Conference Paper

Game-theoretic analysis of IPv4-IPv6 migration process


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Suggested Citation: Trinh, Tuan Anh; Sallai, Gyula (2011) : Game-theoretic analysis of IPv4-IPv6 migration process, 22nd European Regional Conference of the International Telecommunications Society (ITS2011), Budapest, 18 - 21 September, 2011: Innovative ICT Applications - Emerging Regulatory, Economic and Policy Issues, ITS, Budapest

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Tuan Anh Trinh and Gyula Sallai  
Game-Theoretic Analysis of IPv4-IPv6 Migration Process

Abstract
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JEL codes: C72, C73
Keywords: IPv6 transition, game theory

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Abstract—The rate of deployment and adoption issues of new network technologies, IPv6 in particular, have recently been hotly debated in the research community. However, the question of how protocols migrate, especially the dynamics of migration, to new paradigms is still largely open. In this paper, we address the issue from a game theoretic point of view. We model and analyze the profit maximizing strategies of Autonomous Systems (ASes); both the properties of ASes and the topology of the Internet is considered. The contribution our work is threefold. First, we propose an economic model of the ASes and their relations from the IPv4-IPv6 migration viewpoint. Second, we show—after analysing the strategies of ASes using game-theoretic tools—that under the proposed economic model, the all IPv6 AS topology is a Nash equilibrium on any network topology. Third, we apply the findings of evolutionary dynamics on the problem of migration by incorporating Internet-specific properties to the evolutionary model, namely the size of the ASes and the cost of migration. The analyses show that even if IPv6 has higher payoff than IPv4, the whole migration does not happen always fast. Finally, extensive simulations are carried out based on the proposed models to illustrate the impacts of different parameters on the IPv6 migration dynamics in realistic scenarios.

I. INTRODUCTION

The development of IPv6 protocol was carried out more than a decade ago and became an Internet standard in 1998 [1]. Since then numerous additional features have been designed including security extensions, mobility management and tunneling methods. Despite all its benefits, its necessity (as the number of unallocated IPv4 addresses rapidly decrease), and widespread supports [2]–[6], the widespread deployment of IPv6 is still yet to be seen. According to some recent reports [7], [8], the occurrence of IPv6 is still dwarfed by IPv4. The number of active BGP entries is 321,686 in case of IPv4 while in case of IPv6 there exist only 2,887 entries (less than 0.9%), the unique AS numbers reflect similar ratio: 34,236 IPv4 and 2,118 IPv6 entries (less than 6.2%). One question arises from these facts: what cause hinders the spread of IPv6?

A recent effort [9] tried to tackle this issue by using adoption modelling analysis. In this model, every single entity of the network can select its architecture with or without conversion capabilities. It showed that overly efficient converting tools can hinder the protocol adoption. However, the analysis did not include IPv6 related properties like the topology of Autonomous Systems (ASes) and the diverse size of the stakeholders. However, in practice, only ASes are large enough to influence the architecture selection of the whole population, i.e. upgrading from IPv4 to IPv6. As a result, we believe that the case of IPv6 is more a migration issue than an adoptability issue as the entire protocol is deprecated; the newly deployed system is not even necessarily able to interconnect with the older one.

The decision about migration is straightforward if only one entity has to select the best solution based on its own interests. However, it gets complicated if more than one stakeholders are involved in the migration. Not only strategic analyses of game-theory help the modelling and investigation of migration. The dynamic aspect of the migration can be handle using tools of evolutionary dynamics [10], [11] which extends game theory to describe stable and dynamic properties of populations, where the individual utilities depend on the structure of the population.

The history of technology provides examples of migration related to communication, including the migration from analog to digital telephony. Contrary to previous telecommunication related migrations, the economic relations between the stakeholders of the Internet are much more complex [12], therefore, the modelling of the IPv6 migration problem is challenging. Models that include the most important aspects of the IP migration problem, such as different sizes and properties of the ASes and the Internet topology itself, have not been proposed yet. In this paper, we model and analyze the IPv4-IPv6 migration using methods of game theory dealing with strategic and evolutionary aspects. We believe the principal contributions of our work are as follows:

- We propose an economic model of the ASes which includes their revenues, expenditures, and also their growth rate. The economic model is applied throughout the paper in the analytical investigations of migration.
- We analyze the migration to IPv6 using strategic tools of game-theory on several AS topologies and present the conditions of Nash equilibrium strategies. We show that the situation, where every AS uses IPv6, can be a Nash equilibrium on general topologies as well.
- We apply evolutionary dynamics, an extension of game-theory, to create a model of IPv6 migration where the protocol versions—IPv4 and IPv6—represent the competing genes in a population created out of ASes. The concept of games in finite population is adapted to include properties of the Internet such as the structure of the topology, the size of the ASes, and the cost of migration.
We present simulation results based on the findings of the analytical analyses. In addition, we quantify the impact of IPv4 address resell on the dynamics of migration to IPv6.

