the reservation price, $p_{\text{max}}$) is, the higher will be the additional benefits arising from this financial criterion.

In contrast, when the government does not impose rollout obligations, the relative benefits arising from the two bidding rules strongly depend on the level of competition. Whilst when bidders do not face a sufficiently high competitive pressure, LP still provides the highest expected value, the reverse applies when $N$ is relatively high. Moreover, when bidders face competitive pressure, the higher $p_{\text{max}}$ is, the higher will be the additional potential benefits arising from HF.

If the government wishes to raise funds, i.e. when concedent authorities choose the concessionaire according to HF, it is never appropriate to impose rollout requirements, although the additional fiscal benefits arising from allowing the winning bidder to decide the date of service launch tend to vanish as $p_{\text{max}}$ increases.

In contrast, if the government wants to maximise consumer surplus, we cannot generally say which concessioning condition provides the highest value. However, the higher $p_{\text{max}}$ is, the more appropriate appears to be imposing rollout commitments and, unlike HF, as $p_{\text{max}}$ increases, benefits also increase from contracts which impose stringent service launch requirements.

Our results appear to be broadly consistent with actual practice. For instance, when concessioning socially relevant services, such as the operation of a water system in a municipality, which do not face competition from other service modes, awarding authorities generally impose stringent service obligations and select the concessionaire according to the lowest-price criterion. By contrast, when allocating spectrum licences, which generally involve less stringent service launch requirements, governments usually grant the concessions to the bidders offering the highest fee.

However, we do have examples of socially significant services which have been concessioned by imposing stringent investment and rollout obligations, according to the HF bidding rule. Our findings suggest that this practice is generally inappropriate, unless concessioning takes place within a broader regulatory framework which includes a relatively low "price cap" for the monopolistic winning bidder, provided the regulated $p_{\text{max}}$ does not significantly affect (reduce) the number of potential investors ($N$).
Appendix

A Proof of Lemma 2

The flow of profits that the concessionaire will receive once the investment is undertaken is simply:

$$\pi(y_t) = py_t \quad \text{for } t = 0, 1$$  \hspace{1cm} (15)

and, according to Assumptions 3 and 4, the time evolution of the instantaneous profit function becomes:

$$\pi_0 \quad \pi^+_1 = (1 + u)py_0 \quad \text{with probability } q \hspace{1cm} (16)$$

$$\pi^-_1 = (1 - d)py_0 \quad \text{with probability } 1 - q$$

If the concessionaire is allowed to postpone the investment decision, the bidder has to consider this option value that must be included as part of the total cost of the investment in evaluating the $NPV$ at time zero. Operatively, the bidder will compare the $NPV^0$ with the $NPV^1$ at $t = 1$ as of today, evaluated only for $\pi^+_1$:

$$NPV^1 = q \left[ \frac{1 + u}{1 + \rho} py_0 - \frac{I}{1 + \rho} \right]$$  \hspace{1cm} (17)

The overall project value is then given by:

$$\max [NPV^0, NPV^1]$$  \hspace{1cm} (18)

By (18), it is possible to calculate the firm’s option value to wait as:

$$OP^0 = \max [NPV^0, NPV^1] - NPV^0 = \max [NPV^1 - NPV^0, 0]$$  \hspace{1cm} (19)

If $NPV^1 - NPV^0 > 0$ it is optimal to wait one period and decide to invest at $t = 1$ only in the case of good news. If, on the contrary, $NPV^1 - NPV^0 < 0$ it is optimal to invest at $t = 0$. Then, by imposing $NPV^0(\hat{p}) = NPV^1(\hat{p})$, (19) can be rewritten as follows:

$$OP^0 = \max [(\hat{p} - p)K_1, 0]$$  \hspace{1cm} (20)
where \( \hat{p} \equiv \frac{1 + \rho - q}{1 + \rho} K_1 \) and \( K_1 \equiv \left[ 1 + (1 - q) \frac{1 - d}{1 + \rho} \right] y_0 \). Substituting (20) back into (19) and solving for \( NPV^1 \) we get:

\[
NPV^1 = NPV^0 + OP^0 = p K_0 - I + \max \left[ \frac{1 + \rho - q}{1 + \rho} I - p K_1, 0 \right]
\]

\( \equiv (p - \hat{p}) K_0 + \max [(\hat{p} - p) K_1, 0] \)

This concludes the proof.

