Das, Satya P.

Working Paper

International trade and polarization in the labor market

Economics Discussion Papers, No. 2011-48

Provided in Cooperation with:
Kiel Institute for the World Economy (IfW)

Suggested Citation: Das, Satya P. (2011) : International trade and polarization in the labor market, Economics Discussion Papers, No. 2011-48, Kiel Institute for the World Economy (IfW), Kiel

This Version is available at:
http://hdl.handle.net/10419/52229

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes. You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public. If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

http://creativecommons.org/licenses/by-nc/2.0/de/deed.en
International Trade and Polarization in the Labor Market

Satya P. Das
Indian Statistical Institute, Delhi

Abstract The paper builds an argument that international trade can be one explanation behind polarization of employment in the labor market observed in developed countries such as U.K. and U.S. It considers a small open economy, having production sectors which use three types of labor: high-skill, middle-skill and low-skill. The economy faces an increase in the relative price of the high-skill intensive sector. Using decision rules for choosing middle-skill and low-skill education, it is shown that such a terms of trade shock can lead to higher shares of high-skill as well as low-skill workers in the total workforce. The effects off-shoring on wages and job composition are also studied. That of low-skill and high-skill tasks, not middle-skill tasks, is shown to contribute towards polarization in job composition.

JEL F16, J21
Keywords Polarization in labor markets; hollowing out; skill biased technical change; terms of trade; off-shoring

Correspondence Satya P. Das, Indian Statistical Institute, Delhi Centre, 7 SJS Sansanwal Marg, New Delhi 110 016, India, e-mail: das@isid.ac.in

The author is thankful to Chia-Hui Lu for initial discussions on this topic. Parts of the research was completed during his visit to the Xavier Institute of Management, India in 2010 and Utkal University, India in 2011.
1 Introduction

In the last two decades a huge literature, both empirical and theoretical, has emerged on the rising global trend of wage inequality since the 70s and the associated changes in employment patterns. This has been attributed mainly to skill-biased technical change (SBTC), but also significantly to freer international trade between developed and developing countries. See, for instance, Bernard and Jensen (1997), Chapter 4 in Feenstra (2004), Bivens (2007) and Krugman (2008), among many others.

Taking into account relatively more recent data, 1987-2004 for the U.S. and 1975-1995 for the U.K., a more specific – and an interesting – pattern of wage distribution and employment has been discerned, compared to the earlier literature: namely, a continuing positive trend of wage inequality in the top half of the distribution (for both U.S. and U.K.), coupled with a slightly declining trend in the U.S. and a mildly positive trend in the U.K. in the bottom half of the distribution. Furthermore, in both economies the shares of high-wage as well as low-wage jobs have increased, while that of middle-wage jobs has declined.

These wage-employment-pattern changes have been documented by Autor et al. (2006) for the U.S. economy (see Autor et al. (2008) also) and by Goos and Manning (2007) for the British economy. Spitz-Oener (2006) finds similar patterns of employment changes for Germany. Goos and Manning has aptly depicted such a change in the employment pattern as a growth of both “Lousy and Lovely Jobs.” It has also been termed as hollowing out or job polarization, “with employment polarizing into high-wage and low-wage jobs at the expense of middle-skill jobs” (taken from Autor et al. (2006)).

The wage and employment patterns for the U.S. are exhibited in Figures 1 and 2, which are extracted (with permission) from Autor et al. (2006). See Figure 1 in Goos and Manning (2007), which illustrates job polarization in Britain.

Just as the ‘old’ overall wage-inequality phenomena, job polarization together with rising wage-inequality in the upper tail has been attributed to SBTC. Autor et al. (2003) and Autor et al. (2006) outline a one-good economy with a three-task technology of production: abstract, routine and manual. All three tasks are complementary to each other. Abstract and manual tasks are performed respectively by high-skilled (‘college’) workers and low-skilled (‘high school’) workers. Routine task is done by middle skill or computer-capital. High-school workers are equally (low) skilled in performing manual work but vary in their ability in the middle-skill

---

1 New York Times (19 February, 2008) reported that according to the 2008 Index of Silicon Valley, from 2002 to 2006, middle-wage jobs fell to 46 percent of the work force from 52 percent. At the same time, the percentage of higher-end jobs rose slightly to 27 percent from 26, and lower-wage jobs expanded to 27 percent from 22 percent of the work force. In all, more than 50,000 middle-income jobs have disappeared over four years.
Figure 1: Male and Female Log Hourly 90/50 and 50/10 Deciles Earnings Ratios

Source: Autor et. al. (2006)

Figure 2: Smoothed Changes in Occupational Employment Shares 1980-2000, with Occupations Ranked by Their 1980 Average Years of Schooling

Source: Autor et. al. (2006)
routine task. Supplies of college and high school workers are fixed. Computer capital supply is perfectly elastic at a given price.

SBTC is represented by a decline in the price of computer capital. It spurs the use of computer-capital, reduces the demand for middle-skill jobs and depresses the wage rate for these jobs. Some middle-skill workers shift to manual jobs. The total demand for routine tasks is higher. This raises the marginal product of abstract task and hence the wage of high-skilled workers (as the three tasks are complementary to each other in production). The ratio of high-skill to middle-skill wage rises. The increase in the total use of routine tasks (by both computers and workers) tends to increase manual wage also; but, on the other hand, increased supply of manual work due to displacement of workers from routine task tend to lower low-skilled wage. The overall impact on the ratio of middle-skill wage to manual or low-skill wage is ambiguous. Employment composition shifts such that middle-skill jobs shrink, low-skill jobs increase, while the number of high-skill jobs remains unchanged. The model partly explains job polarization, because the supply of high-skill jobs is fixed by assumption.

While the existing literature portrays the job polarization phenomenon to be nearly synonymous with SBTC, other factors may be at work too. Goos and Manning provide a peripheral discussion on other possible factors including international trade. The objective of this paper is to offer a trade-based, theoretical explanation behind job polarization.

The paper considers two types of trade in the ‘North-South’ context: that in commodities, and that in production tasks – which is same as off-shoring or international outsourcing. The focus is on the labor market in the North (developed countries). We assume a small open economy, which provides analytical simplicity as well seems to serve well in illustrating the mechanisms at work. In a scenario having three types of labor, high-skill, medium-skill and low-skill, and skill-acquisition being endogenous, freer trade in commodities is interpreted as an improvement of North’s terms of trade in producing high-skill labor intensive goods vis-a-vis low-skill labor intensive goods. It advances the hypothesis that freer trade can lead to job polarization as well as an increase in the wage inequality in the upper tail.

Labor market polarization has also been partly attributed to off-shoring of jobs Goos and Manning (2007)). There is evidence of off-shoring of low-skill, medium-skill and high-skill jobs from North to South. Low-skill labor intensive tasks in garment and footwear production are outsourced to cheap low-skill-labor countries in Asia. India is a favorite destination of high-end-task jobs particularly relating to IT, and routine medium-skill data-entry and programming jobs. See, for instance, Feenstra (1998) and Prahalad (2005). It is typically argued that if middle-skill jobs go overseas, then there will be a hollowing out in the domestic
labor market. In contrast, our analysis yields that it is the off-shoring of low-skill or high-skill jobs that are conducive to job polarization in the North.

In what follows, related literature is briefly reviewed in Section 2. Section 3 provides the basic intuitions behind our results. Formal analysis begins in Section 4, which develops the core micro foundation behind skill-acquisition decisions, common to different economy settings analyzed in subsequent sections. Section 5 explores the effect of freer commodity trade on the labor market in the standard $2 \times 2$ framework – which features that factor prices are not affected factor supplies. The next two sections respectively introduce sector-specificity of factors and non-traded goods, which incorporate the effect of factor supplies on factor prices and income effects. International outsourcing is examined in Section 8. Some generalizations are noted in Section 9. Section 10 concludes the paper.

2 Relation to Existing Literature

The themes and framework of this paper are closely related to five papers: Yeaple (2005), Grossman and Rossi-Hansberg (2008), Blanchard and Willmann (2009), Costinot and Vogel (2010) and Jung and Mercenier (2010).

In a two-skill setting (low- and high-skill) Grossman and Rossi-Hansberg analyze the effects of international outsourcing of low-skill and high-skill labor tasks on domestic wages. The most novel element of their analysis is the productivity effect, apparently counter-intuitive. Through this effect off-shoring of low-skill (respectively high-skill) tasks tends to raise low-skill (respectively high-skill) wage. It is because off-shoring of tasks is akin to technical progress. They also uncover a terms-of-trade effect and a labor supply effect of off-shoring on wages.

Our analysis builds on Grossman and Rossi-Hansberg’s. Their model is limited to two skills and the impact of off-shoring on domestic wages only, whereas our analysis permits three skills as well as the impact of off-shoring on wages and composition of jobs. We find that the off-shoring of low-skill or high-skill tasks, not that of middle-skill tasks, is likely to contribute towards labor market polarization: the rise of lousy and lovely jobs. This is similar, however, in spirit to their productivity effects.

