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## ABSTRACT

### **Pork-Barrel Politics, Discriminatory Policies and Fiscal Federalism**

by Brian Roberson \*

This paper examines the role of discriminatory policy tools in a model of redistributive politics with jurisdictional specific projects. In equilibrium, the ability to tactically target both jurisdictional specific projects, with benefits concentrated within a given jurisdiction, and the costs associated with those projects across multiple jurisdictions, leads to inefficiency in the provision of the “local” projects. In particular, politically motivated discrimination in the provision of local projects and/or their associated costs results in the foregoing of efficient projects. However, greater discriminatory ability in the set of available policies lowers the level of inefficiency in the provision of the local projects.

*Keywords: Fiscal Federalism, redistributive politics, distributive politics, Colonel Blotto*

## ZUSAMMENFASSUNG

### **Kuhhandel in der Lokalpolitik und der fiskalische Föderalismus**

Dieser Aufsatz untersucht in einem Modell umverteilender Politik die Rolle von lokalen öffentlichen Gütern und öffentlichen Projekten in einzelnen Jurisdiktionen. Politiker können aus strategischen Gründen bestimmte lokale Projekte durchführen und dadurch gezielt bestimmte Gebietskörperschaften beeinflussen, welche die Vorteile dieser Projekte genießen, während die damit verbundenen Kosten von mehreren Jurisdiktionen getragen werden. Dies führt zu einer Ineffizienz in der Durchführung solcher lokalen Projekte. Insbesondere führt eine politisch motivierte Ungleichbehandlung verschiedener Gebietskörperschaften bei der Durchführung lokaler Projekte bzw. ihrer Finanzierung dazu, dass manche effizienten Projekte nicht durchgeführt werden. Allerdings geht diese Ineffizienz zurück, wenn es größere Möglichkeiten zu ungleicher Behandlungen verschiedener Jurisdiktionen gibt.

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# 1 Introduction

Economic fiscal federalism examines the economic efficiency of the allocation of resources by a central government and by local governments. Traditionally (i.e. the literature following Musgrave (1959) and Oates (1972)) it is held that inefficiency is the result of the failure of local governments to internalize the interjurisdictional externalities from their local projects, the failure of local governments to coordinate local project production and utilize economies of scale, the uniformity of the central government's policy over a population with heterogeneous preferences for local projects, or asymmetries in the information available to different levels of government (henceforth the traditional sources of inefficiency). Notably absent from this list is one of the more commonly identified sources of the governmental misallocation of resources, namely pork-barrel politics. This paper examines the strategic political targeting of jurisdictional specific projects (or equivalently local public goods) and/or their associated costs, at both the central and local levels, and the nature of the resulting inefficiencies. Political parties face a tradeoff between efficiency and discriminatory ability, and in equilibrium the presence of discriminatory policy tools results in inefficiency in the provision of the local projects. However, as the set of available policies allows for greater discriminatory ability the level of inefficiency in the provision of the local projects decreases.<sup>1</sup> That is, greater discriminatory ability increases the costs of forgoing efficient local projects. Thus, this paper highlights an important and purely strategic role for discriminatory policy instruments such as interjurisdictional transfers that provides a contrast with the view that such policy instruments are solely a tool for fiscal equalization (i.e. the literature following Buchanan (1950)).

The model of redistributive politics examines the strategic allocation of a budget across voters. Each voter votes for the party offering the highest level of utility, and each party's payoff is equal to their vote share. Several variations of the model (Cox and McCubbins (1986), Lindbeck and Weibull (1987), Dixit and Londregan (1995, 1996), Myerson (1993), and Laslier and Picard (2002)) have served as fundamental tools in the analysis of electoral

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<sup>1</sup>This result holds for all parameter configurations in which the local projects are efficient but not unreasonably so. In particular, the result holds for all parameter configurations in which the local projects are efficient at all levels up to 1.5 times more efficient than direct transfers. Once the local projects are excessively efficient uniform provision of local projects leads to greater efficiency.

competition. Closely related to this paper<sup>2</sup> is Lizzeri and Perisco's (2001, henceforth L-P) model of redistributive competition (over a continuum of voters) with public good provision. In L-P political parties compete for vote share by announcing binding commitments as to how they will allocate a budget across voters and to investment in the production of a (global) public good. This paper extends the model of redistributive politics with a finite population of voters to allow for redistributive competition with local projects and examines the cases of: complete uniformity in both local project provision and the assignment of the associated costs, complete discriminatory ability in local project provision and net transfers, discriminatory net transfers with uniform local project provision, and discriminatory local project provision with uniform net transfers.<sup>3</sup>

Oates' Decentralization Theorem states that, absent interjurisdictional externalities and economies of scale in the production of local projects, decentralization yields a level of welfare that is at least as high as that of centralization. In this context, a decentralized system is one in which each jurisdiction independently makes its own local project provision decision and raises its own taxes, and a centralized system is one in which the central government makes the local project provision decision (often uniformly in this literature) for each jurisdiction and raises taxes to cover the associated costs. Conversely, this paper shows that once the incentives of discriminatory policy tools are taken into account Oates' Decentralization theorem fails to hold. That is, in the absence of the traditional sources of inefficiency, the greater discriminatory ability of a centralized system may result in a higher level of welfare than a decentralized system. Closely related to Oates' Decentralization Theorem is the strand of literature on the unification and break up of nations (see for example Alesina and Spolaore (1997, 2003), Bolton, Roland, and Spolaore (1996), Bolton and Roland (1997), and Rutta (2005)) which focuses on the traditional tradeoff between the gains of centralization and the costs of centralization, i.e. economies of scale and the internalization of externalities versus the uniformity of the central government's policy over a population with heterogeneous preferences for local projects. In contrast, this paper highlights that the greater discriminatory ability of the centralized government may provide an additional incentive for unification.

Also related is the model of distributive politics (see for example Riker (1962), Buchanan

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<sup>2</sup>Also related is Dixit and Londregan (1998) who examine a multi-period multi-tiered model of federalism.

<sup>3</sup>The case of decentralized redistributive competition with individual transfer discrimination examined in this paper is the generalization of Theorem 3 in L-P to a finite population of voters.

and Tullock (1962), Weingast (1979), Shepsle and Weingast (1981), or Ferejohn, Fiorina, and McKelvey (1987)) which examines the centralized provision of jurisdictional specific projects with uniform cost sharing across all jurisdictions. The provision of the local projects is determined by a legislature comprised of representatives from each of the jurisdictions. Most closely related to this paper is Lockwood (2002).<sup>4</sup> Among the extensions that Lockwood (2002) makes, that paper allows for the centralized government to choose a uniform level of taxation within each jurisdiction rather than a uniform level of taxation across all jurisdictions. While this paper does not focus on a rigorous modeling of the legislature,<sup>5</sup> it may be viewed as complementary to the distributive politics literature in focus. However, this paper allows for not only the non-uniformity of the central government's provision of local project provision across jurisdictions, but also the non-uniformity of the central government's net transfers both across and within jurisdictions and the non-uniformity of the local government's net transfers across voters within the jurisdiction. In a sense this analysis levels the playing field between centralization and decentralization in that decentralization is not *a priori* efficient in the absence of the traditional sources of inefficiency.

Lastly, this paper is of theoretical interest to the literature on the Colonel Blotto game (Borel (1921), Tukey (1949), Gross and Wagner (1950), Blackett (1954, 1958), Bellman (1969) and more recently Kvasov (2005), Weinstein (2005), Golman and Page (2006), and Roberson (2006)). In particular, the model of redistributive politics with discriminatory local project provision and discriminatory net transfers is equivalent to a Colonel Blotto game in which each player has multiple military technologies each with different levels of discriminatory ability across the various battlefields. This paper's results on the tradeoffs between discriminatory ability and efficiency apply directly in this context.

Section 2 presents the model of redistributive politics with local project provision. Section 3 characterizes the set of Nash equilibrium univariate marginal distributions of the games of redistributive politics with local projects; demonstrates the existence of a joint distribution with the appropriate equilibrium univariate marginal distributions and with the property that the budget is satisfied with probability one; and explores the efficiency and distributional

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<sup>4</sup>See also Besley and Coate (2003) and Grossman and Helpman (2005) who examine related models.