**B Proof of Proposition 2**

Consider firm \( i \)' bidding decision and suppose that all other firms use the symmetric strategy \( p(\tilde{p}_j) \ \forall j \neq i \) that specifies every bidder’s willingness-to-pay.

Since a bid \( p_i \) is a best response at \( \tilde{p}_i \) (i.e. \( I_i \)) by firm \( i \) if it maximizes its expected payoff against the rivals’ strategies \( p(\tilde{p}_j), \ \forall j \neq i \), than for any feasible bid \( (p) \), the pricing rule is given by:

\[
p_i = \arg \max NPV^0(p_i; \tilde{p}_i) \Pr \left[ \min_{j \neq i} p(\tilde{p}_j) \geq p_i \right] \quad (21)
\]

We show that a price strategy for firm \( i \) is a symmetric function \( p(\tilde{p}_i) \) mapping from the set of firm types \( \tilde{P} = [0, \tilde{p}^u] \) to the set of possible prices \( P \subset \mathbb{R}_+ \). Yet, for each firm \( i \) this function is continuously differentiable and strictly increasing with the property that \( p'(\tilde{p}_i) < 1 \) and \( p(\tilde{p}^u) = \tilde{p}^u \).

Let us assume that each bidder makes rational conjectures about the distribution of the rivals’ prices represented by a common distribution function \( F(p) \), which is strictly increasing on the interval \( P \subset \mathbb{R}_+ \), and the hazard rate \( h(p) \equiv \frac{f(p)}{1 - F(p)} \) is increasing in \( p \). This assumption allows definition of \( F^{(N-1)}(p_i) \equiv 1 - (1 - F(p_i))^{N-1} \) as the cumulative distribution (with density \( f^{(N-1)}(p_i) \)) of the minimum of the \( N-1 \) rivals’ price, i.e., the probability that all the other bidders set lower tariffs than \( i \) on the same support \( P \). Consequently, firm \( i \)'s expected payoff (21) is:

\[
(p_i - \tilde{p}_i) K_0 (1 - F(p_i))^{N-1} \quad (22)
\]

Maximizing (22) with respect to \( p_i \) gives the necessary condition:

\[
(1 - F(p_i))^{N-1} [1 - (N - 1) (p_i - \tilde{p}_i) h(p_i)] = 0
\]
from which we get:

\[ p_i = \hat{p}_i + \frac{1}{(N - 1)h(p_i)} \tag{23} \]

By the assumption \( h'(p_i) > 0 \) the second order condition is always satisfied, i.e., \( -(p_i - \hat{p}_i)h'(p_i) - h(p_i) < 0 \).

Since the costs are uniformly distributed on \( I = [0, I^u] \), also \( \hat{p}_i \) are distributed uniformly within the support \( \tilde{P} = [0, \tilde{p}^u] \). Furthermore, the less efficient firm knows for certain that it will lose the auction, then \( h(p) \to \infty \) and from (23) we get \( p_i \to \tilde{p}^u \) : i.e., the firm has a project value that is too low to win and then fixes as price \( p = \tilde{p}^u \). Finally, \( \frac{dp_i}{d\tilde{p}_i} = -\frac{1}{1 + \frac{1}{(N - 1)b(p_i)}} > 0 \) and \( < 1 \).

