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# OPTIMAL SOVEREIGN DEBT: AN ANALYTICAL APPROACH

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## Abstract<sup>1</sup>

This paper develops a model of sovereign debt where governments are myopic. Instead of focusing on the incentives to repay, as in most of the theoretical literature on the topic (which assumes implicitly that governments have long-term objectives), I therefore consider that governments always repay when they can, but also borrow as much as possible, without paying attention to the burden of future repayments. The pattern of debt is then only determined by the willingness of international investors to lend to the country. I characterize the Rational Expectations Equilibria of the credit market. These equilibria behave like rational bubbles: international investors lend a lot because they anticipate that other investors will lend again in the future. Capital flows are procyclical: the government borrows a fixed proportion of its income until a sudden stop occurs, generating default and an economic crisis. I suggest possible remedies to the high volatility of public expenditures that is generated by such borrowing patterns.

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# 1 Introduction

Most of the theoretical literature on sovereign debt has focused on its insurance role (income smoothing) and on the conditions under which a (domestic) central planner would strategically default rather than repay. By contrast, empirical evidence (Gavin and Perotti, 1997) suggests that capital flows are procyclical in developing countries (these countries borrow more when their income is high) and that, in general, democratic governments try to avoid default as much as they can.<sup>2</sup>

This paper offers a fresh look at sovereign debt analysis by considering a model where short sighted politicians try to borrow as much as they can,<sup>3</sup> while international financiers accept to lend a lot because they anticipate that other lenders will lend again in the future. Thus debt accumulates until the country cannot repay, in which case investors refuse to lend anymore (sudden stop). There are thus two sources of inefficiency: governments cannot commit not to borrow again in the future, while lenders cannot commit in either direction (to stop lending or to continue lending in the future).

Section 2 defines a simple model à la Eaton-Gersovitz (1981) with the main modification that growth rates (not primary incomes) are stationary. In the absence of any imperfection (and assuming complete financial markets), the optimal borrowing policy of the government, characterized in Section 3, would be to insure a constant level of public expenditure, the maximum level compatible with the (infinite horizon) budget constraint of the government.

However, since democratic governments (even if they have a long-term horizon) cannot commit on their future decisions, this (first best) optimum cannot be attained. The second best optimum corresponds instead to the maximum welfare that can be attained without commitment. In my model, it is characterized by a procyclical borrowing policy.

Section 4 characterizes the borrowing decisions of a short-sighted politician. The extreme case is that of a politician who has no hope of being reelected and thus borrows as much as he can, so as to maximize his current popularity (a less benevolent alternative would be to consider that he can divert some fraction of available public money). The borrowing policy of such a

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<sup>2</sup> See, for instance, Bulow and Rogoff (1989).

<sup>3</sup> A similar view is developed by Aizenmann and Powell (1998). In their paper, there is a collective action problem within the government, where several pressure groups compete for scarce funds. In the absence of a strong leader who may impose the collectively efficient outcome, opportunistic behavior of the different pressure groups typically leads to maximum borrowing by the government, generating lending booms and procyclical capital flows.

politician is characterized recursively, as the maximum amount that financial markets are ready to lend, taking into account that, at the next period, the future incumbent will also borrow as much as he can.

Section 5 derives some testable implications of government myopia, which generates a cyclical pattern of capital flows: lending booms followed by default, sudden stops and economic crisis. This in turn generates a very volatile (and inefficient) pattern of public expenditures. Section 6 briefly discusses welfare analysis and offers some policy measures that could generate welfare improvements. Section 7 concludes

## 2 The Model

The model presented is a simple model à la Eaton-Gersovitz (1981) where public debt is used to smooth out public consumption  $c_t$  over time. However, to the classical insurance motive (which generates countercyclical borrowing patterns), we add a more fundamental “front-loading” motive by assuming some persistence in the shocks on government income  $y_t$ .