For the most part, the current paper deals with the effects of commodity trade on the labor market. Blanchard and Willmann adopt the continuous-skill distribution framework of Yeaple (2005) and Costinot and Vogel (2010), and introduce skill-acquisition decision. The resulting general equilibrium of the economy is similar to that in the current paper as well as in Das (2006). Insofar as there is a continuum of skills, Blanchard and Willmann’s set-up is more general than the current paper. In their two-country model the basis of comparative advantage lies in the difference
between educational infrastructure. In our small-country analysis such a basis of comparative advantage is implicit.\(^2\)

However, limiting to a small open economy, a finite number of tasks (three) and finite number of sectors permits various asymmetries and generalities in a trackable manner. Unlike in Blanchard-Willmann’s model where education is obtained by sacrificing consumption of a common consumption good, there are education sectors in our model which directly use real resources. More substantively, production technologies across sectors are asymmetric. Our model explores the implications of both traded and non-traded sectors. Use of very specific functional forms of technology is not required to obtain categorical predictions. A main result of Blanchard and Willmann is that a decline in the middle-range jobs (polarization) is a result of a decrease in trade costs of middle-skill range sectors. The prediction of our analysis is sharply different: hollowing of middle-skill jobs results from an increase in the relative price of the most skill intensive traded sector.

Jung and Mercenier (2010) consider technical change as well as globalization, interpreted as off-shoring of middle-skill jobs. Similar to ours, they allow three tasks according to sophistication, and, show that hollowing-out of middle-skill jobs results from international outsourcing of middle-skill tasks. The prediction of our analysis is radically different. As said earlier, such hollowing-out in the labor market is shown to likely result from off-shoring of low-skill or high-skill tasks.

3 Intuitions

In our model labor supply variations result from heterogeneous individuals (in terms of income valuation or natural talent) sorting into medium- or high-level of education. Low-level of education is assumed to be free and synonymous with low-skill labor. Similar to Grossman and Rossi-Hansberg (2008) we use traditional competitive trade models to analyze the channels through which commodity trade and international outsourcing may impact on the labor market.

It is straightforward to understand that unlike in the traditional trade-wage-premium debate, the effect of changes in terms of trade on the upper-tail wage inequality and job polarization does not follow immediately from the celebrated Stolper-Samuelson theorem. If we brand exports as relatively high-skill labor intensive and the import-competing sector as relatively low-skill labor intensive, then a terms of trade improvement would tend to increase high-skill wage and lower the low-skill wage. Assuming positively sloped factor supply curves, it would imply more employment in the high-skill job category but less employment in the

\(^2\) Unlike Costinot and Vogel, both Blanchard and Willmann and the current paper abstract from differences in skill bias as a basis of trade.
low-skill category. It is the latter implication which is at odds with polarization of employment.

In this paper we consider three categories of labor – high-skilled, medium-skilled and low-skilled – whose supplies are endogenous, based on education choice. A small open economy faces a positive terms of trade shock to the sector which is most intensive in the use of high-skill labor. This exerts a derived-demand shock on the supplies of labor into different skill levels.

To fix ideas, suppose the relative price of the high-skilled labor intensive good increases in a small Heckscher-Ohlin economy in which middle- and low-skill labor are perfect substitutes in production. As a result, high-skill wage rises and both middle- and low-skill wages fall, while the ratio of the latter two remains constant. The ratio of high-skill to medium-skill wage increases. Assuming that high-skill education is imparted by high-skill workers and the cost of high-skill education is proportional to high-skill wage, the excess return on high-skill education over medium skill increases with the ratio of high-skill to medium-skill wage. As the latter increases, so does the excess return on high-skill education. There is a movement away from medium-skill education to high-skill education. The employment share of high-skill labor increases.

The return on middle-skill education depends on middle- and low-skill wages, and, the cost of acquiring middle-skill education. As long as middle-skill and low-skill labor are perfect substitutes (more generally highly substitutable) in production, there will be little change in the relative wage between the two, implying that the absolute difference between the two wage rates will shrink as the relative price of the high-skill labor intensive good rises. If medium-skill education is imparted by high-skill labor also (while one unit of high-skill labor can teach/train more individuals seeking medium skill than those seeking high skill) and the cost of medium-skill education rises with high-skill wage, there will be an unambiguous decline the return from medium-skill education compared to no education. It will imply a movement away from medium skill to low-skill jobs. The employment share of low-skill labor increases too. Medium-skill jobs are squeezed from both sides.

Our analysis also shows that even when factor endowments can directly impact on factor prices (such as in a sector-specific economy), the polarization effect of a terms of trade improvement holds as long as the middle- and low-skill labor are highly substitutable.

Less obvious is the polarization effect when the rate of technical substitution between low-skill labor and other categories of labor in the production of tradeables is not high. A lower relative price of low-skill labor intensive imports would tend to increase the middle-skill wage premium, lower the employment of low-skilled workers and thus work contrary to polarization.
Our analysis reveals another channel through which the job market may become more polarized due to such a terms of trade change, which is independent of the rate of substitution between low-skill and middle-skill labor in traded sectors. It is the following. We recognize that there are many non-traded services that are highly low-skill labor intensive. Accordingly, let us think of a small open economy having a non-traded consumable sector, whose production is most low-skill-labor intensive among all production sectors. Assuming that the non-traded good is normal, an improvement in the terms of trade would increase real income, increase the demand for the non-traded good and thereby tend to raise low-skill wage and employment. The net result will be an ambiguous effect on middle-skill wage premium, but a positive effect on the share of high-skill as well as low-skill jobs in total employment.

Turning to off-shoring of production tasks, the intuition behind low-skill and high-skill tasks migration leading to job polarization is similar to Grossman-Rossi-Hansberg’s productivity effect. Once we understand that off-shoring is ‘like’ cost-saving technical progress it is not difficult to see why that of low- and high-skill tasks – not of middle-skill tasks – may lead to hollowing of middle-skill jobs. It is further shown that in case of these two types of off-shoring the productivity effect is supplemented by either the labor supply effect or the variable labor supply effect (to be introduced).

4 Skill Acquisition, Assignment (Sorting) and Supply of Labor

This is the core of our analysis and constitutes the common thread that binds different competitive models explored in this paper. A small open economy has two traded sectors, 1 and 2, and has three factors of production: low-skilled, middle-skilled and high-skilled labor. Let $w_O, w_M$ and $w_H$ denote the respective wages in terms of good 2, the numeraire good.

The total labor force is fixed, normalized to unity. But the supply of labor into each of the three categories is endogenous. Based on competitive wages, a person decides whether to acquire education and if yes which level. Low skill is costless, while acquiring middle or high skill is not. Both levels of education are produced competitively (as non-traded goods) by high-skill labor only, with fixed coefficients, $E_M$ and $E_H$ respectively. Hence acquiring middle- and high-level education respectively costs $w_HE_M$ and $w_HE_H$. We assume $E_H > E_M$.

Following Spitz-Oener (2006), an empirical bench-mark for middle-skill will be completion of some apprenticeship or graduation from some vocational training. Anyone who has a university or technical college degree will be thought as high-skilled. Those with no occupational training will be counted as low-skilled.

This is relaxed in section 9.
A necessary condition for acquiring either level of education is that there is a pecuniary surplus, i.e., \( w_M > E_M w_H \) and \( w_H > E_H w_H \), or, equivalently, \( E_M < w_M / w_H \) and \( E_H < 1 \). Hence, we are led to assume the following education-profitability condition:

\[
0 < E_M < E_H < 1. \tag{R1}
\]

### 4.1 Heterogeneity, Education Decision Rules and Assignment

There is self-sorting of individuals into the three tasks, based on intrinsic heterogeneity among them.

#### Talent Heterogeneity

It is typically assumed that they differ in their innate talent levels in performing a task after the respective education is undertaken and are paid in proportion to their talents. Talent is perfectly observable, indexed by \( \theta \in (a, \infty) \), where \( a > 0 \) and \( F(\theta) \) denote its distribution function. Thus, \( F(a) = 0 \), \( F(\infty) = 1 \) and \( F'(\cdot) > 0 \). Without loss of generality, let its mean, \( E(\theta) \), be unity.

There exist two critical levels of \( \theta \), \( \theta_M \) and \( \theta_H \) (\( \theta_H > \theta_M \)), such that a natural assignment scheme follows: those with \( \theta < \theta_M \) opt to receive no education (and thus remain low-skilled), those with \( \theta \in (\theta_M, \theta_H) \) obtain middle-skill education and those with \( \theta \geq \theta_H \) go for high-skill education.

Wage earnings of a worker are equal to \( \theta w_j \), \( j = O, M, H \). Let the surplus or indirect utility of a worker be \( \pi(p)(\theta w_j - C_E) \), where \( p \) is vector of prices of goods, \( \pi(p) \) is the cost of living and \( C_E \) (equal to 0, \( w_H E_M \) or \( w_H E_H \)) is the cost of education.

The education decision rules are: (a) Prefer medium-level to no education if the surplus from the middle-level education exceeds that from no education.\(^5\) (b) Prefer high education if the surplus from high-level education is greater that from middle-level education.

These rules imply that \( \theta_M \) and \( \theta_H \) satisfy:

\[
\pi(p)(\theta_M w_M - E_M w_H) = \pi(p)\theta_M w_O \quad \text{and} \quad \pi(p)(\theta_H w_H - E_H w_H) = \pi(p)(\theta_H w_M - E_M w_H). \]

The solutions are:

\[
\theta_M = \frac{E_M}{r_M}; \quad \theta_H = \frac{E_H - E_M}{r_H}, \quad \text{where} \quad r_M \equiv \frac{w_M - w_O}{w_H}; \quad r_H \equiv 1 - \frac{w_M}{w_H}. \tag{1}
\]

The terms \( r_M \) and \( r_H \) reflect respectively the returns to medium-skill acquisition relative to no education and those to high-skill acquisition compared to medium-

\(^5\) No education is not to be interpreted literally. It is meant to capture low-skill, obtained from negligible investment in education.
Figure 3: Wage Function When Workers Differ in Talent and are Paid in Proportion to Their Talent in Each Category

skill. By definition, \( r_H \) and \( r_M \) are related:

\[
r_M = (1 - \rho)(1 - r_H), \quad \text{where} \quad \rho \equiv \frac{w_O}{w_M} < 1.
\]

We presume \( E_H \) to be sufficiently high relative to \( E_M \), so that \( \theta_H > \theta_M \).

