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**Working Paper**

## Part-time unemployment and optimal unemployment insurance

IZA Discussion Papers, No. 5540

**Provided in Cooperation with:**

IZA – Institute of Labor Economics

*Suggested Citation:* Ek, Susanne; Holmlund, Bertil (2011) : Part-time unemployment and optimal unemployment insurance, IZA Discussion Papers, No. 5540, Institute for the Study of Labor (IZA), Bonn,  
<https://nbn-resolving.de/urn:nbn:de:101:1-201104133432>

This Version is available at:

<https://hdl.handle.net/10419/51861>

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IZA DP No. 5540

## **Part-Time Unemployment and Optimal Unemployment Insurance**

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February 2011

# Part-Time Unemployment and Optimal Unemployment Insurance

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Discussion Paper No. 5540  
February 2011

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## ABSTRACT

### **Part-Time Unemployment and Optimal Unemployment Insurance**

A significant fraction of the labor force consists of employed workers who are part-time unemployed (underemployed) in the sense that they are unable to work as much as they prefer. This paper develops a search and matching model to study the design of optimal unemployment insurance in an economy with unemployment as well as part-time unemployment. Part-time unemployment provides income insurance and serves as a stepping stone to full-time jobs. Unemployment benefits for part-timers increase the outflow from unemployment to part-time work but reduce the outflow from part-time work to full-time employment. We examine the optimal structure of benefits for unemployed and underemployed workers. The results indicate non-negligible welfare gains associated with time limits for unemployment benefits as well as for part-time benefits. The welfare gains from optimal UI are larger when wages are fixed than when they are flexible.

JEL Classification: J64, J65

Keywords: job search, part-time unemployment, unemployment insurance

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# 1 Introduction

Conventional measures of unemployment do not capture all dimensions of joblessness. A case in point is underemployment among employed workers, also frequently referred to as part-time unemployment. Workers in this category are employed during a survey week but are unable to work as many hours as they wish. An ILO-resolution from 1998 defines underemployment as comprising persons in employment who (i) are willing to work additional hours, (ii) are available to work additional hours, and (iii) worked less than a threshold relating to working time. Underemployment accounts for a non-negligible fraction of the labor force in most countries, although it is typically somewhat lower in magnitude than the conventional measure of unemployment. By the end of 2009, it stood at almost 5 percent of the labor force in the OECD area.<sup>1</sup>

Research on part-time unemployment has been meager compared to the huge literature on “full-time” unemployment, perhaps reflecting the presumption that underemployment represents less of a social problem than complete joblessness. It is clear, however, that part-time unemployment is of a non-trivial magnitude and raises a number of important policy issues. One issue, hotly debated in some countries, concerns labor market regulation and the case for giving part-timers precedence to full-time jobs. Other issues deal with unemployment insurance and concern the design of benefit levels and potential benefit durations for the unemployed and the underemployed. Our paper contributes to the literature on optimal unemployment insurance with special focus on part-time unemployment. To the best of our knowledge, our paper is the first study that examines optimal unemployment insurance for an economy with both unemployment and underemployment.

We study an economy with two sectors where full-time jobs are offered in one sector and part-time jobs in the other sector. One can think of these sectors as representing two industries; indeed, the data show that the prevalence of part-time unemployment is heavily correlated with industry affiliation, being particularly common in service industries. Unemployed workers

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<sup>1</sup>See OECD (2010), Figure 1.5 and the relevant source table.

prefer full-time jobs but are willing to consider part-time jobs as a stepping stone to full-time jobs. The unemployed worker thus searches for part-time as well as full-time jobs and accepts the first offer that comes along. The part-timer searches only for full-time jobs and quits her part-time job as soon as a full-time offer comes along.

Unemployment benefits affect search effort among the unemployed as well as among the part-timers. More generous benefits for part-timers reduce the flow from part-time work to employment as part-time status becomes more attractive relative to employment. But benefits for part-timers also make part-time status more attractive relative to unemployment, thus increasing the outflow from unemployment to part-time work. In that respect, offering benefits to part-timers is akin to in-work benefits, a policy which has received considerable attention in both policy discussions and research.

We study the optimal structure of unemployment benefits focusing on benefit levels (replacement rates) and the potential duration of benefit receipt for the unemployed as well as for part-timers. The model is of the search and matching variety (Pissarides, 2000). We calibrate the model and find non-negligible welfare gains associated with time limits for unemployment benefits as well as for part-time benefits. The welfare gains from optimal UI are larger when wages are fixed than when they are flexible.

## 2 Related literature

Part-time work has attracted much attention in policy debates as well in labor market research. One issue is whether part-time jobs serve as stepping stones to full-time jobs or mainly constitute dead ends that trap some workers into permanent low-income status; see e.g. Tam (1997) and papers in O'Reilly and Fagan (1998). Another issue concerns how one should understand the part-time pay penalty, i.e., the fact that hourly pay is typically found to be lower for part-timers than for full-timers. The policy discussions have also considered how transitions from part-time to full-time jobs can be affected by regulation and reforms of the unemployment insurance (UI) systems.

Many countries have UI systems that extend UI eligibility to part-timers who search for full-time jobs. There is a small empirical literature that studies the effects of such part-time benefits on transitions to employment. An early paper is Munts (1970), who examines how part-time benefits affect search for regular work. He finds that workers adjust their part-time work so as to gain from combined earnings and benefits. The results in Holen and Horowitz (1974) reinforce Munts's hypothesis. A more recent paper by McCall (1996) studies the effect on job search behavior of an increase in the "disregard", i.e. the highest allowed amount of earnings to still be eligible for unemployment benefits. He finds that an increase in the disregard lowers expected unemployment duration and increases the conditional probability of part-time reemployment. However, his analysis sheds no light on how part-time benefits affect the transitions to full-time work.

Some recent Nordic studies examine how part-time benefits influence transitions from unemployment to full-time employment. Using data from Finland, Kyyrä (2010) finds that part-time benefits increase the probability of finding regular jobs. Using data from Denmark, Kyyrä et al. (2010) find evidence of a significant lock-in effect: being on part-time benefits lowers the transition rate out of part-time employment, although there are positive effects for some subgroups. Månsson et al. (2008) find that part-time unemployed Swedish benefit recipients have around 20 percent lower chance of landing a full time job within two years compared to part-time unemployed persons without benefits.<sup>2</sup>

The paper also relates to the literature on optimal unemployment insurance design. This literature has focused on issues such as the case for benefit variation over the spell of unemployment and the interaction between UI and active labor market policy.<sup>3</sup> Most papers have considered economies with only unemployment and employment, ignoring involuntary part-time work. New issues arise when there is part-time unemployment in the economy. Part-time benefits subsidize part-time work and thus increase the flow

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<sup>2</sup>The problem of separating causal effects from selection effects is addressed ("timing of events approach") in Kyyrä (2010) and Kyyrä et al (2010) but not in the Swedish study.

<sup>3</sup>See Fredriksson and Holmlund (2006) for a survey.

out of unemployment. However, such benefits are also bound to reduce the outflow from part-time status to full-time employment.