The paper is structured as follows. We present our economic model in Section II. Afterwards, we apply the proposed economic model in strategic (Section III) and evolutionary (Section IV) analyses. Extensive simulation results are presented on real-world AS topologies, both at the level of individual decisions and populations in Section V. The paper concludes in Section VI.

II. ECONOMIC MODEL

The migration from IPv4 to IPv6 has multiple stakeholders, including ASes, end-users, site operators, governments and even Internet registries. All of them impact the migration process, however, the effects of their decisions are diverse. While governments and registries can only motivate the migration, mostly throughout recommendations without financial support, the final decisions are made by the ASes.

Therefore, we model the IP protocol migration based on the interests of Autonomous Systems. In this section, we propose an economic model of the ASes that includes both their revenues and expenditures. Our economic model, which is intensively used in the analyses of the forthcoming sections, holds based on the following assumptions:

- First, the model describes the profit of a single AS: the economic relations of the ASes are only considered implicitly. Thus, we do not model which AS pays to whom for the connectivity—typical business case in a customer-provider relation. However, the topology of the ASes is considered in terms of its impact on the revenues.
- Second, we assume the subscribers, who represent end-users in case of local ISPs and other ASes in case of Tier-1 and Tier-2 ISPs, are identical, i.e. they are charged equivalently.
- Third, the ASes can select their protocol from two options: IPv4 or IPv6. IPv4 represents the good old version of IP, which dominates the Internet, while IPv6 denotes the version 6 with converting capabilities—ASes using IPv4 and IPv6 can communicate with each other. For simplicity reasons, we do not consider a third option, architectures like dual-stack IP [13] or 6to4 [14], in our investigations.

Based on these assumptions, we present our economic model of AS; first the revenues than the expenditures are formalized. The notation used in this paper is described in Table I.

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The notation used in this paper is described in Table I.

In general, the value of $t$ is higher if an AS uses IPv6, due to additional services, like Quality-of-Service (QoS) connection, security, and multicast capabilities, than if the AS runs IPv4. The $t$ value of an AS is a function of the protocols used by the AS, by its neighbouring ASes, and by every ASes on the Internet; the $t_i$ value of AS $i$ can be calculated as follows:

$$ t_i = 1 + x \left( R_i + R_i R_{N(i)} + R_i R_{N(i)} R_{\text{global}} \right), \text{ where (2)} $$

$$ R_i = \frac{C_{6} - C_{4}}{f_{6}}, \text{ and } R_{N(i)} = \frac{C_{6} - C_{4}}{f_{6}} \sum_{l \in L} \frac{f_{6} (l)}{f_{4} (l)} $$

The value of $R_i$ is based on an identity function, i.e. $R_i = 1$ if AS $i$ uses IPv6 otherwise $R_i = 0$. The AS receives additional benefits of the IPv6 usage of the other ASes only if it uses IPv6, therefore, both the local and the global network effect of IPv6 deployment are multiplied with $R_i$. The benefit of neighbour ASes is proportional to the ratio of the ASes who have deployed IPv6. Similarly, the global network effect of the application of IPv6 is based on the fraction of ASes using IPv6 in the network. However, good global IPv6 coverage generates higher revenues only for an IPv6-using AS with capable neighbours. This partitioning of the extra revenues generated by IPv6 usage allows us to model and analyze the IP migration where the scope of network effect is restricted. The revenue calculations would otherwise be analytically intractable—if we would consider the traffic flows between ASes—due to the number of ASes and their connections. We choose the ratio of the terms evenly for simplicity reasons—based on large-scale Internet measurement one would create a more realistic weight distribution.

After the revenues we proceed with the expenditures of the ASes. On the one hand, AS $i$ has expenditures related to the services offered to customers, e.g. transit costs, billing costs, and help centers. The service cost of AS $i$, which depends on the protocol version, is proportional to the number of its