So far we have assumed that \( I_i \) (i.e. \( \hat{p}_i \)) is private information, but used the distribution \( F(.) \) over the rivals' price strategies to derive the firm \( i \) optimal price. To characterize the link between the distribution of \( I_i (\hat{p}_i) \) and the firm’s conjecture on output prices we impose:

\[ F(p_i) = G(\tilde{p}_i) = \frac{\hat{p}_i}{\tilde{p}^u} \equiv \frac{I_i}{I^u} \tag{24} \]

We need to ensure that the function \( p_i(.) \) of the random variable \( I_i \) (i.e. \( \hat{p}_i \)) is itself a random variable and to induce the distribution of \( p_i \) from the distribution of \( I_i \) (i.e. \( \hat{p}_i \)). This procedure is an example of the distributional strategies approach introduced by Milgrom and Weber (1985). Since the investment costs are uniformly distributed over \( I = [0, I^u] \), by (24) and the hazard rate, we get:

\[ h(p_i) = \frac{f(p_i)}{1 - F(p_i)} = \frac{1}{\tilde{p}^u - \hat{p}_i} \frac{d\hat{p}_i}{dp_i} \]

from which:

\[ \frac{dp_i}{d\tilde{p}_i} = \frac{1}{h(p_i)} \frac{1}{\tilde{p}^u - \hat{p}_i} \]

By (23):

\[ (\tilde{p}^u - \hat{p}_i) \frac{dp_i}{d\tilde{p}_i} = \frac{1}{h(p_i)} \equiv (N - 1)(p_i - \hat{p}_i) \tag{25} \]

This equality can be expressed as a first order differential equation in \( p(\tilde{p}) \) as:

\[ p' (\tilde{p}) (\tilde{p}^u - \tilde{p}_i) - p(\tilde{p})(N - 1) + \tilde{p}(N - 1) = 0 \tag{26} \]

with the boundary condition that \( p(\tilde{p}^u) = \tilde{p}^u \). By the linearity of (26) we can try the solution:

\[ p(\tilde{p}) = A\tilde{p} + B \tag{27} \]
Substituting (27) in (26) and rearranging we get:

\[ A(\tilde{p}^u - \tilde{p}_i) - (A\tilde{p} + B)(N - 1) + \tilde{p}(N - 1) = 0 \]
\[ [-A - A(N - 1) + (N - 1)]\tilde{p} + A\tilde{p}^u - B(N - 1) = 0 \]

from which, defining \( A = \frac{N - 1}{N} \) and \( B = \frac{\tilde{p}^u}{N} \), we get:

\[ p(\tilde{p}_i) = (1 - \frac{1}{N})\tilde{p}_i + \frac{1}{N}\tilde{p}^u \quad (28) \]

Finally, substituting (28) into (1), the \( NPV^0_i \) becomes:

\[ NPV^0_i \equiv (p_i - \tilde{p}_i)K_0 \equiv \frac{1}{N}(\tilde{p}^u - \tilde{p}_i)K_0 \quad (29) \]

From (29) the weakest firm does not give any value to the project, i.e. \( NPV^0_i \equiv \frac{1}{N}(\tilde{p}^u - \tilde{p}_i)K_0 = 0 \). This concludes the proof.

**C Proof of Proposition 3**

Since the thresholds \( \tilde{p}_i \) are distributed uniformly within \( \tilde{P} = [0, \tilde{p}^u] \), the bidding problem becomes equivalent to the case where each bidder \( i \) assigns a value to the project which is also distributed uniformly over the interval \([NPV^0_i, NPV^u_i]\). The equilibrium strategy calls upon a firm to bid a constant fraction of its \( NPV \) (Krishna, 2002, p. 19), i.e.:

\[ R^0_i = \frac{N - 1}{N}NPV^0_i \equiv \frac{N - 1}{N}[(p_{\text{max}} - \tilde{p}_i)K_0] \]

This concludes the proof.

