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Modeling Two Macro Policy Instruments
Interest Rates and Aggregate Capital Requirements

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Abstract

We present a simple neoclassical model to explore how an aggregate bank-capital requirement can be used as a macroeconomic policy tool and how this additional tool interacts with monetary policy. Aggregate bank-capital requirements should be adjusted when the economy is hit by cost-push shocks but should not respond to demand shocks. Moreover, an optimal institutional structure is characterized as follows: First, monetary policy is delegated to an independent and conservative central banker. Second, setting aggregate bank-capital requirements is separated from monetary policy.

Keywords: central banks, banking regulation, capital requirements, optimal monetary policy.

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1 Introduction

Modern economies usually operate in a corridor of stability, absorbing and smoothing out shocks continuously. Sometimes, however, they leave this corridor and enter a region of instability and crisis (see Leijonhufvud (1973)). The way in which economic policy should deal with these two facets of an economy in the sphere of money and banking has become a major issue in academia and policy-making.

According to pre-crisis consensus, monetary policy and banking regulation are responsible for two different objectives. Monetary policy is executed by central banks and focuses on stabilizing inflation and output in the stability corridor of the economy. Banking regulation aims at preventing the economy from leaving this corridor.

Monetary policy and banking regulation use different instruments. The central bank’s instrument is a short-term interest rate. While this instrument is perfectly sufficient to stabilize demand shocks, the stabilization of supply shocks, such as cost-push shocks, involves a trade-off. In line with Tinbergen’s rule (see Tinbergen (1952)), the central bank cannot achieve two objectives, output and inflation stabilization, perfectly if only one instrument is available. This problem might be further aggravated if central banks were also assigned a financial-stability objective (see De Grauwe and Gros (2009)). The most important instrument in banking regulation is a bank equity capital requirement. However, this instrument is not varied in response to aggregate fluctuations, so bank regulation concentrates on the microeconomic and bank-specific level.

The situation is complicated by the fact that banking crises, during which the economy leaves the stability corridor, involve major output losses and thus affect the central bank’s objectives. Central banks play an important role in the management of a banking crisis by providing sufficient liquidity. Accordingly, monetary policy and banking regulation will necessarily interact.

The purpose of this paper is twofold. First, we devise a simple model to study how bank-capital requirements can be used as an additional tool for stabilizing the economy and how this policy tool can and should interact with traditional monetary policy.\textsuperscript{1}

\textsuperscript{1}Kashyap et al. (2008) also argue for time-varying capital requirements.
We show that bank-capital requirements should be relaxed when the economy is hit by adverse supply shocks driving up inflation and reducing output. On average, lower capital requirements have a beneficial effect on output but entail a slightly higher probability of a banking crisis. Conversely, a supply shock that lowers inflation and increases output calls for stricter capital requirements in order to reduce the risk of a banking crisis.

Second, we examine whether central banks should also be responsible for bank-capital requirements or whether they should concentrate on monetary policy alone. Even in the absence of a classic inflation bias, it is optimal to delegate monetary policy to a conservative central banker. A conservative central banker does not give in to the temptation of output stabilization, which is ineffective in our model and merely causes socially harmful inflation deviations from its socially optimal level. However, if the conservative central banker were also responsible for the capital requirement, he would opt for an inefficiently high capital requirement, paying insufficient attention to its adverse impact on output. As a consequence, an optimal institutional structure requires the separation of bank-capital policy from the central bank. While an independent central bank is advantageous, our model provides no rationale for the creation of an additional independent banking regulator.

## 2 Relation to the Literature

We propose a simple model of banking regulation and monetary policy with two macroeconomic tools. The corresponding policy framework has been outlined in Gersbach (2011). To illustrate the potential working of both tools, we draw on three basic insights from the banking literature.²

1. **Banking crises are costly in terms of aggregate output.**

   The costs of banking crises are documented by Laeven and Valencia (2008).³

   Output losses amount to 20% of GDP over the first four years of the crisis and

²For an extensive account of financial crises in history see Reinhart and Rogoff (2009).
³Hoggarth et al. (2002) find that cumulative output losses are as high as 15-20% of annual GDP over the crisis period.
can be as large as 98% of GDP. We will model these losses as a drop in natural output. This drop can be the result either of a sharp decline in the bank supply of loans or of medium-term tax increases necessary to finance the bail-out of banks.

2. **Higher levels of bank equity tend to reduce the probability of banking crises.**

A higher level of bank equity reduces incentives for excessive risk-taking and improves the extent to which shocks can be absorbed, so it has a benign impact on the stability of the banking sector. Moreover, debtors are more confident that banks will be able to service their debt, which makes it easier for banks to roll over their debt, thus reducing the risk of liquidity problems.

3. **There are positive costs of high bank equity in terms of output.**

Due to the “debt overhang” problem identified by Myers (1977), banks that have to improve their equity ratio may be reluctant to raise new equity, although this would be socially desirable, because they do not take the positive externality on debt-holders into account. In this case, banks may cut back on lending, which would be socially harmful and entail lower output (for a discussion see Hanson et al. (2011)). In Gorton and Winton (2000), Diamond and Rajan (2000), and van den Heuvel (2008), high levels of bank equity are socially costly because they entail a reduction in banks’ ability to create liquidity. Gersbach (2003) presents a model in which non-financial firms competing for equity face tighter credit constraints when capital requirements are high\(^4\).

