

Siebert, Ralph

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**discussion papers**

**Credible Vertical Preemption**

Ralph Siebert \*

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# Credible Vertical Preemption

Ralph Siebert\*

WZB, Harvard University and CEPR

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## Abstract

This study shows that preemptive investment in product proliferation is subject to a commitment problem that is not constrained to models of horizontal product differentiation, but applies to vertical product differentiation settings as well. We investigate the incentives of firms producing high- or low- quality goods that decide simultaneously to introduce new products in different quality areas. In addition, we analyze whether these keep or withdraw similar existing products from the market. The study shows that the introduction of new products depends on the credibility of firms' innovation strategies. The high-quality firm's strategy to proliferate the product space in order to deter the low-quality firm from introducing a new product is not credible. Innovators always withdraw their existing products from the market in order to reduce price competition or to avoid cannibalizing demand for their own products.

JEL: L11, L13, O31, O32.

Keywords: New Product Introduction, Preemption, Product Innovation, Product Proliferation, Vertical Product Differentiation.

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\*Ralph Siebert, Harvard University, Department of Economics, Littauer Center, Cambridge, MA, e-mail: rsiebert@nber.org.

# 1 Introduction

In many industries incumbents introduce new products of varying qualities. The process of new product introduction usually occurs on a regular basis. We frequently observe the same pattern of new product introduction: incumbents introduce new products of higher quality than their existing products and often withdraw similar products from the market. This phenomenon is extensive in the electronics and telecommunications markets, where technological progress stimulates the introduction of new products. For example, the development in PC hardware generates faster processors, which regularly displace previous processors. Nowadays, almost every new PC introduced by an incumbent is equipped with a version of the latest processor. Other examples, among many others, are the introduction of new computer screens with higher resolution, new application software, e.g., improved versions of Microsoft Windows and Linux, the introduction of new cell phones with longer ‘stand by time’, and the introduction of new DVD or MP3 players. Hence, in markets characterized by vertical product differentiation, firms frequently introduce new higher-quality versions that replace the previous ones.<sup>1</sup>

In contrast, horizontal differentiation models show that firms have the opportunity to proliferate their product space in order to prevent rivals from introducing a new product (Prescott and Visscher, 1977; Schmalensee, 1978; and Eaton and Lipsey, 1979). Hence, innovators may keep existing products on the market and tend to proliferate the product space, in order to preempt their rival’s innovation. Judd (1985) emphasizes the relevance of commitment when product proliferation is used as a deterrence strategy. He shows that firms are not able to credibly preempt the introduction of the rival’s new product through preemptive investment in product proliferation, once the firm is able to withdraw the existing product.

However, it is unclear, whether these arguments also apply in a vertical product differentiation setting. It is well established that models characterized by horizontal differentiation may provide different results than vertical differentiation settings.<sup>2</sup> For instance, Shaked and Sutton (1983) show that in vertical differentiation models an upper bound on the number of firms exists, in contrast to the horizontal models in which the market can support an arbitrarily large number of firms. Prominent examples in the ‘damaged goods’ literature suggest

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<sup>1</sup>Our study focuses on a pure vertical product differentiation setting, as originated by Gabszewicz and Thisse (1979) and Shaked and Sutton (1982). Consumers have identical tastes and rank qualities in the same order. However, consumers decide to buy different goods, since their incomes differ. For more recent contributions in this area, see Hoppe and Lee (2003), Hoppe and Lehmann-Grube (2001), and Lehmann-Grube (1997) plus the literature cited therein.

<sup>2</sup>For a detailed discussion of the distinction between horizontal and vertical product differentiation, see Champsaur and Rochet (1989), Constantatos and Perrakis (1997), and Cremer and Thisse (1992). One difficulty is that solving for quality in closed form is not tractable in this setting.

that firms may not be able to proliferate their product space in vertical differentiation models, see, e.g., Deneckere and McAfee (1996) and Johnson and Myatt (2002).

The question whether firms are able to preempt their rivals from innovating is also of major relevance for policy-making authorities when designing government R&D programs to promote faster paced technological progress and firms' competitiveness in the market, see, e.g., Stiglitz (1986) and Wallsten (2000) for further information.

We analyze the incentives of incumbent firms to simultaneously introduce new products in different quality areas and whether these firms should withdraw their existing products from the market. The goal of this study is to explain the above-mentioned innovation pattern in the context of a vertical differentiation model. The main focus of this study is to analyze firms' decisions regarding their existing products, especially those firms with a background of strategic interaction when following preemption strategies. We analyze if innovators may engage in product proliferation in order to prevent their rivals from introducing a new product and illustrate to what extent the credibility of proliferation strategies plays an important role in models of vertical product differentiation.

For illustrative purposes we introduce the following setting: There are two firms each of which offers one product of different quality. The quality space is characterized by numbers: a higher number refers to a higher product quality. Firms set prices in the product market, produce at the same marginal costs, and no entry occurs. Both firms offer their equilibrium qualities, such that the low-quality firm offers a product with quality, say 1, and the high-quality firm offers a product with quality 2. Then, a technological progress occurs which improves each firm's production technologies. Hence, both firms are able to simultaneously introduce a new product of certain quality.<sup>3</sup> A higher quality requires a higher investment in R&D but also ensures higher profits. The innovators have the choice either to keep or to withdraw their existing products.

Suppose both firms introduce a new product of higher quality, but the high-quality firm still offers the highest quality in the market.<sup>4</sup> The high-quality firm may follow two different innovation strategies: it may *accommodate* or *deter* the low-quality firm's innovation. The high-quality firm *accommodates* the low-quality firm's innovation by offering a new product, say with quality 4, while withdrawing its existing product in order to reduce price competition. The low-

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<sup>3</sup>Most of the models in the area of vertical product differentiation, abstract from new product introduction by incumbent firms in an oligopolistic market, see e.g. Donnenfeld and Weber (1995) and Constantatos and Perrakis (1997). Both studies analyze either single product firms entering an empty market or a monopolist offering more than one product. Our study, however, analyzes a duopoly where incumbent firms may introduce new products as well as withdraw existing ones.

<sup>4</sup>We will focus, for now, on this specific innovation case in order to provide motivation for the main mechanism in this study.

quality firm's best response might be to offer a higher quality product, say 2. Alternatively, and similar to the proliferation strategy in the horizontal setting, the high-quality firm may *deter* the low-quality firm's innovation. It chooses to provide a new product with lower quality than in the *accommodation* case, say 3, and keeps the existing product on the market. The high-quality firm proliferates the upper quality space in order to *deter* the low-quality firm from introducing a new product of higher quality.<sup>5</sup>

The high-quality firm's choice on an *accommodation* or *deterrence* strategy is characterized by two countervailing effects, the demand and the strategic effect. Hence, the high-quality firm is faced with the following trade-off: (i) in the *accommodation* strategy, the high-quality firm introduces a higher quality product to the market which softens price competition (own strategic effect). However, it may also expect the low-quality firm to introduce a new product of higher quality, which in turn increases price competition (rival's strategic effect). Moreover, it withdraws its existing product, yielding a lower demand (demand effect); (ii) in the *deterrence* strategy, the high-quality firm offers a lower product quality, which induces tougher price competition (strategic effect), but yields higher product demand (demand effect) by keeping the existing product on the market. The 'maximal (vertical) product differentiation' principle by Shaked and Sutton (1982) suggests that moving product qualities apart in order to soften price competition (strategic effect) outweighs the increase in demand gained by moving qualities closer and capturing consumers from the high-quality firm's product (demand effect).

