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## Optimal consumption and taxation of housing: A life cycle approach

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**Optimal Consumption and Taxation of Housing -  
A Life Cycle Approach**

**CAWM-Discussion Paper No 3**

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**by**

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**Abstract**

*A simple OLG-model is developed, where housing is the only consumption good and saving is either in terms of financial assets or in terms of owner-occupied or rented dwellings. It is shown that, while without taxes optimal consumption plans and maximum utility are the same for both the tenant and the owner-occupier, conventional income taxes cause a bias in favour of the latter. It is argued that the consumption good approach is always advantageous for the owner-occupier in comparison with the investment approach, even with high interest rates and a high share of borrowing. However, even the investment approach does not entirely remove the taxation bias on the expense of the tenant, unless the latter is allowed to deduct total interest payments from his income. An alternative could be cash-flow taxation. It is also shown that the conventional investment approach is equivalent to - and therefore could be replaced by - a much simpler version, where instead of imputed rent imputed interest on net capital is taxed.*

## 1. Introduction

With the challenge of demographic change, the pros and cons of property ownership as compared to other forms of retirement provision have become a most relevant issue. In Germany, as in many other countries, most people appraise owner-occupied accommodation as their favourite mode of preparing for the age. Some authors claim, however, that this is mostly due to psychology. According to the critics, a hard-headed economic assessment would regularly result in favour of renting rather than buying ones domicile. Others point to the different taxation of rented against owner-occupied residences. In particular, while financial interest payments are taxed regularly, the implicit interest of housing proprietary - resulting from the saved rental charges - is left tax-free in most countries. On the other hand, there use to be also both subsidies and tax relief in the tenement market, which make the balance of advantages at least vague.

This paper aims to shed some more light on the essential question whether or not owner-occupied dwellings are generally advantageous compared with other capital investments. “General advantage” is here not meant in terms of risk, fungibility or liquidity, but refers solely to profitability within a perfect capital market. For example, the building societies use to promote their loan contracts by the slogan “property ownership is the only retirement provision which you can live in”. This raises the question, however, if a loan-funded house for the sake of owner-occupation were not better considered as a pre-drawing of consumption rather than saving. Apparently, issues like this should be clarified in order to design a proper housing taxation scheme.

There is a broad literature on that issue, and also a broad international variety of taxing schemes (van der Hoek/Radloff 2007). Many authors like e.g. Goode (1960, p. 526) state that

the exclusion from taxable income of the imputed net rent of owner-occupied dwellings discriminate against tenants, in particular if mortgage interest is deducted. A frequently offered solution to the problem is the taxation of the imputed rent (Merz 1965, p. 255; Aaron 1970, p.803). Many contributions on the subject rest, however, on mere partial analysis or heuristic arguments. Others are chiefly interested in empirical results, frequently lacking a sophisticated theoretical base. Examinations of owner-occupied vs. rented housing within a more general framework of optimal consumption choice are rare. The present paper aims to broaden the scope into this direction.

In particular, we examine an OLG-model in the tradition of Samuelson(1958), Diamond (1965), Kotlikoff (2006) and at latest Conesa/Kitao/Krueger (2007), where the individuals seek to optimise their lifetime consumption pattern by saving and lending. They do this either in terms of financial resources or by investing and disinvesting in proprietary. The model is examined with and without taxes. Concerning the latter, different taxation schemes are considered to affect house-allocation as little as possible.

## **2. A Simple OLG-Model of Housing**

We assume all individuals living for three periods and having the same incomes and tastes. Let  $\mathbf{w} = (w_1; w_2; w_3)$  be the vector of wage incomes and  $\mathbf{c} = (c_1; c_2; c_3)$  the vector of housing consumption in three subsequent phases of lifetime respectively, where 1 stands for the first third of life, 2 for the middle phase and 3 for the retirement. With all accommodations having the same quality, housing consumption  $c$  can simply be measured by the utilized square meters. Individuals have identical, well behaved temporal utility functions  $U(c_1; c_2; c_3)$ , which they seek to maximize. It is supposed that they are purely self-interested and, hence, that there are no heritages.

