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External Sovereign Debt in a Monetary Union: Bailouts and the Role of Corruption

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Abstract

We build a tractable stylized model of external sovereign debt and endogenous international interest rates. In corrupt economies with rent-seeking groups stealing public resources, a politico-economic equilibrium is characterized by permanent fiscal impatience which leads to excessive issuing of sovereign bonds. External creditors envision the corrupt economy’s fiscal impatience and buy its bonds at higher interest rates. In turn, this interest-rate increase exacerbates the problem of oversupplying debt, leading the economy to a perfect-foresight trap. In incorrupt countries which have entered a high-interest-rate/high debt-GDP-ratio trap because an immediately recent disaster has caused a sudden jump to a high outstanding debt-GDP ratio, we show that bailout plans with controlled interest rates can help in reducing debt-GDP ratios after some time. On the contrary, under corruption, we show that bailouts are ineffective unless rent-seeking groups are eradicated.

Keywords: sovereign debt, world interest rates, international lending, rent seeking.

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1. Introduction

The recent sovereign-debt crisis in the Euro zone has led to sovereign bailouts. One reason behind these bailouts is the fact shown in Figure 1: external-debt-GDP ratios in the Euro area are very high—in fact, among the highest in the world—and have increased substantially following the 2008 financial crisis, thereby triggering fears of default. Another reason for bailouts is the fear of contagion that can be triggered by a sovereign default in the EU through the highly integrated European banking system. A key initiating force that triggers sovereign default pressure and default fears, is a high country-specific interest rate set in international markets. A country that borrows at high interest rates makes the servicing burden of new debt socially unsustainable as it implies higher taxes and/or lower public consumption, reducing welfare. Thus, the stated rationale behind bailouts is the need to make the servicing costs of debt socially and politically bearable. So, bailout plans in the EU have focused on regulating such high interest rates in order to offset the negative impact of the latter on future policies of bailed-out countries. The interplay between international interest rates and the political economy of external-debt issuing is not well understood,

1 For a description of the EU bailout for Greece and further information regarding legal constraints for bailouts in the EU, see Buiter and Rahbari (2010), which is downloadable from: http://www.nber.org/~wbuiter/Greece.pdf

2 A glance at Figure 1 reveals that governments in the Euro area tend to issue external sovereign debt which is, (i) a substantial fraction of GDP, and (ii) increasing in the past few recent years. To one extent, this trend is due to a rapid globalization of capital markets in the past two decades. Yet, since the Euro zone is an almost perfectly integrated banking and capital-markets system, it is perhaps unsurprising that several countries within the Euro zone have the highest external-debt-GDP ratios appearing in Figure 1.

3 Euro area commercial banks typically hold a diversified portfolio of government bonds of several union countries and thus can be severely affected by a default through losses on these bonds. Bolton and Jeanne (2011) provide information on Euro area commercial banks foreign debt exposures as of 2010. Bolton and Jeanne (2011) develop a sovereign debt model that highlights the link from sovereign to banking crisis. The link is facilitated by the fact that government bonds serve as collateral for interbank loans and thus provide a funding source for investment and output. Acharya et. al (2011) argue and provide evidence that a crucial factor behind the rise in sovereign risk was the recent financial sector bailouts.

4 Indeed, Kohlscheen (2010) reports that the likelihood of external-debt defaults is higher than this for domestic-debt defaults. Since the debt-servicing burden is a key factor for both types of defaults, one of the direct sources of pressure for default is the high-level of interest rates formed in international markets.
causing vivid debates about bailout-plan designing.\(^5\) Our purpose here is to fill this gap.

We build a stylized model which emphasizes the political economy of external-sovereign-debt issuing and its interaction with international capital markets. Through this model we contribute to the debate on the effectiveness of bailout plans by making a distinction. We distinguish the permanent and structural nature of sovereign-debt traps in countries which suffer from corruption, from the terminable nature of sovereign-debt traps in incorrupt countries.