Applying the 'maximal product differentiation' principle, we may expect the high-quality firm's innovation strategy to be driven by strategic considerations, e.g., the best reply of the low-quality firm. We may expect the high-quality firm's decision to depend on the extent of the low-quality firm's potential innovation in the *accommodation* case: the high-quality firm may prefer to *accommodate* innovation, if the low-quality firm's potential product innovation in quality is only small. The high-quality firm benefits more from softening price competition by offering a higher product quality (own strategic effect) than it suffers from an increase in price competition and a lower demand effect (rival's strategic effect) determined by the low-quality firm's new product introduction. The high-quality firm may prefer to *deter* the low-quality firm's new product introduction in order to avoid intensive price competition, when the low-quality firm's potential new product quality is relatively high.

Beyond the principle of 'maximal product differentiation', firms account for the impact on their new product, e.g., the cannibalization effect.<sup>6</sup> Applying the cannibalization effect to our setting shows that the high-quality firm cannibalizes

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<sup>5</sup>The literature on 'damaged goods' may support a firm following the *deterrence* strategy. Firms degrade the quality of products in order to proliferate the product space.

<sup>6</sup>The cannibalization effect indicates that consumers who have bought one product switch to buy another product.

its new product demand when it keeps the existing product in the market while following the *deterrence* strategy. With respect to this effect, the high-quality firm may find it less attractive to *deter* the low-quality firm's innovation.

We find that the high-quality firm's strategy to *deter* the rival from introducing a new product is not credible. The high-quality firm is not able to credibly commit to keeping its existing product on the market and to preempting its rival's innovation by proliferating the product space. Instead, the high-quality firm should withdraw its existing product in order to avoid tougher price competition and the cannibalization of its new product's demand. The low-quality firm anticipates this commitment problem and introduces a new product with higher quality than its existing product. This study shows that the credibility of strategies plays an important role in vertical product differentiation models, which is in line with the analysis by Judd (1985) of horizontal models. Hence, existing products have no commitment value to credibly preempt the introduction of their rival's new products. The high-quality firm earns higher profits despite its offering a smaller variety of goods.

The remainder of this study is organized as follows. In Section 2, we describe and analyze the model of new product introduction by incumbent firms. In Section 3, we conclude.

## 2 The Model

We consider an outset in which two firms ( $i = 1, 2$ ) offer one product with quality  $\underline{s}$ ,  $\bar{s} \in \mathbb{R}^+$  and  $\underline{s} < \bar{s}$ . Thus, firm 1 is the low-quality and firm 2 the high-quality provider.<sup>7</sup> A technological progress occurs, that improves the firms' technological apability to produce multiple products. Hence, both firms are able to simultaneously introduce a new product of a certain quality and decide whether to keep or withdraw existing products. We analyze a three-stage duopoly game.

In the *first stage*, both firms simultaneously decide whether to introduce a new product and choose the quality of the new product. We assume that firms choose a quality higher than their existing product quality from the following set of qualities  $s_i \in [s, \infty)$ , for  $i = 1, 2$  and  $s = \underline{s}$  or  $\bar{s}$  if  $i = 1$  or  $2$ , respectively.<sup>8</sup> We also assume that the high-quality firm is able to introduce a new product with highest product quality, due to the 'persistence of leadership' result, shown by

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<sup>7</sup>The outset is based on the model by Choi and Shin (1992) which is a modification of Shaked and Sutton (1982) where the version of Tirole (1992) is used. According to Choi and Shin (1992) we assume that firms offer their equilibrium qualities. The outset and results are shown in Appendix 1. Note, that the assumption on the initial qualities has no implications on the basic results of this paper, which are: innovators introduce new products with higher quality and withdraw others, such that preemption is not a credible innovation strategy.

<sup>8</sup>Note, that the decision to introduce a new product with quality  $s_i \neq \underline{s}$  or  $\bar{s}$  if  $i = 1$  or  $2$ , respectively, is equivalent to simultaneously deciding to introducing a new product and its product quality.

Lehmann-Grube (1997). We strongly want to emphasize that these latter two assumptions are imposed without any loss of generality and do not restrict the set of equilibria.<sup>9</sup>

We can distinguish between three quality areas which depend on where the innovators may locate their new products: a low-quality area,  $s_i < \underline{s}$ ; an intermediate-quality area,  $\underline{s} < s_i < \bar{s}$ ; and a high-quality area,  $s_i > \bar{s}$ .

Product Locations	Case <i>a</i>	Case <i>b</i>	Case <i>c</i>
high-quality area	$s_2 > s_1$	$s_2$	$s_2$
existing high-quality product	$\bar{s}$	$\bar{s}$	$\bar{s}$
intermediate-quality area		$s_1$	
existing low-quality product	$\underline{s}$	$\underline{s}$	$\underline{s}$
low-quality area			$s_1$

**Figure 1:** The innovation cases

Innovators have to invest in R&D for producing higher quality. The R&D costs for firm  $i$ 's new product quality is given by the following cost function

$$F_i(s_i) \begin{cases} > 0 \text{ for } s_i > s, \text{ and } s = \underline{s} \text{ or } \bar{s} \text{ if } i = 1 \text{ or } 2 \\ = 0 \text{ otherwise.} \end{cases}$$

with  $F_1(\cdot) \geq F_2(\cdot)$ ,  $F_1(\underline{s}) = F_2(\bar{s}) = 0$ ,  $F'_i(s_i) > 0$ ,  $F''_i(s_i) > 0$  and  $\lim_{s_i \rightarrow \infty} F'(\cdot) = \infty$ , for  $s_i > s$ . Firm  $i$ 's quality choice is supposed to satisfy the 'best response' property and is determined by the profits  $\pi_i$  (stage 3), it earns in the product market and the R&D costs  $F_i$ ,

$$s_i = r_i(\cdot, s_j) = \arg \max_{s_i} \{ \pi_i(\cdot, s_i) - F_i(s_i) \}, \quad (1)$$

for  $i, j = 1, 2$ , and  $i \neq j$ .

Whether a firm introduces a new product with quality  $s_i$ , is a comparison of profits (stage 1) after having introduced a new product with profits from the outset, when no product is introduced, shown by

$$\pi_i(\cdot, s_i) - F_i(s_i) \geq \pi(\cdot, s), \quad (2)$$

with  $\pi = \underline{\pi}$  or  $\bar{\pi}$  and  $s = \underline{s}$  or  $\bar{s}$  if  $i = 1$  or  $2$ , respectively.

In the *second stage*, both firms decide whether to keep or withdraw their existing product from the market given their quality decision from the first stage. Taking the rival's choice into account, firm  $i$  keeps the existing product in the market, if

$$\pi_i^k(s_i^e, s_i, [s_j^e], s_j) - \pi_i^w(s_i, [s_j^e], s_j) > 0 \quad (3)$$

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<sup>9</sup>The latter two assumptions are imposed only for the sake of transparency and to focus on the main issue of the paper, which is the credibility of the proliferation strategy.



applies, for  $i, j = 1, 2$ ;  $i \neq j$ ;  $s_i^e = \underline{s}$  or  $\bar{s}$  if  $i = 1$  or  $2$ ;  $s_j^e = \underline{s}$  or  $\bar{s}$  if  $j = 1$  or  $2$ ;  $\pi_i^k$  and  $\pi_i^w$  denote firm  $i$ 's profits (stage 3), when it keeps or withdraws the existing product, respectively.<sup>10</sup> In terms of the number of products the following cases may occur: both innovators keep the first product in the market and four products are offered in the market, one of the two innovators withdraws the first product and three products are offered, both innovators withdraw their existing product and two products remain in the market, and a single innovator withdraws the existing product and two products are offered in the market.

