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The size of the underground economy in Germany: A correction of the record and new evidence from the Modified-Cash-Deposit-Ratio approach

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The size of the underground economy in Germany:
A correction of the record and new evidence from the Modified-Cash-Deposit-Ratio
approach
by

Michael Pickhardt and Jordi Sarda

July 2010

Abstract: Based on the Ahumada et al. (2007, Review of Income and Wealth) critique we
revise existing estimates of the size of the German underground economy. Among other
things, it turns out that most of these estimates are untenable and that the tax pressure induced
size of the German underground economy may be much lower than previously thought. To
this extent, German policy and law makers have been misguided during the last three decades.
Therefore, we introduce the Modified-Cash-Deposit-Ratio (MCDR) approach, which is not
subject to the recent critique and apply it to Germany for the period 1960 to 2008.

JEL: O17, Q41, C22, Keywords: underground economy, shadow economy, cash-deposit-
ratio, currency demand approach, MIMIC approach,

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The size of the underground economy in Germany:
A correction of the record and new evidence from the Modified-Cash-Deposit-Ratio approach

1 Introduction

Combating the underground economy requires, among other things, an appropriate law system. Yet, in assessing whether an existing law system and related factors such as law enforcement etc. work effectively or require some adjustments, policy and law makers alike depend on the quality and reliability of data on the underground economy. Although anecdotal evidence from actually discovered cases might be of some help here, estimates of the size of the underground economy and its development over time play an important role in this process. For example, a research induced public debate on the extent of the black labour market and the underground economy in Germany eventually led to a new law on combating black labour activities, which applies since August 2004.2

In fact, since the early 1980s, researchers have developed rather complex econometric approaches to estimate the size of the underground economy (e.g. see Tanzi (1980), Frey and Weck (1983), Pissarides and Weber (1989), Lyssiotou et al. (2004)). Overviews are provided by Schneider and Enste (2000) or Pickhardt and Shinnick (2008), among others. A major advantage of these methods is that they may to some extent explain the causes of the underground economy and, therefore, allow law makers to specifically address such issues. Recently, however, some of these approaches, the Multiple-Indicators-Multiple-Causes (MIMIC) method and the currency demand method (the latter is often used as an input for the MIMIC approach, see Giles 1999), have been heavily criticized on econometric grounds by

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1 We use the term ‘underground economy’ interchangeably with other expressions such as shadow economy, hidden economy, black economy, etc. because in previously published literature almost identical estimation equations have been used for estimating the size of the underground economy, shadow economy, etc. and because we think that the observable use of different terms for identical items in this research area is predominantly due to the fact that the phenomena is known under different labels in different languages. Therefore, the variety of terms seems to reflect translations into English, rather than different definitions. For an overview concerning alternative terms see Kazemier (2006).

2 See Bundesgesetzblatt (2004), ‘Gesetz zur Intensivierung der Bekämpfung von Schwarzarbeit’. 
Breusch (2005a,b,c,d). Moreover, Ahumada et al. (2007) have shown that the currency
demand method only produces coherent estimates if the long run income elasticity of the
demand for currency is equal to unity. Yet, this condition is not fulfilled for a large number of
published estimates. Also, Ahumada et al. (2008) have shown that if the lagged dependent
variable is used in currency demand estimations, calculating cardinal values of the size of the
underground economy requires a known initial value of the size of the underground economy.
But in a large number of relevant published estimates no such initial value was used.
Consequently, many published estimates of the size of the underground economy have
provided misleading information to policy and law makers.

The purpose of this paper is to address this issue in various ways. First, we reconsider
existing estimates of the size of the underground economy in Germany and apply the
Ahumada et al. (2007) correction. It turns out that some previous estimates are untenable with
respect to the size of the German underground economy. Second, we develop and apply a
rather simple method that still offers a ‘reasonable’ estimate of the size of the underground
economy and which is not subject to the critique of Breusch (2005a,b,c,d) and Ahumada et al.
(2008, 2007). In particular, we modify the original cash-deposit-ratio approach which was
pioneered by Cagan (1958) and first applied by Gutmann (1977). Among other things, we
show that the modified-cash-deposit-ratio (MCDR) approach allows for reproducing existing
estimates of the underground economy in Germany to some extent.

We proceed as follows. In the next section we summarize the results of previous studies on
the size of the underground economy in Germany and apply the Ahumada et al. (2007)
correction. In section three we briefly review the original cash-deposit-ratio approach and
introduce the main modification. Next we successively introduce further auxiliary
modifications and present results from applying the MCDR approach to Germany. In section
five we discuss findings of the previous sections. The final section concludes.
2 Previous estimates

Estimates of the size of the underground economy in Germany have been carried out since the 1970s by various researchers using alternative methods. Table 1 provides some selected results of the estimated size of the underground economy in Germany.

Inspection of Table 1 shows that most estimates are based on the currency demand approach, as developed by Tanzi (1980, 1982, 1983), which involves an econometrically estimated currency demand equation based on Cagan (1958). Klovland (1980, 1984), Bhattacharyya (1990), Escobedo and Mauleón (1991), and others have developed variants of the method. In addition, results obtained from the currency demand approach are often used as a calibration input for MIMIC estimations (e.g. see Schneider and Enste 2000; Giles 1999, p. F373), because the MIMIC method just generates relative estimates (see Frey and Weck 1983) and, therefore, obtaining cardinal values of the size of the underground economy requires a benchmark value that must come from another source. For example, Buehn et al. (2009) use a 1983 value, which was estimated by Karmann (1990) with a Tanzi version of the currency demand approach, as a calibration input for their MIMIC model. In contrast, Pickhardt and Sarda (2006) calibrate their MIMIC model with a 1980 value obtained from a currency demand estimation using the Escobedo and Mauleón (1991) approach and, in addition, use a direct combination of this currency demand approach and the MIMIC approach, which they call the ‘joint model’ (JM). Langfeldt (1982, 1989) applies not only the currency demand approach, but also the transactions approach developed by Feige (1979), which essentially amounts to a calculation procedure that does not involve econometrics. Albers (1974) and Petersen (1982) apply a procedure based on national accounting, which is also known as the discrepancy method. Finally, Pedersen (2003) and Feld and Larsen (2005) use data from questionnaire surveys for calculating the size of the black sector in Germany.
Table 1: Size of the German Underground Economy in Percent of GDP (GNP)