Note that unlike pure assignment models which begin with a continuum of skills as primitives – such as Costinot and Vogel (2010) – here, the “continuum” (low-skills, middle-skills, and high-skills) is an outcome of continuum of talent together with sufficient differences in education costs of different skills. The resulting wage function is shown in Figure 3. Observe that there are both inter- as well as intra-category differentials in earnings.

**Heterogeneity in Income Valuation**

However, this paper focuses on *inter*, not *intra*-category differential. To abstract from intra-category wage earnings differential, we presume a different source of heterogeneity. There is a literature arguing that individuals receive utility not just from private consumption but also from demonstrating wealth and income as a signal of their social status (Frank (1984, 1985)), Heffetz and Frank (2010)).

---

6 The actual expressions of returns of return would take into account the cost of education \( E_M w_H \) or \( E_H w_H \), not just \( w_H \).

7 If \( \theta_H < \theta_M \), then no one will choose middle-education and it will be an economy without middle-level skills.
In a static framework and in a simple way, this amounts to supposing that utility depends directly on not just consumption of goods but also income (see, for instance, Glazer and Konrad (1996)). We follow Glazer and Konrad (1996) and assume that a person’s social status depends on his/her wage earnings. Accordingly, let the indirect utility function be

\[ \pi(p)(w_j - C) + \Theta \pi(p)w_j, \quad \Theta > 0. \] (2)

The first term is the (indirect) utility from private consumption, dependent of earnings net of education costs, and the second captures utility from education and social status, signaled by earnings gross of education costs. Intuitively, it is the gross, not net, earnings that would reflect a person’s education and social status.

We further assume that the parameter \( \Theta \) varies continuously across individuals. In other words, there is heterogeneity in terms of income valuation towards social status. Some are relatively high income-valuation individuals than some others.

It is easily seen that \( \theta \) works exactly as \( 1 + \Theta \). The expressions of \( \theta_M \) and \( \theta_H \) are same as in (1). The only difference is that there is no intra-category earnings differential: a person’s wage earnings equal \( w_j \), not \( \theta w_j \). The resulting wage function is a step function, as shown in Figure 4.

It is important to observe that whether the \( \theta \) is interpreted as a talent or an income valuation parameter, education decisions – and therefore supplies of labor

---

8 In a dynamic framework, Long and Shimomura (2004) assume that an agent derives utility from consumption and his wealth relative to the average. Also see Pham (2005).
into the three categories – depend on $\theta_M$ and $\theta_H$. Hence, predictions of our analysis towards the effects of trade shocks on employment composition hold in terms of both interpretations of $\theta$.

But those with respect to relative wage movements would differ. In the income valuation framework which we adopt, relative wage – defined as the ratio of average wage in one category (say $i$) to average wage in another (say $j$) – is simply equal to $w_i/w_j$. If instead, $\theta$ is interpreted as talent and wages are proportional to talent, then wage earnings within any category would vary and consequently changes in the average wage earnings – and therefore changes in relative wage earning – must factor in changes in the mass (supply) of workers in a category. In other words, relative wage movement would be a function of $w_j$’s and factor supplies, which is difficult to tract analytically.\(^9\) The differing-income-valuation framework is thus more tractable.

**Labor Supply Functions**

Supplies of three kinds of skill have the respective expressions:

\[
L_O = F(\theta_M); \quad L_M = F(\theta_H) - F(\theta_M); \quad L_H = 1 - F(\theta_H).
\]

The production levels in the education sectors (the number of workers receiving education) are $F(\theta_H) - F(\theta_M)$ and $1 - F(\theta_H)$ respectively. Hence, the supply of high-skill labor input in the two education sectors combined equals

\[
L_{HE} = E_M[F(\theta_H) - F(\theta_M)] + E_H[1 - F(\theta_H)].
\]

The supply of high-skill labor into production sectors is equal to $L_{HP} = L_H - L_{HE}$.

Note that $\theta_M$ and $\theta_H$ – and therefore labor supplies to each category – are functions of $r_M$ and $r_H$. We have

\[
\begin{align*}
L_O &= L_O(r_M) \\
L_M &= L_M(r_M, r_H) \\
L_H &= L_H(r_H) \\
L_{HP} &= L_{HP}(r_M, r_H).
\end{align*}
\]

\(^9\) Of course, simulations can be used to generate plausible hypotheses.
high-skill labor and a decline in the supply of medium-skill labor. It also implies a
greater demand for high-skill labor for the two education sectors combined. By
virtue of the education profitability condition (R1), the last effect is outweighed by
the overall increase in the supply of high-skill labor. Hence, there is an increase in
the supply of high-skill labor for production of goods.

5 The Baseline Economy

It is a small open economy with two traded sectors and two non-traded education
sectors.

In the ‘nuanced’ SBTC story of Autor et al. (2003), a critical assumption is
that ‘high-school’ workers costlessly shift between manual and middle-skill jobs at
varying levels of private efficiency. We suppose instead that the supplies of labor in
each category are separate and variable. Parallel to the aforementioned assumption
of Autor et al. (2003), we make

Assumption 1: Low- and middle-skilled labor are perfect substitutes in producing
the traded goods.\footnote{By virtue of continuity, same results must hold when the elasticity of substitution between low-
and middle-skill labor is small enough.}

Each traded good is produced by high-skilled labor and the composite, middle-
cum-low-skilled labor. Hence the underlying production model is the $2 \times 2$
Heckscher-Ohlin. Let the respective constant-returns production functions be:

\[
Q_1 = \mathcal{F}_1(L_{H1}, L_{M1} + b L_{O1})
\]

\[
Q_2 = \mathcal{F}_2(L_{H2}, L_{M2} + b L_{O2}), \quad 0 < b < 1,
\]

where the subscripts $H$, $M$ and $O$ mark ‘high’, ‘middle’ and ‘low’. The functions
$\mathcal{F}_i(\cdot)$, $i = 1, 2$, are assumed to be twice continuously differentiable and satisfy
positive but diminishing returns with respect to its argument. Assumption 1 implies
that the middle-to-low skill wage ratio is constant, and, equal to:

$\rho = b$.

It is presumed that the technologies and the terms of trade are such that in
equilibrium $w_{M}/w_{H} < 1$ (which would imply $w_{O}/w_{H} < 1$).

Terms of Trade Improvement

Suppose that sector 1 is more high-skill intensive than sector 2 and the country
(North) has comparative advantage in the high-skill intensive good 1. Consider
now an increase in $p_1$.\footnote{By virtue of continuity, same results must hold when the elasticity of substitution between low-
and middle-skill labor is small enough.}
By the Stolper-Samuelson theorem, high-skill wage rises and middle-skill wage falls. Hence the yield from investing in high-level education \((= w_H - w_M)\) as well as the cost of education increase. Proportionately, the yield rises more, pushing up the return from high-level education. That is, \(r_H\) increases and \(\theta_H\) falls. More individuals in the relatively high-end of \(\theta\) choose high-level education.

In view of Assumption 1, a decline in \(w_M\) implies a decline in the yield from middle education, \(w_M - w_O\). The cost of acquiring middle education is higher too. Thus, the return from middle education relative to no education falls. That is, \(r_M\) falls and \(\theta_M\) increases. More individuals in the relatively lower end of \(\theta\) prefer to remain low-skilled. In summary, middle-skill jobs shrink from both sides.

These implications are illustrated in Figure 5. The effect of an increase in \(p_1\) on \(r_H\) is shown along vertical line upwards. The top-left quadrant depicts the supply function of high-skill labor. An increase in \(r_H\) implies more supply of such labor. The top-right quadrant graphs the linear and negative relation between \(r_M\) and \(r_H\). As \(r_H\) rises, \(r_M\) falls. The bottom-right quadrant shows that as \(r_M\) decreases, more individuals remain low-skilled.

**Proposition 1** Given Assumption 1, an increase in the relative price of the high-skill intensive goods leads to an increase in high-skill wage premium, a decrease in the middle-skill wage premium and an increase in employment shares of low-skilled and high-skilled workers.
6 Supply-Side Effects on Wages

In the baseline model, wages are affected by commodity prices only. There are no supply-side effects on wages. We now consider a sector-specific factor economy which accommodates such feedback effects. The main point here is that Assumption 1 still delivers the job polarization outcome of freer trade.

Let the technology of good 1 be the same as in the previous model, whereas let good 2 be produced by ‘land’ and the composite middle-skill-low-skill labor. Thus high-skill labor and land are specific to sectors 1 and 2 respectively. Suppressing the notation for land we write the production functions as

\[
Q_1 = F_1(L_{H1}, L_{M1} + bL_{O1}); \quad Q_2 = F_2(L_{M2} + bL_{O2}),
\]

where \(F_1(\cdot)\) is linearly homogeneous, \(F_2(\cdot)\) strictly concave and each satisfies Inada conditions.

