Schwarz, Christian; Suedekum, Jens

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Global sourcing of complex production processes

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Global Sourcing of Complex Production Processes

Christian Schwarz
Jens Suedekum

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Global Sourcing of Complex Production Processes

Abstract

We develop a theory of a firm in an incomplete contracts environment which decides on its complexity, organization, and global scale. Specifically, the firm decides i) how thinly it wants to slice its production process by choosing the mass of symmetric intermediate inputs that are simultaneously combined to a final product, ii) if the supplier of each component is an external contractor or an integrated affiliate, and iii) if that component is offshored to a foreign country. We also consider the case of asymmetric inputs. Our model leads to a rich set of novel predictions about the structure of multinational firms that are consistent with stylized facts from the recent empirical literature.

JEL-Code: F120, D230, L230.

Keywords: multinational firms, outsourcing, intra-firm trade, offshoring, vertical FDI.

Christian Schwarz
Mercator School of Management
University of Duisburg-Essen
Lotharstrasse 65
Germany – 47057 Duisburg
christian.schwarz@uni-due.de

Jens Suedekum
Mercator School of Management
University of Duisburg-Essen
Lotharstrasse 65
Germany – 47057 Duisburg
jens.suedekum@uni-due.de

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1 Introduction

Most goods require intermediate inputs, and how thinly the production process for a particular final product is “sliced” is a choice made by firms. Some choose a setting with multiple highly specialized suppliers with very narrowly defined tasks, while other firms from the same industry rely on a lower division of labor with fewer suppliers who provide broader inputs. Table 1 illustrates this with an example from the automotive sector: Ford procures the entire door module for the Fiesta from a single supplier (Faurecia), while Volkswagen collaborates with nine different suppliers who manufacture specific parts of the doors for the Golf VI (like the handle and the hinges) which are then combined in the final assembly of the car. The Volvo XC90 ranges in between, with the door module sliced up into six parts each of which comes from a different supplier.

Table 1: Slicing of the production process in car manufacturing - an example

<table>
<thead>
<tr>
<th>Ford Fiesta</th>
<th>Volvo XC90</th>
<th>Volkswagen Golf VI</th>
</tr>
</thead>
<tbody>
<tr>
<td>modules (Faurecia)</td>
<td>window regulators (Brose)</td>
<td>carcasse modules (Arvin Meritor, Brose)</td>
</tr>
<tr>
<td>lockset (HuF)</td>
<td>control units (Brose)</td>
<td>brackets (Brose, Röchling Automotive)</td>
</tr>
<tr>
<td>glazing (Pilkington)</td>
<td></td>
<td>panels (Röchling Automotive)</td>
</tr>
<tr>
<td>seals (Cooper)</td>
<td></td>
<td>side panels (Peguform)</td>
</tr>
<tr>
<td>lock seals (Polymere)</td>
<td></td>
<td>attaching parts for panels (Polytec Group)</td>
</tr>
<tr>
<td>hinges (Edscha)</td>
<td></td>
<td>foam film side armrest (Benecke-Kaliko)</td>
</tr>
</tbody>
</table>


For each component of the final product, a firm then needs to decide whether to obtain it from a supplier who is integrated into the firm’s boundaries, or to outsource it to an external contractor. As is well known since Grossman and Hart (1986) and Hart and Moore (1990), these organizational decisions (“make or buy”) matter in an environment with incomplete contracts, as they affect the suppliers’ incentives to make relationship-specific investments. Finally, in a globalized world, firms also need to decide on the international scale of their sourcing strategy. Some source only domestically, while others collaborate with foreign suppliers either at arm’s length or through intra-firm trade (Grossman and Helpman, 2002). An example that illustrates those dimensions is the “Swedish” Volvo S40. A substantial
share of the inputs for this car is produced by independent foreign suppliers (e.g., the navigation control by Japanese, the side mirror and fuel tank by German, the headlights by American contractors, etc.), while the airbag and the seats are outsourced domestically within Sweden. Yet other inputs are manufactured inhouse. Of those tasks, some are performed within the Swedish parent plants, while other components are manufactured by foreign subsidiaries of Volvo.¹

In this paper, we develop a theory of a firm which decides on the complexity, the organization, and the global scale of its production process. We build on the seminal approach by Antràs and Helpman (2004), who were the first to study global sourcing decisions under incomplete contracts. Their model is restricted to a setting with a headquarter and one single supplier. We consider multiple suppliers. Our model leads to a rich set of novel predictions about the structure of multinational enterprises (MNEs) that are consistent with stylized facts from the recent empirical literature.

We first consider a scenario where the headquarter (the “producer”) decides on the mass of differentiated but symmetric intermediate inputs that are simultaneously combined to a final product. Each input is provided by a separate supplier. We refer to this endogenous mass of inputs, which is equivalent to the mass of suppliers that the firm deals with, as the level of complexity of the production process.² The more suppliers there are, the more specialized is the task that every single supplier performs. Similar as in Acemoglu et al. (2007), this specialization leads to efficiency gains, but it also necessitates contracting with more parties. As we show below, this dilutes the investment incentives of every single supplier, and it endogenously leads to higher fixed costs for the firm. The producer furthermore decides, separately for each component, if the respective supplier is an external contractor or an integrated affiliate, and if it is offshored to a (low-cost) foreign country. Afterwards, we turn to a related scenario where the producer contracts with a given number of two suppliers who provide asymmetric components that may differ in their input intensities, unit costs, and their degree of “sophistication.”

¹See Baldwin (2009) for a further discussion of this example. Other cases include Nike, which relies heavily on foreign outsourcing, or Intel which mainly engages in vertical foreign direct investment (FDI), see Antràs and Rossi-Hansberg (2009).

²In this terminology, the making of the doors for the Golf VI is a more complex production process than for the Ford Fiesta, because it involves more and more specialized suppliers (see Table 1). We should emphasize that we consider horizontal slicing, i.e., the components enter simultaneously into the production process. We do not consider a sequential setup as in Antràs and Chor (2011) or Costinot et al. (2011) with subsequent stages where intermediate inputs are added and the final product is refined in each stage. Relatedly, we also do not consider possible sourcing decisions of suppliers. That is, we are not interested in whether component manufacturers like Faurecia rely themselves on intermediate inputs, but we focus on the firm structure of the final goods producer.
Our model firstly predicts that firms differ in the complexity of their production processes, both within and across industries. Higher productivity and lower headquarter-intensity tend to increase the mass of suppliers that a firm chooses to contract with. Second, firms may outsource some of their suppliers but vertically integrate others. This “hybrid” sourcing mode is prevalent in firms with medium-to-high productivity from sectors with low-to-medium headquarter-intensity. Third, firms may decide to offshore only some components, and this offshoring share tends to be higher in more productive firms and in less headquarter-intensive industries. Importantly, the possibility to engage in offshoring boosts the slicing of the production process, and it is positively correlated with outsourcing. That is, the same firm chooses more suppliers and a higher outsourcing share in an open economy than in a closed economy context. Finally, the model with two asymmetric inputs predicts that the supplier who provides the component with the higher input intensity and the lower unit costs tends to be outsourced, while the supplier with the more sophisticated input which requires more specific knowledge is likely to be kept within firm boundaries.

The predictions of our model are then discussed in the light of the recent empirical literature on multinational firms. That literature has started to carefully explore the internal structure of MNEs, and also to test particular aspects of the baseline model by Antràs and Helpman (2004). Several predictions are supported by the empirical evidence. Other features of the data are harder to understand with this baseline framework, however, or with other theoretical models about the structure of MNEs. For example, Kohler and Smolka (2009), Jabbour (2008) and Jabbour and Kneller (2010) show that most MNEs collaborate with many suppliers and often choose different sourcing modes for different inputs – as in the Volvo S40 example discussed above. In particular, Tomiura (2007) finds that firms which outsource some suppliers while keeping others integrated tend to be more productive than firms which rely on a single sourcing mode in the global economy.

3Du, Lu and Tao (2009) consider an extension of Antràs and Helpman (2004) where the same input can be provided by two suppliers. “Bi-sourcing” (one supplier integrated and the other outsourced) can arise in their model out of a strategic motive, because it systematically improves the headquarter’s outside option and thus its bargaining power. In our model there is an endogenous mass of suppliers who provide differentiated inputs, and our hybrid sourcing result relies on a different, non-strategic motive.

4Consistent with Antràs and Helpman (2004), the study by Nunn and Trefler (2008) finds that intra-firm trade is most pervasive for highly productive firms in headquarter-intensive sectors, and Defever and Toubal (2007) find that highly productive firms tend to choose foreign outsourcing for components with high input intensity. Consistent with the extension in Antràs and Helpman (2008), who consider partial contractibility and cross-country differences in contracting institutions, the study by Corcos et al. (2009) finds that firms are more likely to offshore in countries with good contracting institutions, and Bernard et al. (2010) report that institutional improvements favor foreign outsourcing. The studies by Feenstra and Hanson (2005), Yeaple (2006), Marin (2006), and Federico (2010), among others, are also concerned with the internal structure of MNEs and obtain empirical findings broadly in line with those baseline models.
Our framework is able to account for those stylized facts, since it allows for multiple suppliers. It also delivers new and empirically relevant results compared to Acemoglu et al. (2007) who focus on a closed economy setting. In particular, although they study the organizational structure of firms with an endogenous mass of intermediate inputs, they do not allow for sectoral differences in headquarter-intensity and cannot generate “hybrid sourcing” where a firm has some integrated and some outsourced suppliers. Turning to the global scale dimension, the framework by Grossman and Rossi-Hansberg (2008) predicts quite naturally that firms may offshore only some but not all inputs. Yet, that approach neglects the repercussions with the complexity and organization decisions that we emphasize in our model. Finally, Alfaro and Charlton (2009) show that firms tend to outsource low-skill inputs from the early stages, while keeping high-skill inputs from the final stages of the production process inside the firm boundaries. Corcos et al. (2009) consistently find that inputs with a higher degree of specificity are less likely to be outsourced. Our model with two asymmetric inputs is consistent with those facts as it predicts that firms tend to keep the more sophisticated input inhouse.

The rest of this paper is organized as follows. In Section 2 we present the basic structure of our model. Section 3 is devoted to the scenario with an endogenous mass of symmetric components, while Section 4 looks at the case with two asymmetric inputs. In Section 5 we conclude and contrast the predictions of our model with stylized facts, and we discuss further testable predictions in order to motivate future empirical research.

2 Model

2.1 Demand and technology

We consider a firm that produces a final good $y$ for which it faces the following iso-elastic demand function:

$$y = Y \cdot p^{1/(\alpha - 1)}.$$  

(1)

The variable $p$ denotes the price of this good, and $Y > 1$ is a demand shifter. The demand elasticity is given by $1/(1 - \alpha)$ and is increasing in the parameter $\alpha \in (0, 1)$. Producing this good requires headquarter services and manufacturing components, which are combined according to the following Cobb-Douglas production function:

$$y = \theta \cdot \left( \frac{h}{\eta^H} \right)^{\eta^H} \cdot \left( \frac{M}{1 - \eta^H} \right)^{1-\eta^H},$$  

(2)
where $\theta > 0$ is a productivity shifter; the larger $\theta$ is, the more productive is the firm.

Headquarter services are denoted by $h$ and are provided by the “producer”. The parameter $\eta^H \in (0, 1)$ is the headquarter-intensity, so that $\eta^M = 1 - \eta^H$ is the overall component-intensity of production. This parameter $\eta^H$ is exogenously given and reflects the technology of the sector in which the firm operates.\(^5\)

There is a continuum of manufacturing components, with measure $N \in \mathbb{R}_+$. Each component is provided by a separate supplier. The supplier $i \in [0, N]$ delivers $m_i$ units of its particular input, and the aggregate component input $M$ is given by the following constant elasticity of substitution (CES) function:

$$
M = \left[ \int_0^N \eta_i \left( \frac{m_i}{\eta_i} \right)^\epsilon \, di \right]^{1/\epsilon}.
$$

The parameter $\epsilon \in (0, 1)$ determines the degree of substitutability of the single components, and the elasticity of substitution, $1/(1 - \epsilon)$, is always above unity. The parameter $\eta_i$ reflects the intensity of component $i$ within the aggregate $M$, with $\int_0^N \eta_i \, dj = 1$.\(^6\) Using equations (1), (2) and (3), total firm revenue can be written as follows:

$$
R = \theta^\alpha \cdot Y^{(1-\alpha)} \cdot \left[ \left( \frac{h}{\eta^H} \right)^{\eta^H} \cdot \left( \frac{M}{\eta^M} \right)^{\eta^M} \right]^\alpha.
$$

### 2.2 Firm structure

The producer decides on the structure of the firm, and this choice involves three aspects: i) complexity, ii) organization, and iii) global scale of production. Complexity refers to the mass of components that simultaneously combined in the production process. If the producer chooses “low” complexity, she relies on a setting with relatively few and broad components with a high average input intensity. An increase in complexity lowers the average input intensity across the single components at constant overall component-intensity $\eta^M$. The inputs then become more specialized. To give an example, a car producer may choose to obtain the complete coachwork from a single supplier, or she may choose to obtain different parts of it (like the doors and the hood) from different suppliers.

---

\(^5\)For example, $\eta^M$ is higher in the automobile than, say, in the software industry. The headquarter services thus account for a fixed share $\eta^H$ of total value added and necessarily have to be performed by the producer herself, i.e., they cannot be unbundled, outsourced or offshored.

\(^6\)If all components are symmetric, as will be assumed in Section 3, then $\eta_i = 1/N$. Each component then has an individual input intensity equal to $\eta^M \cdot \eta_i = (1 - \eta^H)/N$. 
Secondly, turning to the organizational decision, the producer decides separately for each of those components if the respective supplier is integrated as a subsidiary within the boundaries of the firm, or if that component is outsourced to an external supplier. The crucial assumption is that the investments for all inputs are not contractible, as in Antràs and Helpman (2004). This may be due to the fact that the precise characteristics of the inputs are difficult to specify *ex ante* and also difficult to verify *ex post*. As a result of this contract incompleteness, the producer and the suppliers end up in a bargaining situation, at a time when their input investment costs are already sunk. Following the property rights approach of the firm, see Grossman and Hart (1986) or Hart and Moore (1990), we assume that bargaining also takes place within the boundaries of the firm in the case of vertical integration. The bargaining power of the involved parties depends crucially on the firm structure as will be explained below.

Finally, the producer decides on the location where each component is manufactured. The headquarter itself is located in a high-wage country 1, where final assembly of good \( y \) is carried out. Both under outsourcing and vertical integration, the respective input suppliers may either also come from country 1, or from a foreign low-wage country 2. There is an arm’s length relationship if the producer outsources a component to a foreign contractor, and intra-firm trade (vertical FDI) if a foreign supplier is vertically integrated.