To facilitate matters, we disregard all consumption goods other than housing. Houses and apartments are supposed to be durable goods, and at the current stage of analysis we also do not take account of depreciations. Housing consumption can be accomplished by either renting or buying. It is also possible to hire out part of ones proprietary. Moreover, the individuals can borrow from and lend money to a perfect capital market at an interest rate  $i$ , with  $q = (1+i)$  denoting the interest factor. Note, however, that this does not apply to the last section of life, because individuals will no more be alive in the following period and hence will neither save nor obtain any credit. Hence, in order to maximize their utility, they must sell their - indelible - proprietary in period 2 at the latest, in order to rent an appropriate apartment in period 3, maybe in a nursing home .

## 2.1. The Tenant`s Case

As a benchmark for our analysis, we take the utility which a tenant can achieve by realizing his optimal temporal consumption plan. Let  $\mathbf{s} = (s_1; s_2; 0)$  be the vector of his net savings in terms of money. For simplicity, we normalize the price of *buying* one square meter of accommodation to unity. Then, with  $r$  denoting the rental charge as a percentage of the accommodation`s value, the tenant faces the following set of budget constraints:

$$(1) \quad rc_1 = w_1 - s_1$$

$$(2) \quad rc_2 = w_2 + s_1q - s_2$$

$$(3) \quad rc_3 = w_3 + s_2q$$

We generalize Samuelson`s original utility function  $U(\mathbf{c}) = c_1c_2c_3$  as follows:

$$(4) \quad U(\mathbf{c}) = \prod_{j=1}^3 c_j^{\alpha_j}$$

where the  $\alpha_j$  can be interpreted as consumption weights of the respective periods.<sup>1</sup> In the Samuelson case with  $\alpha_j = 1 \forall j$ , there is no explicit time preference, and hence the optimisation problem results only from diminishing marginal utility of consumption in the respective periods. Maximizing (4) with respect to (1) to (3), after some manipulation of terms, yields:

$$(5) \quad c_1 = \frac{\alpha_1}{rq^2} \left( \frac{q^2 w_1 + qw_2 + w_3}{\alpha_1 + \alpha_2 + \alpha_3} \right)$$

$$(6) \quad c_2 = \frac{\alpha_2}{rq} \left( \frac{q^2 w_1 + qw_2 + w_3}{\alpha_1 + \alpha_2 + \alpha_3} \right)$$

$$(7) \quad c_3 = \frac{\alpha_3}{r} \left( \frac{q^2 w_1 + qw_2 + w_3}{\alpha_1 + \alpha_2 + \alpha_3} \right)$$

Since the terms in brackets are the same in every period  $j$ , housing consumption rises at the factor  $q\alpha_j / \alpha_{j-1}$ , if we assume the rental charge to be constant. In the special case where  $\alpha_j = 1 \forall j$ , optimal housing consumption rises simply at the interest rate  $i$ .

## 2.2. The Owner-Occupier Case

If the dwelling is occupied by the owner himself, things are slightly more complicated. Due to the durability of houses, if he buys  $e_1$  and  $e_2$  square meters in period 1 and 2 respectively, his consumption is  $c_1 = e_1$  in period 1 and  $c_2 = e_1 + e_2$  in period 2. Selling all his proprietary in period 3, he will attain an extra revenue of  $e_1 + e_2 = c_2$  in that period. His expenses for consumption in period 3 – now as a tenant – are  $rc_3$ . Hence he faces the following set of temporal budget constraints:

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<sup>1</sup> By rewriting utility function (4) as  $\ln U = \alpha_1 \ln c_1 + \alpha_2 \ln c_2 + \alpha_3 \ln c_3$  it follows that time preference is positive if  $\alpha_j < \alpha_{j-1}$  and vice versa.

$$(8) \quad e_1 = c_1 = w_1 - s_1$$

$$(9) \quad e_2 = c_2 - c_1 = w_2 + s_1q - s_2$$

$$(10) \quad rc_3 = w_3 + s_2q + c_2$$

In equilibrium, the investment in a tenement must exactly yield the market interest rate  $i$ .