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**TABLE I**

<table>
<thead>
<tr>
<th>Notation Used in the Paper</th>
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<tbody>
<tr>
<td>$N$</td>
</tr>
<tr>
<td>$N(i)$</td>
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<tr>
<td>$N_{j}(i)$</td>
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<td>$t$</td>
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<td>$n_i$</td>
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<td>$R_{\text{service}}$</td>
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<tr>
<td>$C_{\text{management}}$</td>
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<tr>
<td>$C_{\text{migration}}$</td>
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<td>$C_{j}$</td>
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<td>$\delta C = C_{6} - C_{4}$</td>
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<td>$f_{4}$</td>
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<td>$f_{j}$</td>
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<tr>
<td>$L$</td>
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<tr>
<td>$L_{i}$</td>
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</tbody>
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subscribers \((n_i)\) and to the unit cost of providing service to a single customer, denoted by \(s\). AS \(i\) has additional costs if it uses IPv6 but its neighbouring ASes have not migrated to IPv6 yet due to the packet conversion; let \(e\) denotes the unit conversion cost. The service expenditures of AS \(i\) are formalized as follows:

\[
C_{service} = n_i \left( s + e \sum_{a \in N(i)} I_6(a) \right) \tag{3}
\]

On the other hand, the management of the network causes expenditures for the AS too. The management costs are proportional to the number of customers—as the size of an AS is based on its customers—and to the unit management cost of a single customer, denoted by \(m\), which depends on the protocol version used in the AS. Contrary to the service costs, the management costs of an AS are not influenced by the protocol selection of other ASes. We model the management expenditures with the following formula:

\[
C_{management} = n_i \cdot m \tag{4}
\]

Applying the proposed revenue and expenditure model, the profit of an AS can be formalized as follows:

\[
\Pi_{AS} = I - C_{service} - C_{management} \tag{5}
\]

The forthcoming notations simplify the expression of some concept. The operation cost of an AS using IP version \(j\) is denoted by \(C_j = s_j + m_j\), while the cost saving of using IPv6 instead of IPv4 is denoted by \(\Delta C = C_6 - C_4\).

Our proposed model can be applied in a snapshot of the market; the behaviour of ASes can be analyzed in a static situation. However, an important aspect of the Internet is its continuous increase in terms of the number of Internet subscribers. This fact has a significant impact on the migration to IPv6 due to the exhaustion of IPv4 addresses. Accordingly, a growth rate could be included into our model, a detailed analysis of this is a future work. In our model, the costs—service and management—are constant; the increased expenditures due to the address exhaustion are based on the increased number of customers. We suppose the costs of IPv4 are larger than that of IPv6, therefore, operating an AS on IPv4 will become more and more expensive. Our assumption of the costs is formalized as \(s_4 > s_6\) and \(m_4 > m_6\).

Not only the presented expenditures have to be paid by the ASes; when an AS decides to migrate from IPv4 to IPv6 it has to cover the migration costs too. We model the cost of protocol switch as a one-time cost, i.e. the AS has to pay the following amount after it decides to migrate where the unit switching cost is denoted by \(S_{4\rightarrow 6}\):

\[
C_{migration} = n S_{4\rightarrow 6} \tag{6}
\]

As a summary, we illustrate our economic model in Figure 1. The parameters of the model belong to an AS, if we analyze the behavior of more than one ASes they can have different prices and costs. In the upcoming sections, we analyze individual decisions and group interest under this economic model and gain insights about the migration from IPv4 to IPv6.

### III. IPv6 Migration Analysis: A Strategic Perspective

In this section, we model and analyze the IP version migration problem based on individual decisions of the ASes. Throughout the section, we analyze the selfish interests of ASes with the help of game-theoretic methods. In particular, we present single-shot games on several AS topologies.

#### A. Single-shot games

We present single-shot games in this section and analyze the decisions of the ASes. We investigate analytically the interactions of the ASes on several network topologies. As the players of the games are the ASes, they have to select their strategies—to use IPv4 or IPv6. We assume, that the ASes currently use IPv4, therefore, they have to pay an additional cost presented in Eq. 6 in case of migration. We apply the proposed economic model of Sec. II, accordingly, the payoff function of the ASes is the profit they realize, shown in Eq. 5. We further assume that the unit service and management costs as well as the protocol migration costs are identical for every AS.

1) Customer-provider protocol migration: As the connections between ASes in the Internet are based on bilateral agreements—customer-provider or peering—we first analyze the protocol migration of two ASes. We suppose a customer-provider relation between the ASes, AS\(_1\) is the customer and AS\(_2\) is the provider. However, we note that the same results can be derived if the ASes sign a peering agreement as our economic model does not incorporates the type of the AS connection.

Based on the payoff functions, we construct the payoff matrix of the game, presented in Table II. The following proposition summarizes the conditions of Nash equilibrium strategies:

**Proposition 1:** In the two-player IP migration game, on the one hand, if \(p_i \leq \frac{1}{2} (\Delta C + e + S_{4\rightarrow 6})\), \(\forall AS_i\) holds then (IPv4,IPv4) is a Nash equilibrium. On the other hand, (IPv6,IPv6) is a Nash equilibrium of the game if \(p_i \geq \frac{1}{2} \frac{1}{10} (\Delta C + S_{4\rightarrow 6})\), \(\forall AS_i\) holds.

