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Integrated Public Education, Fertility and Human Capital

Leonid V. Azarnert

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Integrated Public Education, Fertility and Human Capital

Abstract

This article analyzes the consequences of integration in public education. I show that the flight from the integrated multicultural public schools to private education increases private educational expenditures and, as a result, decreases fertility among more affluent parents whose children flee. In contrast, among less prosperous parents, integration in public education decreases their children’s human capital levels. I demonstrate that the poor, who cannot opt out, incur greater costs than the rich, who can resort to private education. I also analyze the overall society-wide effect of the integration policy and derive a condition that determines precisely whether this policy increases or decreases the average level of human capital in society.

JEL-Code: I200, J100.

Keywords: public education, private education, integration, fertility, human capital.

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1. Introduction

A long line of the research has overwhelmingly shown that white students’ choice between public and private schools is influenced by the racial composition of the local student population. Thus, in the US, starting from the 1970s, numerous studies, such as, for example, Clotfelter (1976; 2001), Coleman, Hoffer and Kilgore (1982), Conlon and Kimenyi (1991), Andrews (2002), Fairlie and Resch (2002), Reber (2005), Lankford and Wyckoff (2006), among others, presented evidence of White Flight from the integrated multicultural public schools with large concentrations of black or minority children into more segregated private education. Fairlie (2002) has provided evidence of the "Latino Flight" from the blacks that is not significantly different from the flight of whites. Finally, Betts and Fairlie (2003) found evidence of the "Native Flight" from minority immigrants. Outside the United States, the desire of white parents to have their children educated in predominantly white schools has been well documented, for example, in the United Kingdom (Bagley 1996). Within this context, it has been also argued that higher levels of family income and parental education have a strong positive effect on the probability that children will attend private schools (Lankford and Wyckoff 2001; Betts and Fairlie 2001; 2003; Fairlie 2002; Fairlie and Resch 2002; Epple et al. 2004).

Although no consensus has been reached in the literature on the causes of the flight, the authors of these studies speculate that White Flight occurs due to the use of the racial composition of the school as a signal of academic quality in response to a lack of other measures of quality. A more extensive list of the reasons include, for instance, expectations about poor management of schools where large groups of minority children are enrolled, lower level of discipline in multiethnic classes, peer group effect of a less advantaged school-student population, the desire to avoid contacts with juveniles with

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1 Although flight to another, less desegregated, public school may also be an option (Reber 2005), as Lankford and Wyckoff (2006) note, in the areas where the open enrollment plans have been established to achieve desegregation, the public school choice available to parents is quite limited. As they note, whites living in school attendance areas having relatively "too few" whites in the local public school have no public school choice at all.

2 Very high rates of private school attendance have been also observed among the US-born Asians (Betts and Fairlie 2001; Fairlie and Resch 2002).
supposingly higher problem behavior, \(^3\) parental fear that teachers may decide to spend additional time helping minority students with limited proficiency in the mainstream language at the expense of other students in the classroom or that the presence of the students with limited language proficiency in public school may lead to wholesale changes in teaching methods used for all students. \(^4\) For any reason that causes parents to expect that multicultural integration in public school is likely to reduce their children’s acquisition of human capital, the effect of integration is the same, and this paper is about the effect, not about the reasons. In any case, it has been decisively demonstrated that the racial composition of suburban public schools appears to be the key in explaining why, as compared to white urban families, relatively few suburban families send their children to private schools (Lankford and Wyckoff 2006).

I discuss the issue in the context of a model with endogenous fertility building on Azarnert (2006; 2010a) that is related to the literature on endogenous fertility and growth. \(^5\) The prediction of the present model that opting out of public education to the expensive private education, which increases the cost of having children, is associated with a reduction in fertility is consistent with the traditional theory of endogenous fertility, which implies that any increase in the cost of rearing children leads to a lower fertility choice. Among empirical studies, Lankford and Wyckoff (2001) demonstrated that the choice of private education for children is associated with lower number of children within the family. De la Croix and Doepke (2009) also found some empirical support for their hypothesis that parents who choose public schools for their offspring have more children than parents who choose costly private schools. Equipped with these findings of the previous theoretical and empirical literature, the present work enriches the analysis with a novel channel, through which public education policy can generate a

\(^3\) For example, Freeman (1994) reports that among African-American males aged 18 – 34 in 1993, 12.7% of the work force were incarcerated and 36.7% of the work force were under supervision of the criminal justice system.

\(^4\) To provide an illustrative example, Betts and Fairlie (2003, p. 989, note 4) refer to an observation that, for instance, in a "methods" class at Cal State Long Beach would-be teachers, who will probably wind up in classrooms with a large number of students not fluent in English, were encouraged to find ways to avoid writing, instead of emphasizing it.