### 2.3 Structure of the game

We consider a game that consists of five stages. The timing of events is as follows:

1. The producer simultaneously decides on: i) the complexity, ii) the organization, and iii) the global scale of the production process. In particular, i) she chooses the mass \( N \) of manufacturing components. ii) For each \( i \in [0, N] \) she chooses the organizational form \( \Xi_i \in \{O, V\} \). Here, \( \Xi_i = O \) denotes “outsourcing” and \( \Xi_i = V \) denotes “vertical integration” of supplier \( i \). We order the mass \( N \) such that each supplier \( j \in [0, N^O] \) is outsourced, and each supplier \( k \in (N^O, N] \) is vertically integrated. Then, \( \xi = N^O/N \) (with \( 0 \leq \xi \leq 1 \)) denotes the outsourcing share, and \( (1 - \xi) = N^V/N \) is the share of vertically integrated suppliers/components. Finally, iii) for each \( i \in [0, N] \) the producer decides on the country \( r = \{1, 2\} \) where that component is manufactured. We order the mass of outsourced suppliers \( N^O \) such that each supplier \( j \in [0, N^O_2] \) is offshored to the low-wage country 2, and each supplier \( k \in (N^O_2, N^O] \) is located in the high-wage country 1. Then, \( \ell^O = N^O_2/N^O \) denotes the offshoring share among all
outsourced suppliers (with \( 0 \leq \ell^O \leq 1 \)). Similarly, \( \ell^V = N^V_2 / N^V \) (with \( 0 \leq \ell^V \leq 1 \)) is the offshoring share among all integrated suppliers, and the total offshoring share of the firm is given by \( \ell = \xi \cdot \ell^O + (1 - \xi) \cdot \ell^V \).  

2. Given the firm structure decisions \( \{ N, \xi, \ell^O, \ell^V \} \), the producer offers a contract to potential input suppliers for every component \( i \in [0, N] \). This contract includes an upfront payment \( \tau_i \) (positive or negative) to be paid by the prospective supplier.

3. There exists a large pool of potential applicant suppliers for each manufacturing component in both countries. These suppliers have an outside opportunity equal to \( w^M_r \) in country \( r = \{1, 2\} \). They are willing to accept the contract if their payoff is at least equal to \( w^M_r \). The payoff consists of the upfront payment \( \tau_i \) and the revenue share \( \beta_i \) that supplier \( i \) anticipates to receive at the bargaining stage, minus the investment costs (which may differ across applicants). Potential suppliers apply for the contract, and the producer chooses one supplier for each component \( i \in [0, N] \).

4. The producer and the suppliers independently decide on their non-contractible input levels for the headquarter service \((h)\) and the components \((m_i)\), respectively.

5. Output is produced and revenue is realized according to (2), (3), and (4). The surplus value is divided between the producer and the suppliers.

Starting with stage 5, the surplus value that has to be divided among the \( N + 1 \) agents is the total revenue \( R \) as given in (4). With \( \beta_i \) we denote the revenue share of component supplier \( i \), and \( \beta^M = \int_0^N \beta_j \, dj \) is the joint revenue share of all component suppliers. The revenue share realized by the producer is written as \( \beta^H \), and we have \( \beta^H + \beta^M = 1 \). We study two different scenarios how the surplus value is divided. First, there is a benchmark scenario where the producer is able to freely decide on the division of revenue (see Section 3.1.1.). Notice that this free division of the surplus does not resolve the hold-up and hence the underinvestment problem inherent in this game structure. In the second scenario (see Section 3.1.2.) we follow Grossman and Hart (1986) and Antràs and Helpman (2004) and assume that the producer cannot freely specify the division of revenue. She rather has to decide on the structure of the firm in order to affect the revenue distribution, as this pins down the bargaining power of the involved agents. In that scenario we assume a simultaneous multilateral bargaining setting and use the Shapley value as the solution concept, similar as in Acemoglu et al. (2007) or in Hart and Moore (1990). The details of the revenue division are analyzed later.
In stage 4, anticipating $\beta_i$, each component supplier $i \in [0, N]$ chooses $m_i$ so as to maximize $\beta_i R - c_{i,r}^M m_i$, where $c_{i,r}^M$ denotes the unit cost level of the supplier for component $i$ that the producer has offered the contract. The producer chooses $h$ in order to maximize $\beta^H R - c^H h$, where $c^H$ denotes the unit cost of providing headquarter services. We show in Appendix A.1. that the agents choose the following levels of input provision:

$$h = \alpha \cdot \eta^H \cdot \left( \frac{\beta^H}{c^H} \right) \cdot R \quad \text{and} \quad m_i = \alpha \cdot \eta^M_i \cdot \int_0^N \eta_i \left( \frac{\beta_i}{\eta^M_i} \right) \frac{1}{1-\epsilon} \, d j .$$

(5)

Using (5) in (4), the total revenue given those input provisions can then be written as:

$$R = \Theta \left[ \left( \frac{\beta^H}{c^H} \right)^{\eta^H} \cdot \int_0^N \eta_j \left( \frac{\beta_j}{\eta^M_j} \right) \frac{1}{1-\epsilon} \, d j \right]^{\frac{1}{1-\epsilon}} ,$$

(6)

where $\Theta \equiv Y \cdot (\alpha \theta)^{\alpha/(1-\alpha)}$ is an alternative productivity measure. Everything else equal, $h$ is increasing in $\beta^H$. An increase of $\beta^H$ lowers the remaining share $\beta^M$ that can be distributed among the suppliers, however, and thereby tends to exacerbate their underinvestment problems. The producer thus faces a trade-off between revenue share and level.

Next, in order to receive applications for each desired component input in stage 3, the producer must offer contracts in stage 2 that satisfy the suppliers’ participation constraints. For supplier $i$ this implies that the payoff from forming the relationship, given (5) and (6), must at least match the outside opportunity:

$$\beta_i R - c_{i,r}^M m_i + \tau_i \geq w_i^M .$$

(7)

In stage 1, the producer then chooses the structure of the firm so as to maximize her individual payoff, $\beta^H R - c^H h - \int_0^N \tau_j \, d j$, subject to the incentive compatibility constraints (5) and (6), and the participation constraints (7). Since the producer can freely adjust the upfront payments $\tau_i$, these participation constraints are satisfied with equality for all suppliers $i \in [0, N]$. Rearranging $\tau_i = w_i^M - \beta_i R + c_{i,r}^M m_i$, substituting this into the individual payoff of the producer, and recalling that $\beta^M = 1 - \beta^H$, it follows that the producer’s problem is equivalent to maximizing the total payoff for all $N + 1$ involved parties, i.e.: $\pi = R - c^H h - \int_0^N c_{j,r}^M m_j \, d j - F$, where $h$, $m_j$ and $R$ are given in (5) and (6). The term $F$ denotes the “fixed costs” of production, which consist of an exogenous overhead.
cost $\bar{f}$ and of the outside opportunities aggregated across all suppliers, $F = \bar{f} + \int_0^N w_j^M dj$. Notice that $F$ is increasing in $N$ as long as $w_j^M > 0$, i.e., the participation constraints generate a fixed cost that is endogenously increasing in complexity, as this necessitates contracting with more suppliers.

3 Symmetric components

In this section we consider the case of symmetric components. We first abstract from the global scale dimension, and focus on the complexity and organization decisions when all suppliers are located in country 1.

3.1 Closed economy

When all components that are part of the production process are technologically equally important, we have $\eta_i = 1/N$ so that the individual input intensity of each component is given by $\eta_i = (1 - \eta^H)/N$. Furthermore, all suppliers have the same unit costs and the same outside opportunities, $c_i^M = c^M$ and $w_i^M = w_i^M$ for all $i \in [0, N]$.

Notice that an increase in the complexity level $N$ is associated with a uniform reduction in the individual input intensities of all suppliers. Economically, if the producer chooses to collaborate with more suppliers, each individual supplier performs a more narrowly defined task. We assume that this specialization leads to efficiency gains, similar as in Acemoglu et al. (2007). Specifically, we assume that the unit costs for the suppliers are described by

$$c^M = c \cdot N^{-s}, \quad \text{with} \quad s \in (0, 1).$$

The cost savings from specialization are thus more substantial the larger $s$ is. Without loss of generality, we normalize the parameter $c$ to unity ($c = 1$).

3.1.1 Optimal mass of suppliers and revenue division

We now first study the benchmark scenario where the producer can freely decide on the division of revenue subject to $\beta^H + \beta^M = 1$. In that case, each supplier receives a revenue share $\beta_i = (1 - \beta^H)/N$ due to symmetry. The resulting input provision levels from (5) and (6) simplify and now read as

$$h = \alpha \cdot \eta^H \cdot \left(\frac{\beta^H}{c^H}\right) \cdot R \quad \text{and} \quad m = \frac{M}{N} = \alpha \cdot \left(\frac{1 - \eta^H}{N}\right) \cdot \left(\frac{1 - \beta^H}{N^{1-s}}\right) \cdot R.$$
The total revenue given those input provision levels can now be written as:

\[ R = \Theta \left( \frac{\beta^H}{c^H} \right)^{\eta^H} \left( \frac{1 - \beta^H}{N^{1-s}} \right)^{1-\eta^H} \left( \frac{1 - \beta^H}{N^{1-s}} \right)^{1-\eta^H} \]  

(9)

Finally, the firm’s total payoff given (8) and (9) is

\[ \pi = R - c^H h - c^M N m - F = \Theta \Psi - N w_1 - \bar{f}, \]

with the variable payoff given by

\[ \Theta \Psi = \Theta \left[ \frac{1 - \alpha}{N^\alpha} \left( \frac{1 - \eta^H}{N^\eta^H} \left( 1 + \alpha \eta^H \right) \right) \left( \frac{1 - \beta^H}{N^{1-s}} \right)^{1-\eta^H} \right]. \]

(10)

a) **Zero outside opportunity.** When setting the suppliers’ outside opportunities to zero \((w_1^M = 0)\), the producer’s problem is equivalent to maximizing the variable payoff as given in (10) simultaneously with respect to \(N\) and \(\beta^H\). We can derive the following unique solution for this maximization problem (see Appendix A.2.1.i):

\[ N^* (w_1^M = 0) = \frac{\rho - s \left( 1 - \eta^H \right) \left( 1 + \alpha \eta^H \right)}{2 \left( 1 - s \right) \eta^H} \equiv N^*_0, \]  

(11)

\[ \beta^{H*} (w_1^M = 0) = \frac{2 \eta^H - \rho + s \left( 1 - \eta^H \right) \left( 1 - \alpha \eta^H \right)}{2 \eta^H} \equiv \beta^H_0, \]  

(12)

with \(\rho = \sqrt{s \left( 1 - \eta^H \right) \left( 1 - \alpha \eta^H \right) (4 \eta^H + s \left( 1 - \eta^H \right) \left( 1 - \alpha \eta^H \right))}\). We have \(0 < \beta^H_0 < 1\) and \(N^*_0 > 0\) for all \(\{\alpha, \eta^H, s\} \in (0, 1)\). Using (11) and (12) we can state

**Proposition 1:** Firms from more headquarter-intensive industries (higher \(\eta^H\)) have a lower optimal mass of suppliers \(N^*_0\) and a higher optimal headquarter revenue share \(\beta^H_0\). A stronger cost saving effect (higher \(s\)) leads to a higher \(N^*_0\) and to a lower \(\beta^H_0\).

The intuition for the result \(\partial \beta^H_0 / \partial \eta^H > 0\) is similar as in Antràs and Helpman (2004, 2008): due to the hold up problem, both the headquarter and the suppliers underinvest in the provision of their respective inputs, and this underinvestment problem is more severe for the headquarter (the mass of suppliers) the smaller (the larger) the revenue share \(\beta^H\).

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7 Notice that neither the input levels \(h\) and \(m\) from (8), nor the firm’s revenue and payoff from (9) and (10) depend on the parameter \(\epsilon\), i.e., on the degree of substitutability across components. This is due to the fact that the production function in (2) and (3) features no aggregate gains from component variety, since we have \(M = N \cdot m\) with symmetrical inputs.
is. Ensuring ex ante efficiency requires that the producer must receive a larger share of the surplus in sectors where headquarter services are more intensively used in production.

The basic trade-off with respect to the complexity choice is novel in our framework. It can be seen from (10) that the impact of \( N \) on \( \Theta \Psi \) is, a priori, ambiguous. Intuitively, higher complexity leads to stronger specialization (i.e., lower unit costs \( c^M \)), which tends to increase the firm’s revenue and payoff. On the other hand, for a given share \( \beta^H \), higher complexity also “dilutes” the investment incentives for every single supplier, because the individual input intensities decrease and the overall revenue share \( \beta^M = 1 - \beta^H \) has to be split among more parties. This negatively impacts on the suppliers’ incentives and on the firm’s payoff. The optimal choice \( N^*_0 \) balances the “cost saving” and the “dilution” effect.

Why do firms from more headquarter-intensive industries have a lower optimal mass of suppliers? The intuition for the result \( \partial N^*_0 / \partial \eta^H < 0 \) is that the optimal joint revenue share for the suppliers, \( \beta^{M*} = 1 - \beta^{H*} \), is decreasing in \( \eta^H \). This jeopardizes the suppliers’ investment incentives. To counteract this problem, the producer can concentrate on relatively few components with a high individual input intensity. Although the gains from specialization are smaller in that case, the resulting increases of \( \beta_i \) and \( \eta_i \) again raise the suppliers’ incentives. It is, thus, not clear if the optimal revenue share of a single supplier (\( \beta^{i*}_0 \)) is increasing or decreasing in \( \eta^H \); there is a larger joint revenue share \( \beta^M \) when \( \eta^H \) is low (“component-intensity effect”), but this share is then split among many suppliers (“complexity effect”). Using (11) and (12), it can be shown that \( \beta^{i*}_0 = (1 - \beta^{H*}_0)/N^*_0 \) is in fact hump-shaped over the range of \( \eta^H \) (see Appendix A.2.1.ii). In other words, single suppliers receive the highest revenue shares in sectors with medium headquarter-intensity.

The stronger the cost savings from specialization are, the more profitable is it to add components to the production process (\( \partial N^*_0 / \partial s > 0 \)). This increase in complexity is then accompanied by a decrease in the optimal revenue share, since the incentives for all component manufacturers must be maintained (\( \partial \beta^{H*}_0 / \partial s < 0 \)).\(^8\) When \( s \) becomes very small, so does \( N^*_0 \). Intuitively, the “cost saving” effect disappears if \( s \) tends to zero. The “dilution effect” for the suppliers is still present, however, so that the optimal mass of components would then also become very small. Notice that this is true even though

\(^8\)We show in Appendix A.2.1.iii that \( N^*_0 = 1 \) if \( s = s_{crit} \). Suppose the set of suppliers \( N \) is discrete, by assuming that the mass of inputs on the unit interval \([0,1]\) is provided by a single supplier. In fact, if \( s = s_{crit} \), choosing a unit mass of inputs is optimal for the producer. The corresponding \( \beta^{H*}_0 (s = s_{crit}) \) in that case is identical to eq. (10) in Antrás and Helpman (2004), where it is exogenously imposed that there is one single manufacturing component provided by a single supplier. Their baseline model is thus included in our framework as a special case. When \( s \) is smaller (larger) than \( s_{crit} \), it is optimal to have less (more) than a unit mass of inputs.
contracting with more suppliers leads to no increase in fixed costs as long as \( w_1^M = 0 \).

Finally, notice that (11) and (12) for the case of zero outside opportunities do not depend on \( \Theta \). Still, a firm needs to be sufficiently productive in order to operate in the market, since the variable payoff must be large enough to cover the overhead costs \( \bar{f} \).

b) Positive outside opportunity. Turning to the case with \( w_1^M > 0 \), recall that there is an additional endogenous “complexity penalty” embedded in our model, since more suppliers lead to a larger fixed costs \( N \cdot w_1^M \).