## 3 The Model

### 3.1 The Labor Market

We consider an economy where part-time status is “involuntary” in the sense that part-time workers prefer to work longer hours at the prevailing hourly wage rate. Unemployed individuals are willing to accept part-time work as a stepping stone to full-time work.<sup>4</sup> Voluntary part-time work, reflecting preferences for leisure or home production, is ignored. A satisfactory treatment of both voluntary and involuntary part-time work would require a model where workers have heterogeneous preferences; our model features workers with identical preferences.

The economy is populated by infinitely lived individuals who are all members of the labor force. Workers can be in one of three mutually exclusive states, i.e., they can be fully employed, fully unemployed (referred to as unemployed), or part-time (un)employed; the third category is interchangeably referred to as part-timers or underemployed. The size of the labor force is normalized to unity and the labor force identity is written as

$$e + p + u = 1 \tag{1}$$

where  $e$ ,  $p$  and  $u$  stands for employment, underemployment and unemployment, respectively. We let index  $j$ ,  $j = e, p, u$ , represent labor force states.

Employed workers can work as much as they prefer whereas part-timers are unable to do so and they can therefore be viewed as involuntary part-time unemployed. Part-timers search for full-time jobs in order to realize their preferred working time. The unemployed search for part-time as well as full-time jobs, recognizing that part-time jobs can provide partial income

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<sup>4</sup>Panel data from the Swedish labor force statistics reveal substantial mobility out of part-time unemployment. Over the period 2005–2009, some 20–25 percent of the part-timers searching for full-time jobs have entered full-time work after one quarter.



insurance as well as offering stepping stones to full-time jobs. Search among the unemployed takes place in an undirected fashion: they search for work and whether they become full-time or part-time employed is determined by a random matching process. The probability that an individual unemployed person ends up as employed depends on the number of vacant full-time jobs relative to the competition she faces from other unemployed persons as well as part-timers (since both groups compete for full-time jobs). Analogously, the probability that she ends up as part-time employed depends on the number of vacant part-time jobs relative to the number of unemployed (as only the unemployed compete for part-time jobs).

There are two types of firms (two sectors) that offer either full-time or part-time jobs. Let index  $k$ ,  $k = E, P$ , represent the type of job where  $E$  and  $P$  stand for full-time and part-time jobs, respectively. The job finding rates are determined via sector-specific and constant returns matching functions,  $M_k = m(v_k, S_k)$ , where  $v_k$  is the number of vacancies in sector  $k$  and  $S_k$  is the effective number of workers competing for jobs in sector  $k$ . Labor market tightness in each sector is given by  $\theta_k \equiv v_k/S_k$ . The rate at which an unemployed worker with search effort  $s_u$  finds a full-time job is given by  $s_u m(v_E, S_E)/S_E = s_u \alpha(\theta_E)$ , where  $S_E \equiv s_u u + s_p p$  is the effective number searchers competing for full-time jobs. The rate at which she finds a part-time job is given by  $s_u m(v_P, S_P)/S_P = s_u \alpha(\theta_P)$ , where  $S_P = s_u u$ . Analogously, a part-timer with search effort  $s_p$  finds a full-time job at the rate  $s_p m(v_E, S_E)/S_E = s_p \alpha(\theta_E)$ . Firms encounter searchers at the rate  $q_k = m(v_k, S_k)/v_k = q(\theta_k)$  and thus we have  $\alpha(\theta_k) = \theta_k q(\theta_k)$ . For ease of notation we will, when convenient to do so, use shorthand notations of the form  $\alpha_E = \alpha(\theta_E)$  and  $\alpha_P = \alpha(\theta_P)$ .

Full-time as well as part-time jobs are destroyed at the exogenous rate  $\phi$ . Job destructions always involve job losses for the workers, i.e., entry into unemployment. The steady state flow equilibrium conditions for  $u$  and  $p$  are given as

$$s_u [\alpha (\theta_P) + \alpha (\theta_E)] u = \phi(1 - u) \quad (2)$$

$$[s_p \alpha (\theta_E) + \phi] p = s_u \alpha (\theta_P) u \quad (3)$$

where the left-hand sides capture the outflows from  $u$  and  $p$  and the right-hand sides the inflows. Employment is obtained residually from the labor force identity as  $e = 1 - u - p$ .

### 3.2 Workers

The individual's instantaneous utility is increasing in consumption and leisure. Workers are risk averse and have identical preferences represented by a logarithmic utility function of the form:

$$v_j = \ln c_j + \delta \ln l_j \quad (4)$$

where  $c_j$  denotes consumption and  $l_j$  stands for leisure. We ignore borrowing and savings so consumption equals income in each state.

Let  $B$  denote unemployment benefits for the unemployed worker and  $b$  benefits for the part-timer. Let  $w_j$  denote the wage rate per unit of labor input and let  $h_j$  be working time. All incomes, including benefits, are taxed at the rate  $t$ . The employed worker thus consumes  $c_e = W_e(1 - t) \equiv w_e h_e(1 - t)$ , whereas the part-timer consumes  $W_p(1 - t) \equiv w_p h_p(1 - t)$ . The part-timer's total consumption is thus given by  $c_p = (W_p + b)(1 - t)$ , whereas the unemployed worker's consumption is  $c_u = B(1 - t)$ .

The individual's time endowment is denoted  $T$ . The time budget restriction for the employed worker is given by  $T = h_e + l_e$ ; the relevant restriction for the part-timer is  $T = h_p + l_p + s_p$ ; and the unemployed worker faces the restriction  $T = l_u + s_u$ . For log utility we thus have

$$v_e = \ln(w_e h_e) + \delta \ln(T - h_e) \quad (5)$$

$$v_p = \ln(w_p h_p + b) + \delta \ln(T - h_p - s_p) \quad (6)$$

$$v_u = \ln B + \delta \ln(T - s_u) \quad (7)$$

We assume that full-timers are free to choose their desired working time. With log utility, this implies  $h_e = T/(1+\delta)$ . Part-timers are unable to realize their desired working time so  $h_p < h_e$ . Working time for part-timers is taken as exogenous.

Consider the intertemporal objective functions associated with the three states. Let  $U$  denote the expected discounted present value of utility for an unemployed worker, let  $P$  be the corresponding value if the person is a part-timer, and let  $E$  be the value if the person is employed. The value functions are written as

$$rE = v_e + \phi(U - E) \quad (8)$$

$$rP = v_p + \phi(U - P) + s_p \alpha(\theta_E)(E - P) \quad (9)$$

$$rU = v_u + s_u [\alpha(\theta_P)(P - U) + \alpha(\theta_E)(E - U)] \quad (10)$$

where  $r$  is the subjective rate of time preference. As usual, the flow value of employment, as given by (8), involves the instantaneous utility  $v_e$  as well as a risk of job loss with an associated present value utility loss as the worker switches from employment to unemployment. The flow value of underemployment, as given by (9), involves the instantaneous utility  $v_p$ , the risk of job destruction (the second term on the right-hand side), and also the prospect of making a utility increasing transition to employment (the third term). Finally, the flow value of unemployment, as given by (10), entails the instantaneous utility,  $v_u$ , as well as the possibility of moving to either employment or part-time status.<sup>5</sup> It is assumed that the parameters of the model are such that the inequalities  $E > U$ ,  $P > U$  and  $E > P$  hold.

