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# Dynamic Efficiency and Reswitching

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by

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## ***Abstract***

*The paper argues that, from a dynamic efficiency perspective, intersections of factor price frontiers are irrelevant to the choice of techniques. Because every change in technique involves a temporary loss or gain in both profit and per capita consumption within the transition period, its profitability should be calculated by applying the present value criterion to the entire change process. With only one transition period, there is generally a unique interest rate at which the change in technique breaks even. This critical interest rate is generally the same for a profit maximizing firm as for a central planner who seeks to maximize consumption per unit of work. This critical interest rate does not generally coincide with either of the interest rates at which the factor price frontiers intersect. Therefore, common proofs of the so-called reswitching phenomenon do not stand up well from a dynamic efficiency perspective.*

JEL-Classification: B16, B5, D2, D5, D9, E1, E4

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## 1. Introduction

The Cambridge capital controversies are now mostly regarded as merely of historical interest, although some of its central points of debate have never really been clarified (Harcourt and Cohen 2003, Hagemann 1997, Kurz and Salvadori 1995). In particular, the so-called reswitching phenomenon remains a challenge. In sharp contrast to the neoclassical paradigm, there is no monotonic relationship between the interest rate and the choice of techniques according to the reswitching paradoxon (Harcourt 1969, 371pp, 386pp, Bruno, Burmeister and Sheshinski 1966, Bliss 1975). This choice is generally derived from a comparison of the factor price frontiers which are related to them, where reswitching is associated with a double or even a multiple intersection of these curves (see e.g. Hagemann and Kurz, 1976, 698). Being frequently downplayed by neoclassical authors as a mere theoretical curiosity of little practical importance, the reswitching phenomenon has recently been proved to have some empirical relevance after all (Han and Schefold 2006).

There is a certain relation between reswitching and the “Austrian” capital theory, where a production technique is viewed as a time consuming process of investment and subsequent returns, as has frequently been stressed before (Bruno, Burmeister and Sheshinski 1966, 545, Solow 1967 and 1970, Hicks 1973, Nuti 1973). Similar to the reswitching phenomenon, there is also a problem with multiple equilibrium interest rates in an ordinary investment project, if the periodic payments and receipts repeatedly alter in sign. However, according to the truncation theorem, the problem can generally be solved by truncating the respective process when the present value reaches its maximum (Arrow and Levhari 1969, Flemming and Wright 1971). Therefore, if a production technique (or the stream of net receipts from changing the production technique) is viewed as a time consuming investment project, there should generally be a monotone relation between the interest rate and the present value of that investment project.

Solow’s concept of the social rate of return (Solow 1967) can be viewed as a “capital theory without capital”, where there is no problem of multiple interest rates. Neither is there any ambiguity in the measurement of capital, because the concept of aggregate capital is simply abandoned in favour of a pure theory of interest (Solow 1970, 424, Hagemann 1997, 149). By adopting the present value calculation and including the respective transition periods, Solow showed that, under certain conditions, the switch from one technique to another breaks even

at an interest rate  $r$  which is the same as the interest rate where the factor price frontiers of the two techniques intersect (Solow 1967, 33).

While Pasinetti's early verdict that Solow's theorem was "tautological" has been rightly rejected, even by other Neo-Ricardian writers (Hagemann 1997, 155), some of his more specific objections were widely accepted. In particular, it was thrown into doubt that the theorem still holds in more general cases than the single "malleable" commodity model. Hagemann (1997, 155) showed that, with heterogeneous goods, it is not ensured that all factors can be fully employed within the transition process. Moreover, whether or not a competitive market economy would also ensure the optimal technique choice remained an open question, because Solow confined his analysis to the maximum present value of per capita consumption and, hence, effectively tackled the issue from a central planner's perspective.

In the discussion below, we pick up Solow's central argument that the issue of technique change must be tackled within a dynamic approach, rather than by pure static comparisons of steady states. The reason is that any change in production technique inevitably generates some intermediate gains or losses in the transition periods, both in terms of per capita consumption and of profits in a competitive equilibrium. Therefore, the complete sequence of future gains and losses, including the transition periods, must be taken into consideration. We will show that the critical interest rate which makes the technique change break even does not differ between a profit maximizing firm and a central planner who seeks to maximize consumption per work unit. However, in contrast to Solow's claim, it does *not* generally coincide with either of the intersection points of the factor price frontiers, if there is more than one physically specified, "non malleable" good. Hence, the intersections of the factor price frontiers are irrelevant for the choice of techniques in a dynamic perspective, and so are common examples of reswitching based on them.

In Section 2, we start with a single commodity model as used by Solow (1967). In Section 3, we generalize the model by distinguishing explicitly between the capital good and the consumption good, still assuming that they are both produced with equal factor input proportions. In Section 4, by dropping this assumption, we allow for multiple intersections of the factor price curves and disprove Solow's theorem for this more general case. In Section 5, we draw some conclusions.