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<td>Kirchgaessner (1983)</td>
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<td>Karmann (1990)</td>
<td>1.5h</td>
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<td>Schneider/Enste (2000)</td>
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<td>Kirchgaessner (1983)</td>
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<td>Pickhardt/Sarda (2006), (JM)</td>
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<td>Buehn et al. (2009), (Tanzi)</td>
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<td>1.6</td>
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<td>12.1</td>
<td>15.3</td>
<td>16.3</td>
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<td>Albers (1974), (Nat. Accounts)</td>
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<td>Langfeldt (1989), (Transactions)</td>
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<td>17.5</td>
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<td>Feld and Larsen (2005), (Quest.)</td>
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<td><strong>Ahumada et al. Corrected Values</strong></td>
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<td>A/Kirchgaessner (1983, Klov.)</td>
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Note: a refers to 1968, b refers to 1974, c refers to 1976, d refers to 1983, e refers to 2001, f refers to 2004, g denotes that the figure is based on the physical input of electricity method, h denotes percent of GNP and all other figures are denoted in percent of GDP. Also, most authors provide additional results, which we have not mentioned here for brevity.

As noted, Ahumada et al. (2007) have shown that the currency demand method produces coherent estimates only if the long run income elasticity of the demand for currency, $\beta$, is equal to unity, $\beta = 1$. Essentially, the condition $\beta = 1$ follows directly from the assumption of equal velocities of cash circulation in both the legal and underground economy, which is one of the crucial assumptions on which the currency demand approach rests (e.g. see Breusch 2005b, p. 396). In addition, Ahumada et al. (2007) provide a correction procedure, which can

$^3$ The condition $\beta = 1$ is generally required, except in the rather unlikely case where the size of the underground economy is exactly equal to the size of the legal economy (see Ahumada et al. 2007, p. 367).
be applied if the long run income elasticity of the demand for currency differs from unity, $\beta \neq 1$. With respect to the estimates shown in Table 1, this applies for example to Kirchgaessner’s (1983, p. 213) estimates based on the Klovland method. In general, the Ahumada et al. (2007, p. 368) correction is:

$$
\left( \frac{Y_U}{Y_L} \right)^{\frac{1}{\beta}} = \frac{Y_U}{Y_L},
$$

(1)

where $Y_U$ denotes the size of the underground economy in national currency, $Y_L$ denotes the size of the legal economy in national currency, the ratio $\frac{Y_U}{Y_L}$ denotes the faulty size of the underground economy, the ratio $\frac{Y_U}{Y_L}$ denotes the correct size of the underground economy and $\beta$ is the long run income elasticity, which is different from unity. As Kirchgaesser’s estimate (1983, p. 212, Table 3, A3) is based on a partial adjustment real currency demand equation, the long run income elasticity is $\beta = \frac{0.129}{(1-0.692)} = 0.41883117$ (see also Ahumada et al. 2008, pp. 98–99). Then, if we consider the faulty size of the underground economy in 1980 (see 1983, p. 213, Table 4, A3, period 1955-1980), which is $\frac{Y_U}{Y_L} = 0.088$, application of (1) yields: $0.088^{\frac{1}{0.41883116}} = 0.0030189$, which gives a corrected size of 0.302 percent of GNP for the German underground economy in 1980. Corrected values for 1970 and 1975 are obtained in the same way and displayed in Table 1, bottom line. Inspection of Table 1 shows that the corrected size of the German underground economy is substantially below the initially published size.\(^4\) Moreover, because Kirchgaessner’s (1983) estimations are also subject to the critique of Ahumada et al. (2008), even the corrected values may not give a good approximation of the German underground economy.

\(^4\) Ahumada et al. (2007, p. 370) correct the estimate of Isachsen and Strom (1985, p. 24) for Norway, which is also based on the Klovland method and find that the corrected size of the underground economy is 1.51 percent of GDP in 1978, rather than 6.3 percent.
In this context it is worth noting that the Ahumada et al. (2008, 2007) critique applies to all other results displayed in Table 1, which are obtained by using the currency demand approach (Tanzi version). Further, because each of these estimates yields a different long run income elasticity of the demand for currency, which in each case also differs from unity, the true size of the underground economy obtained from these estimates must differ more than the published figures. Yet, as these estimates all consider the ratio of currency over a monetary aggregate (C/M2) as dependent variable, rather than real currency, we refrain from applying the Ahumada et al. (2007) correction procedure in these cases. Besides it is also worth mentioning that values provided by Pickhardt and Sarde (2006) are not subject to the Ahumada et al. (2007) correction, because the underlying Escobedo and Mauleón (1991) approach does not rest on the assumption of equal velocities of circulation in the legal and the underground economy and, therefore, does not require $\beta = 1$. However, the approach has other disadvantages, which include that the generated underground economy profile essentially mimics the profile of the fiscal pressure variable that is used in the estimation. Thus, estimates based on the Escobedo and Mauleón approach are equally questionable, though for other reasons.

