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“Grandfather rights in the market for airport slots”

by Gernot Sieg, November 2008
Grandfather rights in the market for airport slots

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Abstract

Grandfather rights are currently used in the European Union to allocate airport slots. This article shows that airports prefer such a use-it-or-lose-it rule to unconditional property rights. Assuming that there are informational asymmetries between airports and air carriers because air carriers have better information on passenger demand, the use-it-or-lose-it rule increases slot use when demand for air transport is low. Airport profits increase and those of the air carriers, together with social welfare, decrease. The profit-maximizing rule is a use-it-g < 1-or-lose-it rule.

JEL: L93,R48,D42

Keywords: Airports; Grandfather rights; use-it-or-lose-it rule; airport slots

1 Introduction

As air traffic is increasing on an almost daily basis, some airports have become congested. If an airport is congested, the right to land or take off during a well-defined time period, called a slot, becomes scarce (Jones et.al. 1993). European Council Regulation No 95/93 on the allocation of slots at Community airports defines the rules that are mandatory for coordinated airports (airports where slots are essential for use of the infrastructure). Foremost among the considerations is the fact that there are no property rights defined: neither the airport, the government, nor the air carrier owns the slot. However, there are grandfather rights: an air carrier that has used a slot in the last summer/winter period can use it in the current summer/winter period. More precisely, an alleviated use-it-or-lose-it rule holds.

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An air carrier only has to use the allocated slot 80 percent of the time to obtain the slot in the next period. Furthermore, air carriers are allowed to exchange slots. The sale of a slot is partly legal in the United States and common in the United Kingdom, but uncommon in the rest of the European Union.

History shows (Sened and Riker 1996) that after the acquisition of governmental control over take off and landing slots in high-density airports, the air-carrier industry has contrived to implement the current self-regulated regime of common-property ownership. Boyfield et.al. (2003) discuss the endowment of individual airlines with private ownership of slots and the right to sell them. If two air carriers exchange or sell slots on a so-called secondary market, the allocative efficiency might be enhanced (Starkie 1998). Because slots are scarce they are valuable. The initial definition of slot ownership, i.e. the allocation of slot property rights to airports, current users or the government, produces windfall profits, and therefore governments, airports, and incumbent airlines equally claim the right of property.

From a welfare economics point of view, pure property rights to slot use, without additional conditions such as a use-it-or-lose-it rule, might be not optimal. For example, if the property right is defined and allocated to an air carrier, there is no longer any need for grandfathering because the property of a slot implies that it can also be used in the next period. Airport-specific investment by an air carrier in a connection is more likely if there are grandfather rights or the air carrier owns the slot. However, if an air carrier owns a slot and does not use it to offer air transport, but merely holds it to deter competitor entry (Dempsey 2001), a use-it-or-lose-it rule may be a welfare-enhancing proposition.

Following the arguments by Coase (1960), all firms affected by slot allocation can privately find an efficient solution. However, the longevity of the struggle in the EU to reform Regulation 793/2004 shows that transaction costs are high. Many groups are affected (Button 2005), such as consumers, airport owners, residents and firms in the vicinity of airports, local and federal governments, and air carriers. Therefore, many different rules, such as limiting grandfather rights to a fixed period of time, returning a certain proportion of all slots, slot fees instead of landing fees, and pools for new entrants, have been proposed to improve the allocation efficiency (Boyfield et al. 2003).

The aim of this paper is to analyze the implementation of slot property rights and whether such slot ownership substitutes or complements grandfathering and the use-it-or-lose it rule. The initial step in identifying a welfare maximizing combination of rules for slot use at airports is to determine who
gains and who loses when these rules are applied. Furthermore, the identification of gaining and losing groups is a prerequisite for a public choice analysis of the implementation process. This article shows that a use-it-or-lose-it rule suits the airport through maximizing profits by reducing demand fluctuations. Airports are interested in full use of slots because there are few variable costs and, in addition to take-off and landing fees, commercial revenues increase with the number of passengers using the airport. If the airport knows the approximate demand for flights, it can charge landing and take-off fees to maximize slot use. Air transport demand increases predictably during the peak season, but there are other changes in demand that are not easy to predict (Doganis 2002, 196–200). For example, when the level of disposable income of customers changes, the level of economic activity changes or travel restrictions may arise. Demand forecasting is important for air carriers, but airports are either not able to do so or it is too expensive to forecast demand. This study assumes information asymmetry between the airports and air carriers regarding the current demand status. Take-off and landing fees are set up front for a period of at least 6 months. Therefore, the airport is not able to maximize slot use by changing landing and take-off fees according to changes in demand. The airport thus needs an alternative.

