

Atukeren, Erdal

Working Paper

Measuring the strength of cointegration and Granger-causality

KOF Working Papers, No. 78

Provided in Cooperation with:

KOF Swiss Economic Institute, ETH Zurich

Suggested Citation: Atukeren, Erdal (2003) : Measuring the strength of cointegration and Granger-causality, KOF Working Papers, No. 78, ETH Zurich, KOF Swiss Economic Institute, Zurich

This Version is available at:

<https://hdl.handle.net/10419/50872>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

Arbeitspapiere/ Working Papers

Erdal Atukeren

Measuring the Strength of Cointegration and Granger-Causality

MEASURING THE STRENGTH OF COINTEGRATION AND GRANGER-CAUSALITY

Erdal Atukeren

Swiss Federal Institute of Technology - Zurich

Swiss Institute for Business Cycle Research

Weinbergstrasse 35, ETH Zentrum,

CH-8092 Zurich, Switzerland

e-mail: atukeren@kof.gess.ethz.ch

ABSTRACT

This paper proposes a methodology that combines the use of Schwarz's BIC in subset autoregression and subset transfer function identification along with the posterior odds ratio test developed by Poskitt & Tremayne (1987) in the context of testing for Granger-causality and cointegration tests. This approach provides a measure for the strength (decisiveness) of causality and cointegration between the variables of interest. As an illustration of our methodology, we re-examine the case of bivariate relationship between money and income in Canada.

JEL Classification: C11, C22, E32

Keywords: Schwarz criterion, Cointegration, Granger-causality, Posterior odds ratio, Money-Income Causality.

I. INTRODUCTION

In this paper, we make use of Jeffrey's (1961) concept of 'grades of evidence' along with Schwarz's (1978) Bayesian Information Criterion and Poskitt & Tremayne's (1987) model portfolio approach in the context of cointegration and Granger-causality tests.¹ Under this approach, it is possible to identify the most probable model and a portfolio of alternative models. Then, one can also test for how strong the case for the best model is by comparing the posterior odds of the 'best' model to the posterior odds of the 'best alternative model with qualitatively different implications'. Thus, we argue that the posterior odds ratio test could provide a measure for the strength (decisiveness) of causality and/or cointegration between the variables of interest. We illustrate our methodology for the case of the bivariate relationship between money and income in Canada. We use a historical data set (Hsiao, 1979) that attracted many researchers and led them to arrive at different conclusions about the nature of money-income causality in Canada.

II. METHODOLOGICAL ASPECTS OF TESTING FOR GRANGER-CAUSALITY

Let X and Y be two distinct covariance stationary stochastic processes. The variable X Granger-causes Y if Y can be better predicted by using the history of X in addition to all the available relevant other information in the universe. In a bivariate case, if a regression of Y on its own lags and the lags of X produces a lower forecast error variance than the regression of Y on its own lags only, then X is a *prima facie* Granger-cause of Y . Granger-causality from Y to X can be defined analogously. In its most common testable form, equations (1) and (2) illustrate the above definitions of Granger-causality for the bivariate case.

$$y_t = \alpha_1 + \sum_{j=1}^p \beta_{1j} y_{t-j} + \sum_{j=1}^q \delta_{1j} x_{t-j} + \varepsilon_{1t} \quad (1)$$

$$x_t = \alpha_2 + \sum_{j=1}^r \beta_{2j} y_{t-j} + \sum_{j=1}^s \delta_{2j} x_{t-j} + \varepsilon_{2t} \quad (2)$$

where α_i are the constant terms, p, q, r, s are the lag-orders, and ε_{it} are well-behaved error terms.

Testing for Granger-causality, first, involves the identification of the lag-orders in equations (1) and (2). In Granger (1969), these lag-orders are of equal length, such that $p=q=r=s$. Unfortunately, imposing equal lag structures on each variable quickly exhausts the degrees of freedom. In order to provide a more parsimonious representation and more efficient parameter estimates, Hsiao (1979) and Kang (1989) developed a flexible-lag-lengths version of Granger's original approach by employing a statistical model selection criterion to determine the optimum p, q, r, and s in equations (1) and (2).² Referring to equation (1) and using Schwarz's BIC (SBIC) as the model selection criterion, the procedure runs as follows:

Step 1: Estimate autoregressions for Y from lag 1 to a pre-specified maximum lag M and calculate the SBIC values from these autoregressions. The autoregression where the SBIC is minimised is retained as the best univariate specification for Y, i.e., the best case for non-causality. This identifies "p".