This persistence implies wealth effects that typically lead to pro-cyclical borrowing patterns, when combined with government short termism. As we explain below, a positive shock to  $y_t$  induces the government to borrow more in order to benefit from the induced increase in expected future incomes. We assume that social welfare is measured by

$$W = E \left( \sum_{t=0}^{\infty} \beta^t u(c_t) \right)$$

where  $\beta = \frac{1}{1+r}$  is the discount factor,  $u(\cdot)$  is an iso-elastic utility function, and  $c_t$  denotes public expenditure at date  $t$ . The budget constraint of the government at date  $t$  is:

$$c_t = y_t + B_t - D_t, \tag{1}$$

where  $B_t$  represents total borrowing and  $D_t$  total outstanding debt at date  $t$ . For simplicity<sup>4</sup> we assume that all debt is short term:<sup>5</sup>  $D_t$  is repaid in full at date  $t$  and  $B_t$  can then be interpreted as new borrowings at date  $t$ . Note that, for the moment, we exclude the possibility of default.

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<sup>4</sup> This assumption is not crucial.

<sup>5</sup> Jeanne (2000) shows that short-term debt can be an optimal reaction to the repayment risk faced by foreign creditors.

Let us say a few words about the interpretation of variables  $y_t$  and  $c_t$ . We assume that the government has no control over primary income  $y_t$ , at least in the short run. To fix ideas, we can consider that it is proportional to the country *GDP*  $Y_t$ :

$$y_t = \tau Y_t, \quad (2)$$

where  $\tau > 0$  is a fixed parameter. Then the growth rate of government income is also equal to the growth rate of *GDP*.

In the absence of default, the government has only one decision to make at each date, namely the level  $c_t$  of government expenditures. In the case of classical debt contracts,  $D_t$  is perfectly predictable at date  $t-1$ :

$$D_t = (1 + R_{t-1})B_{t-1}, \quad (3)$$

where  $R_{t-1}$  denotes the interest rate negotiated at date  $t-1$ .

However, more complex, contingent (or indexed) debt contracts can be envisaged, where date  $t$  repayment  $D_t$  is contingent on  $y_t$  (or  $Y_t$ ). Although appealing in theory, these contingent contracts are rarely seen in practice (at least explicitly)<sup>6</sup> in part for feasibility reasons. It would indeed be difficult for the country and the lenders to agree on an objective measure of primary income  $y_t$  (or *GDP*  $Y_t$ ) that cannot be subject to measurement errors or the possibility of manipulation by the government.

For most of this article we adopt the assumption that  $y_t$  is publicly observable at date  $t$ , but not verifiable by a third party such as an international Court of Justice (see Sachs, 1995). This means that the repayment  $D_t$  (on prior debt) cannot be made contingent on  $y_t$  (such a contingent contract would not be enforceable). However, new borrowing  $B_t$  can depend on  $y_t$  since both the country and the investors know  $y_t$  when they jointly decide on  $B_t$ .

The main objective of this article is to investigate the borrowing policy of a country in a context where there are three kinds of imperfections:

- a) governments are unable to commit on their future borrowing policies (government imperfection),

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<sup>6</sup> It is sometimes considered that debt restructuring episodes are an implicit way to implement state contingent debt contracts.

- b) financial markets are unable to provide complete contingent contracts (market incompleteness),
- c) governments only care about short term consequences of their decisions (short termism).

Before investigating the consequences of these imperfections on the borrowing policy of the government, let us examine, as a first benchmark, the first best borrowing policy that would be implemented in the absence of any of these imperfections.

### 3 The First Best Borrowing Policy

It corresponds to the maximization of the expectation of long-term social welfare:

$$W = E \left( \sum_{t=0}^{\infty} \frac{u(c_t)}{(1+r)^t} \right), \quad (4)$$

under a unique, intertemporal, budget constraint:

$$E^* \left[ \sum_{t=0}^{\infty} \frac{(y_t - c_t)}{(1+r)^t} \right] = D_0, \quad (5)$$

where  $E^*$  represents the expectation operator corresponding to the risk-adjusted probability measure,<sup>7</sup> and  $D_0$  is the initial level of public debt. To simplify the analysis we assume that risk premia are zero,<sup>8</sup> so that  $E^*$  coincides with the “historical” expectation operator  $E$ .