Since sector 2 does not use high-skilled labor, \(L_{H1} = L_{HP}\). The standard properties of this model imply

\[
r_H = \bar{r}(p_1, L_{MO}, L_{HP}), \quad \text{where } L_{MO} \equiv L_{M} + bL_{O}.
\]

For the existence of equilibrium we need

**Assumption 2:** Let \(p_1\) and technologies be such that

\[
\bar{r}(p_1, (1 - E_H)[1 - F(E_H - E_M)], F(E_H - E_M)) \leq 0.
\]

(R2)

It says that if the supply of composite input is scarce enough, the medium-skill wage would exceed the high-skill wage.

From the supply side,

\[
L_{MO} = F(\theta_H) - F(\theta_M) + bF(\theta_M)
\]

\[
= F(\theta_H) - (1 - b)F(\theta_M)
\]

\[
= L_{MO}(r_M, r_H).
\]

(5)

We next substitute the labor supply functions \(L_{HP}(\cdot)\) and \(L_{MO}(\cdot)\) from (3) and (5) into (4) and write it as

\[
r_H = \bar{r}(p_1, L_{MO}(r_M, r_H), L_{HP}(r_M, r_H)) = \bar{r}(p_1, r_M, r_H)
\]

\[
= r^d(p_1, r_H)
\]

(7)
Given \( p_1 \), (7) defines a mapping from the “supply price” \( r_H \) to the “demand price” \( r^d_H \). The solution is a fixed point. It will be proven next that a fixed point exists in the linear subspace \( r_H \in (0, 1) \) and it is unique.

The function \( r^d \) is continuous in \( r_H \) since \( \theta \) has a continuous distribution. As \( r_H \to 0 \), the return from high-skill education approaches zero, so no one will invest in high-skill education. Employment of high-skill labor in sector 1 would tend to zero. By virtue of the Inada condition \( w_H \to \infty \), implying that \( r^d \to 1 \). Next, since \( r_H = 1 - w_M / w_H \to 1, r_M \to 0 \). Thus no one would receive middle-skill education. Those with \( \theta < E_H - E_M \) remain low-skilled and the rest go for high-skill education. The supply of the composite input equals \( L_{MO} = F(E_H - E_M) \). The supply of the high-skill labor for production of goods is equal to \((1 - E_H)[1 - F(E_H - E_M)]\). By Assumption 2, \( r^d \leq 0 \).

These limit properties of the function (7) are illustrated in the top-right panel of Figure 6. It follows that a fixed point exists in the interval \((0, 1)\). We observe that \( r^d_H \) is monotonically declining in \( r_H \). Therefore the solution is unique.

Consider now an increase in the terms of trade. In terms of Figure 6, an increase in \( p_1 \) shifts the \( r^d_H \) curve to right, implying an increase in \( r_H \). Thus \( r_M \) falls, shown in the bottom-right panel. From the top-left and bottom-left quadrants we see that...
shares of high-skill (lovely) jobs and low-skill (lousy) jobs expand. Middle-skill jobs are squeezed from both sides.

**Proposition 2** Under Assumptions 1 and 2, the predictions of Proposition 1 hold in the presence of supply effects on wages.

The labor-supply-side effects on wages do not fully offset the original demand-driven effects on wages. They simply ‘accommodate.’

### 7 Low-Skill Labor Intensive Non-traded Sector

We now demonstrate that high substitutability between low- and medium-skill is not necessary for wage- and job-polarization effects of freer trade.

Given finite substitutability between middle-skilled and low-skilled labor and the import-competing sector (sector 2) being low-skill intensive compared to sector 1, the wage ratio $w_M/w_O$ would tend to rise following a positive terms of trade shock, implying less number of workers preferring to remain low skilled. That is, lousy jobs would not expand.

We now recognize a counter-veiling force that induces polarization in the labor market. Similar to Jung and Mercenier (2010) we note that there are services which are highly low-skilled labor intensive. What we have in mind are jobs like hair-cutting or dressing, house-cleaning, taxi driving and burger-flipping.\(^{11}\) We also note that these jobs are are highly nontraded. As the economy’s terms of trade and real income improve, the demand for these services is likely to increase. **This** would tend to improve low-skill wage and increase employment of low-skill workers.

The model to follow captures these effects, along with finite substitutability between medium- and low-skilled labor. It is shown that in equilibrium the wage ratio $w_H/w_M$ rises, while $w_M/w_O$ may increase or decrease. The employment shares of both high-skill and low-skill workers improve.

In place of Assumption 1, Assumption 3 below is the key.

**Assumption 3:** The economy produces a normal non-traded good or service (beside education goods), whose low-skill labor intensity is higher than those in the traded sectors.

For simplicity, suppose that this sector uses low-skilled labor only, while sector 1 uses high-skill labor only and good 2 is produced by middle and low-skill workers (land is suppressed). That is,

\[
Q_1 = L_{H1}; \quad Q_2 = F_2(L_{M2}, L_{O2}); \quad Q_N = L_{ON}.
\]

\(^{11}\) Table 5 in Goos and Manning (2007) lists bottom ten occupations by wage. They include waiters and waitresses, petrol pump attendants and so on.
The subscript $N$ refers to the non-traded sector and $\mathcal{F}_2(\cdot)$ satisfies constant-returns. Thus, among the two traded sectors one is more high-skill labor intensive and the other more low-skill intensive.

The zero-profit condition in sector 1, $w_H = p_1$, determines the high-skill wage. The equation below is the market-clearing condition for the non-traded good:

$$ C_N(p_1, p_N, Y) = Q_N, \quad (8) $$

where $C_N(\cdot)$ is the demand for good $N$ and $Y \equiv p_1 Q_1 + p_N Q_N + w_H L_{HE}$ is the aggregate income.

As shown in Appendix 1, the above equation and cost minimization in sector 2 together imply

$$ w_O = w^d_O(p_1, L_O, L_M, L_H), \quad (9) $$

where the superscript $d$ denotes the demand side of factors – that is, the equilibrium $w_O$ as dependent on commodity terms of trade as well as exogenous factor supplies.

An increase in $p_1$ increases the demand for good $N$ by both income and substitution effects. This tends to spur the production of good $N$, the demand for low-skilled labor and thus push up $w_O$. An increase in $L_M$ and $L_H$ generates a positive income effect on good $N$, and, thereby tend to raise $w_O$, while an increase in $L_O$ tends to create an excess supply of low-skill labor and pull down $w_O$.

The zero-profit condition for sector 2 implies that $w_M$ and $w_O$ are uniquely and negatively related, given by, say,

$$ w_M = \phi(w_O), \quad \phi' < 0. \quad (10) $$

Consider an increase in the terms of trade $p_1$. The (direct) demand side effects on factor prices are immediate. The zero-profit condition in sector 1 implies that $w_H$ increases proportionately. From (9) and (10) respectively, $w_O$ increases and $w_M$ falls. Thus $r_M$ falls and $r_H$ rises. Consequently, the shares of high-skill and low-skill jobs increase and hence that of middle-skill jobs declines. The labor market becomes more polarized.

Of course, these supply-side responses, in turn, influence factor prices. Let $L_O = F(\theta_M(r_M)) \equiv L_D(r_M)$, $L_H = L_H(\theta_H(r_H)) \equiv L_H(r_H)$ and $L_M = 1 - L_O(r_M) - L_H(r_H) \equiv L_{M}(r_M, r_H)$. Substituting these functions into (9),

$$ w_O = w^d_O(p_1, L_O(r_M), L_M(r_M, r_H), L_H(r_H)) $$

$$ \equiv \tilde{w}^d_O(p_1, r_M, r_H). \quad (11) $$

12 The sign of the partial with respect to $r_H$ is not evident. It is derived in Appendix 2.
By definition, \( r_M = [1 - w_O/\phi(w_O)](1 - r_H) \), implicitly defining

\[
w_O = \kappa(r_M, r_H).
\]  (12)

Also,

\[
r_H = 1 - \frac{w_M}{w_H} = 1 - \frac{\phi(w_O)}{p_1}.
\]  (13)

The last three equations solve \( w_O, w_M \) and \( w_H \).

By substituting (12) into (11) and eliminating \( w_O \), we implicitly obtain

\[
G_1(p_1, r_M, r_H) = 0.
\]  (14)

It defines a negative locus between \( r_M \) and \( r_H \), as shown in Figure 7. As \( p_1 \) increases, the locus shifts to the left.

If we substitute (11) into (13), another locus between \( r_M \) and \( r_H \) results:

\[
G_2(r_M, r_H, p_1) = 0.
\]  (15)

Note that positive income effect on the non-traded goods introduces an element of instability in the labor market. An increase in the return for high-skill education tends to push up the supply of high-skill, real income and demand for the low-skill intensive non-traded good. This tends to raise the low-skill wage. At given terms of trade, the medium-skill wage falls, which, in turn, tends to raise the return from high-skill acquisition. This is reflected by \( \partial G_2/\partial r_H \gtrless 0 \). Appendix 3 proves that labor markets are stable if and only if \( \partial G_2/\partial r_H < 0 \) (which holds if \( m_N \), the
marginal propensity to consume the non-traded good, is not too high). We assume stability, which implies that eq. (15) offers a positive locus between \( r_M \) and \( r_H \), shown in Figure 7. It shifts to the left as \( p_1 \) rises.