Step 2: Introduce the lags of X over the best univariate model for Y, and choose the model where the SBIC is minimised. This procedure identifies "q".

Step 3: If the minimum SBIC from the bivariate model is less than value for the best univariate model for Y, then there are grounds to argue that X Granger-causes Y.

In Hsiao's approach if p is found to be 4, then all the lags from 1 to 4 are included in the univariate specification, while all the lags between p and M are excluded from the equation. This may lead to over-parameterization, inefficiency in estimation, and a possible omitted variable bias if there is a significant lag between p and M ($M > p$). To deal with this problem, Kang (1989) suggested the formation of a subset autoregression in the first step and a subset transfer function in the second step by suppressing any insignificant lags between lag 1 and M .³

III. A POSTERIOR ODDS RATIO TEST FOR GRANGER-CAUSALITY

Suppose that one follows the Hsiao-Kang methodology and obtains an SBIC (in natural logarithms) value of -5.793229 for the best univariate specification for Y in step 1, and -5.845061 when the lags of X are included (Step 2). Since the SBIC in Step 2 is less than the one from Step 1, there are grounds to conclude in favour of Granger-causality from X to Y (Step 3). But, 'how close' are these SBIC values? Conventionally, one would use the F , likelihood ratio, or Wald tests for the joint significance of the lags of X . This approach poses conceptual difficulties since it combines the use of a Bayesian statistical cost function with classical statistical significance testing. Then, the question is whether it is possible to use a test that is based on the SBIC at this stage, too? The answer is a yes, and the necessary framework is provided by Poskitt & Tremayne (1987).

Poskitt & Tremayne (1987) used Jeffreys's (1961) Bayesian concept of the "grades of evidence" to develop a more flexible (in their words: less mechanical) framework for linear time series identification.⁴ The grades of evidence approach questions the uniqueness of the best model (the

model with the minimum SBIC value), by comparing its posterior odds with other models. If there are alternative models with posterior odds close enough (i.e., within a given proximity) to the posterior odds of the model chosen by the minimum of the SBIC, this leads to the formation of a model portfolio. As a decision rule to determine whether the posterior odds of an alternative specification is close enough to the specification chosen by the minimum Schwarz criterion, Poskitt & Tremayne (1987, p. 127) employ the following ratio:

$$R = \exp[\frac{1}{2} T | SBIC_1 - SBIC_0 |] \quad (3)$$

where T is the sample size and $| SBIC_1 - SBIC_0 |$ is the absolute value of the difference between the SBIC values being compared.

As Jeffreys (1961, p. 432) indicates, this ratio has no physical meaning, though "[i]ts function is to grade the *decisiveness* of the evidence [italics added]." Poskitt & Tremayne (1987, p. 127) divide the range of this ratio into three intervals. 1) if $R > 100$, the alternative model is unconditionally discarded. 2) if $\sqrt{10} < R \leq 10$, where $\sqrt{}$ is the square root function, there is "no substantial evidence" in favour of the model minimising the SBIC. There exists a competing model to the specification chosen by the minimum SBIC. 3) if $1 < R \leq \sqrt{10}$, then the alternative model is a "close competitor" to the model chosen by the minimum SBIC. One should also note the fourth case, where $10 < R \leq 100$, where the alternative model can again be discarded as non-competing. In this case, however, one would most likely conclude in favour of the "X Granger-causes Y" hypothesis by using classical significance testing methodology.