## 2. Technique Choice in the Single Commodity Model

Assume a single commodity X which can be produced by either of two linear techniques j, according to

$$(2.1) \quad X_j = \min\left(\frac{N_j}{n_j}, \frac{K_{j,t-1}}{k_j}\right) \quad j = I, II$$

where n and k are the fixed factor coefficients of labour N and capital K respectively. In this basic “corn-model”, capital input  $K_{j,t-1}$  is simply that part of output  $X_{j,t-1}$  which is dedicated to subsequent production, rather than consumed in the preceding period. With X and, hence, also K living for just one period, and with N fully employed, consumption per worker is

$$(2.2) \quad c_j = \frac{1-k_j}{n_j} \quad j = I, II$$

It is assumed that capital is purchased at the beginning of period t, whereas the commodity is sold and labour is paid at the end. Therefore, if the commodity price is normalized to  $p_x = 1$ , the price equations of the two techniques are

$$(2.3) \quad 1 = w_j n_j + k_j (1+r) \quad j = I, II$$

Accordingly, the factor price frontiers are easily derived as

$$(2.4) \quad w_j = \frac{1-k_j(1+r)}{n_j} \quad j = I, II$$

In this simple model, there is only one interest rate  $\tilde{r}$  for which  $w_I = w_{II}$  holds:

$$(2.5) \quad \tilde{r} = \frac{n_I - n_{II}}{n_I k_{II} - n_{II} k_I} - 1$$

It is usually argued that  $\tilde{r}$  not only marks the intersection of the factor price frontiers but, for a profit maximizing firm, is also the critical interest rate for the choice between the two techniques I and II. Moreover, the same criterion is also frequently regarded as efficient from

a social point of view, because it maximizes the wage rate with a given interest rate and vice versa.

As Solow (1967) proved, in this model,  $\tilde{r}$  is indeed equal to the critical interest rate needed to make a technique change from I to II (or vice versa) pay in terms of per capita consumption. In contrast to the common static view, Solow does not simply compare the two steady states with each other. If that were done, one would readily find from (2.2) that there is a unique ranking of the two techniques which is totally independent of the interest rate. However, Solow adopts the present value criterion of the respective technique *change*. Provided that there is only one transition period from technique I to II, the respective critical interest rate is defined by

$$(2.6) \quad PV_{I,II} = c_S - c_I + \sum_{j=1}^{\infty} \frac{c_{II} - c_I}{(1+r^*)^j} = c_S - c_{II} + \frac{c_{II} - c_I}{1 - \frac{1}{1+r^*}} = 0$$

where  $c_j$  denotes consumption per worker with the respective technique, the subscript S denotes the transition period, and  $PV_{I,II}$  is the present value of a technique change from I to II and. Solving (2.5) for the interest rate  $r$  yields

$$(2.7) \quad r^* = \frac{c_I - c_{II}}{c_S - c_I}$$

This is simply the permanent change in per capita consumption, divided by the temporary change within the transition period (Solow 1967, 32). From (2.7), it immediately follows that, with  $r > r^*$ , the change from I to II is profitable, if  $c_{II} - c_I < 0$  and vice versa. Consumption per worker in the transition period is easily calculated as

$$(2.8) \quad c_S = \frac{X_I - K_{II}}{N} = \frac{n_{II} - k_{II}n_I}{n_I n_{II}}$$

Inserting (2.8) into (2.7) finally yields (2.6) again and, hence, the Solow theorem  $\tilde{r} = r^*$  holds for our simple single commodity case (Solow 1967, 35).

Figure i shows a numerical example with  $(n_I, k_I) = (0.1, 0.3)$  and  $(n_{II}, k_{II}) = (0.3, 0.1)$ , which leads to  $\tilde{r} = r^* = 1.5$  and  $w_I(\tilde{r}) = w_{II}(\tilde{r}) \equiv \tilde{w} = 2.5$ . The factor price frontiers (2.4) correspond to the left axis, while the present value according to (2.6) is shown on the right axis of the figure. It is evident that the latter is a monotonically increasing function of  $r$ .