Under these assumptions, the maximization of (4) under constraint (5) is trivial. It coincides with perfect consumption smoothing, both over time and over income shocks:

$$c_t \equiv C. \quad (6)$$

The optimal value of  $C$  is the maximum level of consumption that is compatible with the intertemporal budget constraint. It is deduced from the budget constraint (5), by taking  $c_t \equiv C$ .

We have thus proven:

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<sup>7</sup> As is well known from the mathematical finance literature, this probability measure is a simple way to represent contingent prices. This measure exists because of the absence of arbitrage opportunity on financial markets. It is unique because we have assumed (in this section) complete contingent markets.

<sup>8</sup> This assumption is not at all crucial, and just made for simplifying exposition.



**Proposition 1:** *The first best level of public consumption is constant over time and over income shocks*

$$c_t \equiv C = \frac{r}{1+r} \left[ y_0 + E \left( \sum_{t \geq 1} \frac{y_t}{(1+r)^t} \middle| y_0 \right) - D_0 \right]. \quad (7)$$

*This consumption level is sustained by the following borrowing policy:*

$$B_t = C - y_t + (1+r)B_{t-1}. \quad (8)$$

**Proof of Proposition 1:**

Since there are no risk premia ( $E^* = E$ ) and the country has the same discount factor as the market  $\left( \beta = \frac{1}{1+r} \right)$  strict concavity of  $u$  implies that  $c_t \equiv C$ .  $C$  is determined by the intertemporal budget equation:

$$E \left( \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t} \right) - C \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} = D_0.$$

Since  $E(y_t) = y_0(1+g)^t$ , and  $\sum_{t=0}^{\infty} \frac{(1+g)^t}{(1+r)^t} = \frac{1+r}{r-g}$ ,

$$y_0 \left( \frac{1+r}{r-g} \right) - C \left( \frac{1+r}{r} \right) = D_0,$$

which gives

$$C = y_0 \frac{r}{r-g} - D_0 \frac{r}{1+r}.$$

The budget equation at date  $t$  (equation (1)) together with the condition  $D_t = (1+r)B_{t-1}$  give finally:

$$B_t = C - y_t + (1+r)B_{t-1}.$$

In order to simplify the analysis, I focus from this point forward on the case where shocks to government income  $y_t$  are permanent. Specifically, I assume that government income  $y_t$  satisfies:

$$\forall t \geq 1, \quad y_t = y_{t-1}g_t, \quad (9)$$

where the growth factor  $(g_t)_t$  follows an i.i.d. process such that

$$E(g_t) = 1 + g. \quad (10)$$

The expected growth rate  $g$  is assumed to be smaller than the riskless interest rate<sup>9</sup>  $r$ , which is assumed to be constant over time. With i.i.d. growth rates, optimal consumption  $C$  can be computed explicitly:

$$c_t \equiv C = \frac{ry_0}{r-g} - \frac{rD_0}{1+r}. \quad (11)$$

Applying formula (8) with  $t = 0$ , we obtain, after simplifications:

$$B_0 = \frac{gy_0}{r-g} + \frac{D_0}{1+r}. \quad (12)$$

Thus initial borrowing is positively correlated with initial income, which is in line with the analysis of the Argentine convertibility period by Galiani et al. (2002). These authors argue that policymakers indeed believed that the high growth rates of the early 1990s were permanent, which led them to follow a procyclical fiscal policy.

This result that initial borrowing  $B_0$  is positively correlated with initial income  $y_0$  also holds when income shocks are only partially persistent. Indeed formula (12) can be written more generally as

$$B_0 = \frac{1}{1+r} [-y_0 + E(Y | y_0) + D_0],$$

where  $Y = r \sum_{t \geq 1} \frac{y_t}{(1+r)^t}$  is the “permanent income” of the government.

Thus positive correlation holds whenever  $E(Y | y_0) - y_0$  increases in  $y_0$ , i.e., when a positive shock  $\varepsilon$  to  $y_0$  implies an increase in expected permanent income of more than  $\varepsilon$ . In the case of permanent shocks considered here,  $E(Y | y_0) - y_0 = \frac{g(1+r)}{r-g} y_0$ .