We extend Poskitt & Tremayne's (1987) model portfolio notion to make inferences about Granger (non-) causality. Returning to the previous example, we calculate the R-ratio (given the sample size T) between -5.845061 ($SBIC_1$) and -5.793229 ($SBIC_0$). That is, we compare the proximity of the SBICs from the best "X does not Granger-cause Y" model (-5.793229) and the best "X Granger-causes Y" model (-5.845061), respectively. The R-ratio compares the posterior odds of these two models, and yields information as to 'how close' these two models are. This step replaces the significance tests with a posterior odds ratio test. An R-ratio greater than 100 indicates very strong (decisive) evidence that X Granger-causes Y. Thus, the posterior odds ratio test has the added advantage that it yields information about the decisiveness (or the strength) of (non-) causality between the variables of interest. This procedure can be formalized as follows:

Step 4: Compare the SBIC values from Step 1 and Step 2. Denote the SBIC value from Step 1 as $SBIC_0$ and the value from Step 2 as $SBIC_1$. Two cases, Case A and Case B, may arise.

Case A: $SBIC_0$ is less than or equal to $SBIC_1$. Conventionally, one would conclude that X does not Granger-cause Y. However, there may exist a closely competing or competing model where X Granger-causes Y. To check if this is indeed the case, calculate the R-ratio. If $R > 100$, there is decisive (strong) evidence that X does not Granger-cause Y. If $R < 100$, there are competing or closely competing models where X Granger-causes Y, and the initial finding of non-causality from X to Y lacks decisiveness. The finding of non-causality may be, for example, due to a low signal to noise ratio. Furthermore, since the sample size is a determinant of the R-ratio, our approach also takes sample size into account in grading the strength of causal inferences.

Case B: $SBIC_1$ is less than $SBIC_0$. There are grounds to conclude that X Granger-causes Y. However, there may exist a closely competing or competing model where X does not Granger-cause

Y. The R-ratio can be used to identify if this is indeed the case. If $R > 100$, there is decisive (strong) evidence that X Granger-causes Y. If $R < 100$, there are competing or closely competing no-causality models, and the evidence leaning towards Granger-causality from X to Y lacks decisiveness.⁵

It should be noted that a concern on the sensitivity of the Granger-causality tests to lag-specifications has been expressed previously in the literature. For example, Hsiao (1979) suggested a deliberate under- and over-fitting exercise on the chosen model and testing the significance of the additional lags and the excluded lags by the likelihood ratio test. In a similar context, Thornton and Batten (1985) suggested that perhaps one should be agnostic and conclude in favour of Granger-causality if this result is confirmed at each and every lag up to M. These approaches, however, have also drawbacks. In the case of Hsiao (1979), the choice of the lag lengths in the over- or under-fitting exercise remains *ad hoc*. And, in the case of Thornton & Batten (1985), insignificant lags may lead to insignificant F-statistics, and thus the failure to reject non-causality when it is false.

A related approach to the one in this paper was proposed by George & McCulloch (1993). They employed a model portfolio-based approach for the variable selection problem in regression analysis. They searched for the promising subsets of predictor variables based on their higher frequency of appearance in the Gibbs sample. The resulting subsets are identified as those with higher posterior probability. In addition, Ploberger & Phillips (1996) and Chao & Phillips (1999) developed a new model selection criterion, namely the “posterior information criterion (PIC)”, in the context of testing for cointegration. Their approach along with George and McCulloch’s work have

many similarities to our methodology. Nevertheless, the procedure developed in this paper is computationally simpler and has the advantage of giving information about the decisiveness or the strength of the evidence.

IV. THE CASE OF BIVARIATE MONEY AND INCOME RELATIONSHIP IN CANADA

In this section, we illustrate the application of the methods developed above for the case of money-income causality in Canada. As shown by Hsiao (1982), for a variable X to be a direct Granger-cause of a variable Y , X should Granger-cause Y both in bivariate and multivariate models. Therefore, for illustrative purposes, we focus on the bivariate relationship between money and income.⁶ We use the data provided by Hsiao (1979, Appendix). Since this data set has also been used by other authors, it allows us to compare our results with theirs. The data set contains seasonally adjusted quarterly values of nominal GNP (Y) and money supply ($M1$ and $M2$) running from 1955Q1 to 1977Q4. Using this data set, Hsiao (1979) concluded that there is a feedback between $M1$ and Y , and that there exists a unidirectional causality from Y to $M2$. Kim and Ro (1988) used a causal vector autoregressive moving average model, and obtained the same results. Introducing the interest rate (BR) as an additional variable, Hsiao (1982) found that there is unidirectional causality from Y to $M2$ in bivariate analysis, and in a trivariate setting the causality from Y to $M2$ is indirect via BR . Lütkepohl (1982) obtained a similar indirect causality. Penm & Terrell's (1984) study involved subset autoregressions. They found that the causal relationships between money ($M2$), income, and interest rates (BR) are sensitive to the model selection criterion used. However, their results from the SBIC indicated no causal relationships between $M2$ and Y in the bivariate analysis, and an indirect causality from Y to $M2$ via BR in a trivariate setting.