**Figure i**

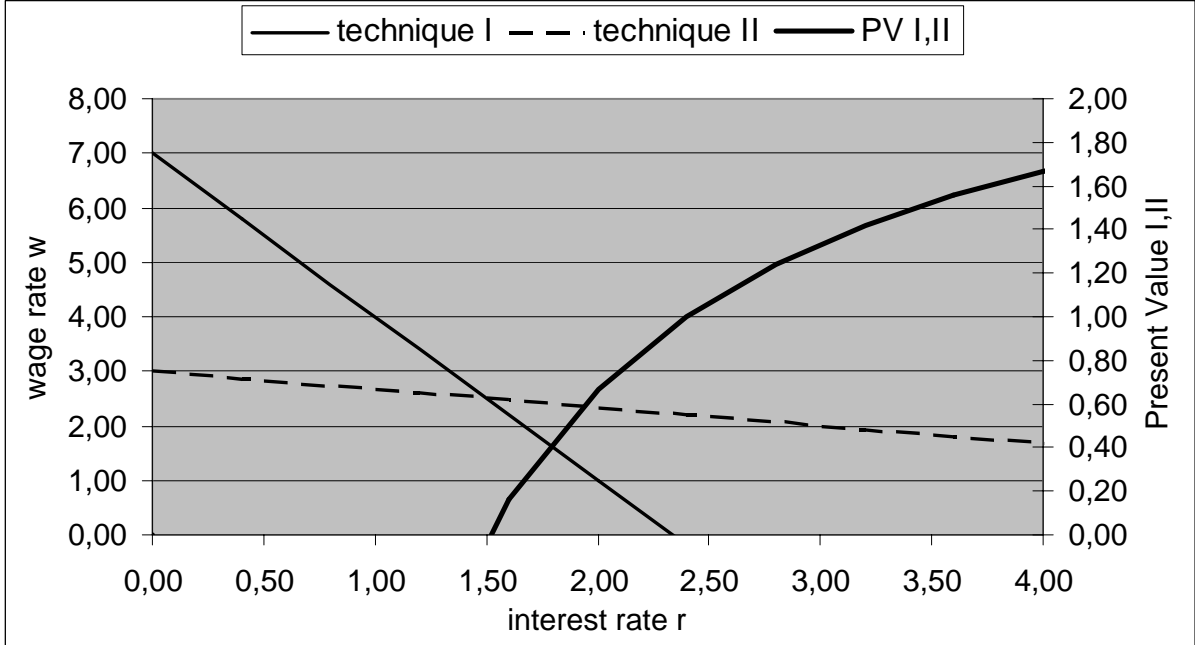


Table i provides some additional detail, with the shaded columns in Table i indicating two switching periods, first from I to II and then back from II to I. It is assumed that the old technique is generally adopted to produce the required amount of capital for the new technique within the switching periods.<sup>1</sup> With all factors being always fully employed in this example, from (2.6), it can easily be calculated that the present value calculation yields the

<sup>1</sup> Otherwise, there would be either idleness or a lack of capital in the switching periods in this example.

same critical interest rate  $r^*$  for both technique changes, where the reverse switch from II to I pays for interest rates *below* 1.5.<sup>2</sup>

**Table i**

Technique Period t	I 0	I 1	I 2	II 3	II 4	II 5	I 6	I 7
N	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00
K input	3,00	3,00	3,00	0,33	0,33	0,33	3,00	3,00
K output	3,00	3,00	0,33	0,33	0,33	3,00	3,00	3,00
X	10,00	10,00	10,00	3,33	3,33	3,33	10,00	10,00
C = c	7,00	7,00	9,67	3,00	3,00	0,33	7,00	7,00
G = g	4,50	4,50	7,17	0,50	0,50	-5,17	4,50	4,50

Solow was rightly criticised by Harcourt and others for failing to show that, if the present value criterion is adopted by a profit maximizing firm, the same solution would result (Hagemann, 1997, 156). However, at least for the simple one-commodity model, it can readily be shown that this is actually true.

Because of the linear production function, the single firm can simply be viewed as a small fraction of the whole economy. In order to calculate the behaviour of the firm correctly, it is crucial to understand that the present value criterion refers to out of pocket expenses and cash receipts, rather than to periodically adjusted costs and revenues.<sup>3</sup> Therefore, when deciding on a switch to another technique, the single firm need only regard the changes in market sales and cash expenses.<sup>4</sup> Moreover, in our simple model, those fractions of output X which serve as capital goods are merely internal intermediate products which are neither bought from nor

<sup>2</sup> The result for the reverse switch is achieved by simply exchanging the subscripts I and II in (2.5) and (2.6).

<sup>3</sup> Calculating a present value from imputed interest would, in fact, be double discounting.

<sup>4</sup> The only exception might be a historic first capital input  $K_0$  which had to be purchased one period before the first receipts accrued to the firm. If so, cash interest must be paid for  $K_0$  in *each* subsequent period. If labour were compensated at the *beginning* of the period, the same would apply to the very first wage expenses. However, all of these interest expenses are constant even in the transition periods and hence cancel out in the calculation of  $r^*$



sold in the market.<sup>5</sup> Because both total labour input and the market wage rate are equal with both techniques,<sup>6</sup> we have

$$(2.9) \frac{g_I - g_{II}}{g_S - g_I} = \frac{(c_I - \tilde{w}) - (c_{II} - \tilde{w})}{(c_S - \tilde{w}) - (c_I - \tilde{w})} = r^*$$

where  $g_j$  denotes the surplus per working place corresponding to the respective type of period defined by (2.6). Note that  $g$ , rather than  $G$ , is the relevant figure, because a reduction in total profits  $G$  due to a decline in total output would clearly be irrelevant for the profitability of a single firm.