Hence, if the investment  $e$  and the first rental yield  $re$  both accrue in the first period,<sup>2</sup> and if the propriety is sold in period 3, the equilibrium relation of the rental charge and the respective propriety price is given by

$$(11) \quad 0 = -e + re + \frac{re}{q} + \frac{e}{q^2}$$

$$\Rightarrow r = 1 - \frac{1}{q} = \frac{i}{1+i}$$

With this inserted into (10), the maximization of utility function (4) with respect to restrictions (8) to (10) yields exactly the same solution for  $\mathbf{c} = (c_1; c_2; c_3)$  as in the tenant's case (see equations 5 to 7).

A first conclusion from the model is, therefore, that owner occupied accommodation does not have a general advantage or disadvantage compared with renting ones home. Starting with the same set of temporal incomes and tastes, both the owner and the tenant realize the same level of utility, consuming exactly the same quantities of accommodation in every period of their lifetime. The only difference is, that the tenant saves by the way of financial investments while the house owner saves by investing in his proprietary. Therefore, it is also false to generally attribute a higher rate of time preference to the tenant as compared to the home owner. Accordingly, at least in a world with perfect capital markets and without taxes, their different decisions on retirement provision cannot be explained that way.

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<sup>2</sup> This assumption saves the symmetry of the landlord case to the owner-occupied accommodation case. It implies that houses need not be built in a time consuming process but can be used instantly after the acquisition.

The following numerical example might be helpful. Assume that  $\alpha_j = (0.9;1.0;1.1)$ , with the interest factor  $q = 2$  and hence, according to (11),  $r = 0.5$ . Then, if the vector of wages is  $\mathbf{w} = (1;3;2)$ , both the tenant and the owner chose the same temporal consumption pattern  $\mathbf{c}$ , thereby realizing the same utility level  $U$  (see table I). Their set of financial saving  $\mathbf{s}$ , however, is different. While the tenant's optimal consumption plan implies positive amount of saving in periods 1 and 2, the owner takes a debt in both periods. Yet their overall consumption-pattern is lastly identical, because of the owner's additional housing investment  $\mathbf{e} = (1.8;2.2;-4)$ .<sup>3</sup>

The owner-occupier could also chose to let a part of his proprietary, as is shown in the last column of table I. In fact, with a rental charge according to (11), he will be indifferent concerning any additional investment volume in terms of tenement. He could, for instance, extend his borrowing to  $\mathbf{s} = (-1.3;-1.5;0)$ , thereby expanding his real estate transactions to  $\mathbf{e} = (2.8;2.6;-4,5)$ . This would bring him an additional rental revenue of  $\mathbf{re} = (0.5;0.7;0)$ , but leaves his total utility as well as his consumption pattern unaffected. The same would apply for the tenant, if he should chose to take a similar – or some other – investment in a tenement himself.<sup>4</sup>

Hence, with the assumptions made above, there is no bias in the housing market to whatever mode of supplying for ones age. Accordingly, a taxation scheme on housing should ideally preserve this symmetry of allocation. In the sequel, it will be shown how this could be achieved.

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<sup>3</sup> Note that, in the first period, the tenant needs less income to realize the same consumption level as the owner, because he must only pay  $re_1$  instead of  $e_1$  for  $c_1$ . Therefore, in spite of the identical  $\mathbf{c}$ , the sum of  $\mathbf{s}$  and  $\mathbf{e}$  is not identical for them in the respective periods.

<sup>4</sup> The same applies for any financial investment which yields the market interest rate.