With \( w_1^M > 0 \), we cannot explicitly solve for \( N^* \) and \( \beta^H^* \). However, using the two first-order conditions for payoff maximization, it is possible to solve \( \partial \pi / \partial \beta^H = 0 \) for \( \beta^H(N) \) with \( \partial \beta^H / \partial N < 0 \), which does not depend on \( w_1^M \) (see Appendix A.2.2.i). Substituting this into the other first-order condition allows us to derive the following function:

\[
\frac{\partial \pi}{\partial N} = \Theta \cdot \left. \frac{\partial \Psi}{\partial N} \right|_{\beta^H = \beta^H(N)} - w_1^M = 0 \iff \Psi' = \frac{w_1^M}{\Theta}. \tag{13}
\]

\( \Psi' \) only depends on \( N \) (and on parameters) and represents the marginal change in the total payoff when raising complexity, taking into account that \( \beta^H(N) \) is optimally adjusted. We know that \( \Psi' = 0 \) is solved by \( N^*_0 \) as given in (12). With \( w_1^M > 0 \), the optimal mass of producers \( N^* \) is determined by setting \( \Psi' \) equal to \( \frac{w_1^M}{\Theta} > 0 \), and since \( \partial \Psi' / \partial N < 0 \) it follows directly that \( 0 < N^* < N^*_0 \) with \( \partial N^* / \partial \Theta > 0 \) and \( \partial N^* / \partial w_1^M < 0 \).

![Figure 1: Optimal complexity with \((N^*)\) and without \((N^*_0)\) increasing fixed costs.](image)

The downward-sloping thick curve in Figure 1 illustrates the function \( \Psi' \). The optimal mass of suppliers is where this curve cuts the horizontal line. An increase of \( w_1^M \) leads to an upward shift, and an increase of \( \Theta \) to a downward shift of this horizontal line. For
given values of $w_1^M$ and $\eta^H$, more productive firms thus collaborate with more suppliers since they can easier cope with the requirement to match their outside opportunities. Still, the complexity choice always remains below $N_0^*$, i.e., $N^*$ is bounded. Furthermore, the $\Psi'$-curve shifts to the left as $\eta^H$ increases. Hence, when comparing equally productive firms from different sectors, those from headquarter-intensive industries have a lower optimal complexity than those from component-intensive industries.

In Figure 2 we illustrate the corresponding optimal headquarter revenue share. The figure firstly depicts the $\beta^*_H$-curve for the benchmark case with $w_1^M = 0$. Since we know from the first-order conditions that $\partial \beta^H / \partial N < 0$, it is clear that the $\beta^*_H$-curve stretches out to the left if $w_1^M > 0$. This implies a higher $\beta^*_H$ throughout the range of $\eta^H$: $0 < \beta_0^* < \beta^*_H < 1$ with $\partial \beta^*_H / \partial w_1^M > 0$. The reason is that an increase in $w_1^M$, by reducing the optimal complexity, leads to a higher individual input intensity $\eta_i = \eta^M / N$ for each supplier. This raises the suppliers’ incentives and thereby allows for a larger optimal revenue share $\beta^*_H$. Yet, this share is lower in firms with higher productivity, i.e., the firm-specific $\beta^*_H$-curve moves closer to the $\beta_0^*$-curve ($\partial \beta^*_H / \partial \Theta < 0$). The intuition is that more productive firms are also more complex, and to maintain the investment incentives, they need to leave a larger share $\beta^M$ for the suppliers.\(^9\)

\(^9\)A stronger cost saving effect $s$ naturally leads to more suppliers (a higher $N^*$) and, thus, to a lower $\beta^*_H$. Graphically, the $\Psi'$-curve in Figure 1 shifts to the right as $s$ increases. In the corresponding Figure 2, both the $\beta_0^*$- and the $\beta^*_H$-curve stretch out to the right. Furthermore, higher productivity implies a higher total payoff, despite the fact that more productive firms have more complex production processes and, thus, higher fixed costs. Higher productivity thus raises the variable payoff stronger than the fixed costs, as is shown in Appendix A.2.2.ii.
Summing up, the results for \( N^*_0 \) and \( \beta_H^* \) from Proposition 1 thus still apply for \( N^* \) and \( \beta_H^* \), and with \( w_1^M > 0 \) we can additionally state

**Proposition 2:** Within an industry, more productive firms (higher \( \Theta \)) have a larger optimal mass of suppliers \( N^* \) and a lower optimal revenue share for the headquarter \( \beta_H^* \). The higher is the suppliers’ outside opportunity \( w_1^M \) the lower is \( N^* \) and the higher is \( \beta_H^* \).

### 3.1.2 The make-or-buy decision under incomplete contracts

We now turn to the incomplete contracts scenario where the producer cannot freely decide on the division of the surplus. The producer now chooses, separately for each component, if the respective supplier is an integrated affiliate or an external (outsourced) contractor, as this affects the bargaining power of the involved agents and thereby the revenue division. In particular, following Grossman and Hart (1986), we show that external suppliers are in a better bargaining position than integrated suppliers vis-a-vis the producer. This is due to the fact that the producer has no ownership of the assets of external suppliers, while she does have residual control rights over the assets of those suppliers that are integrated within the boundaries of the firm.

Assume for the moment that a single outsourced supplier receives a revenue share \( \beta^O \) while a single integrated supplier receives \( \beta^V \), with \( \beta^O \neq \beta^V \). We will shortly derive explicit solutions for these revenue shares. This implies that suppliers are now potentially asymmetric along the organizational dimension, despite symmetric input intensities \( \eta_i = 1/N \), unit costs \( c_M = N^{-s} \), and outside opportunities \( w_1^M \). Using (5) and (6) we can derive the following input provision level for supplier \( i \):

\[
m^i = \alpha \cdot \left( \frac{1 - \eta^H}{N} \right) \cdot \left( \frac{1}{N^{-s}} \right) \cdot \left( \frac{(\beta^V)^{1-x}}{\xi \cdot (\beta^O)^{1-x} + (1 - \xi) \cdot (\beta^V)^{1-x}} \right) \cdot R, \tag{14}\]

It should be recalled that \( \Xi_i \in \{O,V\} \) denotes the organizational form of supplier \( i \), and \( \xi \in [0,1] \) is the outsourcing share of the firm. For the headquarter input we have \( h = \alpha \eta^H \beta^H R/c^H \), with total revenue now given by

\[
R = \Theta \left[ \frac{\beta^H}{c^H} \eta^H \left( \frac{\xi \cdot (\beta^O)^{1-x} + (1 - \xi) \cdot (\beta^V)^{1-x}}{N^{-s}} \right)^{\frac{1-x}{x}} \right]^{\frac{\alpha}{1-\alpha}}. \tag{15}\]
The total payoff is \( \pi = R - c^H h - N^{1-s} \cdot (\xi m^O + (1 - \xi) m^V) - F = \Theta \Psi - N w_1^M - \bar{f}, \) and the variable payoff now reads as

\[
\Theta \Psi = \left[ 1 - \alpha \left( \eta^H \beta^H + \eta^M \left( \frac{\xi \cdot (\beta^O)^{1/\gamma} + \eta^M \cdot (1 - \xi) \cdot (1 - \epsilon)}{\xi \cdot (\beta^O)^{1/\gamma} + \eta^M \cdot (1 - \xi) \cdot (1 - \epsilon)} \right) \right) \right] \cdot R
\]

The producer maximizes the total payoff \( \pi \) with respect to \( N \) and \( \xi \), taking into account that those choices affect the revenue distribution within the firm via the simultaneous multilateral bargaining process in the last stage of the game. We now discuss this bargaining stage where we use the Shapley value as the solution concept. Most formal derivations are deferred to Appendix A.3.1, but we provide here some basic intuition.

In a bargaining game with a finite number of players, a player’s Shapley value is the average of her contributions to all coalitions that consist of players ordered below her in all feasible permutations. Applying this general reasoning to our model, assume for the moment that there is a coalition between the headquarter and \( n \leq N \) symmetric suppliers. We show in Appendix A.3.1. that the joint revenue in this case can be written as

\[
R(n) = R \cdot \left( \frac{n}{N} \right)^\gamma \quad \text{with} \quad \gamma = \alpha(1 - \eta^H)/\epsilon.
\]

Now suppose that one of those supplier drops out of the coalition, so that the new joint revenue is given by \( R(n-1) \). The reduction in the joint revenue is smaller the higher \( N \) is. It is also smaller the lower \( \gamma \) is, i.e., the higher \( \eta^H \) or \( \epsilon \) are. That is, this supplier’s “marginal contribution” is lower when complexity is high, when headquarter services are intensively used in production, or when components are easily substitutable. We shall impose that \( \gamma > 1 \), i.e., that \( \epsilon < \alpha(1 - \eta^H) \). Economically, we thus restrict our attention to situations where single suppliers are sufficiently important, while ruling out those cases where components are both technologically unimportant and easy to substitute.

The difference between outsourced and integrated suppliers is that, if one integrated supplier drops out of a coalition, he cannot threaten to take away the full input provision level. We rather assume, following Antràs and Helpman (2004), that an internal supplier can only threaten to take away the fraction \( \delta \in (0, 1) \) of his input, while the rest stays with the producer owing to her residual control rights. An external supplier, on the other hand, can threaten to take away the entire input level. Following this logic, we derive in Appendix A.3.1. the asymptotic Shapley value of a single supplier in the case of outsourcing and

\[\text{Notice that this parameter restriction on } \epsilon \text{ is stricter in more headquarter-intensive industries. In fact, in the limit with } \eta^H \to 1 \text{ it requires that we contemplate the Cobb-Douglas case with } \epsilon \to 0.\]
introduction, respectively, which is equivalent to that supplier’s realized revenue share:

\[
\beta^O = \frac{\gamma}{(1 + \gamma) N} \quad \text{and} \quad \beta^V = \frac{\gamma}{(1 + \gamma) N} \delta.
\]

(18)

It follows from (18) that every supplier receives a lower revenue share the higher is the complexity level of the firm, the higher is the headquarter-intensity, or the better the component inputs can be substituted. Furthermore, for given values of \(N\), \(\eta^H\) and \(\epsilon\), an outsourced supplier receives a larger revenue share than an integrated affiliate (\(\beta^O > \beta^V\)).

Finally, using (18) we can compute the producer’s residual revenue share \(\beta^H(\xi)\) that can be understood as her effective bargaining power in the multilateral bargaining process:

\[
\beta^H(\xi) = 1 - \xi N \beta^O - (1 - \xi) N \beta^V = \frac{1 + \gamma (1 - \delta) (1 - \xi)}{1 + \gamma}.
\]

(19)

As is clear from (19), the producer’s share is increasing in the headquarter-intensity \(\eta^H\) and in the degree of component substitutability \(\epsilon\), but it is independent of the complexity level \(N\). Most importantly, the producer can increase her effective bargaining power by decreasing the outsourcing share \(\xi\), i.e., by relying more on integrated suppliers. However, for given parameters \(\gamma\) and \(\delta\) she is constrained to the range between

\[
\beta^H_{\min} \equiv \beta^H(\xi = 1) = \frac{1}{1 + \gamma} \quad \text{and} \quad \beta^H_{\max} \equiv \beta^H(\xi = 0) = \frac{1 + \gamma (1 - \delta)}{(1 + \gamma)}.
\]

(20)

To illustrate this available range for the headquarter’s revenue share more specifically, consider first the extreme case with \(\gamma \to \infty\) which results when each component is essential for the production process (if \(\epsilon \to 0\)). In this case the producer’s Shapley value and, thus, her realized revenue share is zero if she only chooses external suppliers \((\beta^H_{\min} = 0)\), while she is able to realize at most a share \(\beta^H_{\max} = (1 - \delta) > 0\) if she chooses complete vertical integration. For an intermediate choice of the outsourcing share, \(\xi \in (0, 1)\), her realized share \(\beta^H(\xi)\) is between 0 and \((1 - \delta)\). Now consider the other extreme case with \(\gamma \to 1\). In that case the producer can achieve a share between \(\beta^H_{\min} = 1/2\) and \(\beta^H_{\max} = (1 - \delta/2)\). Notice that those values of \(\beta^H_{\min}\) and \(\beta^H_{\max}\) are larger than their counterparts under \(\gamma \to \infty\), since the components are now better substitutable so that the suppliers have lower Shapley values. Finally, for intermediate parameter constellations \(\gamma \in (1, \infty)\) the respective \(\beta^H_{\min}(\gamma)\) is between 0 and 1/2, and the respective \(\beta^H_{\max}(\gamma, \delta)\) is between \((1 - \delta)\) and \((1 - \delta/2)\), with \(\partial \beta^H_{\min}/\partial \gamma < 0\), \(\partial \beta^H_{\max}/\partial \gamma < 0\), and \(\partial \beta^H_{\max}/\partial \delta < 0\). Then, given the (exogenous)
values of the respective $\beta^H_{\min} (\gamma)$ and $\beta^H_{\max} (\gamma, \delta)$, the producer can achieve a revenue share $\beta^H (\xi) \in [\beta^H_{\min}, \beta^H_{\max}]$ by the choice of the outsourcing share $\xi \in [0, 1]$. Figure 2 above depicts such an intermediate parameter constellation, in which case the $\beta^H_{\min}$- and the $\beta^H_{\max}$-curves are both upward sloping in $\{\beta^H, \eta^H\}$-space.\footnote{These curves are upward sloping since an increase of $\eta^H$ reduces the value of $\gamma$ for given $\epsilon$ and $\alpha$. Since $\gamma > 1$ is always assumed to hold, those curves then have strictly positive slope only up to $\beta^H_{\min} = 1/2$ and $\beta^H_{\max} = (1 - \delta)/2$, respectively. Notice further that both curves intersect the $\beta^H_{\max}$-curve only once. This single crossing property, which is important for the delineation of component- and headquarter-intensive industries below, is ensured by the parameter restriction $\gamma > 1$. Both the $\beta^H_{\min}$- and the $\beta^H_{\max}$-curve must then cut the $\beta^H_0$-curve from above, since $\beta^H_{0^*} \to 0$ as $\eta^H \to 0$ while $\beta^H_{\max} \to \beta^H_{\min} \to 0$ as $\eta^H \to 0$.}

**a) Zero outside opportunity.** Having clarified the foundations and the solution of the multilateral bargaining process in the ultimate stage of the game, we now turn to the producer’s firm structure decision in the first stage. We start again with the case where the suppliers’ outside opportunities are set to zero ($w_1^M = 0$). In this case, the producer’s problem is to maximize the variable payoff $\Theta \Psi$ as given by (16) simultaneously with respect to $N$ and $\xi$, subject to $\beta^O$ and $\beta^V$ given in (18) and $\beta^H (\xi)$ given in (19).

As shown in Appendix A.3.2., solving the first-order condition $\Theta \Psi_N' = \partial (\Theta \Psi) / \partial N = 0$ yields the following complexity choice for any given outsourcing share $\xi \in [0, 1]$: \begin{equation}
\tilde{N}_0 (\xi) = \frac{(1 - \beta^H (\xi)) \left(1 - \alpha (1 - \eta^H) - \alpha \eta^H\right)}{(1 - s) (1 - \alpha \beta^H (\xi) \eta^H)} \cdot \frac{\beta^H_{\max} (\xi) + \xi}{\delta (1 - \xi) + \xi} (1 - \xi) (1 - \xi) + \xi
\end{equation}
where $\beta^H (\xi)$ comes from the constraint (19). Using (21) we show in Appendix A.3.2. that $\partial \tilde{N}_0 / \partial \xi > 0$, so that $\tilde{N}_0 \equiv \tilde{N}_0 (\xi = 1, \beta^H = \beta^H_{\min}) > \tilde{N}_0 (\xi = 0, \beta^H = \beta^H_{\max}) \equiv \tilde{N}_0^V$. That is, a firm that fully relies on vertical integration is – everything else equal – less complex than a firm with outsourced suppliers only, and an increase in the outsourcing share of the firm is endogenously associated with an increase in complexity. The intuition is similar as in Section 3.1.1.: The producer can countervail the more severe underinvestment problem for integrated suppliers by concentrating on fewer intermediate inputs.