As an interim result, we conclude that, with just one “malleable” good, neither reswitching occurs, nor is there any divergence in the choice of techniques between a profit maximizing firm on the one hand and a benevolent central planner on the other.

### 3. The Two Commodity Case with Equal Factor Input Proportions

It is frequently argued that simple models like that presented in Section 2 do not reflect the questions which capital theory is really about. In particular, the assumption of malleability apparently “charms away” the problem of heterogeneous goods and is, therefore, unsuitable for tackling the central problems of aggregation and constrained versatility of specialized capital goods. In particular, Solow’s contribution was criticised for being inapplicable within a macroeconomic context (Hagemann/Kurz 1976, 703), and also for neglecting the inevitable resource idleness within the transition periods (Pasinetti 1969, Hagemann 1997).

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<sup>5</sup> The revenues from selling the redundant capital, due to the first switch (Period 2), is already included in  $C$ . Correspondingly, the additional capital needed for the second switch (Period 5) is already included in the respective lower  $C$  which is to be sold in that period.

<sup>6</sup> With  $r \neq \tilde{r}$ , the wage rate would be either  $w_I$  or  $w_{II}$  and, hence, remain the same for each individual firm, irrespective of the technique which the latter adopts.

In this section, we generalize the model from Section 2 by distinguishing explicitly between one capital good and one consumption good which are physically different from one another and, therefore, can no longer simply be exchanged for one another.<sup>7</sup> Hence, the required K for the following period must be produced by using N and K according to the input coefficients of the technique in use, the same being true for the respective C. We assume again that Technique I is still in use during the transition from I to II, while Technique II is used during the transition from II to I (see the shaded columns in Table ii). For the moment, we maintain the assumption that the same input coefficients apply for the two commodities with a given technique  $j$ , i.e. we assume  $n_{c,j} = n_{k,j} \equiv n_j$  and  $k_{c,j} = k_{k,j} \equiv k_j$ , where the subscripts  $c$  and  $k$  stand for the factor input coefficients in the production of C and K respectively.

**Table ii**

Technique in use Period	I 1	I 2	I 3	II 4	II 5	II 6	I 7	I 8
K from previous period	3,00	3,00	3,00	0,33	0,33	0,33	3,00	3,00
newly produced K	3,00	3,00	0,33	0,33	0,33	3,00	3,00	3,00
...required N for that purpose	0,30	0,30	0,03	0,10	0,10	0,90	0,30	0,30
...required K for that purpose	0,90	0,90	0,10	0,03	0,03	0,30	0,90	0,90
$C_{\max}$ from idle N	7,00	7,00	9,67	3,00	3,00	0,33	7,00	7,00
$C_{\max}$ from idle K	7,00	7,00	9,67	3,00	3,00	0,33	7,00	7,00
<b>C produced and consumed</b>	<b>7,00</b>	<b>7,00</b>	<b>9,67</b>	<b>3,00</b>	<b>3,00</b>	<b>0,33</b>	<b>7,00</b>	<b>7,00</b>
<b>c (C divided by employed N)</b>	<b>7,00</b>	<b>7,00</b>	<b>9,67</b>	<b>3,00</b>	<b>3,00</b>	<b>0,33</b>	<b>7,00</b>	<b>7,00</b>
<b>G (total surplus)</b>	<b>4,50</b>	<b>4,50</b>	<b>7,17</b>	<b>0,50</b>	<b>0,50</b>	<b>-2,17</b>	<b>4,50</b>	<b>4,50</b>
<b>g (surplus per employed N)</b>	<b>4,50</b>	<b>4,50</b>	<b>7,17</b>	<b>0,50</b>	<b>0,50</b>	<b>-2,17</b>	<b>4,50</b>	<b>4,50</b>
Total N employed	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00

This extended model can be described by the same equations as the simpler version in Section 2, with the exception that (2.1) and (2.2) are modified slightly as follows:

$$(3.1) \quad X_{i,j} = \min\left(\frac{N_{i,j}}{n_j}, \frac{K_{i,j,t-1}}{k_j}\right) \quad j = I, II \quad i = c, k$$

$$(3.2) \quad x_{c,j} \equiv c_j = \min\left(\frac{1-k_j}{n_j}\right) \quad j = I, II$$

<sup>7</sup> A good example may be presented by furniture craftsmanship with the aid of a certain set of tools, the latter in turn being produced by using both the same set of tools and labour.