**D Proof of Proposition 4**

Proposition 4 can be proven following the proof of Proposition 2. The pricing rule is obtained by maximizing the expected project value. In particular, each bidder should maximize the project value as defined in (4):

\[ \max_{p_i} V(p_i)(1 - F(p_i))^{N-1} \]

or equivalently:

\[ \max_{p_i} \{\max[(p_i - \tilde{p}_i)K_0, (p_i - \tilde{p}_i)(K_0 - K_1)]\} (1 - F(p_i))^{N-1}. \]
The optimal price strategy is then given by:

\[ p_i^{\text{option}} = \min \{p(\tilde{p}_i), p(\hat{p}_i)\} \]  

(30)

where \(p(\hat{p}_i)\) is the price when the firm maximizes the \(NPV_i^0\) and \(p(\tilde{p}_i)\) stands for the price when it maximizes the \(NPV_i^1\). Since Lemma 3 provides \(p(\hat{p}_i)\), we need to derive the pricing rule that maximizes:

\[ \max_{p_i} [(p_i - \tilde{p}_i)(K_0 - K_1)](1 - F(p_i))^{N-1} \]

The first order condition for this case is:

\[ (1 - F(p_i))^{N-1}[(K_0 - K_1) - (N - 1)[(p_i - \tilde{p}_i)K_0 + (\hat{p}_i - p_i)K_1]h(p_i)] = 0 \]

from which we get:

\[ p_i = \frac{K_0}{K_0 - K_1}\tilde{p}_i - \frac{K_1}{K_0 - K_1}\hat{p}_i + \frac{1}{(N - 1)h(p_i)} = \tilde{p}_i + \frac{1}{(N - 1)h(p_i)} \]  

(31)

Since \(h'(p_i) > 0\), the second order condition is always satisfied, i.e: \(-[(p_i - \tilde{p}_i)K_0 + (\hat{p}_i - p_i)K_1]h'(p_i) - (K_0 - K_1)h(p_i) < 0\). As costs are uniformly distributed on \(I = [0, I^u]\) also \(\tilde{p}_i\) are distributed uniformly in \(\tilde{P} = [0, \tilde{p}^u]\). The firm with \(\tilde{p}^u\) has a project value that is too low to win, i.e., the less efficient firm knows for certain that it will lose the auction, then \(h(p) \to \infty\) and from \(31\) \(p_i \to \tilde{p}^u\). Finally, we get \(\frac{d}{dp_i} = \frac{-1}{1 + \frac{h'(p_i)}{(N - 1)h(p_i)}} > 0\) and \(< 1\).

Simple verification shows that from \(24\) we obtain a first order differential equation in \(p(\tilde{p})\) similar to \(26\), from which it is easy to get the price rule \(11\) in the text. Substituting \(p(\tilde{p})\) into \(2\) the \(NPV_i^1\) becomes:

\[ NPV_i^1 = (p_i - \tilde{p}_i)(K_0 - K_1) \equiv \frac{1}{N}(\tilde{p}^u - \tilde{p}_i)(K_0 - K_1) \]

(32)

which is also distributed uniformly in \([0, NPV_i^1]\), with \(NPV_i^1 \equiv \frac{1}{N}(\tilde{p}^u - \tilde{p}_i)(K_0 - K_1) = 0\).

Finally, recalling that by assumption 5 we get \(\tilde{p}_i \leq \hat{p}_i \leq \tilde{p}_i\), the following disequality \(p(\tilde{p}_i) < p(\hat{p}_i)\) is always satisfied for all \(i\), i.e.:

\[ (1 - \frac{1}{N})[\phi \tilde{p}_i + (1 - \phi)\hat{p}_i] + \frac{1}{N} [\phi \tilde{p}^u + (1 - \phi)\hat{p}^u] < (1 - \frac{1}{N})\tilde{p}_i + \frac{1}{N}\tilde{p}^u \]

\[(\phi - 1) \left\{ \left[ (1 - \frac{1}{N})\tilde{p}_i + \frac{1}{N}\tilde{p}^u \right] - \left[ (1 - \frac{1}{N})\tilde{p}_i + \frac{1}{N}\tilde{p}^u \right] \right\} < 0 \]