Unfortunately, the other first-order condition $\Theta \Psi' = \partial (\Theta \Psi) / \partial \xi |_{N = \tilde{N}_0 (\xi)} = 0$ cannot be solved explicitly for the optimal outsourcing share that we denote by $\tilde{\xi}_0$. However, it is possible to infer the key properties of $\tilde{\xi}_0$ analytically. In particular, we show in Appendix A.3.2. that $\partial \tilde{\xi}_0 / \partial \eta^H \leq 0$: firms from more headquarter-intensive industries tend to choose less outsourcing. The intuition for this result can be illustrated by using Figure 2. Recall from above that, if the producer were unconstrained in the division of the revenue, she
would choose $\beta^*_0$ as given in (12). In the present context the producer can affect the revenue distribution only via the choice of $\xi$ while being constrained according to (18) and (19). If the firm operates in a sufficiently headquarter-intensive sector, we have $\beta^*_0 > \beta^*_{\max}$ for given parameter values ($\epsilon$, $\delta$, $s$ and $\alpha$). Firms from those sectors with $\eta^H > \tilde{\eta}^H_0$ choose complete vertical integration, $\tilde{\xi}_0 = 0$, as this leads to the maximum possible revenue share $\beta^*_{\max}$ for the headquarter and thus to the closest possible alignment of $\beta^H(\xi)$ with $\beta^*_0$. The corresponding complexity choice is $\tilde{N}_0^V$ as obtained from (21). Analogously, if the firm operates in a sufficiently component-intensive sector ($\eta^H < \tilde{\eta}^H_0$), the producer aims for the highest possible revenue share for the suppliers by choosing complete outsourcing ($\tilde{\xi}_0 = 1$), with the corresponding $\tilde{N}_0^O$.12 Finally, in sectors with medium headquarter-intensity ($\tilde{\eta}^H_0 \leq \eta^H \leq \tilde{\eta}^H_0$) the constraint $\beta^H_{\min} \leq \beta^*_0 \leq \beta^H_{\max}$ from (19) is not binding. The producer can therefore set an outsourcing share $\tilde{\xi}_0 \in [0, 1]$ so as to realign $\beta^H(\tilde{\xi}_0)$ closely to $\beta^*_0$. This outsourcing share is higher in more component-intensive industries within that range ($\partial \tilde{\xi}_0 / \partial \eta^H < 0$), since $\partial \beta^*_0 / \partial \eta^H > 0$ holds as shown above. The complexity level corresponding to $\tilde{\xi}_0$ then follows from (21).13 Summing up, we can state

**Proposition 3:** Firms from more headquarter-intensive industries have less suppliers (lower $\tilde{N}_0$), and a lower outsourcing share ($\tilde{\xi}_0$). Firms from sectors with medium headquarter-intensity choose a coexistence of both organizational forms (hybrid sourcing).

These results are illustrated in Figure 3a. Here we assume fixed values of $\epsilon$, $\delta$, $s$ and $\alpha$, which pin down the sector thresholds $\tilde{\eta}^H_0$ and $\tilde{\eta}^H_0$, and we depict the total realized payoff $\tilde{\pi}_0 = \Theta \cdot \tilde{\Psi}(N = \tilde{N}_0, \xi = \tilde{\xi}_0) - \tilde{f}$ as a function of $\Theta$ and $\eta^H$. A darker color indicates a higher complexity level $\tilde{N}_0$. Within every sector (i.e., moving parallel to the $\Theta$-axis), we see that higher productivity implies a higher total payoff. It does not affect the firms’ complexity or organization decision as long as the suppliers’ outside opportunities are zero, however, as those decisions then have no implications for the firms’ fixed costs. Both complexity and organization then differ only across but not within sectors. Firms from sectors with low headquarter-intensity have a huge mass of suppliers ($\tilde{N}_0^O$), all of which are outsourced.

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12In Appendix A.3.2, we show that these thresholds $\tilde{\eta}^H_0$ and $\tilde{\eta}^H_0$ must exist for given parameter values of $\epsilon$, $\delta$, $s$ and $\alpha$ where it is always understood that the restriction $\gamma > 1$ (i.e., $\epsilon < \alpha(1 - \eta^H)$) is satisfied. These thresholds are then such that $0 < \tilde{\eta}^H_0 < \tilde{\eta}^H_0 < 1$.

13We can also consider the comparative statics with respect to $s$ and $\epsilon$. An increase of $s$ stretches the $\beta^*_0$-curve out to the right, but it does not affect the Shapley values and hence the $\beta^H_{\min}$- and the $\beta^H_{\max}$-curves. The parameter domain where outsourcing is chosen thus becomes larger. Analogously, an increase of $\epsilon$ increases $\beta^H_{\min}$ and $\beta^H_{\max}$, but it does not affect $\beta^*_0$. The parameter domain where outsourcing is chosen thus also becomes larger.
Gradually increasing $\eta^H$, we first see no change in the firms’ organizational structures, since $\tilde{\xi}_0 = 1$ as long as $\eta^H < \tilde{\eta}^H_0$, but a gradually decreasing mass of suppliers. Once we turn to sectors with headquarter-intensity above $\tilde{\eta}^H_0$ there is hybrid sourcing: Firms in those sectors choose to have some outsourced and some integrated suppliers. The outsourcing share $\tilde{\xi}_0$ and the complexity level $\tilde{N}_0$ are both gradually decreasing in $\eta^H$. Finally, once $\eta^H$ goes beyond $\tilde{\eta}^H_0$, firms choose $\tilde{\xi}_0 = 0$ and $\tilde{N}_0$. Firms in the most headquarter-intensive sectors are thus the least complex ones, and fully vertically integrated.

![Figure 3: Total firm payoff, complexity and organization.](image)

**b) Positive outside opportunity.** We now focus on the case where an increase in complexity endogenously leads to higher fixed costs ($w_1^M > 0$). We cannot explicitly solve for $\tilde{N}$ and $\tilde{\xi}$ that maximize the firm’s total payoff $\pi = \Theta \cdot \Psi - w_1^M N - \tilde{f}$ in that case, but similar as in subsection 3.1.1. it is again possible to infer the comparative statics.

First, we can use the first-order condition with respect to $N$ to characterize the complexity choice for a given organizational decision as follows: From (13) we know that the optimal mass of suppliers $N^*$ is determined according to $\Psi' = w_1^M / \Theta$ where $\Psi'$ is as defined in (13). The complexity choice $\tilde{N}$ under the constraint $\beta^H (\xi) \in [\beta^H_{\min}, \beta^H_{\max}]$ follows a similar logic. Using (16), we can define a function $\Psi' (N, \xi)$ that can be represented by a downward-sloping curve in $N$, similarly as in Figure 1. This function depicts the marginal change in the variable payoff when raising complexity, taking the value of $\xi$ as fixed and substituting in for $\beta^H$, $\beta^O$ and $\beta^V$ according to (18) and (19). We show in Appendix A.3.3. that $\partial \Psi' / \partial \xi \geq 0$, so that $\Psi' (N, \xi = 1, \beta^H = \beta^H_{\min}) > \Psi' (N, \xi = 0, \beta^H = \beta^H_{\max})$ with the $\Psi'$-curves for the intermediate cases with $\xi \in (0, 1)$ in between the $\Psi'$ and the $\Psi'$-curve. Furthermore, we show that $\partial \Psi' / \partial \eta^H < 0$ for any given $\xi$, i.e., all of those
curves shift to the left when $\eta^H$ is increased. The complexity choice conditional on the outsourcing share, $\tilde{N}(\xi)$, follows from the first-order condition for payoff maximization $\Psi' = w_1^M/\Theta$. Graphically, it is thus located at the intersection of the respective downward-sloping $\Psi'$-curve with the horizontal line at $w_1^M/\Theta$. We can state the following results:

**Proposition 4:** For given levels of $\eta^H$ and $\xi$, higher firm productivity is associated with a greater mass of suppliers; this complexity level $\tilde{N}(\xi)$ always remains below the respective $\tilde{N}_0(\xi)$. For given levels of $\Theta$ and $\eta^H$, a larger outsourcing share is associated with higher complexity. For given levels of $\Theta$ and $\xi$, higher headquarter-intensity is associated with lower complexity.

Turning to the organizational decision, it should be noted that the first-order condition with respect to $\xi$ does not depend on $w_1^M$, while the term $\pi'_{\xi} = \partial \pi/\partial \xi = \Theta \cdot \partial \Psi/\partial \xi$ depends non-negatively on $N$.

Since $\partial \tilde{N}/\partial \eta^H < 0$ and $\partial \tilde{N}/\partial \Theta > 0$ according to Proposition 4, the optimum condition $\pi'_{\xi} = 0$ then immediately implies that $\partial \tilde{\xi}/\partial \eta^H \leq 0$ and $\partial \tilde{\xi}/\partial \Theta \geq 0$. As before, we thus find that firms from more headquarter-intensive industries tend to choose less outsourcing. Furthermore, with endogenous fixed costs less productive firms also tend to choose less outsourcing, since they try to avoid the higher complexity level that is associated with this organizational structure.

To grasp the intuition for these results, it is again useful to consider that the producer chooses the optimal outsourcing share $\tilde{\xi}$ in such a way that the resulting revenue share $\beta^H(\tilde{\xi})$ from (19) is realigned closely with the payoff-maximizing revenue share $\beta^{H*}$. For given parameters (in particular, the sector-specific $\eta^H$ and the firm-specific $\Theta$), comparing $\beta^{H*}$ with the available range of revenue shares, $\beta^H(\xi) \in [\beta^{H}_{\min}, \beta^{H}_{\max}]$, every firm can thus be classified into one out of the following three groups:

1.) $\beta^{H*}(\eta^H, \Theta) > \beta^{H}_{\max}$,  
2.) $\beta^{H*}(\eta^H, \Theta) < \beta^{H}_{\min}$, 
3.) $\beta^{H}_{\min} \leq \beta^{H*}(\eta^H, \Theta) \leq \beta^{H}_{\max}$.

For firms in the first two groups the constraint $\beta^H(\xi) \in [\beta^{H}_{\min}, \beta^{H}_{\max}]$ is binding, and all firms in group 1 choose complete vertical integration ($\tilde{\xi} = 0$) while all firms in group 2 choose complete outsourcing ($\tilde{\xi} = 1$). For firms in group 3 this constraint is not binding, and they can choose an outsourcing share $\tilde{\xi}$ so that $\beta^H(\tilde{\xi})$ is close to $\beta^{H*}$. In the previous case without endogenous fixed costs ($w_1^M = 0$), it was possible to delineate these three groups by the sectoral headquarter-intensity alone. With $w_1^M > 0$ this is no longer possible,

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14This follows directly from $\partial \Psi'/\partial \xi = \partial^2 \Psi/(\partial N \partial \xi) \geq 0$. 

21
because $\beta^H(\eta^H, \Theta)$ is now firm-specific as it depends on $\Theta$. In other words, firms from the same industry may choose different firm structures when fixed costs matter.

The consequences of endogenous fixed costs for the final firm structure decisions are illustrated in Figure 3b above. First, consider headquarter-intensive sectors with $\eta^H > \bar{\eta}^H$. All firms from those sectors (regardless of their productivity) belong to group 1, and thus choose complete vertical integration. This is for two reasons. This organization leads to the highest possible revenue share for the producer ($\beta^H_{max}$), which in turn maximizes the variable payoff. Now this choice is reinforced, since vertical integration is also associated with fewer suppliers and thus with lower fixed costs. There is, hence, no change in the organizational decision of firms in headquarter-intensive industries compared to the previous case with $w^M_1 = 0$, which is depicted in Figure 3a. Figure 3b also shows that not only the total payoff $\tilde{\pi}$, but also the complexity level $\tilde{N}^V$ is now increasing in $\Theta$. That is, within a given headquarter-intensive sector, more productive firms vertically integrate more suppliers. Furthermore, comparing two equally productive firms from two industries $A$ and $B$ with $\eta^H_A > \eta^H_B > \bar{\eta}^H$, it turns out that the firm in sector $A$ chooses less complexity than the firm in the relatively more component-intensive sector $B$.

Now consider component-intensive sectors where $\eta^H < \bar{\eta}^H$. Without the endogenous “complexity penalty”, all firms in those sectors would belong to group 2 and choose complete outsourcing (see Figure 3a). With $w^M_1 > 0$, we observe that some firms now switch to group 1 and thus choose complete vertical integration in order to keep fixed costs low. This switch is more likely: i) the lower productivity is, since the increase of $\beta^H_{max}$ is then most substantial, and ii) the closer $\eta^H$ is to the upper bound $\bar{\eta}^H$, since the $\beta^H_{max}$ can then easier exceed $\beta^H_{max}$. There are also firms whose $\beta^H_{max}$ increases by less, so that it now falls inside the range between $\beta^H_{min}$ and $\beta^H_{max}$. Those firms then belong to group 3 and now choose hybrid sourcing ($0 < \xi < 1$). This is more likely to occur for firms with medium productivity, and in sectors with headquarter-intensity not too close to the upper bound $\bar{\eta}^H$. For firms with high productivity, the increase of $\beta^H_{max}$ due to $w^M_1 > 0$ is negligible, and they remain in group 2 and continue to choose complete outsourcing. Intuitively, the higher fixed cost under outsourcing play a minor role for these highly productive firms; their main aim is to maximize the residual rights of the suppliers whose inputs are intensively used. Similarly, firms from highly component-intensive sectors are also more likely to remain in group 2, i.e., to choose complete outsourcing. Summing up, the organization of firms in component-intensive industries now varies over the range of $\Theta$. Low productive firms

\footnote{Recall from Figure 2 that the $\beta^H_{max}$-curve stretches out to the left when $w^M_1 > 0$, and that this increase of $\beta^H_{max}$ compared to the benchmark $\beta^H_{max}$ is larger for less productive firms.}
have few suppliers which are fully vertically integrated. With rising productivity, there is a gradual increase of complexity $\tilde{N}$ and the outsourcing share $\tilde{\xi}$, and the most productive firms collaborate with a huge mass of suppliers all of which are outsourced.  

Finally, the organizational decision of firms from sectors with $\eta_H^U \leq \eta^H \leq \eta_H^I$ is now also tilted towards more vertical integration. More precisely, all firms decrease their outsourcing share in response to an increase of $w^M_i$. Firms with low productivity see a larger increase in $\beta^H*$, so they are more likely to become constrained by $\beta^H_{\text{max}}$ and thus choose $\tilde{\xi} = 0$. This switch from group 3 to group 1 is also more likely to happen in sectors where $\eta^H$ is only slightly below $\eta^H$, since the outsourcing share was already low there. Firms with high productivity and with headquarter-intensity relatively close to $\tilde{\eta}^H$ are, in contrast, more likely to continue to choose hybrid sourcing. Yet, since $\beta^H*$ has increased, this necessarily implies a lower outsourcing share even for those firms. Overall, Figure 3b suggests that the coexistence of integration and outsourcing is most pervasive in firms with medium-to-high productivity in sectors with low-to-medium headquarter-intensity.