<sup>5</sup>As is common in models of electoral competition (see for example Myerson (1993) or Grossman and Helpman (1996)), the policy implemented by the legislature may be assumed to be a probabilistic compromise of the parties' equilibrium local project and cost allocation policies.

properties of equilibrium under the different regimes. Section 4 concludes.

## 2 The Model

### Voters

The model extends Laslier and Picard's (2002) two-party model of redistributive politics by including local projects and by examining how the various discriminatory tools available to the political parties affect the resulting electoral competition. The electorate consists of a finite and even number  $n$  of voters, which are denoted by  $z \in \{1, \dots, n\}$ . The set of voters is partitioned into a finite number  $k \geq 2$  of disjoint jurisdictions, where jurisdiction  $j \in \{1, \dots, k\}$  consists of a finite number  $m_j \geq 2$  of voters with  $\sum_{j=1}^k m_j = n$ . Voters are distinguished by the jurisdiction to which they belong, where voter  $z$  in jurisdiction  $j$  is denoted by  $z(j)$ . Let  $N_j$  denote the set of voters in jurisdiction  $j$ . Each voter is endowed with 1 unit of a private homogeneous good.

While the results of this paper apply more generally, in order to simplify the statement and proofs of the results we will focus on the symmetric case in which for all jurisdictions  $j, -j \in \{1, \dots, k\}$ ,  $m \equiv m_j = m_{-j}$ ,  $m$  is even,  $k$  is even, and each local project provides a common benefit of  $G$  to each voter within the jurisdiction to which it is provided. In each jurisdiction the production of the local project is a zero-one decision which is denoted by  $t_i^j \in \{0, 1\}$  for party  $i$  in jurisdiction  $j$ . The production of the local project in jurisdiction  $j$  requires  $p \leq m$  units of the homogeneous good. Thus, local project provision is efficient if  $G > \frac{p}{m}$ .

Each voter in each jurisdiction receives an offer of a tax or transfer from each party. For voter  $z$  in jurisdiction  $j$ , let  $t_i^{z(j)} \in \mathbb{R}_+$  denote the amount of the private homogeneous good available for consumption by the voter after party  $i$ 's commitment to any tax or transfer to that voter. We assume that voters' utilities are additively separable in the private homogeneous good and the local project. Thus, the utility that voter  $z$  in district  $j$  receives from party  $i$  who offers them  $(t_i^j, t_i^{z(j)})$  is

$$u_{z(j)}(t_i^j, t_i^{z(j)}) = t_i^{z(j)} + t_i^j G.$$

Each citizen votes for the party that provides them with the higher level of utility. In the



case that the parties provide the same level of utility to a voter, the parties win the voter with equal probability.

## Political Competition

Two parties,  $A$  and  $B$ , make simultaneous offers of transfers to each of the  $n$  voters and production commitments for the local projects in each of the  $k$  jurisdictions. Each party's payoff is its vote share. The maximum tax that can be imposed upon a voter is equal to one unit of the private homogeneous good. Thus, each voter's allocation of the private homogeneous good, after any taxes or transfers, is nonnegative.

As in the model of redistributive politics (Laslier and Picard (2002)), there are no pure strategy equilibria in the game of redistributive politics with discriminatory local projects and/or transfers. A mixed strategy for party  $i$  in the game with discriminatory local project provision and transfers which are discriminatory both across and within each jurisdiction, which we call a *redistributive-lp schedule*, is an  $n + k$ -variate distribution function  $P_i : \{0, 1\}^k \cup \mathbb{R}_+^n \rightarrow [0, 1]$ . The vector of party  $i$ 's net transfers to the  $n$  voters and the  $k$  local project choices is a random  $n + k$ -tuple drawn from  $P_i$  with the set of univariate marginal distributions  $\left\{ L_i^j, \left\{ F_i^{z(j)} \right\}_{z \in N_j} \right\}_{j=1}^k$ . Since the production decision for each local project is zero-one, the  $k$  univariate marginal distribution functions,  $\{L_i^j\}_{j=1}^k$ , one univariate marginal distribution function for each district  $j$ , are each Bernoulli distributions. The probability that party  $i$  provides the local project in district  $j$ ,  $E_{L_i^j}(x)$ , is denoted by  $\alpha_i^j$ . The remaining  $n$  univariate marginal distribution functions,  $\left\{ F_i^{z(j)} \right\}_{z=1}^n$ , one univariate marginal distribution function for each voter  $z$ , are the univariate marginal distributions of the allocations that result from party  $i$ 's net transfers to each voter  $z$ .

Each party's redistributive-lp schedule must satisfy the aggregate budget constraint. The set of budget balancing completely discriminatory redistributive-lp schedules is denoted by,

$$\mathfrak{B} = \left\{ \left\{ \left\{ t^j \right\}_{j=1}^k, \left\{ t^{z(j)} \right\}_{z=1}^n \right\} \mid \sum_{j=1}^k t^j p + \sum_{z=1}^n t^{z(j)} \leq n \right\}.$$

The support of any feasible redistributive-lp schedule is contained in  $\mathfrak{B}$ .

The two types of discriminatory ability, transfer and local project, result in four different types of discriminatory abilities: complete uniformity in transfers (both across and

within jurisdictions) and local projects; complete discriminatory ability in transfers and local projects; uniform local project provision with discriminatory transfers, uniform transfers with discriminatory local projects.

## 2.1 Complete Uniformity

There are only two possible budget-balancing pure strategies for each party  $i$  in the game under uniform project provision and uniform taxation (henceforth complete uniformity) either uniformly produce the local projects in each of the jurisdictions and spread the costs uniformly across all jurisdictions ( $c_i^j = 1$  for all  $j$  and  $t_i^z = 1 - \frac{p}{m}$  for all  $z$ ) or do nothing ( $c_i^j = 0$  for all  $j$  and  $t_i^z = 1$  for all  $z$ ).

The *game of complete uniformity*, which we label

$$CU \{G, p, m, n\},$$

is the one-shot game in which parties attempt to maximize their vote shares by simultaneously announcing completely uniform policies, under the assumption that each voter votes for the party that provides the higher utility (with ties broken by fair randomization).

## 2.2 Discriminatory Local Project Provision and Transfers

We now examine the opposite extreme in which neither the provision of the local projects nor the individual transfers are required to be uniform.

The *game of completely discriminatory redistributive politics with local projects*, which we label

$$CD \{G, p, m, n\},$$

is the one-shot game in which parties attempt to maximize their vote shares by simultaneously announcing aggregate budget balancing completely discriminatory redistributive-lp schedules, under the assumption that each voter votes for the party that provides the higher utility (with ties broken by fair randomization).

## 2.3 Uniform Project Provision and Discriminatory Transfers

As in the case of completely discriminatory redistributive politics with local projects, there are no pure strategy equilibria in the game of redistributive politics with uniform local

projects and discriminatory transfers. However, since local project provision is uniform, the  $k$  univariate marginal distributions  $\{L_i^j\}_{j=1}^k$  can be represented by a single univariate marginal distribution function,  $\{L_i\}$ . The probability that party  $i$  provides the local project,  $E_{L_i}(x)$ , is denoted by  $\alpha_i$ .

The *game of transfer discriminating redistributive politics with local projects*, which we label

$$TD\{G, p, m, n\},$$

is the one-shot game in which parties attempt to maximize their vote share by simultaneously announcing budget balancing transfer discriminating redistributive-lp schedules, under the assumption that each voter votes for the party that provides the higher utility (with ties broken by fair randomization).

## 2.4 Discriminatory Project Provision and Uniform Transfers

Similar to the two previous cases, unless local projects are sufficiently inefficient, there are no pure strategy equilibrium in the game of redistributive politics with discriminatory local projects and uniform transfers. However, since transfers are uniform, the  $n$  univariate marginal distributions  $\{F_i^{z(j)}\}_{z=1}^n$  can be represented by a single univariate marginal distribution function,  $\{F_i\}$ .

The *game of local project discriminating redistributive politics with local projects*, which we label

$$LPD\{G, p, m, n\},$$

is the one-shot game in which parties attempt to maximize their vote share by simultaneously announcing budget balancing local project discriminating redistributive-lp schedules, under the assumption that each voter votes for the party that provides the higher utility (with ties broken by fair randomization).

