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# Arbeitspapiere/ Working Papers

Andrzej Malawski and Martin Wörter

Diversity Structure of the  
Schumpeterian Evolution.  
An Axiomatic Approach



# DIVERSITY STRUCTURE OF THE SCHUMPETERIAN EVOLUTION. AN AXIOMATIC APPROACH<sup>1</sup>

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## **Abstract**

*The purpose of the paper is to impose the diversity structure on the Schumpeterian evolution modelled axiomatically within a dynamical system approach to the Arrow-Debreu theory of general equilibrium. In this framework two basic forms of economic life studied by Schumpeter, i.e., the circular flow and economic development are investigated as specific properties of a dynamic system, where single production systems being the parts of the Debreu models change in the definite environment according to the rules of the Schumpeterian evolution.*

*Moreover, the Sterling diversity-triplet concept based on subcategories of variety, balance and disparity, is slightly modified and applied to serve as a tool for rigorous analysis of innovative and cumulative changes in production system with respect to diversity structure.*

*Finally, integrating diversity aspects into a dynamic system approach, the study identifies innovative processes as the source of growing diversity in the Schumpeterian vision of economic development.*

**JEL Classification: D50, O30**

**Keywords:** diversity, Schumpeterian evolution, Arrow-Debreu theory, dynamic system approach

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## 1. INTRODUCTION

The history of economic thinking quite often shows that notions from biology had an influence on economic thinking. This could have been observed e.g. in the field of “competition” (Morgan, 1993) or especially in evolutionary economics (Nelson and Winter, 1982). Certainly, aspects from other sciences may provide an inspiring background in search of an useful concept of “diversity” as well. However differences remain, as has already been stated by Schumpeter (1954).

In a Schumpeterian framework innovation and technology generation and its diffusion are very important for economic evolution, by “pushing” the economy “out of equilibrium” or switching “equilibrium”. Hereby the diversity of actors plays an important role. Loch and Hubermann (1999) show that the resistance of switching equilibrium is weakened by the diversity of actors with respect to their appreciation of new technologies. Thus diversity affects competition and is an important mechanism for economic evolution. Furthermore Saviotti (1996) points at the importance of “variety” in Schumpeterian thinking and emphasises the innovative behaviour of entrepreneurs as a “variety”-enhancing factor in the economy. Metcalf and Miles (1994) state that the joint distribution of relevant differences across firms’ behaviour is important; the process of evolution depends upon it. As a consequence evolution is connected to diverse behaviour. But what influences diverse behaviour of a firm and leads to more or less advanced innovations that promote new market products or commodities, influences firms’ profits, their production plans and thus moves the economy?

The diverse behaviour of different firms and as a result a diverse economy can be traced back to firm specific interactions between organisation principles, technology and learning abilities and personal perception (see Woerter 2001). Firstly, organisational principles, expressed in firm routines (see Nelson 1995) are differing among firms and are bounded e.g. according to the “satisficing” principle of Simon (1956). They are very seldom fundamentally questioned and remain unchanged even if the economic environment may suggest a quite different behaviour (see Simon 1981). Secondly, firm’s knowledge base, its technology and learning abilities are also very often bounded to prevailing paradigms (see Dosi 1988) or focused on a dominant design (see Utterback 1996) and thus limiting firms abilities to react upon or adapt to new market circumstances. Thirdly, personal rule dependent perception as analysed by Holland et al. (1986) is an important reason for “path dependent” behaviour. This way, important environmental signals, pointing at a change in behaviour, maybe overseen or simply ignored.

Differences in these parameters and their interactions are decisive for a firms' innovative behaviour. The innovative behaviour in turn, impacts production plans, market products and finally fosters economic evolution, as will be shown in a "Schumpeterian" framework modelled within a dynamic system approach to the Arrow-Debreu theory (see section 3). Since firms differ in these parameters, intuitively a greater number of innovative firms, new products and technologies are signs of a rather diverse economic behaviour.

In this context, the purpose of the paper is to impose the diversity structure on the Schumpeterian evolution modelled axiomatically within a dynamical system approach (Malawski 1995, 2005) to the Arrow-Debreu theory of general equilibrium (Debreu, 1959). In that framework two basic forms of economic life studied by Schumpeter (1934), i.e., the circular flow and economic development are investigated as specific properties of a dynamical system, where single production systems being the parts of the Debreu models change in the definite environment according to the rules of the Schumpeterian evolution.

Moreover, the Sterling (1998, 2004) diversity-triplet concept based on subcategories of variety, balance and disparity, is slightly modified and applied to serve as a tool for rigorous analysis of innovative and cumulative changes in production system with respect to diversity structure. In particular, in our Arrow-Debreu world the hierarchical formulation of the diversity-triplet concept is proposed, where variety refers to the number of commodities, markets or products, balance indicates the rate of change in the variety components, and disparity measures the share of innovators and remaining producers

In other words and more concretely our modelling of diversity within Schumpeterian evolutionary economics, as rooted in the dynamic approach to the Arrow-Debreu theory of general equilibrium, considers the following three new factors. Firstly, we distinguish two aspects of the problem, as referred to diversity where?, and diversity of what? Secondly, we assume that it is possible to assign to components defining diversity-triplet concept some characteristics of the formal model. Thirdly, we postulate the hierarchical structure of diversity. Based on these new model specifications we focus mainly on three hypotheses:

#### Hypothesis 1.

The first perspective means that the diversity problem will be studied in the space of production systems defined in Malawski (1999, 2005), and it is based on the conviction that in our evolutionary approach the key concept of diversity should be related to the changes to occur in this space. These are referred as the extensions, cumulative, innovative and related to creative destruction, of production systems.

Hypothesis 2. Secondly, we suggest to assigning to the subcategories defining the diversity-triplet concept the respective parameters characterising the model of production system in our formal set-up.

Hypothesis 3.

Thirdly, the literature on the diversity concept suggests that the problem of the internal structure of this idea has not found yet any satisfactory solution. Therefore we postulate the hierarchical structure of diversity, borrowed, as mentioned above, from the Sterling diversity-triplet concept.

