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## Cross-hedging of correlated exchange rates

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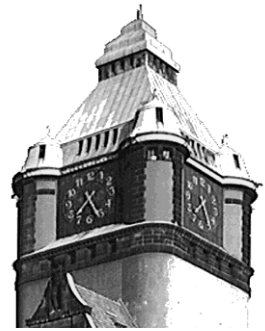
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# Cross-Hedging of Correlated Exchange Rates

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## Abstract:

This paper examines the behavior of a competitive exporting firm that exports to two foreign countries under multiple sources of exchange rate uncertainty. The firm has to cross-hedge its exchange rate risk exposure because there is only a forward market between the domestic currency and one foreign country's currency. When the firm optimally exports to both foreign countries, we show that the firm's production decision is independent of the firm's risk attitude and of the underlying exchange rate uncertainty. We show further that the firm's optimal forward position is an over-hedge or an under-hedge, depending on whether the two random exchange rates are positively or negatively correlated in the sense of expectation dependence.

JEL-Classification: D21; D24; D81; F31

Keywords: Correlated exchange rates; Cross-hedging; Exports; Production

# Cross-Hedging of Correlated Exchange Rates

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## Abstract

This paper examines the behavior of a competitive exporting firm that exports to two foreign countries under multiple sources of exchange rate uncertainty. The firm has to cross-hedge its exchange rate risk exposure because there is only a forward market between the domestic currency and one foreign country's currency. When the firm optimally exports to both foreign countries, we show that the firm's production decision is independent of the firm's risk attitude and of the underlying exchange rate uncertainty. We show further that the firm's optimal forward position is an over-hedge or an under-hedge, depending on whether the two random exchange rates are positively or negatively correlated in the sense of expectation dependence.

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## 1. Introduction

The literature on international firms under exchange rate uncertainty has extensively studied how currency forward/futures hedging affects the behavior of these firms (see, e.g., Katz and Paroush, 1979; Benninga et al., 1985; Kawai and Zilcha, 1986; Broll and Zilcha, 1992; Broll et al., 1999; to name just a few). Two notable results, the separation and full-hedging theorems, emanate. The separation theorem states that the optimal production decisions are independent of firms' risk attitude and of the underlying exchange rate uncertainty if there are currency forward/futures markets for hedging purposes. The full-hedging theorem states that firms should completely eliminate their exchange rate risk exposure by adopting a full-hedge if

currency forward/futures markets are unbiased.

While currency hedging is useful for international firms, forward/futures markets need not be readily available for all currencies in general, and are typically absent in many less developed countries in particular (see Eiteman et al., 2009).<sup>1</sup> International firms may as such have to avail themselves of forward contracts on related currencies to cross-hedge their exchange rate risk exposure (see, e.g., Anderson and Danthine, 1981; Eaker and Grant, 1987; Broll, 1997; Broll and Eckwert, 1999; Chang and Wong, 2003).

The purpose of this paper is to examine the optimal export and hedging decisions of a competitive exporting firm in a cross-hedging context. Following the expected utility model of Battermann et al. (2006), we consider the firm that exports to two foreign countries under multiple sources of exchange rate uncertainty. There are no hedging instruments between the domestic currency and one foreign country's currency. The firm, however, has access to an unbiased forward market between the home currency and the other foreign country's currency for cross-hedging purposes.

We show that the separation theorem holds when the firm optimally exports to the foreign country with the currency forward market. The full-hedging theorem, on the other hand, holds only when the firm exports exclusively to the foreign country with the currency forward market. In the more interesting case wherein the firm exports to both foreign countries, we show that the firm's optimal forward position is an over-hedge or an under-hedge, depending on whether the two random exchange rates are positively or negatively correlated in the sense of expectation dependence (Wright, 1987). Our results thus refine those of Battermann et al. (2006) by introducing the expectation dependence structure to describe the multiple sources of exchange rate uncertainty.

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<sup>1</sup>Even if some less developed countries have currency forward contracts, these contracts are deemed to be forward-cover insurance schemes that are not governed by market forces (see Jacque, 1996).