**Proof:** First, we prove that the (IPv4,IPv4) protocol selection is a Nash equilibrium. AS\(_1\) does not migrate to IPv6

![Fig. 1. The economic model of an AS](image)
if it has a higher profit with IPv4:

\[ n_1(p_1 - C_4) \geq n_1((1 + x)p_1 - C_6 - e - S_{4\rightarrow6}) \]

\[ p_1x \leq C_6 - C_4 + e + S_{4\rightarrow6} \]

Similar condition can be computed for AS\textsubscript{2} with \(p_2\) as price due to the elimination of the number of customers, therefore, (IPv4,IPv4) is a Nash equilibrium.

Second, we compute the conditions for the (IPv6,IPv6) Nash equilibrium. AS\textsubscript{1} will choose IPv6 if the following holds:

\[ n_1(p_1 - C_4) \geq n_1((1 + 3x)p_1 - C_6 - S_{4\rightarrow6}) \]

\[ 3xp_1 \geq C_6 - C_4 + S_{4\rightarrow6} \]

As similar condition can be computed for the other AS, (IPv6,IPv6) is a Nash equilibrium of the game.

The implication of the proposition is that IPv6 is an optimal protocol selection if the additional cost of migration can be covered by the extra revenues of IPv6 usage. However, if IPv6 cannot provide value added services then the protocol migration only occurs if its operational costs are lower than IPv4.

2) Provider with two customer ASES: An AS, who provides transfer services for other ASES, has more than one customers in general. Therefore, we analyze the IP migration on a network topology where the provider has two customers. AS\textsubscript{1} and AS\textsubscript{2} are customers of AS\textsubscript{3}, therefore they have similar payoff in this game. The payoffs of the ASES are presented in Table III. The revenues of the ASES are implications of the IPv6 penetration in the network. If only one AS uses IPv6 then its revenue is \((1 + x)p_1\). However, if two ASES use IPv6 they realize higher payoffs: the ASES with only one neighbouring AS have \((1 + x)(1 + 1 + \frac{x}{2})p_1 = (1 + \frac{x}{2})p_1 \) while the provider AS has \((1 + x)(1 + \frac{x}{2} + \frac{x^2}{2})p_1 = (1 + \frac{x}{2} + \frac{x^2}{2})p_1\), where the additional revenue is a consequence of the own, local, and global IPv6 usage. This game with the shown payoffs has a unique equilibrium; its conditions are presented in the following proposition:

**Proposition 2:** In a network, where a provider AS has two customers, there exists only one Nash equilibrium—(IPv6,IPv6,IPv4)—if the following holds for every AS:

\[ p_i \geq \frac{1}{x} \left( \Delta C + e + S_{4\rightarrow6} \right) \]  

\[ \text{(7)} \]

**Proof:** Similar to the proof of the previous proposition, the all-IPv4 network is not a Nash equilibrium if at least one of the ASES has higher payoff using IPv6. Formally, \((1 + x)p_1 - C_6 - e - S_{4\rightarrow6} \geq p_1 - C_4\), the condition of the proposition.

Moreover, if only one AS uses IPv6 there does not exist a Nash equilibrium. If AS\textsubscript{1} has IPv6 in its network then AS\textsubscript{2} migrate to IPv6 if \(n_2((1 + x)p_2 - C_6 - e - S_{4\rightarrow6}) \geq n_2(p_2 - C_4)\) holds, which is the condition of the proposition. In the other case, AS\textsubscript{3} changes its protocol if \(n_3((1 + \frac{11}{8}x)p_3 - C_6 - \frac{1}{2}e - S_{4\rightarrow6}) \geq n_3(p_3 - C_4)\); this inequality holds if the condition of the proposition is true because \((1 + \frac{11}{8}x) > (1 + x)\) and \(-\frac{1}{2}e > e\). The proof of the case where AS\textsubscript{2} has already deployed IPv6 follows the same steps as before. If the provider has IPv6 network, AS\textsubscript{1} chooses IPv6 if \(n_1((1 + \frac{3}{8}x)p_1 - C_6 - S_{4\rightarrow6}) \geq n_1(p_1 - C_4)\) which holds if the condition of the proposition holds. The condition for the migration of AS\textsubscript{2} is analogous.