**Table I: Optimal Consumption Pattern without Taxes**

	Tenant	Owner-occupier	Owner-occupier, renting part of his proprietary
consumption $c_1$	1.800	1.800	1.800
consumption $c_2$	4.000	4.000	4.000
consumption $c_3$	8.800	8.800	8.800
savings $s_1$	0.1000	-0.8000	-1.3000
savings $s_2$	1.2000	-0.8000	-1.5000
savings $s_3$	0	0	0
Utility level U	74.2568	74.2568	74.2568

### 3. Efficient Housing Taxation

Introducing taxation into the model, we neglect wage taxes, by simply assuming that  $w$  already denotes net wage incomes. Interest income is assumed to be taxed by a proportional income tax  $t$  which is the same in all periods. According to taxing schemes in many countries, private interest payments on debt are supposed to be irrelevant for taxation, however. It is assumed that rental receipts are taxed by the same rate as interest income. This implies that the relation of rental charge and the value of a dwelling must rise from  $r$  to  $r_t$ , according to

$$(1+i)0 = -e + r_t e(1-t) + \frac{r_t e(1-t)}{1+i(1-t)} + \frac{e}{[1+i(1-t)]^2}$$

$$\Rightarrow r_t = \frac{i}{1+i(1-t)}$$

This is of particular relevance for the tenant, who has to bear a double tax-burden: Not only a part of his interest receipts is taxed away, but he also faces a higher rental charge for a given amount of housing consumption. Accordingly, his set of restrictions changes as follows:

$$(1i) r_t c_1 = w_1 - s_1 - T_1$$

$$(2i) r_t c_2 = w_2 + s_1 q - s_2 - T_2$$

$$(3i) r_t c_3 = w_3 + s_2 q - T_3$$

with

$$(12) T_j = s_{j-1} i t \quad \text{if } s_{j-1} > 0$$

$$T_j = 0 \quad \text{if } s_{j-1} \leq 0$$

If, for example, the tax rate is  $t = 0.4$ , the relative rental charge rises from  $r = 0.5$  to  $r_t = 0.625$ .

The tenant's optimal consumption pattern then changes to  $c = (1.6000; 3.2381; 5.6991)$  in the example from table I. This is due to both the increase from  $r$  to  $r_t$  and his tax payments  $T = (0; 0; 0.3905)$ , which leave him with a utility of merely 33.5259 instead of formerly 74.2568 units. Moreover, unlike in the model without taxes, it is no longer true that consumption rises by the factor  $q^* \alpha_j / \alpha_{j-1}$ .<sup>5</sup>

The corresponding effects on an owner-occupier's position depend on the particular taxation scheme which is applied to him. In the sequel, we analyse the pure consumer-good approach as compared to the pure investment approach, leaving out the various intermediate solutions to be adopted in reality.

### 3.1. The Consumer Good Approach

If living in one's own house is viewed as mere consumption, it appears natural to leave it tax-free. From that it follows, unlike the landlord's case, that also interest on debt should not be allowed for in determining the tax payment. Formally, the resulting change in the budget constraints for the owner-occupier is equivalent to the tenant's case (see equations 1i to 3i):

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<sup>5</sup> In the special case where all  $\alpha = 1$  consumption rises by  $q$  in period 2 and by the *net* interest factor  $1+i(1-t)$  in period 3.

$$(8i) \quad e_1 = c_1 = w_1 - s_1 - T_1$$

$$(9i) \quad e_2 = c_2 - c_1 = w_2 + s_1q - s_2 - T_2$$

$$(10i) \quad r_t c_3 = w_3 + s_2q + c_2 - T_3$$

with

$$(12i) \quad T_j = s_{j-1}it \quad \text{if } s_{j-1} > 0$$

$$T_j = 0 \quad \text{if } s_{j-1} \leq 0$$

Economically, however, the owner-occupier is in tendency better off than the tenant with this taxation scheme (see table II below). Although he is also negatively affected by the rising rental fee in period 3, he can still achieve a higher level of both temporal consumption and maximum utility as compared with the tenant.<sup>6</sup> As a matter of fact, the owner-occupier does not pay taxes at all, because his returns from saving are only in tax-free terms of accommodation use instead of financial interest receipts. This advantage is the higher, the more he would have to save as a tenant in order to optimise his temporal consumption pattern.