Our paper contributes to the discussion on the optimal institutional structure for banking supervision and monetary policy. Whether central banks should be responsible for banking supervision is still a contentious issue. While central banks like the Bank of England have been granted independence, they have also been stripped of their responsibilities for banking supervision in return. However, the central bank’s role as a lender of last resort may make it necessary to invest it with some authority with regard to bank supervision (see Goodhart (2002) for a review of these arguments). Peek et al.\(^4\)

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\(^4\)Admati et al. (2010) provide an extensive critical review of the arguments supporting the prevailing view that high levels of bank equity involve social costs.
(1999) provide evidence that information from banking supervision may be valuable for the conduct of monetary policy. Adrian and Shin (2009) argue that monetary policy and policies aiming at financial stability are inseparable, notably because of the link between short-term interest rates and the credit supply of financial intermediaries.

Finally, our paper is related to the discussion on whether financial stability should be an additional goal for central banks. This has been argued by De Grauwe and Gros (2009), who maintain that, at times, there is a tradeoff between price stability and financial stability. Cecchetti et al. (2000) have insisted that central banks adjust their instruments not only in response to their forecasts about future inflation and the output gap, but in response to asset prices as well (see also Borio and Lowe (2002)). Leaning against asset price bubbles may reduce the probability of these bubbles occurring in the first place. However, the conventional wisdom summarized in Mishkin (2001) is that monetary policy should only respond to asset price bubbles to the extent that bubbles have an effect on output and inflation through their impact on households’ wealth and thus consumption demand. First, it is inherently difficult for central banks to identify bubbles. Second, raising interest rates in response to asset price bubbles may not be very effective in containing bubbles and would endanger the other objectives of monetary policy.

3 Model

3.1 Motivation

The purpose of this exercise is to analyze how monetary policy interacts with equity capital requirements in a simple model. Our starting point is a standard neoclassical model in the tradition of Kydland and Prescott (1977) and Barro and Gordon (1983). In its standard form, the model has been derived from microeconomic foundations by Neiss (1999). To this model, we add equity capital of banks on an ad hoc basis. Nevertheless this combination of banking regulation from a macroeconomic perspective

5Schwartz (1988) argued that price stability is conducive to financial stability.
and monetary policy can offer several insights and outlines several directions of future inquiry.

First, our model is both analytically tractable and captures essential features of monetary policy and banking regulation. In particular, we are careful about adding equity capital and banking crises to the model in a way that is consistent with the existing banking literature, as detailed in the previous section. Second, our model identifies in the simplest way the conflicts of interest that may arise between bank-equity policies and monetary policy. Third, it points to essential features that more elaborate and fully-fledged microfounded models of monetary policy and banking regulation should capture.\(^6\)

To sum up, we propose the simplest yet plausible aggregate model with the moderate objective of studying monetary policy and financial regulation as well as the optimal institutional structure for both policies.

### 3.2 Set-up

The economy is populated by three actors: a central bank, a financial regulator, and the public. Demand is described by an IS curve:

\[
y = y_0 - \alpha (i - \pi^e) + \mu, \tag{1}
\]

where \(y\) denotes demand, \(y_0\) natural output, \(i\) the nominal interest rate,\(^7\) and \(\pi^e\) the inflation expectations of the public.\(^8\) Parameter \(\alpha\) is strictly positive, and demand is subject to a shock \(\mu\) with expected value 0 drawn from an otherwise arbitrary distribution.

\(^6\)However, one might also argue that general equilibrium analysis based on microeconomic foundations, despite its numerous advantages (see Ljungqvist and Sargent (2004, pp. xxvi-xxvii)), may be less appropriate to model times of banking crises than periods of relative economic tranquility. This would be in line with the view that severe crises cannot be adequately described by markets cleared by a Walrasian auctioneer (see Leijonhufvud (1981)).

\(^7\)We have normalized the natural real rate of interest to zero, so \(i\) should be interpreted as the difference between the nominal interest rate and the natural real rate of interest. If the natural real rate of interest were different in a banking crisis than in normal times, this would have no impact on our findings.

\(^8\)An interesting future extension to our model would take into account the fact that the interest rate may reach the zero lower bound in the event of a banking crisis.
Supply is described by a Phillips curve

\[ \pi = \pi^e + \beta(y - y_0) + \varepsilon, \]  

(2)

where \( \pi \) denotes the rate of inflation and \( \beta \) is a strictly positive parameter. The supply shock \( \varepsilon \) has an expected value of zero and a distribution function with finite support \( [\bar{\varepsilon}, \bar{\varepsilon}] \) (\( \bar{\varepsilon} < 0 < \bar{\varepsilon} \)).