Equation (3.1) takes into account that total capital and labour input must now be devoted explicitly to the production of either the capital good or the consumption good, while (3.2) is a mere copy of (2.2), only intended to simplify the notation. Not surprisingly, therefore, by again assuming  $(n_I, k_I) = (0.1, 0.3)$  and  $(n_{II}, k_{II}) = (0.3, 0.1)$ , this model generates the same results as the simplified version from Section 2. In particular, again  $\tilde{r} = r^* = 1.5$  and  $w_I(\tilde{r}) = w_{II}(\tilde{r}) \equiv \tilde{w} = 2.5$ , such that Figure I still applies.

Nonetheless, the explicit distinction between the capital good and the consumption good provides some important new insights. First of all, with two goods C and K and two technologies I and II, we now have, in principle, four rather than two possible production scenarios (which we refer to as techniques rather than technologies). It is only because of our preliminary assumption of equal factor input coefficients with both goods that only two of the four possible techniques are effectively relevant.<sup>8</sup> Moreover, there are, in principle, also two technologies to choose between within the transition period for each of the commodities, such that the total number of technology combinations (i.e. techniques) rises to 16. If a multitude of firms is assumed to produce each commodity, even more combinations are conceivable, unless we require all firms to behave equally.

In the following analysis, we continue to confine ourselves to the choice between just two techniques. The simplest case is to assume that C and K, although physically different goods, are produced in vertically integrated firms which do not specialize in one of the commodities. It is also assumed that each firm can choose only between Technique I and Technique II to produce *both* C and K. With these assumptions, there are just two techniques available to each individual firm, as for the economy as a whole, which is the assumption underlying Table ii.

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<sup>8</sup> If, for example, C were produced with Technology I, and K with Technology II, the  $n_j$  and the  $k_j$  would apparently be different, which would violate our assumption from above.

Concerning the change in technique, the present value calculation of the representative firm must again be based on cash expenses and receipts. If we allow for market purchases of K instead of pure in-house production, there are now some capital expenses in addition to wage payments. However, because the capital expenses of one firm are the purchases of another firm at the same moment, they cancel out in the calculation for the representative firm.

Therefore, we have analogously to (2.9)

$$(3.3) \quad \frac{g_I - g_{II}}{g_S - g_I} = \frac{\left( \frac{k_I}{n_I} + c_I - \tilde{w} - \frac{k_I}{n_I} \right) - \left( \frac{k_{II}}{n_{II}} + c_{II} - \tilde{w} - \frac{k_{II}}{n_{II}} \right)}{\left( \frac{k_S}{n_S} + c_S - \tilde{w} - \frac{k_S}{n_S} \right) - \left( \frac{k_I}{n_I} + c_I - \tilde{w} - \frac{k_I}{n_I} \right)} = \frac{c_I - c_{II}}{c_S - c_I} = r^*$$

For the reverse switch from II to I it is easily verified that the same critical interest rate  $r^* = 1.5$  results from both the consumption and the profit approach in our example (see Table ii). Analogous results are obtained if we allow the firms to specialize either in the production of C or of K. Note that  $p_K = p_C$  still holds, because of the identical technologies used for both products. Therefore, for the single K-producer, exactly the same figures apply as shown in Table ii, with the only qualification that his final product is not C but K. The same is essentially true for the single C-producer, although he buys his capital input instead of producing it himself. By employing 1 unit of labour and 3 units of capital with Technique I, he both produces and sells 10 units of C in each period, but must buy 3 units of new capital for the preceding period. Hence, his surplus per worker is again 4.5. When switching to Technique II, he starts with 3 units of capital from the previous period, but buys only 0.33 units of K for the needs of the preceding period. Hence, his surplus in the switching period is

again 7.17, declining to 0.5 in the new steady state with Technique II.<sup>9</sup> An analogous calculation applies to the switch back from Technique II to Technique I.

Therefore,  $r^*$  is not only the critical interest rate for a switch from a central planner's perspective, but also the result of independent market decisions. However, because of our assumption of equal factor proportions in the production of C and K, there is still no double intersection of the factor price curves. Therefore, in order to deal with the reswitching argument, an even more generalized model is required.

#### 4. Two Commodities with Factor Input Proportions Unequal

In order to consider the reswitching phenomenon, we now extend our model to the more general approach as, for example, used by Bruno et al. (1966). Steady state equilibrium is then characterized by the following set of equations, all of which are generalizations of the respective equations from Section 2:

$$(4.1) \quad X_{i,j} = \min\left(\frac{N_{i,j}}{n_{i,j}}, \frac{K_{i,j,t-1}}{k_{i,j}}\right) \quad j = I, II \quad i = c, k$$

$$(4.2) \quad x_{c,j} \equiv \frac{X_{c,j}}{N} \equiv c_j = \frac{1}{\frac{n_{k,j}n_{c,j}}{1-k_{k,j}} + n_{c,j}} \quad j = I, II$$

$$(4.3) \quad p_{i,j} = w_j n_{i,j} + k_{i,j}(1+r) \quad j = I, II \quad i = c, k$$

$$(4.4) \quad w_j = \frac{p_{c,j}[1-k_k(1+r_j)]}{(n_{k,j}k_{c,j} - n_{c,j}k_{k,j})(1+r_j) + n_{c,j}} \quad j = I, II$$