## 3 Results

We begin with the most basic case of complete uniformity in both the provision of local projects and their associated costs. While this is a trivial case, it provides a point of comparison with more discriminatory policy spaces.

**Proposition 1:** In any pure strategy Nash equilibrium of the game  $CU \{G, p, m, n\}$  each party must employ a strategy that uniformly produces the local projects with the necessary revenue raised through uniform taxation ( $t_i^j = 1$  for all  $j$  and  $t_i^z = 1 - \frac{p}{m}$  for all  $z$ ) if  $G \geq \frac{p}{m}$  and does nothing ( $t_i^j = 0$  for all  $j$  and  $t_i^z = 1$  for all  $z$ ) otherwise.

It is important to note that, in the absence of the traditional sources of inefficiency complete uniformity (or equivalently the absence of discriminatory policy tools) results in efficient provision of the local projects.

### 3.1 Optimal Univariate Marginal Distributions

Theorem 1 provides a set of equilibrium univariate marginal distributions for the completely discriminating game, and section 3.2 establishes the existence of  $n + k$ -variate distributions that generate the required set of equilibrium univariate marginal distributions and allocate the aggregate budget with probability one.<sup>6</sup> To avoid integer issues we will focus on the case that  $\frac{pk}{2m} \in \{0, 1, 2, \dots, \frac{k}{2}\}$ .<sup>7</sup> In the case that  $\frac{pk}{2m}$  is not an integer, a similar result holds by replacing each  $p$  in Theorem 1 with  $\frac{2m}{k} \lceil \frac{pk}{2m} \rceil$ , where the ceiling function,  $\lceil x \rceil$ , provides the smallest integer greater than or equal to  $x$ .

#### 3.1.1 Equilibrium in a Completely Discriminating System

**Theorem 1:** A Nash equilibrium of the game  $CD \{G, p, m, n\}$  is for each party to employ a mixed strategy that produces the local projects and offers net transfers according to the following univariate marginal distribution functions:

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<sup>6</sup>Given the connection between the model of redistributive politics with local projects and the Colonel Blotto game it is conjectured that the result on the uniqueness of the univariate marginal distributions in the Colonel Blotto game (Roberson (2006)) can be extended to cover this game. However, the extension of this result to the case of redistributive politics with local projects is nontrivial.

<sup>7</sup>Note that the support of any optimal completely discriminating redistributive-lp schedule,  $P$ , is contained in the boundary of  $\mathfrak{B}$ , and thus,  $\sum_{z=1}^n t^{z(j)} = n - \sum_{j=1}^k \iota^j p$  with probability one. Recall that  $E_P \left( \sum_{j=1}^k \iota^j \right) = \sum_{j=1}^k \alpha^j$ . It follows directly that in any strategy in which  $\sum_{j=1}^k \alpha^j$  is not an integer  $\sum_{j=1}^k \iota^j$  must take on at least two different integer values with positive probability. It is then straightforward to establish that any strategy in which  $\sum_{j=1}^k \alpha^j$  is not an integer is strictly dominated by a strategy in which  $\sum_{j=1}^k \alpha^j$  is an integer.

1. If  $G \geq \frac{p}{m}$ , then for each party  $i$  and jurisdiction  $j$

$$\begin{aligned} \forall z \in N_j \quad F_i^{z(j)}(x|l_i^j = 0) &= \frac{xm}{p} & x \in [0, \frac{p}{m}] \\ F_i^{z(j)}(x|l_i^j = 1) &= \frac{x}{2 - \frac{p}{m}} & x \in [0, 2 - \frac{p}{m}] \\ L_i^j(y) &= \begin{cases} \frac{p}{2m} & y = 0 \\ 1 & y = 1 \end{cases} \end{aligned}$$

2. If  $G < \frac{p}{m}$ , then for each party  $i$  and jurisdiction  $j$

$$\begin{aligned} \forall z \in N_j \quad F_i^{z(j)}(x|l_i^j = 0) &= \frac{x}{2} & x \in [0, 2] \\ L_i^j(y) &= \begin{cases} 1 & y = 0 \\ 1 & y = 1 \end{cases} \end{aligned}$$

In each case, the expected payoff for each party is  $\frac{1}{2}$  of the vote share.

**Proof:** We begin by showing that in case 1,  $G \geq \frac{p}{m}$ , these univariate marginal distributions are part of an equilibrium joint distribution. First, in any optimal strategy the budget is expended with probability one and thus in expectation. Assuming that there exists a sufficient  $n + k$ -variate distribution (which is established in section 3.2), the univariate marginals given for case 1,  $G \geq \frac{p}{m}$ , are part of a feasible joint distribution since:

$$\begin{aligned} \sum_{j=1}^k \alpha_i^j p + \sum_{j=1}^k \sum_{z \in N_j} (1 - \alpha_i^j) E_{F_i^{z(j)}|l_i^j=0} \left( x_i^{z(j)} \right) \\ + \sum_{j=1}^k \sum_{z \in N_j} \alpha_i^j E_{F_i^{z(j)}|l_i^j=1} \left( x_i^{z(j)} \right) = n \end{aligned} \quad (1)$$

The following proof is for case 1,  $G \geq \frac{p}{m}$ . Case 2,  $G < \frac{p}{m}$ , follows directly. Party  $A$ 's expected payoff from an arbitrary strategy,  $\bar{P}_A$ , in which party  $A$ 's transfers are contained in the support of party  $B$ 's transfers and with the set of univariate marginal distributions  $\left\{ \bar{L}_A^j, \left\{ \bar{F}_A^{z(j)} \right\}_{z \in N_j} \right\}_{j=1}^k$  is:

$$\begin{aligned} \frac{1}{2n} \sum_{j=1}^k \alpha_A^j p + \frac{1}{2n} \sum_{j=1}^k \sum_{z \in N_j} (1 - \alpha_A^j) \int_0^{\frac{p}{m}} x d\bar{F}_A^{z(j)}(x|l_A^j = 0) \\ + \frac{1}{2n} \sum_{j=1}^k \sum_{z \in N_j} \alpha_A^j \int_0^{2 - \frac{p}{m}} x d\bar{F}_A^{z(j)}(x|l_A^j = 1) \end{aligned}$$

But from the expectation of the budget constraint, equation (1), it follows that

$$\begin{aligned} \frac{1}{2n} \sum_{j=1}^k \alpha_A^j p + \frac{1}{2n} \sum_{j=1}^k \sum_{z \in N_j} (1 - \alpha_A^j) \int_0^{\frac{p}{m}} x d\bar{F}_A^{z(j)}(x|l_A^j = 0) \\ + \frac{1}{2n} \sum_{j=1}^k \sum_{z \in N_j} \alpha_A^j \int_0^{2 - \frac{p}{m}} x d\bar{F}_A^{z(j)}(x|l_A^j = 1) \leq \frac{1}{2} \end{aligned} \quad (2)$$

which holds with equality if and only if party  $A$  uses a completely discriminatory redistributive-lp schedule that expends the entire budget in expectation, as does the equilibrium strategy. Thus party  $A$ 's vote share cannot be increased by deviating to another strategy in which party  $A$ 's transfers are contained in the support of party  $B$ 's transfers.