Note that the first best borrowing policy defined by (10) has several aspects that are rejected by empirical evidence, at least in developing countries:

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<sup>9</sup> See Hellwig and Lorenzoni (2006) for an exploration of the consequences of the reverse hypothesis  $r < g$ .

- It is countercyclical: conditionally on  $y_{t-1}$ ,  $B_t$  and  $y_t$  are perfectly anti-correlated

$$\text{cor}_t(B_t, y_t | y_{t-1}) = -1.$$

- Debt is always repaid (no default).
- Investors are always willing to lend (no sudden stops).
- There is no spread on interest rates:

$$R_t \equiv r.$$

A natural question is therefore which of the imperfections mentioned above may explain the discrepancies between observed and theoretical borrowing policies of developing countries. We now discuss this question.

Note first that market incompleteness is not the (sole) explanation. Indeed, the borrowing policy defined by (10) does not rely on contingent contracts, but can be implemented by standard debt contracts:

$$D_t = (1+r)B_{t-1}. \quad (13)$$

However, it requires that new borrowings  $B_t$  be perfectly anti-correlated with new income  $y_t$ , which is clearly not time consistent. To see this, take for example  $t=0$  and  $t=1$  in (10):

$$\begin{aligned} B_0 &= C - y_0 + (1+r)B_{-1}, \\ B_1 &= C - y_1 + (1+r)B_0. \end{aligned}$$

Now  $(1+r)B_{-1} = D_0$  and  $C = \frac{ry_0}{r-g} - \frac{rD_0}{1+r}$ .

Thus we can write:

$$B_0 = \frac{ry_0}{r-g} - \frac{rD_0}{1+r} - y_0 + D_0 = \frac{gy_0}{r-g} + \frac{D_0}{1+r}, \quad (14)$$

and

$$B_1 = \frac{ry_0}{r-g} - \frac{rD_0}{1+r} - y_1 + (1+r)B_0. \quad (15)$$

To compare (15) with (14) let us introduce

$$D_1 = (1+r)B_0 = (1+r)\left[\frac{gy_0}{r-g}\right] + D_0.$$

We thus can write:

$$D_0 = (1+r)\frac{gy_0}{r-g} - (1+r)B_0,$$

and

$$B_1 = \frac{ry_0(1+g)}{r-g} - y_1 + \frac{D_1}{1+r}. \quad (16)$$

We now show that this borrowing policy is in general time-inconsistent. Indeed, if the government re-optimizes at date 1, based on  $y_1$  and  $D_1$ , the adaptation of formula (14) gives:

$$\hat{B}_1 = \frac{gy_1}{r-g} + \frac{D_1}{1+r}. \quad (17)$$

Comparing with (15) we see that

$$\hat{B}_1 - B_1 = \frac{r}{r-g} [y_1 - (1+g)y_0]. \quad (18)$$

Thus, except in the very peculiar case where  $y_1 = (1+g)y_0$  (realized growth equals expected growth) the government will not stick to its initial borrowing policy. More precisely, (17) shows that new borrowings will indeed exhibit some form of procyclicality: additional net borrowings are positive (negative) whenever realized growth exceeds (is lower than) expected growth.

This shows that lack of commitment power by the government is a primary reason why the first best borrowing policy cannot be attained.<sup>10</sup>

However, this lack of commitment power could be counteracted if the government had access to complete contingent contracts. Suppose indeed that the government could sign at date  $t-1$  a contingent contract whereby it would have to repay  $\hat{D}_t$  at date  $t$ , where  $\hat{D}_t$  is defined by:

$$\hat{D}_t = \frac{1+r}{r-g} (y_t - y_0) + D_0. \quad (19)$$

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<sup>10</sup> Strategic default could also be a consequence of the lack of commitment power by the government. This aspect will not be considered in this paper, where governments are supposed to be myopic. Since default is typically followed by sanctions, it is likely that a myopic government will try its best to avoid default.