As a next step, we integrate capital requirements for banks into this otherwise completely standard model. As mentioned before, capital requirements have two effects on our economy. First, higher capital requirements make banking crises less likely. To model the relationship between capital requirements and the probability of a banking crisis, we introduce the indicator variable \( \delta \), which is identical to one in the case of a banking crisis and to zero otherwise. For simplicity, we assume a linear relationship between the probability of a banking crisis and the capital requirement \( E \):

\[ \delta = \begin{cases} 
0 & \text{with probability } 1 - \sigma + \phi E \\
1 & \text{with probability } \sigma - \phi E 
\end{cases} \]

(3)

Parameter \( \sigma \) (\( 0 < \sigma < 1 \)) represents the probability of a banking crisis in the absence of a capital requirement (\( E = 0 \)). Parameter \( \phi \) (\( \phi > 0 \)) describes how strongly an increase in the capital requirement affects the probability of a banking crisis. The capital requirement can be chosen from the interval \( [0, \sigma/\phi] \) by the regulator. Henceforth we will refer to an economy in a banking crisis as an economy in state \( B \) (bad). We will use \( G \) (good) to describe a situation without a banking crisis. As is well-documented, banking crises involve substantial output losses. We assume that a banking crisis causes natural output to drop by \( \Delta \) (\( \Delta > 0 \)).

Second, we assume that higher capital requirements also involve a social cost. More specifically, an increase in capital requirements leads to a proportional decrease in

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\(^9\)The assumption of finite support is made for analytical convenience. For appropriately chosen \( \varepsilon \) and \( \bar{\varepsilon} \), the regulator will always choose an interior value for the capital requirement.

\(^{10}\)We assume that all shocks are independent. One might argue that a supply shock that boosts output but not inflation will make banking crises more likely. This would not affect our findings in Section 4, because one would merely have to re-interpret the probability of a banking crisis in (3) as the respective probability conditional on a particular realization of the supply shock.
natural output, where the factor of proportionality is \( b \) \((b > 0)\). According to these considerations, natural output can be written in the following way:

\[
y_0 = \overline{y} - bE - \delta \Delta,
\]

(4)

where \( \overline{y} \) is the level of natural output without capital requirements and without a banking crisis.

### 3.3 Loss functions

As is standard in the neoclassical framework, we assume that the social loss function captures deviations of both inflation and output from their socially optimal levels. More specifically, we adopt the following quadratic specification:

\[
\tilde{L} = \pi^2 + \tilde{a}(y - \tilde{y}^*)^2,
\]

(5)

where \( \tilde{a} \) is the relative weight on output stabilization and \( \tilde{y}^* \) is the output target. Without loss of generality, we set \( \tilde{y}^* \) to zero. As a consequence, output is measured in terms of deviations from the socially optimal level. In order to study optimal delegation, we allow for the possibility of the financial regulator (denoted by subscript \( R \)) and the central bank (without subscript) having different objective functions

\[
L = \pi^2 + a(y - y^*)^2,
\]

(6)

\[
L_R = \pi^2 + a_R(y - y^*_R)^2,
\]

(7)

with weights \( a, a_R > 0 \) and output targets \( y^* \) and \( y^*_R \). We do not make assumptions on the signs of \( y^* \) and \( y^*_R \). For example, \( y^* < 0 \) would imply that the central bank is targeting an output level below the socially optimal level of output. In the tradition of Rogoff (1985), a benevolent government can delegate monetary policy and banking supervision to authorities with objectives that are different from the social ones.

### 3.4 Sequence of events

We adopt the following assumption about the sequence of events:
1. Shocks $\varepsilon$ and $\mu$ materialize.

2. The regulator chooses the capital requirement $E$.

3. The public forms its inflation expectations $\pi^e$.

4. A banking crisis may occur. Then the economy is in state $B$, otherwise it is in state $G$.

5. The central bank chooses the interest rate $i$.

A few comments are in order regarding this timing structure. Our aim is to capture the effect that monetary policy can help to alleviate the consequences of a banking crisis. Hence we place the stage in which nature determines whether a crisis will occur before the central bank’s move but after the formation of inflation expectations. In addition, we want to describe the consequences of banking regulation for risk-taking and thus for the probability of a crisis. In line with this objective, the regulator moves before nature decides on whether a crisis will occur. Shocks materialize in the initial stage because we are out to describe the optimal response of the regulator to these shocks.

### 3.5 Parameter restrictions

We complete the description of our model by imposing three restrictions on the set of admissible parameter values. These restrictions ensure that the regulator will choose an interior solution of $E$. First, we require

$$b > \frac{a + 2\beta^2}{a + \beta^2} \phi \Delta.$$ (8)

This assumption ensures that the first-order condition of the regulator’s optimization problem does indeed correspond to a minimum. Otherwise the regulator would either choose the minimum level $E = 0$ or the maximum level $E = \sigma / \phi$.

Condition (8) has the implication $b > \phi \Delta$, which entails that expected natural output conditional on $E$, which can be immediately derived from (4) as

$$y^*_e = \bar{y} - b E - (\sigma - \phi E) \Delta,$$ (9)
is a decreasing function of $E$. Hence, setting the regulatory equity requirement involves a tradeoff. A higher level of $E$ leads to a lower probability of a banking crisis. At the same time, it reduces the expected level of output.