### 3.2 Open Economy

We now incorporate the global scale dimension into the producer’s problem, who now also decides on the country $r \in \{1, 2\}$ where each component $i \in [0, N]$ is manufactured. We assume that unit costs of foreign suppliers are lower than for domestic suppliers, namely $c^M_2 = \phi \cdot c^M_1$ where $\phi \in (0, 1)$ reflects the cross-country cost difference. Recalling that unit costs of domestic suppliers are given by $c^M_1 = N^{-s}$ we thus have $c^M_2 = \phi \cdot N^{-s}$.

#### 3.2.1 Optimal mass of suppliers, revenue division, and offshoring share

As in the closed economy case, we first analyze the scenario where the producer can freely assign the revenue distribution. Due to symmetry, all domestic input suppliers receive

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16 Antràs and Helpman (2004) obtain the opposite result, namely that headquarter-intensive sectors are those where organizational structures are different across the productivity spectrum. That result is driven by the ad-hoc assumption that integration is associated with exogenously higher fixed costs than outsourcing. Grossman, Helpman and Szeidl (2005) consider the alternative ad-hoc assumption that outsourcing is associated with exogenously higher fixed costs. Our model is qualitatively more consistent with the latter, but it is important to note that in our model fixed cost differences between organizational modes emerge endogenously as they imply different optimal complexity levels. We could generate a similar sourcing pattern as in Antràs and Helpman (2004) when assuming that $f$ is sufficiently higher under integration than under outsourcing. We refrain from doing so, however, as the consequences of such an ad-hoc assumption are well understood.

17 If an increase of $w^M_i$ overall leads to more or less hybrid sourcing is unclear, since there is exit from group 3 to group 1 but also entry from group 2 to group 3.
an equal revenue share that we denote by $\beta_1$. Analogously, since all foreign suppliers are
also symmetric, each of them receives a revenue share $\beta_2$. Let $\beta_2 = \nu \beta_1$. The parameter
$\nu$ reflects the revenue division within the group of input suppliers, to be chosen by the
producer. With $\nu > 1$ a foreign low-cost suppliers receives a larger revenue share than a
domestic high-cost supplier, and vice versa for $\nu < 1$. Using (5), it then follows that the
input provision levels of domestic and foreign suppliers are given by

$$m_1 = \frac{\alpha (1 - \eta H)}{N} \cdot \left( \frac{\beta_1}{c_1} \right) \cdot \frac{1}{1 - \ell + \ell \left( \frac{\nu}{\phi} \right)^{1/(1-\epsilon)}} \cdot R_{\text{open}} \quad \text{and} \quad m_2 = \left( \frac{\nu}{\phi} \right)^{1/(1-\epsilon)} m_1,$$

(22)

where we impose $\nu > \phi$ so that $m_2 > m_1$. Since $\left( 1 - \beta^H \right) / N = (1 - \ell) \beta_1 + \ell \beta_2$ must hold,
and since $h_{\text{open}} = \left( \alpha \eta H c^H / c^H \right) R_{\text{open}}$, the firm’s total revenue is given by

$$R_{\text{open}} = \Lambda_R \cdot \Theta \left[ \left( \frac{\beta^H}{c_H} \right)^{\eta^H} \left( \frac{1 - \beta^H}{N^{1-s}} \right)^{1-\eta^H} \frac{1}{\beta^H} \right] \cdot R_{\text{closed}},$$

(23)

where $R_{\text{closed}}$ is the expression for total revenue from the closed economy case as given in
(9). The variable payoff is $\Theta \Psi_{\text{open}} = R_{\text{open}} - c^H h_{\text{open}} + (1 - \ell) N c_1 m_1 + \ell N \phi c_1 m_2$, where
$m_1$ and $m_2$ are given by (22), and it can be written as

$$\Theta \Psi_{\text{open}} = \Lambda_R \cdot \left[ 1 - \alpha \left( \beta^H \eta^H + \Lambda_C \cdot \frac{(1 - \eta^H) (1 - \beta^H)}{N} \right) \right] \cdot R_{\text{closed}},$$

(23)

where $\Lambda_C = \frac{1}{1 - \ell + \nu \ell} \left[ \frac{1 - \ell + \ell \phi (\nu/\phi)^{1/(1-\epsilon)}}{1 - \ell + \ell (\nu/\phi)^{1/(1-\epsilon)}} \right].$

When the outside opportunity in both countries is equal to zero ($w_1^M = w_2^M = 0$),
the producer’s problem is equivalent to maximizing the firm’s variable payoff as given in
(23) simultaneously with respect to $N$, $\beta^H$, $\ell$ and $\nu$. We show in Appendix B.1.i that the
following solutions are obtained:

$$N_{0,\text{open}}^* = N_{0,\text{closed}}^*, \quad \beta_{0,\text{open}}^H = \beta_{0,\text{closed}}^H, \quad \ell_0^* = 1.$$

(24)

That is, with zero outside opportunities, the firm would choose the same complexity level
and revenue distribution as in the closed economy, which are given in (11) and (12). Fur-
thermore, the firm would fully offshore all components to the foreign country. This is
intuitive, because offshoring only has advantages (lower unit costs of foreign suppliers) but
no disadvantages when endogenous fixed costs play no role.\footnote{Notice that the choice of $\nu$ becomes immaterial with $\ell^* = 1$, as there is then no asymmetry across suppliers in equilibrium. All suppliers are foreign and receive the same share $\beta_2^* = (1 - \beta_{0, \text{open}}^*)/N_{0, \text{open}}^*$.} Given those optimal choices, the firm then realizes a payoff equal to $\Theta \Psi_{0, \text{open}}^* = (1/\phi)^{(1-\eta^H)/(1-\alpha)} \cdot \Psi_{0, \text{closed}}^* > \Psi_{0, \text{closed}}^*$. Now suppose that $w_1^M = w_2^M > 0$, i.e., endogenous fixed costs matter but there are no cross-country differences in the “complexity penalty”. In that case, the producer maximizes $\pi_{\text{open}} = \Theta \Psi_{\text{open}} - w_2^M N$. Since the fixed cost term does not depend on the offshoring share, it is easy to see that the firm would still offshore all components, since it still has no disadvantages to choose foreign component manufacturing. Put differently, the firm chooses $\ell^* = 1$ as this maximizes the variable payoff $\Theta \Psi_{\text{open}}$ as shown before.

To determine $N_{\text{open}}^*$ and $\beta_{H, \text{open}}^*$ for the case with $w_1^M = w_2^M > 0$, we can adopt the same solution approach as in the closed economy. That is, we can solve the first-order condition $\partial \pi_{\text{open}} / \partial \beta^H = 0$ for $\beta^H(N)$ and substitute this into the other first-order condition $\partial \pi_{\text{open}} / \partial N = 0$ to derive a function $\Psi'_{\text{open}}$ that depends negatively on $N$. With $\ell^* = 1$, it is shown in Appendix B.1.ii that $\beta^H(N)$ is identical to its closed economy counterpart, while we have $\Psi'_{\text{open}} = (1/\phi)^{(1-\eta^H)/(1-\alpha)} \cdot \Psi'_{\text{closed}} > \Psi'_{\text{closed}}$. The optimal complexity is determined according to $\Psi'_{\text{open}} = w_2^M / \Theta$. Comparing the structure of firms within and across industries in the open economy, the results spelled out in Propositions 1 and 2 therefore still hold.

In particular, more productive firms and firms from more component-intensive industries have more suppliers and a lower headquarter revenue share. Yet more importantly, with the above condition we can also compare the structure of the same firm in the open and in the closed economy. This corresponds to the standard thought experiment where an economy opens up to trade, which in our context means that we move from an autarky scenario with domestic suppliers only (as described in Section 3.1.1.) to the present scenario where offshoring to a foreign low-cost country is feasible. Since $\Psi'_{\text{open}} > \Psi'_{\text{closed}}$ for given parameters $\Theta$, $\eta^H$, $s$, and $w_1^M = w_2^M > 0$, this comparison immediately implies $N_{\text{open}}^* > N_{\text{closed}}^*$ and $\beta_{H, \text{open}}^* < \beta_{H, \text{closed}}^*$. We can thus state

**Proposition 5:** Provided that $w_1^M = w_2^M > 0$, all firms increase their complexity level and decrease their headquarter revenue share when the economy opens up to trade.

What is the intuition for this result? Recall that the optimal complexity level is determined in a trade-off with the cost saving effect on the one side, and the dilution effect and the fixed cost increase on the other side. Since $N_{0, \text{open}}^* = N_{0, \text{closed}}^*$ with zero outside opportunities, we know that the balance between the cost saving and the dilution effect is
unaffected by the lower unit costs of the foreign suppliers, although the absolute magnitude of both effects is amplified. When the fixed cost channel matters, this implies higher costs of adding complexity (so that $N_{\text{open}}^* < N_{\text{closed}}^*$), but in relative terms those costs increase by less than in the closed economy (so that $N_{\text{open}}^* > N_{\text{closed}}^*$) since the fixed cost effect is not amplified. Economically, this result implies that globalization boosts the slicing of the production process, i.e., firms collaborate with more suppliers than under autarky.

Finally, we consider the case where offshoring has both advantages and disadvantages for the firm. In fact, as is widely known, offshoring often leads to higher communication and transportation costs, more expensive managerial oversight, and so on. To take this into account, we may assume that there is an extra fixed cost $f_X > 0$ per offshored component, capturing those higher transaction costs for the firm. Overall fixed cost are then given by $w_M^1 \cdot (1 - \ell)N + (w_M^2 + f_X) \cdot \ell N + \bar{f}$, and we assume that $\Delta \equiv w_M^2 + f_X - w_M^1 > 0$ which allows us to rewrite fixed costs as $(w_M^1 + \ell \Delta)N + \bar{f}$. When it comes to the maximization of the total payoff, which now reads as $\pi_{\text{open}} = \Theta \Psi_{\text{open}} - (w_M^1 + \ell \Delta)N - \bar{f}$, there is henceforth a trade-off: offshoring generates a higher variable payoff, but also larger fixed costs.

Due to this trade-off, it may therefore be optimal for a firm to offshore only some but not all components, in which case it would collaborate both with high-cost (domestic) and with low-cost (foreign) suppliers. This raises the issue how to divide the revenue among these asymmetric component manufacturers, i.e., how to choose $\nu$ optimally given the choices of $\ell$, $N$ and $\beta^H$. For the case of a unit elasticity of substitution across components ($\epsilon \to 0$), it is possible to show analytically that the producer would always set $\nu^* = 1$, i.e., she would divide the joint revenue share $\beta^M = 1 - \beta^H$ equally among all suppliers (see Appendix B.2.). Furthermore, for that case we can formally show that $\partial \ell^*/\partial \Theta \geq 0$ and $\partial \ell^*/\partial \eta^H \leq 0$, with strict inequalities if $0 < \ell^* < 1$. In the more general case with $\epsilon > 0$, we cannot solve analytically but only numerically for the optimal $\nu^*$. These numerical simulations then suggest, in particular, that the comparative static results for $\ell^*$ with respect to $\Theta$ and $\eta^H$ remain robust. We hence state:

**Proposition 6:** Within every sector, more productive firms have a higher optimal offshoring share. For a given productivity level, the optimal offshoring share is lower in more headquarter-intensive industries.

The intuition for these results is straightforward: The positive effect of offshoring on the variable payoff is multiplied by the firm’s productivity level, while the fixed cost increase

\[\text{Suppliers from country 1 may have a higher outside opportunity than those from the poor country 2. Assuming } \Delta > 0 \text{ ensures that the offshoring cost } f_X \text{ outweighs the difference in outside opportunities.}\]
does not depend on $\Theta$. Sufficiently highly productive firms therefore still set $\ell^* = 1$, as they are relatively little affected by the higher offshoring fixed costs (particularly if $\Delta$ is not too large). Firms with sufficiently low productivity solely rely on domestic suppliers ($\ell^* = 0$) in order to keep fixed costs low. Partial offshoring ($0 < \ell^* < 1$) is chosen by firms with intermediate productivity. Furthermore, firms from more headquarter-intensive industries tend to offshore less, since they choose lower complexity levels and thus benefit relatively less from lower unit costs of their (fewer) suppliers.

### 3.2.2 The make-or-buy decision under incomplete contracts

Turning now to the incomplete contracts environment where the producer cannot freely decide on the revenue distribution, first suppose that fixed cost considerations play no role at all (i.e., $w^M_1 = w^M_2 = f^X = 0$). In that case, the producer would offshore all components ($\tilde{\ell}^O_0 = \tilde{\ell}^V_0 = 1$) while making the same complexity and organization decisions as in the closed economy (see Figure 3a).\footnote{This follows from the facts that: i) $N^*_0$ and $\beta^H_0$ are the same as in the closed economy, and ii) that the Shapley values in (18) do not depend on the suppliers’ costs. Hence there is also no change in the producer’s effective bargaining power (19) or in the available range $\beta^H \in [\beta^H_{min}, \beta^H_{max}]$.} That is, with $\eta^H < \tilde{\eta}^H_0$ firms would completely rely on arm’s length transactions, with $\eta^H > \tilde{\eta}^H_0$ on intra-firm trade, and with $\tilde{\eta}^H_0 \leq \eta^H \leq \tilde{\eta}^H_0$ on a combination of the two sourcing modes (“hybrid global sourcing”).

Similarly, with $w^M_1 = w^M_2 > 0$ and $f^X = 0$, all firms would have foreign suppliers only. Comparing the complexity and organization decisions across firms and industries, a similar pattern as in Figure 3b applies. Moreover, it can again be shown that all firms raise their complexity level when the economy opens up to trade, as stated in Proposition 5. Furthermore, since $\tilde{N}_{open} > \tilde{N}_{closed}$, the firm’s optimal headquarter revenue share decreases while the producer’s effective bargaining power (19) remains constant. This, in turn, implies that $\tilde{\xi}_{open} \geq \tilde{\xi}_{closed}$ (also see Appendix B.3.). In other words, we have

**Proposition 7:** Provided that $w^M_1 = w^M_2 > 0$ and $f^X = 0$, no firm decreases and some firms increase the outsourcing share when the economy opens up to trade.

Economically, this implies that the possibility to engage in offshoring is positively correlated with outsourcing. Notice that this “time series” correlation (identical firms tend to choose more outsourcing after the economy has opened up to trade) is consistent with a “cross-sectional” pattern as shown in Figure 3b, where many low productive firms choose vertical integration in order to keep fixed costs low.
Finally, with $w^M_1 > 0$ and $\Delta > 0$ we again have the trade-off between higher fixed costs and higher variable payoffs under offshoring. The higher $\Theta$ is, the more important is the latter aspect, hence productivity and offshoring are positively related, i.e., $\partial \tilde{\ell} / \partial \Theta \geq 0$. Summing up, the overall sourcing pattern in the open economy can be described as follows:

1. **Headquarter-intensive industries**: All firms choose complete vertical integration. The least productive firms collaborate with few suppliers and only source domestically. As productivity rises, firms gradually increase complexity and the offshoring share. The most productive firms collaborate with a huge mass of integrated foreign suppliers.

2. **Component-intensive industries**: The least productive firms have few suppliers, all of which are domestic and vertically integrated. As productivity increases, firms tend to increase complexity, the outsourcing share, and the offshoring share. The most productive firms collaborate with a huge mass of external foreign suppliers.