In the *third stage*, firms maximize profits by simultaneously choosing prices in the product market having observed the number of products and the product qualities offered in the market.<sup>11</sup> We distinguish between R&D costs for quality improvement and production costs, the latter being independent of quality. No entry is assumed to occur.

Consumers' preferences are described by  $U = \theta s - p$  if they buy a good and zero otherwise. Every consumer is allowed to buy at most one of the products and each consumer has the same ranking of qualities, as it prefers higher quality for a given price  $p$ . Consumers differ in their income. Their income parameter  $\theta$  is uniformly distributed over the interval  $[0, 1]$ .<sup>12</sup> The assumption on the income parameter implies that the market is not covered. Hence, some consumers do not buy any one of these products.

We look for pure strategies and solve the model backwards, according to subgame perfection. In order to simplify the following analysis, we rule out several subgames, in which the low-quality firm considers to introduce (keep) a second product adjacent to its existing product quality, as shown by cases *b* and *c* in Figure 1.

**Lemma 1** *If the low-quality firm offers the lowest two product qualities in the market, its dominant strategy is to withdraw the product with lowest quality.*

The proof is shown in Appendix 2. Lemma 1 allows us to explicitly rule out the stage 2 subgame in case *b*, in which the low-quality firm considers to keep the existing product with lowest quality, as well as the stage 1 subgame in case *c*, in which the low-quality firm may introduce a new product in the low-quality area. In the following, we analyze case *b*, as shown in Figure 1.

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<sup>10</sup>A product in indicated in brackets represent the firm's option to either keep or withdraw the existing product.

<sup>11</sup>When the innovator keeps its existing product in the market it is allowed to internalize price competition among its own products. More precisely, it takes into account that a price change of one of its products has an impact on its other product.

<sup>12</sup>Another common assumption is the bimodular distribution, which is, especially used in the "damaged goods" literature. The uniform distribution is an appropriate assumption for the electronics and telecommunications market. Otherwise, it might be difficult to explain very low preference for intermediate-quality goods for the industries under consideration.

## 2.1 Intermediate-Quality Innovation by the Low-Quality Firm

In innovation case  $b$ , the high-quality firm may introduce a new product with highest product quality, whereas the low-quality firm considers to introduce a new product in the intermediate-quality area. According to Lemma 1, the low-quality firm follows its dominant strategy and withdraws the existing product with quality  $\underline{s}$  from the market. Three products with qualities  $s_1 < \bar{s} < s_2$  will be considered in this innovation scenario. We obtain the following result.

**Proposition 1** *The high-quality firm's strategy to deter the low-quality firm from introducing a new product is not credible. The high-quality firm accommodates the low-quality firm's new product introduction. Both innovators withdraw their existing products from the market.*

In accordance to subgame perfection, we solve this innovation case by applying backward induction. First, we derive prices, demand, and profits in the product market (stage 3) given the stage 2 subgames. We then analyze the decision to keep or withdraw their existing product from the market (stage 2). Finally, we investigate firms' incentives to introduce a new product in the quality areas under consideration (stage 1).

*Product Market Competition - Stage 3:* When the high-quality firm introduces a new product in the high-quality area and the low-quality firm offers a new product with intermediate-quality, three products with qualities  $s_1 < \bar{s} < s_2$  are offered in the market. Consequently, there are three indifferent consumers prevalent in the market. One of them is indifferent between buying the product with highest quality  $s_2$ , or with second highest quality  $\bar{s}$  from the high-quality firm. The income parameter of this consumer is given by  $\theta_3 = \frac{(p_2 - \bar{p})}{(s_2 - \bar{s})}$ . The consumer who is indifferent between buying the high-quality firm's existing product with quality  $\bar{s}$  and the low-quality firm's new product with quality  $s_1$  is described by the income parameter  $\theta_2 = \frac{(\bar{p} - p_1)}{(\bar{s} - s_1)}$ , whereas the income parameter  $\theta_1 = \frac{p_1}{s_1}$  represents the consumer who is indifferent between buying the new product with quality  $s_1$  from the low-quality firm and not buying at all. For the demand functions, we get

$$D_2(\bar{p}, p_2, \bar{s}, s_2) = \int_{\theta_3}^{\theta=1} f(\theta) d\theta = 1 - \frac{(p_2 - \bar{p})}{(s_2 - \bar{s})}, \quad (4)$$

$$\bar{D}(p_1, \bar{p}, p_2, s_1, \bar{s}, s_2) = \int_{\theta_2}^{\theta_3} f(\theta) d\theta = \frac{(p_2 - \bar{p})}{(s_2 - \bar{s})} - \frac{(\bar{p} - p_1)}{(\bar{s} - s_1)}, \quad (5)$$

and

$$D_1(p_1, \bar{p}, s_1, \bar{s}) = \int_{\theta_1}^{\theta_2} f(\theta) d\theta = \frac{(\bar{p} - p_1)}{(\bar{s} - s_1)} - \frac{p_1}{s_1}. \quad (6)$$

Firms' profit functions for the product market are as follows

$$\begin{aligned} \pi_1^w(p_1, D_1) &= p_1 D_1(\cdot), \text{ and} \\ \pi_2^k(\bar{p}, \bar{D}, p_2, D_2) &= \bar{p} \bar{D}(\cdot) + p_2 D_2(\cdot). \end{aligned}$$

Each firm maximizes the profit function with respect to its own product price. The first order condition for the low-quality firm, is

$$\frac{\partial \pi_1^w(p_1, D_1)}{\partial p_1} \equiv 0 \implies p_1(\bar{p}) = \frac{\bar{p} s_1}{2\bar{s}}.$$

The first order condition for the high-quality firm with respect to the price of the high-quality product, is as follows

$$\frac{\partial \pi_2^k(\bar{p}, \bar{D}, p_2, D_2)}{\partial p_2} \equiv 0 \implies p_2(\bar{p}) = \frac{2\bar{p} - \bar{s} + s_2}{2},$$

and with respect to the price of its existing product, is given by

$$\frac{\partial \pi_2^k(\bar{p}, \bar{D}, p_2, D_2)}{\partial \bar{p}} \equiv 0 \implies \bar{p}(p_1) = \frac{p_1 - s_1 + \bar{s}}{2}.$$

The reaction functions are strictly monotone. Solving the first order conditions yields the corresponding prices

$$\begin{aligned} p_1(s_1, \bar{s}) &= \frac{s_1(\bar{s} - s_1)}{4\bar{s} - s_1}, \quad \bar{p}(s_1, \bar{s}) = \frac{2\bar{s}(\bar{s} - s_1)}{4\bar{s} - s_1}, \text{ and} \\ p_2(s_1, \bar{s}, s_2) &= \frac{4\bar{s}s_2 - s_1(s_2 + 3\bar{s})}{2(4\bar{s} - s_1)}. \end{aligned}$$

Substituting these into equations (4), (5), and (6) gives us the equivalent demand

$$D_1(s_1, \bar{s}) = \frac{\bar{s}}{4\bar{s} - s_1}, \quad \bar{D}(s_1, \bar{s}) = \frac{s_1}{2(4\bar{s} - s_1)}, \text{ and } D_2 = \frac{1}{2}.$$

Similarly, firms' profits in the product market are

$$\pi_1^w(s_1, \bar{s}) = \frac{s_1 \bar{s} (\bar{s} - s_1)}{(4\bar{s} - s_1)^2}, \text{ and}$$

$$\pi_2^k(s_1, \bar{s}, s_2) = \frac{s_1 \bar{s} (\bar{s} - s_1)}{(4\bar{s} - s_1)^2} + \frac{4\bar{s}s_2 - s_1(3\bar{s} + s_2)}{4(4\bar{s} - s_1)}. \quad (7)$$

When the high-quality firm withdraws its existing product from the market, each firm offers only the new product in the market. The results are analogous to the outset (see Appendix 1, adjusted for the corresponding product qualities  $s_1 = \underline{s}$  and  $s_2 = \bar{s}$ ). The different results for the stage 3 subgames show that competition in the product market is determined by the number of products and the qualities offered in the market.