The value functions can be solved for present value differences. When evaluated at  $r \rightarrow 0$ , these expressions take the form:

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<sup>5</sup>The value functions above imply that a job loss for a part-timer results in the same income as is available for a full-time employed worker if laid off. This is realistic for some countries to the extent that the part-timer has a history of full-time work.

$$E - U = \frac{1}{A} [(\phi + s_p \alpha_E)(v_e - v_u) + s_u \alpha_P(v_e - v_p)] \quad (11)$$

$$P - U = \frac{1}{A} [(\phi + s_u \alpha_E)v_p - (\phi + s_p \alpha_E)v_u - (s_u - s_p)\alpha_E v_e] \quad (12)$$

$$E - P = \frac{v_e - v_p}{\phi + s_p \alpha_E} \quad (13)$$

where  $A \equiv (\phi + s_u \alpha_E + s_u \alpha_P)(\phi + s_p \alpha_E)$ . It is clear that a proportional tax on all labor income, including benefits, will have no effect on the present value differences given by (11), (12) and (13).

The part-timer chooses search effort,  $s_p$ , in order to maximize  $rP$ ; likewise, the unemployed person chooses her search effort,  $s_u$ , in order to maximize  $rU$ . The first-order conditions are then given as:

$$s_p : \frac{\delta}{T - h_p - s_p} = \alpha(\theta_E)(E - P) \quad (14)$$

$$s_u : \frac{\delta}{T - s_u} = \alpha(\theta_P)(P - U) + \alpha(\theta_E)(E - U) \quad (15)$$

These conditions state that the marginal cost of increasing search effort should be equated to the expected marginal gain of doing so. It is clear from the expressions that the marginal gain of increasing search effort is higher for an unemployed worker than for a part-timer. The unemployed worker's returns from additional search involve a chance of finding a part-time job as well as a full-time job whereas the part-timer's search is confined to full-time jobs. It follows immediately that an unemployed person allocates more effort to job search than a part-timer, i.e.,  $s_u > s_p$ . Indeed, it follows that  $s_u > h_p + s_p$ , implying  $l_u < l_p$ : an unemployed person enjoys less leisure than a part-timer.

### 3.3 Firms

Firms operate under constant returns to labor, an assumption that allow us to treat a job as a stand in for the firm (Pissarides, 2000). As already noted,

we consider an economy with two sectors (industries), where one consists of full-time jobs and the other includes part-time jobs. There is free entry of firms in either sector and the number of jobs in each sector is endogenously determined. Jobs of either type are destroyed at the rate  $\phi$ . A full-time job is never transformed into a part-time job so a worker who loses a full-time job becomes unemployed.

Workers and jobs are randomly matched: a firm with a vacant full-time job hires a job searcher at the rate  $q(\theta_E)$  whereas a firm with a vacant part-time job finds a worker at the rate  $q(\theta_P)$ . Let  $V_k$  denote the present value of opening a vacancy and  $\kappa$  the flow cost of a vacancy. The value functions for vacancies then take the form:

$$rV_E = -\kappa + q(\theta_E)(J_E - V_E) \quad (16)$$

$$rV_P = -\kappa + q(\theta_P)(J_P - V_P) \quad (17)$$

Labor productivity, denoted  $y$ , is exogenous and uniform across workers and firms. The flow values of occupied full-time and part-time jobs are then written as:

$$rJ_E = (y - w_e)h_e + \phi(V_E - J_E) \quad (18)$$

$$rJ_P = (y - w_p)h_p + \phi(V_P - J_P) + s_p\alpha(\theta_E)(V_P - J_P) \quad (19)$$

The second value function is slightly non-standard since it incorporates on-the-job search,  $s_p\alpha(\theta_E)$ : part-timers search for full-time jobs which they land at the rate  $s_p\alpha(\theta_E)$ . When the part-time worker quits, the part-time job becomes vacant. Free entry implies  $V_k = 0$  and the resulting two key job creation equations can thus be written as:

$$J_E : \frac{(y - w_e)h_e}{\phi} = \frac{\kappa}{q(\theta_E)} \quad (20)$$

$$J_P : \frac{(y - w_p)h_p}{\phi + s_p\alpha(\theta_E)} = \frac{\kappa}{q(\theta_P)} \quad (21)$$

where  $r \rightarrow 0$  is assumed. Free entry and zero discounting imply that the steady-state flows of profits are equal to zero for both types of firms, i.e.,  $\pi_k = e(y - w_e) - \kappa v_E = p(y - w_p) - \kappa v_P = 0$ .

### 3.4 Wage Determination

In search and matching models, wages are typically determined by decentralized worker-firm Nash bargaining. This approach leads to a high degree of real wage flexibility in response to shocks, a feature that has generated some controversy in the recent literature; see for example Shimer (2005), Hall (2005), Hall and Milgrom (2008), and Pissarides (2009). Some authors have argued that the baseline model generates too much wage flexibility and have considered models where the relevant threat points in the wage bargain are the payoffs during delays rather than payoffs available if the parties separate from each other. These models generate more wage rigidity than the standard formulation.

We proceed under two polar assumptions regarding wage determination. In the *fixed-wage* regime, we simply impose completely rigid real wages. In the *flex-wage* case, we adopt the standard approach where wages are determined by decentralized worker-firm Nash bargaining with the outside options taken as threat points. By considering both fixed-wages and flex-wages, we can also shed light on how optimal UI policies are affected by allowing for incentives in wage setting. The standard model implies that higher UI benefits lead to an increase in wage pressure, a mechanism that represents a source of moral hazard in addition to the impact on search effort. This mechanism may suggest that optimal UI with flexible wages would be less generous than optimal UI with fixed wages. This logic may not necessarily carry over to our model, however; as we shall see, generous benefits for part-timers may actually provide incentives for wage moderation.

Consider, then, Nash bargaining with both full-time and part-time workers. The relevant threat point for both categories is taken to be unemployment,  $U$ . The Nash product for full-time firms is then

$$\Omega(w_e) \equiv (E - U)^\beta (J_E - V_E)^{1-\beta}$$

where  $\beta \in (0, 1)$  denotes the worker's bargaining power. The first-order condition evaluated at  $V_E = 0$  is

$$(1 - \beta)(E - U)h_e = \beta J_E \frac{\partial v_e(w_e)}{\partial w_e} \quad (22)$$

where  $J_E = \kappa/q(\theta_E)$ . We note that the first-order condition (22) is independent of the tax rate when the utility function is logarithmic; we have  $\partial v_e(w_e)/\partial w_e = 1/w_e$ .

The relevant Nash product for part-time bargaining is

$$\Omega(w_p) \equiv (P - U)^\beta (J_P - V_P)^{1-\beta}$$

and the first-order condition evaluated at  $V_P = 0$  is

$$(1 - \beta)(P - U)h_p = \beta J_P \frac{\partial v_p(w_p)}{\partial w_p} \quad (23)$$

where  $J_P = \kappa/q(\theta_P)$ . Again we note that the first-order condition is independent of the tax rate. The marginal utility of a wage increase to the part-time worker is  $\partial v_p(w_p)/\partial w_p = h_p/(w_p h_p + b)$ .