Furthermore if party  $B$  is following the equilibrium strategy, then party  $A$  cannot increase its voteshare by providing transfers outside the support of party  $B$ 's transfers. Let  $\bar{s}_i^j | \iota_i^j$  be the upper bound of candidate  $i$ 's distribution of transfers for district  $j$  conditional on the public project provision. Clearly, it is never a best response for  $\bar{s}_A^{z(j)} | \iota_A^j = 1 > \bar{s}_B^{z(j)} | \iota_A^j = 1$ . By way of contradiction suppose that there exists at least one  $z(j)$  such that  $\bar{s}_A^{z(j)} | \iota_A^j = 0 > \bar{s}_B^{z(j)} | \iota_A^j = 0$  where  $\bar{s}_B^{z(j)} | \iota_A^j = 0 \leq G$ . If  $\bar{s}_A^{z(j)} | \iota_A^j = 0 \leq G$  for all  $z$ , then the result follows directly from the arguments given above. The remaining case is that for each  $z$  in which  $\bar{s}_A^{z(j)} | \iota_A^j = 0 > \bar{s}_B^{z(j)} | \iota_A^j = 0$  it is also the case that  $\bar{s}_A^{z(j)} | \iota_A^j = 0 > G$ . Let  $\bar{Z}$  denote the set of  $z(j)$  for which  $\bar{s}_A^{z(j)} | \iota_A^j = 0 > G$ . The expected vote share for party  $A$  from an arbitrary strategy  $\bar{P}_A$ , in which party  $A$ 's transfers are contained in the support of party  $B$ 's transfers except for the set  $z \in \bar{Z}$  is:

$$\begin{aligned} & \frac{1}{2n} \sum_{j=1}^k \alpha_A^j p + \frac{1}{2n} \sum_{j=1}^k \sum_{z \in N_j} (1 - \alpha_A^j) \int_0^{\frac{p}{m}} x d\bar{F}_A^{z(j)} (x | \iota_A^j = 0) \\ & \quad + \frac{1}{2n} \sum_{j=1}^k \sum_{z \in N_j} (1 - \alpha_A^j) \frac{p}{m} \left( 1 - \bar{F}_A^{z(j)} \left( \frac{p}{m} | \iota_A^j = 0 \right) \right) \\ & \quad + \frac{1}{2n} \sum_{j=1}^k \sum_{z \in N_j} (1 - \alpha_A^j) \int_G^{\bar{s}_A^{z(j)} | \iota_A^j = 0} (x - G) d\bar{F}_A^{z(j)} (x | \iota_A^j = 0) \\ & \quad + \frac{1}{2n} \sum_{j=1}^k \sum_{z \in N_j} \alpha_A^j \int_0^{2 - \frac{p}{m}} x d\bar{F}_A^{z(j)} (x | \iota_A^j = 1) \end{aligned}$$

From the expectation of the budget constraint, equation (1), it follows that

$$\begin{aligned} & \frac{1}{2n} \sum_{j=1}^k \alpha_A^j p + \frac{1}{2n} \sum_{j=1}^k \sum_{z \in N_j} \alpha_A^j \int_0^{2 - \frac{p}{m}} x d\bar{F}_A^{z(j)} (x | \iota_A^j = 1) \\ & \quad + \frac{1}{2n} \sum_{j=1}^k \sum_{z \in N_j} (1 - \alpha_A^j) \left[ \int_0^{\frac{p}{m}} x d\bar{F}_A^{z(j)} (x | \iota_A^j = 0) + \int_G^{\bar{s}_A^{z(j)} | \iota_A^j = 0} x d\bar{F}_A^{z(j)} (x | \iota_A^j = 0) \right] \\ & \quad + \frac{1}{2n} \sum_{j=1}^k \sum_{z \in N_j} (1 - \alpha_A^j) \left[ \frac{p}{m} \left( 1 - \bar{F}_A^{z(j)} \left( \frac{p}{m} | \iota_A^j = 0 \right) \right) - G \left( 1 - \bar{F}_A^{z(j)} (G | \iota_A^j = 0) \right) \right] \\ & \leq \frac{1}{2} - \frac{1}{2n} \sum_{j=1}^k \sum_{z \in N_j} (1 - \alpha_A^j) \int_{\frac{p}{m}}^G (x - \frac{p}{m}) d\bar{F}_A^{z(j)} (x | \iota_A^j = 0) \end{aligned}$$

Thus, given that party  $B$  is following the equilibrium strategy, party  $A$  cannot increase its voteshare by providing transfers outside the support of party  $B$ 's transfers. The argument for player  $B$  is symmetric.

Q.E.D.

Remarkably, a modified form of Theorem 1 applies in the case that transfers are required to be uniform within each jurisdiction but are allowed to be discriminatory across jurisdictions (i.e. for each voter  $z$  and jurisdiction  $j$ ,  $t^{z(j)} = t^j$ ). Thus, the presence of interjurisdictional transfers, or equivalently the ability to discriminate transfers across jurisdictions, is sufficient to generate the sort of equilibrium discriminatory policies given in Theorem 1. This result, stated in Corollary 1 below, follows immediately from the proof of Theorem 1.

**Corollary 1:** In the modified form of the game  $CD\{G, p, m, n\}$  in which transfers are uniform within each jurisdiction but are allowed to be non-uniform across jurisdictions, a Nash equilibrium is for each party to employ a mixed strategy that produces the local projects and offers net transfers according to the following univariate marginal distribution functions:

1. If  $G \geq \frac{p}{m}$ , then for each party  $i$  and jurisdiction  $j$

$$\begin{aligned} \forall z \in N_j \quad F_i^j(x | \nu_i^j = 0) &= \frac{xm}{p} & x \in [0, \frac{p}{m}] \\ F_i^j(x | \nu_i^j = 1) &= \frac{x}{2 - \frac{p}{m}} & x \in [0, 2 - \frac{p}{m}] \\ L_i^j(y) &= \begin{cases} \frac{p}{2m} & y = 0 \\ 1 & y = 1 \end{cases} \end{aligned}$$

2. If  $G < \frac{p}{m}$ , then for each party  $i$  and jurisdiction  $j$

$$\begin{aligned} \forall z \in N_j \quad F_i^j(x | \nu_i^j = 0) &= \frac{x}{2} & x \in [0, 2] \\ L_i^j(y) &= \begin{cases} 1 & y = 0 \\ 1 & y = 1 \end{cases} \end{aligned}$$

In each case, the expected payoff for each party is  $\frac{1}{2}$  of the vote share.

### 3.1.2 Equilibrium in a Transfer Discriminating System

The following Theorem provides a set of equilibrium univariate marginal distributions for the game of transfer discriminating redistributive politics with local projects. This result extends Lizzeri and Persico's (1998, 2001) model of redistributive politics with project provision to allow for a finite population of voters. In L-P, as in Myerson (1993), each party's strategy space for net transfers is a single univariate distribution function over  $\mathbb{R}_+$ , where the measure

over each interval is interpreted as the fraction of voters for whom a party's transfers have value in that interval. Conversely in this paper, as in Laslier and Picard (2002), each party's strategy space for net transfers is an  $n$ -variate distribution function over  $\mathbb{R}_+^n$  where the set of  $n$  univariate marginal distributions is interpreted as the randomization in transfers individually targeted at each of the  $n$  voters. Section 3.2 establishes the existence of  $n + 1$ -variate distributions for each jurisdiction  $j$  that generate the required set of equilibrium univariate marginal distributions and allocate the jurisdiction's budget with probability one.

**Theorem 2:** A Nash equilibrium of the game  $TD\{G, p, m, n\}$  is for each party to employ a mixed strategy which produces the local project and offers net transfers according to the following univariate distribution functions:

1. If  $\frac{p}{m} \leq G \leq \frac{2p}{m}$ , then for party  $i$

$$L_i(y) = \begin{cases} \frac{2p}{m} - G & y = 0 \\ 1 & y = 1 \end{cases}$$

$\forall z$

$$F_i^{z(j)}(x|l_i = 0) = \begin{cases} \frac{1}{2} \left( \frac{x}{\frac{2p}{m} - G} \right) & 0 \leq x < \frac{2p}{m} - G \\ \frac{1}{2} & \frac{2p}{m} - G \leq x < G + 2 - \frac{2p}{m} \\ \frac{1}{2} \left( 1 + \frac{x - G - 2 + \frac{2p}{m}}{\frac{2p}{m} - G} \right) & G + 2 - \frac{2p}{m} \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$

and

$$F_i^{z(j)}(x|l_i = 1) = \frac{x}{2 - \frac{2p}{m}} \quad x \in \left[0, 2 - \frac{2p}{m}\right]$$

2. If  $G > \frac{2p}{m}$ , then both parties provide the local project with certainty, where  $F_i^{z(j)}(x|l_i = 1)$  is given above.

3. If  $G < \frac{p}{m}$ , then with certainty, neither party provides the local project and

$\forall z$

$$F_i^{z(j)}(x|l_i = 0) = \frac{x}{2} \quad x \in [0, 2]$$

In all cases, the expected payoff for each party is  $\frac{1}{2}$  of the vote share.