\(^5\) For a survey of a recent literature on endogenous fertility and growth see Galor (2005); cf. also Azarnert (2008; 2009).
different effect on the level of fertility among groups that differ from each other with respect to their average rates of participation in public education.

In this paper, I assume that the basic amount of education provided in public school is financed by taxes levied outside the economic environment that is being examined and thereby is free for families. This simplification assumption allows us to abstract from the negative effect of taxation on individuals’ decisions with respect to the optimal investments in the quantity and quality of their offspring and concentrate on the pure effect of the multicultural integration in public education. It is also consistent with the situation after the school finance reform that launched in the mid-1970s in the US, where governmental grants to schools are now often provided at a simple per-student base (Card and Payne 2002). Assuming that public schools are financed by an endogenously determined tax, which implies a reallocation of resources from the rich, who can resort to the White Flight strategy, to the poor, will increase the threshold level of income (human capital), above which parents decide in favor of opting out of public education, without altering the qualitative nature of this paper’s results.

The basic idea of this paper may be stated as follows. Assume an economy populated by two different groups: $A$-type individuals and $B$-type individuals. Assume that for some reason, for instance, expectations about poor management of schools where large groups of minority children are enrolled, low level of discipline in multiethnic classes, peer group effect of a less advantaged student population, or simply general difficulties for teaching in more heterogeneous classes, $A$-type parents expect that in the integrated school their children will not devote their entire time to the acquisition of human capital. If parents expect that in the integrated school their children are likely to spend a fraction of their time unproductively, this generates an incentive to opt out and

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6 In this case the particular tax levied in order to finance public education is irrelevant for the analysis. For example, it could be a lump sum tax or a local property tax along with direct aid received exogenously from the government (as e.g. in Nechyba 2003); cf. also Azarnert (2010a; 2010b).

7 The disincentive effect of taxation has been well recognized in the literature (see Azarnert (2004) and references therein).

8 Some references to the large literature on this subject can be found in Epple et al. (2004), de la Croix and Doepke (2009), Azarnert (2010a); cf. also Benabou (2002).

9 This is an approximation to the situation in the integrated urban areas where the exit of whites has not been complete. In contrast, in the segregated suburban areas, where students’ population in public schools is almost entirely white, whites show much lower interest in private education (Lankford and Wyckoff 2006, among others), which is consistent with the prediction of the present model.
flee to the exclusive private school. Provided that the exclusive private education is more expensive, only more affluent parents can afford opting out of the public school.

The flight to more expensive private education increases parental private expenditures on the education of their offspring and, as a result, increases the total cost of having children. Provided that children are viewed as a normal good, this increase in the cost of children decreases fertility among more affluent parents whose children flee integrated public schools. In contrast, among relatively less skilled (poor) parents, who cannot afford private education for their offspring, multicultural integration in public education decreases their children’s human capital levels.

In this work, I derive the threshold level of human capital (income) that divides the \(A\)-type population into two groups: the more educated (wealthy), for whom opting out is optimal, and the less educated (poor), who cannot afford to resort to private education. Then I demonstrate that the poor, who cannot opt out, incur greater negative costs than the rich, who can resort to private education. Moreover, the human-capital losses of the offspring of the poor who remain in integrated public schools coupled with the per-capita human-capital gains of the offspring of the rich who opt out to private education, may have long-lasting consequences for the persistence of inequality among the \(A\)-type individuals in the future.

Following the classical approach in the literature (e.g., Hoxby 2000 and references therein), I model the peer effects associated with the integration as externalities in the production of human capital that work in the opposite directions for the two types of students. This allows us to compare the losses from the integration for \(A\)-type students to the benefits for \(B\)-type students. I also analyze the overall society-wide effect of the integration policy on the level of human capital in the society as a whole and derive a condition that guarantees an increase/ decrease in the society-wide average level of human capital as a result of integration in public education.

2. The Basic Structure of the Model

Consider an overlapping-generations economy, in which activity extends over an infinite discrete time. In every period the economy produces a single homogenous good using a
constant-returns-to-scale technology with human capital as the only input. In each generation, agents live for two periods: childhood and adulthood. During childhood, individuals acquire human capital. During adulthood, they work, become parents and bring up their offspring. As parents, adult individuals allocate a positive fraction of their time to feeding and raising their children and invest in the education of their children.

The economy is populated by two different groups: $A$-type individuals and $B$-type individuals. Suppose that for exogenous reasons, $B$-type school students have different norms of behavior, which are associated with less favorable peer effects. Suppose also that $B$-type parents are poorer and cannot afford private education for their children. In all other respects, individuals of both types are similar. Children of both groups can acquire human capital in public schools. If $A$-type children flee the integrated public school, they acquire education in a more expensive private school.