To summarize, inspection of Table 1 shows that various applications of the currency demand approach arrive at roughly comparable values for the size of the German underground economy. However, it follows from Ahumada et al. (2007) that all these values and their alleged similarity are questionable. By applying the Ahumada et al. (2007) correction procedure we were able to correct one of these estimates. The corrected value is rather low and substantially lower than the initially published size of the underground economy, but may still be faulty according to Ahumada et al. (2008) and Breusch (2005b,c). Besides, the

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5 In fact, we selected the Kirchgaessner (1983) estimations for correction because of several reasons. For example, Kirchgaessner (1983) considers real currency as a dependent variable, which makes our correction comparable to those made by Ahumada et al. (2007) for other published estimates. Also, he applies both the Tanzi and the Klovland version of the currency demand approach and presents his econometrical findings with all relevant details.
transactions approach yields much larger values and, therefore, represents the upper bound of the spectrum, whereas the questionnaire survey method offers values that are comparatively low and, thus, represent the lower bound of the spectrum. In any case, the brief literature review demonstrates that there is some faulty and mixed evidence concerning the size and development of the underground economy in Germany. Possible ways of addressing this issue include developing alternative methods and conducting some plausibility tests. Therefore, in a first step, we proceed with the MCDR approach.

3 The modified-cash-deposit-ratio approach

As noted, the original cash-deposit-ratio approach was pioneered by Cagan (1958) and first applied by Gutmann (1977). Formal representations and critical reviews of the original approach are provided by Thomas (1999, pp. F382–F383), Feige (1989, pp. 36–44), and Blades (1982, p. 43), among others.

The original approach is based on the following assumptions (see Ahumada et al. 2008, pp. 97–98; Thomas 1999, pp. F382–F383; Feige 1989, pp. 36–44; Trockel 1987, pp. 103–106, Tanzi 1982, p. 73, among others). First, total output of the economy \( Y \) in period \( t \) may consist of recorded or legal output \( Y_L \) and unrecorded or underground output \( Y_U \). Second, it is assumed that all cash in circulation outside banks can be separated logically into one part that circulates exclusively in the official or legal economy \( C_L \) and another part that circulates exclusively in the underground economy \( C_U \). Third, it is assumed that agents in the legal economy \( Y_L \) may use both cash and sight deposits (i.e., demand deposits) for their transactions, whereas agents in the underground economy \( Y_U \) are effectively restricted to the use of cash because the use of sight deposits may allow the authorities to trace their activities. Fourth, all cash holdings and all sight deposits are held exclusively for transaction motives, all other motives are disregarded. Fifth, agents in the legal economy wish to maintain a constant
proportion $\lambda$ of cash holdings $C$ versus sight deposits $D$ over time. These five assumptions allow us to write out the following definitions and functions.

\begin{align*}
Y_t &= Y_{Lt} + Y_{Ut}, \\
C_t (Y_t) &= C_{Lt} (Y_{Lt}) + C_{Ut} (Y_{Ut}), \\
M1_t (Y_t) &= C_t (Y_t) + D_t (Y_{Lt}), \\
\lambda_t &= C_{Lt0} / D_0, \quad t = 1, \ldots, Z,
\end{align*}

where $C_t$ denotes cash in circulation outside banks in period $t$, $C_{Lt0}$ denotes cash used in the legal economy in the base year or period, $D_0$ denotes sight deposits held in the base year or period, $M1$ is the standard monetary aggregate, and $Z$ is the final period under consideration.

Next, Irving Fisher’s (1911) quantity theory of money may be considered, that is:

\begin{equation}
M \cdot v = P \cdot T,
\end{equation}

where $M$ denotes money in terms of M1, $v$ is the velocity of money circulation, $P$ refers to the price level and $T$ indicates the number of transactions. Substituting $T$ for real output $Y_r (= Y_n/P)$ in (6) yields, $M \cdot v = Y_n (6')$, where $Y_n$ denotes nominal output. Based on the assumptions made so far, we can then express the size of the legal and underground economy by using (6’) separately for each case and rearranging allows for calculating the hypothetical velocity of circulation $v$ in both the legal and the underground economy.

\begin{align*}
Y_{Lt} / (C_{Lt} + D_t) &= v_{Lt}, \\
Y_{Ut} / C_{Ut} &= v_{Ut},
\end{align*}
where $C_{Lt} + D_t$ represents $M_{Lt}$ and $C_{Ut}$ represents $M_{Ut}$ with $M_L = M_{Lt} + M_{Ut}$, $Y_{Lt}$ and $Y_{Ut}$ are denoted in nominal terms, but the index $n$ is dropped to simplify notation, $\nu_L$ is the velocity of money circulation in the legal economy, and $\nu_U$ denotes the velocity of cash circulation in the underground economy.

The sixth assumption then consists of assuming that the velocity of circulation $\nu$ is identical in both the legal and the underground economy, that is, $\nu_{Lt} = \nu_{Ut}$, with $t = 1, \ldots, Z$ (e.g. see Tanzi 1980; Isachsen and Strom 1985, p. 24). Next, a base year or a base period must be chosen and it is assumed that the underground economy does not exist in this base year or base period. This is the seventh assumption and the base year is indexed $t = 0$, so that: $Y_{U0} = 0$. Hence, based on (5) we can state that the demand for cash holdings in the legal economy in period $t$ is given by:

$$C_{Lt} = \lambda_t \cdot D_t,$$  \hspace{2cm} (9)

and that the demand for cash holdings in the underground economy in period $t$ is given by:

$$C_t - \lambda_0 \cdot D_t = C_{Ut}. \hspace{2cm} (10)$$

Equating (7) and (8), substituting $C_{Lt}$ and $C_{Ut}$ according to (9) and (10), respectively, and rearranging yields:

$$\frac{C_t - \lambda_0 \cdot D_t}{\lambda_t \cdot D_t + D_t} \cdot Y_{Lt} = \frac{C_t - C_{L0} \cdot \frac{D_t}{D_0}}{C_{L0} \cdot \frac{D_t}{D_0} + D_t} \cdot Y_{Lt} = Y_{Ut}. \hspace{2cm} (11)$$
As data is usually available for all variables on the left hand side or middle term of (11), \( \bar{Y}_{Ut} \) can be calculated from (11). Hence, equations (2) to (11) fully describe the original cash-deposit-ratio approach, although Gutmann (1977) did not write out these assumptions explicitly nor did he provide any formal representation or based the approach on Fisher’s quantity theory of money.

