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Nonparametric Identification of Auction Models with Non-Separable Unobserved Heterogeneity

Yingyao Hu David McAdams Matthew Shum

Abstract

We propose a novel methodology for nonparametric identification of first-price auction models with independent private values, which accommodates auction-specific unobserved heterogeneity and bidder asymmetries, based on recent results from the econometric literature on nonclassical measurement error in Hu and Schennach (2008). Unlike Krasnokutskaya (2009), we do not require that equilibrium bids scale with the unobserved heterogeneity. Our approach accommodates a wide variety of applications, including settings in which there is an unobserved reserve price, an unobserved cost of bidding, or an unobserved number of bidders, as well as those in which the econometrician fails to observe some factor with a non-multiplicative effect on bidder values.

1 Introduction

In this short paper, we propose a methodology for nonparametric identification of first-price auctions with unobserved heterogeneity, in the independent private values framework. By unobserved heterogeneity, we mean auction-specific factors that are observed by the bidders and affect their equilibrium bids. However, these factors are not observed by the econometrician, leading to spurious correlation between the bids for a given auction, even under the assumption of independent private values.

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Krasnokutskaya (2009) proposed an ingenious approach to identify and estimate such a model, as long as the unobserved heterogeneity affects bidders’ valuations multiplicatively and is independent of the bidder-specific component of each bidder’s valuation. Under these assumptions, Krasnokutskaya shows that results from the classical measurement error literature can be applied, so that two bids per auction are sufficient to identify and estimate the structural components of the model, which consist of the marginal distributions of the unobserved heterogeneity and of bidders’ valuations. Subsequently, Krasnokutskaya’s results have been used in applied analyses of timber auctions (Athey, Levin, and Seira (2005)), stamp auctions (Asker (2009)), and procurement auctions (Decarolis (2009)).

In this paper, we apply findings from the more recent literature on nonclassical measurement error in Hu and Schennach (2008). Using these more powerful tools, we obtain non-parametric identification of bidder values under much weaker assumptions. In particular, we allow the unobserved heterogeneity to affect bidders’ valuations in arbitrary nonlinear fashion, and we do not assume that bidders’ private signals are independent of the unobserved heterogeneity. We show that, using two bids per auction plus a third instrument (which could be a third bid), we can identify the marginal distribution of the unobserved heterogeneity, as well as the distributions of bidder valuations conditional on the unobserved heterogeneity.

Our identification results are very similar in spirit to those in d’Haultfoeille and Fevrier (2008), who focus on conditionally independent common value (“mineral rights”) models. Roberts (2009) takes a control-function approach to identify an ascending auction model with unobserved heterogeneity, using two bids and the reserve price as an instrument. Our identification approach, which is based on measurement error results, is quite distinct from these two papers.

The rest of the paper is organized as follows. In Section 2, we present the model, develop our main identification result, discuss an extension to settings with both unobserved heterogeneity and endogenous entry, and compare our approach with that of Krasnokutskaya (2009). Section 3 then discusses various additional examples in which this paper’s methods can be applied, including settings with an unobserved reserve price, an unobserved cost of bidding, and an unobserved number of bidders.
2 Model and identification

A fixed set of bidders \( i = 1, \ldots, n \) participates in a first-price auction \( t \), where \( n \geq 3 \). Bidder \( i \)'s value is distributed as \( V_{it} = v(X_{it}, Y_t) \), where \( X_{it} \) is privately observed, \( Y_t \) is one-dimensional “unobserved heterogeneity” that is commonly observed by all bidders but not by the econometrician, and \( v(\cdot, \cdot) \) is some deterministic function.\(^1\) \( X_{it} \) are independent across bidders conditional on \( Y_t \), so that our model is one of asymmetric independent private values (IPV). For all \( Y_t \), \( X_{it}|Y_t \) has a well-defined, continuous density over the same support for all \( i \).

Given these assumptions, there exists a unique Bayesian equilibrium of the bidding game in strictly increasing and differentiable strategies that depend on \( Y_t \) (Lebrun (1999)). We assume that bidders play these equilibrium strategies, generating random bids \( B_t = (B_{it} : i = 1, \ldots, n) \) observed by the econometrician.