3. **Industries with medium headquarter-intensity**: Low productive firms collaborate with few suppliers and tend to choose vertical integration and domestic sourcing. Highly productive firms have many suppliers and completely rely on foreign suppliers; they choose a combination of foreign outsourcing and intra-firm trade (“hybrid global sourcing”). For a given headquarter-intensity, increasing productivity is associated with higher complexity, more outsourcing and more offshoring.

4. **Asymmetric components**

In this last step of the analysis we consider a discrete setting with two asymmetric suppliers, which we denote by $a$ and $b$.\footnote{It is straightforward to consider a discrete version of our model by dividing the range $[0, N]$ into $X$ equally spaced subintervals, where all inputs in each subinterval $N/X$ are performed by a single supplier.} We focus on the organizational decision in this last part of the paper, which can be written as a tuple $\Xi$ that can take four possible realizations:\footnote{We abstract from the complexity and the global scale decision. Complexity is now exogenously given by $N = 2$, so that we also neglect the cost saving effect $s$. We also do not consider the offshoring decision of the producer. However, we do allow for exogenous marginal cost differences across the two suppliers, so that the low-cost (high-cost) supplier may be considered as foreign (domestic).}

$$\Xi \in \{\{O, O\}, \{O, V\}, \{V, O\}, \{V, V\}\},$$

where the first (second) element depicts whether the supplier of input $a$ (input $b$) is outsourced or vertically integrated.
The two suppliers can differ exogenously along three dimensions: i) with respect to the input intensities $\eta^M \cdot \eta_i$ for $i = a, b$ (with $\eta_a + \eta_b = 1$), which measure the technological importance of the respective input for the production process, ii) with respect to marginal costs $c_i^M$ for $i = a, b$, and iii) with respect to the “thread points” $\delta_i$ for $i = a, b$ under vertical integration, i.e., with respect to shares that they threaten to take away in the bargaining process. We believe that this latter asymmetry is a useful measure for the sophistication or knowledge specificity of the respective input, because it captures how well the producer can deal with the leftovers of the input when the (vertically integrated) supplier $i$ refuses to collaborate. If the input is difficult (easy) to handle, we expect $\delta_i$ to be high (low). Both suppliers $a$ and $b$ still threaten to take away their entire input provision levels if they are external subcontractors, since they maintain asset ownership in that case.

The organizational decision $\xi = \{\xi_a, \xi_b\} \in \Xi$, together with the exogenously given asymmetries, determine the Shapley values of the suppliers and hence their revenue shares $\beta_{\xi_i}$. The producer’s revenue share then follows residually as $\beta_{H\xi} = 1 - \beta_{\xi a} - \beta_{\xi b}$. Her problem is to maximize the firm payoff with respect to $\xi$ as follows

$$\max_{\xi \in \Xi} \Theta\Psi = \left[1 - \alpha \left(\frac{\eta^H \beta H \xi \eta^M}{\eta a \left(\frac{\beta \xi}{c_a^M}\right)^{\frac{1}{1+\epsilon}} + \eta b \left(\frac{\beta \xi}{c_b^M}\right)^{\frac{1}{1+\epsilon}}}\right)\right] \cdot R$$

where

$$R = \Theta \left[\left(\frac{\beta H \xi \eta^M}{c^M}\right) \cdot \left(\eta a \left(\frac{\beta \xi}{c_a^M}\right)^{\frac{1}{1+\epsilon}} + \eta b \left(\frac{\beta \xi}{c_b^M}\right)^{\frac{1}{1+\epsilon}}\right)^{\frac{1}{1+\epsilon}}\right]^{\frac{\alpha}{1-\alpha}}$$

In Appendix C we show how the Shapley values can be computed numerically. We find that, everything else equal, the Shapley value of supplier $i$ and hence his revenue share $\beta_{\xi_i}$ is: i) higher if he is outsourced than if he is vertically integrated, ii) increasing in the input intensity $\eta^M \cdot \eta_i$, iii) decreasing in the unit costs $c_i^M$, and iv) increasing in the “thread” point $\delta_i$. Intuitively, in all cases supplier $i$ has a higher bargaining power because his marginal contribution to every possible coalition increases.

We now discuss the payoff maximizing organizational decision. First, we consider the simplest case where the components $a$ and $b$ differ only in their input intensities, while the suppliers have identical unit costs and thread points ($c_a^M = c_b^M$ and $\delta_a = \delta_b$). In Figure 4a, we depict the headquarter-intensity of production on the horizontal and the technological asymmetry across components on the vertical axis (with $\eta_a = 1/2$ we have symmetrical
inputs). The different colors indicate which organizational mode is payoff-maximizing. As the graph shows, the producer would vertically integrate (outsource) both suppliers for sufficiently high (low) values of $\eta_H$. Hybrid sourcing (one integrated and one outsourced supplier) is chosen in sectors with intermediate headquarter-intensity, and within this range the producer tends to choose $\{O,V\}$ if $\eta_a > 1/2$ and $\{V,O\}$ if $\eta_a < 1/2$. That is, under hybrid sourcing, she tends to outsource the technologically “more important” component in order to properly incentivize the respective supplier.

![Figure 4: Organizational decision with two asymmetric components](image)

Now suppose that the components differ in their input intensities and unit costs, while the thread points are still identical ($\delta_a = \delta_b$). We assume that $c_a^M > c_b^M$, i.e., supplier $a$ is the high-cost type. The impact of this cost asymmetry on the firm structure decision is illustrated in Figure 4b. As before, we find that the producer chooses outsourcing (vertical integration) of both suppliers if headquarter-intensity is sufficiently low (high). Yet, for intermediate values of $\eta^H$ where hybrid sourcing is chosen, we now see that the choice $\{V,O\}$ becomes more prevalent. That is, given that there is hybrid sourcing, the cost asymmetry favors vertical integration of the high-cost and outsourcing of the low-cost supplier.\(^{23}\) This is due to the fact that the low-cost supplier $b$ contributes a lot and is, thus, highly valuable to the firm. The producer therefore outsources supplier $b$ more easily, as it is relatively more important to incentivize this supplier. Notice, however, that there

\(^{23}\)This can be seen by comparing the firm structure decisions for values of $\eta^H$ close to $1/2$ and $\eta_a$ slightly above $1/2$ in Figure 4. Without the cost asymmetry (Figure 4a) the producer chooses $\{O,V\}$, and with the cost asymmetry (Figure 4b) she chooses $\{V,O\}$.
is still a range in Figure 4b where the producer chooses \( \{O, V\} \), namely if \( \eta_a \) is sufficiently large. In that case, it becomes relatively more important to incentivize the supplier of the technologically highly important input \( a \), rather than the low-cost supplier \( b \).

A qualitatively similar picture as in Figure 4b emerges in the case where components/suppliers differ in their input intensities and thread points, while now assuming that unit costs are the same \( (c^M_a = c^M_b) \). Specifically, we assume that \( \delta_a > \delta_b \). Input \( a \) can then be thought of as the “sophisticated” component that requires more specific knowledge to be usable by the producer when the supplier refuses to collaborate. As Figure 4b shows, given that we are in the intermediate range of \( \eta^H \) where hybrid sourcing is chosen, and given that \( \eta_a \) is not too large, the producer indeed vertically integrates the sophisticated input \( a \) and outsources the simple input \( b \). The intuition is that the headquarter can relatively easily incentivize supplier \( a \) even as an integrated affiliate, because the supplier still threatens to take away a large share if the coalition breaks down. For supplier \( b \) it is relatively more difficult to be incentivized within the boundaries of the firm, so that outsourcing of that supplier is a more effective device to reduce the underinvestment problem.

Summing up, we infer the following Proposition from Figure 4:

**Proposition 8:** Suppose there are two asymmetric components \( a \) and \( b \). If headquarter-intensity \( \eta^H \) is sufficiently low (high), the producer outsources (vertically integrates) both suppliers. For intermediate headquarter-intensity, the producer outsources one and vertically integrates the other supplier (hybrid sourcing). Given that the producer chooses hybrid sourcing, she tends to outsource the component with the higher input intensity \( \eta_i \) and the lower unit costs \( c^M_i \). She tends to vertically integrate the supplier who has the higher thread point \( \delta_i \), i.e., the supplier of the “more sophisticated” input.

5 Conclusions

In this paper, we have developed a theory of a firm which decides on the complexity, the organization, and the global scale of its production process. Our model leads to several novel predictions about the structure of multinational firms that are consistent with stylized facts from the recent empirical literature. For example, studies by Jabbour (2008), Jabbour and Kneller (2010), and Kohler and Smolka (2009) show that MNEs, in practice, are characterized by multiple suppliers and a variety of different sourcing modes. In particular, Tomiura (2007) shows that firms which rely on a mixture of affiliates and external contractors (i.e., on hybrid sourcing) tend to be more productive than firms which
rely on a single sourcing mode in the global economy. This finding is consistent with our framework for the case of intermediate headquarter-intensity, which is likely to encapsulate many industries in the data. Our model version with two asymmetric inputs may provide a rationale for the empirical findings by Alfaro and Charlton (2009) and Corcos et al. (2009), that firms tend to keep high-skill inputs or components with a higher degree of specificity within their boundaries. Our model may also motivate future empirical research, as it leads to several predictions that have – to the best of our knowledge – not been confronted with data yet. For example, it would be interesting to explore if trade integration has indeed led to a stronger slicing of the production process, or if (conditional on productivity) firms from headquarter-intensive industries systematically have fewer suppliers than firms from component-intensive sectors.

The model in this paper is about single firms. It could potentially be embedded into a general equilibrium framework where firm interactions within and across industries are taken into account. Such a framework would be useful to explore more fully the repercussions of trade integration with cross-country differences in market conditions, factor prices and incomes, as well as their implications for global sourcing decisions. Furthermore, our model is based on a static bargaining scenario. In practice, suppliers may care about long-term relationships, or may try to collude with other suppliers in order to induce pressure on the headquarter. Exploring those and other extensions is left for future research.

References


Appendix A: Closed Economy

To simplify notation, we denote the first-order partial derivative of a function $f$ with respect to $x$ as $f'_x$. Analogously, the second-order partial derivative with respect to $y$ is denoted by $f''_{xy}$.

A.1. Input provision.

Supplier $i \in [0, N]$ chooses the level of input provision $m_i$ so as to maximize $\pi_i = \beta_i R - c_i^M m_i$. Using (3) and (4), the first-order-condition (FOC) for the maximization problem of supplier $i$ can be written as follows:

$$\pi'_{m_i} = \beta_i \cdot R'_{m_i} - c_i^M = \alpha \cdot \beta_i \cdot \eta^M \cdot \left( \frac{\eta_i}{m_i} \right)^{1-\epsilon} \cdot R \cdot \left[ \int_0^N \left( \frac{m_j}{\eta_j} \right) \epsilon \right]^{-1} - c_i^M = 0.$$

Using (5) for $m_i$ and (6) for $R$ yields $\pi'_{m_i} = 0$. It remains to be shown that the second-order-condition (SOC) is satisfied:

$$\pi''_{m_i,m_i} = -\alpha \cdot \beta_i \cdot \eta^M \cdot R \cdot \left[ (1+\epsilon) \int_0^N \eta_i \left( \frac{m_j}{\eta_j} \right) \epsilon \right] < 0$$

Analogously, it can be shown that $h = \alpha \cdot \eta^H \cdot \beta^H \cdot R/c^H$ maximizes the producer payoff $\pi^H = \beta^H R - c^H h$.

A.2. Complexity and revenue division.

A.2.1. Zero outside opportunity.

i.) Maximization problem: The FOCs are given by $\pi'_N = \Theta \Psi'_N = 0$ and $\pi'_{\beta^H} = \Theta \Psi'_{\beta^H} = 0$. Using (10), the terms $\Psi'_N$ and $\pi'_{\beta^H}$ are given by

$$\Psi'_N = \alpha [N (1-s) (1-\alpha \beta^H \eta^H) - \beta^M (1-s \alpha \eta^M - \alpha \eta^H)]$$

and

$$\Psi'_{\beta^H} = \alpha \left[ \beta^H (N - \beta^M) - N (1 - \beta^H (\beta^M - \alpha) - \beta^M (\alpha + \beta^H) \eta^H - \alpha (\beta^M - N \beta^H) (\eta^H)^2) \right].$$

respectively. With eqs. (27) and (28) it is straightforward to show that $N^*_0$ and $\beta^*_0$ as given in eqs. (11) and (12) solve the FOCs. The matrix of SOCs can be expressed as follows:

$$\Gamma = \left[ \begin{array}{c} \pi''_{NN} \quad \pi''_{N\beta^H} \\ \pi''_{\beta^H N} \quad \pi''_{\beta^H \beta^H} \end{array} \right] = \Theta \cdot \left[ \begin{array}{c} \Psi''_{NN} \quad \Psi''_{N\beta^H} \\ \Psi''_{\beta^H N} \quad \Psi''_{\beta^H \beta^H} \end{array} \right]$$

and it can be shown that for the first diagonal element $\Psi''_{NN} < 0$ holds while for the determinant $|\Gamma| > 0$. Hence, the matrix $\Gamma$ is negative definite.

ii.) Revenue share of a single supplier: The revenue share $\beta^*_0 = (1 - \beta^*_0) / N^*_0$ is given by $\beta^*_0 = \{(1-s) \left( s (1-\eta^H) (1-\alpha \eta^H - \rho) \right) \} / \left\{ s (1-\eta^H) (1+\alpha \eta^H) - \rho \right\}$. It can be verified that $\beta^*_{\eta^H \beta^H} > 0$ for $\eta^H < \eta^H_{crit}$ and $\beta^*_{\eta^H \beta^H} < 0$ for $\eta^H > \eta^H_{crit}$, where the threshold is given by
We can solve

\[ n^H_{\text{crit}} \equiv \left\{ 2 - \sqrt{(4 (1 - \sqrt{s}) + s (1 - \alpha)) (1 - \alpha) - \sqrt{s} (1 + \alpha)} \right\} / \left\{ 2 \alpha (1 - \sqrt{s}) \right\}, \]

which is decreasing in \( s \). Hence, \( \beta^*_{00} \) is hump-shaped over the range of \( n^H \).

iii.) Antràs and Helpman (2004): We claim in footnote 8 that there exists a \( s_{\text{crit}} \) such that \( N^*_0 = 1 \). This critical level is given by

\[ s_{\text{crit}} \equiv \eta^H / \left\{ (1 - \alpha (1 - \eta^H)) \eta^H + \sqrt{(1 - \alpha (1 - \eta^H)) (1 - \eta^H) (1 - \alpha \eta^H)} \right\}, \]

and it can be verified that \( \beta^H_{0s} (s = s_{\text{crit}}) \) is identical to eq. (10) in Antràs and Helpman (2004):

\[ \beta^H_{0s} (s = s_{\text{crit}}) = \frac{\eta^H (\alpha \eta^H + 1 - \alpha) - \sqrt{\eta^H (1 - \eta^H) (1 - \alpha \eta^H) (\alpha \eta^H + 1 - \alpha)}}{2 \eta^H - 1}. \]

A.2.2. Positive outside opportunity.

i.) Maximization problem: The FOCs are given by \( \pi'_N = \Theta \Psi'_N - w^M_1 = 0 \) and \( \pi''_{\beta\mu} = \Theta \Psi''_{\beta\mu} = 0 \). We can solve \( \Psi''_{\beta\mu} = 0 \) for

\[ \beta^H (N) = \frac{N - 1 + (1 + N) (1 - \alpha) \eta^H + (1 + N) \alpha (\eta^H)^2 - \hat{\rho}}{2 (\eta^H (1 + N) - 1)} \quad \text{where} \quad (29) \]

\[ \hat{\rho} \equiv \sqrt{(1 - \eta^H) (1 - \alpha \eta^H) \left[ (1 - N)^2 - (1 + N) (1 + N (-3 + \alpha) + \alpha) \eta^H + (1 + N)^2 \alpha (\eta^H)^2 \right]}. \]

Note that \( \beta^H (N) \) as stated in (29) does not depend on \( w^M_1 \). Furthermore, it directly follows that \( \beta^H_{N^*} < 0 \). Using \( \beta^H (N) \) in \( \Psi''_N = 0 \) allows us to derive the condition (13), which uniquely determines \( N^* \). It then directly follows from Appendix A.2.1.i, and from the fact that \( \Psi''_{NN} < 0 \) in the relevant domain, that \( N^* \) solves the first-order conditions. This \( N^* \) is then associated with an optimal headquarters revenue share \( \beta^H_{N^*} = \beta^H (N = N^*) \) from (29) that solves \( \Psi''_{\beta\mu} = 0 \).

ii.) Total profits: The optimal mass of suppliers is implicitly given by \( \pi'_N = \Theta \Psi'_N - w^M_1 = 0 \). It then directly follows that \( \pi'_\theta = \Psi + N^*_0 (\Theta \Psi'_N - w^M_1) = \Psi > 0 \).