Yet, application of (11) would yield negative sizes of the underground economy in Germany and virtually all other industrialized countries (e.g. see Frey and Pommerehne 1984, p. 16), but for brevity results are not displayed here. Essentially, negative results emerge because the fifth assumption is no longer valid. That is, agents in the legal economy apparently do not wish to maintain a constant proportion \( \lambda \) of cash holdings \( C \) and sight deposits \( D \) over time. Despite some country specific differences, agents in industrialized countries seem to have changed their preferences over time toward a substantially higher share of deposits and inspection of (11) shows that this change would generate negative values of the underground economy.

Therefore, the main modification we propose consists of dropping the fifth assumption that agents wish to hold a fixed ratio of currency to deposits over time, because empirical evidence clearly shows that this assumption is untenable. Instead, we assume that all currency in circulation in the base year, \( C_0 \), represents the entire cash agents wish to hold in any year after the base year for the set of legal transactions they prefer to carry out in cash. Moreover, we assume that all additional transactions in the legal economy are carried out via sight deposits by using cheques, debit and credit cards, etc. Otherwise, however, we continue to apply the remaining assumptions of the original cash-deposit-ratio approach. Therefore, by definition, any cash holdings in excess of those in the base year can be fully attributed to the underground economy. Formally, the main modification changes (9) and (10), respectively, to:
\[ C_{Lt} = C_0, \quad t = 1, \ldots, Z. \]  

(12)

\[ C_t - C_0 = C_{Ut}, \quad t = 1, \ldots, Z. \]  

(13)

Equating (7) and (8), substituting \( C_{Lt} \) and \( C_{Ut} \) according to (12) and (13), respectively, and rearranging now yields:

\[ \frac{C_t - C_0}{C_0 + D_t} \cdot Y_{Lt} = Y_{Ut}. \]  

(14)

Comparison with equation (11) shows that the main modification effectively removes the \( D_t/D_0 \) ratio from equation (11). Therefore, equation (14) yields positive values of the underground economy. In particular, application of (14) generates underground economy profile \( G_0 \) (see Table 2 and Figure 1), where \( C_t \) denotes currency in circulation outside banks at the end of the year, \( C_0 \) denotes currency in circulation outside banks at the end of 1960 and \( D_t \) denotes sight deposits held by domestic non-banks (non-MFIs) at the end of the year.

Finally, it should be emphasized that the currency demand approach essentially rests on an equation like (14) with respect to obtaining cardinal values of the size of the underground economy, except that the nominator, \( C_{Ut} \), is generated by an econometric estimation procedure (e.g. see Breusch 2005b, p. 396; Tanzi 1983, pp. 293–294; Tanzi 1982, p. 86).  

\[ \text{4 Auxiliary modifications} \]

Although the main modification solves the major problem of the original cash-deposit-ratio approach (i.e., negative results), it must be emphasized that all criticism put forward with respect to the original cash-deposit-ratio approach, in particular, regarding its heroic

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6 The version of (14) used within the currency demand approach often amounts to: \( C_{Lt} \cdot v_t = Y_t \cdot v_t \). Note, however, that obtaining the velocity of circulation \( v_t \) from a third source, say from the national bank, will inevitably lead to a faulty size of the underground economy, unless \( M1 \) is corrected for \( C_{Ut} \), that is: \( v_t = Y_t / (M1 - C_0) \).
assumptions, its arbitrariness and its inability to explain causes of underground activities, applies to the modified version as well. To some extent, however, this criticism can be addressed with further modifications.

For example, inspection of the underground profile $G0$ reveals an extraordinarily sharp drop of the size of the underground economy in 2001. It follows from the German data set that the introduction of Euro coins and notes on January 1, 2002, has caused a sharp drop with respect to currency in circulation outside banks at the end of 2001, $C_t$, and according to (14) a sharp decrease of $C_t$ would ceteris paribus cause a sharp drop in the calculated underground economy profile. To accommodate this special Euro zone aspect, we estimate $C_t$ for Germany, with data from the period 1960 to 1999, using a variant of the method proposed by Seitz (1995). For simplicity, details of the estimation procedure, relevant results and diagnostic statistics are provided in the appendix. Based on this estimation we then forecast $C_t$ over the period 1960 to 2006, which allows us to bridge the drop in $C_t$ due to the Euro coins and notes introduction. Yet, to rule out any deviations after 2006, we use the original $C_t$ data for the years 2007 and 2008. This procedure yields $FC_t$, which is the forecasted currency in circulation outside banks and replacing $C_t$ by $FC_t$ in (14) yields (15).

$$\frac{FC_t - C_0}{C_0 + D_t} Y_{L_t} = Y_{Lt}.$$  \hspace{1cm} (15)

Applying (15) yields underground economy profile $G0_2$ for Germany in Table 2.\footnote{The profile $G0_2$ and all following profiles are subject to the standard error of the estimation procedure, which is: 0.019521. But for simplicity alone, we refrain from taking this explicitly into account with respect to calculating the size of the underground economy.} Inspection of the period 2001 to 2006 in Table 2 and Figure 3 makes it clear that this modification does solve the Euro coins and notes distortion problem to a large extent. Also, Figure 3 compares to Figure 1 of Buehn et al. (2009, p. 711), who apply the Seitz method for the same purpose.
Additional examples for relevant auxiliary modifications include: 1) that inflation may require increasing \( C_0 \) over time to allow agents to carry out their preferred set of cash transactions, 2) that changes in the size of the population may require to adjust \( C_0 \) over time, 3) that a certain fraction of \( C_t \) may be held abroad, 4) likewise, that a certain fraction of \( C_t \) may be hoarded by national agents, 5) that the number and set of transactions, which agents wish to carry out in cash, may change over time, for example, due to the evolution of new non-cash payment methods and facilities, 6) that some proceeds from underground activities may in fact be held as sight deposits, for example, because of money laundering or because the illegal transactions did not involve any cash payments at all, so that \( D_t \) may have to be reduced accordingly to \( D_{Lt} \) in the denominator of (14) and \( D_{Ut} \) may have to be added to \( C_{Ut} \) in the nominator, with \( D_t = D_{Lt} + D_{Ut} \), 7) that the size of the underground economy may not have been close to zero in the base year or base period. In the following we address some of these issues step by step and show how these additional modifications change the size of the underground economy.