We demonstrate that, when rent-seeking groups are capable of consuming public resources, the politicoeconomic outcome exhibits permanent fiscal impatience: more resources are spent today and taxes are postponed for later. As a consequence, the rate at which the corrupt economy issues external sovereign debt is higher than the rate dictated by the rate of time preference of external creditors. Within a monetary union, this impatience discrepancy leads to higher country-specific real interest rates for the corrupt country. With higher cost of borrowing, excessive sovereign-debt issuing is exacerbated, leading the corrupt country to an excessively high sovereign-debt-GDP ratio which may be socially unsustainable.

The number of noncooperative rent-seeking groups plays a crucial role. More than one rent-seeking groups lead to higher fiscal impatience, due to a commons problem which aggravates the voracity effect on public resources.\(^6\) For example, if rent seeking is tightly attached to a polarized multi-partisan system, the impact of corruption on excessive sovereign debt issuing is stronger. If all rent-seeking groups cooperate in order to formulate a single “big mob”, then the fiscal impatience problem may be avoided. However, even under one “mob”, and even without the fiscal impatience problem, a high outstanding debt-GDP ratio gener-

\(^5\) Debates are not unjustifiable, since advanced economies in the EU have been, until recently, perceived as being insulated from sovereign default risk. As Reinhart and Rogoff (2008) report, the latest incidence of sovereign default in Europe is Austria in 1940, Germany in 1939, Poland in 1940 and Hungary in 1941.

\(^6\) For an introduction to the “voracity effect” caused by the presence of rent-seeking groups see, for example, Lane and Tornell (1996) and Tornell and Lane (1999).
ates higher utility losses for non-rent-seekers due to a higher discrepancy between taxes paid and public benefits enjoyed. So, even with a single rent-seeking group in action, corruption is a source of social dissatisfaction and fragility in cases of emergency debt-issuing due to disasters. Since our analysis is performed under perfect foresight, our model serves as a tool for demonstrating that corruption is a structural problem in a monetary union, needing to be treated as such a type of problem.

In incorrupt countries (with no rent-seeking groups), optimal fiscal policy is aligned with the rate of time preference of external creditors. This alignment implies no structural fiscal-policy pathologies. Yet, even without structural problems, if the outstanding debt-GDP ratio is exceptionally high, fragility may still arise. If external creditors think that the social-pressure burden of servicing a certain outstanding debt-GDP ratio is excessive, then they may expect a unilateral haircut as a means for alleviating this burden. On the one hand, it is such an anticipation of a haircut that leads to an increase in interest rates, in order to compensate investors. On the other hand, high interest rates make equilibrium debt-GDP ratios to increase even further, leading the economy to a trap.

Two key ingredients may trigger such a trap. First, it is a high debt-GDP ratio. One reason for inheriting a high initial outstanding debt-GDP ratio may be that a disaster occurred in the immediately recent past, requiring substantially high emergency public spending/borrowing. Second, a critical informational asymmetry between external creditors and a country’s government may arise after a disaster, leading to a miscoordination. Our model is deterministic, so we do not model explicitly information and we do not explain how a

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A recent study showing that expectations after a disaster can be over-reactive is Koulovatianos and Wieland (2011). In a model of rational learning about rare disasters, Koulovatianos and Wieland (2011) show that after a disaster occurs beliefs jump to pessimistic levels, and that they slowly recover to optimism after some time that disasters do not occur. In such a framework of learning, slight informational asymmetries at times of optimism can lead to major differences in optimism pessimism after a disaster occurs and after beliefs jump to new levels. If beliefs of others are unknown, the possibility of market failures is open.
country enters such a trap. Nevertheless, we describe the mechanics of such a trap: our model implies that even under optimal policy, high interest rates make debt-GDP ratios to increase over time, leading to fiscal divergence within the monetary union, and calling for some bailout plan.