*Number of Products - Stage 2:* In this stage, we investigate the innovators' decision to keep or withdraw the existing product from the market. The low-quality firm follows its dominant strategy to withdraw the existing product, as shown in Lemma 1. Therefore, we only need to analyze the high-quality firm's choice on its existing product with quality  $\bar{s}$ , according to equation (3). In the following, we relate the high-quality firm's decision to keep or withdraw the existing product to its decision whether to follow the *accommodation* or *deterrence* strategy. In doing so, we want to emphasize the credibility of firms' innovation strategies and stress the link towards horizontal product differentiation settings.

*Accommodation versus Deterrence:* The high-quality firm may *accommodate* the low-quality firm's new product introduction by withdrawing the existing product from the market. As mentioned above, each firm offers one new product in the market, similar to the outset, with  $s_1 = \underline{s}$  and  $s_2 = \bar{s}$ .

The high-quality firm may *deter* the low-quality firm's new product introduction by keeping the existing product in the market, which hampers the low-quality firm from introducing a new product.<sup>13</sup>

The high-quality firm's decision to *accommodate* or *deter* the low-quality firm's innovation is determined by the following effects<sup>14</sup>

$$\underbrace{\frac{\partial \pi_2^k}{\partial s_2}}_{\text{Deterrence}} \underset{<}{\geq} \underbrace{\frac{\partial \pi_2^w}{\partial s_2} + \frac{\partial \pi_2^w}{\partial s_1}}_{\text{Accommodation}}$$

$$\underbrace{\frac{\partial \pi_2^k}{\partial \bar{D}} \frac{\partial \bar{D}}{\partial s_2} + \frac{\partial \pi_2^k}{\partial D_2} \frac{\partial D_2}{\partial s_2}}_{\text{Demand effect in deterrence case}} \underset{<}{\geq} \underbrace{\frac{\partial \pi_2^w}{\partial D_2} \frac{\partial D_2}{\partial p_1} \frac{\partial p_1}{\partial s_2}}_{\text{Own strategic effect in accom.}}^+ + \underbrace{\frac{\partial \pi_2^w}{\partial D_2} \frac{\partial D_2}{\partial s_2}}_{\text{Demand effect in accom.}}$$

<sup>13</sup>Equation (14), adjusting for  $s_1 = \underline{s}$ , shows that the low-quality firm already offers the optimally chosen product quality  $\underline{s} = \frac{4}{7}\bar{s}$ . Therefore, the low-quality firm has no incentive to introduce a new product into the intermediate-quality area. Note also, that the low quality firm replacing its product coincides with not introducing a new product.

<sup>14</sup>Variables indicated by a superscript  $k$  ( $w$ ) refer to the *deterrence* (*accommodation*) case, respectively.

$$+ \left[ \underbrace{\frac{\partial \pi_2^w}{\partial D_2} \frac{\partial D_2}{\partial p_1} \frac{\partial p_1}{\partial s_1} + \frac{\partial \pi_2^w}{\partial D_2} \frac{\partial D_2}{\partial s_1}}_{\text{Rival's strategic effect in accom. case}} \right] \quad (8)$$

In the *accommodation* case, the high-quality firm offers a product with higher quality than in the *deterrence* case.<sup>15</sup> The high-quality firm benefits by offering a higher product quality, because price competition towards the low-quality firm's product is softened (see *own strategic effect* in equation (8)). On the other hand, the high-quality firm loses part of its profits (stage 3) in the *accommodation* case, since the low-quality firm is able to introduce a new product quality in the intermediate-quality area. This enforces price competition and takes over some of the product demand from the high-quality firm (see *rival's strategic effect*).

In the *deterrence* case, the high-quality firm benefits by a higher *demand effect* than in the *accommodation* case, through keeping the existing product in the market and avoiding the *rival's strategic effect* caused by the low-quality firm's new product introduction in the *accommodation* case, see equation (8).

At first glance, we may expect the high-quality firm's decision to *deter* or *accommodate* the low-quality firm's new product introduction being characterized by the extent of the low-quality firm's potential innovation in the *accommodation* case: the high-quality firm may prefer to *accommodate* innovation when it benefits more from softening price competition by offering a higher product quality (*own strategic effect*) than it suffers from an increase in price competition and a lower demand effect (*rival's strategic effect*) determined by the low-quality firm's new product introduction. On the other hand, the high-quality firm may prefer to *deter* the low-quality firm's new product introduction, when the low-quality firm's new product quality would be relatively high such that it intensifies price competition, see *rival's strategic effect* in equation (8).

However, the high-quality firm's strategy to *deter* the low-quality firm's new product introduction is not credible. The decision to keep or withdraw the existing product from the market is a comparison of profits under both scenarios, as shown in equation (3). Given the low-quality firm's dominant strategy to withdraw the existing product, as shown in Lemma 1, we focus on the high-quality firm's stage 2 decision. The main problem in solving this stage is given by the difficulty to explicitly solve for quality, as the terms are often characterized by polynomials of high degrees. Consequently, we are not able to compare reduced-form profits (as in equation (3)) when the high-quality firm keeps the existing

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<sup>15</sup>From Appendix 1, (15) with  $s_1 = \underline{s}$  and  $s_2 = \bar{s}$ , as well as Appendix 3, equation (17), we see that marginal profits (stage 3) in the *accommodation* case are higher than in the *deterrence* case because  $\frac{\partial \pi_2^w(s_1, s_2)}{\partial s_2} - \frac{\partial \pi_2^k(\underline{s}, \bar{s}, s_2)}{\partial s_2} = \frac{s_2^2(s_1 + 20s_2)}{4(4s_2 - s_1)^3} \geq 0$  applies. As firms set marginal profits (stage 3) equal to marginal costs, and marginal costs are identical in both cases, it follows that the equilibrium quality in the *accommodation* case will be higher than in the *deterrence* case.

product in the market with the profits when it withdraws the product from the market. Therefore, we implicitly solve the system by parceling out the total effects in several parts which makes the analysis computationally tractable. Taking the total derivative of the high-quality firm's profit functions (equation (7)) with respect to the existing product quality  $\bar{s}$ , gives<sup>16</sup>

$$\frac{d\pi_2^k}{d\bar{s}} = \underbrace{\frac{\partial \pi_2^k}{\partial \bar{D}} \frac{\partial \bar{D}}{\partial p_1} \frac{dp_1}{d\bar{s}}}_{\text{strategic effect}} + \underbrace{\frac{\partial \pi_2^k}{\partial \bar{D}} \frac{\partial \bar{D}}{\partial \bar{s}}}_{\text{demand effect}} + \underbrace{\frac{\partial \pi_2^k}{\partial D_2} \frac{\partial D_2}{\partial \bar{s}}}_{\text{cannibalization effect}} \quad (9)$$

$$= \frac{s_1^2 (s_1 + 20\bar{s})}{4 (s_1 - 4\bar{s})^3} > 0. \quad (10)$$

The total derivative of the high-quality firm is positive, indicating that its profits continuously increase in the quality of the existing product. The high-quality firm benefits from softening price competition and avoiding to cannibalize its new product demand. The extreme case when both the high-quality firm's product qualities are identical, is equivalent to withdrawing the existing product from the market. We get the following result.