**MOD1**: The first auxiliary modification consists of adjusting \( C_0 \) with the annual inflation rate, as measured by the consumer price index. This adjustment is necessary because the main modification introduced in the preceding section implicitly rests on stable preferences for cash payments, that is, the set and number of transactions which each agent wishes to carry out in cash do not change over time. Therefore, we need to transform \( C_0 \) into \( IC_{0t} \) according to: 

\[ IC_{0t} = IC_{0,t-1} \cdot (1 + \text{Inf}_{t}/100), \]

where \( \text{Inf}_t \) refers to the inflation rate of year \( t \) in percent and \( IC_{0t} \) is the inflation adjusted value of \( C_0 \) in period \( t \). Replacing \( C_0 \) accordingly in (15) yields:

\[
\frac{FC_t - IC_{0t}}{IC_{0t} + D_t} \cdot Y_{Lt} = Y_{Ut}. \tag{16}
\]
Other things being equal, applying (16) now yields underground economy profile $G1$, again shown in Table 2 and Figure 1. Inspection of Table 2 reveals that this modification reduces the initially calculated size of the underground economy substantially.

**MOD2**: Furthermore, the base version of the MCDR implicitly assumes that the preferred average cash per capita ratio remains constant over time. Hence, if the size of the population ($POP$) changes during the relevant period, this change must be taken into account. To do so, we construct a population index ($POP_t/POP_0$) and multiply the inflation adjusted $C_0$ figures, $IC_{0t}$, in each year with the relevant population index number. This procedure yields population and inflation adjusted values of $C_0$, which we denote as $PIC_{0t}$ in (17). Applying (17) gives underground economy profile $G2$ in Table 2 and Figure 1.

$$\frac{FC_t - PIC_{0t}}{PIC_{0t} + D_t} \cdot Y_L = Y_{Ut}. \quad (17)$$

As the German population has grown since 1960, $G2$ figures of the underground economy are below those of $G1$. Moreover, the modification allows for incorporating the German reunification effect.

**MOD3**: Next, we need to take into account that a substantial share of the German currency in circulation outside banks was held outside Germany during the Deutsche Mark period. Essentially the same is true for the Euro period. To estimate this amount, we can use again the variant of the Seitz (1995) method. In fact, the estimation which was already used for forecasting currency in circulation outside banks, $FC_t$, can be used to calculate the amount of forecasted currency in circulation outside banks, inside Germany, $INFC_t$ (see appendix for details). Note, however, that we also need to replace the base year value $C_0$ of $PIC_{0t}$ in (13) by the $INC_0$ value and re-apply the modifications **MOD1** and **MOD2**, which yields $INPIC_{0t}$. 
Given these modifications, equation (18) emerges from (17) and applying (18) yields underground economy profile $G3$, again displayed in Table 2 and Figure 1.

\[
\frac{INFC_t - INPIC_{0t}}{INPIC_{0t} + D_t} \cdot Y_{Lt} = Y_{Ut}. \tag{18}
\]

As noted, the next modification concerns the issue of hoarded currency in Germany. Any amount of hoarded currency would reduce the size of $INFC_t$ and, therefore, would \textit{ceteris paribus} reduce the size of the underground economy. However, we have been unable to obtain relevant time series data and, thus, we must currently refrain from applying this modification to the $G3$ profile. This notwithstanding, the Bundesbank (2009) claims that at the end of 2007 about 14.09 billion of Deutsche Mark currency was still in circulation outside banks (about 6.99 billion in banknotes and 7.10 billion in coins), which amounts to about 7.2 billion Euro. This currency might be hoarded as a souvenir or because of distrust in the Euro system or may simply be lost or destroyed. Deducting this amount from $INFC_t$ in (18) and reapplying (18) \textit{ceteris paribus} for $t = 2007$ yields a size of the underground economy of 8.5 percent instead of 9.36 percent (see Table 2, G3, 2007).

Moreover, the modifications introduced so far are based on the implicit assumption that cash payment preferences are stable over time. However, for a number of reasons this may actually not be the case. Therefore, the fifth auxiliary modification we suggest above deals with possible changes regarding cash payment preferences. On the one hand, if a society gets relatively richer over time, agents may develop a preference for replacing some household production by market exchanges. This may include going out for dinner rather than preparing food at home, having some washing done by a laundry rather than at home, etc. Such developments may lead to a higher number of cash transactions and may enlarge the set of transactions agents wish to carry out in cash. One the other hand, however, the evolution of
new non-cash payment methods and facilities may reduce the number of cash transactions and may reduce the set of transactions agents wish to carry out in cash. Unfortunately, we did not find sufficient data on these two developments and, therefore, we have been unable to include this modification.

For the same reason, we have been unable to adjust the value of sight deposits, as suggested in auxiliary modification six. In fact, there is no time series data on the extent of money laundering in Germany or on the extent of underground economy activities that involve payments via sight deposits. This notwithstanding, there is growing scientific interest in determining the extent of money laundering. For example, Unger (2007) provides an overview on methods for quantifying money laundering, and Schneider and Windischbauer (2008) try to quantify the extent of money laundering for Germany. But as already noted, inspection of (18) makes it clear that any reduction of $D_t$ to $D_L$, and a subsequent adding of $D_{Ut}$ to $C_{Ut}$ would *ceteris paribus* lead to higher values of the underground economy.