### 3.5 Equilibrium

All ingredients of the model are now in place. The key relationships are the two job creation equations, two first-order conditions for optimal search, and three equations for present value differences, i.e., (11), (12) and (13). When wages are endogenous, we add the two first-order conditions for wage bargaining. We reproduce the equations for job creation, optimal search and Nash bargaining:

$$\frac{(y - w_e) h_e}{\phi} = \frac{\kappa}{q(\theta_E)} \quad (24)$$

$$\frac{(y - w_p) h_p}{\phi + s_p \alpha(\theta_E)} = \frac{\kappa}{q(\theta_P)} \quad (25)$$

$$\frac{\delta}{T - h_p - s_p} = \alpha(\theta_E)(E - P) \quad (26)$$

$$\frac{\delta}{T - s_u} = \alpha(\theta_P)(P - U) + \alpha(\theta_E)(E - U) \quad (27)$$

$$\beta(E - U)h_e = (1 - \beta) \frac{\kappa}{q(\theta_E)} \frac{\partial v_e(w_e)}{\partial w_e} \quad (28)$$

$$\beta(P - U)h_p = (1 - \beta) \frac{\kappa}{q(\theta_P)} \frac{\partial v_p(w_p)}{\partial w_p} \quad (29)$$

Equations (24) – (29) determine  $\theta_E, \theta_P, s_p, s_u, w_e$  and  $w_p$ , using also (11), (12) and (13). The unemployment and underemployment rates,  $u$  and  $p$ , are obtained from (2) and (3), noting that employment follows from  $e = 1 - p - u$ . One can use  $S_E \equiv s_u u + s_p p$  and  $S_P = s_u u$  to determine  $S_E$  and  $S_P$ . Finally, by invoking  $\theta_k = v_k/S_k$  we obtain the number of vacancies of either type,  $k = E, P$ .

As we have noted, the bargained (gross) wages are independent of the tax rate; the tax is thus completely borne by the workers and the tax can be determined residually from the government's budget restriction. Government revenues are given by

$$T \equiv t[(1 - u - p)W_e + uB + p(W_p + b)]$$

and total benefit expenditures are given by  $TB \equiv uB + pb$ . The tax rate can then be obtained from the budget restriction  $T(t) = TB$ . A proportional tax on income including benefits at the rate  $t$  is equivalent to a proportional wage tax on firms at the rate  $t/(1 - t)$

$$\tau \equiv \frac{t}{1 - t} = \frac{uB + pb}{(1 - u - p)W_e + pW_p} \quad (30)$$

## 3.6 Comparative Statics

### 3.6.1 Fixed Wages

We are mainly interested in the impact of benefits,  $b$  and  $B$ . It is useful to start the analysis with exogenous wages, in which case the model has a simple recursive structure.  $\theta_E$  is determined from (24). Hence  $s_p$  is obtained from (26) using (13). Using these results,  $\theta_P$  is obtained from (25) and finally  $s_u$  from (27) using (11) and (12). It is helpful to make use of envelope properties that follow from the fact that workers optimally choose search efforts. Hence  $\partial U/\partial s_u = \partial P/\partial s_p = 0$ . From (8) follows that  $\partial E/\partial s_u = 0$  when evaluated at the optimal  $s_u$ . Moreover,  $\partial P/\partial s_u = 0$  follows from (9). When examining the impacts of  $b$  and  $B$ , it is thus sufficient to look at the impacts via  $v_p(b)$  and  $v_u(B)$ . Table 1 summarizes some comparative statics results where starred



signs are numerical results based on a calibrated version of the model; see Appendix for details about the calibration.

Table 1. Comparative statics, exogenous wages.

	$\theta_E$	$\theta_P$	$s_u$	$s_p$	$e$	$p$	$u$
$B$	0	0	-	0	-	-	+
$b$	0	+	+*	-	-*	+*	-*

When wages are fixed, an increase in  $B$  has no impact on  $\theta_E$  and hence no impact on  $s_p$  and therefore, via (25), no impact on  $\theta_P$ . From (24) also follows that  $\partial\theta_E/\partial b = 0$ . An increase in  $b$  reduces the utility difference ( $E - P$ ) and thus the returns to search while part-time unemployed; thus  $\partial s_p/\partial b < 0$ . Using also (25) we get  $\partial\theta_P/\partial b > 0$ . To understand this result, note that a decline in  $s_p$  increases the value to firms of part-time jobs since workers stay longer as part-timers; hence the incentive to open more part-time vacancies increases.

The impact on  $s_u$  is slightly more involved. Consider the first-order condition for  $s_u$  as given by (27) and note that  $B$  enters via  $(P - U)$  and  $(E - U)$ . Clearly, a higher  $B$  reduces both utility differences and thus  $\partial s_u/\partial B < 0$ . The impact of  $b$  works via  $\alpha(\theta_P)$ ,  $(P - U)$  and  $(E - U)$ , where  $\theta_P$  increases and  $E - U$  decreases with a rise in  $b$ . Note that  $(P - U)$  increases with a rise in  $b$ , a fact that reflects that part-time benefit is akin to an in-work subsidy. It is difficult to sign  $\partial s_u/\partial b$  but the ambiguity is of some interest since it suggests that high benefits for part-timers may have negligible consequences for unemployment. Indeed, for all calibrations of the model we find that  $\partial s_u/\partial b > 0$ , i.e., higher part-time benefits increase search efforts among the unemployed.

The ambiguity of  $\partial s_u/\partial b$  makes it impossible to determine how an increase in  $b$  affect  $u$  and  $p$ . However, if  $\partial s_u/\partial b > 0$ , it follows that  $\partial u/\partial b < 0$ . Indeed, this is what the calibrated model implies. A rise in part-time benefits reduces unemployment as it encourages search efforts among the unemployed. However, there is also a concomitant decline in search efforts among part-timers that will reduce the pace at which they locate full-time jobs. Higher part-time benefits therefore tend to increase part-time unemployment.

An increase in  $B$  increases  $u$  via reduced search efforts among the unemployed. This decline in  $s_u$  implies a concomitant fall in  $p$  via lower inflow into part-time work. The net effect on employment is negative: the rise in  $u$  is bigger than the decline in  $p$ .

### 3.6.2 Flexible Wages

To see how benefits affect wage bargaining, it is useful to examine the partial equilibrium effects by inspecting the first-order conditions. We take labor market tightness as given and use the envelope property that  $E - U$  and  $P - U$  are invariant to derivative changes of search effort when search is optimally determined. From (11) and (22), it is clear that a rise in  $B$  as well as a rise in  $b$  leads to higher wages for full-timers. The reason is that either type of benefit hike reduces the value of employment relative to the value of unemployment. From (12) and (23) follow that a rise in  $B$  leads to higher wages for part-timers, an implication of the fact that the utility difference between part-time work and unemployment is reduced. From (12) and (23) follow also that a rise in  $b$  leads to *lower* wages for part-timers. This result flows from the fact that a rise in  $b$  *increases* the utility difference between part-time work and unemployment while at the same time reducing the marginal utility to the worker of a wage hike. Higher part-time benefits thus encourage wage moderation among part-timers.