The rest of the paper is organized as follows. Section 2 delineates the model of a competitive exporting firm in a cross-hedging context. Section 3 derives the firm's optimal export and hedging decisions. The final section concludes.

## 2. The model

Consider a competitive exporting firm under exchange rate uncertainty. To begin, the firm produces a single commodity according to a deterministic cost function,  $c(x)$ , in the domestic country, where  $x \geq 0$  is the output level. The firm's production technology exhibits decreasing returns to scale so that the cost function,  $c(x)$ , satisfies that  $c'(0) = c''(0) = 0$ , and  $c'(x) > 0$  and  $c''(x) > 0$  for all  $x > 0$ . The firm exports its entire output,  $x$ , to two foreign countries, indexed by  $i = 1$  and 2. Let  $x_i$  be the amount of exports sold in country  $i$ , where  $x_i \geq 0$  for  $i = 1$  and 2, and  $x_1 + x_2 = x$ . The selling price of the commodity in country  $i$  is exogenously fixed at  $p_i$  per unit, where  $p_i > 0$  is denominated in country  $i$ 's currency for  $i = 1$  and 2.

The exchange rate uncertainty comes from two sources,  $\tilde{e}_1$  and  $\tilde{e}_2$ , that denote the random exchange rates expressed in units of the domestic currency per unit of country 1's currency and per unit of country 2's currency, respectively.<sup>2</sup> Let  $F_i(e_i)$  be the marginal cumulative distribution function (CDF) of  $\tilde{e}_i$  over support  $[\underline{e}_i, \bar{e}_i]$  with  $0 < \underline{e}_i < \bar{e}_i$  for  $i = 1$  and 2, and  $G(e_1, e_2)$  be the joint CDF of  $\tilde{e}_1$  and  $\tilde{e}_2$  over support  $[\underline{e}_1, \bar{e}_1] \times [\underline{e}_2, \bar{e}_2]$ . Cross-hedging is modeled by allowing the firm to trade infinitely divisible forward contracts between the domestic and country 1's currencies at the forward rate,  $e_1^f$ , expressed in units of the domestic currency per unit of country 1's currency. To focus on the firm's pure hedging motive, we assume that the forward contracts are unbiased in that  $e_1^f = E(\tilde{e}_1)$ , where  $E(\cdot)$  is the expectation operator

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<sup>2</sup>Throughout the paper, we use a tilde ( $\sim$ ) to denote a random variable.

with respect to  $G(e_1, e_2)$ .<sup>3</sup> There are, however, no direct hedging instruments for the random exchange rate,  $\tilde{e}_2$ .

The firm's profit, denominated in the domestic currency, is given by

$$\tilde{\pi} = \tilde{e}_1 p_1 x_1 + \tilde{e}_2 p_2 x_2 - c(x_1 + x_2) + [E(\tilde{e}_1) - \tilde{e}_1]h, \quad (1)$$

where  $h$  is the number of the forward contracts sold (purchased if negative) by the firm. We say that the forward position,  $h$ , is an under-hedge, a full-hedge, or an over-hedge, depending on whether  $h$  is less than, equal to, or greater than the amount of sales in country 1,  $p_1 x_1$ , denominated in country 1's currency, respectively.

The firm is risk averse and possesses a von Neumann-Morgenstern utility function,  $u(\pi)$ , defined over its domestic currency profit,  $\pi$ , with  $u'(\pi) > 0$  and  $u''(\pi) < 0$ . The firm's ex-ante decision problem is to choose amounts of exports,  $x_1$  and  $x_2$ , and a forward position,  $h$ , so as to maximize the expected utility of its domestic currency profit:

$$\max_{x_1, x_2, h} E[u(\tilde{\pi})], \quad (2)$$

where  $\tilde{\pi}$  is given by equation (1). The Kuhn-Tucker conditions for program (2) are given by

$$E\{u'(\tilde{\pi}^*)[\tilde{e}_1 p_1 - c'(x_1^* + x_2^*)]\} \leq 0, \quad (3)$$

$$E\{u'(\tilde{\pi}^*)[\tilde{e}_2 p_2 - c'(x_1^* + x_2^*)]\} \leq 0, \quad (4)$$

and

$$E\{u'(\tilde{\pi}^*)[E(\tilde{e}_1) - \tilde{e}_1]\} = 0, \quad (5)$$

---

<sup>3</sup>If  $e_1^f > (<) E(\tilde{e}_1)$ , the firm would have a speculative motive to sell (purchase) the forward contracts.