The assumption here that there are at least three bidders is important, as the third bid plays the role of an instrument for the unobserved heterogeneity \( Y_t \). If there are only two bidders, our analysis still applies if an appropriate alternative instrument can be found. Loosely speaking, such an instrument must be correlated with the bids but independent of the bids conditional on \( Y_t \). See Assumptions 2-4 below for precise conditions.

2.1 Identification

In this paper, we provide a novel approach to identify both the distribution of the unobserved heterogeneity \( Y_t \) and the distribution of bids \( B_t \) conditional on \( Y_t \) from the distribution of bids \( B_t \). Given these distributions, identification of the distribution of bidder values \( V_t = (V_{it} : i = 1, \ldots, n) \) is straightforward from existing results. To see why, let \( G_{it}(b|y) = Pr(\max_{j \neq i} B_{jt} \leq b | Y_t = y) \) denote the cdf of the highest bid submitted by any of bidder \( i \)'s competitors conditional on \( Y_t = y \), and let \( g_{it}(b|y) \) be the associated pdf. The first-order condition of equilibrium bidding implies that bidder \( i \)'s realized private value \( v_{it} \) when he bids \( b_{it} \) in equilibrium can be expressed simply as his bid plus a mark-up that depends on

\(^1\)Random variables are capitalized while realizations are in lower case. For simplicity, we assume that the function \( v(\cdot, \cdot) \) is identical across bidders. Our approach allows \( v(\cdot, \cdot) \) to differ across bidders but, in any case, this distinction is not very important because we will not be able to identify \( v(\cdot, \cdot) \).
the elasticity of his probability of winning:

\[ v_{it} = b_{it} + \frac{G_{it}(b_{it}|y)}{g_{it}(b_{it}|y)}. \]  \hspace{1cm} (1)

(See Guerre, Perrigne, and Vuong (2000) and Athey and Haile (2002) for more details on this standard step.) Finally, given the distributions of both \( V_t|Y_t \) and \( Y_t \), one may recover the latent distribution of bidder values \( V_t \).

Fix any three bidders \( i,j,k \). We shall provide conditions given which both the distribution of \( Y_t \) and the joint distribution of \( (B_{it}, B_{jt}, B_{kt}) \) conditional on \( Y_t \) are identified from the joint distribution of \( (B_{it}, B_{jt}, B_{kt}) \). When these conditions hold for all triplets of bidders, then our methods allow one to identify both the distribution of \( Y_t \) and of \( B_t|Y_t \) from the distribution of \( B_t \), as desired.

**Assumption 1** The joint density of \( (B_{it}, B_{jt}, B_{kt}, Y_t) \) exists and is bounded away from zero and infinity.

Let \( f_i(b_{it}|Y_t), f_j(b_{jt}|Y_t), f_k(b_{kt}|Y_t) \), and \( f(Y_t) \) denote the marginal densities and \( B_i, B_j, B_k \), and \( Y \subset \mathbb{R} \) the supports of \( b_{it}|Y_t, b_{jt}|Y_t, b_{kt}|Y_t \), and \( Y_t \), respectively.

**Assumption 2** (i) \( f_i(b_{it}|Y_t, b_{jt}, b_{kt}) = f_i(b_{it}|Y_t) \) and (ii) \( f_j(b_{jt}|Y_t, b_{kt}) = f_j(b_{jt}|Y_t) \).

That is, \( (b_{it}, b_{jt}, b_{kt}) \) are independent conditional on \( Y_t \). This assumption is satisfied in the IPV bidding model.

**Assumption 3** For all bounded functions \( h : \mathbb{R} \to \mathbb{R} \), \( E[h(b_{kt})|b_{it}] = 0 \) for all \( b_{it} \in B_i \) implies that \( h(b_{kt}) = 0 \) for all \( b_{kt} \in B_k \).

**Assumption 4** For all bounded functions \( h : \mathbb{R} \to \mathbb{R} \), either \( E[h(Y_t)|b_{jt}] = 0 \) for all \( b_{jt} \in B_{jt} \) or \( E[h(Y_t)|b_{kt}] = 0 \) for all \( b_{kt} \in B_{kt} \) implies that \( h(y) = 0 \) for all \( y \in Y \).