The comparative statics for the general equilibrium with endogenous wages are more involved. The system has no longer a simple recursive structure and analytical results are hard to come by. We have therefore resorted to a computational model; see Appendix for details. Table 2 shows numerical comparative statics results for the model with endogenous wages.

Table 2. Comparative statics, endogenous wages.

	$\theta_E$	$\theta_P$	$s_u$	$s_p$	$w_e$	$w_p$	$\ln(w_e/w_p)$	$e$	$p$	$u$
$B$	-	-	-	-	+	+	+	-	+	+
$b$	-	+	+	-	+	$\pm$	$\pm$	-	+	-

We note that higher part-time benefits increase search effort among the

unemployed, as in the fixed-wage case. Unemployment as well as employment is reduced whereas part-time unemployment increases. The partial equilibrium wage moderation result of a rise in  $b$ , i.e.,  $\partial w_p / \partial b < 0$ , does not generally carry over to the general equilibrium with endogenous labor market tightness. Note that a rise in  $b$  reduces  $\theta_E$  and increases  $\theta_P$ , thus increasing  $q(\theta_E)$  and reducing  $q(\theta_P)$ . By invoking the first-order conditions (28) and (29), it follows that  $E - U$  must decline relative to  $P - U$  which would suggest an increase in  $w_p$  relative to  $w_e$ . The negative partial equilibrium wage response may therefore be offset by wage responses to induced changes in tightness.

This completes the positive analysis and we turn to normative issues.

## 4 Optimal Unemployment Insurance

### 4.1 Optimal Policy with Indefinite Benefit Payments

We first consider a UI system with indefinite benefit payments. Benefits are financed by a proportional tax on wage income and benefits. We focus on steady states and ignore discounting, i.e., we let  $r \rightarrow 0$ . The relevant utilitarian welfare objective will then be the worker's expected utility which is a function of net income (consumption) in the three states:

$$\Lambda = \ln(1 - t) + uv_u(B) + pv_p(W_p + b) + (1 - u - p)v_e(W_e) \quad (31)$$

where wage incomes are given as  $W_e = w_e h_e$  for full-timers and  $W_p = w_p h_p$  for part-timers. Benefits can also be expressed in terms of replacement rates,  $\rho_j$ , i.e.,  $B = \rho_e W_e$  and  $b = \rho_p w_p (h_e - h_p)$ . The replacement rates capture the fraction of income losses covered.<sup>6</sup> Substitute the government budget restriction into the welfare objective and obtain:

$$\Lambda = -\ln[1 + \tau(\cdot)] + uv_u(B) + pv_p(W_p + b) + (1 - u - p)v_e(W_e) \quad (32)$$

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<sup>6</sup>Wages generally differ between full-timers and part-timers. Our results are virtually independent of the choice of wage concept in the definition of replacement rate.

where we have used  $\tau \equiv t/(1-t)$  and  $\tau(\cdot)$  is given by (30). Absent moral hazard, i.e., with exogenous search effort and exogenous wages, it is straightforward to confirm that optimal insurance is full insurance, i.e., equal incomes in the three states:  $W_e = B = W_p + b$ . However, search effort and wages respond to benefits and full insurance will not be optimal.

The welfare effect of a policy is measured relative to a baseline. It is expressed as the equivalent of a consumption tax that equalizes welfare across policy regimes. Let  $\Lambda^U$  represent welfare associated with the benchmark and  $\Lambda^A$  welfare associated with an alternative policy. The measure of the welfare gain of policy  $A$  relative to policy  $U$  is given by the value of the tax rate  $x$  that solves  $\Lambda^A [(1-x)w; \cdot] = \Lambda^U$ . With logarithmic utility functions we have  $\Delta\Lambda \equiv \Lambda^A - \Lambda^U = -\ln(1-x) \approx x$ .

Our calibrated model replicates some key features of the Swedish labor market. We consider a baseline case with unemployment at 6 percent, part-time unemployment at 2.5 percent and a full-time wage premium of 5 percent. The statutory replacement ratio can be as high as 80 percent in Sweden but coverage of UI is relatively low: only around a third of the unemployed in 2009 are covered by UI according to the labor force surveys. We set the replacement rate to 0.3 for the unemployed as well as for part-timers to capture incomplete coverage as well as finite benefit duration. Table 3 shows the baseline outcome along with the outcomes associated with optimal uniform replacement rates,  $\rho_u = \rho_p$ , as well as optimally differentiated benefits. When wages are treated as fixed, they are frozen at the baseline levels.

The optimal uniform system involves a replacement rate slightly higher than the benchmark 30 percent for both wage regimes. The rise in the replacement rate leads to a decline in unemployed search as well as in part-time search. Unemployment as well as part-time unemployment increases as the outflow rates decline. The welfare gain relative to the baseline is trivial when wages are negotiated but amounts to 0.24 percent of consumption in the fixed-wage case. The optimally differentiated system, displayed in the last two columns, yields further welfare gains. The replacement rate for the unemployed increases whereas it declines for part-timers compared to the optimal uniform system.

Table 3. Optimal UI, indefinite benefit payments.

	Baseline case	Optimal uniform		Optimally differentiated	
$\rho_u$	0.30	0.34	0.31	0.40	0.37
$\rho_p$	0.30	0.34	0.31	0.29	0.22
$\theta_E$	0.198	0.198	0.191	0.198	0.168
$\theta_P$	0.005	0.009	0.006	0.004	0.001
$s_u$	0.994	0.927	0.977	0.798	0.862
$s_p$	0.299	0.199	0.242	0.318	0.270
$w_e$	0.922	0.922	0.923	0.922	0.928
$w_p$	0.876	0.876	0.887	0.876	0.945
$\ln(w_e/w_p)$	0.050	0.050	0.040	0.050	-0.018
$u$	0.060	0.061	0.061	0.074	0.078
$p$	0.025	0.044	0.033	0.022	0.017
$e$	0.915	0.895	0.906	0.904	0.905
$t$	0.023	0.030	0.025	0.035	0.033
Wages		Fixed	Flexible	Fixed	Flexible
Welfare $\Delta\Lambda$ (%)		0.24	0.05	0.65	0.38

We also note that overall UI is less generous with flexible wages than with fixed wages. This is as expected given a presumption that wage responses reinforce moral hazard. Indeed, we find that the optimally differentiated system with flexible wages involves wage increases relative to the baseline. Moreover, the welfare gains from optimal UI are smaller with flexible wages; generous UI is more costly when the forces of moral hazard carry more weight.