This is usually viewed as a discrimination of financial investments compared with investing in an owner-occupied dwelling. Another - and possibly even more relevant - misallocation is, however, the implicit discrimination of renting compared with living in ones own house. For, while there is principally the option for both the tenant and the owner-occupier for additional financial investments, one cannot be a tenant and an owner-occupier at the same time. Hence, this “either-or”-decision should not at all be affected by the taxing scheme.

### 3.2. The Investment Approach

According to the so called investment approach, the imputed rent of an owner-occupied dwelling should be taxed equally to financial interest receipts. The usual proposal is for the taxation of the – hypothetical - rent which the owner could achieve by letting his

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<sup>6</sup> The results of this and the following maximization problems were mainly achieved by numerical methods. More general proofs can hopefully be provided at the conference.

accommodation, minus necessary expenses of ownership, in particular mortgage debt and depreciation ( Goode 1960, p. 505). Neglecting depreciation and all repair and maintenance costs, and also leaving out all complications of assessment, the restrictions (8i) to (10i) would be unaltered, but the taxing formula (12i) would change to

$$(12ii) T_j = t(r_j e_j + s_{j-1} i)$$

At first glance, this approach appears rather reasonable. But, apparently, there are still fundamental differences to the taxation of a tenant. Firstly, in contrast to the tenant, the owner-occupier pays a tax already in period 1. Secondly, his tax payment  $T_3$  is negative, because no further rental receipts but only interest payments occur in that period. While these two effects exactly outweigh each other and hence lastly cancel out, this does not apply to the third and, therefore, crucial difference. For, in contrast to the tenant, the owner-occupier can deduct *all* his interest payments for housing from his taxable income, while the tenant must pay his private interest payments from fully taxed income, although it is used solely for accommodation too. This gives the owner-occupier an advantage in comparison with the tenant, even if the investment approach is realized. In our example from above, for instance, the owner-occupier realizes still both a consumption bundle  $\mathbf{c}$  and a utility level  $U$  which is slightly above the maximum utility which the tenant can achieve (see table II).

Only if the initial vectors  $\mathbf{w}$  and  $\alpha \equiv (\alpha_1; \alpha_2; \alpha_3)$  are such that no debt must be taken by the tenant for realizing his optimal consumption bundle  $\mathbf{c}$ , the advantage of the owner-occupier disappears and, hence, the investment approach seems appropriate. For example, with  $\mathbf{w} = (3; 3; 2)$  and  $\alpha \equiv (0.9; 1.0; 1.1)$  both the renter and the owner-occupier would realize a consumption bundle  $\mathbf{c} = (2.72; 4.83; 8.49)$  and a utility level  $U = 124.77$ .

**Table II: Optimal Consumption Pattern with different Taxing Schemes**

	Tenant	Owner-occupier, Consumption good approach	Owner-occupier, investment approach (imputed rent)	Owner-occupier, investment approach (imputed interest)
consumption $c_1$	1.6000	1.8000	1.7550	1.7550
consumption $c_2$	3.2381	4.0000	3.1200	3.1200
consumption $c_3$	5.6991	7.0400	5.4910	5.4910
savings $s_1$	0.0000	-0.8000	-1.1937	-0.0969
savings $s_2$	0.9762	-0.8000	-1.0550	0.8949
savings $s_3$	0	0	0	0
Utility level U	33.5259	58.0945	33.7006	33.7006

The same results as with taxing imputed rent can be achieved by an alternative investment approach, where, instead of a hypothetical rent, an imputed interest receipt from net capital is taxed in the respective period. The budget constraints are still the same as (8i) to (10i), but now the taxing-formula changes to

$$(12iii) \quad T_j = ti(k_{j-1} + e_j + s_j - s_{j-1})$$

where  $k$  denotes net capital, which is initially zero and rises by  $e_j$  plus the respective net financial saving  $s_j - s_{j-1}$  in each period  $j$ . Note that no imputed rent must be assessed in this version of the investment approach. With this taxing scheme and the example from above with  $t = 0.4$ ,  $q = 2$ ,  $\mathbf{w} = (1;3;2)$  and  $\alpha \equiv (0.9;1.0;1.1)$ , the owner-occupier realizes the same consumption bundle  $\mathbf{c}$  and the same utility level  $U$  as with the imputed rent approach (see the last column in table II). There remains, of course, also the same advantage compared to the tenant.