where an asterisk (\*) signifies an optimal level. If  $x_1^* > 0$ , condition (3) holds with equality. Likewise, if  $x_2^* > 0$ , condition (4) holds with equality.<sup>4</sup>

### 3. Optimal export and hedging decisions

As a benchmark, we first consider the case that the firm does not export to country 1, i.e.,  $x_1 \equiv 0$ . In this case, the first-order conditions for program (2) become

$$\mathbb{E}\{u'(\tilde{\pi}^0)[\tilde{e}_2 p_2 - c'(x_2^0)]\} = 0, \quad (6)$$

and

$$\mathbb{E}\{u'(\tilde{\pi}^0)[\mathbb{E}(\tilde{e}_1) - \tilde{e}_1]\} = 0, \quad (7)$$

where  $\tilde{\pi}^0 = \tilde{e}_2 p_2 x_2^0 - c(x_2^0) + [\mathbb{E}(\tilde{e}_1) - \tilde{e}_1] h^0$ , and a nought (<sup>0</sup>) indicates an optimal level.

Let  $\text{Cov}(\cdot, \cdot)$  be the covariance operator with respect to  $G(e_1, e_2)$ . We have

$$\text{Cov}[u'(\tilde{\pi}^0), \tilde{\pi}^0] = \text{Cov}[u'(\tilde{\pi}^0), \tilde{e}_2] p_2 x_2^0 - \text{Cov}[u'(\tilde{\pi}^0), \tilde{e}_1] h^0 < 0, \quad (8)$$

where the inequality follows from risk aversion. We can write equations (6) and (7) as<sup>5</sup>

$$c'(x_2^0) - \mathbb{E}(\tilde{e}_2) p_2 = \frac{\text{Cov}[u'(\tilde{\pi}^0), \tilde{e}_2] p_2}{\mathbb{E}[u'(\tilde{\pi}^0)]}. \quad (9)$$

and

$$\text{Cov}[u'(\tilde{\pi}^0), \tilde{e}_1] = 0, \quad (10)$$

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<sup>4</sup>The second-order conditions for program (2) are satisfied given risk aversion and the strict convexity of  $c(x)$ .

<sup>5</sup>For any two random variables,  $\tilde{x}$  and  $\tilde{y}$ , we have  $\text{Cov}(\tilde{x}, \tilde{y}) = \mathbb{E}(\tilde{x}\tilde{y}) - \mathbb{E}(\tilde{x})\mathbb{E}(\tilde{y})$ .

respectively. It then follows from equations (8), (9), and (10) that  $c'(x_2^0) < E(\tilde{e}_2)p_2$ .

Resume now the original case that the firm can export to both countries. We state and prove our first proposition.

**Proposition 1.** *The competitive exporting firm has access to the unbiased forward contracts between the domestic and country 1's currencies for hedging purposes. There are three cases.*

(i) *If  $E(\tilde{e}_1)p_1 \geq E(\tilde{e}_2)p_2$ , the firm chooses the optimal output level,  $x^* = x_1^*$ , that solves  $c'(x_1^*) = E(\tilde{e}_1)p_1$ , and the optimal forward position,  $h^* = p_1x_1^*$ , is a full-hedge. In this case, the firm exports its entire output to country 1, i.e.,  $x_2^* = 0$ .*

(ii) *If  $c'(x_2^0) < E(\tilde{e}_1)p_1 < E(\tilde{e}_2)p_2$ , the firm chooses the optimal output level,  $x^* = x_1^* + x_2^*$ , that solves  $c'(x^*) = E(\tilde{e}_1)p_1$ , and exports to both countries, i.e.,  $x_1^* > 0$  and  $x_2^* > 0$ . The optimal amounts of exports,  $x_1^*$  and  $x_2^*$ , and the optimal forward position,  $h^*$ , solve conditions (3) and (4) with equality and equation (5) simultaneously.*

(iii) *If  $E(\tilde{e}_1)p_1 \leq c'(x_2^0) < E(\tilde{e}_2)p_2$ , the firm chooses the optimal output level,  $x^* = x_2^0$ , and the optimal forward position,  $h^* = h^0$ , that solve equations (6) and (7) simultaneously. In this case, the firm exports its entire output to country 2, i.e.,  $x_1^* = 0$ .*

*Proof.* See Appendix A.