---

<sup>9</sup> The surplus in each period is calculated as  $g_{c,j} = c_j(n_{c,j}, k_{c,j}) - w_j n_{c,j} - k_{c,t+1}$ , where, in the steady state,  $k_{c,t+1} = k_{c,j}$ , while  $k_{c,t+1} = k_{c,II}$  in the switching period from I to II.

$$(4.5) \quad \tilde{r}_{1,2} = -\frac{\omega}{2} \pm \sqrt{\left(\frac{\omega}{2}\right)^2 - q - 1}$$

with

$$\omega \equiv -\frac{D_{II} - D_I + k_{k,II}n_{c,I} - k_{k,I}n_{c,II}}{k_{k,II}D_I - k_{k,I}D_{II}}$$

$$q \equiv \frac{n_{c,II} - n_{c,I}}{k_{k,II}D_I - k_{k,I}D_{II}}$$

$$D_j \equiv n_{k,j}k_{c,j} - n_{c,j}k_{k,j}$$

Due to the quadratic form of (4.5), there is now the possibility of a double intersection of the factor price frontiers at two positive values of  $\tilde{r}$ , which is usually regarded as reswitching. However, as in the models above, again a present value calculation including the transition period is required.

It is easily demonstrated that, in the more general model, Solow's theorem of  $\tilde{r} = r^*$  no longer holds, as Pasinetti (1969) has shown. Suppose, for example, that

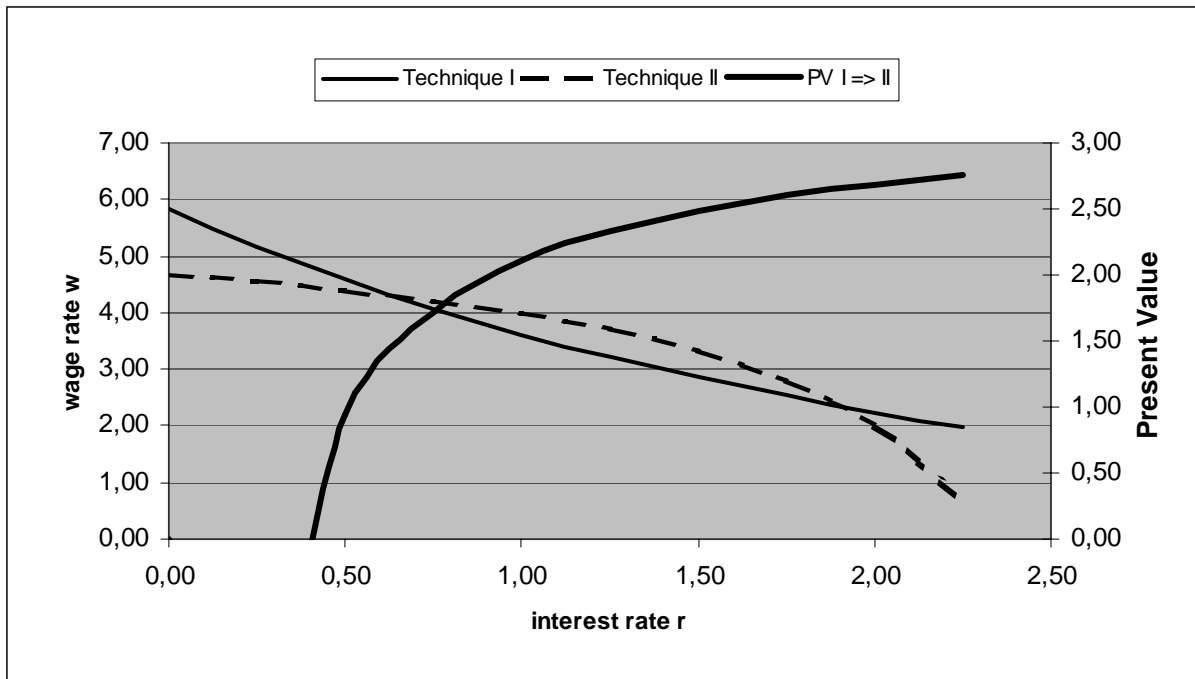
$$(n_{k,I}, n_{c,I}, k_{k,I}, k_{c,I}) = (0.1, 0.1, 0.16, 0.6) \text{ and } (n_{k,II}, n_{c,II}, k_{k,II}, k_{c,II}) = (0.1, 0.2, 0.3, 0.1).$$