This article shows that an use-it-or-lose-it provision improves slot use. Air carriers that confront temporary decreases in demand offer more flights and attract more airport customers when the use-it-or-lose-it rules holds to avoid losing the slot. This babysitting behavior decreases demand fluctuations and transfers some of the negative effects of the drop in demand from the airport to the air carriers. Airport profits increase and those of air carriers decrease. Social welfare decreases in magnitude if a use-it-or-lose-it rule holds. However, the losses are less severe if revenues from non-aviation activities are large.

The focus of the current literature on airport pricing is the internalization of congestion as well as secondary markets but not grandfathering. The improvement in efficiency of the market for airport slots by establishing a secondary market has been analyzed by Starkie (1998), Abeyratne (2000), and Barbot (2004). Auctions may improve slot allocation (Button 2008). Boyfield et al. (2003) discuss the question of slot ownership and secondary markets in detail and provide reform options to enhance competition. They compare both gainers and losers of the current system (pp. 82–84), but do not analyze the position of airports as in the present study. The gains and losses of airports and their attitude to reform of slot property rights are essential for institutional change, as demonstrated by Riker and Sened (1991) and Sened and Riker (1996) in their analyses of the political origin
of property rights. Button (2008) analyzes the allocation of rents under different slot allocation approaches but does not analyze who gains and how social welfare is affected by grandfathering, which is addressed here.

The outline of the paper is as follows. Section 2 presents prices and quantities of slot use in the subgame perfect equilibria of the game between airport, airline and customers with and without grandfathering. In Section 3 the gainers and losers of grandfathering are identified. Section 4 analyzes social welfare and Section 5 concludes.

2 The model

For simplicity, in this model a monopolist airport sells slots to an air carrier that is a monopolistic supplier of air transport to consumers.1 Because of changes in demand that are not easy to predict (Doganis 2002, 196–200), consumer demand is stochastic and two levels of demand, \( i = l \) (low) and \( i = h \) (high), occur with probability \( 0 < w = w_h < 1 \) and \( w_l = (1 - w) \), respectively. The stochastic demand for tickets \( x_i \) is represented by:

\[
  x_i = D_i - d \cdot p_i^c,
\]

where \( 0 < d \) is the slope of the linear demand curve and \( 0 < D_l < D_h \) the ordinate intercept.

This study assumes asymmetry of information between the airport and the air carrier regarding the current demand status. In contrast to the airport, the carrier knows the type of consumer demand it faces.

Airport revenues consist of the landing fee \( p_a \) charged by the airport and exogenous non-aviation revenues \( s > 0 \) such as rent from shops in the airport or parking fees for cars.2 For simplicity, it is assumed as in Barbot (2004) that \( p_a \) is both the landing fee and the price-cost margin over operational costs (because they are assumed to be zero) and that the capital costs of the airport are fixed costs that do not influence the pricing strategy. The airport maximizes the expected profits

\[
  E\Pi = E[(p_a + s) \cdot x]
\]

1 This is a simple version of the vertical structure approach initiated by Brueckner (2002) and used by, among others, Pels and Verhoef (2004) and Zhang and Zhang (2006) and compared to the traditional approach by Basso and Zhang (2008).