To see the intuition of Proposition 1, we recast equation (1) as

$$\tilde{\pi} = E(\tilde{e}_1)p_1x_1 - c(x_1 + x_2) + \tilde{e}_2p_2x_2 + [E(\tilde{e}_1) - \tilde{e}_1](h - p_1x_1). \quad (11)$$

Given the forward hedge via the contracts between the home and country 1's currencies, it is evident from equation (11) that the marginal revenue from exports to country 1 is locked in at the deterministic level,  $E(\tilde{e}_1)p_1$ . Since the marginal revenue from ex-

ports to country 2 is  $\tilde{e}_2 p_2$ , which is stochastic, the risk-averse firm sells exclusively in country 1 if the expected marginal revenue from exports to country 2 does not exceed the deterministic marginal revenue from exports to country 1, i.e.,  $E(\tilde{e}_2)p_2 \leq E(\tilde{e}_1)p_1$ . In this case, equation (11) reveals that the firm could have completely eliminated its exchange rate risk exposure had it chosen  $h = p_1 x_1$  within its own discretion. Alternatively put, the degree of exchange rate risk exposure to be assumed by the firm should be totally unrelated to its production decision. The firm as such chooses the optimal output level,  $x^* = x_1^*$ , that maximizes  $E(\tilde{e}_1)p_1 x - c(x)$ , which gives rise to  $c'(x_1^*) = E(\tilde{e}_1)p_1$ . Since the unbiased forward contracts offer actuarially fair “insurance” to the firm, the risk-averse firm optimally opts for full insurance by choosing  $h^* = p_1 x_1^*$ , which completely eliminates its exchange rate risk exposure. These results are simply the celebrated separation and full-hedging theorems emanated from the literature on international firms under exchange rate uncertainty.

If  $E(\tilde{e}_1)p_1 < E(\tilde{e}_2)p_2$ , the firm finds it optimal to export to country 2. Consider first that  $c'(x_2^0) < E(\tilde{e}_1)p_1 < E(\tilde{e}_2)p_2$ . In this case, selling in country 1 is optimal and the firm equates the marginal cost of production to the deterministic marginal revenue from exports to country 1. The optimal levels of exports,  $x_1^*$  and  $x_2^*$ , and the optimal forward position,  $h^*$ , are uniquely determined by solving conditions (3) and (4) with equality and equation (5) simultaneously. While the firm’s optimal output level,  $x^*$ , is independent of its risk attitude and of the underlying exchange rate uncertainty, the optimal amounts of exports,  $x_1^*$  and  $x_2^*$ , are not, rendering the partial collapse of the separation theorem. Furthermore, the firm may or may not opt for a full-hedge, i.e.,  $h^*$  may or may not be equal to  $p_1 x_1^*$ , without knowing the specific joint probability distribution function of  $\tilde{e}_1$  and  $\tilde{e}_2$ . Thus, the full-hedging theorem fails to hold.

Consider now that  $E(\tilde{e}_1)p_1 \leq c'(x_2^0) < E(\tilde{e}_2)p_2$ . In this case, the deterministic marginal revenue from exports to country 1 is not enough to cover the marginal cost

of production at  $x = x_2^0$ , the optimal output level should the firm sell exclusively in country 2. Hence, the firm finds it optimal to sell exclusively in country 2 so that  $x_1^* = 0$  and  $x_2^* = x_2^0$ . The optimal output level,  $x^* = x_2^0$ , and the optimal forward position,  $h^* = h_0$ , are uniquely determined by solving equations (6) and (7) simultaneously, from which we can see that neither the separation theorem nor the full-hedging theorem holds.