Only from this bias in taxation it follows, that individuals with a strong preference for present consumption and a relatively high income in their early periods of life will prefer to buy rather than renting their accommodation, while the others will be indifferent. For only the former will have the need to borrow and hence are better off as an owner-occupier, then

being allowed to deduct total interest payments. Due to this effect, there is also a bias towards investing in owner-occupied dwellings on the account of other, economically possibly more efficient investments.

It is often argued that the investment approach is advantageous for the owner-occupier as compared to the consumer good approach, if interest rates are high and preferences are such that high debt must be taken to optimise lifetime consumption. However, in the light of our analysis, this statement appears to be wrong. Rather the consumption good approach is definitely the favourable taxation scheme from the owner's point of view. This can immediately be seen from the imputed interest version of the investment approach given above. Because the internal interest rate of the owner-occupied dwelling must be equal to the market interest rate, from that approach it follows that the tax burden for the owner-occupier is the higher, the higher is the interest rate.<sup>7</sup>

### **3.3. Solution I: Deduction of Interest Payments also for the Tenant**

A very simple solution of the problem would be to abandon any taxation of interest, which would immediately remove the bias in favour of owner-occupied vs. rented accommodation. Although this solution is sometimes supposed actually, we do not pursue it here in more detail. Implying both a massive breakdown of tax revenues and a violation of fundamental principles of justness in taxation, it appears to be far away from real political options.

A much easier and more realistic approach would be to adopt the investment approach to the owner-occupier and to make interest payments completely deductive also for the tenant. This would put him on par with the owner-occupier even if his optimal saving plan implies a pre-

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<sup>7</sup> Note that also the imputed rent is positively related to the interest rate because of equation (11).

drawing of consumption in any period. The only change in his budget restrictions which one would have to make is to replace (12i) by

$$(12i) T_j = s_{j-1}it$$

which is symmetrical on positive and negative interest receipts. For example, with  $q = 2$ ,  $t = 0.4$ ,  $w = (1;3;2)$  and  $\alpha \equiv (0.9;1.0;1.1)$ , we achieve the optimal consumption plans shown in table III, all of which yielding the same level of utility.

**Table III: Optimum consumption plans with solution I**

	Tenant, with interest Payments fully deductive	Owner-occupier, Investment approach with taxing imputed rent	Owner-occupier, Investment approach with taxing imputed interest receipts
consumption $c_1$	1.7550	1.7550	1.7550
consumption $c_2$	3.1201	3.1201	3.1201
consumption $c_3$	5.4910	5.4910	5.4910
savings $s_1$	-0.0969	-1.1937	-0.7550
savings $s_2$	0.8949	-1.0550	-0.2750
savings $s_3$	0	0	0
Tax payment $T_1$	0	0.4387	0
Tax payment $T_2$	-0.0388	0.3025	0.4000
Tax payment $T_3$	0.3580	-0.4220	1.1380
Utility level $U$	33.7006	33.7006	33.7006

One might object that it could be difficult to separate hypothecary from other credit in practice. On the other hand, it is not at all clear that this would really be sensible. Why not pre-draw consumption by purchasing a car instead of a dwelling? In the light of our analysis, the only thing that counts is to treat the tenant's interest payments the same as the owner-occupier's. Moreover, even in the latter's case, it lastly cannot be controlled for which purpose his credits are really used. Hence, this solution appears to be both elegant and reasonable also from a pragmatic point of view.