## 4.2 Optimal Policy with Time Limits

So far we have assumed that benefits last forever. Now let us consider time limits for  $B$  and  $b$ . This is modeled along the lines of Fredriksson and Holmlund (2001). Benefits expire randomly at the rate  $\lambda$  for full-time unemployed and the rate  $\mu$  for part-timers. When UI benefits expire, workers have ac-

cess to unemployment assistance and are referred to as non-insured. The value function for  $U$  is modified so as to distinguish between those insured ( $I$ ) and those non-insured ( $N$ ). Insured full-timers get  $B^I = \rho_u^I W_e$  whereas those not insured get  $B^N = \rho_u^N W_e$ . Wages for part-timers are taken to be “quasi-fixed” in the sense that they remain constant for the duration of the job, an assumption that rules out wage differences between insured and non-insured part-timers. Part-timers receive  $b^I = \rho_p^I w_p (h_e - h_p)$  if insured and  $b^N = \rho_p^N w_p (h_e - h_p)$  if not insured.

We need to distinguish between search effort among those insured and those not insured. Moreover, we have to recognize that there are two groups of full-time unemployed, insured and not insured, as well as two groups of part-timers:

$$u = u^I + u^N \quad (33)$$

$$p = p^I + p^N \quad (34)$$

The tightness concepts are defined as before, i.e.,  $\theta_k \equiv v_k/S_k$ ,  $k = E, P$ . Note, however, that the total number of searchers in efficiency units is now given as:

$$S_E = s_u^I u^I + s_u^N u^N + s_p^I p^I + s_p^N p^N \quad (35)$$

$$S_P = s_u^I u^I + s_u^N u^N \quad (36)$$

The modified value functions are obtained as:

$$rE = v_e + \phi(U^I - E) \quad (37)$$

$$\begin{aligned} rU^I &= v_u^I + s_u^I [\alpha(\theta_P)(P^I - U^I) + \alpha(\theta_E)(E - U^I)] \\ &\quad + \lambda(U^N - U^I) \end{aligned} \quad (38)$$

$$rU^N = v_u^N + s_u^N [\alpha(\theta_P)(P^I - U^N) + \alpha(\theta_E)(E - U^N)] \quad (39)$$

$$rP^I = v_p^I + \phi(U^I - P^I) + s_p^I \alpha(\theta_E)(E - P^I) + \mu(P^N - P^I) \quad (40)$$

$$rP^N = v_p^N + \phi(U^I - P^N) + s_p^N \alpha(\theta_E)(E - P^N) \quad (41)$$

Employed workers as well as part-timers are immediately eligible for UI when laid off. The flow value of being insured unemployed,  $rU^I$ , includes a term that captures the risk of benefit expiration and the associated

change in the value of unemployment,  $\lambda(U^N - U^I)$ . When the non-insured unemployed worker finds a part-time job, she becomes immediately eligible for part-time benefits; the change in the state values is thus given by  $P^I - U^N$ . The flow value of being an insured part-timer,  $rP^I$ , involves the term  $\mu(P^N - P^I)$  that captures the risk of benefit termination and the related change in the value of being part-time unemployed.

The relevant utility functions are:

$$\begin{aligned} v_u^I &= \ln B^I + \ln(1-t) + \delta \ln(T - s_u^I) \\ v_u^N &= \ln B^N + \ln(1-t) + \delta \ln(T - s_u^N) \\ v_p^I &= \ln(w_p h_p + b^I) + \ln(1-t) + \delta \ln(T - h_p - s_p^I) \\ v_p^N &= \ln(w_p h_p + b^N) + \ln(1-t) + \delta \ln(T - h_p - s_p^N) \\ v_E &= \ln(w_e h_e) + \ln(1-t) + \delta \ln(T - h_e) \end{aligned}$$

Optimal search efforts are obtained as:

$$s_p^I : \frac{\delta}{T - h_p - s_p^I} = \alpha(\theta_E)(E - P^I) \quad (42)$$

$$s_p^N : \frac{\delta}{T - h_p - s_p^N} = \alpha(\theta_E)(E - P^N) \quad (43)$$

$$s_u^I : \frac{\delta}{T - s_u^I} = \alpha(\theta_P)(P^I - U^I) + \alpha(\theta_E)(E - U^I) \quad (44)$$

$$s_u^N : \frac{\delta}{T - s_u^N} = \alpha(\theta_P)(P^I - U^N) + \alpha(\theta_E)(E - U^N) \quad (45)$$

The usual optimality conditions apply: the marginal cost of increasing search effort should be equated to the expected marginal gain of doing so. A comparison of (42) and (44) reveals that the marginal gain of increasing search effort is higher for an unemployed insured worker than for an insured part-timer as long as  $P^I > U^I$ ; indeed,  $P^I > U^I$  must hold in order to induce the unemployed worker to accept part-time jobs. It follows that the insured unemployed worker allocates more effort to job search than the insured part-timer, i.e.,  $s_u^I > s_p^I$ . An analogous argument implies that the non-insured unemployed worker allocates more effort to job search than the non-insured part-timer, i.e.,  $s_u^N > s_p^N$ . We also note that  $s_p^I < s_p^N$  as well as  $s_u^I < s_u^N$ .

holds under the assumptions that the optimal policy implies  $U^I > U^N$  and  $P^I > P^N$ : benefit cuts boost search effort by increasing the marginal gain from additional search.

There will be six policy parameters in the most general case, viz. four replacement rates,  $\rho_u^I, \rho_u^N, \rho_p^I, \rho_p^N$ , as well as two parameters determining the potential duration of benefit receipt,  $\lambda$  and  $\mu$ . We proceed by focusing on two special cases. The first case involves indefinite unemployment benefits whereas part-time benefits are subject to time limits. In this case, there are four policy parameters of interest, viz.  $\rho_u, \rho_p^I, \rho_p^N$  and  $\mu$ . The second case we consider entails indefinite part-time benefits but unemployment benefits with time limits. The four policy parameters of interest are thus  $\rho_p, \rho_u^I, \rho_u^N$  and  $\lambda$ .

#### 4.2.1 Time Limits for Part-time Benefits

The flow equilibrium condition for unemployment remains intact for the first special case. Flow equilibrium for part-timers implies:

$$p^I = \frac{s_u \alpha(\theta_P) u}{\phi + s_p^I \alpha(\theta_E) + \mu} \quad (46)$$

$$p^N = \frac{\mu p^I}{\phi + s_p^N \alpha(\theta_E)} \quad (47)$$

Time limits pertaining to part-time benefits affect the value to firms of having part-timers employed. When an insured part-timer loses benefits, her search effort is affected and thereby the probability of landing a full-time job and thus leaving the firm. The value functions take the form:

$$rJ_P^I = (y - w_p) h_p + \phi (V_P - J_P^I) + s_p^I \alpha(\theta_E) (V_P - J_P^I) + \mu (J_P^N - J_P^I) \quad (48)$$

$$rJ_P^N = (y - w_p) h_p + \phi (V_P - J_P^N) + s_p^N \alpha(\theta_E) (V_P - J_P^N) \quad (49)$$

$$rV_P = -\kappa + q(\theta_P) (J_P^I - V_P) \quad (50)$$

where the term  $\mu (J_P^N - J_P^I)$  in (48) captures the fact that the insured part-timer loses benefits at the rate  $\mu$  which brings about a change in the value



to the firm of having a part-timer employed. The values of filled part-time jobs, evaluating at  $V_P = 0$  and  $r \rightarrow 0$ , are then obtained as:

$$J_P^I = \frac{(y - w_p) h_p}{\phi + s_p^N \alpha(\theta_E)} \left( \frac{\mu + \phi + \alpha(\theta_E) s_p^N}{\mu + \phi + \alpha(\theta_E) s_p^I} \right) \quad (51)$$

$$J_P^N = \frac{(y - w_p) h_p}{\phi + s_p^N \alpha(\theta_E)} \quad (52)$$

where  $J_P^I > J_P^N$  since  $s_p^N > s_p^I$  holds when the optimal policy implies  $P^I > P^N$ . An insured part-timer is more valuable to the firm than a non-insured one since she is likely to stay longer in the firm.