In Sections 2.1 to 2.5, the analysis concentrates on the results of the integration policy in public education for $A$-type individuals only. In section 2.6, I analyze the results of the integration policy for $B$-type individuals. Next, in Section 2.7, I discuss the effect of the integration policy on the average level of human capital in the society as a whole.

### 2.1. Human Capital Production

In the first period of life children are endowed with one unit of time. In the exclusive private school children devote their entire time to the acquisition of human capital. In the integrated public school children spend a fraction $1 - \theta$ of their time unproductively, as follows from, for example, poor management of schools where large groups of minority children are enrolled, lower level of discipline in integrated classes, peer group effect of a less advanced student population, general difficulties for teaching in more heterogeneous classes, or simply a perceived threat from $B$-type school students with supposedly different norms of behavior. As a result, they devote to the acquisition of human capital a fraction $\theta$ of their time only; $\theta \in (0,1)$. Therefore, the variable $\theta$ is inversely related to the fraction of $B$-type school students in the integrated school: $\theta_B' < 0$.

In the public school a certain amount of human capital – equal for all children – is provided at zero cost for their parents. This basic public education is assumed to be
financed by taxes levied outside the economic environment that is being examined. In addition, to increase their children’s human capital levels, parents supplement this basic public educational expenditure with their own private investments in their children’s human capital. In the exclusive private school all the costs of the acquisition of human capital are assumed to be financed by parents themselves.

The human capital level of a child, who becomes an adult at period \( t+1 \) \((h_{t+1})\), is therefore an increasing function of the public per-child expenditure \((\alpha)\) if a child acquires education in the public school, the parental real expenditure on the child’s education in private or public school in period \( t \) \((e_t)\), as well as the child’s time investment \((\theta)\):

\[
h_{t+1} = h(\alpha, \theta, e_t).
\]

A particular form of human capital production function is specified below in Eq. (8).

### 2.2. The Optimization of Parents

Under both scenarios (in the case of flight from public school and in the case of no flight from public school), agents derive utility from their own consumption in adulthood and from the total future income of their children. The utility function of an individual born at time \( t - 1 \) is therefore

\[
U_t = (1 - \beta) \log C_t + \beta \log (I^{N,j}_{t+1}),
\]

where \( C_t \) is an individual’s own consumption, \( I^{N,j}_{t+1} \) is the future income of the individual’s offspring and \( \beta \in (0, 1) \) captures the relative weight given to children. In the case of flight \( j = F \) and in the case of no flight \( j = NF \). The conditions that lead to the decision to resort to flight are analyzed below in Section 2.5.

In every period \( t \), \( A \)-type adults are characterized by a skill level \( h_t \) that is distributed according to the cumulative density function \( F_t^{A}(\cdot) \) over the strictly positive support \([h_{t,A}^{\min}, h_{t,A}^{\max}]\) and are endowed with one unit of time, which they allocate between childbearing and labor force participation. The cost of feeding and raising children is measured in terms of work time foregone at \( \delta \) per child. The cost of acquiring human capital is measured in units of the wage per efficiency unit of labor \((w)\).
Under each scenario \(( j = F \) or \( j = NF \)), in order to maximize utility, an adult simultaneously chooses a current consumption, \( C_t \), the number of children, \( N^j_t \), and invests \( e^j_t \) units of \( w \) in each child’s education subject to the following budget constraint:\(^{10}\)

\[
C_t + w(\delta h_t + e^j_t)N^j_t \leq wh_t,
\]

while the total future income of the individual’s offspring is:

\[
I_{t+1}^{N,j} = N_t h^{j}_{t+1}w.
\]

The right-hand side of Eq. (3) represents an adult’s income, which is allocated between consumption and the total cost of rearing children. Under each scenario \(( j = F \) or \( j = NF \)), the amount of resources invested in the education of each child \(( e^F_t \) or \( e^{NF}_t \)) and hence the children’s levels of human capital \(( h^F_{t+1} \) or \( h^{NF}_{t+1} \)), as well as the total number of children \(( N^F_t \) or \( N^{NF}_t \)), may be different. The wage per efficiency unit of labor \(( w \)) is fixed over time, as follows from, for instance, the assumption of a CRS technology with a single factor of production.

### 2.3. Quantity - Quality Tradeoff

From optimization, regardless of the choice of public or private education for his children, an adult’s consumption is

\[
C_t = (1 - \beta)wh_t.
\]

That is, a fraction \( 1 - \beta \) of an adult’s full income is devoted to consumption and hence a fraction \( \beta \) is devoted to childrearing.

In order to allocate resources between children’s quantity and quality, an adult makes two simultaneous decisions. First, he decides how much consumption to forego during his adulthood to rear a family. Second, he decides what amount of resources to invest privately in the education of his children to increase their skill level.