This concludes the proof.
E Proof of Proposition 5

Since the thresholds $\tilde{p}_i$ and $\bar{p}_i$ are distributed uniformly within $\tilde{P} = [0, \tilde{p}^u]$ and $\bar{P} = [0, \bar{p}^u]$, the bidding equilibrium strategy requires reporting of a fee that is a constant fraction of the $\max(NPV_0, NPV_1)$. That is:

$$R_{i, \text{option}} = \frac{N - 1}{N} \max[NPV_i^0, NPV_i^1] \equiv \max[R_i^0, R_i^1]$$

where $R_i^0 = \frac{N - 1}{N} [(p_{\text{max}} - \tilde{p}_i)K_0]$ and $R_i^1 = \frac{N - 1}{N} [(p_{\text{max}} - \bar{p}_i)(K_0 - K_1)]$. In addition, taking the difference $R_i^0 - R_i^1$, we get:

$$R_i^0 - R_i^1 = \frac{N - 1}{N} [(p_{\text{max}} - \tilde{p}_i)K_0] - \frac{N - 1}{N} [(p_{\text{max}} - \bar{p}_i)(K_0 - K_1)] \quad (33)$$

where the last equality follows from the fact that $\bar{p}_i \equiv \phi \tilde{p}_i + (1 - \phi)\hat{p}_i$, where $\phi \equiv \frac{K_0}{K_0 - K_1} > 1$ and $(1 - \phi) \equiv \frac{-K_1}{K_0 - K_1} < 0$.

From (33) each will report $R_i^0 > R_i^1$ if $p_{\text{max}} > \tilde{p}_i$ and on the contrary it will report $R_i^1$ if $p_{\text{max}} < \bar{p}_i$. Therefore if $p_{\text{max}} > \bar{p}^u$ than $R_i^0 > R_i^1$ for all $i$ and $R_{i, \text{option}} = R_i^0$. This concludes the proof.

F Proof of Lemmas 3 and 4

Let us first consider the expected consumer surplus. We need to distinguish between the LP bidding rule format with and without time flexibility. Indicating the surplus for the first and second cases by $S^1$ and $S^0$ respectively, we get:

$$S^0 = E \left\{ \sum_{t=0}^{1} \frac{1}{(1 + \rho)^t} \int_{p_{i,\text{opt}}}^{p_{\text{max}}} y_t dp \right\} = \sum_{t=0}^{1} \frac{1}{(1 + \rho)^t} \int_{p_i(\tilde{p}_i)}^{p_{\text{max}}} E(y_t) dp$$

$$= (p_{\text{max}} - p_i(\tilde{p}_i))(y_0 + \sum_{t=1}^{1} \frac{1}{(1 + \rho)^t} E(y_t)) = (p_{\text{max}} - p_i(\tilde{p}_i))K_0$$

where it must be $p_{\text{max}} > \tilde{p}^u$ (assumption 7) to guarantee that whoever the winner is the surplus $S^0$ is always positive.

For $S^1$ we get:

$$S^1 = q \left\{ \frac{1}{(1 + \rho)} \int_{p_i(\tilde{p}_i)}^{p_{\text{max}}} y_i^+ dp \right\} = (p_{\text{max}} - p_i(\tilde{p}_i))q \frac{y_i^+}{(1 + \rho)}$$

$$= (p_{\text{max}} - p_i(\tilde{p}_i))(K_0 - K_1)$$

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where it must be $p^\max > \hat{p}^u$ (obviously so by assumption 7) to guarantee that whoever the winner is the surplus $S^1$ is always positive. Since the consumers do not know the winning bidder, the ex-ante surplus is given by:

$$E[S^0] = (p^\max - E[p_i(\tilde{p}_i)])K_0 \equiv (p^\max - \frac{2}{N+1}\tilde{p}^u)K_0$$

and:

$$E[S^1] = (p^\max - E[p_i(\tilde{p}_i))(K_0 - K_1) \equiv (p^\max - \frac{2}{N+1}\tilde{p}^u)(K_0 - K_1)$$

where $E[p_i(\tilde{p}_i)] = \frac{2}{N+1}\tilde{p}^u$ and $E[p_i(\bar{p}_i)] = \frac{2}{N+1}\bar{p}^u$ (Wolsftetter, 1999, p. 236).