We need to introduce two additional assumptions to ensure an interior solution of the regulator’s optimization problem:

\[(a + \beta^2)^2(b - \phi \Delta)(\bar{y} - y_{av}^*) + \phi \left( (a + 2\beta^2)\sigma a + \frac{1}{2}\beta^4 \Delta^2 \right) > b(a + \beta^2)^2 \sigma \Delta, \quad (10) \]

\[(a + \beta^2)^2(b - \phi \Delta)(\bar{y} - y_{av}^*) + \frac{1}{2}\beta^4 \phi^2 \Delta^2 < (a + \beta^2)^2(b - \phi \Delta)\sigma b, \quad (11) \]

where we have introduced $y_{av}^*$, the weighted average of the output targets $y^*$ and $y_R^*$, as

\[y_{av}^* := \frac{a^2 y^* + a_R \beta^2 y_R^*}{a^2 + a_R \beta^2}. \quad (12)\]

Effectively, (10) represents an upper bound for $y_{av}^*$ and thus also for $y_R^*$. This guarantees that the regulator will not choose the corner solution $E = 0$ in order to boost expected output as much as possible. By contrast, (11) imposes a lower bound on $y_R^*$. If $y_R^*$ were lower than this bound, then the regulator would strive for extreme safety and choose the maximum possible value of $E$ to eliminate the possibility of a banking crisis.\(^{11}\)

### 4 Equilibrium

In Appendix A we show that optimal monetary policy, conditional on a particular value of $E$, results in

**Proposition 1**

In cases $B$ (bad) and $G$ (good), inflation as a function of $E$ amounts to

\[\pi_B = \frac{a}{\beta} \left( y^* - \bar{y} + bE + \frac{a(\sigma - \phi E) + \beta^2}{a + \beta^2} \Delta + \frac{\varepsilon}{\beta} \right), \quad (13)\]

\[\pi_G = \frac{a}{\beta} \left( y^* - \bar{y} + bE + \frac{a(\sigma - \phi E) + \Delta + \frac{\varepsilon}{\beta}}{a + \beta^2} \right), \quad (14)\]

\(^{11}\)It is straightforward to show that for every admissible combination of the other parameters, the set of $y_{av}^*$ for which both (10) and (11) jointly hold is non-empty.
In both cases, output, conditional on $E$, is given by
\begin{align}
y_G &= \overline{y} - bE - \frac{a(\sigma - \phi E)}{a + \beta^2} \Delta - \frac{\varepsilon}{\beta}, \\
y_B &= \overline{y} - bE - \frac{a(\sigma - \phi E) + \beta^2}{a + \beta^2} \Delta - \frac{\varepsilon}{\beta}.
\end{align}

At stage 3, expected inflation and output are
\begin{align}
\pi^e &= \frac{a}{\beta} \left( y^* - \overline{y} + bE + (\sigma - \phi E) \Delta + \frac{\varepsilon}{\beta} \right), \\
y^e &= \overline{y} - bE - (\sigma - \phi E) \Delta - \frac{\varepsilon}{\beta}.
\end{align}

It is instructive to look at these findings more closely. We focus on the impact of the following factors on monetary policy: the demand shock $\mu$, the shock to the Phillips curve $\varepsilon$, the level of equity $E$ required by the regulator, the realization of the state $B$ or $G$, and parameter $y^*$ of the central bank’s loss function.

First, it is apparent that the demand shock $\mu$ does not show up in equations (13)-(18). This is plausible, as the central bank can perfectly stabilize demand shocks because they do not involve a tradeoff between output and inflation stabilization. As a result, the demand shock has an impact on the level of interest rates $i$ but not on output and inflation.

Second, we discuss the impact of supply shock $\varepsilon$. A positive realization drives up inflation and causes a decline in output for both cases $B$ and $G$. This is completely standard. It is worth mentioning that the central bank cannot dampen the effect of shock $\varepsilon$ on output. Irrespective of the value of $a$, which represents the weight on output stabilization in the central bank’s loss function, the impact of $\varepsilon$ on $y$ is given by $\varepsilon/\beta$. This is a consequence of our assumption that the shock realization is known when inflation expectations are formed. Thus the central bank cannot deliver shock-dependent deviations of inflation from its expected value $(\pi - \pi^e)$, which would enable it to moderate the shock. By contrast, the consequences of $\varepsilon$ on inflation depend on the size of $a$. This results from the central bank’s futile attempts to moderate the shock, which lead to a varying inflation bias. From the point of view of minimizing the fluctuations of inflation and output created by shock $\varepsilon$, it would thus be optimal...
to delegate monetary policy to a conservative central bank that is indifferent to the stabilization of output \((a = 0)\).

Third, we discuss the role of \(E\). As we have seen, (8) guarantees that the expected value of natural output \(y_0\) will be a decreasing function of \(E\), as the harmful impact of equity requirements on \(y_0\) is stronger in expected terms than the beneficial effect arising from declines in the probability of a banking crisis. This relationship is also reflected by Proposition 1, as \(y_B, y_G,\) and \(y^e\) are decreasing in \(E\). Because higher levels of \(E\) shift output away from the central bank’s target, the central bank has a stronger incentive to boost output by increasing inflation. The public sees through such attempts, which are therefore unsuccessful in increasing output. But as a result inflation is higher, the stricter the capital requirement \(E\) is (it can be verified directly that the derivatives of (13), (14), and (17) with respect to \(E\) are strictly positive).