**Lemma 2** *Both firms withdraw their existing product from the market, if they introduce a new product in the quality areas under consideration.*

We now turn to analyze firms' decision on product quality.

*Quality Choice - Stage 1:* Firm  $i$ 's objective is to choose a product quality, according to equation (1), taking into account that both firms withdraw their existing product from the market. Marginal profits (stage 3) with respect to quality  $s_i$  are positive for both firms (see Appendix 1, equation (14) and (15), adjusting for  $s_1 = \underline{s}$  and  $s_2 = \bar{s}$ ). Moreover, by definition of the convex profit function (stage 3) and the concave R&D cost function, it follows that firms are always better off to increase quality. As this study primarily focuses on analyzing the credibility of preemptive innovation strategies and the number of products offered in the market, we refer to other literature in terms of properties on R&D cost functions, see e.g. Lehmann-Grube (1997) and Caplin and Nalebuff (1991) regarding the existence and uniqueness of the equilibria. We get the following result.

**Lemma 3** *The high (low) quality firm always has an incentive to introduce a new product in the high (intermediate) quality area.*

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<sup>16</sup>Second-stage optimization, implies  $\frac{\partial \pi_2^k}{\partial p_2} = 0$  and  $\frac{\partial \pi_2^k}{\partial \bar{p}} = 0$ . Thus, the effect of  $\bar{s}$  on  $\pi_2^k$  through the high-quality firm's price change can be ignored by applying the envelope theorem.

We could show that the credibility of strategies plays an important role in determining firms' incentives to introduce a new product in the market. As the high-quality firm is not able to credibly preempt the low-quality firm's new product introduction through product proliferation, or *deter* the low-quality firm's product introduction, it is better off to withdraw the existing product from the market. Both firms introduce a new product of higher quality and withdraw their existing product from the market in order to avoid cannibalizing its new product demand and to reduce price competition. We now turn to the innovation case *a*, as shown in Figure 1.

## 2.2 High-Quality Innovation by the Low-Quality Firm

In the following, we turn to analyze case *a*, in which both firms may introduce a new product in the high-quality area. We obtain the following result.

**Proposition 2** *The high-quality firm always has an incentive to introduce a new product in the high-quality area, whereas the low-quality firm may introduce a new product in the high-quality area only, when R&D costs are sufficiently small. Both innovators withdraw their existing product from the market.*

Again, we solve the game by applying backward induction and begin with the product market.

*Product Market Competition - Stage 3:* Prices, demand, and profits are given as follows. When both firms keep the existing product in the market, results are shown in Appendix 4; when only firm 1 withdraws, see Appendix 5; when only firm 2 withdraws, see Appendix 6. When both firms withdraw, see Appendix 1, setting  $s_1 = \underline{s}$  and  $s_2 = \bar{s}$ .

We continue analyzing the low-quality firm's decision whether to keep or withdraw the existing product from the market, according to equation (2).

*Number of Products - Stage 2:* As polynomials of high degrees prevent us from solving for qualities, we analyze firms' marginal profits with respect to their existing product quality. When firms keep their products in the market, the low-quality firm's marginal profits with respect to quality  $\underline{s}$ , are given by<sup>17</sup>

$$\frac{d\pi_1^k}{d\underline{s}} = \underbrace{\frac{\partial \pi_1^k}{\partial \underline{D}} \frac{\partial \underline{D}}{\partial \underline{p}} \frac{\partial \underline{p}}{\partial \underline{s}}}_{\text{first strategic effect}} + \underbrace{\frac{\partial \pi_1^k}{\partial D_1} \frac{\partial D_1}{\partial \underline{p}} \frac{\partial \underline{p}}{\partial \underline{s}}}_{\text{second strategic effect}} + \underbrace{\frac{\partial \pi_1^k}{\partial D_1} \frac{\partial D_1}{\partial p_2} \frac{\partial p_2}{\partial \underline{s}}}_{\text{third strategic effect}} + \underbrace{\frac{\partial \pi_1^k}{\partial \underline{D}} \frac{\partial \underline{D}}{\partial \underline{s}}}_{\text{demand effect}} < 0. \quad (11)$$

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<sup>17</sup>Again, second-stage optimization, implies  $\frac{\partial \pi_1^k}{\partial \underline{p}} = 0$  and  $\frac{\partial \pi_1^k}{\partial p_1} = 0$ . Thus, the low-quality firm's price change can be ignored by applying the envelope theorem.

The total effect in equation (11) is negative, because the *second strategic effect* dominates the *demand effect* (see Appendix 7, equation (23)). The low-quality firm earns higher profits through reducing the quality of its existing product in order to soften price competition towards the high-quality firm's existing product. It benefits more by softening price competition (*second strategic effect*) instead of attracting more consumers (*demand effect*). Therefore, the low-quality firm's best response is to withdraw the existing product.

When the high-quality firm withdraws its first product from the market, the low-quality firm withdraws as well, see Lemma 1.

Given the low-quality firm's dominant strategy to withdraw, the high-quality firm is better off to withdraw the existing product, as well. The total derivative of the high-quality firm's profit function with respect to  $\bar{s}$  (see Appendix 5, equation (19)) is given as follows

$$\frac{d\pi_2^k}{d\bar{s}} = \underbrace{\frac{\partial \pi_2^k}{\partial D_2} \frac{\partial D_2}{\partial p_1} \frac{\partial p_1}{\partial \bar{s}}}_{\text{first strategic effect}} + \underbrace{\frac{\partial \pi_2^k}{\partial \bar{D}} \frac{\partial \bar{D}}{\partial p_1} \frac{\partial p_1}{\partial \bar{s}}}_{\text{second strategic effect}} + \underbrace{\frac{\partial \pi_2^k}{\partial \bar{D}} \frac{\partial \bar{D}}{\partial \bar{s}}}_{\text{demand effect}} < 0. \quad (12)$$

The total effect is negative as the first strategic effect dominates the demand effect,

$$\underbrace{\frac{3(\bar{s} - s_1)^2 s_1^2 (\bar{s}(3s_1 + s_2) - 4s_1 s_2)}{2(2\bar{s}s_1 + s_1^2 + \bar{s}s_2 - 4s_1 s_2)^3}}_{\text{first strategic effect}} + \underbrace{\frac{3(\bar{s} - s_1)^2 s_1^2 (\bar{s}(3s_1 + s_2) - 4s_1 s_2)}{4(s_1(s_1 - 4s_2) + \bar{s}(2s_1 + s_2))^2}}_{\text{demand effect}} =$$

$$\frac{s_1^2 (s_2 - s_1)^2 (44s_1 s_2 + s_1^2 - 11\bar{s}s_2 - 34\bar{s}s_1)}{4(2\bar{s}s_1 + s_1^2 + \bar{s}s_2 - 4s_1 s_2)^3} < 0.$$

The high-quality firm increases profits by withdrawing the existing product from the market, as price competition is softened.