Finally, we present that if only one AS does not use IPv6, it is not a Nash equilibrium. AS\textsubscript{1}, using IPv4 realizes higher payoff with the migration if \(n_1((1 + 3x)p_1 - C_6 - \frac{1}{2}e - S_{4\rightarrow6}) \geq n_1(p_1 - C_4)\) holds, therefore the condition of the proposition guarantees this network situation is not a Nash equilibrium.

A consequence of the last condition is that the all-IPv6 network situation is a Nash equilibrium because none of the ASES can have higher payoff changing their protocol.

The implication of the proposition is that if the additional advantage of IPv6 is large enough then every AS will migrate to IPv6, otherwise, it will realize lower profit.

Additional connections in the network have an impact on the payoff of the ASES. Let us suppose, that AS\textsubscript{1} and AS\textsubscript{2} have a peering agreement with each other. Only three rows of Table III are modified as a consequence of the peering connection. If both AS\textsubscript{1} and AS\textsubscript{2} use IPv6, their payoffs are increased to \(n_1((1 + \frac{11}{16}x)p_1 - C_6 - \frac{1}{2}e - S_{4\rightarrow6})\) due to the additional edge in the topology. In the other cases, when one customer AS and AS\textsubscript{3} migrate to IPv6, the payoff of the IPv6-using customer decreases slightly to \(n_1((1 + \frac{11}{16}x)p_1 - C_6 - \frac{1}{2}e - S_{4\rightarrow6})\). However, if the condition of Proposition 2 holds, there exists still only a unique Nash equilibrium in the modified network.

3) General case: Next, we generalize our finding for any network topology. We assume there exist \(N\) ASES in the network from that \(q\) use IPv6. AS\textsubscript{4} has \(k_i\) IPv6 using neighbouring AS out of \(k_i\) adjacent ASES. If the following condition hold, none of the mixed populations, where both IPv4 and IPv6 ASES exist, can be a Nash equilibrium:

**Theorem 1:** In a general AS topology, only all-IPv6 is a Nash equilibrium, if the following holds for every AS:

\[ p_i \geq \frac{1}{x} \left( \Delta C + e + S_{4\rightarrow6} \right) \]  

\[ \text{(8)} \]

**Proof:** AS\textsubscript{4} has \(n_4(p_i - C_4)\) as payoff if it uses IPv4; the migration to IPv6 results in \(n_4(1 + x)(1 + \frac{11}{16}(1 + \frac{3}{8}))p_i - C_6 - \frac{k_i - k_i}{k_i}e - S_{4\rightarrow6})\) payoff. Therefore, if the following hold, it is worth to choose

\begin{table}
\centering
\caption{Payoff matrix in the two-player migration game}
\begin{tabular}{|c|c|c|}
\hline
 & IPv4 & IPv6 \\
\hline
IPv4 & \((n_1(p_1 - C_4), n_2(p_2 - C_4))\) & \((n_1(p_1 - C_4), n_2(p_2 - C_6 - e - S_{4\rightarrow6}))\) \\
IPv6 & \((n_1((1 + x)p_1 - C_6 - e - S_{4\rightarrow6}), n_2(p_2 - C_4))\) & \((n_1((1 + 3x)p_1 - C_6 - S_{4\rightarrow6}), n_2((1 + 3x)p_2 - C_6 - S_{4\rightarrow6}))\) \\
\hline
\end{tabular}
\end{table}
IPv6.

If \( \hat{k}_i = 0 \) the left-hand side expression has its minimal value while the right-hand side expression has its maximum. Therefore, if the condition of the theorem holds for every AS the all-IPv6 network is a unique Nash equilibrium.

This assures the complete dominance of IPv6 in the network. The implication of the theorem is that the first migrating ASes who migrate to IPv6 later, realize higher revenues and hence, the impact of the theorem can be analyzed using evolutionary game theory [15].

In an evolutionary dynamics model there exist two different strategies—in our analysis IPv4 and IPv6—that are applied by the members of a population. Consider a game between IPv4 and IPv6 strategies in a population, where \( N \) identical ASes exist playing with each other randomly; the payoffs of the individual ASes are as follows. If both AS uses IPv4 they realize \( \Pi_4 = p - C_4 \), if one of them uses IPv6 and the other uses IPv4 they have \( \Pi_{46} = (1 + x)p - C_6 - e - S_{4\rightarrow6} \) and \( \Pi_4 = p - C_4 \) payoffs, respectively, and if both AS apply IPv6 their payoff is \( \Pi_6 = (1 + 3x)p_2 - C_6 \).