As is shown in Proposition 1, the optimal forward position,  $h^*$ , is a full-hedge, i.e.,  $h^* = p_1 x_1^*$ , if  $E(\tilde{e}_1)p_1 \geq E(\tilde{e}_2)p_2$ . On the other hand, if  $E(\tilde{e}_1)p_1 < E(\tilde{e}_2)p_2$ , we only know that  $h^*$  is characterized by equation (5), which reduces to

$$\text{Cov}\left\{u'\{\tilde{e}_1 p_1 x_1^* + \tilde{e}_2 p_2 x_2^* - c(x_1^* + x_2^*) + [E(\tilde{e}_1) - \tilde{e}_1]h^*\}, \tilde{e}_1\right\} = 0. \quad (12)$$

To determine whether  $h^*$  is an under-hedge, a full-hedge, or an over-hedge, we need to impose some tractable dependence structure on  $\tilde{e}_1$  and  $\tilde{e}_2$ . To this end, we define the CDF of  $\tilde{e}_2$  conditional on the event that  $\tilde{e}_1 \leq e_1$  as

$$F_2(e_2|\tilde{e}_1 \leq e_1) = \frac{G(e_1, e_2)}{F_1(e_1)}, \quad (13)$$

over support  $[e_2, \bar{e}_2]$  for all  $e_1 \in [e_1, \bar{e}_1]$ . Let  $E(\tilde{e}_2|\tilde{e}_1 \leq e_1)$  be the expected value of  $\tilde{e}_2$  with respect to  $F_2(e_2|\tilde{e}_1 \leq e_1)$ . The following bivariate dependence structure, known as expectation dependence, is due to Wright (1987).

**Definition 1.** *The exchange rate,  $\tilde{e}_2$ , is said to be positively (negatively) expectation dependent on the exchange rate,  $\tilde{e}_1$ , if*

$$ED(\tilde{e}_2|e_1) = E(\tilde{e}_2) - E(\tilde{e}_2|\tilde{e}_1 \leq e_1) \geq (\leq) 0, \quad (14)$$

for all  $e_1 \in [e_1, \bar{e}_1]$ , where the inequality is strict for some non-degenerate intervals.

To see how Definition 1 defines dependence, we write equation (14) as

$$\begin{aligned} \text{ED}(\tilde{e}_2|e_1) &= \int_{\underline{e}_2}^{\bar{e}_2} e_2 \, dF_2(e_2) - \int_{\underline{e}_2}^{\bar{e}_2} e_2 \, dF_2(e_2|\tilde{e}_1 \leq e_1) \\ &= \int_{\underline{e}_2}^{\bar{e}_2} [F_2(e_2|\tilde{e}_1 \leq e_1) - F_2(e_2)] \, de_2, \end{aligned} \quad (15)$$

where the second equality follows from integration by parts. According to Lehmann (1966), we can write  $\text{Cov}(\tilde{e}_1, \tilde{e}_2)$  in terms of the CDFs,  $G(e_1, e_2)$ ,  $F_1(e_1)$ , and  $F_2(e_2)$ :

$$\begin{aligned} \text{Cov}(\tilde{e}_1, \tilde{e}_2) &= \int_{\underline{e}_1}^{\bar{e}_1} \int_{\underline{e}_2}^{\bar{e}_2} [G(e_1, e_2) - F_1(e_1)F_2(e_2)] \, de_1 \, de_2 \\ &= \int_{\underline{e}_1}^{\bar{e}_1} \left\{ \int_{\underline{e}_2}^{\bar{e}_2} [F_2(e_2|\tilde{e}_1 \leq e_1) - F_2(e_2)] \, de_2 \right\} F_1(e_1) \, de_1 \\ &= \int_{\underline{e}_1}^{\bar{e}_1} \text{ED}(\tilde{e}_2|e_1) F_1(e_1) \, de_1, \end{aligned} \quad (16)$$

where the second equality follows from equation (13), and the last equality follows from equation (15). From Definition 1 and equation (16), we have  $\text{Cov}(\tilde{e}_1, \tilde{e}_2) > (<) 0$  if  $\tilde{e}_2$  is positively (negatively) expectation dependent on  $\tilde{e}_1$ .