Given the trap scenario described above, we study the success potential of an EU-type bailout. We focus on the level of interest rates that bailout plans may offer. For countries outside the trap, the key determinant of free-market international interest rates is the rate of time preference.\textsuperscript{8} We call these outside-trap free-market interest rates, “normal”. For countries inside a trap scenario, our model identifies a crucial threshold level of initial external-debt-GDP ratios: below this threshold, a short-duration bailout plan can offer above-“normal” interest rates and still be successful; above this threshold, a bailout plan must offer lower than “normal” interest rates, which implies a transfer of resources from the monetary union to the indebted country. By “success” of a bailout plan we mean that the external-debt-GDP ratio is reduced to a socially sustainable level after a period of time, which allows the domestic economy to return to free international bond markets.

In the case of corruption with more than one rent-seeking groups, the structural nature of fiscal impatience does not allow a government to be in international markets for a long period of time. Even if a bailout plan with lower than “normal” interest rates is implemented, and even if the corrupt country’s debt-GDP ratio falls to low levels, if left alone in free markets for some time, excessive debt-GDP ratios will return. A bailout plan in the case of corruption should aim at eliminating rent-seeking groups.

Our model is stylized. We assume away physical capital and uncertainty. Although it seems unnatural to study bailouts in a deterministic environment, our framework rules out

\textsuperscript{8} In our model we assume that the rate of time preference is the same between domestic taxpayers and external creditors.
known technical complexities. These technical complexities relate to optimal debt-maturity setting, state contingency, and other issues related to the arbitrage of risky assets. In turn, we are able to analyze the commons problem arising from the Markovian-Nash strategic behavior of rent-seeking groups while international interest rates are endogenous.

Another simplifying assumption we adopt is exogenous productivity growth. Although a substantial body of literature studies a potential two-way causality between external sovereign debt and growth, our model focuses on the impact of exogenous growth rates on the dynamics of sovereign debt and on international-market interest rates of government bonds.

A few recent papers explicitly explore various policy responses to sovereign default concerns in a monetary union. Daniel and Shiamptanis (2010) argue that the recent EU debt crisis primarily reflects a potential fiscal solvency crisis where the present value of fiscal surpluses are inconsistent with a sustainable long run debt trajectory. In their analysis, partial sovereign default arises as a policy response in order to restore fiscal solvency—though this response is ineffective without fiscal reforms that raise fiscal surpluses. Roch and Uhlig

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9 There is a substantial body of literature studying sovereign debt and risk of default, which makes simplifying assumptions in order to cope with these complexities. Most of these studies have built on the model of Eaton and Gerzowitz (1981) with a debt maturity of one period and financial autarky as a default penalty. Using this framework several papers try to explain the factors involved in government default, particularly the role of interest rates and output fluctuations. These papers include Aguiar and Gopinath (2006), Arellano (2008) and Guimaraes (2011). Two key studies explaining optimal maturity structure in economies without capital are Stokey and Lucas (1983) and Angeletos (2002).

10 A recent study focusing on the impact of rent-seeking politics and fiscal debt is Caballero and Yared (2010). In their model, rent-seeking governments alternate exogenously (randomly), and they borrow at a fixed international interest rate, following the example of other papers which use this fixed-interest-rate assumption. For example, Cuadra et.al (2010) analyze fiscal policy in the presence of sovereign default risk in emerging economies in a dynamic stochastic small open economy with constant interest rates. Other recent work studying fiscal policy in the presence of external debt under constant “world” interest rates, is one by Aguiar et. al (2009), who investigate optimal taxation of foreign capital and optimal sovereign debt policy without commitment where governments are more impatient than the private sector. Finally, another literature strand focuses on political instability and sovereign default in emerging countries. Examples are Cuadra and Sapriza (2008) and Alesina and Tabellini (1988).

11 External sovereign debt and its effects on growth have been studied by Cohen and Sachs (1986). A key empirical study on the interplay between external and growth is Reinhart and Rogoff (2010), while Reinhart and Rogoff (2008) contributes an impressive set of historical data set of sovereign default crises.
(2011) show that the existing EU bailout plans cannot be successful without strong fiscal retrenchment—their ultimate effect is simply to postpone default. The reason for this ineffectiveness is two-fold: the bailout allows governments to keep borrowing to finance public consumption and is offered at a relatively steep price (compared to the risk-free rate).