The property of the population, i.e. the deployment of IPv6, can be described with only one variable due to the identical ASes; the number of ASes using IPv6 is denoted by \( i \) while \( N - i \) represents the number of IPv4-using ASes. Based on the individual payoff of the ASes, we compute the expected payoff of the strategies in the population. \( i \) member of the population uses IPv6, therefore one of them can interact with \( i - 1 \) IPv6-using ASes, thus realize high profit. Contrary, the AS can play with one of the \( N - i - 1 \) IPv4-using ASes which results lower payoff. As the members of the population interact with each other randomly the expected payoff the IPv4 and IPv6 can be formulated as follows:

\[
\begin{align*}
\Pi_4^{(i)} &= \Pi_4 \\
\Pi_6^{(i)} &= \frac{\Pi_{66} (i - 1) + \Pi_{46} (N - i - 1)}{N - 1}
\end{align*}
\]

The fitness of a strategy describes its worth, e.g. if IPv6 has higher fitness then it is better for an AS to use IPv6 because it will realize higher payoff. The fitness of a strategy depends on its expected payoff, however, the payoff is weighted with \( \omega \). Let \( \omega \) denotes the intensity of selection, if \( \omega = 0 \) the

**Table III**

<table>
<thead>
<tr>
<th>Protocol version</th>
<th>Payoff</th>
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<tbody>
<tr>
<td>AS1</td>
<td>AS2</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
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<td>6</td>
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protocols are identical—they have the same fitness—where \( \omega = 1 \) presents a game where only the expected payoff of the strategies determines the value of the fitness. Accordingly, the fitness of IPv4 and IPv6 is represented by the following expressions:

\[
\begin{align*}
    f_4^{(i)} &= (1 - \omega) + \omega \Pi_4^{(i)} \\
    f_6^{(i)} &= (1 - \omega) + \omega \Pi_6^{(i)}
\end{align*}
\]  

The evolutionary game is modelled as a Moran process, a stochastic model—similar to a birth-death process—describing selection in population between IPv4 and IPv6. In a Moran process, first a parent individual (an AS) is selected based on the ratio of the fitness values to be the sample for reproduction. After that, another AS is selected randomly who changes its protocol based on the sample. We note that it is not necessary that the composition of the population changes, e.g. both selected ASes use the same protocol. We include a new stochastic parameter into our analysis; \( M \) denotes the impact of migration cost on the fitness. If \( M = 1 \) then the migration takes place always while in case of \( M = 0 \) no migration happens, thus the migration is proportional to the value of \( M \). The probabilities that the number of IPv6-using ASes increases, decreases, or remains the same can be expressed using the fitness values and the migration variable:

\[
\begin{align*}
    p_{i,i+1} &= \frac{if_6^{(i)}}{if_6^{(i)} + (N - i)f_4^{(i)}} N - i \quad M \\
    p_{i,i-1} &= \frac{(N - i)f_4^{(i)}}{if_6^{(i)} + (N - i)f_4^{(i)}} i \quad M \\
    p_{i,i} &= 1 - p_{i,i+1} - p_{i,i-1}
\end{align*}
\]

The formulas imply that \( p_{0,0} = p_{N,N} = 1 \) because if all the ASes uses the same protocol there will not be a parent AS with the other protocol. Techniques of queuing theory can be applied as this is a birth-death process; the \( y = Py \) equation holds where \( P \) is the stochastic transition matrix and \( y^{(i)} \) represents the asymptotic probability of reaching the all IPv6 AS population, if the starting state is \( i \). In evolutionary dynamics, the probability that one of the strategies takes over the whole population is called fixation probability. The fixation probability of IPv6 can be expressed as \( P_6 = \frac{1}{1 + \sum_{k=1}^{N-1} \prod_{i=1}^{k} \frac{f_4^{(i)}}{f_6^{(i)}}} \) [10]; selection favours IPv6 replacing IPv4 if \( P_6 > \frac{1}{N} \). This exact analytical result gives a condition of the migration to IPv6, however, important aspects of the migration process are not included in this simple model.

Therefore, we next extend the evolutionary model to include Internet related properties like the topology and the different size of the ASes. In our extended model, on the one hand, the ASes have different sizes in terms of number of their customers, on the other hand, an AS can interact with and influence its neighbouring ASes.

The payoff of the ASes is based on our economic model without the migration cost. The expected payoff of the strategies can be expressed based on the individual payoffs and the topology as follows, where \( L \) denotes the actual population—\( L_4 \) and \( L_6 \) denotes the set of IPv4 and IPv6 using ASes:

\[
\begin{align*}
    \Pi_4^{(L)} &= \frac{\sum_{i \in L_4} n_i \Pi_4^{(i)}}{\sum_{i \in L_4} n_i} \\
    \Pi_6^{(L)} &= \frac{\sum_{i \in L_6} n_i \Pi_6^{(i)}}{\sum_{i \in L_6} n_i}
\end{align*}
\]

The expected values of the protocols are the average of the individual AS payoffs weighted with the size of the ASes. The economic model includes the impacts of topology, thus, we do not have to compute the expected payoffs based on the protocol versions used by the neighbouring ASes. The fitnesses of the protocols are computed based on Eq. 11 and 12.