Fourth, what is the impact of the realization of state \(B\) or \(G\) for a given level of \(E\)? Comparing (15) and (16) reveals that output is lower in state \(B\), which is an obvious consequence of our assumption that a banking crisis leads to a drop in natural output of size \(\Delta\). Importantly, the difference between \(y_G\) and \(y_B\) amounts to \(\frac{(\beta^2}{a + \beta^e}\Delta\) and is thus smaller than \(\Delta\). Hence the central bank is successful in moderating the impact of banking crises on output to some extent. This is an implication of our assumption that inflation expectations are formed prior to the stage where nature determines whether a banking crisis will occur. As a result, the central bank can engineer a somewhat higher than expected rate of inflation in the event of a crisis, thus increasing output, and somewhat lower inflation in the absence of a crisis, thus entailing a decrease in output. We conclude our discussion of states \(B\) and \(G\) by emphasizing the plausible assertion that the difference between output in the good state and the bad state is a decreasing function of \(a\). From the perspective of moderating the adverse consequences of banking crises for output variance, a conservative central bank \((a = 0)\) would thus be detrimental.

Finally, we explore the relevance of the output target \(y^*\) for the outcomes of monetary policy. In a neoclassical framework with rational expectations, the central bank cannot
systematically increase output. This is reflected by the fact that (15), (16), and (18) are independent of $y^*$. By contrast, higher levels of $y^*$ make it more attractive for the central bank to attempt to increase output by inflationary policy. This leads to an inflation bias. In line with these considerations, (13), (14), and (17) are increasing functions of $y^*$.

Having outlined the optimal response of the central bank to shocks and the regulator’s choice, we turn in the next proposition to the capital requirement set by the regulator.

**Proposition 2**

In equilibrium, the regulator’s choice of $E$ can be written as

$$E = C_1 \left( \bar{y} - y^*_{av} - \frac{\varepsilon}{\beta} \right) + C_2,$$

where $C_1$ and $C_2$ are constants independent of $\bar{y}$, $y^*_{av}$, and $\varepsilon$ with $C_1 > 0$.

The proposition is proved in Appendix A.

Proposition 2 shows that the capital requirement $E$ is unaffected by demand shock $\mu$. This is intuitive, as the regulator anticipates that the central bank will neutralize this shock’s effect on output and inflation. By contrast, the capital requirement is a negative function of the Phillips curve shock $\varepsilon$. A positive realization of the shock will tend to reduce output, which will induce the regulator to relax capital requirements in order to increase output in expected terms (compare (18)). On the downside, this also raises the probability of a banking crisis.

Next we focus on some comparative statics. A higher output objective of the regulator and thus a higher value of $y^*_{av}$ results in lower capital requirements. In this case, the regulator will be more inclined to take a higher risk of a crisis in exchange for a higher average output level. By contrast, a rise in the maximum possible value $\bar{y}$ of natural output will make the regulator more cautious in the sense of opting for a higher capital requirement. If $\bar{y}$ is large so that output is high anyway, further increasing output by relaxing capital requirements is less attractive. In addition, we observe that a higher value of the central bank’s output target $y^*$ also entails a higher value of $y^*_{av}$ and
accordingly lower capital requirements. The negative relationship between the central bank’s output target and the regulator’s choice of \( E \) can be explained in the following way: If the central bank has a high output target, this will create large incentives for the central bank to increase output by inflationary policies. Obviously, this will not lead to output gains but to high inflation rates. The regulator anticipates this and opts for loose capital requirements, which raise output on average and thus dampen the central bank’s incentives to choose high inflation rates.

Finally, we note that parameters \( y^*_R \) and \( a_R \) in the regulator’s loss function only impact on the equity requirement chosen by the regulator through their impact on \( y^*_\text{av} \) (see (19)). Consequently, all combinations of \( y^*_R \) and \( a_R \) leading to the same value of \( y^*_\text{av} \) are observationally equivalent and above all involve the same levels of social welfare. In particular, it would be possible to assume, without loss of generality, that the financial regulator only has an output target \((a_R \to \infty)\). Then \( y^*_\text{av} = y^*_R \) would hold (see (12)).

Combining our findings from Propositions 1 and 2, we can derive equilibrium inflation and output in the contingencies \( B \) and \( G \) and the respective levels expected at stage 3 of the game if the regulator chooses \( E \) optimally:

**Corollary 1**

In equilibrium, inflation and output are given by

\[
\begin{align*}
  y_G &= \frac{(a + \beta^2)(b - \phi\Delta)y^*_\text{av} + \beta^2\phi\Delta \left( \frac{\varepsilon}{\beta} - \bar{y} \right) - \frac{1}{2} \frac{\phi\Delta^2 \beta^4}{a + \beta^2} + \Delta\beta^2 \sigma b}{b(a + \beta^2) - \phi\Delta(a + 2\beta^2)}, \\
  y_B &= y_G - \frac{\beta^2\Delta}{a + \beta^2}, \\
  y^e &= \frac{\phi^2 \beta^3 \Delta^2 \left( \frac{\varepsilon}{\beta} - \bar{y} \right) + (b - \phi\Delta)^2(a + \beta^2)^2y^*_\text{av} + (\sigma b - \frac{1}{2}(b - \phi\Delta)) \phi\beta^4 \Delta^2}{((a + \beta^2)b - a\phi\Delta)((a + \beta^2)b - (a + 2\beta^2)\phi\Delta)}, \\
  \pi_G &= -\frac{a(y_G - y^*)}{\beta}, \\
  \pi_B &= -\frac{a(y_B - y^*)}{\beta}, \\
  \pi^e &= -\frac{a(y^e - y^*)}{\beta}.
\end{align*}
\]