Assumption 3 implies that the bid \( b_{it} \) is correlated with \( b_{kt} \) through the unobserved heterogeneity \( Y_t \), while Assumption 4 implies that the bids \( b_{jt}, b_{kt} \) are correlated with \( Y_t \). In Krasnokutskaya (2009)’s convolution setting, Assumptions 3, 4 are implied by her assumption that \( Y_t \) and \( X_{it} \) have non-vanishing characteristic functions.
**Assumption 5** There exists a known functional $M$ such that $M[f(\cdot|y)] = y$ for all $y \in \mathcal{Y}$.

Examples of functionals $M$ that could be used to satisfy Assumption 5 include those corresponding to any location of the distribution of $b|Y_t$, such as the mean, i.e., $M[f] = \int xf(x)dx$, the mode, i.e., $M[f] = \arg \max_x f(x)$, or the $\tau$-th quantile, i.e., $M[f] = \inf \{x^* : \int_{x^*}^\infty f(x)dx \geq \tau\}$. For instance, suppose that the mean of $b|Y_t$ is known to be strictly monotone in $Y_t$. If so, one can normalize $Y_t$ via some monotone transformation so that $E[b|Y_t = y] = y$ for all $y \in \mathcal{Y}$ and Assumption 5 is satisfied.

As a simple illustration, consider a symmetric example in which bidders’ valuations take the form $V_{it} = X_{it} \ast Y_t$, where $X_{it} \sim U[0,1]$ i.i.d. across bidders. In equilibrium, $b_{it} = Y_t \ast \frac{n-1}{n}X_{it}$. Moreover, $\text{med}(b_{it}|Y_t) = aY_t$, for the constant $a \equiv \frac{n-1}{2n}$. Hence, the functional $M[f(\cdot|y)] \equiv \text{med}(b_{it}|Y_t = y)/a = y$ satisfies Assumption 5. See Section 3 for a variety of additional examples.

Let $f(b_{it}, b_{jt}, b_{kt}) = \int f_i(b_{it}|Y_t) f_j(b_{jt}|Y_t) f_k(b_{kt}|Y_t) f(Y_t) dY_t$ denote the joint density of $(b_{it}, b_{jt}, b_{kt})$. The main result of the paper is that, under these assumptions, one may identify the distributions of $b_{it}|Y_t$, $b_{jt}|Y_t$, $b_{kt}|Y_t$, and $Y_t$ from the distribution of $(b_{it}, b_{jt}, b_{kt})$.

**Theorem 1** Under Assumptions 1, 3, 5 above, the density $f(b_{it}, b_{jt}, b_{kt})$ uniquely determines $f_i(b_{it}|Y_t)$, $f_j(b_{jt}|Y_t)$, $f_k(b_{kt}|Y_t)$, and $f(Y_t)$.

**Proof.** We prove the identification by showing that all the assumptions in Theorem 1 in Hu and Schennach (2008) are satisfied. Their Assumptions 1, 2, 5 are directly assumed. As discussed in their paper, Assumption 3 and the $(b_{kt}, \mathcal{B}_k)$ part of Assumption 4 here together imply their Assumption 3. Their Assumption 4 requires that for all $y_1, y_2 \in \mathcal{Y}$, the set $\{b : f_j(b|y_1) \neq f_j(b|y_2)\}$ has positive probability whenever $y_1 \neq y_2$. This can be shown as follows. Suppose their Assumption 4 does not hold in our model. Then there exist two $y_1, y_2 \in \mathcal{Y}$ such that $f_j(\cdot|y_1) = f_j(\cdot|y_2)$, i.e., the two distributions are the same. We may then construct a function $h$ such that $\int f_j(b|y) h(y)dy = 0$ for all $b$ and the function $h(\cdot)$ not equal to zero at $y_1$ and $y_2$. Therefore, their Assumption 4 is implied by the $(b_{jt}, \mathcal{B}_j)$ part of Assumption 4 here. Finally, their Theorem 1 implies our identification results.