2 The landing fee \( p_a \) is usually a function of frequency and tickets \( x \) and non-aviation revenues \( s \) are a function of passengers. In this model it is assumed that tickets, non-aviation revenues and landing fees share as unit a fully booked aircraft that minimizes landing fees depending on maximum take-off weight, noise (Brueckner and Girvin 2008), emissions, etc.
by charging an optimal landing fee, \( p_a \), from the airline. Because the airport
does not know whether demand is high or low, it charges \( p_a \) independently
of the status of demand.

The carrier maximizes its profit knowing the demand status. For sim-
plecticity, the only costs to the carrier to operate a flight and transport a
passenger are assumed to be the take-off and landing fees. Therefore, the
carrier demands the ticket price \( p^c_i \) from its passengers depending on the
type of demand and maximizing profits as follows:

\[
(p^c_i - c) \cdot x_i (p^c_i),
\]

where the constant total costs per flight \( c \) of the carrier correspond to the
landing fee charged by the airport, i.e., \( c = p_a \). This assumption follows the
model by Zhang and Zhang (2006) but simplifies it even more by assuming
that all the variable costs except for landings fees are zero.\(^3\) If there is exactly
only one air carrier, as in this study, no congestion externality exists, and
imposing a congestion fee cannot improve efficiency. Therefore, this study
does not assume delay costs for consumers or extra costs for airlines due to
congestion.

The timing of events is as follows. The airport has to determine the take-
off and landing fees in advance and allocate slots at the beginning of each
summer or winter period. The air carrier can decide day by day how many
slots to use and what price to charge for a ticket. Therefore, the game is an
extensive-form game and the airport is assumed to be the first mover and the
air carrier the second mover, and the prices and quantities discussed in the
following are results of subgame perfect equilibria determined by backward
induction.

### 2.1 No use-it-or-lose-it rule

In the absence of a use-it-or-lose-it rule, the slot-use ratio has no effect on
further possibilities for the use of slots. If there are property rights for
slots, the carrier that owns the slots can use them in the future even if they
are not used in the current period. The same holds true if there are no
grandfather rights at all; i.e., even after 100% use the slot is not guaranteed
for the next period. Therefore, the carrier optimizes its operations and
profits independently of future periods.

\(^3\)If the carrier decides to operate a flight (even for babysitting), optimal yield manage-
ment results in selling all tickets. Therefore, the total costs for a flight only operated to
maintain the slot are equal to the costs of all other flights considered in the model and
therefore can be assumed to be zero without changing the qualitative results of the model.
The air carrier searches for a price vector \((p_h, p_l)\) that maximizes profits

\[ \Pi_h = (p_h - c)(D_h - dp_h) \]  

in the case of high demand and

\[ \Pi_l = (p_l - c)(D_l - dp_l) \]  

when demand is low. A simple calculation shows that the optimal ticket price is

\[ p_i^* = \frac{D_i + cd}{2d} \]  

and the derived demand for tickets is then represented by

\[ x_i^* = \frac{D_i}{2} - \frac{cd}{2}. \]

In contrast to the carrier, the airport does not know whether demand is high or low. The airport rationally expects the demand situation and anticipates the price decision of the carrier and the derived demand. Profit maximization by the risk neutral airport without knowing the current demand status leads to the optimal landing fee charged to the airline:

\[ p^*_a = \frac{ED}{2d} - \frac{s}{2}, \]

where \(ED = w_h D_h + w_l D_l\). Consequently, the airport earns the expected profit

\[ \Pi = \frac{(ED + sd)^2}{8d}. \]

### 2.2 Airport profits with grandfather rights

Let \(0 \leq g \leq 1\) be the ratio of times that the carrier has to use a slot, for example 80% in the EU, to retain the slot for itself. If the optimal quantity of slots in the event of low demand is greater than \(g\) percent of the number of slots during high demand, the carrier behaves similarly to the process outlined in the previous section. However, if the optimal slot use in the event of low demand could result in the loss of some slots, i.e., if \(x_l < g \cdot x_h\), equivalently represented as

\[ g > 1 - \frac{2(D_h - D_l)}{2D_h - ED + ds}, \]
the carrier chooses to babysit the slots if it is more profitable to do this and to hold the slots rather than use fewer slots in the event of low demand and, as a consequence, lose some slots. If \( g > D_l/D_h \), then \( x_l < g \cdot x_h \), and the slot-use ratio becomes large enough to be binding.