From Figure 7 it is clear that \( r_M \) declines as terms of trade improve. In view of (12) and (13), it is straightforward to argue that \( r_H \) must rise: suppose \( r_H \) falls or remain unchanged as \( p_1 \) increases; from (12) we see that \( w_O \) rises; thus the r.h.s. of (13) increases; it implies that \( r_H \) increases, a contradiction.

Hence, the demand-side effects remain in tact qualitatively. Similar to previous model, the supply side effects are accommodative. A terms of trade improvement leads to a hollowing-out of middle-skill jobs.

Furthermore, as \( r_M \) decreases and \( r_H \) increases, \( \rho = w_O/w_M \) may increase or decrease, i.e., the impact on low-to-middle-skill wage premium is ambiguous (even though the return from middle-skill education is unambiguously less). It is because, while the terms of trade tends to push \( w_M \) down, the middle-skill supply squeeze from both sides tends to raise \( w_M \).

In summary,

**Proposition 3** Given Assumption 3, as the relative price of the high-skill labor intensive sector increases, the high-skill wage premium increases, while the middle-skill wage premium may increase or decrease. The employment shares of high-skill and low-skill labor increase unambiguously and thus that of middle-skill labor falls.

This proposition is consistent with job polarization together with an increase in wage inequality in the upper tail of the distribution, without having to assume that middle- and low-skill tasks are close substitutes in production.

8 Off-shoring and Polarization

As mentioned in the Introduction, polarization in the labor market has been alluded as consequence of offshoring of jobs also. This section extends Grossman and Rossi-Hansberg’s framework to ours and examine the effects of offshoring on wages as well as composition of jobs. Compared to two tasks in their framework, low-skilled and high-skilled, our model features three: high-skill, middle-skill and low-skill tasks.\(^{13}\)

Since labor supplies vary, in addition to a relative price effect, a productivity effect and a labor supply effect, there is a new effect through changes in labor supplies when wage rates change. This is different from the labor supply effect

\(^{13}\) As Grossman and Rossi-Hansberg do, we define production as output produced by quantities of different tasks, and, in turn, the tasks are undertaken by respective workers.
of Grossman and Rossi-Hansberg. For want of better terminology, let it be called a *variable labor supply effect*. Altogether then, there are four effects. Of course, as we consider a small open economy, by assumption, there is no relative price effect. In what follows we consider the effects of an economy-wide offshoring of any particular task.\(^\text{14}\)

However, to keep the analysis somewhat tractable, we simplify Grossman and Rossi-Hansberg’s ‘micro’ structure by assuming that an exogenous fraction, say \(\mu\), of total quantity of a particular task used in the production sectors is off-shored and an increase in this fraction represents a higher extent of off-shoring. While simplistic, this way of modeling is capable of bringing out various effects off-shoring in a concise way.

### 8.1 Productivity Effects

This is fleshed out most simply in terms of the \(2 \times 2\) economy we have extensively used thus far, in which low-skilled and medium-skilled tasks are perfect substitutes. Because factor prices are determined by product prices only, there are only productivity effects but no labor-supply or variable labor-supply effects on wages.

We begin with off-shoring of low-skill tasks. As a fraction \(\mu\) of unskilled tasks is off-shored, the average cost of low-skilled labor is \((1 - \mu)w_O + \mu w^*_O\), where \(w^*_O < w_O\) is the unskilled wage cost abroad, assumed to be exogenous. Perfect substitutability between low-skill and high-skill tasks implies

\[
bw_M = (1 - \mu)w_O + \mu w^*_O. \tag{16}\]

Notice that even if low- and medium-skill tasks are perfect substitutes, the ratio of the two domestic wages is *not* constant and \(\rho \equiv w_O/w_M > b\). The total cost of medium-skill and low-skill workers in any particular sector \(i\) is:

\[
L_{Mi}w_M + L_{Oi}[(1 - \mu)w_O + \mu w^*_O] = (L_{Mi} + bL_{Oi})w_M.
\]

Hence the effective price of the composite input consisting of low-skill and medium-skill labor is \(w_M\), and, the unit cost function has the form: \(c_i(w_H, w_M)\). Therefore, the zero-profit conditions in the two traded sectors imply that off-shoring of low-skill tasks does not affect \(w_H\) or \(w_M\). It follows from (16) that \(w_O\) rises, which is the productivity effect.

Since \(w_O\) increases while \(w_M\) and \(w_H\) remain unchanged, \(r_M\) falls and \(r_H\) is unchanged. Thus, low-skill labor employment increases and there is no change in high-skill employment remains. It follows that middle-skill jobs are squeezed. The result is a *partial* hollowing-out of middle-skill jobs in the economy.

\(^{14}\) It is straightforward to analyze sector-specific offshoring.
If middle-skill jobs are off-shored, we have the arbitrage condition

\[ b[(1 - \mu)w_M + \mu w^*_M] = w_O. \]  

(17)
The total cost of medium and low-skilled labor in sector \( i \) is equal to

\[ L_{Mi}[(1 - \mu)w_M + \mu w^*_M] + L_{Oi}w_O = \frac{(L_{Mi} + bL_{Oi})w_O}{b}. \]

Thus \( w_O/b \) is the effective price of the composite input and the unit cost functions have the form: \( c_i(w_H, w_O/b) \). The implication is that \( w_O \) and \( w_H \) remain unchanged, while \( w_M \) increases. Middle-skill jobs expand in the economy, whereas both low and high skill jobs shrink. Similar to the ‘paradoxical’ increase in the middle-skill wage due to off-shoring of middle-skill jobs, there is a de-polarization impact on employment composition due to the productivity effect.

In case of off-shoring of high-skill jobs, perfect substitutability between low- and middle-level tasks in production implies that the ratio or \( w_O \) to \( w_M \) is constant. The unit cost function takes the form \( c_i[(1 - \mu)w_H + \mu w^*_H, w_M] \). An increase in \( \mu \), results in an increase in \( w_H \), whereas medium-skill and low-skill wages remain unaffected.\(^{15} \) Hence \( r_H \) rises and \( r_M \) falls, implying a higher share of lousy and lovely jobs and a smaller share of middle-skill jobs.

In summary, the ‘counter-intuitive’ productivity effects on wages extend to our three-task framework and are accompanied by similar effects on employment composition. Polarization is fostered by off-shoring of low-skill and high-skill tasks, not by that of middle-skill tasks.

### 8.2 Supply Effects

These effects are brought in via a sector-specific model of the kind presented in section 6. Consider the off-shoring of low-skill tasks. The arbitrage condition (16) holds and \( w_M \) is the price of the composite input. We can write the zero-profit condition for sector 1 as

\[ c_1(w_H, w_M) = 1, \]

(18)
where we have normalized product prices to unity.

Furthermore, the domestic supply of low-skill labor must equal \( (1 - \mu)L_O \), where \( L_O \) is the total quantity of low-skill tasks used in production. Hence, \( (1 - \mu)L_O = F(\theta_M) \), implying

\[ L_{MO} \equiv L_M + bL_O = F(\theta_H) - \left(1 - \frac{b}{1 - \mu}\right)F(\theta_M) = L_{MO}(r_M, r_H, \mu). \]

\(^{15} \) The two zero-profit conditions are: \( c_i[(1 - \mu)w_H + \mu w^*_H, w_M] = p_i, i = 1, 2 \). Thus \( (1 - \mu)w_H + \mu w^*_H \) and \( w_M \) remain unchanged as \( \mu \) changes.
While $\partial L_{MO}/\partial r_M \gtrless 0$ generally, in case of ‘marginal’ off-shoring ($\mu \simeq 0$) it is positive.

In the sector-specific model, the price of the mobile factor is dependent on factor supplies; hence

$$w_M = \psi(L_{HP}(r_M, r_H), L_{MO}(r_M, r_H, \mu)).$$ \hspace{1cm} (19)

Eqs. (18) and (19) together with the arbitrage condition (16) solve $w_O$, $w_M$ and $w_H$.\(^\dagger\)

Unfortunately, the overall effects of off-shoring are generally ambiguous. But it is useful to understand various effects individually, and they can be discerned.

Productivity effects on wages are captured by (i) $\mu$ in eq. (16) and (ii) treating $L_{HP}$ and $L_{MO}$ as exogenous in (19). These are already discussed.

**Labor-Supply Effects**

These effects can be isolated by varying $\mu$ in eq. (19) only, and, treating $L_{HP}$ and $L_{MO}$ to be unaffected by $r_H$ or $r_M$. (This amounts to assuming that factor endowments are given.) We see that $w_M$ falls. Thus, (a) $w_H$ increases (see eq. (18)) and (b) $w_O$ falls (see eq. (16)). It is clear that $r_H$ increases. From (16) it follows that if offshoring is ‘marginal,’ i.e., $\mu \simeq 0$,

$$d(w_M - w_O) = \left(1 - \frac{\mu}{b} - 1\right) dw_O < 0.$$

Thus $r_M$ falls. The changes in $r_M$ and $r_H$ imply that both lousy and lovely jobs increase, while middle-skill jobs tend to shrink.

**Variable Labor-Supply Effects**

Unlike productivity or labor supply effects, the variable labor supply effect of off-shoring is not a direct one. As $r_H$ increases and $r_M$ declines, the supply of high-skill labor to the production sectors, $L_{HP}$, increases and that of the composite medium-skill and low-skill labor, $L_{MO}$ falls. The effects of these supply changes on wages are the variable labor-supply effects on wages. From (19), we see that $w_M$ increases. Hence, in view of eqs. (16) and (18) respectively. Thus $r_H$ decreases and $r_M$ increases. These changes imply de-polarization: a decline in lousy and lovely jobs.