**Lemma 4** *Both firms withdraw their existing product given they introduce a new product in the high-quality area.*

We now turn to investigate the innovators' quality choice (stage 1).

*Quality Choice - Stage 1:* The innovator's (firm  $i$ 's) objective is to choose a product quality which maximizes profits, as shown by equation (3). For the same reasons as in the previous case  $b$ , setting  $s_1 = \underline{s}$  and  $s_2 = \bar{s}$ , the high-quality firm always introduces a new product in the high-quality area. The low-quality



firm introduces a new product in the market when R&D costs for quality are sufficiently small. Otherwise, it will introduce a new product in the intermediate-quality area, see Appendix 1.

**Lemma 5** *The high-quality firm has an incentive to always introduce a new product in the high-quality area, whereas the low-quality firm may introduce a new product in the high-quality area only, when R&D costs for quality are sufficiently small.*

We now turn to the last innovation case.

### 2.3 Low-Quality Innovation by the Low-Quality Firm

In the following, we turn to case *c*, in which the high-quality firm may introduce a new product with highest product quality, whereas the low-quality firm considers to introduce a new product in the low-quality area. We obtain the following result.

**Proposition 3** *The high-quality firm always has an incentive to introduce a new product in the high-quality area and withdraws the existing product from the market. The low-quality firm will not introduce a new product in the low-quality area.*

According to Lemma 1, we know that the low-quality firm will not introduce a new product in the quality area. Therefore, three products with qualities  $\underline{s} < \bar{s} < s_2$  will be considered in this innovation scenario. Again, we begin with the product market.

*Product Market Competition - Stage 3:* Prices, demand and profits for the different subgames are given as follows. When the high-quality firm keeps the existing product, the sequence of product qualities offered in the market is given by  $\underline{s} < \bar{s} < s_2$ . The results for the product market are shown in case *b*, setting  $\underline{s} = s_1$ ; when the high-quality firm withdraws its existing product, the results are shown in Appendix 1, with  $s_2 = \bar{s}$ .

Next, we analyze the high-quality firm's decision to keep or withdraw their existing product from the market, given the low-quality firm follows its dominant strategy and does not introduce a new product in the low-quality area.

*Number of Products - Stage 2:* According to equations (9) and (10) with  $\underline{s} = s_1$ , the high-quality firm is better off withdrawing the existing product from the market.

*Quality Choice - Stage 1:* Applying the same argument on the properties of profit and R&D functions as above to this case, implies that the high-quality firm

always has an incentive to increase quality. We can confirm our Proposition 3 from above.

After analyzing the innovation cases  $a$ ,  $b$ , and  $c$ , we can conclude with the following result.

**Proposition 4** *The high-quality firm always has an incentive to introduce a new product in the high-quality area and withdraws the existing product from the market. The low-quality firm introduces a new product with higher quality, depending on the R&D costs for quality and withdraws the existing products from the market.*

As the high-quality firm always has an incentive to introduce a new product of higher quality and to withdraw the existing product from the market, it is not credible to preempt the low-quality firm's innovation through proliferating the product space. Hence, the high-quality firm's *deterrence* strategy is not credible.

### 3 Conclusion

We have analyzed a model characterized by vertical product differentiation in which firms may simultaneously introduce new products in certain quality areas and decide whether to keep or to withdraw their existing products from the market.

This study explains the innovation process we often observe in electronics and telecommunications markets. We could show that firms introduce a new product with higher quality than their existing products in order to concentrate sales on high income consumers. Moreover, innovators always withdraw their existing product in order to reduce price competition and to avoid cannibalizing their new product's demand.

We could also show that the introduction of new products depends on the credibility of firms' innovation strategies. The high-quality firm's strategy to *deter* the low-quality firm's new product introduction is not credible. Preempting the low-quality firm's new product introduction through product proliferation is not credible, once the high-quality firm is allowed to withdraw its existing product. The high-quality firm always chooses a quality according to the *accommodation* strategy and withdraws the existing product from the market in order to reduce price competition and to avoid cannibalizing its new product demand. Existing products have no commitment value in order to credibly hamper their rivals from introducing a new product, an outcome which is similar to Judd (1985) for horizontal differentiation models.

It is interesting to note that our results also contribute to the literature on 'damaged goods', as we show that the distribution of quality preferences plays an important role in determining firms' decisions on their existing products. The literature on 'damaged goods', often based on a bimodal distribution of preferences for quality, supports firms decisions to follow the *deterrence* strategy by proliferating their product space. This study, based on the assumption of a uniform distribution of preferences for quality is consistent with firms following the *accommodation* strategy.

## 4 APPENDIX

### Appendix 1: The Outset

The outset is based on Choi and Shin (1992) which is a modification by Shaked and Sutton (1982) where the existing version by Tirole (1992) is used. The model represents a noncooperative two-stage game in which two firms ( $i = 1, 2$ ) simultaneously choose their qualities in the first stage and given their qualities they compete in the second stage with prices in the product market.

Product qualities  $\underline{s}, \bar{s}$  with  $\underline{s} < \bar{s}$  are chosen from the following set of qualities defined as  $s \in [0, \tilde{s}]$  where  $\tilde{s}$  is any finite number. Firm 1 is supposed to be the low-quality firm and firm 2 is the high-quality provider. We focus on pure strategies. Consumers' preferences are the same as described in the model section above. After deriving the corresponding demand functions, we get for the corresponding prices

$$\underline{p}(\underline{s}, \bar{s}) = \frac{\underline{s}(\bar{s} - \underline{s})}{4\bar{s} - \underline{s}} \text{ and } \bar{p}(\underline{s}, \bar{s}) = \frac{2\bar{s}(\bar{s} - \underline{s})}{4\bar{s} - \underline{s}}.$$

For demand, we get

$$\underline{D}(\underline{s}, \bar{s}) = \frac{\bar{s}}{4\bar{s} - \underline{s}} \text{ and } \bar{D}(\underline{s}, \bar{s}) = \frac{2\bar{s}}{4\bar{s} - \underline{s}}.$$

Profits are

$$\underline{\pi}(\underline{s}, \bar{s}) = \frac{\underline{s}\bar{s}(\bar{s} - \underline{s})}{(4\bar{s} - \underline{s})^2} \text{ and } \bar{\pi}(\underline{s}, \bar{s}) = \frac{4\bar{s}^2(\bar{s} - \underline{s})}{(4\bar{s} - \underline{s})^2}. \quad (13)$$

Reduced-form profit functions are continuous and differentiable, given by

$$\frac{\partial \underline{\pi}(\underline{s}, \bar{s})}{\partial \underline{s}} = \frac{\bar{s}^2(4\bar{s} - 7\underline{s})}{(4\bar{s} - \underline{s})^3} \geq 0 \text{ for } \underline{s} \leq \frac{4}{7}\bar{s} \text{ and} \quad (14)$$

$$\frac{\partial \bar{\pi}(\underline{s}, \bar{s})}{\partial \bar{s}} = \frac{4\bar{s}(2\underline{s}^2 - 3\underline{s}\bar{s} + 4\bar{s}^2)}{(4\bar{s} - \underline{s})^3} > 0. \quad (15)$$

From equation (14) we see that the low-quality firm's profits first increase in quality since more consumers buy the new product (demand effect). The closer the product quality moves towards the competitor's product the higher is the price competition (*strategic effect*) which decreases the low-quality firm's profits. When both product qualities are identical Bertrand competition drives firms' profits to zero. The low-quality provider's optimal distance to the high-quality product is given by the point where the demand effect and the strategic effect are balancing each other. The high-quality firm increases profits by offering a higher product quality. We get the result of 'maximal product differentiation' where in equilibrium firms maximally differentiate

their products. The low-quality firm offers the lowest feasible product quality and the high-quality firm offers the highest feasible product quality, according to  $\underline{s} = \frac{4}{7}\bar{s}$ .