I assume that babysitting is always more profitable than not to do so.\(^4\) This assumption holds true if it is assumed that a carrier loses all unused slots for good when it does not use the slots \( g \) percent of the time and if the lost slots are allocated to its competitors, with market entry that consequently destroys the market power of the incumbent (Dempsey 2001). Let \( x_h^* \) be the optimal quantity when demand is high. Babysitting means that the carrier has to operate \( g \cdot x_h^* \) slots in the case of low demand in order not to lose all unused slots. The optimal babysitting price for low demand sells all \( (g \cdot x_h^*) \) tickets and therefore has to satisfy the following equation:

\[
D_l - d \cdot p_l^* = g \cdot x_h^* \quad (5)
\]

and thus the optimal price is

\[
p_l^* = D_l/d - g/d \cdot D_h + g \cdot p_h^*. \quad (6)
\]

If the air carrier does not babysit, it searches for a price vector \((p_h, p_l)\) that maximizes expected profits

\[
E \Pi = w_h ((p_h - c)(D_h - dp_h)) + w_l ((p_l - c)(D_l - dp_l)), \quad (7)
\]

a problem solved in Section 2.1. However, if the air carrier does babysit, the number of high-demand slots and low-demand slots are linked (equation 5). Using more slots in the high-demand case results in more slots to babysit and a lower price (equation 6). Expected profits of the carrier thus are represented by

\[
E \Pi = w_h ((p_h - c)(D_h - dp_h)) + w_l ((p_l^* - c)(D_l - dp_l^*)), \quad (8)
\]

which can be simplified to

\[
E \Pi = (w_h p_h + w_l p_l^* g - c \tilde{w}) (D_h - dp_h) \quad (9)
\]

with \( \tilde{w} = w_h + gw_l \). The optimal high-demand price is

\[
p_h^* = \frac{(w_h + 2wg^2) D_h - w_l g D_l + cd \tilde{w}}{2d (w_h + wg^2)},
\]

\(^4\)This is the case if the discounted sum of monopoly profits is higher than the discounted sum of profits when not babysitting.
and the number of tickets sold is

\[ x^*_h = \frac{w_h D_h + w_l g D_l - cd \tilde{w}}{2 (w_h + w_l g^2)}. \]

Therefore, a babysitting air carrier uses fewer slots under high-demand circumstances and more slots in the low-demand case than an air carrier that optimizes each demand state separately.

The airport, anticipating the pricing and babysitting policy of air carriers, maximizes its profit

\[ \Pi = (p_a + s) [w_h x^*_h (p_a) + w_l x^*_l (p_a)] \]

by charging the optimal landing fee

\[ p_a = \frac{\tilde{D}}{2d \tilde{w}} - \frac{s}{2}, \]

where \( \tilde{D} = w_h D_h + w_l g D_l \). The combination of an optimal landing fee and the air carrier’s pricing policy leads to the following number of tickets sold in the high-demand case:

\[ \tilde{x}^*_h = \frac{\tilde{D} + s d \tilde{w}}{4 (w_h + w_l g^2)}. \] (10)

To summarize, the airport earns

\[ \Pi^g = (p_a + s) \tilde{x}^*_h \tilde{w} = \frac{(\tilde{D} + s d \tilde{w})^2}{8d (w_h + w_l g^2)}. \]

3 Comparison of profits

**Theorem 1** To compare profits, let the difference in profits \( \Psi \) be represented as

\[ \Psi(s) = \Pi^g - \Pi = \frac{(\tilde{D} + s d \tilde{w})^2}{8d (w_h + w_l g^2)} - \frac{(E D + s d)^2}{8d}. \]

For the function \( \Psi \), the following results hold:

1. If

\[ \frac{D_l}{D_h} < \frac{1 + g}{2} \]

and \( w_h > 1/2 \), then \( \Psi(0) > 0 \);
2. \( \lim_{s \to \infty} \Psi(s) = -\infty \); 

3. if \( D_l/D_h > g \), then \( \Psi' < 0 \); and 

4. If \( D_l/D_h < g \), then \( \Psi \) is unimodal with its maximum at 
\( s^* = \left( \frac{D_h g - D_l}{d(1-g)} \right) \).