Four implications of this corollary are worth mentioning. First, an increase in \( y^*_R \), which is the regulator’s output target, leads to higher output \((y_G, y_B, \text{ and } y^e \text{ are increasing in } y^*_\text{av} \text{ and thus in } y^*_R)\). Second, and somewhat surprisingly, output is decreasing in \( \bar{y} \), which is the maximum possible level of natural output. An increase in \( \bar{y} \) has two
effects on output in our model. On the one hand, it increases output for a given level of $E$. On the other, as explained in the discussion following Proposition 2, it also raises the regulator’s choice of $E$ and thus reduces the average level of output. On balance, the second effect is stronger, which explains the negative relationship between equilibrium output and $\bar{\gamma}$. Third, it is instructive to consider simultaneous increases of $\bar{\gamma}$, $y^*$, and $y_*^R$ of the same size. Plausibly, this raises $y_G$, $y_B$, and $y^e$ by the same amount, leaving inflation constant. Fourth, and again surprisingly, a positive realization of shock $\varepsilon$ increases output but lowers inflation, which contrasts with the behavior in Proposition 1, where, for a given level of $E$, a positive shock leads to lower output and higher inflation. The intuition for this finding is related to the one given for the negative relationship between output and $\bar{\gamma}$, as the effect of an increase in $\varepsilon$ is analogous to a decrease in $\bar{\gamma}$, which can be confirmed from Proposition 1.

5 Optimal Delegation

In this section we analyze two questions about optimal delegation and the optimal institutional structure for central banking and banking supervision. We take the perspective that policies can be delegated to independent bodies whose objectives can be determined either by the selection of a policy-maker with appropriate preferences from a pool of candidates\textsuperscript{12} or by incentive contracts (see Walsh (1995)).

First, we examine whether a single authority or two different bodies should be responsible for monetary policy and bank-equity policy. We show that, in an optimal institutional structure, bank-equity policy is separated from central banking. Second, the previous finding raises the follow-up question of whether it is advantageous to assign bank-equity policy to an additional independent authority or to leave it under the auspices of elected politicians. We demonstrate that the creation of an additional independent authority for banking regulation involves no benefits for society.

We start with the comparison of two scenarios, one with separate bodies responsible for the two policy tools considered in this paper, the other with a single authority.\textsuperscript{12}

\textsuperscript{12} The literature on delegation of monetary policy to a conservative central banker goes back to Rogoff (1985).
In the first case, we assume that the central bank and the banking regulator have loss functions characterized by different parameters, all of which are chosen optimally from a perspective of ex-ante welfare. In the second case, both policies are assigned to a single authority whose preferences are described by a loss function with optimally chosen parameters. Formally, optimal delegation corresponds to the determination of optimal values for $a$, $a_R$, $y^*$, and $y^*_R$. In the scenario in which a single authority is responsible for both policies, the optimal values are chosen subject to the additional restrictions $a = a_R$ and $y^* = y^*_R$.

By construction, delegation to a single authority can never yield superior values of welfare. However, we have observed that all combinations of $a_R$ and $y^*_R$ that result in the same value of $y^*_av$ (see (12)) lead to equivalent levels of welfare. This flexibility in choosing $a_R$ and $y^*_R$ makes it plausible that delegation to a single authority may not involve welfare losses. In Appendix B, we show that this conjecture is incorrect.

**Proposition 3**

*Delegating monetary policy and bank-equity policy to a single authority is strictly inferior to the delegation of these policies to two different bodies.*

Intuitively, it is optimal to appoint a conservative central banker in our model. Even in the absence of a classic inflation bias, a conservative central banker is less tempted to stabilize the impact of supply shocks $\varepsilon$ on output. These attempts are ultimately futile because shock $\varepsilon$ is known when the public forms its expectations about inflation. However, they lead to a high variance of inflation and are therefore socially costly. So while it is beneficial to make a conservative central banker responsible for monetary policy, it is socially costly to endow this conservative central banker with the additional task of choosing capital requirements because he would choose too restrictive a value of $E$.

As a next step, we ask whether delegation of banking regulation to a separate authority is advantageous. To address this question, we examine whether the optimal choice of $a$, $a_R$, $y^*$, and $y^*_R$ is consistent with $a_R = \tilde{a}$ and $y^*_R = 0$. This is indeed the case.
Proposition 4

The optimal institutional structure does not require delegating banking regulation to an independent authority whose preferences differ from those compatible with social welfare.

The proof is given in Appendix C. The proposition confirms our previous claim that it is not optimal to let a conservative central banker decide on equity-capital requirements. While a multitude of combinations of $a_R$ and $y_R^*$ would entail the optimal value of $y_{av}^*$, one solution is $a_R = \tilde{a}$ and $y_R^* = 0$, in which case the banking regulator shares society’s preferences.

6 Conclusions

In this paper we have proposed a model with two policy instruments: a conventional short-term interest-rate and an aggregate equity requirement for the banking sector. First, we have shown how both instruments can be used in the event of shocks. In particular, a supply shock that reduces output and increases inflation requires lower capital requirements, which on average have a benign effect on output but increase the risk of a banking crisis. Conversely, a shock that boosts output and lowers inflation induces stricter capital requirements. Second, we have characterized the optimal institutional structure for monetary policy and banking regulation. In this optimal structure, the power to set the aggregate equity requirement has to be separated from monetary policy. Moreover, while it is advantageous to delegate monetary policy to an independent central bank, our model provides no rationale for the delegation of the equity capital tool to an independent authority.
A Derivation of the Equilibrium

The equilibrium in our economy can be derived by backward induction. First, we derive the central bank’s optimal monetary policy. Second, we compute the public’s inflation expectations. Finally, we determine the optimal capital requirement set by the regulator.