### 2.2 Extension: endogenous participation

Consider an augmented version of our model in which there is a universe $\{1, ..., \bar{n}\}$ of potential bidders, of which only a random subset $N_t$ choose to bid. Theorem 1 provides
conditions under which to identify the distributions of the unobserved heterogeneity and of bidder values conditional on \( N_t \) for all \( N_t \subset \{1, \ldots, \pi\} \). In this way, our analysis can accommodate models of endogenous entry, in which the set of bidders \( N_t \) is associated with the unobserved heterogeneity \( Y_t \). Recently, there has been interest in such models (e.g. Li and Zheng (2009), Marmer, Shneyerov, and Xu (2009)).

Indeed, across a random sample of auctions in which all bids and bidders are observed, one can estimate directly from the data the probability that \( N_t \) is the set of active bidders, for all \( N_t \). Hence, given our estimates of \( f(Y_t|N_t) \), one can also form the joint distribution \( f(Y_t, N_t) = f(Y_t|N_t) \cdot f(N_t) \), and hence also the conditional distribution \( f(N_t|Y_t) \). This suggests that, in the IPV context, a model with non-separable unobserved heterogeneity and endogenous number of bidders can be estimated.

### 2.3 Comparison with Krasnokutskaya (2009)

In a path-breaking paper, Krasnokutskaya (2009) provided conditions under which results from the literature on classical measurement error can be used to identify IPV first-price auction models in the face of unobserved heterogeneity. Our paper is similar in spirit, but differs in that we bring to bear more powerful results from the recent literature on non-classical measurement error.

These results allow us to weaken most of the restrictive assumptions imposed by Krasnokutskaya (2009). First, Krasnokutskaya assumes that the unobserved heterogeneity has the same multiplicative effect on all bidders’ valuations, i.e. \( V_{it} = X_{it} \cdot Y_t \), with the implication that equilibrium bids also scale multiplicatively with \( Y_t \). By contrast, we require only that some location of the distribution of equilibrium bids be increasing in \( Y_t \). This condition is automatically satisfied in her setting, since the mean of each bidder’s equilibrium bid is not only increasing but linear in \( Y_t \). Second, Krasnokutskaya assumes that the unobserved heterogeneity \( Y_t \) is independent of the idiosyncratic components \( X_{it} \) of bidders’ values, as well as that \( X_{it} \) are independent conditional on \( Y_t \). By contrast, we only require that \( X_{it} \) are independent conditional on \( Y_t \). Also, as noted earlier in the text, the completeness Assumptions 3-4 are implied by Krasnokutskaya’s assumption that \( Y_t \) and \( X_{it} \) have non-vanishing characteristic functions. On the other hand, Krasnokutskaya makes weaker demands on the data. Whereas we require the observation of three bids, she requires only two.

By insisting only on a monotone relationship between bids and unobserved heterogeneity, rather than a multiplicative one, our approach opens up a wide variety of new applications.
For instance, suppose that bidders are symmetric and each bidder faces a cost $C_t$ of bidding which varies across auctions. While equilibrium bids will certainly not be multiplicative in $C_t$, the probability that each bidder chooses not to submit a bid is increasing in $C_t$. Using our approach, one may therefore non-parametrically identify both the distribution of $C_t$ and the distribution of bidder values conditional on $C_t$. (See Section 3.2 for details.)

3 Examples

This section provides a variety of examples in which this paper’s identification approach may be applied. In each example, one object is sold via first-price auction to risk-neutral bidders having independent private values. Except as otherwise specified, the econometrician observes all bids and the set of bidders is fixed and known to the econometrician.

3.1 Unobserved reserve price

Suppose that each auction $t$ has a reserve price $R_t$ that is random and unobserved by the econometrician, common knowledge among the bidders before the bidding, and independent of bidders’ private valuations. For example, if the seller lacks the power to commit to a reserve price, then $R_t$ would equal his opportunity cost of selling the good. Or, if the seller has the power to set an optimal reserve price based on the true distribution of bidder values, then $R_t$ is some strictly increasing function of that seller cost. Let $\emptyset$ denote the “null bid” ($\emptyset < b$ for all $b \geq 0$) made in equilibrium by any bidder whose value is less than the reserve.