Note that, contrary to all contracts considered so far,  $\hat{D}_t$  depends on current income  $y_t$ , which may pose feasibility problems as we argued above. If these feasibility problems are overlooked, the ex-post optimal policy of the government is given by the adaptation of formula (9):

$$\hat{C}_t = y_t \frac{r}{r-g} - \hat{D}_t \frac{r}{1+r}.$$

Given (19) this is also equal to:

$$\hat{C}_t = y_t \frac{r}{r-g} - \frac{r}{r-g} (y_t - y_0) - \frac{rD_0}{1+r} = C.$$

Thus if complete contingent contracts were accessible, the first best policy could be attained even in the absence of commitment power by the government.

The following proposition summarizes the above results.

**Proposition 2:** *In our model, procyclicality of capital flows results from the combination of the lack of commitment power of the government and market incompleteness:*

- a) *If the government could commit on its future borrowing policy, it could implement the first best by a perfectly anti-cyclical borrowing policy (10) and standard debt contracts (11).*
- b) *Conversely if contingent debt contracts (18) were available, even a government without commitment power could implement the first best.*

## 4 Bubbling Debt and Sudden Stops

We now consider the case of extreme political instability, where governments only last for one period and maximize the current level of public consumption, without any consideration of future outcomes.

Under this extreme assumption, governments always borrow as much as they can. Indebtedness is thus completely determined by markets' willingness to lend, which itself depends on their expectations of the government's future income, as well as markets' willingness to lend again in the future. Note that the stationarity of growth factors implies that the borrowing capacity  $B_t$  of the country at date  $t$  is proportional to current government income  $y_t$ . We denote by  $b_t$  the proportionality coefficient:  $B_t = b_t y_t$ . Then the assumption of competitive financial

markets implies that  $D_{t+1}$  is also proportional to  $y_t$ :  $D_{t+1} = d_t y_t$ . For a current level of government income  $y_t$  and a promised debt repayment of  $D_{t+1} = d_t y_t$ , the anticipated probability of failure is

$$\Pr((1 + b_{t+1})y_{t+1} < d_t y_t) = F\left(\frac{d_t}{1 + b_{t+1}}\right), \quad (20)$$

where  $b_{t+1}y_{t+1}$  is the maximum amount that the government will be able to borrow in the future (as anticipated at date  $t$  by the market) and  $F(x) = \Pr(\tilde{g} \leq x)$  is the cumulative distribution function of the growth factor  $\tilde{g}$ . Assuming a zero recovery rate in case of default<sup>11</sup> and competitive financial markets, the maximum amount that markets agree to lend at date  $t$  (for a promised repayment  $d_t y_t$ ) is determined by the zero profit condition:

$$(1 + r)B(d_t, b_{t+1})y_t = d_t y_t \left[ 1 - F\left(\frac{d_t}{1 + b_{t+1}}\right) \right]. \quad (21)$$

The intuition behind (21) is simple: financial markets anticipate that the next government will be able to borrow  $b_{t+1}y_{t+1}$  at date  $t+1$ . Given that strategic default is not in its interest,<sup>12</sup> debt  $d_t y_t$  will be repaid whenever  $(1 + b_{t+1})y_{t+1} \geq d_t y_t$ . Since  $\frac{y_{t+1}}{y_t} \equiv g_t$  is stationary with c.d.f.  $F$ , the probability of repayment is then  $1 - F\left(\frac{d_t}{1 + b_{t+1}}\right)$ . Competitive lenders are then ready to lend as much as

$$B(d_t, b_{t+1})y_t = \frac{d_t y_t}{1 + r} \left[ 1 - F\left(\frac{d_t}{1 + b_{t+1}}\right) \right],$$

which gives condition (19).

As we saw, a myopic government will borrow as much as possible, that is, it will choose

$$b_t = \max_{d_t} B(d_t, b_{t+1}). \quad (22)$$

<sup>11</sup> This assumption is not crucial but it simplifies the exposition. With a positive recovery rate, the amount of sustainable debt would be increased.

<sup>12</sup> We exclude situations where the new government would try to boost its popularity by defaulting strategically. We assume instead that default has immediate costs that a myopic government wants to avoid as much as possible.