1. The central bank’s optimal choice of $i$: As (1) is a one-to-one relationship between $i$ and $y$ for given $\pi^e$, $\mu$, and $y_0$, the central bank can achieve any value of $y$ by selecting the appropriate value of $i$. Thus the central bank’s optimization problem is equivalent to a minimization of (6) with respect to $y$, subject to (2). This yields the following first-order condition:

$$0 = 2\pi \frac{d\pi}{dy} + 2a(y - y^*) = 2\beta\pi + 2a(y - y^*), \quad (20)$$

where we have utilized $\frac{d\pi}{dy} = \beta$, which follows from (2). Using (2) again to replace $y$ in (20) and solving for $\pi$, we obtain the value of inflation as a function of $\pi^e$, $\varepsilon$, and $y_0$:

$$\pi = \frac{a}{a + \beta^2} \cdot (\pi^e + \beta(y^* - y_0) + \varepsilon) \quad (21)$$

We observe that $\mu$ does not appear in this equation. This is a consequence of the fact that demand shocks can be perfectly stabilized by the central bank.

2. Derivation of inflation expectations: Next we compute the public’s inflation expectations as a function of $\varepsilon$ and $E$. For this purpose, we note that the public expects natural output $y_0$ to amount to

$$y_0^e = \overline{y} - bE - (\sigma - \phi E)\Delta, \quad (22)$$

which relies on (4) and the observation that the indicator variable $\delta$, which is one for a banking crisis and zero otherwise, has an expected value of $\sigma - \phi E$. Taking expectations for (21) yields

$$\pi^e = \frac{a}{a + \beta^2} \cdot (\pi^e + \beta(y^* - y_0^e) + \varepsilon), \quad (23)$$
Replacing $y^e_0$ by the expression in (22) and re-arranging gives

$$
\pi^e = \frac{a}{\beta} \left( y^* - \overline{y} + bE + (\sigma - \phi E)\Delta + \frac{\varepsilon}{\beta} \right).
$$

(24)

3. Derivation of the optimal value of $E$ for a given realization of $\varepsilon$: To determine the optimal equity requirement as a function of the supply shock $\varepsilon$, a few preliminary steps are necessary. First, we determine inflation for the two different realizations of $\delta$. Inserting (4) and (24) into (21) and simplifying gives

$$
\pi = \frac{a}{\beta} \left( y^* - \overline{y} + bE + \frac{\varepsilon}{\beta} + \frac{a(\sigma - \phi E) + \beta^2 \delta}{a + \beta^2} \Delta \right).
$$

(25)

We introduce subscript $B$ for a bad realization of $\delta$, i.e. a banking crisis ($\delta = 1$), and $G$ for a good realization, where there is no banking crisis ($\delta = 0$). Evaluating (25) at $\delta = 0$ and $\delta = 1$ results in the following expressions for $\pi_G$ and $\pi_B$:

$$
\pi_G = \frac{a}{\beta} \left( y^* - \overline{y} + bE + \frac{\varepsilon}{\beta} + \frac{a(\sigma - \phi E) + \beta^2 \delta}{a + \beta^2} \Delta \right),
$$

(26)

$$
\pi_B = \frac{a}{\beta} \left( y^* - \overline{y} + bE + \frac{\varepsilon}{\beta} + \frac{a(\sigma - \phi E) + \beta^2 \delta}{a + \beta^2} \Delta \right).
$$

(27)

Output can be determined by solving (2) for $y$ and inserting (4), (24), and (25):

$$
y = \overline{y} - bE - \frac{\varepsilon}{\beta} - \frac{a(\sigma - \phi E) + \beta^2 \delta}{a + \beta^2} \Delta
$$

(28)

The good and bad realizations of output $y_G$ and $y_B$ correspond to (28) for $\delta = 0$ and $\delta = 1$:

$$
y_G = \overline{y} - bE - \frac{\varepsilon}{\beta} - \frac{a(\sigma - \phi E) + \beta^2 \delta}{a + \beta^2} \Delta
$$

(29)

$$
y_B = \overline{y} - bE - \frac{\varepsilon}{\beta} - \frac{a(\sigma - \phi E) + \beta^2 \delta}{a + \beta^2} \Delta
$$

(30)

Because the expected value of $\delta$ is $\sigma - \phi E$, (28) implies the expression for $y^e$ given in (18). After these preliminary steps, we can formulate the regulator’s optimization problem in the following way:

$$
\min_E \left\{ (1 - \sigma + \phi E) \left( \pi_G^2 + aR(y^*_R - y_G)^2 \right) + (\sigma - \phi E) \left( \pi_B^2 + aR(y^*_R - y_B)^2 \right) \right\},
$$

(31)