Finally, integrating diversity aspects into a dynamic system approach, the study identifies innovative processes as the source of growing diversity in the Schumpeterian vision of economic development.

The paper is organised as follows. Section 2 presents the applied diversity concept as well as the key components of the Schumpeterian vision on economic evolution. In Section 3 the model of a Debreu economy is presented in a multirange relational system, and interpreted as a state of a state-space constructed of such models, where a respective (quasi)-semidynamical system is defined. There also the hypotheses, mentioned above, are formally examined in both static and dynamic framework. The main results are presented in the form of respective theorems. Conclusion summarizes the results and suggests new research perspective in this area.

## **2. DIVERSITY AND SCHUMPETERIAN EVOLUTION**

There are, among others, two important issues discussed widely in the contemporary literature on evolutionary economics, i.e., the debates on the role of diversity in economic evolution and on the influence of the Schumpeterian legacy on the current studies in this discipline. Both of them are discussed controversial. However, basically these two points of discussion are interrelated.

Indeed, like mentioned above, biological notions are quite useful in conceptualising diversity in an economic framework. For instance, Solow, Polasky and Broadus (1993) and M. Weitzman (1992, 1993) focus their diversity measure on “genetic” distances between species, stating that the longer ago a species separated from another one (“age”), the greater is its “genetic” distance and the greater is its meaning for diversity. To measure diversity, based on such distances and age might be quite inspiring for measuring economic diversity as well. However there are limitations.

Firstly, we know from several studies that industries and technologies are exposed to certain development cycles. Radical innovations set these cycles in motion, which make

them indispensable for economic evolution. As a consequence of the cyclic development of an economy, the age of an industry or the number of different industries alone are not a satisfying indicator for diversity. Heuss (1965) characterises market development cycles based on different types of entrepreneurs. Initiative types of entrepreneurs dominate early development phases, while in more mature phases initiative and mimicking types of entrepreneurs are increasingly substituted by conservative types of entrepreneurs, who finally dominate the market. Tushman and Rosenkopf (1992) analyse industrial or technological development cycles based on four phases: variation, turbulence, selection and persistence. They are characterised through different organisational behaviour. Utterback (1996) found that market development goes along with an increasing number of competitors until a “dominant design” emerges. The “dominant design” shifts the focus of innovation, process innovations and economies of scale get more important, barriers to entry rise and the number of firms decrease. The respective cyclic economic developments are set in motion or accompanied by innovative behaviour of firms.

Secondly, we have to know more about the structure of the industry and the incidence of specific characteristics (e.g. firm size) in order to reasonably measure the diversity of an economy. What is the value of a character and its incidence? In contrast to biology (see the discussion about weighting of species in Humphries et al. 1995) an economic characteristic like a specific firm size is – even if once totally lost – recoverable. Firms’ characteristics are a result of the market circumstances and up to the regulatory framework of the economic system. In biology an extinct species is lost forever. What is more important in an economic competitive framework is that characteristics are more or less equally distributed, since a certain number of similar characteristics intensifies competition and make it likely that it provides the society with an acceptable result. Thus in contrast to biology, we are less worried if certain firm characteristics disappear, as a result of “workable” competition processes. In fact, we are worried if we have only few representatives of a certain characteristic and if they collude. This way the society can not be sure that resources are allocated in an “optimal” way. Thus in economics a greater incidence of characteristics and the allocation of characteristics is important for measuring diversity in an economic system.

There are several concepts of measuring diversity in an economic framework. Jacobs (1969) and Glaeser et al. (1992) focus on “externalities” and “spillovers” to conceptualise the basically positive impact of diversity on innovative behaviour and growth of firms in specific regions e.g. cities. Saviotti (1996) emphasises the importance of variety for economic development. He defines variety based on “the number of distinguishable types of actors, activities and outputs required to characterise an economic system” (Saviotti,



1996: 92). Our concept of diversity is strongly based on Sterling's achievements in this field of investigation. Sterling (1998, 2004) introduced a measure of diversity considering variety, balance and disparity of a "subsystems". Furthermore he integrated the different parts in a multi-criteria diversity index. In order to apply his diversity-triplet concept in an Arrow-Debreu framework, we had to modify it to some extent. Most importantly we propose a hierarchical formulation of the triplet-diversity concept. However, the basic idea of the concept remains unchanged.

Variety expresses the number of specific characteristics. In our Arrow-Debreu world the measure refers to the number of commodities, markets or products. Balance indicates the rate of change in the variety components respectively, and disparity takes into account the share of innovators (of different degrees) and remaining producers when looking at the different characteristics identified in the variety component.

In order to apply diversity in a dynamic Arrow-Debreu framework, the different parts of diversity were integrated into a "coherent" index. In contrast to the multi-criteria index of Sterling (1998), we formulate a hierarchical concept of diversity as follows: In comparing two subsystems in relation to diversity aspects, we assume that the impact of the variety component is greatest, followed by balance and disparity. Thus the diversity of a subsystem increases if variety increases independent of the way the other components develop. If variety remains unchanged, the balance aspect gains in importance. If balance increases, diversity decreases and if balance decreases, diversity increases. In case, variety and balance remain unchanged, disparity will have an effect on the overall diversity development. If disparity increases, diversity increases as well and contrary.

This way the diversity-triplet concept allows us to control for a number of factors related to diversity. Beside the sheer number of characteristics it is possible to investigate differences in the structure of these characteristics as well. However the aim of this paper is to impose the diversity structure on the Schumpeterian evolution model following a dynamical system approach based on the Arrow-Debreu theory. To our knowledge it is the first trial to introduce a rather comprehensive measure of diversity into this framework.

Therefore, the main motive of our construction results from the fact that the idea of diversity, although present in Schumpeter's works as mentioned before, does not play any role as a key-mechanism of his vision of economic evolution. This allows us to equip economic evolution with diversity structure to explain the relationship between these two important themes in today evolutionary economics.