**Proof:** We begin by showing that in case 1,  $\frac{p}{m} \leq G \leq \frac{2p}{m}$ , these univariate marginals are part of an equilibrium joint distribution. First, in any optimal strategy the budget is expended with probability one and thus in expectation. Assuming that there exists a sufficient  $n$ -variate distribution (which is established in section 3.2), the univariate marginals specified for case 1,  $\frac{p}{m} \leq G \leq \frac{2p}{m}$ , are part of a feasible joint distribution since:

$$\begin{aligned} kp + \sum_{z=1}^n E_{F_i^{z(j)}|l_i=1} \left( x_i^{z(j)} \right) &= n \\ \text{and } \sum_{z=1}^n E_{F_i^{z(j)}|l_i=0} \left( x_i^{z(j)} \right) &= n \end{aligned} \quad (3)$$

The following proof is for case 1,  $\frac{p}{m} \leq G \leq \frac{2p}{m}$ . Cases 2 and 3,  $G > \frac{2p}{m}$  and  $G < \frac{p}{m}$  respectively, follow directly. Given that party  $B$  is following the equilibrium strategy, it is never a best response for party  $A$  to provide transfers outside the support of party  $B$ 's transfers. Thus, party  $A$ 's expected payoff from an arbitrary strategy  $\bar{H}_A$ , with the set of univariate marginal distributions  $\left\{ \bar{L}_i, \left\{ \bar{F}_i^{z(j)} \right\}_{z=1}^n \right\}$ , is

$$\begin{aligned} &\alpha_A \left( 1 + G - \frac{2p}{m} \right) \frac{1}{n} \sum_{z=1}^n \int_0^{2-\frac{2p}{m}} \frac{x}{2-\frac{2p}{m}} d\bar{F}_A^{z(j)}(x|l_A = 1) \\ &\quad + \alpha_A \left( \frac{2p}{m} - G \right) \frac{1}{2} \\ &+ (1 - \alpha_A) \left( 1 + G - \frac{2p}{m} \right) \frac{1}{m} \sum_{z=1}^n \left[ 1 - \bar{F}_A^{z(j)} \left( G + 2 - \frac{2p}{m} | l_A = 0 \right) \right] \\ &+ (1 - \alpha_A) \left( \frac{2p}{m} - G \right) \frac{1}{m} \sum_{z=1}^n \int_0^2 F_B^{z(j)}(x|l_B = 0) d\bar{F}_A^{z(j)}(x|l_A = 0) \end{aligned}$$

Simplifying, party  $A$ 's expected payoff is

$$\begin{aligned} &\alpha_A \left( 1 + G - \frac{2p}{m} \right) \frac{1}{2(m-p)} \sum_{z=1}^n \int_0^{2-\frac{2p}{m}} x d\bar{F}_A^{z(j)}(x|l_A = 1) \\ &\quad + \alpha_A \left( \frac{2p}{m} - G \right) \frac{1}{2} \\ &+ (1 - \alpha_A) \frac{1}{2n} \sum_{z=1}^n \int_0^{\frac{2p}{m}-G} x d\bar{F}_A^{z(j)}(x|l_B = 0) \\ &+ (1 - \alpha_A) \frac{1}{2n} \sum_{z=1}^n \int_{G+2-\frac{2p}{m}}^2 x d\bar{F}_A^{z(j)}(x|l_B = 0) \end{aligned}$$

But from the expectation of the budget constraint, equation (3), it follows that player  $A$ 's expected payoff is less than or equal to  $\frac{1}{2}$  and holds with equality if and only if party  $A$  uses a transfer discriminating redistributive-lp schedule that does not provide transfers outside the support of the equilibrium strategy and expends the entire budget in expectation, as does the equilibrium strategy. Thus party  $A$ 's vote share cannot be increased by deviating to another strategy. The argument for party  $B$  is symmetric.

Q.E.D.

As was the case with both discriminatory projects and transfers, a modified form of Theorem 2 applies in the case that transfers are required to be uniform within each jurisdiction but are allowed to be discriminatory across jurisdictions (i.e. for each voter  $z$  and jurisdiction  $j$ ,  $t^{z(j)} = t^j$ ). Thus, the presence of interjurisdictional transfers, or equivalently the ability to discriminate transfers across jurisdictions, is sufficient to generate the sort of equilibrium policies given in Theorem 2, and this result, stated in Corollary 2 below, follows immediately from the proof of Theorem 2.

**Corollary 2:** In the modified form of the game  $TD\{G, p, m, n\}$  in which transfers are uniform within each jurisdiction but are allowed to be non-uniform across jurisdictions, a Nash equilibrium is for each party to employ a mixed strategy which produces the local project and offers net transfers according to the following univariate distribution functions:

1. If  $\frac{p}{m} \leq G \leq \frac{2p}{m}$ , then for party  $i$

$$L_i(y) = \begin{cases} \frac{2p}{m} - G & y = 0 \\ 1 & y = 1 \end{cases}$$

for each  $j$  and  $\forall z \in N_j$

$$F_i^j(x|\iota_i = 0) = \begin{cases} \frac{1}{2} \left( \frac{x}{\frac{2p}{m} - G} \right) & 0 \leq x < \frac{2p}{m} - G \\ \frac{1}{2} & \frac{2p}{m} - G \leq x < G + 2 - \frac{2p}{m} \\ \frac{1}{2} \left( 1 + \frac{x - G - 2 + \frac{2p}{m}}{\frac{2p}{m} - G} \right) & G + 2 - \frac{2p}{m} \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$

and

$$F_i^j(x|\iota_i = 1) = \frac{x}{2 - \frac{2p}{m}} \quad x \in \left[0, 2 - \frac{2p}{m}\right]$$

2. If  $G > \frac{2p}{m}$ , then both parties provide the local project with certainty, where  $F_i^j(x|\iota_i = 1)$  is given above.
3. If  $G < \frac{p}{m}$ , then with certainty, neither party provides the local project and for each  $j$  and  $\forall z \in N_j$

$$F_i^j(x|\iota_i = 0) = \frac{x}{2} \quad x \in [0, 2]$$

In all cases, the expected payoff for each party is  $\frac{1}{2}$  of the vote share.

### 3.1.3 Equilibrium in a Local Project Discriminating System

The following Theorem provides the characterization of equilibrium joint distributions for the game of local project discriminating redistributive politics with local projects.

**Theorem 3:** In all Nash equilibria of the game  $LPD \{G, p, m, n\}$  each party employs a mixed strategy which produces the local project and offers net transfers according to the following characterization of the joint distribution function:

If  $G > \frac{p}{m}$ , then for party  $i$

$$L_i(y) = \begin{cases} \frac{1}{2} & y = 0 \\ 1 & y = 1 \end{cases}$$

$$F_i\left(x \mid \sum_{j=1}^k \iota_i^j\right) = \begin{cases} 0 & 0 \leq x < 1 - \frac{p}{n} \frac{\sum_{j=1}^k \iota_i^j}{n} \\ 1 & x \geq 1 - \frac{p}{n} \frac{\sum_{j=1}^k \iota_i^j}{n} \end{cases}$$

and letting the distribution of  $\sum_{j=1}^k \iota_i^j$  by denoted by  $G_i$

$$G_i(z) = \begin{cases} 0 & z < 0 \\ \delta_0 & z = 0 \\ \delta_0 + \delta_1 & z = 1 \\ \vdots & \vdots \\ \sum_{j=0}^{\phi} \delta_j & z = \phi \\ \vdots & \vdots \\ 1 & z \geq k - 1 \end{cases}$$

where  $\delta_j \geq 0$  for all  $j \in \{0, 1, 2, \dots, k\}$  and  $\sum_{j=0}^k \delta_j = 1$ . The exact value of each  $\delta_j$  depends critically on the number of jurisdictions,  $k$ , and is summarized in the Appendix.

The expected payoff for each party is  $\frac{1}{2}$  of the vote share.

The proof of this theorem is provided in the Appendix.