### 3.4. Solution II: Cash Flow Taxation

Another option is a cash-flow tax, making all outflows deductible and all inflows taxable for both financial and real investments. As is well known, a cash-flow tax leaves the after-tax-rate of return and, hence, also the interest rate unaffected (see e.g. Katz 1999, p.5). Therefore, condition (11) can now be written as

$$(11ii) \quad 0 = (-e + r_t e)(1-t) + \frac{r_t e(1-t)}{q} + \frac{e(1-t)}{q^2}$$

$$\Rightarrow r_t = 1 - \frac{1}{q} = \frac{i}{1+i}$$

For the tenant, a cash flow tax on his non-wage-income would mean that the taxation formula in his set of restrictions ((1i) to (3i)) must be rewritten as follows:

$$(12iv) \quad T_j = t((1+i)s_{j-1} - s_j)$$

For the owner-occupier the set of restrictions is still (8ii) to (10ii), but instead of (12iii) the new taxation formula adopted to him is also (12iv). In other words, both the tenant and the owner-occupier pay taxes for every inflow of interest or reflux from former savings, while they get a tax credit for every interest payment or new financial saving. Note that, other than in the investment approaches discussed above, both house purchases  $e$  and imputed rent  $re$  are no longer relevant for taxation. With the cash-flow taxing scheme and the example from the last section, we get the consumption patterns shown in table IV:

Not surprisingly, total utility is higher than with solution I, because a cash-flow tax lastly means that financial receipts are not taxed at all (the present value of taxes being zero). For this reason, a change towards a cash-flow tax would at best be achievable in small steps, maybe starting with the housing sector. However, from a theoretical point of view, this solution is equally efficient as the one discussed above.



**Table IV: Optimum consumption plans with solution II**

	Tenant, with cash-flow tax on financial transactions	Owner-occupier, with cash-flow tax on financial transactions	Owner-occupier, without taxes on financial receipts at all
consumption $c_1$	1.8000	1.8000	1.8000
consumption $c_2$	4.0000	4.0000	4.0000
consumption $c_3$	8.8000	8.8000	8.8000
savings $s_1$	0.1667	-1.3333	-0.8000
savings $s_2$	2.0000	-1.3333	-0.8000
savings $s_3$	0	0	0
Tax payment $T_1$	-0.0667	0.5333	0
Tax payment $T_2$	-0.0667	-0.5333	0
Tax payment $T_3$	1.6000	-1.0667	0
Utility level U	74.2569	74.2569	74.2569

#### 4. Concluding remarks

Within the limits of our assumptions, it has been shown that

- without taxes, there is no difference in either the optimal consumption pattern or the utility level which can be achieved concerning the rent or buy decision in housing;
- with a proportional income tax on wages and interest receipts, there accrues a bias in favour of owner-occupied dwellings as compared to renting;
- this bias is much greater in case of the consumption good approach, but it also exists with the investment approach;
- the consumption good approach is definitely more advantageous for the owner-occupier than is the investment approach, if the optimal consumption pattern is chosen;
- concerning the investment approach, taxing the imputed interest on net capital in each period would be a perfect equivalent to the conventional tax on both actual interest receipts and imputed rent;

- equal taxation of the tenant and the owner-occupier could be accomplished by either imposing a cash flow tax, or by applying the investment approach to the owner-occupier and making interest payments completely deductible also for the tenant.

These results might be helpful in designing a real world taxing scheme on housing. However, the limits of our analysis must be regarded. In particular, yet no allowance was made for depreciation, maintenance and repair. We also did not allow for inflation, and the analysis was confined to perfect capital markets and to a proportional tax. On the other hand, the chosen theoretical approach is quite general, integrating the problem in the broader context of optimal lifetime consumption choice.

From a more general point of view, it could be doubted that equal taxing of owner-occupied dwellings and financial investments is desirable at all. There are many examples where the principle of equal taxing is violated and yet no one cares. For instance, the services which a housewife provides for her husband are left tax-free, whereas the same services, being bought on the market, have to bear a tax burden. Although this might lead to substantial misallocations from a pure economic point of view, no one would think on taxing private marriage duties. Other examples are the use of private cars in comparison with taking a taxi, providing for ones own garden instead of hiring a gardener or do it yourself rather than employing a craftsman. Therefore, it could reasonably be argued that the state should on principle not interfere into private self-services, which would also let the consumption good approach appear in a different light.

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