Now this expression can be transformed:

$$\max_{d_t} B(d_t, b_{t+1}) = (1 + b_{t+1}) \max_x \frac{x[1 - F(x)]}{1 + r},$$

where  $x = \frac{d_t}{1 + b_{t+1}}$  denotes the default threshold. Thus (20) is equivalent to

$$b_t = (1 + b_{t+1})\gamma, \quad (23)$$

where

$$\gamma \equiv \max_x \frac{x(1 - F(x))}{1 + r}. \quad (24)$$

Note that  $\gamma$  is the maximum amount that the country could borrow (per unit of income) if it were unable to borrow again in the future. When lenders take into account the future borrowing capacity of the country, the maximum amount that the country can borrow becomes

$$b_t = \gamma + \gamma b_{t+1},$$

hence formula (23).

A Rational Expectations Equilibrium (REE) of the credit market (with a myopic government) is thus characterized by a sequence of borrowing ratios  $b_t$  that satisfies (23), or equivalently:

$$b_{t+1} = \frac{b_t}{\gamma} - 1. \quad (25)$$

These REE behave like (rational) bubbles. Investors lend at  $t$  because they anticipate that other investors (possibly themselves) will lend again at  $t + 1$ . This is ex-ante rational because investors charge a default premium that covers the expected loss in case of default.

**Lemma 1:**  $\gamma < 1$ .

**Proof:** For any  $x$  in the support of  $\tilde{g}$  we have that:

$$x(1 - F(x)) = \int_x^\infty x f(t) dt \leq \int_x^\infty t f(t) dt \leq E(\tilde{g}) = 1 + g.$$

Now by assumption  $g < r$ . Thus

$$\gamma = \max_x \frac{x(1 - F(x))}{1 + r} < 1.$$

Since  $\gamma \in ]0,1[$  (by Lemma 1) all such sequences of borrowing ratios are unbounded except the one that is stationary:<sup>13</sup>

**Proposition 3:** *a) The only REE with a bounded sequence of borrowing rates is stationary:*

$$b_t \equiv b^* \equiv \frac{\gamma}{1-\gamma}. \quad (26)$$

*b) The associated debt ratio is also constant:*

$$d_t \equiv d^* = (1+b^*)g^*, \quad (27)$$

where  $g^* = \arg \max x(1-F(x))$ .

In the stationary REE, the government always borrows at date  $t$  a fixed proportion of its income  $b^* y_t$  and, whenever possible, repays at date  $t+1$  its debt  $d^* y_t$ . The interest factor is constant:

$$R_t \equiv \frac{d^*}{b^*} = \frac{1+r}{1-F(g^*)}. \quad (28)$$

This comes from the fact that the probability of default  $F(g^*)$  is also constant. As long as  $g_t \geq g^*$  the government repays its debt and borrows again, leading to a level of public consumption determined by:

$$c_t = (1+b^*)(y_t - g^* y_{t-1}). \quad (29)$$

The first time  $g_t < g^*$ , the country is not able to repay, therefore investors stop lending (sudden stop), generating a crisis. For simplicity we assume that  $y_t$  is dissipated by the crisis and lost both for the country and the investors. In particular we have in this case:

$$c_t = 0. \quad (30)$$

Immediately after the crisis, we assume that the country is able to borrow again and that previous debt is forgiven.<sup>14</sup> Therefore the REE of the credit market is characterized by periods of

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<sup>13</sup> Unbounded sequences correspond to “irrational” bubbles (or lending booms) where myopic investors do not realize immediately that their expectations are not consistent: at some future date, the borrowing needs of the country will exceed any reasonable bound.



steady (and procyclical) capital inflows, ending in crisis periods where the country defaults and investors stop lending for a time.

## 5 Testable Implications

The empirical implications of this simple model of “bubbling debt” are interesting. In our RE equilibrium with a myopic government we have indeed found that:

- The government borrows always (excluding crisis periods) a fixed proportion of its current income, thus net capital inflows are pro-cyclical:

$$c_t - y_t = b^* y_t - d^* y_{t-1}.$$

- The debt ratio  $b^*$  and the probability  $F(g^*)$  of default may be substantially different from one country to another, if the means and variances of the growth factors are themselves substantially different.