## 3.2 Existence of Equilibrium Joint Distributions

Subject to the constraint that there exist joint distribution functions that expend the respective budgets with probability one and that provide the respective sets of equilibrium univariate marginal distributions, Theorems 1 and 2 provide sets of equilibrium univariate marginal distributions in the games of completely discriminating and transfer discriminating redistributive politics with local projects. The following two subsections establish the existence of sufficient joint distributions in case 1 of Theorem 1,  $G \geq \frac{p}{m}$ , and case 1 of Theorem 2,  $\frac{p}{m} \leq G \leq \frac{2p}{m}$ . The remaining cases follow directly.<sup>8</sup>

### 3.2.1 Completely Discriminating System

We begin with the  $k$ -variate marginal distribution function of  $P_i$ , the completely discriminating redistributive-lp schedule, with the set of univariate marginal distribution functions  $\{L_i^j\}_{j=1}^k$ . This  $k$ -variate marginal distribution specifies the provision of the local projects across the  $k$  jurisdictions. Given that  $\frac{pk}{2m} \in \{0, 1, 2, \dots, \frac{k}{2}\}$ , it is clear that there are  ${}_k C_{\frac{pk}{2m}}$  partitions of the  $k$  jurisdictions into a disjoint  $\frac{pk}{2m}$ -subset and  $k - \frac{pk}{2m}$ -subset. A sufficient  $k$ -variate marginal distribution function for  $P_i$  is formed by the  $k$ -variate distribution function which places equal probability on each of these partitions and provides the local project in each jurisdiction contained in the  $k - \frac{pk}{2m}$ -subset.

To determine an  $n$ -variate marginal distribution function of  $P_i$ , which specifies the net transfers across the  $n$  voters generating the set of univariate marginal distribution functions  $\{F_i^{z(j)}\}_{z=1}^n$ , note that in any realization of the  $k$  production decisions in the  $k$ -variate marginal distribution described above  $\sum_{j=1}^k v_i^j = k - \frac{pk}{2m}$  with probability one. Given that  $m$  is even, in each of the  $k - \frac{pk}{2m}$  jurisdictions in which the local projects are produced the voters may be partitioned into  $\frac{km}{2} - \frac{pk}{4}$  disjoint subsets with 2 voters each. In each subset of 2 voters,  $z$  and  $z'$  in jurisdiction  $j$ , suppose that each party independently applies the

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<sup>8</sup>In the modified form of the game  $CD\{G, p, m, n\}$  in which transfers are uniform within each jurisdiction but are allowed to be non-uniform across jurisdictions (as in Corollary 1), a similar result applies. However, this case requires a slightly more complicated construction that follows along the lines of Roberson (2006). In the modified form of the game  $TD\{G, p, m, n\}$  in which transfers are uniform within each jurisdiction but are allowed to be non-uniform across jurisdictions (as in Corollary 2), the result follows directly.

Fréchet-Hoeffding lower bound 2-copula,<sup>9</sup>

$$W \left( F_i^{z(j)} \left( x_i^{z(j)} | \iota_i^j = 1 \right), F_i^{z'(j)} \left( x_i^{z'(j)} | \iota_i^j = 1 \right) \right) = \max \left\{ F_i^{z(j)} \left( x_i^{z(j)} | \iota_i^j = 1 \right) + F_i^{z'(j)} \left( x_i^{z'(j)} | \iota_i^j = 1 \right) - 1, 0 \right\},$$

where for each party  $i$  and voter  $z$  in jurisdiction  $j$ ,  $F_i^{z(j)} \left( x_i^{z(j)} | \iota_i^j = 1 \right) = \frac{x_i^{z(j)}}{2 - \frac{p}{m}}$  for  $x_i^{z(j)} \in [0, 2 - \frac{p}{m}]$ .<sup>10</sup> Recalling that the support of a bivariate distribution function,  $H$ , is the complement of the union of all open sets of  $\mathbb{R}^2$  with  $H$ -volume zero, it follows directly that the support of the bivariate distribution, formed by the 2-copula  $W$  and the univariate distribution  $F_i^{z(j)} \left( x_i^{z(j)} | \iota_i^j = 1 \right)$ , is given by  $\{(x^1, x^2) \in \mathbb{R}_+^2 | x^1 + x^2 = 2 - \frac{p}{m}\}$ . Thus, each of these  $\frac{km}{2} - \frac{pk}{4}$  bivariate distributions generates transfers that sum to  $2 - \frac{p}{m}$ .

Similarly, in each of the  $\frac{pk}{2m}$  jurisdictions in which the local project is not produced the voters may be partitioned into  $\frac{pk}{4}$  disjoint subsets with 2 voters each. In each subset of 2 voters,  $z$  and  $z'$  in jurisdiction  $j$ , suppose that each party independently applies the Fréchet-Hoeffding lower bound 2-copula,

$$W \left( F_i^{z(j)} \left( x_i^{z(j)} | \iota_i^j = 0 \right), F_i^{z'(j)} \left( x_i^{z'(j)} | \iota_i^j = 0 \right) \right) = \max \left\{ F_i^{z(j)} \left( x_i^{z(j)} | \iota_i^j = 0 \right) + F_i^{z'(j)} \left( x_i^{z'(j)} | \iota_i^j = 0 \right) - 1, 0 \right\},$$

where for each party  $i$  and voter  $z$  in jurisdiction  $j$ ,  $F_i^{z(j)} \left( x_i^{z(j)} | \iota_i^j = 0 \right) = \frac{x_i^{z(j)}}{\frac{p}{m}}$  for  $x_i^{z(j)} \in [0, \frac{p}{m}]$ . It follows directly that the support of the bivariate distribution, formed by the 2-copula  $W$  and the univariate distribution  $F_i^{z(j)} \left( x_i^{z(j)} | \iota_i^j = 0 \right)$ , is given by  $\{(x^1, x^2) \in \mathbb{R}_+^2 | x^1 + x^2 = \frac{p}{m}\}$ . Thus, each of these  $\frac{pk}{4}$  bivariate distributions generate transfers that sum to  $\frac{p}{m}$ . Given these  $\frac{pk}{4}$  bivariate distributions, the  $\frac{km}{2} - \frac{pk}{4}$  bivariate distributions described above, and that all  $\frac{n}{2}$  of these bivariate distributions are independent, it is clear that the support across all  $n$  voters is contained in  $\{\mathbf{x} \in \mathbb{R}_+^n | \sum_{i=1}^n x^i = n - pk + \frac{p^2k}{2m}\}$ . Thus, we have the following.

**Proposition 2:** Let  $P_i^*$  denote the  $n + k$ -variate distribution function of party  $i$  with support contained in  $\left\{ \left\{ \iota^j, \{t^{z(j)}\}_{z \in N_j} \right\}_{j=1}^k \mid \sum_{j=1}^k \left( \iota^j p + \sum_{z \in N_j} t^{z(j)} \right) = n \right\}$  induced by the construction outlined above. Then,  $P_i^*$  generates the equilibrium

<sup>9</sup>For an introduction to copulas see Nelsen (1999)

<sup>10</sup>See Kvasov (2005), who in a related problem discusses the issue of partitioning joint distributions into independent sets of lower dimension.

univariate marginal distributions in Theorem 1 and expands the aggregate budget with probability one.  $(P_1^*, P_2^*)$ , therefore, constitutes a Nash equilibrium of the completely discriminating game of redistributive politics with local projects,  $CD(G, p, m, n)$ .

### 3.2.2 Transfer Discriminating System

Recall that, by assumption,  $n$  is even. Beginning with the case that the local project is provided in each jurisdiction, there are  $\frac{n}{2}$  disjoint subsets with 2 voters each. In each subset of 2 voters,  $z$  and  $z'$ , suppose that each party independently applies the Fréchet-Hoeffding lower bound 2-copula,

$$W(F_i^z(x_i^z | \iota_i = 1), F_i^{z'}(x_i^{z'} | \iota_i = 1)) = \max\{F_i^z(x_i^z | \iota_i = 1) + F_i^{z'}(x_i^{z'} | \iota_i = 1) - 1, 0\},$$

where for each party  $i$  and voter  $z$ ,  $F_i^z(x_i^z | \iota_i = 1) = \frac{x_i^z}{2 - \frac{2p}{m}}$  for  $x_i^z \in [0, 2 - \frac{2p}{m}]$ . It follows directly that the support of the bivariate distribution, formed by the 2-copula  $W$  and the univariate distribution  $F_i^z(x_i^z | \iota_i = 1)$ , is given by  $\{(x^z, x^{z'}) \in \mathbb{R}_+^2 | x^z + x^{z'} = 2 - \frac{2p}{m}\}$ . Given that each of the  $\frac{n}{2}$  bivariate distributions generate net transfers that sum to  $2 - \frac{2p}{m}$ , and that these bivariate distributions are independent, the support across all  $n$  voters is contained in  $\{\mathbf{x} \in \mathbb{R}_+^n | \sum_{z=1}^n x^z = n - pk\}$ .