This paper’s approach allows the econometrician to identify the distribution of $R_t$ as well as the distribution of each (asymmetric) bidder’s valuation, as long as (i) there are at least three bidders and (ii) the probability that the reserve price $R_t$ binds on bidder $i$ is strictly increasing in $R_t$ over its support.\footnote{Condition (ii) is satisfied in a broad set of circumstances. For example, suppose that the seller is able to set an optimal reserve, and that the seller’s private cost $C_t$ has support $[0, \bar{c}]$ while each bidder $i$’s value has support $[0, \bar{\tau}_i]$ for some $\bar{\tau} < \max\{\bar{\tau}_1, \ldots, \bar{\tau}_N\}$. Condition (ii) fails for bidder $i$ only if his maximal value is low enough that he is sometimes “priced out” of the auction, i.e. $\Pr(R_t > \bar{\tau}_i) > 0$. An optimally chosen reserve price will never price out all bidders, so (ii) must be satisfied for at least one bidder. Furthermore, should bidders be symmetric, it will be satisfied for all bidders.} In particular, condition (ii) implies that the functional $M[f_i(\cdot|r)] = \inf\{b : \Pr(B_{it} > b|R_t = r) < \Pr(B_{it} > \emptyset|R_t = r)\}$ corresponding to the minimal submitted bid equals $r$. Thus, Assumption 5 is satisfied without any need for a
normalization of $R_t$. The other assumptions are trivial to check, given that bidders have independent private values that are uncorrelated with the reserve price. The distribution of $R_t$ and of bidders’ values are therefore non-parametrically identified.

### 3.2 Unobserved cost of bidding

Suppose that bidders are symmetric and the reserve price is zero, but some bidders choose not to participate because of a common cost $C_t$ of submitting a bid that is random and independent of bidders’ valuations.

It is straightforward to show that, in the unique (symmetric) equilibrium, the probability that each bidder chooses not to submit a bid is increasing in $C_t$. Thus, the functional $M[f_t(\cdot | c)] = \Pr(b_i = \emptyset)$ corresponding to the probability of non-participation is strictly increasing in the unobserved heterogeneity $C_t$. Hence, Assumption 5 is satisfied after an appropriate normalization, i.e. when $Y_t = \gamma(C_t)$ for some strictly increasing $\gamma(\cdot)$. All other assumptions are again trivial to check. Thus, the distribution of bidder values conditional on $Y_t$ and conditional on having made a bid are non-parametrically identified.

Since $\gamma(\cdot)$ is an unknown normalization, however, more work is necessary in order to back out the bidding costs $C_t$ corresponding to each conditional distribution of values. Let $b_t(y)$ denote the minimal bid submitted by each bidder when $Y_t = y$, and let $v_t(y)$ denote the corresponding bidder value. In equilibrium, a bidder having value $v_t(y)$ must be indifferent between bidding $b_t(Y_t)$ and not bidding at all:

$$C_t = (v_t(Y_t) - b_t(Y_t)) \Pr(\max_{j \neq i} b_{jt} = \emptyset | Y_t). \tag{2}$$

Through this indifference condition, the distribution of $C_t$ is also identified.

### 3.3 Unobserved number of bidders

Suppose that $N_t$ symmetric bidders choose to participate in an auction with zero reserve price, where $N_t$ is random and common knowledge among the participating bidders before the bidding, and each participating bidder’s value is drawn iid from the same distribution regardless of $N_t$. The econometrician observes detailed bid-data for three bidders $i, j, k$, i.e. whether they chose to participate and what they bid, but does not observe the bids made by
other bidders, nor the total number of bidders. Such a scenario could arise if a researcher has acquired data from individual bidders only.

The condition here that the distribution of bidder values does not depend on $N_t$ is naturally satisfied in several sorts of settings. (See Section 2.2 for discussion of an alternative setting in which bidder values are correlated with $N_t$.) For example, if participation is costly and bidders observe their values before deciding to participate, then in equilibrium each bidder will participate if his value exceeds a symmetric threshold. In this case, the distribution of participating bidders’ values is just a truncation of the original distribution of values. Or, if bidders do not observe their values until they decide to participate, then obviously the distribution of participating bidders’ values will just be the original distribution. Or, finally, participation might simply be exogenous, as when participants are comprised of “random passers-by”.