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\(^{10}\) The time constraint requires that \( 0 \leq 1 - (\delta + e_t^j/h_t)N^j_t \leq 1.\)
Under each scenario, in the case of a non-corner solution, the standard condition of setting the marginal rate of substitution between quality and quantity equal to the price implies that

$$\frac{h_{t+1}^j}{N_t^j} - \frac{\partial h_t^j + e_t^j}{N_t^j / (dh_{t+1}^j / de_t^j)} = 0$$  if  \( e_t^j > 0 \),  \( j = 1 \) \( j = 1 \) \( \frac{N_t^j}{dh_{t+1}^j / de_t^j} \)

where \( h_{t+1}^j / N_t^j \) is the marginal rate of substitution between quality and quantity, \( w(\partial h_t^j + e_t^j) \) is the cost of an additional child for a given level of parental private investment in the child’s education and \( wN_t^j / [dh_{t+1}^j / de_t^j] \) is the marginal cost of children’s quality (human capital) for a given number of children.

From Eq. (6), optimization with respect to child’s quality thus implies that

$$h_{t+1}^j = (\partial h_t^j + e_t^j) \frac{dh_{t+1}^j}{de_t^j}.$$  \( j = 1 \) \( j = 1 \) \( \frac{dh_{t+1}^j}{de_t^j} \)

The next subsection discusses the solution for the parents’ optimization problem for a particular form of the human capital production function and analyzes the effect of integration in public education on parental educational expenditures, children’s human capital levels and fertility.

### 2.4. Choice of Fertility and Investment in Education

To characterize optimal choices of fertility and investment in schooling, suppose that public and private schools have access to the same technology of human capital production:

$$h_{t+1}^j = (\theta(\alpha + e_t^j))^{\gamma}, \quad 0 \leq \theta \leq 1, \quad \alpha \geq 0, \quad 0 < \gamma < 1.$$  \( j = 1 \) \( j = 1 \) \( \theta \) \( \alpha \) \( \gamma \)

In this particular learning technology the variable \( \theta \) captures the major difference between the integrated and exclusive education. As has been assumed in Section 2.1, in the exclusive school, children devote their entire unit of time to the acquisition of human capital \( (\theta = 1) \), whereas in the integrated multicultural school they devote a fraction \( \theta \) of their unit of time only \( (0 < \theta < 1) \).

The difference between public and private education is captured here by the variable \( \alpha \) that measures the level of public educational expenditures per child in the
public school \((\alpha > 0)\), which in this work are assumed to be financed by taxes levied outside the model. In contrast, all of the expenditures in the private school are financed by parents themselves, so that in the case of the private school \(\alpha = 0\).

Given the differences between public and private education, as captured by \(\theta\) and \(\alpha\), this human capital production technology can be re-formulated as:

\[
h^j_{i+1} = \begin{cases} 
(\theta(\alpha + e^j_t))^\gamma, & \text{if } j = \text{NF} \\
(e^j_t)^\gamma, & \text{if } j = \text{F} 
\end{cases}, \quad 0 < \theta < 1, \quad \alpha > 0, \quad 0 < \gamma < 1. \tag{8'}
\]

Given \((8')\), the optimal choice of an \(A\)-type individual's private investment in the children's education in the integrated and private schools is\(^{11}\)

\[
e^j_t = \begin{cases} 
\frac{\gamma \partial h_t - \alpha}{1 - \gamma}, & \text{if } j = \text{NF} \\
\frac{\gamma \partial h_t}{1 - \gamma}, & \text{if } j = \text{F}, 
\end{cases} \tag{9}
\]

so that, according to \((7)\),

\[
h^j_{i+1} = \begin{cases} 
(\theta(\partial h_t - \alpha))^{\gamma}, & \text{if } j = \text{NF} \\
(\frac{\gamma \partial h_t}{1 - \gamma})^{\gamma}, & \text{if } j = \text{F}. 
\end{cases} \tag{10}
\]

Given the amount of resources allocated to children’s education in each of the cases, the desired fertility is

\[
N^j_t = \begin{cases} 
\frac{\beta(1 - \gamma)}{\delta - \alpha / h_t}, & \text{if } j = \text{NF} \\
\frac{\beta(1 - \gamma)}{\delta}, & \text{if } j = \text{F}. 
\end{cases} \tag{11}
\]

The following lemma summarizes the main result concerning the effect of White Flight on \(A\)-type individuals’ expenditures on the education of their children, the children’s human capital levels and fertility.

**Lemma 1:** The flight of \(A\)-type children from the integrated multicultural public school

1. Increases their parents’ private expenditures on the children’s education.
Proof. Eq. 9.

(2) Increases the children’s human capital levels.
Proof. Eq. 10.

(3) Decreases fertility among the parents whose children flee.
Proof. Eq. 11.

In the next section, I derive conditions that lead to the choice of flight in preference to no flight and analyze the losses of $A$-type individuals from integration.