Similarly, in the case that the local project is not provided, in each subset of 2 voters,  $z$  and  $z'$ , suppose that each party independently applies the Fréchet-Hoeffding lower bound 2-copula,

$$W(F_i^z(x_i^z | \iota_i = 0), F_i^{z'}(x_i^{z'} | \iota_i = 0)) = \max\{F_i^z(x_i^z | \iota_i = 0) + F_i^{z'}(x_i^{z'} | \iota_i = 0) - 1, 0\},$$

where for each party  $i$  and voter  $z$ ,

$$F_i^z(x_i^z | \iota_i = 0) = \begin{cases} \frac{1}{2} \left( \frac{x_i^z}{\frac{2p}{m} - G} \right) & 0 \leq x_i^z < \frac{2p}{m} - G \\ \frac{1}{2} & \frac{2p}{m} - G \leq x_i^z < G + 2 - \frac{2p}{m} \\ \frac{1}{2} \left( 1 + \frac{x_i^z - G - 2 + \frac{2p}{m}}{\frac{2p}{m} - G} \right) & G + 2 - \frac{2p}{m} \leq x_i^z < 2 \\ 1 & x \geq 2 \end{cases}$$

It follows directly that the support of the bivariate distribution, formed by the 2-copula  $W$  and the univariate distribution  $F_i^z(x_i^z | \iota_i = 0)$ , is contained in  $\{(x^z, x^{z'}) \in \mathbb{R}_+^2 | x^z + x^{z'} = 2\}$ .

Given that each of the  $\frac{n}{2}$  bivariate distributions generate transfers that sum to 2, and that these bivariate distributions are independent the support across all  $n$  voters is contained in  $\{\mathbf{x} \in \mathbb{R}_+^n \mid \sum_{z=1}^n x^z = n\}$ . Given this we have the following.

**Proposition 3:** Let  $H_i^*$  denote the  $n + 1$ -variate distribution function of party  $i$  with support contained in  $\{\{\iota, \{t^z\}_{z=1}^n\} \mid \iota p + \sum_{z=1}^n t^z = n\}$  induced by the construction outlined above. Then,  $H_i^*$  generates the equilibrium univariate marginal distributions in Theorem 2 and spends the aggregate budget with probability one. Therefore,  $(H_1^*, H_2^*)$  constitutes a Nash equilibrium of the transfer discriminating game of redistributive politics with local projects,  $TD(G, p, m, n)$ .

### 3.3 Effects of Discriminatory Tools on Efficiency

We now apply the equilibrium characterizations of the systems with various discriminatory abilities to compare the inefficiencies that arise in each. The primary criterion for comparing the efficiency of these systems is the likelihood that each party commits to the production of the local project in each jurisdiction when the provision of the local project is efficient. The related criterion of ex-ante expected utility of each voter yields similar results. Corollary 3 compares the inefficiencies that arises under discriminatory local project provision and net transfers to those which arise under discriminatory local project provision and uniform transfers.

**Corollary 3:** Under discriminatory local project provision and net transfers the probability that each party commits to the production of the local project in a given jurisdiction when its provision is efficient is  $1 - \frac{p}{2m}$ . Under local project discrimination and uniform net transfers the probability that each party commits to the production of the local project when its provision is efficient is  $\frac{1}{2}$ . Thus, the probability that each party commits to the production of the local project when its provision is efficient is always higher in the more discriminatory system.

In all systems with discriminatory ability, the political parties face a tradeoff between the efficiency of local projects and the pork-barrel incentive to “cultivate favored minorities.” In the case that local project provision is discriminatory and net transfers are uniform, the tactical targeting of local projects is the only available means of strategic discrimination.

Moving from that system to one in which both local projects and net transfers are discriminatory reduces the effectiveness of local project discrimination, or equivalently raises the costs of forgoing efficient local projects. Thus, the greater discriminatory ability actually increases the efficiency of local project provision.

Corollary 4 compares the inefficiencies that arises under discriminatory local project provision and net transfers to those which arise under discriminatory net transfers and uniform local project provision.

**Corollary 4:** Under discriminatory local project provision and net transfers the probability that each party commits to the production of the local project in a given jurisdiction when its provision is efficient is  $1 - \frac{p}{2m}$ . Under transfer discrimination and uniform local project provision the probability that each party commits to the production of the local project when its provision is efficient is  $1 + G - \frac{2p}{m}$ . Thus, the probability that each party commits to the production of the local project when its provision is efficient is higher in the more discriminatory system if  $\frac{p}{m} < G < \frac{3p}{2m}$  and higher in the less discriminatory system if  $G > \frac{3p}{2m}$ .

In the case that net transfers are discriminatory and local project provision is uniform, the tactical targeting of net transfers is the only available means of strategic discrimination. Barring the case that local projects are excessively efficient, moving from that system to one in which both local projects and net transfers are discriminatory also reduces the effectiveness of local project discrimination, or equivalently raises the costs of forgoing efficient local projects. Thus, the greater discriminatory ability actually increases the efficiency of local project provision. However, once the benefits from the local projects are sufficiently large ( $G > \frac{3p}{2m}$ ), uniformity of local project provision leads to efficiency gains. This follows from the fact that as the benefits provided by the local projects become sufficiently large, the costs of not offering the efficient local projects become increasingly larger in the case of uniform of local project provision. In the case in which both local projects and net transfers are discriminatory, there is no corresponding change in the cost of forgoing efficient local projects.

Together Corollaries 3 and 4 demonstrate that, barring excessively efficient local projects, greater discriminatory ability actually increases the efficiency of local project provision by weakening the incentives for pork-barrel politics. As noted in Proposition 1 the absence of the



traditional sources of inefficiency and any discriminatory ability results in complete efficiency in the provision of local projects. Thus, discriminatory policy tools such as interjurisdictional transfers create an incentive for pork-barrel politics that results in inefficiency, but greater discriminatory ability actually weakens these incentives and results in gains in efficiency.

## 4 Conclusion

This paper extends Laslier and Picard's (2002) model of redistributive politics with a finite population of voters to allow for redistributive competition with local projects and shows that the level of discriminatory ability is an important determinant in the efficiency of local project provision. Remarkably, little has been done to examine the incentives for pork-barrel politics in a fiscal federalism setting. This paper finds that, in such a setting, political parties face a tradeoff between efficiency and discriminatory ability, and in equilibrium the presence of discriminatory policy tools results in inefficiency in the provision of the local projects. However, as the set of available policies allows for greater discriminatory ability the level of inefficiency in the provision of the local projects decreases. That is, greater discriminatory ability increases the costs of forgoing efficient local projects. Thus, this paper highlights an important and purely strategic role for discriminatory policy instruments such as interjurisdictional transfers that provides a stark contrast with the view that such policy instruments are solely a tool for fiscal equalization.

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## Appendix

This appendix contains the proof of Theorem 3. We begin by showing that if,  $G \geq \frac{p}{m}$ , the univariate marginals given in Theorem 3 are part of an equilibrium joint distribution. First, in any optimal strategy the budget is expended with probability one. The univariate marginals specified are part of a feasible joint distribution since for each player  $i$  and each random  $k + 1$ -tuple drawn from  $R_i$ :

$$p \sum_{j=1}^k t_i^j + \sum_{i=1}^n \left[ 1 - \frac{p \sum_{j=1}^k t_i^j}{n} \right] = n.$$

Additionally, it is clear that there exist  $k + 1$ -variate distribution functions which provide these univariate marginal distributions, the distribution for  $\sum_{j=1}^k t_i^j$ , and that satisfy the aggregate budget constraint with probability one.