To start our analysis, we first ran Phillips & Perron's (1988) unit root tests on the natural logarithms of each series in order to establish the correct method of detrending.⁷ The results showed that all series are integrated of order one, $I(1)$.⁸ Given that $\log(\text{GNP})$, $\log(\text{M1})$, and $\log(\text{M2})$ are all $I(1)$ variables, we used Johansen's (1990,1991) trace test to check whether $\log(\text{GNP})$ and $\log(\text{M1})$ and $\log(\text{GNP})$ and $\log(\text{M2})$ are cointegrated. In doing so, we extended the use of posterior odds ratio (R-ratio) and the model portfolio concepts into the context of testing for cointegration. This approach allows one to select the type of the model, optimal lag length, and the rank simultaneously. This is indeed similar to the approach provided by Chao & Phillips (1999), but it is computationally simpler.⁹

We set the maximum lag length to eight and selected the model with the minimum SBIC among Johansen's five specifications.¹⁰ For the case of $\log(\text{GNP})$ and $\log(\text{M1})$, this yielded a model that allowed for quadratic deterministic trend in the data, and SBIC was minimum at lag 1 (SBIC = -16.84123) with the likelihood ratio (LR) equal to 23.98, where the 1% critical value is 23.46 and the 5% critical value is 18.17. Thus, there is evidence of cointegration between $\log(\text{GNP})$ and $\log(\text{M1})$ according to Johansen's test. The closest SBIC from a model that indicated "no cointegration" (the same model but with rank = 0) yielded an SBIC value of -16.79010. As the next step, we tested the "strength" of cointegration by using Poskitt & Tremayne's posterior odds ratio test. This resulted in an R-ratio of 8.56. Therefore, the finding of cointegration between $\log(\text{GNP})$ and $\log(\text{M1})$ cannot be said to be decisive. Using the same procedure described above, we examined the relationship

between $\log(\text{GNP})$ and $\log(\text{M2})$. This resulted in a model with a quadratic deterministic trend in the data, with lag 1, and a minimum SBIC value of -16.94379 . In this model, $\log(\text{GNP})$ and $\log(\text{M2})$ are not cointegrated. Indeed, Johansen's test for this specification leads to a likelihood ratio of 12.75, where the 5% and 1% critical values are 18.17 and 23.46, respectively. Thus, hypothesis of no cointegration cannot be rejected. The closest model where $\log(\text{GNP})$ and $\log(\text{M2})$ are found to be cointegrated is the case with one lag in the VAR with no trend in the data and no constant term in the model. The SBIC value for this model is -16.90705 . The R-ratio between this model and the best 'no cointegration' model is 4.68.¹¹ That is, there is no strong or decisive evidence for the lack of cointegration between $\log(\text{GNP})$ and $\log(\text{M2})$. It leans towards no cointegration, but a model where $\log(\text{GNP})$ and $\log(\text{M2})$ are cointegrated is a close competitor. However, the posterior odds ratio test does not replace Johansen's methodology, but rather helps to choose the type of model and the lag specification while shedding light on how strong or decisive the findings are. Thus, keeping in mind how strong the evidence is, we model the relationship between $\log(\text{GNP})$ and $\log(\text{M1})$ as a vector error correction model, but choose a VAR specification for the relationship between $\log(\text{GNP})$ and $\log(\text{M2})$.