Proof: See the Appendix.

The theorem shows that if \( s \) is not too large, then the airport prefers a use-it-or-lose-it rule. Because take-off and landing fees have to be paid only if the air carrier uses the slot, airport revenues are lower when demand is low. A use-it-or-lose-it rule leads to higher slot use in the event of low demand, but lower slot use during high demand. In this study the airport capacity is assumed to be endogenous. This assumption means that, in the long run, airport capacity equals slot use during high demand. However, different institutions (with or without use-it-or-lose-it stipulations) have different capacities. Therefore, the use-it-or-lose-it rule dampens demand fluctuations by inducing the air carrier to increase prices in the event of high demand and to decrease prices when there is low demand to a higher degree. As a consequence, the profits of the airport are proportionally higher if the use-it-or-lose-it rule holds validity.

If the use-it-or-lose-it rule holds, then the air carrier offers more tickets for a lower than optimal price in the event of low demand and fewer tickets for a higher than optimal price when demand is high. Therefore, the profits of the air carrier are lower for a use-it-or-lose-it rule compared to the situation for slot ownership or when grandfather rights do not exist at all. However, if this situation is compared to one in conjunction with grandfather rights, the air carriers prefer a use-it-\( g \)-percent-or-lose it rule to a pure grandfathering proposal because it is more profitable to use \( g < 100\% \) of the slots than to lose all unused slots.

Airports claim the ownership of slots because they provide the infrastructure that can be used during the time slot. What type of contract would airports prefer if they really owned the slots? Because airports prefer a use-it-or-lose-it rule to a system whereby the air carrier has an unconditional option, especially slot ownership by air carriers, airports should not sell the slot to air carriers but should rent it out and hold on to the use-it-or-lose-it rule; the renting contract ends if slot use is not at least \( g \) percent. The status quo, grandfathering combined with a use-it-or-lose it rule, is equivalent to renting the slot for an amount equal to zero, with a user fee if the slot is used and an option for the same contract in the next period if the slot use is great enough. Therefore, the establishment of property rights for slots and
their initial allocation to airports might not destroy the use-it-or-lose-it rule, and the resulting contract might be analogous to the status quo. Instead of paying take-off and landing fees, the air carrier has to pay a rental fee for a slot, independent of slot use.

Similar to the prediction of the model, the professional association of airport operators, the Airport Council International (ACI 2007), suggests the levy of a slot reservation fee that should be accompanied by a decrease in landing and take-off charges. If the slot is used, the reservation fee should be deducted from the airport charges. If the slot is not used, the air carrier would not be reimbursed the reservation fee.

Furthermore, if an airport owns a slot and sells it to an air carrier, the possibility arises that the carrier may not use or sell the slot, but merely hold it to prevent another carrier from offering competitive flights. If a carrier owns a slot, the carrier is sure that it can use the slot in the next period even if it is not used in the current period. This effect is not modeled here because the market structure is assumed to be fixed. However, the effect supports the claim that airports are not interested in selling slots to air carriers without additional rules for slot use.

If air carriers can define the slot-use ratio \( g \) required for further slot use, they can demand the profit-maximizing slot-use ratio from airports. Because

\[
\frac{\partial \Pi_g}{\partial g} = \frac{(D_l + ds - g(D_h + ds))w(1 - w)[\bar{D} + \bar{w}ds]}{4d(w + (1 - w)g^2)^2},
\]

the optimal slot-use ratio is represented by the equation

\[
g^* = \frac{D_l + ds}{D_h + ds},
\]