The value functions imply that the job creation condition for part-time jobs takes the form:

$$\frac{(y - w_p) h_p}{\phi + s_p^N \alpha(\theta_E)} \left( \frac{\mu + \phi + \alpha(\theta_E) s_p^N}{\mu + \phi + \alpha(\theta_E) s_p^I} \right) = \frac{\kappa}{q(\theta_P)} \quad (53)$$

There are two groups of workers who bargain over wages: full-time workers and insured part-timers. The relevant threat point for all workers is insured unemployment  $U^I$ . The Nash first-order condition for a full-timer is similar to (22):

$$(1 - \beta)(E - U^I)h_e = \beta J_E \frac{\partial v_e(w_e)}{\partial w_e} \quad (54)$$

where  $J_E = \kappa/q(\theta_E)$ . When an unemployed worker lands a part-time job she is immediately eligible for part-time benefits. The part-time wage is taken as constant for the duration of the job and is obtained from a Nash first-order condition of the form:

$$(1 - \beta)(P^I - U^I)h_p = \beta J_P^I \frac{\partial v_p(w_p)}{\partial w_p} \quad (55)$$

where  $J_P^I$  is given by (51).<sup>7</sup>

The equilibrium of this economy is obtained from three first-order conditions for optimal search ( $s_u$ ,  $s_p^I$  and  $s_p^N$ ) along with the free entry conditions

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<sup>7</sup>We have also considered the case where part-time wages are renegotiated once a part-timer switches from the insured to the non-insured state. The welfare results are very similar.

for firms, the Nash bargaining equations, and the flow equilibrium conditions. To determine optimal search, we also need to make use of the present value differences  $P^I - U^I$ ,  $E - U^I$ ,  $E - P^I$  and  $E - P^N$  which are obtained from the value functions given by (37)–(41). (Recall that  $\lambda = 0$  in this analysis.)

The welfare function is slightly modified and takes the form:

$$\begin{aligned}\Lambda = & \ln(1-t) + u [\ln B + \delta \ln (T - s_u)] + (1-u-p) [\ln (W_e) + \delta (T - h_e)] \\ & + p^I [\ln (W_p + b^I) + \delta \ln (T - h_p - s_p^I)] \\ & + p^N [\ln (W_p + b^N) + \delta \ln (T - h_p - s_p^N)]\end{aligned}\quad (56)$$

Benefits are financed by a proportional tax on all income, including benefits. The budget constraint for the government then implies:

$$\frac{t}{1-t} = \frac{uB + p^I b^I + p^N b^N}{(1-u-p)W_e + pW_p}\quad (57)$$

#### 4.2.2 Time Limits for Unemployment Benefits

We now consider a policy where unemployment benefits are subject to time limits whereas part-time benefits are paid indefinitely. The relevant policy parameters are thus  $\rho_p$ ,  $\rho_u^I$ ,  $\rho_u^N$  and  $\lambda$ . The flow equilibrium equations for the two unemployment states and part-time unemployment are as follows:

$$u^I : \quad \phi(1 - u^I - u^N) = (\lambda + s_u^I [\alpha(\theta_P) + \alpha(\theta_E)]) u^I \quad (58)$$

$$u^N : \quad \lambda u^I = s_u^N [\alpha(\theta_P) + \alpha(\theta_E)] u^N \quad (59)$$

$$p : \quad s_u^I \alpha(\theta_P) u^I + s_u^N \alpha(\theta_P) u^N = [\phi + s_p \alpha(\theta_E)] p \quad (60)$$

The job creation conditions are given by (24) and (25), exactly as in a regime without any time limits. Wages are obtained from Nash first-order conditions of the form given by (54). The welfare function is:

$$\begin{aligned}\Lambda = & \ln(1-t) + u^I [\ln B^I + \delta \ln (T - s_u^I)] + u^N [\ln B^N + \delta \ln (T - s_u^N)] \\ & + p [\ln (W_p + b) + \delta \ln (T - h_p - s_p)] \\ & + (1-u-p) [\ln (W_e) + \delta (T - h_e)]\end{aligned}\quad (61)$$

and the government's budget constraint implies:

$$\frac{t}{1-t} = \frac{u^I B^I + u^N B^N + pb}{(1-u-p)W_e + pW_p}\quad (62)$$

### 4.2.3 Results

Table 4 presents numerical results for exogenous wages and Table 5 the results for endogenous wages. The previous results for indefinite and uniform benefit payments are reproduced in the first two columns. The third columns show the effects of introducing time limits pertaining to part-time benefits while maintaining indefinite benefits payments for the unemployed. The fourth columns show the effects of time limits for unemployment benefits. Finally, the fifth columns allow for time limits for part-time benefits as well as unemployment benefits. The time unit is a day so  $1/\mu$  gives the expected potential duration of benefit days for part-timers and  $1/\lambda$  the analogous measure for unemployed persons. A number of observations can be made.

The optimal UI systems involve steeply declining replacement rates over the spell of unemployment as well as over the spell of part-time work. The replacement rates in the first tiers are on average roughly twice as large as the levels in the second tiers. It is noteworthy that the first-tier replacement rate for a part-timer is at least as high (or higher) as the corresponding replacement rate for an unemployed person; the second-tier replacement rates are somewhat lower for part-timers. The potential duration of benefit receipt is longer for the unemployed. The number of non-insured unemployed is substantially higher than the number of insured ones. Moreover, the number of non-insured part-timers exceeds the number of insured ones by a large magnitude.

We also note that the overall welfare gain from benefit differentiation is mainly driven by the gain associated with time limits for unemployment benefits, a result apparent from comparisons of the third and fourth columns in the tables. Finally, the welfare gain from optimal UI is smaller when wages are flexible (0.62 percent of consumption relative to 0.92 percent in the fixed-wage case); this confirms results from the analysis of indefinite benefit payments. However, it is no longer obvious that the optimal UI system is less generous with flexible wages than with fixed wages; we note that the potential duration of benefit receipt is longer in the flex-wage regime.

Figure 1 illustrates the optimal after-tax income profiles for part-timers

and unemployed persons when wages are endogenously determined. The incomes are measured relative to net incomes among full-time employees and the spell durations are measured in days.

Why do the optimal replacement (and income) profiles decline over spells of job search? By introducing time-dependent benefits, incentives for active job search are strengthened. Consider an unemployed person whose benefits have expired. By finding a job, entitlement to future benefit receipt in case of unemployment is secured. The bigger the difference is between unemployment benefits and unemployment assistance, the stronger the search incentives to requalify for UI benefits via employment.