### 2.5. To Flee Or Not To Flee? The Losses of $A$-type Individuals from Integration

In this section, I first analyze the tradeoff between the multicultural public school and the exclusive private school. Next, I analyze the losses of $A$-type individuals from integration.

Recall that, on the one hand, a significant fraction of the expenditures in the integrated public school is financed by the government, whereas all the expenditures in the exclusive private school are financed by parents themselves. As a result, a certain amount of children’s education in the integrated public school is provided for free for parents. On the other hand, in the integrated public school, children devote less time to the acquisition of skills.

In order to establish conditions that lead to the choice of flight over no flight, compare the levels of parental utility derived under both scenarios. As long as $U^{NF}_t \geq U^F_t$, it is optimal to remain in the integrated public school. Once this inequality is reversed, it is optimal to leave public school in favor of the exclusive private education. From optimization, as determined in Eq. (5), adults’ consumption remains unaffected whether their children attend public school, or opt out for private education. Therefore, the level of parental utility in the case of flight ($U^F_t$) is higher than the level of parental utility in the case of no flight ($U^{NF}_t$) if the total future income of the one’s children in the case of flight ($I^{NF,F}_{t+1}$) is higher than the corresponding total children’s income in the case of no flight.

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11 An assumption that $h^{4,min}_i > a/\gamma \delta$ ensures that all parents invest in the education of their children if $j = NF$. 
From Eq. (4), given the optimal levels of fertility and the children’s human capital, as shown in Eqs. (10) and (11),

\[ I_{t+1}^{N,F} = N_t^{N,F} h_{t+1}^{F} w \geq I_{t+1}^{N,NF} = N_t^{N,NF} h_{t+1}^{NF} w, \text{ if} \]

\[ h_t \geq \hat{h} = \frac{\alpha}{\delta(1 - \theta^{\beta-\gamma})}. \tag{12} \]

Given Eq. (12), Proposition 1 determines precisely when it is optimal to send children to the multicultural public school and when it is optimal to opt out and educate children in the exclusive private school.\(^{12}\)

**Proposition 1:** For parents with human capital levels below the threshold \( \hat{h} = \alpha / \delta(1 - \theta^{\beta-\gamma}) \), the no-flight strategy is optimal, whereas for parents with human capital levels above that threshold, the flight strategy is optimal.

**Proof.** Substituting the optimal levels of \( C_t, h_t^{j}, \text{ and } N_t^{j} \) into (2) yields that \( U_t^{NF} \geq U_t^{F} \), if \( h_t \leq \alpha / \delta(1 - \theta^{\beta-\gamma}) \) and \( U_t^{NF} \leq U_t^{F} \), if \( h_t \geq \alpha / \delta(1 - \theta^{\beta-\gamma}) \).

Therefore, this allows us to summarize the major effect of the multicultural integration in public education on the number and human capital levels of children that have been born to \( A \)-type parents with different levels of human capital.

**Proposition 2:** (1) Multicultural integration in public education that causes relatively skilled (wealthy) parents with human capital levels above \( \hat{h} \) to resort to private education decreases fertility among these parents with human capital levels above \( \hat{h} \).

**Proof.** Proposition (1) in conjunction with Lemma 1(3).

(2) In contrast, among relatively less skilled (poor) parents with human capital levels below \( \hat{h} \), who cannot afford private education for their offspring, multicultural integration in public education decreases their children’s human capital levels.

**Proof.** Proposition (1) in conjunction with Eq. (10), if \( j = NF \). Note that \( \theta \in (0,1) \).

Therefore, the integration policy in public education that has been designed for the benefit of less advantaged \( B \)-type children generates a negative effect on the opportunities of the other weak segment of society: the offspring of the \( A \)-type poor.
Given that for parents with human capital levels above \( \hat{h} \) their resort to the flight strategy implies an increase in the private parental expenditures on their children’s education that, although at the expense of a reduction in the number of children, more than offsets the lost public education, this also allows us to shed new light on the effect of the integration on inequality among \( A \)-type individuals.

**Proposition 3:** Multicultural integration in public education increases inequality among \( A \)-type individuals in the children’s generation.

*Proof.* Proposition (1) in conjunction with Lemma 1(2).