The next step is to test for Granger-causality between Y and M1 and Y and M2. In line with Step 1 in Section III, we fitted the best univariate for each variable. Hsiao (1979) set the maximum lag-length as $M=14$. Since the criteria used by Hsiao (the final prediction error criterion) selects longer lags than the SBIC, we set $M=12$ in order to gain more degrees of freedom. For Y and M1, we also took into account that they are cointegrated and included the error correction term (ECT) into the specifications where relevant. Table 1 shows the results. The resulting SBIC values for the

univariate specifications indicate that, without allowing for subset autoregressions, one would choose the optimal lag-order (p) for GNP (without ECT), GNP (with ECT), M1 (with ECT), and M2 as 3, 3, 0, and 2, respectively.

< Table 1 >

A further examination of the SBIC values for GNP (without the ECT) in Table 1 indicates that the introduction of the second lag over the first lag increases the SBIC value. Therefore lag 2 can be suppressed. The introduction of lag 3 over the first two lags decreased the SBIC. That is, the third lag term should not be suppressed. The Schwarz BIC values following lag 3 continue to increase. Thus, further lags can be suppressed. Therefore, our subset autoregression for Y includes the first and the third lags.¹² As shown on the last line of Table 1, the resulting SBIC from this subset autoregression is lower than the best specification that would otherwise be obtained (-5.793229 versus -5.762252). The subset autoregressions for the other cases in Table 1 can be similarly identified.

What is more important is that if one does not include the significant (in a statistical cost function sense) lags between the minimum Schwarz BIC and lag M , an omitted variable bias is committed. The insignificant lags and omitted variables, yielding in larger Schwarz BIC values than the subset autoregressions, provide a weaker case for non-causality (univariate model). In testing for Granger-causality, this could bias the results towards detecting causality when it is false.

Table 2 shows the SBIC values from (subset) transfer functions of M1 on GNP, GNP on M1, M2 on GNP, and GNP on M2. The methodology here follows the one described in Step 2 in Section III. To test whether M1 causes GNP, we introduced the lags of M1, up to M=12, over the best univariate specification for GNP. The resulting SBIC values are shown under the heading "M1 causes GNP" in Table 2. The SBIC is minimised at lag 2. The first lag of M1 is also significant in our sense, since its introduction lowered the Schwarz BIC from the univariate model for Y. Lag 3 of M1 produces a higher Schwarz BIC than lag 2, thus lag 3 can be suppressed. This is not the case for lag 4, which produced a lower Schwarz BIC over the first 3 lags. Other lags of M1 can all be suppressed following a similar logic. As a result, we retain the first, second and fourth lags of M1 in our transfer function specification. The Schwarz BIC from this subset transfer function is lower (-5.824348) than the one which did not allow subsetting (-5.800771).

< Table 2 >

Using a similar procedure for "GNP causes M1", "M2 causes GNP", and "GNP causes M2", we obtained the following results. The first, the fifth, and the sixth lags of Y are significant when the lags of Y are regressed over the best specification for M1. Similarly, the first and the third lags of M2 was found to be significant when we regressed the lags of M2 over the best univariate specification for GNP. We also found that only the first lag of GNP is significant when the lags of GNP are regressed over the best subset autoregression for M2. Based on Table 1 and Table 2, we can now compute the R-ratios between the univariate models and bivariate models. The results are shown in Table 3.

< Table 3 >

Strikingly, the only decisive causal evidence ($R > 100$) is obtained for the hypothesis "GNP causes M1". Although there is some evidence for causality (especially) from M1 to GNP and from M2 to Y, the no-causality models are competing (but not closely-competing) cases. The case for (non-) causality from GNP to M2 is very weak. Indeed, the no-causality case is a close-competitor to the causality model, and *vice versa*. We can reinstate this evidence as follows. Table 3 suggests that there is a case for a feedback between M1 and GNP, but no causality in any direction can be (decisively) established between GNP and M2.

These findings support our interpretation of the decisiveness of the evidence from tests of cointegration. In the case of M1 and Y, we concluded in favour of cointegration by using Johansen's procedure - but with a reservation of the strength of the cointegrating relationship. After modelling the M1 and Y as cointegrated series, we find that the causal relationship indeed strongly runs from Y to M1, while there is still a weaker argument for a feedback. In the case of Y and M2, we did not find a decisive or strong case for neither a cointegrating nor a Granger-causal relationship.