subject to the condition that \( g \) is binding. The optimal \( g \) is less than 1, and therefore the optimal rule is not pure grandfathering. The reason why \( g = 1 \) is not optimal is the endogeneity of the airport capacity, and therefore the eventual endogeneity of the number of slots in the long run analyzed here. An air carrier that maximizes profits considers that each slot used when demand is high must be babysat if demand is low. Therefore, a higher \( g \) ratio increases the proportion of low-demand versus high-demand slot use, but might decrease high-demand slot use. In the long run, optimal slot use by air carriers during high demand is equivalent to the handling capacity of the airport. Therefore, a high \( g \) ratio decreases the airport capacity and, by implication, airport profits.
4 COMPARISON OF SOCIAL WELFARE

If there are no commercial revenues, i.e., if \( s = 0 \), then the optimal slot-use ratio is \( g^* = D_l/D_h \). Because \( \partial g^*/\partial s > 0 \), higher commercial revenues lead to greater optimal slot-use ratios. The commercial profits of airports are increasingly projected as being more important. As the current model predicts, the professional association of airport operators, the ACI, suggests “strengthening of the use-it-or-lose-it-rule to 90/10” (ACI 2007).

4 Comparison of social welfare

Social welfare is defined as the sum of the profits of air carriers \( (E(p^c - p_a)x) \) and airports \( (E(p_a + s)x) \) and the consumer rent \( (E[\int_0^x D_i - q \, dq - (p^c + s)x]) \) and equals

\[
W = E[\int_0^x \frac{D_i - q}{d} \, dq] = E[\frac{2D_i x - x^2}{2d}]
\]

because slot revenues are expenditure for air carriers but revenue for airports, and ticket revenues and non-aviation revenues are expenditure for consumers. The non-aviation good is therefore assumed to be a pure by-product of passenger transport and the price \( s \) in the competitive market equals the willingness of consumers to pay.

Without grandfather rights, \( x_i = (D_i - cd)/2 \) and \( c = (ED - sd)/(2d) \). Therefore, \( x_i = (2D_i - ED + sd)/4 \).

As a result,

\[
W_o = \frac{1}{d}[w_h(D_h x_h - x_h^2/2) + (1 - w_h)(D_l x_l - x_l^2/2)]
\]

and therefore

\[
W_o = \frac{12(w_h D_h^2 + (1 - w_h) D_l^2) - 4ED(ED - ds) - (ED - ds)^2}{32d}.
\]

If the use-it-or-lose-it rule holds true, the high-demand slot use is \( \tilde{x}_h^* \), as calculated in Equation 10, and the low-demand slot use is \( g \cdot \tilde{x}_h^* \). Therefore,

\[
W_g = \frac{7D_l^2 + 6Ds\hat{w} - (sd\hat{w})^2}{32d(w_h + (1 - w_h)g^2)}.
\]

Theorem 2 If \( 0 \leq s \leq ED/d \), then a use-it-or-lose-it rule reduces social welfare. Welfare losses decrease with increasing values of \( s \), i.e., welfare losses are most severe when commercial revenues are small.
Proof: See the Appendix.

Because commercial revenue is increasingly important, losses in social welfare and therefore possible gains in efficiency predicted for abandonment of the rule decrease in theory.

If $s$ is large, i.e., $s \geq \hat{D}/(d\hat{w})$, then the main source of profits is non-aviation revenue. In this case it would be optimal for the airport to pay a subsidy by levying negative take-off or landing fees to increase traffic. However, this model assumes that congested airports charge positive fees. Therefore, the assumption $s < \hat{D}/(d\hat{w})$ is sensible and because $\hat{D}/(d\hat{w}) < ED/d$ the condition for Theorem 2 is fulfilled whenever an airport charges landing and take-off fees.

5 Conclusion

Comparison of a use-it-or-lose-it rule and an unrestricted slot-ownership plan revealed that the rule is profitable for airports but decreases carrier profits and social welfare. If airports owned well-defined property rights to slots, they would not sell such slots because it would be more profitable to substitute take-off and landing fees by a slot rent (independent of their use). Furthermore, the option to renew a rental agreement should depend on the effective use of slots: a use-it-$g < 1$-or-lose-it rule should be enforced.