\(^\dagger\) In solving the sector-specific model, eq. (16) essentially substitutes eq. (6) used earlier.
8.3 Sum-Up

Various individual effects of off-shoring of low-skill tasks are summarized in the initial rows in Table 1. While overall effects are ambiguous, notice that two out of three effects, namely, productivity and labor supply effects, tend to imply polarization.\textsuperscript{17} Hence, there seems to be a fair presumption that such outsourcing leads to job polarization.

Table 1 also summarizes the effects of middle-skill and high-skill tasks off-shoring, whereas Appendix 4 lays down the relevant system equations. Note that in case of international outsourcing of high-skill tasks, the productivity effects and the variable labor supply effects work in the direction of job polarization. Job polarization is somewhat likely to result.

It is the off-shoring of middle-skill tasks which is associated with much ambiguity.

\textsuperscript{17} The only unambiguous result when all three effects are taken into consideration is that the effect of a marginal offshoring of low-skilled tasks on \( r_M \) is negative. Thus the employment share of low-skilled workers increases. The zero-profit condition (18), yields \( w_M = m(r_H) \) and \( w_H = n(r_H) \), where \( m' < 0 < n' \). By definition,

\[
 r_M = \frac{m(r_H) - w_O}{n(r_H)}.
\]

Thus \( w_O = m(r_H) - r_M n(r_H) \equiv w_O(r_H, r_M) \).

Using these, we now treat eqs. (19) and (16) as having two variables, \( r_H \) and \( r_M \). Eq. (19) spells a negative schedule.

Totally differentiating (16),

\[
 [bm' - (1 - \mu)(m' - r_M n')]dr_H = (1 - \mu)ndr_M + (w_O - w_O^*)d\mu.
\]

If \( \mu \approx 0 \) initially, the coefficient of \( dr_H \) equals \( -(1 - b)m' + r_M n' > 0 \). The above equation thus implies a positive locus between \( r_H \) and \( r_M \).

Shifts of curves as \( \mu \) increases show that \( dr_M/d\mu < 0 \), while \( dr_H/d\mu \) ambiguous in sign.
<table>
<thead>
<tr>
<th>Low-Skill Tasks Offshored</th>
<th>( w_O )</th>
<th>( w_M )</th>
<th>( w_H )</th>
<th>( r_H )</th>
<th>( r_M )</th>
<th>Employment Composition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Productivity Effect</td>
<td>↑</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>↓</td>
<td>Partial Polarization</td>
</tr>
<tr>
<td>Labor-Supply Effect</td>
<td>↓</td>
<td>↓</td>
<td>↑</td>
<td>↑</td>
<td>↓*</td>
<td>Polarization</td>
</tr>
<tr>
<td>Variable Labor-Supply Effect</td>
<td>↑</td>
<td>↑</td>
<td>↓</td>
<td>↓</td>
<td>↑</td>
<td>De-Polarization</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Middle-Skill Tasks Offshored</th>
<th>( w_O )</th>
<th>( w_M )</th>
<th>( w_H )</th>
<th>( r_H )</th>
<th>( r_M )</th>
<th>Employment Composition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Productivity Effect</td>
<td>0</td>
<td>↑</td>
<td>0</td>
<td>↓</td>
<td>↑</td>
<td>De-Polarization</td>
</tr>
<tr>
<td>Labor-Supply Effect</td>
<td>↓</td>
<td>↓</td>
<td>↑</td>
<td>↑</td>
<td>↑</td>
<td>↑*</td>
</tr>
<tr>
<td>Variable Labor-Supply Effect</td>
<td>↑</td>
<td>↑</td>
<td>↑</td>
<td>↑</td>
<td>↑</td>
<td>↑*</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>High-Skill Tasks Offshored</th>
<th>( w_O )</th>
<th>( w_M )</th>
<th>( w_H )</th>
<th>( r_H )</th>
<th>( r_M )</th>
<th>Employment Composition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Productivity Effect</td>
<td>0</td>
<td>0</td>
<td>↑</td>
<td>↑</td>
<td>↓</td>
<td>Polarization</td>
</tr>
<tr>
<td>Labor-Supply Effect</td>
<td>↑</td>
<td>↑</td>
<td>↓</td>
<td>↓</td>
<td>↑</td>
<td>De-Polarization</td>
</tr>
<tr>
<td>Variable Labor-Supply Effect</td>
<td>↓</td>
<td>↓</td>
<td>↑</td>
<td>↑</td>
<td>↓</td>
<td>Polarization</td>
</tr>
</tbody>
</table>

“0”: means ‘no effect’  
*: assumes \( \mu = 0 \) initially

To sum up, the productivity effects on wages uncovered by Grossman and Rossi-Hansberg extend to our three-task economy, and, in case of off-shoring of low-skilled and high-skilled tasks, the attendant productivity effects on job composition – that is, polarization – is complemented by either the labor-supply effects or the variable labor supply effects.

While terms of trade or relative price effects of off-shoring are outside the scope of this paper, we may informally infer the likely impacts. Just as Grossman and Rossi-Hansberg, assume a world economy with two countries, Home and Foreign, and let in all production sectors the Hicks-neutral productivity be higher in Home, so that in the free trade equilibrium but in the absence of offshoring, wages of all three categories are higher than their counterparts in the foreign country. It is then the firms in Home who would have an incentive to outsource. Let each country’s production structure be \( 2 \times 2 \).

Consider now outsourcing of low-skill or medium-skill tasks. This would tend to increase the world output of the low-skill-cum-medium-skill intensive sector.
The relative price of this sector will fall, i.e., that of the high-skill intensive good will increase. In view of our analysis in previous sections, employment composition will tend to be polarized. By the same reasoning, outsourcing of high-skill task would tend to imply de-polarization via the relative price effect.

9 Some Generalizations

9.1 Education Cost Not Proportional to High-Skill Wage

In our analysis a common factor contributing towards an increase in returns from high-skill education and a decrease in return from middle-skill education is the proportionate increase in the cost of both levels of education as high-skill wage rises. But it is not critical to the argument. Suppose for instance that, along with high-skill labor, there are fixed factors (like infrastructure) in the education sector and as a result, education cost rises less than proportionately with respect to high-skill wage. The return on high-skill education varies with \((w_H - w_M) / f_H(w_H)\), where \(f_H(\cdot)\) is a strictly concave function of \(w_H\). We can write this as

\[ r_H = \frac{w_H}{f_H(w_H)}. \]

We see that as \(r_H\) and \(w_H\) increase, the above term increases and hence the return from high-skill education increases. A greater fraction of labor force would acquire high-skill education.

Now consider the return from middle-skill education, which is proportional to \((1 - \rho)w_M / f_M(w_H)\). As \(w_M\) falls and \(w_H\) rises, this return decreases. A greater fraction of labor force chooses to remain low-skilled.

Indeed, the same argument goes through even if the cost of education is completely unrelated to high-skill wage, i.e., \(f_H(w_H)\) and \(f_M(w_H)\) are constant and given in terms of the numeraire good (as in Galor and Zeira (1993)).

9.2 Credit Markets

Our analysis has not permitted any credit market. It is all too well-known that this market plays a very important role in skill acquisition decision or human capital accumulation. The literature on this issue, especially credit market imperfection in relation to investment in human capital, is just huge. See, for instance, Galor and Zeira (1993) and Galor (2000) among many others. How does such imperfection in the credit market affect skill-acquisition decision, thereby the equilibrium composition of jobs?

A previous version of this paper considers the case of a country which is small (an interest rate taker) in the world credit market. Demand for credit is introduced
by allowing for another dimension of heterogeneity across individuals, namely, an initial endowment in terms of the numeraire good. Those who are relatively poorly endowed may borrow to finance their education. It shows that all results continue to hold as long as the credit market is perfect. Initial endowment does not affect the education choice.

A la Galor and Zeira (1993), credit market imperfection is introduced through a wedge between the borrowing rate \( i_b \) and the lending rate \( i_l \), where \( i_b > i_l \). Since the borrowing rate is higher than the lending rate, it is not optimal for an individual to finance education by borrowing if her initial endowment can cover the (respective) cost of education.

We consider education decision among three sets of individuals: (a) those whose endowment levels are so small that neither level of education can be self-financed, (b) those whose endowment can finance medium-skill education but not high-skill education and (c) those who are sufficiently endowed to finance high-skill education (and medium-skill education).

The added element in the presence of such credit market imperfection is that initial endowment may figure in the education choice problem. More specifically, it is the value of the initial endowment in terms of the education good, i.e. the ratio of initial endowment to high-skill wage, that matters.

For instance, consider those who are poor enough and cannot afford either level of education. Denoting initial endowment of person \( h \) by \( a_h \), no education is the optimal choice if

\[
(a_h - w_H E_H)(1 + i_b) + \theta w_M \leq a_h (1 + i_l) + \theta w_O. \tag{20}
\]

This solves a critical \( \theta \):

\[
\tilde{\theta}_M = \frac{E_M(1 + i_b) - a_h(i_b - i_l)/w_{HR}}{r_M},
\]

such that if \( \theta < \tilde{\theta}_M \), the person does not receive any education.