A.3. The make-or-buy decision under incomplete contracts.

A.3.1. Shapley Value.

Remark: In the text we assume a continuum of intermediate inputs and each intermediate input is provided by a separate input supplier. In the following we derive the discrete Shapley value for the case of \( M \) suppliers, each controlling a range \( \kappa = N / M \) of the continuum of intermediate inputs. Similar to Acemoglu, Antràs and Helpman (2007) we then transform the Shapley value with a finite number of players into the asymptotic Shapley value by assuming that each supplier’s controlled range of inputs becomes infinitesimally small (\( \kappa \to 0 \)).

Step 1: Marginal contribution of a supplier. To compute the Shapley value for a component supplier \( j \), we need to determine the marginal contribution of this supplier to a given coalition size.
Consider a situation in which \( n \) suppliers contribute inputs equal to \( m \) and the firm contributes \( h \) as given by (8). The joint revenue of coalition size \( n \) is given by

\[
R(n, N) = Y^{1-\alpha} g^\alpha \left[ \left( \frac{h}{\eta^H} \right)^{\eta^H} \cdot \left( \frac{1}{\eta^M} \right) \left( \frac{n \cdot \eta_i \cdot \left( \frac{m}{\eta_i} \right)^{\gamma}}{n} \right) \right]^{\alpha} R \left( \frac{n}{N} \right)^\gamma.
\]

where \( R \) is given by (9). Note that in the bargaining stage \( \eta^H, \eta^M, \eta_i = 1/N \) and \( c^M \) are already determined. The marginal contribution of supplier \( j \) is the increase in revenue if the supplier is part of the coalition versus if he is not part of the coalition. It is also important to note that the firm must be part in any coalition, otherwise the joint revenue is zero. For a given \( N \) and coalition size \( n + 1 \) (\( n \) suppliers plus the firm), the marginal contribution of supplier \( j \) is given by

\[
\Delta R(n, N) = \frac{R}{N^\gamma} \cdot \left[ n^\gamma - (n - 1)^\gamma \right].
\]

**Step 2: Average over all coalitions.** Along the lines of Acemoglu et al. (2007) we consider a bargaining game with the firm and \( n \) suppliers such that \( n + 1 \) players bargain over the surplus value. We denote a permutation where player 0 is the headquarter and players 1, 2, ..., \( n \) are the suppliers by \( g = \{g(0), g(1), ..., g(n)\} \). The set of feasible permutations is given by \( G \). Now let \( z'_g = \{j'| g(j) > g(j')\} \) be the set of players ordered below \( j \) in the permutation of \( g \). Finally, the joint surplus of the coalition consisting of any subset of the \( n + 1 \) players is given by \( v : G \rightarrow \mathbb{R} \). Using this notation the Shapley value of player \( j \) is given by

\[
s_j = \frac{1}{(M + 1)!} \sum_{g \in G} \left[ v(z'_g \cup j) - v(z'_g) \right].
\]

**Step 3: Permutations.** The probability that \( g(j) = i \) is \( 1/(M + 1) \) for every \( i \). However, if the supplier is the first \( g(j) = 0 \) player in a permutation the firm is necessarily ordered after \( j \). In this case the marginal contribution of the supplier \( j \) is zero. If \( g(j) = 1 \) then the firm is ordered before \( j \) with probability \( 1/M \) and after \( j \) with probability \( 1 - 1/M \). If the firm is ordered after supplier \( j \) it follows \( v(z'_g \cup j) = 0 \) while \( v(z'_g \cup j) = R(1, N) \) if the firm is ordered before. Therefore, for \( g(j) = 1 \) the conditional expected value of \( v(z'_g \cup j) \) is given by \((1/M) \cdot R(1, N)\). Similar, for \( v(z'_g) \) the conditional expected value is given by \((1/M) \cdot R(0, N)\). Repeating this argument for \( g(j) = i \) with \( i > 1 \), the conditional expected value of \( v(z'_g \cup j) \) is given \((1/M) \cdot R(i, N)\) while the conditional expected value of \( v(z'_g) \) is \((i/M) \cdot R(i - 1, N)\). Using eqs. (30) and (31) we can simplify:

\[
s_j = \frac{1}{(M + 1) M} \sum_{i=1}^{M} i \left[ R(i, N) - R(i - 1, N) \right] = \frac{1}{(M + 1) M} \sum_{i=1}^{M} i \Delta R(i, N).
\]

**Step 4: Asymptotic Shapley value.** Using \( \kappa = N/M \) and (30), we rewrite the discrete Shapley value as

\[
s_j = \frac{1}{(N + \kappa) N} \sum_{i=1}^{M} i \kappa^2 \Delta R(i, N) = \frac{R}{(N + \kappa) N^{1+\gamma}} \sum_{i=1}^{M} i \kappa^2 \left[ i^\gamma - (i - \kappa)^\gamma \right].
\]

Using the first-order Taylor expansion of \( i^\gamma - (i - \kappa)^\gamma = \gamma i^{\gamma-1} \kappa + o(\kappa) \) yields
\[
\frac{s_j}{\kappa^{2-\gamma}} = \frac{R}{(N + \kappa) N^{1+\gamma}} \sum_{i=1}^{M} \gamma (i \kappa)^\gamma \kappa + \frac{o(\kappa)}{\kappa^{2-\gamma}}.
\]

Taking the limit as \(\kappa \to 0\) and \(M \to \infty\), the sum becomes a Riemann integral:

\[
\lim_{\kappa \to 0} \left( \frac{s_j}{\kappa^{2-\gamma}} \right) = \frac{R}{N^{2+\gamma}} \int_{0}^{N} \gamma z^\gamma dz.
\]

Evaluating the integral leads to the asymptotic Shapley value for a supplier \(j\) which is given by

\[
\beta^O = \frac{s_j}{R} = \frac{\gamma}{(1 + \gamma) N}.
\]

**Step 5: Outsourcing vs. integration.** We denote \(\beta^O\) as the revenue share of an outsourced supplier. This is the case since we assume in (30) that the supplier \(j\) contributes \(m\) in terms of input provision. If all suppliers are outsourced (\(\xi = 1\)), the firm’s revenue share is the residual and given by \(\beta^H (\xi = 1) = 1 - N \beta^O = \frac{1}{1+\gamma}\). In the case of vertical integration the supplier can only threaten to take away \(b\) with \(0 < \delta < 1\). Using this assumption, the expression in (30) is given by \(\Delta_R (n, N) = R \cdot [\gamma - (n - \delta)\gamma] / N^\gamma\) and the Shapley value of supplier \(j\) can be written as

\[
s_j = \frac{R}{(N + \kappa) N^{1+\gamma}} \sum_{i=1}^{M} \gamma i^{\gamma} \alpha \beta^{H}(\xi = 1 - \xi) \delta^{1-\gamma} \xi (1 - \xi) + \xi^2 \delta^{1-\gamma} \xi (1 - \xi) + \xi^2.
\]

Comparing the extreme cases \(\xi = 1\) and \(\xi = 0\) reveals \(\beta^H > 0\) from (19), for \(\xi \in (0, 1)\) we thus have:

\[
\frac{\partial \tilde{N}_0}{\partial \xi} = \frac{(1 - \beta^H \eta^H) \alpha (1 - \alpha^H) + (1 - \alpha^H) \left( \delta^{1-\gamma} (1 - \xi) + \xi \right) \delta^{1-\gamma} (1 - \xi) + \xi^2}{(1 - s \alpha (1 - \eta^H) - \alpha^H)^{-1} (1 - s) (1 - \beta^H \eta^H) \left( \delta^{1-\gamma} (1 - \xi) + \xi \right)^2 (\xi - (1 - \xi))^2} > 0.
\]

**A.3.2. Zero outside opportunity.**

\(i.)\) **Complexity:** We claim that solving \(\Theta \Psi'_N = \partial \Psi / \partial N = 0\) leads to \(\tilde{N}_0 (\xi)\) as given by (21). To derive \(\tilde{N}_0 (\xi)\) consider \(\Psi'_N = 0\) which can be rearranged to

\[
\left( \frac{1 - \alpha}{1 - s} + \alpha \left( 1 - \eta^H \right) \right) (1 - \beta^H) \left( \delta^{1-\gamma} (1 - \xi) + \xi \right) (1 - s) = \frac{N \left( 1 - \alpha^H \eta^H \right) \left( \delta^{1-\gamma} (1 - \xi)^2 + \delta (1 - \xi) \xi \delta - \delta^{1-\gamma} (1 - \xi) \xi + \xi^2 \right)}{\left( 1 - \beta^H \eta^H \right) \left( 1 - \alpha^H \eta^H \right) + (1 - \alpha^H) \left( \delta^{1-\gamma} (1 - \xi) + \xi \right) \delta^{1-\gamma} (1 - \xi) + \xi^2}.
\]

Solving for \(N\) and simplifying yields \(\tilde{N}_0 (\xi)\). Comparing the extreme cases \(\xi = 1\) and \(\xi = 0\) reveals \(\tilde{N}_0 (\xi = 1) > \tilde{N}_0 (\xi = 0)\). Since \(\beta^H > 0\) from (19), for \(\xi \in (0, 1)\) we thus have:

\[
\frac{\partial \tilde{N}_0}{\partial \xi} = \frac{(1 - \beta^H \eta^H) \alpha (1 - \alpha^H) + (1 - \alpha^H) \left( \delta^{1-\gamma} (1 - \xi) + \xi \right) \delta^{1-\gamma} (1 - \xi) + \xi^2}{(1 - s \alpha (1 - \eta^H) - \alpha^H)^{-1} (1 - s) (1 - \beta^H \eta^H) \left( \delta^{1-\gamma} (1 - \xi) + \xi \right)^2 (\xi - (1 - \xi))^2} > 0.
\]

\(ii.)\) **Outsourcing share:** Ideally we would solve \(\partial (\Theta \Psi |_{N = \tilde{N}_0 (\xi)}) = 0\) for \(\xi_0\) in explicit form. Unfortunately, this is not possible. We thus use an indirect approach to show that \(\partial \xi_0 / \partial \eta^H < 0\). To illustrate this approach, consider the case of \(\gamma \to \infty \Leftrightarrow \epsilon \to 0\). Solving \(\Theta \Psi |_{N = \tilde{N}_0 (\xi)} = 0\) for \(\xi\)
is then equivalent to solving the following equation:
\[
\text{LHS} = \frac{(1 - \delta) \eta^H (1 - \alpha (1 - \eta^H) - \alpha \eta^H)}{(1 - \eta^H)(1 - \alpha (1 - \delta) \eta^H (1 - \xi))} + \frac{\eta^H}{(1 - \eta^H)(1 - \xi)} + \frac{(1 - s) (1 - \delta)}{\xi + \delta (1 - \xi)} = -\ln[\delta] \equiv \text{RHS}
\]
Note that the RHS is independent of \(\eta^H\) and \(\xi\). For a given \(\eta^H\), the LHS cuts the RHS uniquely from below at some \(\xi = \xi_0\). Furthermore, \(\partial \text{LHS}/\partial \eta^H > 0\). Hence, \(\partial \xi_0/\partial \eta^H < 0\). For \(0 < \epsilon < 1\) (i.e., for \(\gamma > 1\)) this indirect approach can be repeated, but it now involves extensive expressions. However, numerical simulations reveal that the same comparative static result holds.

iii.) Sector cutoffs: In the following we show that there exist two thresholds \(\bar{\eta}^H\) and \(\tilde{\eta}^H\) (with \(0 < \bar{\eta}^H < \tilde{\eta}^H < 1\)) such that firms from sectors with \(\eta^H > \bar{\eta}^H\) choose \(\xi_0 \in (0, 1)\) with \(\partial \xi_0/\partial \eta^H < 0\), firms with \(\eta^H < \bar{\eta}^H\) choose \(\xi_0 = 1\), and firms with \(\eta^H > \tilde{\eta}^H\) choose \((\xi_0 = 0)\). To see this, consider \(\Psi\) for \(\xi = 0\) and \(\xi = 1\) at the boundaries \(\eta^H \to 0\) and \(\eta^H \to 1\), given the corresponding \(\tilde{N}_0\). For the most component-intensive sector with \(\eta^H \to 0\) we have:
\[
\Psi(\xi = 1) - \Psi(\xi = 0) = \Theta (1-\alpha) (1 - \delta s\alpha) (1 - s) \left(\frac{\alpha}{\alpha + \epsilon}\right) > 0,
\]
while for \(\eta^H \to 1\) we have:
\[
\Psi(\xi = 1) - \Psi(\xi = 0) = \Theta \left(\frac{\Theta}{2}\right) (c^H)^{-\frac{\alpha}{1-\alpha}} \left[2 - \frac{\alpha}{1-\alpha} (2 - \alpha) - 2 (2 - 2\alpha + \alpha\delta) (1 - \delta/2)^{\frac{\alpha}{1-\alpha}}\right] < 0
\]
since \(\alpha, \delta \in (0, 1)\). This comparison together with the continuity of \(\Psi\) in \(\eta^H\) implies, that the aforementioned cutoffs \(\bar{\eta}^H\) and \(\tilde{\eta}^H\) must exist such that \(0 < \bar{\eta}^H < \tilde{\eta}^H < 1\).

A.3.3. Positive outside opportunity.