In this context it should be emphasised that our study takes into account three pairs of fundamental oppositions defining the Schumpeterian economic evolution (Schumpeter, 1934, p. 82):

1. of two real processes: the circular flow as a tendency towards equilibrium and a spontaneous change in the channels of economic routine,
2. of theoretical apparatuses: statics and dynamics,
3. of two types of conduct as two kinds of behavioural patterns: mere managers and entrepreneurs.

This set-up produces two connecting paths of thinking running through those pairs of oppositions: circular flow – statics – managers, and spontaneous change – dynamics – entrepreneurs.

Indeed, in the core of the Schumpeterian economic thinking there lie distinguishable two forms of economic activity which can be further analyzed (Schumpeter 1934): circular flow and economic development. The former refers to the periodicity of economic life and is based on the Walrasian general equilibrium model and interpreted as comparative statics within which an economic system tends to an equilibrium defining commodities' prices and quantities. In this approach, economic life goes smoothly along the beaten tracks, keeping the governing rules unchanged year after year.

On the other hand, economic life goes beyond these narrow patterns of continuous evolution taking shape of a development, which is described by Schumpeter in the following way (ibid., p. 64): „*Development in our sense is a distinct phenomenon, entirely foreign to what may be observed in the circular flow or in the tendency towards equilibrium. It is spontaneous and discontinuous change in the channels of the flow, disturbance of equilibrium, which forever alters and displaces the equilibrium state previously existing*”. Consequently, the driving force and the key to the understanding of the phenomenon of economic development is the creative destruction rule (Schumpeter, 1942). The rule is the synthesis of two opposite tendencies present in the economic life development: creative innovations and eliminating existing, old products and organizational structures by new ones.

To grasp this core of Schumpeterian economic evolution in a way that the principles of scientific rationality dominating in today's economic theory such as rigor, generality and simplicity of the analysis are preserved, two paths of thinking in the contemporary economic science have been synthesized (Malawski, 1995, 2005): the general equilibrium theory in its Arrow-Debreu set-up (Debreu, 1959) and the evolutionary approach represented by the Schumpeterian theory of economic development (Schumpeter, 1934) modeled in these categories. Consequently, the static Arrow-Debreu theory of general

economic equilibrium has been dynamized to model the Schumpeterian vision of economic evolution.

In this framework the first thread running through the oppositions mentioned above is modeled as a cumulative extension of a production system being a part of a Debreu economy with the private ownership, where producers play the passive, managerial role, adopting optimal production plans to given technologies and existing prices. The second one is studied as a discontinuous semidynamical production system which preserves in time an innovative extension of a production system. This type of change results from the operations of existing entrepreneurs who play an active role as innovators.

In the following we try to conceptualise the concepts of diversity and Schumpeterian evolution in a more rigorous and general way, and introduce them in a dynamic Arrow-Debreu model, stating that innovation impacts diversity, which has a favourable effect on economic evolution. In other words, it is suggested to analyse both categories under study together in a more formal, axiomatic framework.

### 3. THE MODEL

#### 3.1. The static analysis

A dynamical system approach to the Arrow-Debreu theory of general equilibrium (Malawski, 1999, 2005) enables us to study the model of a private ownership economy as describes in Debreu (1959) in the form of a multi-range relational system which includes, in combination, the production system and the consumption one.

The production system is represented by a two-range relational system

$P = (B, \mathbf{R}^l; \gamma, p, \eta, \pi)$ , where:

- $B = \{b_1, \dots, b_n\}$  is a finite set of the producers,
- $\mathbf{R}^l$  is an  $l$ -dimensional commodity/price space,
- $\gamma \subset B \times \mathbf{P}_0(\mathbf{R}^l)$  is a correspondence of production sets which to every producer  $b \in B$  assigns a production set  $Y_b \subset \mathbf{R}^l$  being a compact subset of the commodity space and representing the producer's feasible production technology,
- $p \in \mathbf{R}^l$  is a price system,
- $\eta \subset B \times \mathbf{P}_0(\mathbf{R}^l)$  is a correspondence of supply which to every producer  $b \in B$  assigns a compact set  $\eta(b)$  of the production plans maximizing its profit  $py_b$  in a price system  $p$ , it is to say:  $\eta(b) = \eta_b(p) = \{y_b' \in Y_b : py_b' = \max_{y_b \in Y_b} py_b\}$ ,
- $\pi: B \rightarrow \mathbf{R}$  is a maximal profit function which measures the maximum profit value in the set of plans  $\eta(b)$ , i.e., for every  $b \in B$ :  $\pi(b) = \pi_b(p) = \max_{y \in \eta(b)} py_b$ .

Similarly, the consumption system is represented by a three-range relational system

$C = (A, \mathbf{R}^l, \mathbf{P}; x, e, \varepsilon, p, \beta, \varphi)$ , where

- $A = \{a_1, \dots, a_m\}$  is a finite set of the consumers,
- $\mathbf{R}^l$  is an  $l$ -dimensional commodity/price space,
- $\mathbf{P} \subset P(\mathbf{R}^{2l})$  is the family of all preference relations defined on the commodity space  $\mathbf{R}^l$ ,

- $x \subset A \times \mathbf{P}_0(\mathbf{R}^l)$  is a correspondence of consumption sets which to every consumer  $a \in A$  assigns a consumption set  $x(a)$  being a compact subset of the commodity space  $\mathbf{R}^l$  and representing the consumer's feasible consumption plans with respect to his psychophysical structure,

- $e \subset A \times \mathbf{R}^l$  is an initial endowment mapping which to every consumer  $a \in A$  assigns some initial endowment vector  $e(a) \in x(a)$ ,

- $\varepsilon \subset A \times P(\mathbf{R}^{2l})$  is a correspondence which to every consumer  $a \in A$  assigns a preference relation  $\preceq_a \in \mathbf{P}$ , restricted to the consumption set  $x(a)$ ,

- $p \in \mathbf{R}^l$  is a price system,

- $\beta \subset A \times \mathbf{P}_0(\mathbf{R}^l)$  is a correspondence of budget sets which to every consumer  $a \in A$  assigns his set of budget constraints  $\beta(a) \subset x(a)$  with the price system  $p$  and the initial endowment  $e(a)$ , i.e., for every  $a \in A$ :

$$\beta(a) := \beta_{(p, e(a))}(a) := \{x \in x(a) : px \leq pe(a)\},$$

- $\varphi \subset A \times \mathbf{P}_0(\mathbf{R}^l)$  is a demand correspondence which to every consumer  $a \in A$  assigns his consumption plans maximizing preferences on the budget set  $\beta(a)$ , i.e., for every  $a \in A$ :  $\varphi(a) := \varphi_{(e(a), p, e(a))}(a) := \{x \in \beta_{(p, e(a))}(a) : \forall x' \in \beta_{(p, e(a))}(a) : x' \preceq_a x\}$ .