Likewise, a person would prefer high- to middle-skill education if

\[
(a_h - w_H E_H)(1 + i_b) + \theta_H w_H \geq (a_h - w_H E_M)(1 + i_b) + \theta_H w_M. \tag{21}
\]

It yields

\[
\tilde{\theta}_H = \frac{(E_H - E_M)(1 + i_b)}{r_H}.
\]

Thus if \( \theta > \tilde{\theta}_H \), the person will prefer high-skill education. This critical \( \theta \) is not dependent on \( a_h \), because she has to borrow – and thus face the same interest rate – to finance either level of education.

The upshot is that an increase in high-skill wage tends to reduce the value of endowment in terms of the education good and thus has an additional negative

---

As well-known, moral hazard problems facing borrowers imply this wedge.
impact on preference towards education for those individuals who have to weigh between lending and borrowing. This effect tends to discourage even those seeking high-skill education by borrowing but can self-finance medium-skill education (and remain as lenders).

The implication is now clear. Compared to a situation of no credit market imperfection, if credit market imperfection in the form of a wedge between the borrowing and the lending rate is present but not large enough, for the $2 \times 2$ production economy laid out in sections 5, the positive impact of an increase in the relative price of high-skill-labor intensive good on the employment share of high-skill labor is weakened, while that on the employment share of low-skill labor is strengthened.

9.3 Medium-Skill Workers Imparting Medium-Skill Education

We have assumed that high-skill workers teach or train high-skill as well as medium-skill tasks. A reader of this paper has rightly pointed out that it may be more realistic to suppose, for instance, that carpentry is best taught or trained by a carpenter. If medium-level education is imparted by medium-skill workers, the cost of such education in our model will be proportional to $w_M$. So,

$$r_M = \tilde{r}_M = 1 - \frac{w_O}{w_M};$$

high-skill wage does not enter the expression of rate of return from medium-level education.

Therefore, when medium- and low-skill labor are perfect substitutes, a terms of trade change does not affect $\tilde{r}_M$ and thus there is no effect on the employment of low-skill jobs. The medium-skill jobs are squeezed from one side only.

But, in our model with finite substitutability and the presence of a non-traded-good sector which is most unskilled labor intensive, it is shown that a relative price improvement of the high-skill intensive traded good leads to an increase in $w_0$ and a decrease in $w_M$. Hence, $\tilde{r}_M$ would fall and employment level of lousy jobs would increase. Medium-skill jobs would be squeezed from both sides.

9.4 Heterogeneous Non-Traded Sector

Among various economic environments considered in this paper, perhaps the non-traded good scenario explored in Section 7 comes closest to explaining relative wage increase in the upper-tail distribution, combined with no clear tend in the lower-tail distribution and polarization in job composition. In that section, the non-traded-good sector as a whole was assumed to be more low-skilled intensive than the traded sectors. This assumption can relaxed however.
The polarization effect of trade would hold (actually become stronger) as long as the non-traded sector is populated by both highly unskilled-labor jobs and highly skilled-labor jobs or services (e.g. specialized health and nursing care). As terms of trade improve and real income rises, the demand for these non-traded goods will increase, which would tend to raise the demand for both low-skill and high-skill labor and this would tend to imply squeezing of medium-skill jobs.

10 Concluding Remarks

As long as markets clear, any rationale behind job composition shifts in the economy must take into account supply side changes. Autor et al. (2006) incorporate such changes of low-skill and medium-skill jobs in furnishing their SBTC rationale behind job polarization.

This paper has developed an international-trade and international outsourcing based explanation behind job polarization. The standard, competitive trade models are capable of providing insights into how a relative price increase of high-skill intensive goods in the basket of traded sectors may lead to a higher wage inequality of a particular kind, and job polarization. Different (competitive) models are able to highlight different effects or mechanisms through which wages and employment composition are affected. Our purpose is not to identify or formulate a single theoretical model to explain the observed changes in the wage and employment patterns noted in the introduction.

Blanchard and Willmann (2009) and Jung and Mercenier (2010) have respectively shown that a lowering of trade costs of middle-skill intensive products and off-shoring of middle-skill tasks are the factors contributing towards polarization. In sharp contrast, this paper has argued how a terms of trade improvement of high-skill labor intensive goods and off-shoring of low-skill and high-skill tasks may be a causal factor behind labor market polarization.

Given normal supply side responses, the conditions identified behind job polarization due to a terms of trade improvement of high-skill intensive goods are: (a) relatively high substitutability in production between low-skill and medium-skill jobs or (b) a terms-of-trade induced real income effect and an increase in demand for non-traded, highly low-skill intensive jobs.

We have extended the international outsourcing analysis of Grossman and Rossi-Hansberg (2008). By allowing for three skills instead of two and variable labor supplies as opposed to fixed factor endowments, our model permits to examine the impact of off-shoring not only on wages but also on the composition of high-skill, medium-skill and low-skill jobs. Variability of labor supplies generate a new effect per se, in addition to Grossman and Rossi-Hansberg’s terms of trade effect, productivity effect and labor supply effect. We have termed it as the variable labor
supply effect. It is shown that labor-supply and variable labor-supply effects are opposite to each other in case of off-shoring of low-skill and high-skill tasks. But one of these effects works in tandem with the productivity effect, indicating a fair presumption of labor-market polarization due to off-shoring of low-skill and high-skill tasks. An obvious next step would be obtain a sense of magnitude of these effects.

Trade-based argument behind labor market polarization is not to be regarded as an explanation ‘competing’ with SBTC. Other channels may be at work too. For instance, a rise in total real income accompanied by a higher labor force participation, especially by women, is likely to increase the demand for low-skilled household services as well as the demand for luxury goods, appliances and gadgets that are high-skill intensive. This would tend to imply polarization in the labor market.
Appendices

Appendix 1: Derivation of Partials of the $w^d_O(\cdot)$ Function in Eq. (9)

Totally differentiating (8) and using the Slutsky’s decomposition theorem,

$$\frac{\partial C_N}{\partial p_1} dp_1 + \frac{\partial C_N}{\partial p_N} dp_N + \frac{\partial C_N}{\partial Y} d(p_1 Q_1 + Q_2 + w_H L_{HE}) = (1 - m_N) dQ_N, \quad (A.1)$$

where $m_N \equiv p_N \frac{\partial C_N}{\partial Y}$ is the marginal propensity to consume good $N$. All three goods are assumed normal, implying $0 < m_N < 1$.

We have

$$d(p_1 Q_1 + Q_2 + w_H L_{HE}) = Q_1 dp_1 + w_H dL_H + w_M dL_M + w_O dL_O + w_H dL_{HE} + L_{HE} dw_H$$
$$dQ_N = dL_O - dL_{O2}$$
$$dp_N = dw_O.$$

Substituting these into (A.1),

$$\left( \frac{\partial C_N}{\partial p_1} + L_H \frac{\partial C_N}{\partial Y} \right) dp_1 + \frac{\partial C_N}{\partial p_N} dw_O + \left( \frac{\partial C_N}{\partial Y} w_O + 1 - m_N \right) dL_{O2}$$
$$+ \frac{\partial C_N}{\partial Y} (w_H dL_H + w_M dL_M) = dL_O.$$

But $\frac{\partial C_N}{\partial Y} w_O = \frac{\partial C_N}{\partial Y} p_N = m_N$. Hence

$$\left( \frac{\partial C_N}{\partial p_1} + L_H \frac{\partial C_N}{\partial Y} \right) dp_1 + \frac{\partial C_N}{\partial p_N} dw_O + dL_{O2} + \frac{\partial C_N}{\partial Y} (w_H dL_H + w_M dL_M) = dL_O.$$

Cost minimization in sector 2 yields $w_O$ as an one-to-one function in the ratio $L_M/L_{O2}$, i.e., $L_{O2} = g(w_O, L_M)$. Substituting this into the above equation and eliminating $L_{O2}$, we obtain

$$\left( \frac{\partial C_N}{\partial p_1} + L_H \frac{\partial C_N}{\partial Y} \right) dp_1 + \left( \frac{\partial C_N}{\partial p_N} + \frac{\partial g}{\partial w_O} \right) dw_O$$
$$+ \frac{\partial C_N}{\partial Y} w_H dL_H + \left( \frac{\partial C_N}{\partial Y} w_M + \frac{\partial g}{\partial L_M} \right) dL_M = dL_O, \quad (A.2)$$

which forms the basis of the equation (9) in the text.
Appendix 2: Sign of $\frac{\partial w^d_Q}{\partial r_H}$ in Eq. (11)

We use the properties of $w^d_Q$ function. For convenience, let us denote $|\partial w^d_Q / \partial L_O|$ by $\xi$. In view of (A.2),

$$\frac{\partial w^d_Q}{\partial L_M} = \frac{\xi m_N}{\rho} > 0$$
$$\frac{\partial w^d_Q}{\partial L_H} = \frac{\xi m_N}{(1 - r_H)\rho} > 0.$$  \hspace{1cm} (A.3)

Thus,

$$\frac{\partial w^d_Q}{\partial r_H} = \frac{\xi m_N}{(1 - r_H)\rho} \frac{dL_H}{dr_H} + \frac{\xi m_N}{\rho} \frac{\partial L_M}{dr_H}$$
$$= \left[ \frac{\xi m_N}{(1 - r_H)\rho} - \frac{\xi m_N}{\rho} \right] \frac{dL_H}{dr_H}$$
$$= \frac{\xi m_N}{(1 - r_H)\rho} \frac{dL_H}{dr_H}$$
$$= \frac{\xi m_N}{r_H(1 - r_H)\rho} f(\theta_H) > 0. \hspace{1cm} (A.4)$$

Appendix 3: Proof that Labor Market Stability implies $\frac{\partial G^2}{\partial r_H} > 0$ in Eq. (15)

Using (A.3),

$$\frac{\partial w^d_Q}{\partial r_H} = -\xi \left(1 + \frac{m_N}{\rho}\right) \frac{dL_O}{d\theta_M} \frac{d\theta_M}{dr_H} = \xi \left(1 + \frac{m_N}{\rho}\right) \frac{E_M}{r_M} f(\theta_M). \hspace{1cm} (A.5)$$

Now, at given $p_1$, if we totally differentiate (13) and substitute (A.4) and (A.5), we obtain

$$\left[ w_H - \xi \eta \frac{m_N(E_H - E_M)}{\rho r_H(1 - r_H)} f(\theta_H) \right] dr_H - \xi \eta \left(1 + \frac{m_N}{\rho}\right) \frac{E_M}{r_M} f(\theta_M) dr_M = 0. \hspace{1cm} (A.6)$$

It is shown below that labor markets are stable if and only if the coefficient of $r_H$ in the above equation is positive, i.e.,

$$\forall = \frac{\eta \xi m_N}{(1 - r_H)\rho} - \frac{r_H w_H}{f(\theta_H)(E_H - E_M)} < 0. \hspace{1cm} (A.7)$$

and thus $\frac{\partial G^2}{\partial r_H} > 0$ in (15).