The seventh auxiliary modification concerns the size of the underground economy in the base year. This issue might be addressed by taking the size of the underground economy in the base year from other studies or by assuming some arbitrary positive value. However, we refrain from adopting such procedures on the grounds that any such choice is as arbitrary as the initial assumption of no underground economy in the base year. Furthermore, it must be emphasized that the size of the underground economy according to any of the profiles $G_0$ to $G_3$ also depends on the (arbitrarily) chosen base year or base period. For example, by choosing 1965 as base year, instead of 1960, we would get the following values for $G_3$: (1970: 3.76%), (1980: 8.41%), (1995: 12.64%), (2008: 9.87%). Comparison with relevant $G_3$ values in Table 2 shows that the difference is rather large at the beginning, but diminishes over time. Besides, choosing 1960 as a base year does make some sense, for example,
### Table 2: Size of the German Underground Economy in Percent of GDP or GNP

<table>
<thead>
<tr>
<th>G0</th>
<th>G0_2</th>
<th>G1</th>
<th>G2</th>
<th>G3</th>
<th>Year</th>
<th>K(T)</th>
<th>K(K)</th>
<th>BKS</th>
<th>PS</th>
<th>V</th>
<th>K(A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1960</td>
<td>2.00</td>
<td>1.7</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1961</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>16P</td>
<td>---</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1962</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

Note: G0 denotes the uncorrected size of the German underground economy according to equation (14). G0_2, G1, G2, and G3 denote the size of the German underground economy due to auxiliary modifications according to (15), (16), (17), and (18), respectively. G0 to G3
are denoted in percent of GDP. \( K(T) \) denotes figures by Kirchgässner (1983, p. 203) based on the Tanzi method, \( K(K) \) denotes figures by Kirchgässner (1983, p. 213, A3, 1955-1980) based on the Klovland method, \( BKS \) denotes results by Buehn et al. (2009, p. 719, Table 5, column 4, H-DIY model), \( PS \) denotes results presented by Pickhardt and Sarda (2006, p. 1711, Table 4, joint model). \( V \) denotes results presented by various authors according to Table 1, where \( A \) denotes Albers (1974), \( F \) denotes Friedman et al. (2000, p. 466, share2 column) which is based on the electricity method, \( FL \) denotes Feld and Larsen (2005), \( KA \) denotes Karmann (1990), \( L \) denotes Langfeldt (1989) transactions method, \( P \) denotes Petersen (1982), \( PE \) denotes Pedersen (2003), \( SE \) denotes Schneider and Enste (2000) and \( FS \) denotes Feld and Schneider (2010). \( K(A) \) denotes values derived from a recalibration of the BKS MIMIC index. This index (not displayed) is derived from dividing figures shown in the BKS column by the original calibration value 8.5 for 1983. The resulting BKS MIMIC index in then recalibrated with the 1980 value of Ahumada et al. (2007) corrected values of Kirchgässner (1983, Klovland), where the recalibration value is 0.282 percent of GDP in 1980. Figures in all columns on the right hand side of the column year are denoted in percent of either GDP or GNP, according to Table 1. GDP denotes cross domestic product and GNP denotes cross national product.

Figure 1: Size of the Underground Economy in Germany 1961-2008 in Percent of GDP based on MCDR approach

because the Deutsche Mark became convertible in 1958, the federal state of Saarland became part of Germany again so that its data is included since 1959 in Bundesbank time series data
and the macroeconomic environment, which included full employment, was most favourable. In this context it is also important to stress that the problem of the initial size condition is, contrary to conventional beliefs, not solved if the lagged dependent variable is considered in one way or another in currency demand estimations, as Ahumada et al. (2008) have shown.

Finally, it is worth noting that the underground economy profile $G3$ in Table 2 and Figure 1 still exhibits some particularities. First, there is an extraordinary jump in the period 1987 to 1989, which is mainly due to withholding tax effects (see Seitz 1995, p. 11). Also, the reunification period 1990 to 1991 certainly exhibits some distortions. Therefore, during the period 1987 to 1991 the $G3$ profile may not be fully attributable to the development of the German underground economy. Another extraordinary development occurs with respect to the formation of the European Monetary Union (EMU) in 1999, which led to some changes in the Bundesbank statistics. For example, until 1998 the time series sight deposits (OU0221) included time deposits with less than and up to one month, but since 1999 these time deposits are included in the time deposits series (OUA192). Moreover, as of 2000 the amount of cash held outside Germany is forecasted and the same is true for the period 2001 to 2006 with respect to cash in circulation (see appendix for details).

To summarize, by a stepwise application of various modifications we eventually obtained underground economy profile $G3$. Although further modifications are considered necessary, a lack of relevant data currently prevents any additional quantitative adjustments. This notwithstanding, some qualitative statements are possible. At least four aspects have been identified that might have an influence on the variables in (18). But because two may cause an increase of the size of the underground economy, whereas two might lead to a decrease, the sign of the net effect remains unpredictable. Therefore, the $G3$ profile may represent a lower or upper bound of the size of the German underground economy, or may even represent a

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8 For example, comparing the 1986 and 1992 values of $G3$ in Table 2, 13.27 percent and 13.61 percent, respectively, suggests that the corrected $G3$ values during the period 1987 to 1991 may have been in this range as well.
rough estimate of its true size in case that the four influences and possible other influences balance out. Also, some periods have been identified in which the data set exhibits extraordinary distortions so that in these periods the $G3$ profile cannot be fully attributed to the underground economy.

5 Discussion

In the preceding sections we have developed the MCDR approach and applied it to Germany for the period 1960 to 2008, which produced underground economy profile $G3$ (see Table 2). The procedure has demonstrated that the MCDR approach has some appealing aspects from a methodological perspective. In particular, the approach allows for incorporating an unlimited number of auxiliary modifications, which may or may not be of an econometrical nature. Further, the stepwise application of additional modifications keeps the adjustment process transparent and traceable at each stage.