In the evolutionary game, the parent AS is selected randomly on its weighted fitness value. The influenced AS may be selected based on its size if we focus on the additional revenues and decreased cost. On the contrary, selecting the influenced AS inversely proportional to its size can also be motivated as smaller AS can change their protocol easier. However, we can not judge which model describes better the reality, therefore, we decide to select the influenced AS randomly out of neighbouring ASes. The probability that the number of IPv6-using ASes increases by one can be expressed as follows; the probability of IPv6 can be formulated similarly:

\[
\begin{align*}
    p_{|L_6|=|L_4|+1} &= \frac{\sum_{i \in L_4} n_i f_6^{(i)} + \sum_{i \in L_6} n_i f_4^{(i)}}{\sum_{i \in L_4} n_i f_6^{(i)} + \sum_{i \in L_6} n_i f_4^{(i)} \sum_{j \in N(i)} i} M
\end{align*}
\]

In the extended version of the evolutionary game, the fixation probability can not be computed from the transition matrix as above. On the one hand, the construction of the transition matrix is hard because the members of the network are not equivalent. Neither the size nor the neighbours of the ASes are the same, therefore, a state would have been created for every possible population structure—resulting exponential matrix size. On the other hand, the transition matrix is dynamic; the fitness of the strategies and therefore the transition probabilities are altered after every step where a protocol change happened. Accordingly, we do not provide analytical results for the extended version of the evolutionary game; however, we present extended simulation results based on the proposed evolutionary model to gain insight about the stochastic behaviour of the migration to IPv6.

V. SIMULATION RESULTS

In this section, we present migration cases based on extensive simulations carried out on different real world topologies (US, Japan, and world topology). We use data provided by CAIDA [16], especially the structure of the AS topology and the properties of ASes, in terms of size of allocated IPv4 address space, are utilized. The simulators are implemented in Python. First, we show the migration to IPv6 process based on the strategic decisions of the ASes. Next, we investigate the protocol change in a stochastic way by applying the methods of evolutionary dynamics. Finally, we illustrate the impact of the resell of the IPv4 addresses on the duration of the migration.
A. The impact of selfish decisions on migration

In this section, we present simulation results where the migration decisions are made based on the self-interest of the ASes. In the simulations, the topology of US ASes is applied where 11,968 ASes exist with 24,419 connections in total; only connections between two US ASes are considered. The country information of the ASes is also based on the data provided by CAIDA.

Every AS starts to use IPv6 when the AS can realize higher profit with the new protocol. The revenues and expenditures determine the process of migration. However, initiatives may influence the protocol selection of some ASes; these directives have an impact on the migration. We suppose that ASes on a fix level in the AS hierarchy have been already switched to IPv6—based on the recommendation of the initiatives.

The size of the ASes is multiplied by 1.5 as a growth rate in every round. The ASes decide in every round based on their payoff function, where the followings parameters are set. The price of the services is \( p = 100 \), the service and management costs of IPv4 and IPv6 are \( s_4 = m_4 = 120 \) and \( s_6 = m_6 = 100 \), the cost of migration to IPv6 is \( S_{4\rightarrow6} = 100 \), the conversion cost is \( e = 10 \), while the ASes look forward for 1 rounds with 0.9 as discount value. We select these parameters for the ASes identical but we include an addition logarithmic, in the size of the AS, expenditure in order to explore the impact of the hierarchy. Otherwise, every AS would migrate to IPv6 in the same round as both revenues and costs are proportional to the size. We present the number of IPv6 using ASes in Figure 2 where three cases are presented: the ASes in the lowest(14), average(12), and high hierarchy level (2) have IPv6 already at the beginning of the simulation.

The first group of bars shows the number of ASes at the specific hierarchy level. If the local ASes, positioned in the lowest hierarchy level, migrate to IPv6 then the whole IPv6 network is reached the fastest (in the fourth round) while at the other cases even in the fifth round exist ASes who realize higher profit with IPv4. The network effect, i.e. higher revenues and lower costs, is the cause of this phenomena. The percentages of IPv6 using ASes at round 5 are nearly identical in case of high level and medium level ASes, however, the migration process is slower if the migration starts from the top. The implication of these results is that initiatives have to motivate first the local ASes for a faster migration.