At any given $p_1$, $w_H$ is fixed. Assume the following Marshallian adjustment processes.

$$L_O = \alpha \left[ \bar{w}^d_O(L_O, L_M) - w^d_O(L_O, L_M) \right]$$
$$L_M = \beta \left[ \bar{w}^d_M(L_O, L_M) - w^d_M(L_O, L_M) \right], \hspace{1cm} (A.8)$$
where the dot denotes the derivative with respect to time. These processes are expressed in
the \((L_O, L_M)\) space, using the identity \(L_H = 1 - L_O - L_M\), where “1” is equal to \(LE(\theta)\). The
function \(\dot{w}_O^d(\cdot)\) is derived from the function \(w_O^d(\cdot)\): \(\dot{w}_O^d(\cdot) = w_O^d(p_1, 1 - L_O - L_M, L_M, L_O)\).
Recall that \(w_M\) and \(w_O\) are related uniquely via (10). Hence the function \(w_O^d(p_1, 1 - L_O - L_M, L_M, L_O)\) leads to the function, \(w_M^d(p, 1 - L_O - L_M, L_M, L_O)\).

In view of (A.2),

\[
\frac{\partial \dot{w}_O^d}{\partial L_O} = -\xi \left( 1 + \frac{m_N}{(1 - r_H)\rho} \right) < 0; \quad \frac{\partial \dot{w}_O^d}{\partial L_M} = -\xi \frac{m_N r_H}{(1 - r_H)\rho} < 0.
\]

Using (10), and, recalling that \(\eta \equiv |\phi'(\cdot)|\),

\[
\frac{\partial \dot{w}_M^d}{\partial L_O} = \eta \xi \left( 1 + \frac{m_N}{(1 - r_H)\rho} \right) > 0; \quad \frac{\partial \dot{w}_M^d}{\partial L_M} = \eta \xi \frac{m_N r_H}{(1 - r_H)\rho} > 0.
\]

The partial \(\partial \dot{w}_M^d / \partial L_M\) is of unintuitive sign – as it says that an increase in \(L_M\) tends to
increase the reward to middle skill. The reason is that an increase in \(L_M\) impacts on \(w_M\) in
two ways. On one hand, it tends to increase the demand for good \(N\) and hence raise \(w_O\). This implies a higher proportion of middle-skill to low-skill used in sector 2. In turn, it
tends to lower \(w_M\). This part is expected. But, on the other hand, at given \(L_O\), an increase
in \(L_M\) implies a decline in \(L_H\), which tends to push \(w_M\) up. Since \(w_H > w_M\), the latter effect
 dominates.

From the supply side,

\[
dL_M = f(\theta_H) d\theta_H - f(\theta_M) d\theta_M = -f(\theta_H) \frac{E_H - E_M}{r_H^2} d\theta_H - dL_O
\]

\[
= f(\theta_H) \frac{E_H - E_M}{r_H^2} d\theta_M - dL_O.
\]

Hence \(w_M = w_M^d(L_O, L_M)\), and, it has the properties:

\[
\frac{\partial \dot{w}_M^d}{\partial L_O} = \frac{\partial \dot{w}_M^d}{\partial L_M} = \frac{r_H^2 w_H}{f(\theta_H)(E_H - E_M)} > 0.
\]

Also from the supply side,

\[
dL_O = -\frac{f(\theta_M) E_M}{(1 - r_H)^2 (1 - \rho)^2 w_H} (d w_M - d w_O)
\]

\[
\implies d w_O = d w_M + \frac{(1 - r_H)^2 (1 - \rho)^2 w_H}{f(\theta_M) E_M} d L_O.
\]

Hence, \(w_O = \psi(w_M, L_O)\). Given the function \(w_M = w_M^d(L_O, L_M)\),

\[
\psi(w_M, L_O, L_M) = w_O^d(L_O, L_M)\),

having properties

\[
\frac{\partial \dot{w}_O^d}{\partial L_O} = \frac{\partial \dot{w}_O^d}{\partial L_M} = \frac{(1 - r_H)^2 (1 - \rho)^2 w_H}{f(\theta_M) E_M} > 0; \quad \frac{\partial \dot{w}_O^d}{\partial L_M} = \frac{\partial \dot{w}_M^d}{\partial L_M} > 0.
\]
This completes the derivation of the expressions of various partials and their signs. In summary,

\[ w^d_O = \tilde{w}^d_O(L_O, L_M); \quad w^s_O = w^s_O(L_O, L_M) \]
\[ w^d_M = \tilde{w}^d_M(L_O, L_M); \quad w^s_M = w^s_M(L_O, L_M) \]

Result: The dynamic system (A.8) is stable if (A.7) holds, i.e., if \( \mathcal{A} < 0 \).

Proof: The stability of the dynamic system (A.8) requires

\[
\begin{align*}
\frac{\partial \tilde{w}^d_O}{\partial L_O} - \frac{\partial w^s_O}{\partial L_O} &< 0 \\
\frac{\partial \tilde{w}^d_M}{\partial L_O} - \frac{\partial w^s_M}{\partial L_M} &< 0
\end{align*}
\]

(S1)

\[
B = \left| \frac{\partial \tilde{w}^d_O}{\partial L_O} - \frac{\partial \tilde{w}^d_M}{\partial L_O} \right| > 0.
\]

(S3)

From the sign structure, condition (S1) is met. Further, condition (S2) is met, given (A.7), i.e., if \( \mathcal{A} < 0 \). Finally, using various expressions, we have

\[
B = -\mathcal{A} r_H \left[ \xi \left( 1 + \frac{m_N}{(1-r_H)\rho} \right) + \frac{r^2_H w_H}{f(\theta_H)(E_H-E_M)} + \frac{(1-r_H)^2(1-\rho)^2 w_H}{f(\theta_H)E_M} \right] \\
+ \left[ \eta \xi \left( 1 + \frac{m_N}{(1-r_H)\rho} \right) - \frac{r^2_H w_H}{f(\theta_H)(E_H-E_M)} \right] \left[ \frac{\xi m_N r_H}{(1-r_H)\rho} + \frac{r^2_H w_H}{f(\theta_H)E_M} \right] \\
- \mathcal{A} r_H \left[ \frac{\xi m_N}{(1-r_H)\rho} + \frac{r_H w_H}{f(\theta_H)(E_H-E_M)} \right] \\
+ \eta \xi \left( 1 + \frac{m_N}{\rho} \right) r_H \left[ \frac{\xi m_N}{(1-r_H)\rho} + \frac{r_H w_H}{f(\theta_H)(E_H-E_M)} \right] \\
= -\mathcal{A} r_H \left[ \frac{\xi m_N}{(1-r_H)\rho} + \frac{(1-r_H)^2(1-\rho)^2 w_H}{f(\theta_H)E_M} \right] \\
+ \eta \xi \left( 1 + \frac{m_N}{\rho} \right) r_H \left[ \frac{\xi m_N}{(1-r_H)\rho} + \frac{r_H w_H}{f(\theta_H)(E_H-E_M)} \right] \\
> 0 \text{ as long as } \mathcal{A} < 0.
\]

Note that the stability condition \( \mathcal{A} < 0 \) holds if \( m_N \) is low enough.
Appendix 4: Equation Systems in Case of Off-shoring of Middle-Skill and High-Skill Tasks

In case of off-shoring of middle-skill tasks, the equations are

\[ c_1(w_H, w_O/b) = 1 \]
\[ w_O = \psi(L_{HP}(r_H, r_M), L_{MO}(r_H, r_M, \mu)) \]
\[ b[(1 - \mu)w_M + \mu w^*_M] = w_O. \]

If high-skill tasks are off-shored, the relationship between \( w_O \) and \( w_M \) remains unaffected and thus \( r_H \) and \( r_M \) are monotonically related, unaffected by off-shoring. The relevant equations are:

\[ c_1((1 - \mu)w_H + \mu w^*_H, w_M) = 1 \]
\[ w_M = \Psi(r_H, \mu). \]
References


Please note:

You are most sincerely encouraged to participate in the open assessment of this discussion paper. You can do so by either recommending the paper or by posting your comments.

Please go to:


The Editor