By making the rather strong assumption that all other conceivable influences on $G3$ balance out and by taking into account that during the period 1987 to 1991 the $G3$ profile may not be fully attributed to the underground economy for reasons mentioned above, we may compare the $G3$ profile with previously published estimates of the German underground economy. In Table 2, right hand side, we provide more detailed results from previous studies, of which some are already mentioned in Table 1. A comparison of the $G3$ profile with values of Kirchgaessner (1983), Table 2, columns K(Tanzi) and K(Klovland), shows a close correspondence, except for the year 1970. A comparison with values of Buehn et al. (2009), Table 2, column BKS, reveals that the $G3$ profile exceeds the BKS profile by about three percentage points before German reunification, and that the $G3$ profile is rather close to the BKS profile during the early 1990s, whereas a gap between the two profiles develops as of 1996, where the $G3$ profile is up to seven percentage points below the BKS profile. A comparison with values of Pickhardt and Sarda (2006), Table 2, column PS, yields almost the
same result, although the $G3$ profile is much closer to the PS values in the early 1980s. Further, the $G3$ profile compares well to a number of values obtained by various other researchers for different years (see Table 2, column V). For example, the relevant $G3$ values compare in 1970, 1995 and 2000 with the Schneider and Enste (2000) values.

Yet, in section two we made clear that many of these previously published results are faulty, that is, those obtained by using the currency demand method, including MIMIC model results, if they were obtained by calibrating the MIMIC index with results from the currency demand method. To this extent it follows from the comparison that previously published and popularized figures on the size of the German underground economy can be roughly reproduced only with the MCDR approach, and only under the rather strong assumptions that led to underground economy profile $G3$. But it is important to keep in mind that the currency demand approach usually estimates additional cash holdings due to tax pressure, whereas the MCDR approach would cover, in addition, cash holdings due to underground economy activities that are independent from tax pressure, such as illegal drug dealing, human trafficking, etc. Hence, other things being equal, the MCDR approach should yield a larger size of the underground economy than the currency demand approach.

To further investigate this claim, we proceed with some plausibility tests. For example, if it is assumed that a MIMIC index as such is correct, the index may be used for plausibility testing. In particular, the index may be used for testing whether two or more independently obtained estimates in different years comply with each other or not. To demonstrate this, we assume that the Buehn et al. (2009) MIMIC index is a correct estimate of the development of the tax and regulation induced underground economy in Germany during the relevant period of time. Next we consider the Kirchgaessner (1983, Klovland) estimate of for the year 1980, corrected according to the Ahumada et al. (2007) procedure, which yields 0.30 percent of GNP, or 0.282 percent of GDP. By recalibrating the Buehn et al. (2009) MIMIC index with the 0.282 percent of GDP value of 1980, we obtain the underground economy profile shown
in column K(A) of Table 2. Inspection of the results shows that the size of the underground economy according to the recalibration values in column K(A) is comparatively close to the values of Feld and Larsen (2005) for the years 2001 and 2004 (see Table 1 and Table 2, column V).

Also, the rather low size of the underground economy according to the Ahumada et al. (2007) corrected Kirchgassner values and according to Feld and Larsen (2005) complies well with views expressed by Graf (2009, 2008, 2007), Koch (2008, 2007) and others who argue that previously published values of the size of the German underground economy are far too high (see Table 1). In addition, these low values are more in line with findings from the Bundesrechnungshof (2007, p. 147) according to which the special task force ‘black labour’ of the federal ministry of finance (Finanzkontrolle Schwarzarbeit) was able to detect a damage of about 554 Million Euro in 2005 (0.025 percent of GDP) and about 602 Million Euro in 2006 (0.026 percent of GDP). Further, as noted, by taking into account that the $G3$ profile covers all illegal activities that use cash and, therefore, covers not only black labour activities paid in cash but also illegal drug trade, illegal prostitution, human trafficking, etc., it follows that the size of the German underground economy according to the $G3$ profile should indeed exceed the size according to the K(A) calibration (see Table 2) or Feld and Larsen (2005). In summarizing, evidence provided above seems to suggest that the size of the German underground economy is predominantly driven by abnormally high profit rates in illegal activities, rather than by high tax pressure and/or by regulations.

6 Concluding remarks

The critique of Breusch (2005a,b,c,d) and Ahumada et al. (2008, 2007) has made it plain clear that most of the existing estimates of the underground economy around the world suffer from

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9 The Buehn et al. (2009) MIMIC index is derived by dividing the BKS values shown in Table 2 by 8.5 (i.e. the original calibration value). The resulting index can then be recalibrated with the 1980 value of 0.282 percent of GDP, which yields the values shown in column K(A), Table 2.
serious econometrical and mathematical flaws and, therefore, are faulty. In this paper we applied the Ahumada et al. (2007) correction procedure for the first time to published estimates of the size of the German underground economy. It turns out that the corrected size of the German underground economy, according to the currency demand approach, is much lower than previously thought. However, according to Ahumada et al. (2008) and Breusch (2005b,c) even these corrected values may be faulty because several other issues remain unaddressed. To this extent, German policy and law makers have been misguided during the last three decades.