From a macroeconomic point of view, this finding is significant since we find that there is no strong causal flow from monetary growth to output growth even in a bivariate model. From an economic policy point of view, the absence of a decisive causal link from M1 to Y, coupled with a strong causal link from Y to M1, may also help explain the failure of M1 targeting in Canada.

V. CONCLUSIONS

In this paper, we offered a framework for the consistent use of Schwarz Bayesian information criterion in the tests of Granger-causality and cointegration. The Schwarz BIC has been increasingly popular as a model selection criterion due to its optimal asymptotic and finite sample properties. Previous approaches for testing Granger-causality mostly combined the use of a statistical cost function with significance testing. Such a strategy poses difficulties if one uses a Bayesian cost function, such as the Schwarz BIC. To overcome this difficulty, we developed a consistent approach that is based on the use of Schwarz BIC for model selection (subset autoregression and transfer function identification) and making inferences about causality as well as testing for cointegration. Our approach has a Bayesian motivation with the further advantage that one can now grade the decisiveness (strength) of the evidence. As an application of our procedures, we re-examined the case of Canadian money-income causality. Contrary to previous findings, our results indicated that the only decisive causal link is from nominal GNP growth to M1 growth. This may also help explain the failure of monetary targeting (M1) in Canada.

FOOTNOTES

1. The Schwarz BIC (SBIC) results from a Bayesian procedure of seeking the most probable (*a posteriori*) model. The SBIC is given by:

$$\text{Schwarz BIC}(p,q) = (\text{RSS}/T) T^{(p+q+1)/T}$$

In practice, the natural logarithm of Schwarz BIC is reported. That is,

$$\text{Log Schwarz BIC}(p,q) = \text{Log} (\text{RSS}/T) + [(p+q+1)/T] \text{Log} (T).$$

where p and q are lag orders, T is the sample size, and RSS is the residual-sum-of-squares.

Mills & Prasad (1992), for example, based on their Monte-Carlo experiments, recommend the Schwarz BIC as the first choice of applied researchers. See also Nickelsburg (1985), Lütkepohl (1985), and Yi & Judge (1988).

2. Hsiao (1979) employed the final prediction error criterion. The FPE is given by:

$$\text{FPE}(p,q) = (\text{RSS}/T) [(T+p+q+1)/(T-p-q-1)].$$

where p and q are lag orders, T is the sample size, and RSS is the residual-sum-of-squares.

3. An estimated coefficient at lag k is considered "significant", if the associated SBIC value is lower than the SBIC at lag $k-1$. See also Penm & Terrell (1984) and Marin (1992).

4. See Mills (1990, pp. 140-142).

5. For the example considered in this section ($\text{SBIC}_0 = -5.793229$ and $\text{SBIC}_1 = -5.845061$), the posterior odds ratio (R) is 7.75 for $T = 79$. That is, there is no decisive evidence for the model where 'X Granger-causes Y'. The case where 'X does not Granger-cause Y' is a closely competing model. See also Table 3. Note that for the evidence for 'X Granger-causes Y' to be considered decisive, the sample size (T) would have to be at least 178, given the same SBIC values. At $T=79$, the noise prevents one to reach decisive or strong conclusions.

6. See Cagan (1989), Stock & Watson (1989), and Blanchard (1990) for surveys of the literature on the relationship between money and income.

7. Hsiao (1979, 1982) used second differencing on the natural logarithms (logs) of each variable. Lütkepohl (1982) and Penm & Terrell (1984) took first differences of the log-levels. Kim and Ro (1988) conducted their analyses on the log-levels of the variables.

8. The resulting modified t-statistics from a model where a constant and a trend term are included are -0.31, -1.05, and -0.26 for $\log Y$, $\log M1$, and $\log M2$, respectively. The critical value is -3.13. The truncation lag is set at 4. Phillips and Perron's modified t-statistics on the first differences of the above variables are -7.41, -6.38, and -5.74, respectively.

9. See also Dorfman (1995) and So & Li (1999) for the use of a similar posterior odds ratio concept in the context of testing for unit roots and cointegration.

10. The five models are specified as follows: 1) no trend in the data with no trend and no constant term in the model 2) no trend in the data with no trend but with a constant term in the model, 3)

linear trend in the data with no trend but with a constant term in the model, 4) linear trend in the data with trend and a constant term in the model, 5) quadratic trend in the data with trend and a constant term in the model.