Let  $\alpha(\cdot)$  and  $\beta(\cdot)$  be functions of bounded variation. Cuadras (2002) proves that  $\text{Cov}[\alpha(\tilde{e}_1), \beta(\tilde{e}_2)]$  can be written in terms of the CDFs,  $G(e_1, e_2)$ ,  $F_1(e_1)$ , and  $F_2(e_2)$ :

$$\text{Cov}[\alpha(\tilde{e}_1), \beta(\tilde{e}_2)] = \int_{\underline{e}_1}^{\bar{e}_1} \int_{\underline{e}_2}^{\bar{e}_2} [G(e_1, e_2) - F_1(e_1)F_2(e_2)] \, d\alpha(e_1) \, d\beta(e_2). \quad (17)$$

Evaluating the left-hand side of Eq. (12) at  $h^* = p_1 x_1^*$  yields

$$\begin{aligned} &\text{Cov}\{u'[\mathbb{E}(\tilde{e}_1)p_1 x_1^* + \tilde{e}_2 p_2 x_2^* - c(x_1^* + x_2^*)], \tilde{e}_1\} \\ &= \int_{\underline{e}_1}^{\bar{e}_1} \int_{\underline{e}_2}^{\bar{e}_2} [G(e_1, e_2) - F_1(e_1)F_2(e_2)] \\ &\quad \times u''[\mathbb{E}(\tilde{e}_1)p_1 x_1^* + \tilde{e}_2 p_2 x_2^* - c(x_1^* + x_2^*)] p_2 x_2^* \, de_1 \, de_2 \end{aligned}$$

$$= \int_{\underline{e}_2}^{\bar{e}_2} \mathbb{E}D(\tilde{e}_2|e_1)u''[\mathbb{E}(\tilde{e}_1)p_1x_1^* + \tilde{e}_2p_2x_2^* - c(x_1^* + x_2^*)]p_2x_2^*F_2(e_2) \, de_2, \quad (18)$$

where the first equality follows from Eq. (17) with  $\alpha(\tilde{e}_1) = \tilde{e}_1$  and  $\beta(\tilde{e}_2) = u'[\mathbb{E}(\tilde{e}_1)p_1x_1^* + \tilde{e}_2p_2x_2^* - c(x_1^* + x_2^*)]$ , and the second equality follows from Eq. (16). Since  $u''(\pi) < 0$  and  $x_2^* > 0$  given that  $\mathbb{E}(\tilde{e}_1)p_1 < \mathbb{E}(\tilde{e}_2)p_2$ , Eq. (18) is negative (positive) if  $\tilde{e}_2$  is positively (negatively) expectation dependent on  $\tilde{e}_1$ . It then follows from Eq. (12) and the second-order conditions for program (2) that the optimal forward position,  $h^*$ , must be greater (smaller) than  $p_1x_1^*$ , thereby invoking the following proposition.

**Proposition 2.** *Given that the forward contracts between the domestic and country 1's currencies are unbiased, and that  $\mathbb{E}(\tilde{e}_1)p_1 < \mathbb{E}(\tilde{e}_2)p_2$ , the competitive exporting firm optimally opts for an over-hedge (under-hedge), i.e.,  $h^* > (<) p_1x_1^*$ , if the exchange rate,  $\tilde{e}_2$ , is positively (negatively) expectation dependent on the exchange rate,  $\tilde{e}_1$ .*

The intuition for Proposition 2 is as follows. Given that covariances can be interpreted as marginal variances, Eq. (12) implies that the optimal forward position,  $h^*$ , is the one that minimizes the variance of the firm's marginal utility. If the exchange rates,  $\tilde{e}_1$  and  $\tilde{e}_2$ , are positively (negatively) correlated in the sense of expectation dependence, a full-hedge that completely eliminates the risk due to  $\tilde{e}_1$  is suboptimal because the firm's marginal utility remains volatile as  $e_2$  varies. In this case, an over-hedge (under-hedge) reduces the firm's profit as  $e_1$  increases (decreases), which is more likely when  $e_2$  is higher. Given risk aversion, such a forward position is more effective in reducing the variability of the firm's marginal utility, thereby rendering the optimality of an over-hedge (under-hedge) if  $\tilde{e}_2$  is positively (negatively) expectation dependent on  $\tilde{e}_1$ .