The model derived above shows that arrangements that increase airport profits do not necessarily improve social welfare. Therefore, the suggestions proposed by airports to reform European Council Regulation 95/93 on common rules for the allocation of slots at Community airports may or may not enhance the efficiency of airport use.

Appendix

Proof of Theorem 1

1.

\[ \Psi(0) > 0 \]

\[ \iff \hat{D}^2 > ED^2(w_h + (1 - w_h)g^2) \]

5 If the state owns the airport, subsidies may be considered as illegal state aid in the EU.
\( \iff \]
\[
(w_h D_h + g(1 - w_h) D_l)^2 > (w_h D_h + (1 - w_h) D_l)^2 (w_h + (1 - w_h) g^2) \\
\iff \]
\[
w_h^2 D_h^2 (1 - w_h - g^2 (1 - w_h)) > (w_h D_h) 2 w_h (1 - w_h) D_h D_l (w_h + g^2 (1 - w_h) - g) + (1 - w_h)^2 D_l^2 (w_h + g^2 (1 - w_h) - g^2).
\]

Because \( D_h > D_l \) and \( w_h > 1 - w_h \), \( \Psi(0) \) is positive if
\[
(2 w_h - 1)(1 - g^2) D_h^2 > 2 D_h D_l (w_h - g + g^2 (1 - w_h))
\]
\[
\iff \]
\[
\frac{D_h}{D_l} > \frac{2(w_h - g + g^2 - w_h g^2)}{(2w_h - 1)(1 - g^2)} = 1 + \frac{1 - g}{(1 + g)(2w_h - 1)}.
\]

If \( D_l / D_h < (1 + g) / 2 \), then
\[
\frac{D_l}{D_h} < \frac{(1 + g)(2w_h - 1)}{2 + 2g - 2g} = \frac{(1 + g)(2w_h - 1)}{2(1 + g) - 2g} < \frac{(1 + g)(2w_h - 1)}{2w_h(1 + g) - 2g}
\]
and therefore
\[
\frac{D_h}{D_l} > 1 + \frac{1 - g}{(1 + g)(2w_h - 1)}.
\]

2. Statement 2 holds true because \( sd > sd \tilde{w} \).

3. Differentiating \( \Psi \) with respect to \( s \) yields
\[
\Psi' = \frac{(-1 + g)(D_l - D_h g - d(-1 + g) s)(-1 + w_h)w_h}{4g^2(-1 + w_h) - 4w_h}.
\]

Therefore,
\[
\Psi' < 0 \iff D_l - D_h g - d(-1 + g) s > 0 \iff s > \frac{D_h g - D_l}{d(1 - g)}.
\]

If \( D_l / D_h > g \), then \( D_l > D_h g \) and \( D_h g - D_l < 0 < s \). Therefore, \( \Psi' < 0 \) and statement 3 holds true.
Proof of Theorem 2

To compare the welfare, let

$$\Phi(s) = W_g - W_o.$$ 

Assume that $D_l/D_h < g$. For the function $\Phi$, the following results hold:

1. $\Phi(0) < 0$;
2. $\Phi'(0) > 0$;
3. $\lim_{s \to \infty} \Phi(s) = \infty$; and
4. $\Phi(ED/d) < 0$.

Proof:

1. $\Phi(0) < 0$ if $W_g < W_o$, and therefore, if and only if

$$12(w_hD_h^2 + (1 - w_h)D_l^2) - 4ED^2 - ED^2(w_h + (1 - w_h)g^2) > 7\tilde{D}^2$$