In particular, comparing the actual level of human capital acquired by the offspring of the \( A \)-type poor in the integrated public school, as shown in Eq. (10); \( j = NF \) (denoted by \( h_{t+1}^{NF} \)), to the potential level of human capital they could acquire in a non-integrated public school, as computed from the same equation with \( \theta = 1 \) (denoted by \( h_{t+1}^{PNI} \)), the per-child loss of human capital as a fraction of the child's potential human capital level is:

\[
\frac{\Delta h_{t+1}^{NF}}{h_{t+1}^{PNI}} = \frac{h_{t+1}^{PNI} - h_{t+1}^{NF}}{h_{t+1}^{PNI}} = 1 - \theta^\gamma.
\]  \hspace{1cm} (13)

Similarly, comparing the actual level of human capital acquired by the offspring of the \( A \)-type rich in the private school, as shown in Eq. (10); \( j = F \) (denoted by \( h_{t+1}^F \)), to the potential level of human capital they could acquire in a non-integrated public school (\( h_{t+1}^{PNI} \)), the per-child gains of human capital as a fraction of the child's potential human capital level are:

\[
\frac{\Delta h_{t+1}^F}{h_{t+1}^{PNI}} = \frac{h_{t+1}^F - h_{t+1}^{PNI}}{h_{t+1}^{PNI}} = \left( \frac{\partial h_t}{\partial h_t - \alpha} \right) - 1.
\]  \hspace{1cm} (14)

In addition, in view of the positive relationship between the parental human capital levels and their children’s human capital levels, as shown in Eq. (10), this effect of integration on inequality between the offspring of the \( A \)-type rich and the poor may have long-lasting consequences.

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\(^{12}\) Notice that from Eq. (12), it is immediately clear that in the potential case of no integration in public education (\( \theta = 1 \)), it is optimal for everyone to remain in the public school.
Proceed now to the analysis of the losses of adult $A$-type individuals. Given the utility function in Eq. (2) and the results of optimization with respect to an adult’s consumption in Eq. (5), these losses are calculated in terms of the lost potential future income of an $A$-type individual’s children as a fraction of their total potential incomes.

In order to calculate the lost potential income of an $A$-type individual’s children, compute first the total potential income of his children in a potential case of a non-integrated public education. Denoting the potential non-integrated public education by $j = PNI$, the total potential income of the one’s offspring in this case can be computed by multiplying the potential per-child level of human capital, as shown in Eq. (10), if $j = NF$ and $\theta = 1$, by the number of the individual’s children, as shown in Eq. (11), if $j = NF$. Subtracting from this potential level of the total income the real total income of the one’s offspring in the case of flight or in the case of no flight $(I_{t+1}^{N.PNI} - I_{t+1}^{N,j})$ and then dividing it by the potential total income of all that person’s children in the potential case of a non-integrated public education $(I_{t+1}^{N,PNI})$, one can compute the fraction of the total potential income of an $A$-type individual’s children that has been lost as a result of integration:

$$\frac{\Delta I_{t+1}^{N,j}}{I_{t+1}^{N,PNI}} = \frac{I_{t+1}^{N,PNI} - I_{t+1}^{N,j}}{I_{t+1}^{N,PNI}} = \begin{cases} 1 - \theta^\gamma, & \text{if } j = NF, \\ 1 - (1 - (\alpha / \partial h_i))^{1-\gamma}, & \text{if } j = F. \end{cases}$$ (15)

Clearly, since the wage per one unit of human capital is fixed in this model, the above equation also implies that the fraction of the total potential human capital of an $A$-type individual's children that has been lost due to integration is the same as the fraction of the total children's human capital.

Therefore, given Eq. (15), I emphasize that:

**Proposition 4:** The fraction of total potential income (human capital) of an $A$-type individual's children that has been lost due to integration is higher among parents with human capital levels below $\hat{h}$ than among parents with human capital levels above $\hat{h}$, and among the latter it is higher the lower is the individual’s level of human capital.

Proof. Note that for any $h_i \geq \hat{h}$, $1 - \theta^\gamma \geq 1 - (1 - (\alpha / \partial h_i))^{1-\gamma}$ and $1 - (1 - (\alpha / \partial h_i))^{1-\gamma}$ is decreasing in $h_i$. 
Therefore, this allows us to conclude that among A-type individuals the poor, who cannot afford to avoid integration, incur the greater costs than the rich, who can resort to private education. This result may partly explain why negative sentiments toward several minorities are particularly strong among the less educated, as has been widely argued.

2.6. B-type Individuals

In this section, I introduce B-type individuals and analyze the effect of the integration in public education on this group of population.

Recall that in this model the difference between the two types of population stems from:

1. Different norms of behavior of B-type school students,
2. Poverty of B-type parents who cannot afford private education for their children.

To capture the first distinctive feature of B-type individuals, I assume that B-type children devote a lower fraction of their time to the acquisition of human capital relative to A-type children. Furthermore, I assume that if B-type children are enrolled in a "B-type-only" public school, they devote to the acquisition of human capital only a minimum amount of time $\theta_{\text{min}}$. If however they are enrolled in an integrated public school, owing to human-capital spillovers and better peer effects of A-type students, they devote to the acquisition of human capital a greater share of their time, $\tilde{\theta} > \theta_{\text{min}}$, which, as in the case of A-type students, is also inversely related to the fraction of B-type students in the school; $\tilde{\theta}_{B} < 0$. Therefore, integration is modeled here as a positive externality for B-type group's human capital accumulation.