Then, by employing (4.5), we find a double intersection of the two factor price frontiers at the interest rates  $\tilde{r}_1 = 0.6243$  and  $\tilde{r}_2 = 1.9040$ . However, there is still just one critical interest rate  $r^* = 0.3568$  from the present value calculation according to (2.6) and (2.7), which is *not* equal to any of the switching points (see Figure ii). Even if there is no intersection of the factor price frontiers at all, there is still a critical interest rate  $r^*$  at which a technique change breaks even in terms of the present value criterion.<sup>10</sup>

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<sup>10</sup> A suitable demonstration is found, for example, by substituting  $k_{c,I} = 0.6$  by  $k_{c,I} = 0.4$  in our example, which leads to a new  $r^* = 0.8644$ , although the factor price frontier with Technique I is now above of the factor price frontier with Technique II over the entire range of possible interest rates.

**Figure ii**



**Table iii**

relevant wage rate $w$ for single firm	4,90	4,90	4,90	4,90	4,81	4,81	4,81	4,81
interest rate $r$	0,36	0,36	0,36	0,36	-0,38	-0,38	-0,38	-0,38
<b>Technique in use</b>	I	I	I	II	II	II	I	I
<b>Period</b>	1	2	3	4	5	6	7	8
K from previous period	4,17	4,17	4,17	0,67	0,67	0,67	4,17	4,17
newly produced K	4,17	4,17	0,67	0,67	0,67	4,17	4,17	4,17
...required N for that purpose	0,42	0,42	0,07	0,07	0,07	0,42	0,42	0,42
...required K for that purpose	0,67	0,67	0,11	0,20	0,20	1,25	0,67	0,67
$C_{max}$ from idle N	5,83	5,83	9,33	4,67	4,67	2,92	5,83	5,83
$C_{max}$ from idle K	5,83	5,83	6,77	4,67	4,67	-5,83	5,83	5,83
<b>C produced and consumed</b>	<b>5,83</b>	<b>5,83</b>	<b>6,77</b>	<b>4,67</b>	<b>4,67</b>	<b>-5,83</b>	<b>5,83</b>	<b>5,83</b>
<b>c (C divided by employed N)</b>	<b>5,83</b>	<b>5,83</b>	<b>9,10</b>	<b>4,67</b>	<b>4,67</b>	<b>7,78</b>	<b>5,83</b>	<b>5,83</b>
<b>g (surplus per employed N)</b>	<b>0,93</b>	<b>0,93</b>	<b>4,20</b>	<b>-0,24</b>	<b>-0,15</b>	<b>2,96</b>	<b>1,02</b>	<b>1,02</b>
Total N employed	1,00	1,00	0,74	1,00	1,00	-0,75	1,00	1,00

Table iii shows why the Solow-Theorem breaks down in the more general model. Due to the diverging factor input proportions of C and K with both technologies, an amount of idle labour in the switching period from I to II now accrues (Period 3). Therefore, as Hagemann (1997, 150) points out, the implicit full employment assumption in Solow's proof is no longer fulfilled. Moreover, in our example, there it is no longer possible to return to Technique I, once Technique II has been chosen, at least not within a single switching period. The reason is that the capital requirement of Technique I exceeds the potential capital output with Technique II and, hence, negative values of C and N result (Period 6 in Table iii).

Nethertheless, our general result that  $r^*$  is the relevant figure for the technique choice for both a central planner and the individual firm, holds true even in the generalized model. We still assume that the central planner maximizes the present value of per capita consumption, while the profit-maximizing firm switches to another technique if the present value of the change is positive at the given interest rate. While steady state per capita consumption is given by equation (4.2) as before, potential idleness of either labour or capital within the transition period must now be taken into account. If the transition moves from I to II, we therefore have

$$(4.6) \quad c_S \equiv \frac{C_S}{N_S} = \min \left[ \frac{(\bar{N} - n_{k,I} K_{II}) / n_{c,I}}{N_S}, \frac{(K_I - k_{k,I} K_{II}) / k_{c,I}}{N_S} \right]$$

where  $N_S = n_{k,I} K_{II} + n_{c,I} C_S$  is the amount of labour used during the transition period. The first term in the squared brackets denotes maximum per capita consumption to be achieved after allowing for the labour requirements to produce the required capital for the new technology. The second term is defined equivalently with respect to the capital requirements for the same purpose. In contrast to the steady state, the two restrictions do not coincide in the transition periods, as can be seen from the shaded columns in Table iii, and, hence, either idleness or a lack of input factors accrues. Moreover, the critical interest rate  $r^*$ , which applies to the switch from I to II, is no longer the same as the critical interest rate  $r^{**}$  which applies to the reverse switch from II to I. Instead there is now a form of path dependency, which even precludes an instant switch back to Technique II in our example (Period 6 in Table iii).