## 4. Conclusion

In this paper, we have examined the behavior of a competitive exporting firm that exports to two foreign countries under multiple sources of exchange rate uncertainty. While there are no hedging instruments between the domestic currency and one foreign country's currency, the firm has access to an unbiased forward market between the home currency and the other foreign country's currency for cross-hedging purposes. We have shown that the separation theorem holds when the firm optimally exports to the foreign country with the currency forward market. However, the full-hedging theorem holds only when the firm exports exclusively to the foreign country with the currency forward market. When the firm exports to both foreign countries, we have shown that the firm's optimal forward position is an over-hedge or an under-hedge, depending on whether the two random exchange rates are positively or negatively correlated in the sense of expectation dependence (Wright, 1987).

## Appendix A

We formulate program (2) as a two-stage optimization problem. In the first stage, the firm chooses the optimal amount of exports to country 1,  $x_1(x_2)$ , and the optimal forward position,  $h(x_2)$ , for a given amount of exports to country 2,  $x_2$ . In the second stage, the firm chooses the optimal amount of exports to country 2,  $x_2^*$ , taking  $x_1(x_2)$  and  $h(x_2)$  as given. The complete solution to program (2) is thus  $x_2^*$ ,  $x_1^* = x_1(x_2^*)$ , and  $h^* = h(x_2^*)$ .

The solution to the first-stage optimization problem must satisfy the following Kuhn-Tucker conditions:

$$\mathbb{E}\left\{u'[\tilde{\pi}(x_2)]\{\tilde{e}_1 p_1 - c'[x_1(x_2) + x_2]\}\right\} \leq 0, \quad (\text{A.1})$$

and

$$\mathbb{E}\{u'[\tilde{\pi}(x_2)][\mathbb{E}(\tilde{e}_1) - \tilde{e}_1]\} = 0, \quad (\text{A.2})$$

where  $\tilde{\pi}(x_2) = \tilde{e}_1 p_1 x_1(x_2) + \tilde{e}_2 p_2 x_2 - c[x_1(x_2) + x_2] + [\mathbb{E}(\tilde{e}_1) - \tilde{e}_1]h(x_2)$ . If  $x_1(x_2) > 0$ , condition (A.1) holds with equality. Multiplying  $p_1$  to equation (A.2) and adding the resulting equation to condition (A.1) yields

$$\mathbb{E}(\tilde{e}_1)p_1 - c'[x_1(x_2) + x_2] \leq 0, \quad (\text{A.3})$$

since  $u'(\pi) > 0$ . For  $x_2$  sufficiently small such that  $c'(x_2) < \mathbb{E}(\tilde{e}_1)p_1$ , it follows that  $x_1(x_2) > 0$  and inequality (A.3) holds with equality. Thus, when  $x_2 = 0$ , we have

$$\mathbb{E}(\tilde{e}_1)p_1 - c'[x_1(0)] = 0. \quad (\text{A.4})$$

In this case,  $h(0) = p_1 x_1(0)$  solves equation (A.2) since  $\pi(0) = \mathbb{E}(\tilde{e}_1)p_1 x_1(0) - c[x_1(0)]$ , which is non-stochastic.