A private ownership economy  $E_p$  is a combination of a production system  $P$  and a consumption system  $C$  such that the consumers share in the producers' profits (the shares are measured by a mapping  $\theta$ ) and some fixed (initial) total resource  $\omega$  of the economy  $E_p$  is the consumers' property. It means, that a budget set correspondence  $\beta \subset A \times \mathbf{P}_0(\mathbf{R}^l)$  is modified for the economy  $E_p$ , so that the expenditures of  $a$ th consumer do not exceed the value  $w_a = pe(a) + \sum_{b \in B} \theta_{ab} \pi_b(p)$ .

Then  $\beta(a) := \beta_{(p, e(a))}(a) := \{x \in x(a) : px \leq w_a\}$ .

Thus the private ownership economy  $E_p$  can concisely be described in the form:

$E_p = (P, C, \theta, \omega)$ , where

- $P$  is a production system,
- $C$  is a consumption system,
- $\theta \subset (A \times B) \times \mathbf{R}_+$  is a function describing the consumers' shares in the producers' profits, i.e., for every  $(a, b) \in A \times B$  the number  $\theta_{ab} := \theta(a, b) \in [0, 1]$  measures the consumer  $a$ 's share in the producer  $b$ 's profit and there is, for every  $b \in B$ ,  $\sum_{a \in A} \theta_{ab} = 1$ ,  $\omega = \sum_{a \in A} e(a)$ , so that  $\omega \in \mathbf{R}^l$ .

Such a formulation of the Debreu equilibrium theory in the form of a system  $E_p$  makes it possible to construct a whole space of models of the type; on this space a (quasi)-semidynamical system can then be defined. Individual models as elements of the state space are subsequently interpreted as states of a definite dynamics, which models the temporal evolution of the model examined. The suggested formulation retains all the results of this component of the Debreu general equilibrium theory and simultaneously, in a natural and mathematically elegant way, “puts them in motion” in time, which makes our formulation a more accurate tool to analyze real economic processes.

However, in the context of the Schumpeterian vision of economic evolution (Schumpeter, 1934) generality of our setting can be restricted to the analysis of production system, because of innovators to play the crucial role in the economic development.

Therefore, we shall precede as follows. A fixed group of producers acting in a fixed commodity space  $\mathbf{R}^l$  is endowed with different activity characteristics, which leads first to a definition of spaces of production characteristics, and then to the construction of spaces of systems with fixed ranges, as follows.

Let us shortly denote a production system  $P$  by  $P = (B, \mathbf{R}^l, Ch_p)$ ,

where  $Ch_p = (y, p, \eta, \pi)$  is the production characteristic of the system  $P$  and let  $\mathbf{CH}_p$  denote the space of all characteristics of the type, i.e.,

$\mathbf{CH}_p = \{Ch_p: Ch_p = (y, p, \eta, \pi) \text{ is a production characteristic}\}$ .

We now define the space  $\mathbf{P}$  of all production systems with the fixed ranges  $B$  and  $\mathbf{R}^l$  as follows:  $\mathbf{P} := \mathbf{P}(B, \mathbf{R}^l) := \{P : P = (B, \mathbf{R}^l, Ch_p), Ch_p \in \mathbf{CH}_p\}$ .

In the Schumpeterian framework, a fundamental task is to define a cumulative extension and an innovative one so that these ideas would model the rules of circular flow and economic development, respectively, the best way possible. A quasi-semidynamical system of production, which preserves (in time) the rules governing those two forms of economic life, undergoes an axiomatic-deductive analysis from its diversity angle.

Let two production systems be given:

$P = (B, \mathbf{R}^l, Ch_p)$  where  $Ch_p = (y, p, \eta, \pi)$  and

$P' = (B, \mathbf{R}^l, Ch_{p'})$  where  $Ch_{p'} = (y', p', \eta', \pi')$ .

**Definition 3.1.** A production characteristic  $Ch_{p'}$  is a cumulative extension of a production characteristic  $Ch_p$  (shortly,  $Ch_p \subset_c Ch_{p'}$ ) if

- 1)  $y \subset_c y'$ , i.e.,  $Y_b \subset Y'_b$  for every  $b \in B$  where  $Y_b = y(b)$ ,  $Y'_b = y'(b)$ ;
- 2)  $p \leq p'$ ;
- 3)  $\eta \subset_c \eta'$ , i.e.,  $\eta_b(p) \subset \eta'_b(p')$  for every  $b \in B$ ;
- 4)  $\pi \leq_c \pi'$ , i.e.,  $\pi_b(p) \leq \pi'_b(p')$  for every  $b \in B$ .

Definition 3.1 makes the following definition possible.

**Definition 3.2.** A production system  $P' = (B, \mathbf{R}^l, Ch_{P'})$  is a cumulative extension of a production system  $P = (B, \mathbf{R}^l, Ch_P)$  (shortly,  $P \subset_c P'$ ) if  $Ch_P \subset_c Ch_{P'}$ .