and only if

$$12(w_hD_h^2 + (1 - w_h)D_l^2) - 5ED^2(w_h + (1 - w_h)g^2) >$$

$$7w_h^2D_h^2 + 14w_h(1 - w_h)gD_hD_l + 7g^2(1 - w_h)^2D_l^2,$$

which is equivalent to

$$2D_hD_l(7g + 5g^2(1-w)+5w) < D_h^2(5w+g^2(12-5w))+D_l^2(7+5(1-w)g^2+5w)$$

$$\iff$$

$$D_hD_l(7g + 5g^2(1-w)+5w) + D_hD_l(7g + 5g^2(1-w)+5w) <$$

$$D_l^2(7g+5g^2(1-w)+5w+7(1-g)) + D_h^2(7g+5g^2(1-w)+5w+7g^2-7g)$$

$$\iff$$

$$D_l(7g+5g^2(1-w)+5w)(D_h-D_l)+D_h(7g+5g^2(1-w)+5w)(D_l-D_h) <$$

$$D_l^2(1-g) + D_h^2(7g-1)$$

$$\iff$$

$$(D_h - D_l)(D_l - D_h)(7g + 5g^2(1-w)+5w) < 7(1-g)(D_l^2 - gD_h^2)$$


\[ (D_h - D_l)^2(7g + 5g^2(1 - w) + 5w) > 7(1 - g)(gD_h^2 - D_l^2). \]

Because \( D_l/D_h < g \iff gD_h > D_l \), it follows that \( D_h - D_l > D_h - gD_h = D_h(1 - g) \), and therefore \( W_g < W_o \) if

\[ D_h^2(1 - g)^2(7g + 5g^2(1 - w) + 5w) > 7(1 - g)(gD_h^2 - D_l^2) \]

\[ \iff \]

\[ D_h^2(1 - g)(7g + 5g^2(1 - w) + 5w) > 7gD_h^2 - 7D_l^2 \]

\[ \iff \]

\[ D_h^2(1 - g)(7g + 5w(1 - g^2) + 5g^2) > 7gD_h^2 - 7D_l^2 \]

\[ \iff \]

\[ D_h^2(-7g + 7g - 7g^2 + (1 - g)(5w(1 - g^2)) + 5g^2 - 5g^3) + 7D_l^2 > 0 \]

\[ \iff \]

\[ D_h^2(-2g^2 - 5g^3 + (1 - g)(5w(1 - g^2))) + 7D_l^2 > 0. \]

Because \( D_h > D_l/g \), it follows that \( D_h^2 > D_l^2/g^2 \), and therefore \( W_g < W_o \) if

\[ \frac{D_l^2}{g^2}(-2g^2 - 5g^3 + (1 - g)(5w(1 - g^2))) + 7D_l^2 > 0 \]

\[ \iff \]

\[ D_l^2(7 - 2 - 5g + \frac{(1 - g)}{g^2}(5w(1 - g^2))) > 0, \]

which is true.

2.

\[ \Phi'(s) = \frac{3\hat{w}^2 - 3ED(w + (1 - w)g^2)}{16(w + (1 - w)g^2)} = \frac{3(1 - g)(gD_h - D_l)(1 - w)w}{16(w + (1 - w)g^2)} > 0. \]

3. Because \( \hat{w}^2 - (w + (1 - w)g^2) = -(1 - g)^2(1 - w)w < 0 \),

\[ \lim_{s \to \infty} \Phi'(s) = -\frac{\hat{w}^2 - (w + (1 - w)g^2)}{16(w + (1 - w)g^2)} > 0. \]
4. For all $0 \leq s < 3ED/d$, the functions $W_g(s)$ and $W_o(s)$ are monotonically increasing. Therefore,

$$\Psi(ED/d) = W_g(ED/d) - W_o(ED/d) < W_g(\tilde{D}/(d\tilde{w}))) - W_o(ED/d)$$

$$= \frac{12\tilde{D}^2}{32d(w + (1-w)g^2)} - \frac{12(wD_h^2 + (1-w)D_l^2)}{32d} < 0$$

if and only if

$$\tilde{D}^2 - (wD_h^2 + (1-w)D_l^2)(w + (1-w)g^2) = -(gD_h - D_l)^2(1-w)w < 0,$$

which is true.

From the above statements 1–4 it follows that if $0 \leq s < ED/d$, a use-it-or-lose-it rule reduces social welfare.

References


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5 CONCLUSION


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