## Appendix 2: Two Adjacent Products offered by the Low-Quality Firm

Let us begin with case *b*, in which the low-quality firm may introduce a new product in the intermediate-quality area, such that it offers two adjacent products with lowest quality,  $\underline{s} < s_1 < \bar{s}$  at prices  $\underline{p} < p_1 < \bar{p}$ . The low-quality firm's decision to withdraw or keep its product with lowest quality  $\underline{s}$ , depends on the difference in firms' profits, respectively, given by

$$\pi_1^w = \left[ \frac{(\bar{p} - p_1)}{(\bar{s} - s_1)} - \frac{p_1}{s_1} \right] p_1$$

and

$$\pi_1^k = \left[ \frac{(\bar{p} - p_1)}{(\bar{s} - s_1)} - \frac{(p_1 - \underline{p})}{(s_1 - \underline{s})} \right] p_1 + \left[ \frac{(p_1 - \underline{p})}{(s_1 - \underline{s})} - \frac{\underline{p}}{\underline{s}} \right] \underline{p}.$$

The difference is,

$$\pi_1^w - \pi_1^k = \frac{(s_1 \underline{p} - \underline{s} p_1)^2}{s_1 \underline{s} (s_1 - \underline{s})} > 0. \quad (16)$$

As shown in equation (16), the low-quality firm earns higher profits when it withdraws its product with lower quality  $\underline{s}$ . Moreover, we see that the rival's offer does not affect the difference of the low-quality firm's profits. As this result holds even under the assumption that the same prices are charged under both regimes, we can abstract from any strategic effects and it is the low-quality firm's dominant strategy to withdraw the product with lowest quality. This result also applies to case *c*, in which the low-quality firm considers to introduce a new product in the low-quality area.

## Appendix 3: Low-Quality Innovation by Low-Quality Firm (Case *a*)

The total derivative of the high-quality firm's reduced-form profits (equation (7)) with respect to its new product quality is given by

$$\begin{aligned} \frac{d\pi_2^k(\underline{s}, \bar{s}, s_2)}{ds_2} &= \underbrace{\frac{\overbrace{\partial \pi_2^k}^+}{\partial \bar{D}} \frac{\underbrace{\partial \bar{D}}^-}{\partial s_2}}_{\text{cannibalization effect}} + \underbrace{\frac{\overbrace{\partial \pi_2^k}^+}{\partial D_2} \frac{\overbrace{\partial D_2}^+}{\partial s_2}}_{\text{demand effect}} \\ &= \frac{\bar{s}(\bar{s} - \underline{s})}{(4\bar{s} - \underline{s})(\underline{s} - \bar{s})} + \frac{4\bar{s}s_2 - \underline{s}(3\bar{s} + s_2)}{4(4\bar{s} - \underline{s})(s_2 - \bar{s})} = \frac{1}{4}. \end{aligned} \quad (17)$$

As we see, the high-quality firm's marginal profits increase in the new product's quality.

#### Appendix 4: High-Quality Innovation by Low-Quality Firm (Case a)

Both firms offer a new product in the high-quality area, whereby the high-quality firm still offers the highest product quality. Both firms keep their existing products in the market. We get the following sequence of qualities  $\underline{s} < \bar{s} < s_1 < s_2$  offered in the market.

For demand we get

$$\underline{D}(\underline{p}, \bar{p}, \underline{s}, \bar{s}) = \frac{(\bar{p}-\underline{p})}{(\bar{s}-\underline{s})} - \frac{\underline{p}}{\underline{s}}, \quad \bar{D}(\underline{p}, \bar{p}, p_1, \underline{s}, \bar{s}, s_1) = \frac{(p_1-\bar{p})}{(s_1-\bar{s})} - \frac{(\bar{p}-\underline{p})}{(\bar{s}-\underline{s})},$$

$$D_1(\bar{p}, p_1, p_2, \bar{s}, s_1, s_2) = \frac{(p_2-p_1)}{(s_2-s_1)} - \frac{(p_1-\bar{p})}{(s_1-\bar{s})}, \text{ and } D_2(p_1, p_2, s_1, s_2) = 1 - \frac{(p_2-p_1)}{(s_2-s_1)}.$$

Both firms maximize their profits with respect to their product prices. For the reaction functions we get

$$\underline{p}(\bar{p}) = \frac{\bar{p}\underline{s}}{2\bar{s}}, \quad \bar{p}(p_1, \underline{p}) = \frac{p_1(\bar{s}-\underline{s}) + \underline{p}(s_1-\bar{s})}{2(s_1-\bar{s})},$$

$$p_1(p_2, \bar{p}) = \frac{p_2(s_1-\bar{s}) + \bar{p}(s_2-s_1)}{2(s_2-\bar{s})}, \text{ and } p_2(p_1) = \frac{p_1+s_2-s_1}{2}.$$

The reaction functions are strictly monotone. For the corresponding prices we get

$$\underline{p}(\underline{s}, \bar{s}, s_1, s_2) = \frac{\underline{s}(\underline{s}-\bar{s})(\bar{s}-s_1)(s_1-s_2)}{\Omega},$$

$$\bar{p}(\underline{s}, \bar{s}, s_1, s_2) = \frac{2\bar{s}(\underline{s}-\bar{s})(\bar{s}-s_1)(s_1-s_2)}{\Omega},$$

$$p_1(\underline{s}, \bar{s}, s_1, s_2) = \frac{(\bar{s}-s_1)(4\bar{s}s_1-\underline{s}(3\bar{s}+s_1))(s_1-s_2)}{\Omega},$$

$$p_2(\underline{s}, \bar{s}, s_1, s_2) = \frac{1}{2} \left\{ s_2 - s_1 + \frac{(\bar{s}-s_1)(4\bar{s}s_1-\underline{s}(3\bar{s}+s_1))(s_1-s_2)}{\Omega} \right\}.$$

For demand we get

$$\underline{D}(\underline{s}, \bar{s}, s_1, s_2) = \frac{\bar{s}(\bar{s}-s_1)(s_1-s_2)}{\Omega}, \quad \bar{D}(\underline{s}, \bar{s}, s_1, s_2) = \frac{2\bar{s}(\underline{s}-s_1)(s_1-s_2)}{\Omega},$$

$$D_1(\underline{s}, \bar{s}, s_1, s_2) = \frac{(\underline{s}(3\bar{s}+s_1)-4\bar{s}s_1)(\bar{s}-s_2)}{\Omega},$$

$$D_2(\underline{s}, \bar{s}, s_1, s_2) = \frac{2(\underline{s}(3\bar{s}^2-2\bar{s}s_2-s_1s_2)-\bar{s}(\bar{s}(3s_1+s_2)-4s_1s_2))}{\Omega},$$

where

$$\Omega = \underline{s}(9\bar{s}^2 + 2\bar{s}(s_1 - 4s_2) + s_1(s_1 - 4s_2)) - 4\bar{s}(s_1(s_1 - 4s_2) + \bar{s}(2s_1 + s_2)).$$