According to the definition, transition from the production system  $P$  to its cumulative extension  $P'$  is reduced to a cumulative extension of the system characteristic. In particular, neither new firms or commodities appear, nor are the old ones eliminated from the production process. All old technologies are still being used and, with non-decreasing prices, the firms optimal production plans remain optimum and give not less profits. All this means that the idea of a cumulative extension can be interpreted as one modeling circular flow of the production sphere as it follows the rules ascribed by J. Schumpeter to just this sphere of economic life. It is also evident, that the producers play a purely passive, managerial role adopting actions to the given prices and technologies.

Let us now consider a different form of a production system extension, called an innovative extension. So, let two production systems,  $P = (B, \mathbf{R}^l, Ch_P)$  and  $P' = (B', \mathbf{R}^l, Ch_{P'})$  be given. To compare their characteristics we employ for the respective elements of “new” characteristic  $Ch_{P'}$  the formal concept of projection on the “old” commodity/price space  $\mathbf{R}^l$ , as follows.

**Definition 3.3.** A production system  $P' = (B', \mathbf{R}^l, Ch_{P'})$  is an innovative extension of a production system  $P = (B, \mathbf{R}^l, Ch_P)$  (shortly,  $P \subset_{in} P'$ ) if

- 1)  $l < l'$ ;
- 2)  $\exists b' \in B' \forall b \in B$  :
  - (2.1)  $\text{proj } Y'_{b'} / \mathbf{R}^l \not\subset Y_b$ ,
  - (2.2)  $\text{proj } p' / \mathbf{R}^l = p$ ,
  - (2.3)  $\text{proj } \eta'_{b'}(p') / \mathbf{R}^l \not\subset \eta_b(p)$ ,
  - (2.4)  $\pi_b(p) < \pi'_{b'}(p')$ .

According to the definition, there appears in the production system  $P'$  at least one new product or commodity, which can be interpreted as better way of meeting the needs present earlier in the system  $P$ . In the same time, the definition does not say, whether new products are put out by brand new firms or by ones already existing, but modernized. All that is demanded here is that in the production system  $P'$  there is at least one producer  $b'$  whose technological abilities go beyond the abilities of all producers acting within the production system  $P$ . Hence, the optimal (i.e., maximizing the profit) production plans of the producer  $b'$  cannot be reduced to the analogous plans being realized by the producers in the production system  $P$ . Moreover, although the prices of “old” products do not change, the fixed producer’s maximum profit is greater than the one of the producers in the system  $P$  can make. Thus, according to the Schumpeterian terminology, the producer  $b'$  can be treated as an innovator and entrepreneur, who

makes the profit resulting from his initiative and who should be given a leading role in the production system  $P'$ .

It is also easy to see, that the definition 3.3 covers at least four cases of five internal changes characterized by Schumpeter as development (Schumpeter 1934, p. 66), i.e.,

- (1) the introduction of a new good – the condition 1,
- (2) the introduction of a new method of production – the condition 2.1,
- (3) the opening of a new market – the condition 1,
- (4) the carrying out of the new organization of any industry – the condition 2 as a whole.

Let us also see that, in consequence, the new product or commodity created can be recognized as better in the frame of the existing competitive technologies. As a result, old, less demanded and less competitive commodities, get eliminated. The phenomenon is strictly defined in the following definition.

**Definition 3.4.** *A production system  $P'$  is a creative destruction of a production system  $P$  (shortly,  $P \subset_{cd} P'$ ) if*

- 1)  $P \subset_{in} P'$ ;
- 2)  $\exists i \in \{1, \dots, l\} \forall b' \in B' : y_{b'} = (y_1^{b'}, \dots, y_i^{b'}, \dots, y_l^{b'}) \in Y_{b'} \Rightarrow y_i^{b'} = 0$ .

This way, the transition from a production system  $P$  to its creative destruction  $P'$  takes both into account: the phenomenon of innovation and that of elimination. Consequently, this principle of creative destruction is a carrier of structural changes in the production system.

Now, three problems should be considered to impose the diversity structure on the Schumpeterian evolution modelled axiomatically within a dynamical system approach to the Arrow-Debreu theory of general economic equilibrium. These problems represent in a formal way the hypotheses suggested in Section 1.

1. To establish the relationship between the diversity concept as a key mechanism for economic evolution and precondition for functioning competition (see: Section 2), and the creative destruction principle being an engine to set in motion the development of capitalistic economy within the Schumpeterian vision of economic evolution.
2. To assign to the subcategories defining the diversity-triplet concept the respective parameters characterising the model  $P \in \mathbf{P}$ .
3. To define the internal structure of the diversity-triplet concept, i.e., the relationships between the subcategories of variety, balance and disparity.

Problem 1. Diversity where? and of what?

The diversity approach to the analysis of economic life suggests that the diversity concept should be referred to the changes to occur in economy. In the context of the Schumpeterian vision of economic evolution it means that these changes can be studied in the space  $\mathbf{P}$  of production systems as defined above, and identified as two basic forms of the extension of production systems, i.e., cumulative and innovative ones to model the circular flow and economic development, respectively. Moreover, the innovative extension leads to the principle of creative destruction. Therefore, three kinds of diversity, cumulative, innovative and one referred to creative destruction, related to the forms of extensions mentioned above, will be discussed.

In the Schumpeterian framework the diversity in the economy as a whole can be reduced to the technological diversity, what means in our setting the restriction to the production system, where the technological changes are studied. Therefore, we will formally write:  $\text{div}(\Delta E_p) = (\text{div}(\Delta P), \text{div}(\Delta C))$  and  $\text{div}_{\text{tech}}(\Delta E_p) = \text{div}(\Delta P)$ , where symbol  $\Delta$  stands for a change in respective system, so that the analysis of changes in consumption system  $\Delta C$  goes beyond the scope of our setting.

To study diversity of structural changes in the production system  $P$  we will use the following denotations:

$\Delta_c P = (P, P', \subset_c)$ , such that  $P \subset_c P'$  for the cumulative change in  $P$ ,

$\Delta_{\text{in}} P = (P, P', \subset_{\text{in}})$ , such that  $P \subset_{\text{in}} P'$  for the innovative change in  $P$ , and

$\Delta_{\text{cd}} P = (P, P', \subset_{\text{cd}})$ , such that  $P \subset_{\text{cd}} P'$  for the change related to the principle of creative destruction.