11. Detailed results are available upon request.

12. The first lag is included in the specification since its introduction led to a decrease in the Schwarz BIC over the specification where Y is regressed on a constant term only.

REFERENCES

- Akaike, H. (1969) Fitting autoregressive models for prediction, *Annals of the Institute of Statistical Mathematics*, **21**, 243-247.
- Blanchard, O.J. (1990) Why Does Money Affect Output? A Survey, in *Handbook of Monetary Economics* (Eds.) B.M. Friedman and F.H. Hahn, North-Holland, Amsterdam.
- Cagan, P. (1989) Money-Income Causality -- A Critical Review of the Literature since 'A Monetary History', in: (Ed.) Bordo, M.D. *Money, History, and International Finance: Essays in Honor of Anna J. Schwartz*, NBER Conference Report Series, University of Chicago Press.
- Chao, J.C. and Phillips, P.C.B. (1999) Model selection in partially nonstationary vector autoregressive processes with reduced rank structure, *Journal of Econometrics*, **91**(2), 227-71.
- Dorfman, J.H. (1995) A numerical Bayesian test for cointegration of AR Processes, *Journal of Econometrics*, **66**, 289-324.
- George E.I. and McCulloch, R.E. (1993) Variable selection via Gibbs sampling. *Journal of the American Statistical Association*, **88**, 881-889.
- Granger, C.W.J. (1969) Investigating causal relationships by econometric models and cross-spectral methods, *Econometrica*, **36**, 424-438.
- Hsiao, C. (1979) Autoregressive modeling of Canadian money and income data, *Journal of the American Statistical Association*, **74**, 553-560.
- Hsiao, C. (1982) Autoregressive modelling and causal ordering of economic variables, *Journal of Economic Dynamics and Control*, **4**, 243-259.
- Jeffreys, H. (1961). *Theory of Probability*, 3rd edition, Oxford University Press, London.

- Johansen, S. and Juselius, K. (1990) Maximum likelihood estimation and inference on cointegration with application to demand for money, *Oxford Bulletin of Economics and Statistics*, **52**, 169-210.
- Johansen, S. (1991) Estimation and hypothesis testing of cointegrating vectors in Gaussian vector autoregressive models, *Econometrica*, **59**, 1551-1580.
- Kang, H. (1989) The optimal lag selection and transfer function analysis in Granger-causality tests. *Journal of Economic Dynamics and Control*, **13**, 151-169.
- Kim, W. and Ro, K. (1988) A causal VARMA model analysis with an application to Canadian money and income data, *Applied Economics*, **20**, 1167-1183.
- Lütkepohl, H. (1982) Non-causality due to omitted variables, *Journal of Econometrics*, **19**, 367-378.
- Lütkepohl, H. (1985) Comparison of criteria for estimating the order of a vector autoregressive process, *Journal of Time Series Analysis*, **6**, 35-52.
- Marin, D. (1992) Is the export-led growth hypothesis valid for industrialized countries?, *Review of Economics and Statistics*, **54**, 678-688.
- Mills, J.A. and Prasad, K. (1992) A comparison of model selection criteria, *Econometric Reviews*, **11**, 201-233.
- Mills, T.C. (1990) *Time Series Techniques for Economists*, Cambridge University Press, Cambridge, England.
- Nickelsburg, G. (1985) Small-sample properties of dimensionality statistics for fitting VAR models to aggregate economic data: a Monte-Carlo study, *Journal of Econometrics*, **28**, 183-192.
- Penm, J.H.W. and Terrell, R.D. (1984) Multivariate subset autoregressive modelling with zero constraints for detecting 'overall causality', *Journal of Econometrics*, **24**, 311-330.
- Phillips, P.C.B. and Perron, P. (1988) Testing for a unit root in time series regression, *Biometrika*, **75**, 335-346.
- Phillips, P.C.B and Ploberger, W. (1996) An asymptotic theory of Bayesian Inference for time series, *Econometrica*, **64**, 381-412.
- Poskitt, D.S. and Tremayne, A.R. (1987) Determining a portfolio of linear time series models, *Biometrika*, **74**, 125-137.
- Schwarz, G. (1978) Estimating the dimension of a model, *Annal of Statistics*, **6**, 461-464.