Let  $EU$  be the objective function of program (2) with  $x_1 = x_1(x_2)$  and  $h = h(x_2)$ . Totally differentiating  $EU$  with respect to  $x_2$ , using the envelope theorem, and evaluating the resulting derivative at  $x_2 = 0$  yields

$$\left. \frac{dEU}{dx_2} \right|_{x_2=0} = u'[\pi(0)]\{\mathbb{E}(e_2)p_2 - c'[x_1(0)]\}. \quad (\text{A.5})$$

Substituting equation (A.4) into the right-hand side of equation (A.5) yields

$$\left. \frac{dEU}{dx_2} \right|_{x_2=0} = u'[\pi(0)][\mathbb{E}(\tilde{e}_2)p_2 - \mathbb{E}(\tilde{e}_1)p_1]. \quad (\text{A.6})$$

If  $\mathbb{E}(\tilde{e}_1)p_1 \geq \mathbb{E}(\tilde{e}_2)p_2$ , equation (A.6) implies that  $x_2^* = 0$ . We then know from equation (A.4) that  $x_1^*$  solves  $c'(x_1^*) = \mathbb{E}(\tilde{e}_1)p_1$  and  $h^* = p_1 x_1^*$ . This proves part (i) of Proposition 1.



If  $E(\tilde{e}_1)p_1 < E(\tilde{e}_2)p_2$ , equation (A.6) implies that  $x_2^* > 0$ . In this case, inequality (4) holds with equality. Let us reformulate program (2) as a two-stage optimization problem. In the first stage, the firm chooses the optimal amount of exports to country 2,  $x_2(x_1)$ , and the optimal forward position,  $h(x_1)$ , for a given amount of exports to country 1,  $x_1$ . In the second stage, the firm chooses the optimal amount of exports to country 1,  $x_1^*$ , taking  $x_2(x_1)$  and  $h(x_1)$  as given. The complete solution to program (2) is thus  $x_1^*$ ,  $x_2^* = x_2(x_1^*)$ , and  $h^* = h(x_1^*)$ .

The solution to the first-stage optimization problem must satisfy the following first-order conditions:

$$E\left\{u'[\tilde{\pi}(x_1)]\{\tilde{e}_2p_2 - c'[x_1 + x_2(x_1)]\}\right\} = 0, \quad (\text{A.7})$$

and

$$E\{u'[\tilde{\pi}(x_1)][E(\tilde{e}_1) - \tilde{e}_1]\} = 0, \quad (\text{A.8})$$

where  $\tilde{\pi}(x_1) = \tilde{e}_1p_1x_1 + \tilde{e}_2p_2x_2(x_1) - c[x_1 + x_2(x_1)] + [E(\tilde{e}_1) - \tilde{e}_1]h(x_1)$ . Let  $EU$  be the objective function of program (2) with  $x_2 = x_2(x_1)$  and  $h = h(x_1)$ . Totally differentiating  $EU$  with respect to  $x_1$ , using the envelope theorem, and evaluating the resulting derivative at  $x_1 = 0$  yields

$$\left.\frac{dEU}{dx_1}\right|_{x_1=0} = E\{u'(\tilde{\pi}^0)[\tilde{e}_1p_1 - c'(x_2^0)]\}, \quad (\text{A.9})$$

where  $x_2^0$  and  $h^0$  are defined in equations (6) and (7). Substituting equation (7) into the right-hand side of equation (A.9) yields

$$\left.\frac{dEU}{dx_1}\right|_{x_1=0} = E[u'(\tilde{\pi}^0)][E(\tilde{e}_1)p_1 - c'(x_2^0)]. \quad (\text{A.10})$$

If  $c'(x_2^0) \geq E(\tilde{e}_1)p_1$ , equation (A.10) implies that  $x_1^* = 0$ . Thus, in this case we have  $x_2^* = x_2^0$  and  $h^* = h^0$ . This proves part (iii) of Proposition 1.

Finally, if  $c'(x_2^0) < E(\tilde{e}_1)p_1$ , equation (A.10) implies that  $x_1^* > 0$ . In this case, condition (3) holds with equality:

$$E\{u'(\tilde{\pi}^*)[\tilde{e}_1 p_1 - c'(x^*)]\} = 0. \quad (\text{A.11})$$

Multiplying  $p_1$  to equation (5) and adding the resulting equation to equation (A.11) yields  $c'(x^*) = E(\tilde{e}_1)p_1$ , since  $u'(\pi) > 0$ . The optimal amounts of exports,  $x_1^*$  and  $x_2^*$ , and the optimal forward position,  $h^*$ , then solve conditions (3) and (4) with equality and equation (5) simultaneously. This proves part (ii) of Proposition 1.

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