In more general terms we assume:

$\Delta P = (P, P', \subset)$ , such that  $P \subset P'$ , where  $\subset = \subset_c \vee \subset_{\text{in}} \vee \subset_{\text{cd}}$  for the change cumulative, innovative or based on creative destruction in  $P$ .

Problem 2.

Formally, diversity is designed as a mapping (function) from the space of changes in  $\mathbf{P}$  to  $\mathbf{R}_+^3$ , symbolically:  $\text{Div}: \mathbf{P} \times \mathbf{P} \times \mathbf{P}^2 \rightarrow \mathbf{R}_+^3$ , such that for every  $P \in \mathbf{P}$

$\text{div}(\Delta P) := (\text{var}(\Delta P), \text{bal}(\Delta P), \text{disp}(\Delta P))$ , where the components, variety of  $\Delta P$ , balance of  $\Delta P$  and disparity of  $\Delta P$ , are defined by some parameters characterising them, as follows:

- $\text{var}(\Delta P) := \max(l, l')$ ,
- $\text{bal}(\Delta P) := (l' - l) / l$ ,
- $\text{disp}(\Delta P) := \text{card } B_{\text{in}}'$ , where  $B_{\text{in}}' \subset B'$  is the set of innovators,

i.e., formally:  $B_{\text{in}}' = \{b' \in B'; \text{the conditions 2.1-2.4 of definition 3.3 hold for } b'\}$ .

The above formalisation can be interpreted as follows:

1. Variety measures the number of commodities, markets or products.
2. Balance indicates the rate of change in variety.



3. Disparity takes into account the partition of a group of agents into innovators and remaining producers, as well as it measures the number of innovators.

In particular, the above arrangement can be related to cumulative and innovative extensions, denoted by  $\text{div}(\Delta_c P) := (\text{var}(\Delta_c P), \text{bal}(\Delta_c P), \text{disp}(\Delta_c P))$  and  $\text{div}(\Delta_{\text{in}} P) := (\text{var}(\Delta_{\text{in}} P), \text{bal}(\Delta_{\text{in}} P), \text{disp}(\Delta_{\text{in}} P))$ , and referred as cumulative and innovative diversity, respectively. Similarly,  $\text{div}(\Delta_{\text{cd}} P) := (\text{var}(\Delta_{\text{cd}} P), \text{bal}(\Delta_{\text{cd}} P), \text{disp}(\Delta_{\text{cd}} P))$  denotes diversity related to creative destruction.

This implies immediately:

**Lemma 3.1.**  $\text{div}(\Delta_c P) = (l, 0, 0)$

**Lemma 3.2.**  $\text{div}(\Delta_{\text{in}} P) = (l', (l' - l) / l, \text{card } B_{\text{in}}') = \text{div}(\Delta_{\text{cd}} P)$

Problem 3.

The study we present in Section 2 shows that the problem of the internal structure of the diversity-triplet concept, i.e., the relationships between the subcategories of variety, balance and disparity has not found yet any satisfactory solution. Therefore, we suggest a comparative analysis of diversity of changes in production systems  $P \in \mathbf{P}$ , assuming the hierarchical structure of the concept of diversity.

This means, that we formally proceed as follows.

Let be given three production systems  $P, P', P'' \in \mathbf{P}'$ .

Now, under assumption on the hierarchical structure of the triplet  $\text{div}(\Delta P)$  we are able to compare diversity for two changes  $\Delta P', \Delta P''$  in production systems  $P', P'' \in \mathbf{P}$  starting with  $P$ , employing the lexicographic ordering, as follows:

$$\begin{aligned} \text{div}(\Delta P') < \text{div}(\Delta P'') &\Leftrightarrow 1) \text{var}(\Delta P') < \text{var}(\Delta P''), \text{ or} \\ &2) \text{var}(\Delta P') = \text{var}(\Delta P'') \wedge \text{bal}(\Delta P') < \text{bal}(\Delta P''), \text{ or} \\ &3) \text{var}(\Delta P') = \text{var}(\Delta P'') \wedge \text{bal}(\Delta P') = \text{bal}(\Delta P'') \wedge \\ &\quad \text{disp}(\Delta P') < \text{disp}(\Delta P''). \end{aligned}$$

The hierarchical structure of diversity means here that we rank variety as the (most) dominating component in the diversity-triplet concept. If these characteristics are equal, we compare balances, and finally, disparity plays the least significant role.

Now, the basic properties of diversity can be established. Under lemmas 3.1, 3.2, we have:

**Lemma 3.3.** Let  $P, P', P'' \in \mathbf{P}'$  such that  $P \subset_c P'$  and  $P \subset_{\text{in}} P''$ .

Then  $\text{div}(\Delta_c P') < \text{div}(\Delta_{\text{in}} P'')$ .

Moreover, diversity turns out to be an invariant to cumulative changes (extension). Indeed, by definition we have immediately:

**Lemma 3.4.** If  $P \subset_c P'$ ,  $P \subset_c P''$ , then  $\text{div}(\Delta_c P') = \text{div}(\Delta_c P'')$ .

Similarly, we obtain:

**Lemma 3.5.** If  $P \subset_{in} P'$ ,  $P \subset_{in} P''$  and  $P' \subset_c P''$ , then  $\text{div}(\Delta_c P') = \text{div}(\Delta_c P'')$ .

Proof.  $l' = l'' \Rightarrow (l' - l)/l = (l'' - l)/l$ . Also,  $B_{in}' = B_{in}'' \Rightarrow \text{card } B_{in}' = \text{card } B_{in}''$ .

Finally, we can conclude by lemma 2 that the innovative extension  $\subset_{in}$  can be replaced by one related to creative destruction  $\subset_{cd}$  in lemmas 3 and 5.