Three important expressions that will notationally simplify the following analysis are given below. These expressions represent the expected payoffs to party  $A$  from playing an arbitrary pure strategy,  $\left\{ \{ \bar{t}_A^j \}_{j=1}^k, 1 - \frac{p \sum_{j=1}^k \bar{t}_A^j}{n} \right\}$ , when party  $B$  is using a mixed strategy in which there exists a  $\beta \in \{0, 1, \dots, k\}$ , such that for all  $j$

$$L_B^j(y) = \begin{cases} 1 - \frac{\beta}{k} & y = 0 \\ 1 & y = 1 \end{cases}$$

and

$$G_B(z) = \begin{cases} 0 & z \leq \beta \\ 1 & z \geq \beta \end{cases}$$

where  $G_B$  denotes the distribution of  $\sum_{j=1}^k t_B^j$ . Thus, the probability that each jurisdiction  $j$  receives the local project from party  $B$  is  $\frac{\beta}{k}$  and  $\sum_{j=1}^k t_B^j = \beta$  with probability one.

Let  $\alpha = \sum_{j=1}^k \bar{t}_A^j$  if  $\alpha < \beta$  then  $1 - \frac{p\alpha}{n} > 1 - \frac{p\beta}{n}$ . Thus party  $A$  wins every jurisdiction in which they provide the local project and all jurisdictions in which both parties do not provide the local project, and party  $A$  loses every jurisdiction in which they do not provide the local project and party  $B$  does (i.e.  $1 - \frac{p\alpha}{n} < G + 1 - \frac{p\beta}{n}$  since  $G \geq \frac{p}{m}$ ). Thus the expected payoff to player  $A$  when  $\alpha < \beta$  is:

$$\pi_A|_{\alpha < \beta} = \frac{1}{k^2} [\alpha\beta + \alpha(k - \beta) + (k - \alpha)(k - \beta)] = \frac{1}{k^2} [\alpha\beta + k(k - \beta)] \quad (4)$$

Similarly if  $\alpha > \beta$  then  $1 - \frac{p\alpha}{n} < 1 - \frac{p\beta}{n}$ . Thus party  $A$  loses every jurisdiction in which they do not provide the local project and all jurisdictions in which both parties do provide the

local project, and party  $A$  wins every jurisdiction in which they do provide the local project and party  $B$  does not (i.e.  $G + 1 - \frac{p\alpha}{n} > 1 - \frac{p\beta}{n}$  since  $G \geq \frac{p}{m}$ ). Thus the expected payoff to player  $A$  when  $\alpha > \beta$  is:

$$\pi_A|_{\alpha>\beta} = \frac{1}{k^2} [\alpha(k - \beta)] \quad (5)$$

Finally, if  $\alpha = \beta$  then  $1 - \frac{p\alpha}{n} = 1 - \frac{p\beta}{n}$ . Clearly party  $A$  ties in every jurisdiction in which both parties provide the local project and all jurisdictions in which both parties do not provide the local project. Party  $A$  wins every jurisdiction in which they provide the local project and party  $B$  does not, and party  $A$  loses in every jurisdiction in which they do not provide the local project and party  $B$  does. Thus the expected payoff to player  $A$  when  $\alpha = \beta$  is:

$$\pi_A|_{\alpha=\beta} = \frac{1}{k^2} \left[ \frac{1}{2}\alpha\beta + \alpha(k - \beta) + \frac{1}{2}(k - \alpha)(k - \beta) \right] = \frac{1}{2} \quad (6)$$

Suppose that there exists a symmetric mixed strategy equilibrium  $G$  ( $G = G_A = G_B$ ), over  $\alpha$  and  $\beta$  respectively,

$$G(z) = \begin{cases} 0 & z < 0 \\ \delta_0 & z = 0 \\ \delta_0 + \delta_1 & z = 1 \\ \vdots & \vdots \\ \sum_{j=0}^{\phi} \delta_j & z = \phi \\ \vdots & \vdots \\ 1 & z \geq k - 1 \end{cases}$$

where  $\delta_j \geq 0$  for all  $j \in \{0, 1, 2, \dots, k\}$  and  $\sum_{j=0}^k \delta_j = 1$ .

Given that party  $B$  is following such a strategy, party  $A$ 's expected payoff from an arbitrary pure strategy  $\left\{ \{\tilde{t}_A^j\}_{j=1}^k, 1 - \frac{p\alpha}{n} \right\}$  where  $\alpha = \sum_{j=1}^k \tilde{t}_A^j$ , is

$$\pi_A|_{\alpha} = \sum_{j<\alpha} \delta_j \pi_A|_{\alpha>j} + \delta_{\alpha} \frac{1}{2} + \sum_{j>\alpha} \delta_j \pi_A|_{\alpha<j}.$$

Furthermore, the equilibrium payoffs must be attained over the support of the strategy. Thus, for each  $i$ ,  $\pi_i|_z = \frac{1}{2}$  for each  $z$  in the support of the symmetric equilibrium strategy. It must also be the case that  $\pi_i|_z \leq \frac{1}{2}$  for each  $z$  not in the support of the symmetric equilibrium strategy.

Suppose that there exists an equilibrium in which  $\pi_i|_z = \frac{1}{2}$  for all  $z$ ,  $\sum_{j=0}^k \delta_j \binom{j}{k} = \frac{1}{2}$ , and  $\delta_0 = \delta_k = 0$ . The remainder of the appendix establishes that there exists such an equilibrium.<sup>11</sup>

Given that  $\pi_i|_z = \frac{1}{2}$  for all  $z$ , it follows that for all  $z < k$ ,  $\pi_i|_z - \pi_i|_{z+1} = 0$  or equivalently

$$\sum_{j < z} \delta_j \left(1 - \frac{j}{k}\right) + \sum_{j > z+1} \delta_j \binom{j}{k} = (\delta_z + \delta_{z+1}) \left(\frac{k}{2} - (z+1) + \frac{z(z+1)}{k}\right).$$

Similarly, for all  $z > 0$ ,  $\pi_i|_{z-1} - \pi_i|_z = 0$  or equivalently

$$\sum_{j < z-1} \delta_j \left(1 - \frac{j}{k}\right) + \sum_{j > z} \delta_j \binom{j}{k} = (\delta_{z-1} + \delta_z) \left(\frac{k}{2} - z + \frac{(z-1)z}{k}\right).$$

Recalling that  $\sum_{j=0}^k \delta_j \binom{j}{k} = \frac{1}{2}$  and  $\delta_0 = \delta_k = 0$ , for  $z = 0$ ,  $\sum_{j > 1} \delta_j \binom{j}{k} = (\delta_0 + \delta_1) \left(\frac{k}{2} - 1\right)$ . Solving for  $\delta_1$  yields  $\delta_1 = \frac{1}{2\left(\frac{k}{2}-1+\frac{1}{k}\right)}$ . Similarly, for  $z = k$ ,  $\sum_{j < k-1} \delta_j \left(1 - \frac{j}{k}\right) = (\delta_{k-1} + \delta_k) \left(\frac{k}{2} - k + \frac{(k-1)k}{k}\right)$ . Thus,  $\delta_{k-1} = \delta_1 = \frac{1}{2\left(\frac{k}{2}-1+\frac{1}{k}\right)}$ . A similar argument establishes that for all  $j$ ,  $\delta_j = \delta_{k-j}$ .

Given that  $\delta_j = \delta_{k-j}$  for all  $j$  and recalling that  $k$  is even, it follows that

$$\sum_{j=0}^k \delta_j \binom{j}{k} = \sum_{j=0}^{\frac{k}{2}-1} \delta_j \left[\frac{j}{k} + \frac{k-j}{k}\right] + \delta_{\frac{k}{2}} \left[\frac{k}{2}\right] = \sum_{j=0}^{\frac{k}{2}-1} \delta_j + \frac{1}{2} \delta_{\frac{k}{2}} = \frac{1}{2} \sum_{j=0}^k \delta_j = \frac{1}{2}.$$

Lastly, note that for  $k = 2$ ,  $\delta_0 = \delta_2 = 0$ ,  $\delta_1 = \frac{1}{2\left(\frac{2}{2}-1+\frac{1}{2}\right)} = 1$ , and, thus,  $\sum_{j=1}^k \delta_j = 1$  for  $k = 2$ . A straightforward proof by induction establishes that  $\sum_{j=1}^k \delta_j = 1$  for all even  $k > 2$ .

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<sup>11</sup>In the case of a continuum of jurisdictions, this equilibrium would be the solution to a first-order linear differential equation with standard boundary conditions. Given that there are a finite number of jurisdictions, the equilibrium is modified accordingly.