Let us now turn to the expected revenue. Defining $V_i = \max [NPV_i^0, NPV_i^1]$, the bidder $i$’s expected payment is given by:

$$E[R_i] = R_i Pr(\text{win}) \equiv \frac{N-1}{N}V_i \left( \frac{V_i - V^i}{V^u - V^l} \right)^{N-1}$$

The government earns from each bidder an expected payment $E[R_i]$. Since the government does not know the bidders’ valuations, it takes an expected value:

$$E[E[R_i]] = \int_{V^l}^{V^u} E[R_i(V_i)] \frac{1}{V^u - V^l} dV_i \equiv \frac{N-1}{N} \left( \frac{1}{V^u - V^l} \right)^N \int_{V^l}^{V^u} V_i(V_i - V^l)^{N-1} dV_i$$

$$\equiv \frac{N-1}{N} \left( \frac{1}{V^u - V^l} \right)^N \left[ \frac{V_i^u(V_i - V^l)^N}{N} - \int_{V^l}^{V^u} \frac{V_i(V_i - V^l)^N}{N} dV_i \right]$$

$$\equiv \frac{N-1}{N} \left[ \frac{NV_i^u + V^l}{N(N+1)} \right] = \frac{N-1}{N(N+1)} V^u + \frac{N-1}{N^2(N+1)} V^l$$

from which we get:

$$E[R^\text{option}] = NE[E[R_i]] = \frac{N-1}{N+1} V^u + \frac{N-1}{N(N+1)} V^l$$

(34)

$$\equiv \frac{N-1}{N+1} \max [NPV_i^0, NPV_i^1] + \frac{N-1}{N(N+1)} \max [NPV_i^0, NPV_i^1]$$

Without time flexibility or for $p^\max > \tilde{p}^u$, (34) reduces to

$$E[R^0] = \frac{N-1}{N+1} NPV_i^0 + \frac{N-1}{N(N+1)} NPV_i^0$$

$$\equiv \frac{N-1}{N+1} p^\max K_0 + \frac{N-1}{N(N+1)} (p^\max - \tilde{p}^u)K_0$$

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While for \( \hat{p}^u < p^\text{max} < \hat{p}^u \), (34) reduces to:

\[
E[R^\text{option}] = \frac{N - 1}{N + 1} NPV_u^0 + \frac{N - 1}{N(N + 1)} NPV_l^1 \\
\equiv \frac{N - 1}{N + 1} (p^\text{max} - \hat{p}^u) K_0 + \frac{N - 1}{N(N + 1)} (p^\text{max} - \hat{p}^u)(K_0 - K_1) \\
\equiv E[R^0] - \frac{N - 1}{N(N + 1)} (p^\text{max} - \hat{p}^u) K_1 > E[R^0]
\]

This concludes the proof.

**G  Proof of Proposition 8**

Under the LP bidding rule, the difference between the two consumer surpluses becomes:

\[
E[S^1] - E[S^0] \equiv (p^\text{max} - \frac{2}{N + 1} \hat{p}^u)(K_0 - K_1) - (p^\text{max} - \frac{2}{N + 1} \hat{p}^u) K_1
\]

\[
\equiv \left[ -p^\text{max} + \frac{2}{N + 1} \hat{p}^u \right] K_1
\]

which is always negative for \( p^\text{max} > \hat{p}^u \). Under HF, by Lemma 4, the difference between the two expected fee is:

\[
E[R^\text{option}] - E[R^0] = 0 \quad \text{if} \quad p^\text{max} \geq \hat{p}^u
\]

and

\[
E[R^\text{option}] - E[R^0] = -\frac{N - 1}{N(N + 1)} (p^\text{max} - \hat{p}^u) K_1 > 0 \quad \text{if} \quad \hat{p}^u < p^\text{max} < \hat{p}^u
\]

This concludes the proof.
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