### 3.2. The dynamical approach

Let us now note a production system  $P$  as  $P = (B, \mathbf{R}^l, Ch_p)$ , where  $Ch_p = (y, p, \eta, \pi)$  is the production characteristic of the system  $P$ , let also  $\mathbf{CH}_p$  and  $\mathbf{P}$  as usual stand for the spaces of all such characteristics and production systems, respectively. Moreover, let  $\mathbf{P}'$  stand for the space of all production systems  $P$ , in which the ranges  $B$  and  $\mathbf{R}^l$  can also vary. We shall apply the idea of a quasi-semidynamical system in a dynamic formulation of the phenomena examined as defined in Malawski (1999, 2005).

Now following the general idea of a quasi-semidynamical system, we shall say that a correspondence  $f_P : \mathbf{P} \times \mathbf{R}_+ \rightarrow \mathbf{P}_0(\mathbf{P})$  defines a quasi-semidynamical production system if for every  $P \in \mathbf{P}$  and every  $t_1, t_2 \in \mathbf{R}_+$  there is:

- 1)  $f_P(P, 0) = \{P\}$ ,
- 2)  $f_P(f_P(P, t_1), t_2) = f_P(P, t_1 + t_2)$ , where  $f_P(S, t) := \bigcup_{P \in S} f_P(P, t)$  for  $S \subset \mathbf{P}$ .

In particular, a quasi-semidynamical production system  $f_P$  is called:

- single-valued if every value of  $f_P$  is a one-element set so that we may think of a mapping  $f_P : \mathbf{P} \times \mathbf{R}_+ \rightarrow \mathbf{P}$ ;
- cumulative if  $t_1 < t_2 \Rightarrow f_P(P, t_1) \subset_c f_P(P, t_2)$ , i.e., for every  $P' \in f_P(P, t_1)$  there is a  $P'' \in f_P(P, t_2)$  such that  $P' \subset_c P''$ ;
- a semidynamical production system if  $\lim_{t \rightarrow t_0} \delta(f_P(P, t), f_P(P, t_0)) = 0$  for every  $P \in \mathbf{P}$  and every  $t_0 \in \mathbf{R}_+$ , where  $\delta$  denotes a metric defined on the space  $\mathbf{P}$ .

Similarly, a single-valued quasi-semidynamical production system  $f_{P'} : \mathbf{P}' \times \mathbf{R}_+ \rightarrow \mathbf{P}'$  is called:

- innovative if  $t_1 < t_2 \Rightarrow f_{P'}(P, t_1) \subset_i f_{P'}(P, t_2)$ , i.e., for every  $P' \in f_{P'}(P, t_1)$  there is a  $P'' \in f_{P'}(P, t_2)$  such that  $P' \subset_i P''$ ;
- Schumpeterian if  $t_1 < t_2 \Rightarrow f_{P'}(P, t_1) \subset_{cd} f_{P'}(P, t_2)$ , i.e., for every  $P' \in f_{P'}(P, t_1)$  there is a  $P'' \in f_{P'}(P, t_2)$  such that  $P' \subset_{cd} P''$ .

The above definitions mean that a single-valued quasi-semidynamical production system  $f_P$  - cumulative preserves in time the rules governing the circular flow of economic life, while one  $f_{P'}$  - Schumpeterian preserves those characteristics of the rule of the creative destruction which defines economic development.

Now, the properties of diversity established in the static analysis can be transferred to the dynamic one. In particular, we will say that diversity for a quasi-semidynamical production system  $f_P$  is growing in time if  $t_1 < t_2 \Rightarrow \text{div}(\Delta P^{t_1}) < \text{div}(\Delta P^{t_2})$ , where  $P^t = f_{P'}(P, t)$  for  $t_1, t_2 \in \mathbf{R}_+$ .

Thus, we have the following theorems.

**Theorem 3.1.** In the cumulative quasi-semidynamical production system  $f_P$  diversity is fixed (does not change).

Proof. Definition  $f_P$  – cumulative, lemma 3.4.

**Theorem 3.2.** In the innovative quasi-semidynamical production system  $f_{P'}$  diversity is growing in time.

Proof. Definition  $f_{P'}$  – innovative, lemma 3.3.

Finally, we conclude that the same result can be established for the Schumpeterian quasi-semidynamical production system  $f_{P'}$ .

#### 4. CONCLUSION

The aim of this paper is to impose the diversity structure on the Schumpeterian evolution modelled axiomatically within a dynamical system approach to the Arrow-Debreu theory of general equilibrium. To this end we modified the diversity concept from Sterling (1998, 2004), i.e. a diversity-triplet concept consisting of variety, balance and disparity. Instead of a multi-criteria-index (Sterling 1998) we introduced diversity in a hierarchical formulation. Moreover, in focusing on the production side of the Arrow-Debreu axiomatic model, we are addressing the question of “diversity of what” and “diversity where”. Based on

Malawski (1999, 2005) we see that in an evolutionary approach the concept of diversity should be related to changes that occur in a parameter space, referring to the extensions – cumulative, innovative, as well as to the principle of creative destruction - of production systems. In a further step we assign the parameters of a dynamical Arrow-Debreu economy to the hierarchical diversity concept.

The results obtained in the paper support in a simple, rigorous, general and elegant way the following viewpoints:

1. In the Schumpeter's evolutionary economics the diversity is not changing in the circular flow, but it is growing in the economic development.
2. Diversity of innovative change and diversity referring to creative destruction are indistinguishable.
3. Growing diversity is the necessary condition for innovation.

First of them seems to be intuitively evident by Schumpeter's understanding of those two forms of economic life.