We cannot solve \(\pi_N' = \Theta \Psi_N' - w_1 = 0\) for \(\tilde{N}(\xi)\) but as in Appendix A.3.2 we consider \(\gamma \to \infty\):
\[
\frac{\partial \Psi_N'}{\partial N} \bigg|_{N=\tilde{N}_0} = -\frac{\alpha (1 - \eta^H) (1 - \alpha \beta^H \eta^H) \Theta \delta \frac{\alpha \beta^H (1 - \eta^H)}{(1 - \eta^H) (1 - \alpha \beta^H \eta^H)} \left(\frac{1}{1 - \alpha \beta^H \eta^H}\right) - 2 + \alpha \theta H^H}{\left(1 - \alpha\right) \left(c^H \beta^H\right)^{\alpha \eta^H}} < 0,
\]
which implies that: i.) \(\tilde{N}_{\epsilon_1} < 0\), ii.) \(\tilde{N}_0 > 0\), and iii.) \(\tilde{N} \to \tilde{N}_0\) as \(\Theta \to \infty\). Next, we have
\[
\frac{\partial \Psi_N'}{\partial \eta^H} \bigg|_{N=\tilde{N}_0} = \left(\frac{\partial \Psi_N'}{\partial N} \bigg|_{N=\tilde{N}_0}\right)^{-1} \frac{\alpha (1 - \beta^H)^2}{(1 - \beta^H \eta^H)^2} < 0,
\]
which implies iv.) \(\tilde{N}_{\eta^H} < 0\). For \(\xi = 0\) and \(\xi = 1\) with \(\beta^H = \beta_{\min}^H\) and \(\beta^H = \beta_{\max}^H\) we have
\[
\frac{\partial \Psi_N'}{\partial \beta^H} \bigg|_{N=\tilde{N}_0} = \left(\tilde{N}_0 \frac{\partial \Psi_N'}{\partial N} \bigg|_{N=\tilde{N}_0}\right)^{-1} \frac{(1 - \alpha \eta^H)}{(1 - \beta^H) (1 - \alpha \beta^H \eta^H)} < 0
\]
Hence, v.) \(\tilde{N}_{\beta^H} < 0\). This implies \(\Psi' > \Psi''\), and for the intermediate cases \(\xi \in (0, 1)\):
\[
\frac{\partial \Psi_N'}{\partial \xi} \bigg|_{N=\tilde{N}_0} = -\left(\frac{\partial \Psi_N'}{\partial N} \bigg|_{N=\tilde{N}_0}\right)^{-1} \frac{(1 - \delta) (1 - \alpha \eta^H)^2}{(1 - \alpha \beta^H \eta^H)^2} > 0,
\]
39
Appendix B: Open Economy

B.1. Optimal mass of suppliers, revenue division and offshoring share

i.) Zero outside opportunities: The first two FOCS are given by $\pi'_{\text{open},N} = \Theta \Psi'_{\text{open},N} = 0$ and $\pi'_{\text{open},\beta H} = \Theta \Psi'_{\text{open},\beta H} = 0$ which can be written as follows:

$$\frac{\Psi'_{\text{open},N}}{\Psi_{\text{open}}} = \frac{\alpha \eta M \left[ N (1-s) (1 - \alpha \beta H \eta^H) - \beta^M \Lambda_C \left( 1 - s \alpha M - \alpha \eta H \right) \right]}{N (1-\alpha) \left[ \Lambda_C \alpha \beta M \eta^M - N (1-\alpha \beta H \eta^H) \right]},$$

(32)

$$\frac{\Psi'_{\text{open},\beta H}}{\Psi_{\text{open}}} = \frac{\alpha \left[ \beta^M \Lambda_C \eta^M \left( \alpha \eta^H - \beta^H \right) - \Lambda_C \beta^H \eta^H \left( 1 - \alpha \beta^H \eta^H \right) \right]}{(1-\alpha) \beta^H \Lambda_C \eta^M \left( \alpha \eta^H - \beta^H \right) - \Lambda_C \beta^H \eta^H \left( 1 - \alpha \beta^H \eta^H \right)},$$

(33)

Solving (32) and (33) yields $N^*_0 = \Lambda_C \cdot N^*_0$ and $\beta^H = \beta^H$. Next, we consider the optimal offshoring share $\ell^*$. Using $N^*_0$ and $\beta^H$ the variable payoff $\Theta \Psi_{\text{open}}$ can be written as $\Theta \Psi_{\text{open}} = \Lambda_{\Psi} \cdot \Theta \Psi_{\text{closed}}$, where the term $\Lambda_{\Psi}$ is given by

$$\Lambda_{\Psi} = \frac{\Lambda_R}{\alpha (1-\eta^H (1-\alpha))} = \left[ \frac{1 - \ell + \ell \left( \frac{\nu}{\phi} \right)^{\frac{1}{1-s}}} {(1-\ell + \nu \ell) s \left( 1 - \ell + \phi \ell \left( \frac{\nu}{\phi} \right)^{\frac{1}{1-s}} \right)^{1-s}} \right]^{\alpha (1-\eta H (1-\alpha))}. \quad (34)$$

Notice that $\Lambda_{\Psi} (\ell = 0) = 1 < \Lambda_{\Psi} (\ell = 1) = (1/\phi)^{\alpha (1-\eta^H (1-\alpha))}$. Hence, the variable payoff is higher with $\ell = 1$ than with $\ell = 0$. Furthermore, it can be shown that $\Lambda_{\Psi} (0 < \ell < 1)$ is never above $\Lambda_{\Psi} (\ell = 1)$. This can be most easily seen if $\nu = 1$, because in that case we have

$$\frac{\partial \Lambda_{\Psi}}{\partial \ell} (\nu = 1) = \left( 1 + \lambda (1/\phi)^{\frac{1}{1-s}} - \lambda \right)^{\alpha (1-\eta^H (1-\alpha))} > 0, \quad (35)$$

but also in all other cases we find that $\ell = 1$ maximizes $\Lambda_{\Psi} (\ell)$. This implies $\ell^*_0 = 1$. Noting that $\Lambda_C (\ell = 1) = 1$, it follows that $N^*_0 = N^*_0$.

ii.) Equal outside opportunities. With $w^1_M = w^2_M > 0$, solving $\partial \pi_{\text{open}} / \partial \beta H = 0$ for $\beta H$ yields

$$\beta^H (N) = \frac{N - \Lambda_C - (1-\alpha) (N + \Lambda_C) \eta^H \alpha (N + \Lambda_C) \eta^H + \hat{\rho}_{\text{open}}}{2 (N + \Lambda_C) \eta^H - 2 \Lambda_C}, \quad \text{with} \quad (36)$$

$$\hat{\rho}_{\text{open}} = \frac{2 (N + \Lambda_C^2) \eta^H - 2 \Lambda_C}{2 (N + \Lambda_C) \eta^H - 2 \Lambda_C}. \quad \text{The expression in} \quad (36) \quad \text{reduces to the} \quad \beta^H (N) \quad \text{as given by} \quad (29) \quad \text{if} \quad \ell = 1, \quad \text{since this implies} \quad \Lambda_C = 1.

Furthermore $\ell = 1$ also implies $\Lambda_R (\ell = 1) = \Lambda_{\Psi} (\ell = 1) = (1/\phi)^{\alpha (1-\eta^H (1-\alpha))} > 1$. Using this, it follows that: $\frac{\partial (\Theta \Psi_{\text{open}})}{\partial N} \bigg|_{\beta H = \beta^H (N)} = \Theta \cdot (1/\phi)^{\alpha (1-\eta^H (1-\alpha))} \cdot \Psi'_{\text{closed}}.$
B.2. Asymmetric outside opportunities.

Assume that \( w^M_1 > 0 \) and \( \Delta > 0 \). We know from Appendix B.1. that we can solve \( \pi'_{\beta H} = \Theta \Psi'_{\text{open}, \beta H} = 0 \) for \( \beta H (N) \) as given in (36). The other FOCs are then:

\[
\pi'_{\text{open}, N} = \Theta \Psi'_{\text{open}, N} - (w^M_1 + \ell \Delta) = 0, \quad \pi'_{\text{open}, \ell} = \Theta \Psi'_{\text{open}, \ell} - \Delta N = 0, \quad \pi'_{\text{open}, \nu} = \Theta \Psi'_{\text{open}, \nu} = 0.
\]

The FOC \( \pi'_{\text{open}, \nu} = 0 \) cannot be solved in general for \( \nu \), however for \( \epsilon \to 0 \) it simplifies to

\[
\lim_{\epsilon \to 0} \Lambda \psi = \left( \frac{(\nu/\phi)^{\ell}}{1 - \lambda + \nu \lambda} \right)^{\alpha (1 - \eta^H)} \quad \text{and} \quad \psi_{\text{open}, \nu} = \frac{\partial \Lambda \psi}{\partial \nu} = \frac{1}{\Lambda \psi} \left[ \frac{\alpha (1 - \ell \nu) (1 - \nu)}{(1 - \alpha) (1 - \ell + \nu \ell \nu)} \right]
\]

which implies that the FOC is solved by \( \nu^* = 1 \). This simplifies the analysis as it implies \( \Lambda_C = 1 \) and \( \partial \Lambda \psi / \partial \ell > 0 \), see (35). Note that independently of this assumption we still have \( \Lambda \psi (\ell = 0) < \Lambda \psi (\ell = 1) \), see Appendix B.1. Substituting \( \beta H (N) \) as given by (36) and \( \nu^* = 1 \) into the remaining two FOCs leads to:

\[
\pi'_{N} = \Theta \Psi'_{\text{open}, N} |_{\beta H = \beta H (N)} - (w^M_1 + \ell \Delta) = 0, \quad \pi'_{\ell} = \Theta \Psi'_{\ell} |_{\beta H = \beta H (N)} - \Delta N = 0.
\]

For sufficiently productive firms we have \( \pi'_{\ell} > 0 \) for all \( \ell \in [0, 1] \), since \( \Psi'_{\ell} > 0 \) and \( N^*_{\text{open}, 0} \) approaches \( N^*_{\text{open}, 0} \) and is bounded from above. Hence, the global maximum is given by \( \ell^* = 1 \). Vice versa, for firms with sufficiently low productivity, \( \pi'_{\ell} < 0 \) and hence \( \ell^* = 0 \). Finally, we use the implicit function theorem to derive the comparative statics (to alleviate notation we drop the subscript \( \text{open} \)). First, the matrix of SOCIs is:

\[
K = \begin{bmatrix}
\pi''_{NN} & \pi''_{NL} \\
\pi''_{LN} & \pi''_{LL}
\end{bmatrix} = \begin{bmatrix}
\Theta \Psi''_{NN} & \Theta \Psi''_{N\ell} - \Delta \\
\Theta \Psi''_{L\ell} - \Delta & \Theta \Psi''_{\ell\ell}
\end{bmatrix},
\]

which is negative definite since \( \Psi''_{NN} < 0 \) and \( |K| > 0 \). Using \( K \) we have:

\[
\ell^* = \begin{bmatrix}
\Theta \Psi''_{NN} & -\Psi''_{N} \\
\Theta \Psi''_{L\ell} - \Delta & -\Psi''_{\ell}
\end{bmatrix} = \frac{-\Theta \Psi''_{NN} \psi + \psi_N (\Theta \Psi''_{NN} - \Delta)}{|K|} \geq 0,
\]

\[
\ell''_{\eta H} = \begin{bmatrix}
\Theta \Psi''_{NN} & -\Theta \Psi''_{N\eta} \\
\Theta \Psi''_{L\ell} - \Delta & -\Theta \Psi''_{\ell\eta}
\end{bmatrix} = \frac{-\Theta \Psi''_{NN} \psi_{\eta} + \Theta \Psi''_{NN} (\Theta \Psi''_{NN} - \Delta)}{|K|} \leq 0.
\]

B.3. The make-or-buy decision under incomplete contracts.

With \( w^M_1 = w^M_2 > 0 \) we have \( \tilde{\ell} = 1 \) and thus \( \Lambda_C = 1 \). The variable payoff can be written as \( \Theta \Psi_{\text{open}} = \Lambda_R \Theta \Psi_{\text{closed}} \), and using the approach as in Appendix A.3.3 with \( \gamma \to 1 \), we have \( \frac{\partial \Psi'_{\text{open}}}{\partial \xi} |_{N = N_0} = \Lambda_R \frac{\partial \Psi'_{\text{closed}}}{\partial \xi} |_{N = N_0} > 0 \). Since \( \Lambda_R > 1 \) is independent of \( \xi \), it follows that \( \tilde{N}_{\text{open}} > \tilde{N}_{\text{closed}} \) which implies \( \tilde{\xi}_{\text{open}} \geq \tilde{\xi}_{\text{closed}} \).
Appendix C: Asymmetric components

In the following we provide an algorithm for the derivation of the discrete Shapley values in case of two asymmetric suppliers.

Step 1: Marginal contribution of a supplier: The coalition size is either \( n = 2 \) (headquarter and one supplier) or \( n = 3 \) (producer and both suppliers \( a \) and \( b \)). A coalition that does not contain the producer earns zero total revenue. Note that with \( n = 3 \) the marginal contribution of supplier \( i \) also depends on the other supplier’s organizational form. For the different coalition sizes \( n = 2 \) and \( n = 3 \) and organizational forms we can derive the following marginal contributions of supplier \( a \) (those for supplier \( b \) are analogous):

\[
\begin{align*}
MC_{a}^{O,ξ_b}(n = 2) &= \hat{H} \left( m_{a}^{O,ξ_b} \right)^{\alpha(1-\eta^H)} \frac{\alpha(1-\eta)^{1-\eta}}{\eta}.
MC_{a}^{V,ξ_b}(n = 2) &= \hat{H} \left( m_{a}^{V,ξ_b} \right)^{\alpha(1-\eta^H)} \frac{\alpha(1-\eta)^{1-\eta}}{\eta} - \left( (1 - \delta_{a}^{V}) m_{a}^{V,ξ_b} \right)^{\alpha(1-\eta^H)} \frac{\alpha(1-\eta)^{1-\eta}}{\eta}.
MC_{a}^{O,ξ_b}(n = 3) &= \hat{H} \left( \left( m_{a}^{O,ξ_b} \right)^{\epsilon \eta_{a}^{1-\epsilon}} + \left( m_{b}^{O,ξ_b} \right)^{\epsilon \eta_{b}^{1-\epsilon}} \right)^{\alpha(1-\eta^H)} \frac{\alpha(1-\eta)^{1-\eta}}{\eta}.
MC_{a}^{V,ξ_b}(n = 3) &= \hat{H} \left( \left( m_{b}^{V,ξ_b} \right)^{\epsilon \eta_{b}^{1-\epsilon}} \right)^{\alpha(1-\eta^H)} \frac{\alpha(1-\eta)^{1-\eta}}{\eta} - \hat{H} \left( \left( (1 - \delta_{a}^{V}) m_{a}^{V,ξ_b} \right)^{\epsilon \eta_{a}^{1-\epsilon}} + \left( m_{b}^{V,ξ_b} \right)^{\epsilon \eta_{b}^{1-\epsilon}} \right)^{\alpha(1-\eta^H)} \frac{\alpha(1-\eta)^{1-\eta}}{\eta}.
\end{align*}
\]

with \( \hat{H} \equiv \Theta \left( 1 - \eta^H \right)^{-\alpha(1-\eta^H)} (h/\eta)^{\alpha\eta^H} \).

Step 2: Average over all coalitions. For both coalition sizes there exist six permutations. For \( n = 2 \) the probability that player \( a \) is ordered after the headquarter is given by \( 1/6 \). For \( n = 3 \) the probability that player \( a \) is ordered after the headquarter and supplier \( b \) is \( 1/3 \). Hence, for a given organizational choice \( ξ \) the Shapley value of supplier \( a \) is

\[
\begin{align*}
\phi_{a}^{ξ_a,ξ_b}(n = 2) &= \frac{1}{6} MC_{a}^{ξ_a,ξ_b}(n = 2) + \frac{1}{3} MC_{a}^{ξ_a,ξ_b}(n = 3).
\end{align*}
\]

Step 3: Outsourcing vs. integration. As is clear from (37), the Shapley values depend on the input contributions which themselves depend on the normalized Shapley values via the revenue shares. This yields a system of equations that cannot be solved in closed form. However, using the implicit condition given by (37) we can conduct numerical simulations. Upon request we provide a mathematica file with the algorithm to conduct those numerical simulations.