These developments not only call for revising existing estimates of the underground economy, but also for revised and new methods. Therefore, in a first attempt, we developed the MCDR approach and applied it to Germany for the period 1960 to 2008. Despite the fact that the approach suffers from a number of serious shortcomings, it must be emphasized that it does not suffer from the critique put forward by Breusch (2005a,b,c,d) or Ahumada et al. (2008, 2007). Rather, as demonstrated, the MCDR approach has some appealing aspects from a methodological perspective. Thus, the current version of the approach may at least be used as a simple plausibility test, whereas a more refined version may even have some potential to give a rough estimate of the size of the underground economy and the relative importance of its major causes.
Appendix

Data on currency in circulation outside banks (series TXI300 and printed matter ‘Monatsberichte’), sight deposits held by domestic non-banks (series OU0221) and time deposits of one year or less held by domestic non-banks (series OUA192) was collected from the Bundesbank. Data on population was collected from Statistisches Bundesamt (Statistical Yearbook). Data on household consumption (series 13496FCZF, 13496FCZW) and on the exchange rate (series 134RFZF, 163RFZF) was collected from International Financial Statistics online. The consumer price index (2000 = 100) was collected from International Financial Statistics online (series 1346DZF, 1959-1989) and the Bundesbank (series UUFA01, 1990-2008). We used the EViews 5.1 software package for our estimations.

With respect to the estimation procedure proposed by Seitz (1995) we introduced three changes. First, we used annual data because quarterly data for currency in circulation outside banks was not available for the period before 1970. Second, we have used inflation instead of the interest rate to measure the opportunity cost of cash holding. Third, with respect to cointegration we tested alternative methods and found that the Engle-Granger procedure performed best. In particular, we estimated the following error correction currency demand equation (A1):

\[
\Delta \ln(C_r)_{t-1} = 0.163 \Delta \ln(\text{PHC}_r)_{t-1} + 0.476 \Delta \ln(\text{PHC}_r)_{t-1} - 0.016 \Delta \ln(\text{INF})_{t-1} \\
+ 0.172 \Delta \ln(\text{ER})_{t-1} + 0.056 \Delta \ln(\text{ER})_{t-1} - 0.664 \Delta \ln(C_r)_{t-1} - 12.764 - 0.437 \Delta \ln(\text{PHC}_r)_{t-1} - 0.010 \Delta \ln(\text{INF})_{t-1} \\
- 0.165 \Delta \ln(\text{ER})_{t-1} - 0.132 \Delta \ln(D87)_{t-1} - 0.195 \Delta \ln(D90)_{t-1} + 0.082 \Delta \ln(D91)_{t-1} - 0.009 \Delta \ln(Trend)_{t-1} 
\]

where $C_r$ is real currency in circulation outside banks, $\text{PHC}_r$ denotes real private household consumption, $\text{INF}$ denotes inflation, $\text{ER}$ denotes the Dollar/EUR (DM) exchange rate, $D87$ is a dummy that takes the value 1 in 1987, 1988, 1989 and 0 otherwise to capture withholding tax effects (Seitz 1995, p. 11), $D90$ is a dummy that takes the value 1 from 1990 onwards and
0 otherwise to capture reunification, $D91$ is a dummy that takes 1 in 1991 and 0 otherwise to capture the reunification shock, $Trend$ is a deterministic time trend, $\Delta$ denotes first differences and $t$ denotes the time period.

Relevant $t$-statistics are given in parenthesis below the coefficients and diagnostic statistics are: $Adj. R^2 = 0.75$, standard error = 0.019521, normality $\chi^2_{Norm}(2) = 0.71\ [0.70]$, no residual serial correlation $\chi^2_{SC}(1) = 0.24\ [0.62]$, no autocorrelation in the error term $\chi^2_{ARCH}(1) = 0.25\ [0.61]$, heteroscedasticity $\chi^2_{Hetero}(1) = 7.82\ [0.73]$ and no misspecification $\chi^2_{RESET}(1) = 0.15\ [0.69]$, with p-values given in brackets. To rule out misspecification due to parameter instability, we have applied the cumulative sum of recursive residuals CUSUM (results not displayed) and the CUSUM of squares tests (see Figure 2). Both tests indicate the absence of parameter instability because the test statistics are within the 5% critical bounds.

Actual real currency $C_{rt}$ and forecasted real currency $FC_{rt}$ are displayed in logarithms in Figure 3. The consumer price index (CPI) was then used to transform $FC_{rt}$ into forecasted nominal currency $FC_{t}$, for the period 1960 to 2006. Data for the years 2007 and 2008 in $FC_{t}$ corresponds again to nominal actual currency, $C_{t}$, to avoid deviations. Hence, $FC_{t}$ data for the period 1960 to 2008 is used in equation (15) of the main text.
Figure 2: CUSUM of Squares Test

Figure 3: Actual and Forecasted Real Currency in Logarithms, Germany 1960-2006
Regarding the amount of forecasted currency in circulation outside banks, inside Germany, $INFC_t$ in equation (18) we used the following procedure. The error correction model in (A1) can be expressed in logarithms as:

$$\ln C_{rt} = \alpha_0 + \alpha_1 \ln PHC_{rt} + \alpha_2 INF_t + \alpha_3 ER_t + \epsilon_t, \quad (A2)$$

Reversing logarithms yields:

$$C_{rt} = PHC_{rt}^{\alpha_1} \cdot e^{(\alpha_0 + \alpha_2 INF_t + \alpha_3 ER_t + \epsilon_t)} \quad (A3)$$

Next, we assume that real currency held inside Germany, $INC_{rt}$, does not depend on the exchange rate $ER_t$, which yields:

$$INC_{rt} = PHC_{rt}^{\alpha_1} \cdot e^{(\alpha_0 + \alpha_2 INF_t + \epsilon_t)} \quad (A4)$$

Dividing (A3) by (A4) yields:

$$\frac{C_{rt}}{INC_{rt}} = e^{(\alpha_3 ER_t)} \quad (A5)$$

Because $C_{rt}$ and $ER_t$ are known and $\alpha_3$ can be obtained from (A1), which is 0.164820, we can calculate $INC_{rt}$ from (A5) and replacing $C_{rt}$ and $INC_{rt}$ by $FC_{rt}$ and $INFC_{rt}$ yields the relevant values for (18). Note, however, that $\alpha_3$ is obtained from an estimation covering the period 1960 to 1999 and, thus, may not be a good proxy for years after 1999. This should be taken into account with respect to the interpretation of $G3$ in Table 2.
References