As an illustration, suppose that  $\tilde{g}$  is uniform on  $[1 + g - \sigma, 1 + g + \sigma]$ . On this interval we have that

$$1 - F(x) = \frac{1}{2\sigma}(1 + g + \sigma - x).$$

Thus the maximum of  $x(1 - F(x))$  is obtained for

$$\begin{aligned} x = g^* &= \frac{1 + g + \sigma}{2} && \text{when } 1 + g < 3\sigma, \\ \text{and } x = g^* &= 1 + g - \sigma && \text{when } 1 + g \geq 3\sigma. \end{aligned}$$

The equilibrium probability of default is

$$\begin{cases} F(g^*) = \frac{3}{4} - \frac{1+g}{4\sigma} & \text{when } 1 + g < 3\sigma, \\ = 0 & \text{when } 1 + g \geq 3\sigma. \end{cases}$$

It decreases with  $g$  and increases with  $\sigma$ . The equilibrium borrowing ratio is

$$b^* = \frac{\gamma}{1 - \gamma}, \text{ where } \gamma = \frac{(1 + g + \sigma)^2}{8\sigma(1 + r)}$$

(when  $1 + g < 3\sigma$ ). It is increasing in  $g$  and  $U$ -shaped in  $\sigma$ . Thus countries with a large  $g$  and a small  $\sigma$  can have simultaneously a large debt ratio and a small default probability.

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<sup>14</sup> This assumption is innocuous given the myopia of the government. Since policy makers are not concerned by what will happen in the future, maintaining sanctions on a defaulting country would be pointless.

The property that a higher level of volatility of growth rates leads to a lower level of sustainable debt is the opposite to what Eaton and Gersovitz (1981) find. Indeed in their model, strategic default occurs when the welfare level under autarky exceeds the continuation level when the country repays its debt (and thus benefits further from the insurance provided by income smoothing. Thus the higher the volatility of government income, the lower the welfare level obtained under autarky, and thus the higher the cost of defaulting.

Empirical findings (see for example Catao and Kapur, 2004) seem to be more in line with our result. The higher the risk on the country's future incomes, the lower the "one-shot" borrowing capacity  $\gamma$  and thus by induction the lower the sustainable debt level at the stationary REE.

## 6 Welfare Analysis and Policy Implications

Formula (27) shows that public consumption is more volatile when the country has access to capital markets than under autarky. The variance of  $c_t$ , conditionally on  $y_{t-1}$  (which summarizes all the information available at date  $t-1$ ), is multiplied by  $(1+b^*)^2$ , when compared with the autarkic situation.<sup>15</sup> Moreover there is a constant probability of crisis at each date, namely  $F(g^*)$ . This "bubble-like" pattern of sovereign debt comes from the multiplicity of lenders and their collective inability to commit not to lend again in the future. This is related to the "common-agency problem" identified by Tirole (2002).<sup>16</sup>

Due to this extreme volatility of capital flows, it is not clear that the country is better off than under autarky. Indeed in our model, there are two justifications for sovereign debt: income smoothing as in Eaton and Gersovitz (1981) and "front-loading," or benefiting early from future growth of government income. Because of the short termism of politicians the first objective cannot be obtained (quite the opposite, since income volatility is increased by procyclical borrowings!), but the second objective is partially attained.

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<sup>15</sup> This is in line with the empirical results of Prasad et al. (2004), who find that financial integration has resulted in an *increase* in consumption volatility for some emerging countries.

<sup>16</sup> Tirole (2002) also identifies a "dual-agency problem" in the relations between private domestic borrowers, foreign investors and the domestic government. This problem only concerns private borrowing and is therefore outside the scope of our paper.

However, less extreme forms of policy intervention than restrictions on international mobility of capital flows are possible. We conclude this section with a brief discussion of some of these policy interventions or political reforms.