B. The impact of group utility on migration

Another aspect of the IPv6 migration can be examined based on the methods of evolutionary dynamics; the simulations are carried out on the AS topology of Japan, which consists of 481 ASes and 964 connections. As we mentioned before, in this case the strategies—IPv4 and IPv6—compete with each other; the members of the populations are playing games with their neighbours and realize profit. We use the same cost and revenue parameters as in the strategic analyses, however, the switching cost is replaced with \( M = 0.99 \) as the probability of migration. We note that the ASes are not able to switch back to IPv4 contrary to the analytical analyses. The weight of evolution is set as \( \omega = 0.9 \). An iteration of the simulation represents a moment when an AS may migrate to IPv6 based on examples of adjacent ASes.

We illustrate the stochastic behaviour of the population in Figure 3. The results of four diverse runs are selected to plot with 25 percent as the initial ratio of IPv6 ASes. The stochastic behaviour has an impact on the dynamics of the IPv6 migration. The effect can be quantified based on the horizontal axis, e.g. the number of necessary iterations until the 50 percent IPv6 usage can be almost tripled. Moreover, the figure quantifies the uncertainty of decisions on the vertical axis. In some cases, the difference of IPv6 penetration can reach even 20 percent. The topology of the ASes influences the outcome of the migration process as an AS migrates to IPv6 based on a positive example; one of its neighbouring AS using IPv6 can convince the AS due to the larger payoff realized by the neighbour.

In order to reduce the variation of the results, we execute 50 runs for every parameter set. Based on these simulations, we compute the cumulative distribution function of the cases where at least 60% of ASes use IPv6 after different number of iterations, presented in Figure 4. The initial percentages of IPv6-using ASes are 10, 25, and 50; the IPv6 ASes are selected randomly for the initial population setting. Despite the lower costs of IPv6, the population that has low initial IPv6 deployment does not tend to migrate to IPv6 fast as after 2500 iterations only in less than 40 percent of cases becomes
IPv6, the dominant protocol. However, in case of higher initial IPv6 ratios, the IPv6 penetration exceeds 60% after several interactions. The plot also reveals the uncertainties of the migration as the 60 percent IPv6 ratio is not always reached after a given number of iterations, e.g. in case of 25% initial deployment, 500 iterations are rarely enough to reach 60% penetration. This finding shows that even if IPv6 has higher revenues and lower costs than IPv4, the migration process can be delayed by the uncertainties of human decisions.

C. The impact of IPv4 address resell on migration

Throughout the paper, we do not considered the impact of the resell of IPv4 addresses on the migration process. An AS may sell its IPv4 address space after it has migrated to IPv6; these deallocated addresses can be utilized by other ASes using still IPv4. We quantify the impact of the address resell in this section based on simulation results carried out on the world AS topology of 31,273 ASes. We assume in our simulations that the number of the users of ASes increases in every round, e.g. every month, based on the growth rate resulting exponential growth. Thus, if an AS can not purchase additional IPv4 addresses that covers its increased needs then the AS will migrate to IPv6. The available free IPv4 address space is estimated based on [17].

Figure 5 shows the fraction of IPv4-using ASes at different rounds. If the ASes can not sell their addresses, the whole network migrate to IPv6 after a few rounds. However, if the ASes are allowed to sell their IPv4 addresses, the migration process significantly slows down, the complete migration happens only after more than one hundred rounds. Similar delay occurs in case of larger growth rate, similarly, the time difference between the resell and non resell case is not negligible. The finding of these results is that the migration to IPv6 can happen much later—the difference might be expressed in years—than expected if the ASes are able to resell their IPv4 addresses. However, we note that the migration may be motivated, in case of larger IPv6 penetration, not only with the address exhaustion but also with the exclusive services of IPv6.

VI. Conclusions

In this paper, we have addressed the issue of modelling the migration to IPv6 using game-theoretical tools. We have shown under our proposed economic model that IPv6 usage is a Nash equilibrium strategy under some circumstances, even on general topologies. In addition, we presented an evolutionary game model, where the two versions of IP compete with each other in a population; based on the model we have quantified the impact of uncertainties on the dynamics of migration, the same IPv6 penetration can be reach after up to three times longer periods as well. Along with the analytical findings, simulation results have also been shown. We have presented that the migration happens faster if the ASes at the lower levels of the AS hierarchy are incentivized to use IPv6. Furthermore, we have quantified the impact of the resell of IPv4 address spaces; the migration process can be delayed significantly if the ASes sell their unused IPv4 addresses after their migration. Therefore, we suggest authority initiatives—whose identity are not obvious in the eco-system of Autonomous Systems—should restrict the resell of IPv4 addresses if they want to promote the widespread deployment of IPv6.

REFERENCES