However, none of these complications ultimately leads to any form of paradox or inefficiency. Note that the temporary reduction of labour input in Period 3 need not result in unemployment, but could also be absorbed by a reduction in working time. Therefore, if labour is seen in principle as a burden rather than as a good, the central planner would be



perfectly right to maximize productivity instead of total consumption.<sup>11</sup> According to equation (2.7), he will then arrive at a critical interest rate of  $r^* = 0.3568$  in our example, which still constitutes the unique solution to his decision problem, as far as the potential switch from I to II is concerned.

With respect to the calculation made by a profit maximizing firm, we confine ourselves to the case of a vertically integrated firm which produces both C and K.<sup>12</sup> The single firm is not subject to any rationing of sales and, hence, need not care about idle or absolutely limited resources. Due to the linear technology, we can again concentrate on the surplus per worker for an appropriate calculation of its profits. As the vertically integrated firm produces its capital input itself, there are no expenses apart from wages. Moreover, if the individual firm is small enough in relation to the entire market, it will not take into account any changes in wages or prices with respect to her own decisions. Therefore, the present value calculation of the technique change from I to II must be based on the simple surplus function

$$(4.7) \quad g = c - \bar{w}$$

where  $c$  is the amount of the consumption good *sold* and  $w$  is the prevailing wage rate, all variables being defined per worker and referring to the same period. Therefore, equation (2.9) still holds and the switch from I to II will be undertaken exactly at the central planner's critical interest rate  $r^* = 0.3568$ .

The same equality occurs with a switch back to technique II, which is indicated in Periods 5 to 8 in Table iii. With the critical rate  $r^{**} = -0.3750$ , the present value of the technique change is exactly zero, not only for the central planner, but also for the vertically integrated

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<sup>11</sup> If the loss of coachmen due to the emergence of the automobile had been viewed as inefficient, we would still be drawn by horses instead of driving cars.

<sup>12</sup> To allow for specialized firms would require discussing all possible combinations of Technology I and II in both the consumption good industry and the capital good industry, which is beyond the scope of this paper.

firm, as can readily be verified by alluding (2.9) to Periods 5 to 7 in the Table.<sup>13</sup> This is even true in our extreme example of double-intersecting factor price frontiers, but it can also be verified by means of any other numerical example which avoids the negative signs for  $r^{**}$ ,  $c$  and  $g$ .<sup>14</sup>

## 5. Conclusions

The main result emerging from the above analysis is that intersections of the factor price frontiers are irrelevant to the choice of techniques in a dynamic view, both for the central planner and for the profit maximizing firm. Following Solow's early argument, a present value calculation is required instead, which might even lead to a switch towards a technique with a factor price frontier below that of a competing technique over the entire range of positive interest rates. The only requirement is a single, instant gain from the change which is large enough to outweigh the permanent, but discounted losses which follow. We therefore conclude, that, while there can certainly be multiple intersections of factor price frontiers, no phenomenon like reswitching can be derived from such intersections.

One might object that, with more than two periods being necessary for the transition, a problem with multiple interest rates could arise also in the present value calculation. As is familiar from investment theory, the latter generally occurs if the periodical revenues change in sign during an investment period. Because the transition to another technique is nothing other than a multi-period investment project, this problem could be relevant here indeed.

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<sup>13</sup> Note that with  $r^{**}$  both the wage rates and the prices of the capital good with the two techniques are different from the figures with  $r^*$  (see the respective top lines in Table iv).

<sup>14</sup> I could not find an example with a double intersection of the factor price frontiers *and* solely positive signs in the present value calculations for both  $r^*$  and  $r^{**}$ . Presumably, these features are not compatible with each other. No attempt is made here to provide such a proof.

Therefore, it seems that the reswitching problem immediately resurfaces if there are multiple interest solutions of the present value calculation.

However, according to the so-called truncation theorem, each such sequence of periods with alternating positive and negative revenues can be replaced by a shorter process with a unique interest rate solution, which is superior, or at least equally profitable to the initial sequence in terms of present value (Arrow and Levhari 1969). Hence, even with more than one transition period and variable wages and interest rates, reswitching does not occur (Nuti 1973, pp 491). It is true that the multiple intersection of factor price frontiers is a phenomenon different from the case of multiple internal rates of return, because the former refers to the choice of techniques, while the latter deals with a single investment project (Hagemann/Kurz, 1976, 703). However, in a dynamic perspective, the change from one technique to another *is* a single investment project by definition. Hence, if there is anything like reswitching, it must indeed be very closely related to the problem of multiple internal rates of return.

As argued above, a profit maximizing firm will generally switch at the same critical interest rate as a central planner who seeks to maximize the present value of consumption per worker. This was shown, however, only for a vertically integrated firm, but not for single firms which specialize in either the production of capital goods or consumption goods. Moreover, our arguments rest partly on mere numerical examples without a rigorous proof. Therefore, additional theoretical work must still be done to arrive at more general conclusions.

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