- **Reducing Political Instability**

We could enrich our model by assuming that governments have a certain probability of being reelected, but that a financial crisis provokes political change: incumbents are replaced. In such a context, the government would face a trade-off between the immediate benefits of borrowing more and the future benefits of reelection. It would therefore borrow less than the maximum amount that markets are ready to lend (which is still equal to  $b^* y_t$ ). As a result of this more reasonable borrowing policy, the volatility of public consumption, the probability of crisis and the interest rate on debt are all reduced, thus leading to increased welfare.<sup>17</sup>

- **Constitutional Reform**

Suppose that the government is prevented from borrowing more than a certain fraction  $b$  of current income by the Constitution. For the same reasons as above, welfare is presumably increased by the reform, at least up to the point where the “front-loading” motive becomes dominant. It is therefore important to evaluate the “optimal” level of  $b$ , i.e., the one that maximizes social welfare.

- **IMF insurance**

In fact, we want to put forward a combination between the above reform, i.e., a cap on the borrowing rate imposed by the constitution, and an insurance policy by the IMF. Specifically, countries that would agree to include this cap in their constitutions would then benefit from contingent credit lines (or equivalently credit risk insurance) provided by the IMF, and financed ex ante by actuarial premiums. The country could then borrow at a constant rate  $b$ , and would pay at each date  $t$  an insurance premium  $\pi y_t$ . A crisis would occur every time the growth factor  $\tilde{g}$  falls below a certain threshold  $\hat{g}$ , determined by the condition:

$$(1 + b - \pi)y_{t+1} = (1 + r)by_t$$

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<sup>17</sup> This leads to another testable implication, namely that countries with higher political instability have ceteris paribus a lower level of sustainable debt. This is in line with Alesina and Tabellini (2005).

where  $y_{t+1} = y_t \hat{g}$ . Thus:

$$\hat{g} = \frac{(1+r)b}{1+b-\pi}.$$

In such a situation, public consumption would follow a less volatile pattern

$$c_t = (1+b-\pi)y_{t-1}(\tilde{g}_t - \hat{g})_+$$

than in the equilibrium described in Proposition 3 (because  $b-\pi \leq b^*$ ). Crises would occur less often (because  $\hat{g} \leq g^*$ ) and public debt would not suffer from high spreads (because it would be insured by the IMF).<sup>18,19</sup>

## 7 Conclusion

This paper has explored a variant of the sovereign debt model of Eaton and Gersovitz (1981), where some important assumptions are changed:

- shocks on primary incomes are persistent,
- markets are short termist, and
- markets and governments cannot commit on their future actions.

Persistence of shocks to primary incomes generates a “front loading” motive for public debt, namely benefiting early from future growth of government income. This motive adds to the classical insurance motive analyzed by Eaton and Gersovitz (1981). If either governments or markets had long term objectives and could commit on their future actions, they could implement the first best optimum, which consists in borrowing initially a certain fraction of current income (so as to benefit early from future growth) and to insure against income shocks by a perfectly countercyclical borrowing pattern for all subsequent periods. However, such behavior is clearly not time consistent.

We then explore the polar model where political instability is so high that governments only last for one period. In this case, they always borrow as much as they can, and the pattern of

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<sup>18</sup> A similar proposal is put forward by Cohen and Portes (2004), who argue in favor of the IMF behaving as a lender of *first* resort, in exchange for a commitment of the country to refuse borrowing at interest rates above a certain cap.

<sup>19</sup> This is also in line with the role of the IMF as seen by Tirole (2002, pp. 114-1150, who states: “The IMF’s role is to substitute for the missing contracts between the sovereign and individual foreign investors and thereby to help the host country benefit from its capital account liberalization.”

debt is determined by the willingness of financial markets to lend to the country. We analyze the RE equilibria of the credit market. They are all unbounded, except for the one that is stationary. This stationary equilibrium exhibits “bubble-like” patterns:

- the government borrows a fixed proportion of its current income, generating pro-cyclical capital inflows.
- debt ratio and probability of default are determined by the stochastic distribution of the growth factor, as anticipated by financial markets.
- public expenditure is more volatile than under autarky.
- steady lending periods alternate with crisis periods, after which the country's debt is forgiven.

We also analyze possible remedies to these inefficient borrowing patterns: increasing political stability, IMF insurance of loans in exchange for implementation of a constitutional reform that prevents excessive borrowing.

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