To capture the second distinctive feature of B-type individuals, I assume that in period $t$, B-type adults are characterized by a skill level $h_t$ that is distributed over the cumulative density function $F_{t}^{B}(\cdot)$ over a strictly positive support $[h_{t}^{B,\text{min}}, h_{t}^{B,\text{max}}]$ with a strictly lower upper bound than that of the support over the cumulative density function that characterizes the skill level of A-type adults, $F_{t}^{A}(\cdot)$; $h_{t}^{A,\text{max}} > h_{t}^{B,\text{max}}$. Moreover, to ensure that B-type parents will not be able to afford private education for their offspring, I make the technical assumption that $h_{t}^{B,\text{max}} < \alpha/(\delta(1-\bar{\theta}^{\gamma}\gamma))$. 
Therefore, B-type school children can be enrolled either in a "B-type-only" public school (denoted by  \( j = BTO \)), or in an integrated public school (denoted by  \( j = INT \)). The "B-type-only" public school has access to the same technology of human capital production, as shown in Eq. (8), so that the amount of human capital acquired by B-type children in either school is respectively:

\[
\begin{align*}
  h_{t+1}^j &= \begin{cases} 
    (\theta^\text{min} (\alpha + e_t^j))^\gamma, & \text{if } j = BTO \\
    (\tilde{\theta} (\alpha + e_t^j))^\gamma, & \text{if } j = INT.
  \end{cases}
\end{align*}
\]

(16)

Since in all other respects individuals of both types are similar, the optimization problem of B-type parents is similar to the optimization problem of A-type parents, subject to the particular constraints of B-type individuals, as described above.

Therefore, following the same steps as in Sections 2.2 to 2.4, the optimal choice of a B-type individual's private investment in the children's education is

\[
e_t^j = \frac{\gamma \bar{h}_t - \alpha}{1 - \gamma},
\]

(17)

so that,

\[
\begin{align*}
  h_t^j &= \begin{cases} 
    \left(\frac{\theta^\text{min} (\bar{h}_t - \alpha)}{1 - \gamma}\right)^\gamma, & \text{if } j = BTO \\
    \left(\frac{\tilde{\theta} (\bar{h}_t - \alpha)}{1 - \gamma}\right)^\gamma, & \text{if } j = INT,
  \end{cases}
\end{align*}
\]

(18)

and the desired fertility is

\[
N_t = \frac{\beta (1 - \gamma)}{\delta - \alpha/h_t}.
\]

(19)

From equations (17) to (19), it is clear that:

**Lemma 2:** Multicultural integration in public education does not affect B-type parents' fertility and private expenditures on their children's education, but increases their children's human capital levels.

Moreover, from Eq. (18), the per-child gains of human capital, as a fraction of the child's human capital level that could be acquired in the "B-type-only" school are:
Clearly, since these gains result from the positive human capital spillovers that come at a zero cost for $B$-type individuals, the integration policy increases the levels of their utility.

In the next section, I analyze the overall society-wide effect of the integration policy on the average level of human capital in the society as a whole and derive a condition that guarantees an increase/decrease in the society-wide average level of human capital as a result of integration in public education.

### 2.7. Society as a Whole

This section examines the overall society-wide effect of the integration in public education on the average level of human capital in the society as a whole.

As shown previously, integration in public education increases the individual levels of human capital among the offspring of the $B$-type individuals, while decreasing the levels of human capital among the offspring of the $A$-type poor. At the same time, integration causes the flight of the offspring of the $A$-type rich to private education, which, although increasing their per-capita human-capital levels, decreases the number of the offspring of the $A$-type rich, thus slowing down the total accumulation of human capital in this group. The overall effect of the integration on the average level of human capital in the society as a whole is thus uncertain. It can be either positive, if the positive effect on the $B$-type individuals outweigh the negative effect on the $A$-type individuals, or negative, if the latter dominates.

To establish whether the integration affects the society's average level of human capital positively or negatively, suppose first that in period $t$, where the integration takes place, the fraction of $B$-type individuals in the society is $b_t$. Correspondingly, the fraction of the $A$-type individuals is thus $1 - b_t$.