The distribution of each bidder’s bid in the unique equilibrium is easily shown to be strictly increasing in the number of bidders $N_t$, in the sense of first-order stochastic dominance. (Since equilibrium strategies are also symmetric across bidders, we will drop reference to bidder $i$ in what follows.) In particular, the mean of $B_t|N_t$ is strictly increasing in $N_t$. Consequently, Assumption 5 is satisfied for an appropriate normalization $Y_t = \gamma(N_t)$, when we use the functional corresponding to the mean. The other assumptions are again trivial to check, given that bidders have independent private values drawn from the same distribution regardless of the number of bidders. By our results, then, the distribution of $Y_t$ and the distribution of each bidder’s bid conditional on $Y_t$ are non-parametrically identified.

However, more work is required to identify the distribution of bidder values: The mark-up of values over bids in (1) depends on the number of bidders, and $N_t$ cannot be directly inferred from $Y_t$ since $\gamma(\cdot)$ is unknown.

Since $\gamma(\cdot)$ is strictly increasing, each element $y \in \text{supp}(Y_t)$ corresponds to a different number of bidders $n(y) = \gamma^{-1}(y)$ in the support of $N_t$. For any quantile $\alpha \in (0, 1)$, let $v_t(\alpha)$ and $b_t(\alpha|y)$ denote the $\alpha$-th quantiles of the (symmetric) distributions of bidder values and equilibrium bids conditional on $Y_t = y$, respectively. (Recall that, by assumption, the distribution of bidder values does not depend on the number of bidders.) Conditional on $Y_t = y$, the probability that a bidder wins the object with bid $b_t(\alpha|y)$ is simply $G_t(b_t(\alpha|y)|y) = \alpha^{n(y)}$.}

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4See Athey and Haile (2007) Section 6.3 for an excellent discussion of scenarios in which bidders do not observe the number of other bidders. Here, only the econometrician fails to observe $N_t$. 

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Consequently, for each $\alpha \in (0, 1)$, (1) may be re-written as

$$v_t(\alpha) = b_t(\alpha|y) + \frac{\alpha b'(\alpha|y)}{n(y) - 1} \text{ for all } y \in \mathcal{Y}. \quad (3)$$

In particular, for any pair of quantiles $\tilde{\alpha}, \alpha$ and pair $\tilde{y}, y \in \text{supp}(Y_t),$

$$b_t(\tilde{\alpha}|\tilde{y}) + \frac{\tilde{\alpha}b'(%(\tilde{\alpha}|\tilde{y})}{n(\tilde{y}) - 1} = b_t(\tilde{\alpha}|y) + \frac{\tilde{\alpha}b'(\tilde{\alpha}|y)}{n(y) - 1} \quad (4)$$

$$b_t(\alpha|\tilde{y}) + \frac{\alpha b'(\alpha|\tilde{y})}{n(\tilde{y}) - 1} = b_t(\alpha|y) + \frac{\alpha b'(\alpha|y)}{n(y) - 1}. \quad (5)$$

Note that we have already identified all variables in the system of equations (4-5) except for $(n(\tilde{y}), n(y))$. $N_t$ is therefore identified, as long as (4-5) has a unique solution for all $\tilde{y}, y \in \text{supp}(Y_t)$. Given the distribution of $N_t = n(Y_t)$, one may now identify the distribution of bidder values from the first-order condition (3).

In this way, the distribution of the number of bidders $N_t$ and of bidder values can be non-parametrically identified as long as the support of $N_t$ has at least two elements. By contrast, if $N_t$ is unknown but non-random (i.e. the identified distribution of $Y_t$ has singleton support), then bidder values cannot possibly be identified.

References


The $\tilde{\alpha}$-th and $\alpha$-th quantiles of the distribution of bidder values solve (4-5). Thus, this system always has at least one solution under our maintained assumptions about the data generating process. As can be easily shown, it has a unique solution as long as the matrix $A = \begin{bmatrix} \alpha b'(\alpha|y) & -\alpha b'(\alpha|\tilde{y}) \\ \tilde{\alpha} b'(\tilde{\alpha}|y) & -\tilde{\alpha} b'(\tilde{\alpha}|\tilde{y}) \end{bmatrix}$ is invertible.