So, M.K.P. and Li, W.K. (1999) Bayesian unit root Testing in stochastic volatility Models, *Journal of Business and Economic Statistics*, **17**, 491-496.

Stock, J.H. & Watson, M.W. (1989) Interpreting the evidence on money-income causality, *Journal of Econometrics*, **40**, 161-181.

Thornton, D.L. and Batten D.S. (1985). Lag-length selection and tests of Granger causality between money and income, *Journal of Money Credit and Banking*, **17**, 164-178.

Yi, G. and Judge, G. (1988). Statistical model selection criteria, *Economics Letters*, **28**, 47-51.

TABLE 1. Schwarz BIC Values from Univariate Models for Y, M1, and M2.

LAG	LOG(GNP)	LOG(GNP) & ECT	LOG(M1) & ECT	LOG(M2)
C, ECT	-5.70.5941 (C)	-5.655093 (ECT)	-5.323262*** (ECT)	-5.217355 (C)
1	-5.724868*	-5.674090*	-5.314301	-5.353538*
2	-5.719373	-5.666585	-5.261146	-5.358084**
3	-5.762252**	-5.708640**	-5.213782	-5.309677
4	-5.712426	-5.657635	-5.259703*	-5.263622
5	-5.687056	-5.633687	-5.229824	-5.215758
6	-5.631773	-5.578401	-5.203338	-5.164114
7	-5.621530	-5.570073	-5.159923	-5.130300
8	-5.568360	-5.516963	-5.109446	-5.088792
9	-5.516092	-5.465287	-5.102664	-5.035987
10	-5.500739	-5.455246	-5.080769	-5.014955
11	-5.466434	-5.420598	-5.028446	-4.964099
12	-5.413856	-5.368872	-4.984531	-4.964290*
Subset AR	-5.793229***	-5.739376***	-5.316541	-5.363853***

TABLE 2. Schwarz BIC values from Bivariate Models

Lag	M1 Causes GNP	GNP causes M1	M2 causes GNP	GNP causes M2
Min SBIC(.)	-5.739376 (GNP)	-5.325738 (M1)	-5.793229 (GNP)	-5.363853 (M2)
1	-5.761284*	-5.374092**	-5.792255**	-5.362452**
2	-5.800771**	-5.318843	-5.743786	-5.307626
3	-5.745713	-5.263778	-5.759125*	-5.252408
4	-5.780218*	-5.216301	-5.711627	-5.197639
5	-5.753698	-5.368482*	-5.661722	-5.180432
6	-5.698482	-5.373463*	-5.608499	-5.132877
7	-5.678219	-5.329715	-5.553308	-5.087155
8	-5.642724	-5.310434	-5.527172	-5.034800
9	-5.587773	-5.255252	-5.486674	-4.991302
10	-5.532528	-5.210695	-5.434790	-4.982245
11	-5.516659	-5.194322	-5.382719	-4.941231
12	-5.462469	-5.140067	-5.331328	-4.918192
Subset TF	-5.824348***	-5.473354***	-5.845061***	-----

N.B. (**) indicates the minimum SBIC obtained without a subsetting procedure. (*) indicates a significant lag in a statistical cost function sense. TF stands for transfer function. (***) shows that the SBIC from the subset transfer function is smaller than (**).

TABLE 3. Tests of Causal Relationships between GNP, M1 and M2

Hypothesis	Univariate Model:	Univariate SBIC	Entering Variable	Bivariate SBIC	R-Ratio
M1 causes GNP	GNP(ECT,1,3)	-5.739376	M1(1,2,4)	-5.824348	28.69
GNP causes M1	M1(ECT)	-5.325738	GNP(1,5,6)	-5.473354	340.64
M2 causes GNP	GNP(1,3)	-5.793229	M2(3)	-5.845061	7.75
GNP causes M2	M2(1,2,12)	-5.363853	GNP(1)	-5.362452	1.06

N.B. The values in (.) show the lags included in the specification. The R-ratio is calculated from equation (3), where T=79. ECT stands for the error correction term.