The average human capital level in period $t+1$ is defined as

$$
\bar{h}_{t+1} = \int h_{t+1} dF_{t+1}(h) = \int N_{t+1} h_{t+1} dF_t(h) \left/ \int N_{t+1} dF_t(h) \right.
$$

(21)

\[ \frac{\Delta h^{\text{INT}}_{t+1}}{h^{\text{BTO}}_{t+1}} \equiv \frac{h^{\text{INT}}_{t+1} - h^{\text{BTO}}_{t+1}}{h^{\text{BTO}}_{t+1}} = \left( \frac{\bar{\theta}}{\theta_{\text{min}}} \right)^{T} - 1 \]
Given the fractions of $A$-type and $B$-type individuals in the society in period $t$ and distinguishing $A$-type parents with respect to their human capital levels, the average level of the society's human capital in period $t+1$ in the case of integration ($\bar{h}^{\text{INT}}_{t+1}$) is

$$
\bar{h}^{\text{INT}}_{t+1} = \frac{(1 - b_t)(\int_{h > \hat{h}} N_t h_{t+1} dF_t(h^A) + \int_{h \leq \hat{h}} N_t h_{t+1} dF_t(h^B)) + b_t \int N_t h_{t+1} dF_t(h^B)}{(1 - b_t)(\int_{h > \hat{h}} N_t dF_t(h^A) + \int_{h \leq \hat{h}} N_t dF_t(h^A)) + b_t \int N_t dF_t(h^B)}.
$$

(22)

Correspondingly, in the absence of integration, the average level of human capital ($\bar{h}^\text{NI}_{t+1}$) is

$$
\bar{h}^\text{NI}_{t+1} = \frac{(1 - b_t)(\int N_t h_{t+1} dF_t(h^A) + b_t \int N_t h_{t+1} dF_t(h^B))}{(1 - b_t)(\int N_t dF_t(h^A) + b_t \int N_t dF_t(h^B))}.
$$

(23)

Given the number of children and the levels of human capital investment among the two types of agents, as determined in Sections 2.4 and 2.6, the human capital levels in period $t+1$ in both cases are respectively

$$
\bar{h}^\text{INT}_{t+1} = \left(\frac{\gamma}{1 - \gamma}\right)^\gamma \left[(1 - b_t)\left(\int_{h > \hat{h}} \delta^{-1} h_j^{\gamma} dF_i(h^A) + \int_{h \leq \hat{h}} \left(\delta - \frac{\alpha}{h_t}\right) \left(\theta(h_t - \alpha)\right)^\gamma dF_i(h^A)\right) + b_t \int \left(\delta - \frac{\alpha}{h_t}\right) \left(\theta(h_t - \alpha)\right)^\gamma dF_i(h^B)\right]
$$

(24)

$$
\left[\int_{h > \hat{h}} \delta^{-1} dF_i(h^A) + \int_{h \leq \hat{h}} \left(\delta - \frac{\alpha}{h_t}\right) dF_i(h^A) + b_t \int \left(\delta - \frac{\alpha}{h_t}\right) dF_i(h^B)\right]
$$

and
\[
\bar{h}_{t+1}^{NI} = \left( \frac{\gamma}{1 - \gamma} \right)^{\gamma} \left[ (1 - b_t) \int \left( \delta - \frac{\alpha}{h_{t}} \right) \left( \theta (\bar{h}_{t}^{NI} - \alpha) \right) dF_t(h^A) + b_t \int \left( \delta - \frac{\alpha}{h_{t}} \right) \left( \bar{\theta} (\bar{h}_{t}^{NI} - \alpha) \right) dF_t(h^B) \right] \\
\left[ (1 - b_t) \int \left( \delta - \frac{\alpha}{h_{t}} \right) dF_t(h^A) + b_t \int \left( \delta - \frac{\alpha}{h_{t}} \right) dF_t(h^B) \right].
\]

(25)

Comparing the level of human capital in the case of integration (\(\bar{h}_{t+1}^{INT}\)) to that in the absence of integration (\(\bar{h}_{t+1}^{NI}\)), as shown above in equations (24) and (25), respectively, allows us to determine precisely whether the integration increases or decreases the average level of human capital in the society. Thus, if \(\bar{h}_{t+1}^{INT} > \bar{h}_{t+1}^{NI}\), the integration in public education increases the society's average level of human capital. In contrast, if \(\bar{h}_{t+1}^{NI} > \bar{h}_{t+1}^{INT}\), the integration policy decreases the average level of human capital in society.

3. Conclusion

This article analyzes the consequences of integration in public education. I have used a standard model with endogenous fertility to show that the flight from integrated multicultural public schools to private education increases private educational expenditures and, as a result, decreases fertility among more affluent parents whose children flee. In contrast, among less prosperous parents, who cannot afford private education for their offspring, integration in public education decreases their children’s human capital levels. I also demonstrate that the poor, who cannot afford to opt out, incur greater costs than the rich, who can resort to private education. Moreover, the human-capital losses of the offspring of the poor who remain in integrated public schools coupled with the per-capita human-capital gains of the offspring of the rich who opt out may have long-lasting consequences for the persistence of inequality in the future. I also analyze the overall society-wide effect of the integration policy and derive a condition that determines precisely whether this policy increases or decreases the average level of human capital in society.
References

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