Profits are

$$\pi_1^k(\underline{s}, \bar{s}, s_1, s_2) = \frac{\underline{s}\bar{s}(\bar{s}-\underline{s})(\bar{s}-s_1)^2(s_1-s_2)^2}{\Omega^2} + \frac{(s_1-\bar{s})(\underline{s}(3\bar{s}+s_1)-4\bar{s}s_1)^2(\bar{s}-s_2)(s_1-s_2)}{\Omega^2},$$

$$\pi_2^k(\underline{s}, \bar{s}, s_1, s_2) = \frac{4\bar{s}^2(\bar{s}-\underline{s})(\underline{s}-s_1)(\bar{s}-s_1)(s_1-s_2)^2}{\Omega^2} + \frac{4(s_2-s_1)(\underline{s}(2\bar{s}s_2+s_1s_2-3\bar{s}^2)+\bar{s}(\bar{s}(3s_1+s_2)-4s_1s_2))^2}{\Omega^2}.$$

The partial derivative of the high-quality firm's profits with respect to  $\bar{s}$  is given by

$$\frac{\partial \pi_2^k}{\partial \bar{s}} = -\frac{49(9s_1-16s_2)(s_1-s_2)^2(s_1-\underline{s})(s_2-\underline{s})^2}{48(3s_1^2-4s_1s_2-2s_2^2-5s_1\underline{s}+8s_2\underline{s})^3} < 0. \quad (18)$$

## Appendix 5: High-Quality Innovation by Low-Quality Firm (Case *a*) when the Low-Quality Firm withdraws

When both firms introduce a new product in the high-quality area and only the low-quality firm withdraws the existing product, three products with qualities  $\bar{s} < s_1 < s_2$  are offered. Prices are given by

$$\bar{p}(\bar{s}, s_1, s_2) = \frac{\bar{s}(s_1-\bar{s})(s_1-s_2)}{2\Psi}, \quad p_1(\bar{s}, s_1, s_2) = \frac{s_1(s_1-\bar{s})(s_1-s_2)}{\Psi},$$

$$p_2(\bar{s}, s_1, s_2) = \frac{(s_2-s_1)}{2\left(1 + \frac{s_1(s_1-\bar{s})}{3\bar{s}s_1+\bar{s}s_2-4s_1s_2}\right)}.$$

Demand is as follows

$$\bar{D}(\bar{s}, s_1, s_2) = \frac{s_1(s_1-s_2)}{2\Psi}, \quad D_1(\bar{s}, s_1, s_2) = \frac{s_1(\bar{s}-s_2)}{\Psi}, \text{ and}$$

$$D_2(\bar{s}, s_1, s_2) = \frac{(-4s_1s_2 + \bar{s}(3s_1+s_2))}{2\Psi}.$$

Firms' profits in the product market are

$$\pi_2^k(\bar{s}, s_1, s_2) = \frac{\bar{s}(s_1-\bar{s})s_1(s_1-s_2)^2}{4\Psi^2} + \frac{(s_2-s_1)(-4s_1s_2 + \bar{s}(3s_1+s_2))}{4\Psi\left(1 + \frac{s_1(s_1-\bar{s})}{3\bar{s}s_1+\bar{s}s_2-4s_1s_2}\right)} \quad (19)$$

and

$$\pi_1^w(\bar{s}, s_1, s_2) = \frac{s_1^2(s_1-\bar{s})(\bar{s}-s_2)(s_1-s_2)}{\Psi^2}, \quad (20)$$

where  $\Psi = (2\bar{s}s_1 + s_1^2 + \bar{s}s_2 - 4s_1s_2)$ .

## Appendix 6: High-Quality Innovation by Low-Quality Firm (Case *a*) when the High-Quality Firm withdraws

Both firms introduce a new product in the high-quality area and only the high-quality firm withdraws the existing product. Three products with qualities  $\underline{s} < s_1 < s_2$  are offered in the market. Prices are given by

$$\underline{p}(\underline{s}, s_1, s_2) = \frac{\underline{s}(s_2 - s_1)}{4s_2 - s_1}, \quad p_1(s_1, s_2) = \frac{s_1(s_2 - s_1)}{4s_2 - s_1},$$

$$\text{and } p_2(s_1, s_2) = \frac{2s_2(s_2 - s_1)}{4s_2 - s_1}.$$

Demand is as follows

$$\underline{D}(\cdot) = 0, \quad D_1(s_1, s_2) = \frac{s_2}{4s_2 - s_1}, \quad \text{and } D_2 = \frac{2s_2}{4s_2 - s_1}.$$

Firms' profits are shown by

$$\pi_1^k(s_1, s_2) = \frac{s_1 s_2 (s_2 - s_1)}{(4s_2 - s_1)^2}, \quad \text{and} \quad (21)$$

$$\pi_2^w(s_1, s_2) = \frac{4s_2^2 (s_2 - s_1)}{(4s_2 - s_1)^2}. \quad (22)$$

## Appendix 7: High-Quality Innovation by Low-Quality Firm (Case c)

It can be shown that the second strategic effect dominates the demand effect in equation (11), as

$$\underbrace{-\frac{49(\bar{s} - 2s_1)(\bar{s} - s_1)^2(3\bar{s} + s_1 - 4s_2)(s_1 - s_2)^2}{24(3\bar{s}^2 - 4\bar{s}s_1 - 2s_1^2 - 5\bar{s}s_2 + 8s_1s_2)^3}}_{\text{second strategic effect}} +$$

$$\underbrace{\frac{49(\bar{s} - s_1)^2(s_1 - s_2)^2}{144(-3\bar{s}^2 + 2s_1(s_1 - 4s_2) + 2\bar{s}(4s_1 + 5s_2))^2}}_{\text{demand effect}} =$$

$$-\frac{49(\bar{s} - s_1)^2(s_1 - s_2)^2(15\bar{s}^2 - 10s_1(s_1 - 4s_2) - \bar{s}(26s_1 + 19s_2))}{144(3\bar{s}^2 - 4\bar{s}s_1 - 2s_1^2 - 5\bar{s}s_2 + 8s_1s_2)^3} < 0. \quad (23)$$

applies. We begin to illustrate the positive sign of the denominator. Therefore, we take

the derivative of the low-quality firm's new product quality,  $\frac{\partial(3\bar{s}^2 - 4\bar{s}s_1 - 2s_1^2 - 5\bar{s}s_2 + 8s_1s_2)}{\partial s_1} = -4\bar{s} - 4s_1 + 8s_2$ , which is positive. Setting  $s_1 = \bar{s}$ , gives

$3\bar{s}^2 - 4\bar{s}s_1 - 2s_1^2 - 5\bar{s}s_2 + 8s_1s_2|_{s_1=\bar{s}} = -3\bar{s}^2 + 3\bar{s}s_2 > 0$ . As the denominator is positive even for  $s_1 = \bar{s}$ , it must be necessarily positive for any  $\bar{s} < s_1 < s_2$ . Next, we will

show the negative sign of the numerator. As the derivative of  $\frac{\partial(15\bar{s}^2 - 10s_1(s_1 - 4s_2) - \bar{s}(26s_1 + 19s_2))}{\partial \bar{s}} = 30\bar{s} - 26s_1 - 19s_2$  is negative, and also holds for the case when  $\bar{s} = s_1$ ,  $15\bar{s}^2 - 10s_1(s_1 - 4s_2) - \bar{s}(26s_1 + 19s_2)|_{\bar{s}=s_1} = -21s_1(s_1 - s_2)$ , it is necessarily negative for any  $\underline{s} < \bar{s} < s_1$ . It follows that equation (23) must be negative.



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