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where we have taken into account that the good state occurs with probability $1 - \sigma + \phi E$ and the bad one with probability $\sigma - \phi E$. It is tedious but straightforward to solve the first-order condition for $E$: \begin{equation}
E = C_1 \left( \bar{y} - y^*_av - \frac{\varepsilon}{\beta} \right) + C_2, \tag{32} \end{equation}
where \begin{align*}
C_1 &= \frac{(a + \beta^2)^2(b - \phi \Delta)}{((a + \beta^2)b - a\phi \Delta)((a + \beta^2)b - (a + 2\beta^2)\phi \Delta)}, \tag{33} \\
C_2 &= \frac{\phi ((a + 2\beta^2)\sigma a + \frac{1}{2}\beta^4) \Delta^2 - b(a + \beta^2)^2\sigma \Delta}{((a + \beta^2)b - a\phi \Delta)((a + \beta^2)b - (a + 2\beta^2)\phi \Delta)}, \tag{34} \end{align*}
For (32), we have used the definition of the weighted output target $y^*_av$
\begin{equation} y^*_av = \frac{a^2y^* + a_R\beta^2y^*_R}{a^2 + a_R\beta^2}. \tag{35} \end{equation}
We also have to check the second-order condition to confirm that the value of $E$ stated in (32) corresponds to a minimum of the regulator’s expected losses. It is again tedious but not difficult to verify that the second derivative of the regulator’s expected losses amounts to
\begin{equation} \frac{((a + \beta^2)b - a\phi \Delta)((a + \beta^2)b - (a + 2\beta^2)\phi \Delta)(a^2 + a_R\beta^2)}{\beta^2(a + \beta^2)^2}. \tag{36} \end{equation}
Assumption (8) guarantees that this expression is positive. Finally, we have to check whether (32) represents an interior solution for some support of $\varepsilon$, i.e. some combination of $\varepsilon$ and $\tau$. This is the case if two assumptions are fulfilled. First, $E > 0$ must hold for $\varepsilon = 0$, which is equivalent to
\begin{equation} (a + \beta^2)^2(b - \phi \Delta)(\bar{y} - y^*_av) + \phi \left( (a + 2\beta^2)\sigma a + \frac{1}{2}\beta^4 \right) \Delta^2 > b(a + \beta^2)^2\sigma \Delta, \tag{37} \end{equation}
where we have used Assumption (8). Inequality (37) is identical to Assumption (10).
\footnote{The attentive reader may wonder why there is only one solution to the first-order condition. In each case, $B$ and $G$, the regulators’ loss function is quadratic in $E$. Moreover, the probability of $B$ or $G$ occurring is linear in $E$. This suggests that the regulator’s expected losses are a polynomial of degree three. However, the terms in the regulator’s expected losses of the order $E^3$ cancel each other out. As a result, the minimand in (31) is quadratic in $E$ and a unique extremum obtains.}
Second, we must ensure that $E$ does not exceed its maximum possible value $\sigma/\phi$, which implies that the probability of a banking crisis is zero. Inserting (32) into $E < \sigma/\phi$ and re-arranging yields

$$
\phi(a + \beta^2)^2(b - \phi \Delta)(\overline{y} - y_{av}^*) + \phi^2 \left((a + 2\beta^2)\sigma a + \frac{1}{2}\beta^4\right) \Delta^2
$$

$$
< \left((a + \beta^2)b - (a + 2\beta^2)\phi \Delta\right) \left((a + \beta^2)b - a \phi \Delta\right) \sigma + \phi b(a + \beta^2)^2 \sigma \Delta.
$$

(38)

Assumption (11) guarantees that this requirement holds, as can be readily shown.

□

\section*{B Proof of Proposition 3}

In order to consider the optimal choices of $a$, $a_R$, $y^*$, and $y_R^*$, we derive expected social losses (see (5)) from (13)-(16), (19), and the facts that the probability of state $B$ is $\sigma - \phi E$ and the probability of state $G$ is $1 - \sigma + \phi E$. This gives expected social losses as a function of $a$, $y^*$, and $y_{av}^*$. Because the respective expression is unwieldy, we refrain from stating it here. The first-order condition with regard to $y_{av}^*$ yields the following expression:

$$
y_{av}^* = \frac{a^2 y^*}{a^2 + \tilde{a} \beta^2}.
$$

(39)

Recall that $y_{av}^*$ is defined as (see (12))

$$
y_{av}^* = \frac{a^2 y^* + a_R \beta^2 y_R^*}{a^2 + a_R \beta^2}.
$$

(40)

Suppose that optimal delegation were possible with $y_R^* = y^*$. Then (40) simplifies to

$$
y_{av}^* = y^*.
$$

(41)

As a result, we obtain a contradiction because according to (39) $y_{av}^* \neq y^*$. Therefore optimal delegation always requires $y^* \neq y_R^*$.\footnote{Numerical examples are available on request.}
C Proof of Proposition 4

Suppose $a_R = a$ and $y_R^* = 0$, i.e. the regulator’s loss function is identical to social losses. Then the definition of $y_{av}^*$ (see (12)) implies

$$y_{av}^* = \frac{a^2 y^* + a_R \beta^2 y_R^*}{a^2 + a_R \beta^2} = \frac{a^2 y^*}{a^2 + \tilde{a} \beta^2}.$$  

This is equivalent to (39), which guarantees an optimal choice of $y_{av}^*$. Hence the optimal choice of $a$, $a_R$, $y^*$, and $y_R^*$ is consistent with $a_R = \tilde{a}$ and $y_R = 0$, i.e. with a bank regulator sharing the preferences of society. □
References