The second point seems paradoxical only at the first glance. The principle of creative destruction says that old products or technologies get eliminated and respective markets closed. Thus a new technology outperforms one or more older technologies. Consequently variety may decrease and thus economic development would be combined with decreasing diversity. In more real economic terms this is highly unlikely, since a new technology is usually not perceived as competence destroying from the beginning on. In contrast and most of the time, radical new technologies are introduced to the market by market entrants. Thus, the number of firms in a market increases, competition is intensified and the number of technologies increases as well. New and old technologies exist in parallel and variety as well as diversity increase. Actually, the two technologies may exist in parallel for quite a long time, e.g. computer and electronic typewriters, since typewriters show some advantages over computers for specific types of office work. And even if the new technology is clearly competence destroying in character and substitutes the old technology in large parts, it also creates new markets, e.g. the computer is not only used for text processing work. At the beginning it was primarily used for comprehensive calculations. Later it was also used for text processing work or planning processes as well, and thus substitutes the electronic typewriter.

Nevertheless during the competitive battle it may turn out that one technology disappears, e.g. tramways substituted carriages drawn by horses as means of public transport in cities in the nineteenth century. While in this case variety remains unchanged, balance and disparity may cause an overall change in diversity. Since it is very likely that new technologies go along with market extension or new market creation (engines vs.

horsepower), balance would decrease as well as disparity would increase and again economic development goes along with an increase in diversity.

The third point is heuristic in its nature. Please notice that the causality between innovation and diversity could not be addressed explicitly within this framework. However it can be assumed that it is interdependent, i.e. innovative behaviour fosters diversity and a diverse economy makes innovative behaviour more likely, at least as far as more radical innovations are concerned (see Woerter 2001). Investigations of that kind must be left for future research. We also did not address cost aspects of innovations in relation with diversity – it would clearly go beyond the scope of this paper.

## **BIBLIOGRAPHY**

- Debreu G. (1959), *Theory of Value*, New York: Wiley.
- Dosi G. (1988), “Sources, Procedures, and Microeconomic Effects of Innovation”, *Journal of Economic Literature*, Vol. 26, September.
- Glaeser E., Kallal H., Scheinkam J., Shleifer A. (1992), “Growth in cities” *The Journal of Political Economy*, Vol. 100.
- Heuss E. (1965), *Allgemeine Markttheorie*, Tübingen: J.C.B. Mohr (Paul Siebeck).
- Holland J.H., Holyoak K.J., Nisbett R.R., Thagard P.E. (1986), *Induction – Processes of Inference, Learning, and Discovery*, Boston: MIT.
- Humphries C.J., Williams P.H., Vane-Wright R.I. (1995), “Measuring Biodiversity Value for Conservation”, *Annual Review of Ecology and Systematics*, Vol. 26.
- Jacobs J. (1969), *The Economy of Cities*, New York: Random House.
- Loch C.H., Hubermann B.A. (1999), “A Punctuated-Equilibrium Model of Technology Diffusion”, *Management Science*, Vol. 45, No. 2, February.
- Malawski A. (1999), *Metoda aksjomatyczna w ekonomii*. Wrocław: Ossolineum.
- Malawski A. (2005), “A dynamical system approach to the Arrow-Debreu theory of general equilibrium”, *The 9th World Multi-Conference on Systemics, Cybernetics and Informatics, Proceedings 2005, July 10-13, Orlando, Florida, USA*, Vol. VII, pp. 434-439.
- Metcalfe J.S., Miles I. (1994), “Standards, selection and variety, an evolutionary approach”, *Information Economics and Policy*, Vol. 6, No. 3-4, December.
- Morgan M.S. (1993), “Competing Notions of “Competition” in Late Nineteenth-Century American Economics”, *History of Political Economy*, Vol. 25, No. 4.

- Nelson R.R., Winter S.G. (1982), *An Evolutionary Theory of Economic Change*, Cambridge: Belknap Press of Harvard University Press.
- Nelson R.R. (1995), "Recent Evolutionary Theorizing About Economic Change", *Journal of Economic Literature*, Vol. 23, March.
- Saviotti P.P. (1996), *Technological Evolution, Variety and the Economy*, Cheltham, UK, Brookfield, US: Edward Elgar.
- Schumpeter J.A. (1934/1961), *The theory of economic development*, Cambridge Massachusetts: Harvard University Press.
- Schumpeter J.A. (1942), *Capitalism, Socialism and Democracy*, London: George Allen and Unwin.
- Schumpeter J.A. (1954), *History of Economic Analysis*, New York: Oxford University Press.
- Simon H.A. (1956), Rational Choice and the Structure of the Environment, in: *Economics, Bounded Rationality and the Cognitive Revolution*, Herbert Simon, ed.: Egidi M., Marris R. (1992), Aldershot, Brookfield: Edward Elgar Publishing Limited.
- Simon H.A. (1981), *Entscheidungsverhalten in Organisationen*, Verlag Moderne Industrie.
- Sterling A. (1998), "On the Economics and Analysis of Diversity", *SPRU Electronic Working Paper Series*, Paper No. 28, October.
- Sterling A. (2004), "Diverse Design: Fostering Technological Diversity in Innovation for Sustainability", Paper Submittet to Colloquium DEA, Durham Business School and Cranfield School of Management on "The Role of Diversity in Social Systems and its Relation to Innovation", Bologna, 12-13 July.
- Solow A., Polasky St., Broadus J. (1993), "On the Measurement of Biological Diversity", *Journal of Environmental Economics and Management*, Vol. 24.
- Tushman M.L., Rosenkopf L. (1992), "Organizational Determinants of Technological Change: Toward a Sociology of Technological Evolution", *Research in Organizational Behaviour*, Vol. 14.
- Utterback J.M. (1996), *Mastering the Dynamics of Innovation*, Boston (Massachusetts): Harvard Business School Press.
- Weitzman M.L. (1992), "On Diversity", *The Quarterly Journal of Economics*, Vol. 107, No. 2, May.
- Weitzman M.L. (1993), "What to Preserve? An Application of Diversity Theory to Crane Conservation", *The Quarterly Journal of Economics*, Vol. 108, No. 1, February.

Woerter M. (2001), Verborgene Fusionen – symbiotische Forschungs- und Entwicklungskooperationen der Pharmaindustrie aus wettbewerbspolitischer Sicht, Geschichte und Ökonomie (Band 12), Innsbruck, München, Wien, Bozen: Studienverlag.