Unemployed persons receiving unemployment assistance can also qualify for part-time benefits by finding a part-time job. The higher the income associated with part-time insured unemployment is relative to unemployment assistance, the stronger the incentive to actively search for part-time work. It is clear, however, that generous UI benefits for part-timers reduce their incentives to search for full-time jobs. This can be offset by a threat of benefit expiration, i.e., a time limit for UI benefit receipt also for part-timers.

Table 4. Optimal UI with Time Limits (TL), exogenous wages.

	Baseline case	Optimal uniform	TL for part-timers	TL for unemployed	TL for both groups
$\rho_u^I$	0.30	0.34	0.40	0.61	0.60
$\rho_u^N$				0.35	0.35
$\rho_p^I$	0.30	0.34	0.76	0.31	0.62
$\rho_p^N$			0.29		0.31
$1/\mu$	0	0	67	0	40
$1/\lambda$	0	0	0	91	91
$\theta_E$	0.198	0.198	0.198	0.198	0.198
$\theta_P$	0.005	0.009	0.005	0.005	0.006
$s_u^I$	0.994	0.927	0.799	0.741	0.748
$s_u^N$				0.917	0.917
$s_p^I$	0.299	0.199	0.211	0.278	0.231
$s_p^N$			0.318		0.278
$u^I$	0.060	0.061	0.074	0.024	0.024
$u^N$				0.044	0.044
$p^I$	0.025	0.044	0.003	0.028	0.002
$p^N$			0.021		0.026
$e$	0.915	0.895	0.902	0.904	0.904
$t$	0.023	0.030	0.035	0.036	0.036
Welfare $\Delta\Lambda$ (%)		0.24	0.66	0.91	0.92

Table 5. Optimal UI with time limits (TL), endogenous wages.

	Baseline case	Optimal uniform	TL for part-timers	TL for unemployed	TL for both groups
$\rho_u^I$	0.30	0.31	0.38	0.55	0.55
$\rho_u^N$				0.33	0.33
$\rho_p^I$	0.30	0.31	0.74	0.20	0.84
$\rho_p^N$			0.22		0.22
$1/\mu$	0	0	91	0	59
$1/\lambda$	0	0	0	125	125
$\theta_E$	0.198	0.191	0.162	0.155	0.153
$\theta_P$	0.005	0.006	0.003	0.001	0.002
$s_u^I$	0.994	0.997	0.840	0.795	0.791
$s_u^N$				0.947	0.947
$s_p^I$	0.299	0.242	0.230	0.247	0.204
$s_p^N$			0.363		0.317
$w_e$	0.922	0.923	0.929	0.931	0.931
$w_p$	0.876	0.887	0.907	0.968	0.928
$\ln(w_e/w_p)$	0.050	0.040	0.024	-0.040	0.003
$u^I$	0.060	0.061	0.079	0.031	0.031
$u^N$				0.050	0.047
$p^I$	0.025	0.033	0.003	0.013	0.002
$p^N$			0.016		0.018
$e$	0.915	0.890	0.902	0.906	0.902
$t$	0.023	0.025	0.035	0.037	0.037
Welfare $\Delta\Lambda$ (%)		0.05	0.43	0.57	0.62

## 5 Concluding Remarks

We have proposed an equilibrium search and matching model where risk-averse workers occupy one of three mutually exclusive states, namely (full-time) employment, part-time unemployment and (full-time) unemployment.



Figure 1: Net incomes by spell durations

Part-time unemployment is a stepping stone to full-time jobs. The model is used to study the design of optimal unemployment insurance. We examine optimal two-tier benefit schemes for the unemployed as well as for the part-time unemployed, noting that benefits for part-timers may encourage job search among the unemployed but at the same time discourage search for full-time jobs among part-timers. Our results suggest that steeply declining replacement rates over the search spells are optimal for unemployed persons as well as for part-timers.

There are a number of conceivable extensions of the model. We have treated preferences as uniform across individuals, a simplification that is bound to be patently unrealistic. Introducing heterogeneity in preferences, such as heterogeneity in the preference for leisure, raises new issues. When preferences are private information, it will be difficult for the policy maker to distinguish between voluntary and involuntary part-time work. In particular, the policy must be structured so that those who prefer part-time work are excluded from benefits intended for workers who are involuntary unemployed.

These and other issues are left for future work.<sup>8</sup>

## APPENDIX

### The Numerical Model

The calibration is done with an eye on the Swedish labor market. The matching function is Cobb Douglas,  $M = au^\eta v^{1-\eta}$ , where  $\eta = 0.5$  is assumed; this is broadly consistent with most empirical studies. Productivity is normalized to unity for both part-time and full-time employees:  $y = 1$ . The time period is taken to be a day. The rate of interest (equal to the rate of time preference) is set to zero. The annual separation rate is set to 15 percent corresponding to daily rate of  $0.15/365 = 0.000411$ .

UI coverage is relatively low; according to the labor force surveys, around one third of the unemployed received unemployment benefits over the period 2005–2009. We account for this by choosing a benchmark replacement rate of 30 percent for both unemployed and part-timers, i.e.,  $\rho_u = \rho_p = 0.3$ . The choices of  $T$  and  $\delta$  are guided by empirical results regarding the partial-equilibrium responsiveness of job finding to changes in benefits. Estimates of the job finding elasticity with respect to benefits center around 0.5; this would imply  $d \ln s / d \ln B \approx -0.5$  since the job finding rate is given by  $f = s\alpha(\theta)$  and tightness is taken as given in the partial equilibrium context. We end up with  $T = 1.5912$  and  $\delta = 0.72$  using a conventional model with only two states, i.e., employment and unemployment.

Regarding work hours we assume that the full-time employee works as much as she prefers. This implies

$$h_e = \frac{T}{1 + \delta}$$

since this is the worker's preferred working time given the preferences we have assumed. Working time among part-timers is given by  $h_p = 0.5h_e$ , which is

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<sup>8</sup>Preliminary work on a model with worker heterogeneity and both voluntary and involuntary part-time work suggest that positive part-time benefits would be optimal even if the policy maker is unable to observe worker types (see Ek and Holmlund, 2011).



in line with Swedish data: involuntary part-timers in the labor force surveys report that they wish to work twice as much as they actually do.

Regarding wages, empirical work has documented wage penalties for part-timers. A recent Swedish study by Wahlberg (2008) suggests a wage penalty of 20 percent, perhaps implausibly large. We aim for a full-time wage premium of less than 10 percent. When baseline wages are determined by Nash bargaining we obtain  $w_e = 0.922$  and a wage premium for full-time workers of 5 percent.

The matching parameter  $a$ , and the vacancy cost  $\kappa$  are chosen so as to obtain 6 percent unemployment and a part-time rate at 2.5 percent. This part-time rate is lower than the measure of part-time unemployment in the labor force surveys (4 percent) but higher than a part-time measure that includes only those who report active search for work (1.5 percent). We set  $a = 0.0126$  and  $\kappa = 5$  and end up with  $p = 0.025$ .

The parameterized model produces outcomes as given by the first column in Table 3 in the main text.

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