In Section 2 we describe the benchmark model of optimal fiscal policy. We add rent-seeking groups to this benchmark in Section 3. In Section 4 we study sovereign bailouts emphasizing the role of interest rates, and we comment on bailouts and corruption, offering some comments on the bailout of Ireland, Greece, Portugal and on the sovereign-debt prospects of Spain and Italy. We make some concluding remarks in Section 5.

2. Benchmark model of optimal fiscal policy

2.1 The domestic economy

2.1.1 Competitive equilibrium

Consider an economy populated by a large number of identical infinitely-lived agents of total mass equal to 1. A single composite consumable good is produced under perfect competition, using only labor as input through the linear technology,

\[ y_t = z_t \cdot l_t , \]  

in which \( y \) is units of output, \( l \) is labor hours, and \( z \) is productivity. Assume that productivity at time 0 is equal to 1 \( (z_0 = 1) \), and that it grows exogenously at rate \( \gamma \), i.e.,

\[ z_t = (1 + \gamma)^t . \]  

Assume no uncertainty, and assume that the representative agent draws utility from private consumption \( (c) \), leisure \( (1 - l) \), and also from the consumption of a public good, \( G \),
maximizing the life-time utility function

\[
\sum_{t=0}^{\infty} \beta^t \left[ \ln(c_t) + \theta_l \ln(1 - l_t) + \theta_G \ln(G_t) \right]
\]  

(3)

in which \( \beta \in (0, 1) \) is the utility discount factor, while \( \theta_l, \theta_G > 0 \) capture the weight on leisure and public consumption in the utility function respectively. The public good is financed via both income taxes and fiscal debt,

\[ G_t = B_{t+1} - (1 + r_t)B_t + \tau_t Y_t , \]  

(4)

in which \( B_t \) is fiscal debt in period \( t \)—assuming the government issues only one period zero coupon bonds, \( r_t \) is the interest rate for servicing the debt in period \( t \), \( Y_t \) is aggregate production in period \( t \), and \( \tau_t \) is the income tax rate in period \( t \). We assume all fiscal debt is external in period \( 0 \), i.e., that agents in this economy do not hold any government bonds. For simplicity, we also assume that agents cannot have access to domestic government bonds in the future, and that the consumable good is not storable, that there is no other form of capital. Under these assumptions, the budget constraint of an individual household is

\[ c_t = (1 - \tau_t)z_t l_t . \]  

(5)

The representative household maximizes its lifetime utility given by (3), subject to equation (5), by picking the optimal stream of consumption and labor supply throughout the infinite horizon, \( \{(c_t, l_t)\}_{t=0}^{\infty} \), for a given stream of tax rates and provided public-good quantities, \( \{(G_t, \tau_t)\}_{t=0}^{\infty} \). Since the solution to this problem is based on intra-temporal conditions only, we obtain a simple formula, namely,

\[ l_t = \frac{\theta_l}{1 + \theta_l} = L , \quad t = 0, 1, \ldots , \]  

(6)

and notice that market clearing implies that aggregate labor supply is equal to \( L \) as well. The reason why labor supply does not respond to changes in marginal tax rates is that
logarithmic utility makes the income and substitution effects of taxation on leisure cancel out each other. Combining (1) with (2) gives the competitive-equilibrium GDP level,

\[ Y_t = (1 + \gamma)^t L \quad (7) \]

### 2.1.2 Policy setting

The government chooses the optimal sequence of taxes and debt \( \{\{\tau_t, B_{t+1}\}\}_{t=0}^{\infty} \) in order to maximize the lifetime utility of the representative household (or a utilitarian social welfare since everyone is the same in this economy), subject to the fiscal-budget constraint given by (4), and for a given sequence of interest rates, \( \{r_t\}_{t=0}^{\infty} \), determined in international markets. Substituting the competitive-equilibrium solution given by equations (6) and (7) into (3), gives an indirect-utility function of the form,

\[ V(\{\{\tau_t, B_t\}\}_{t=0}^{\infty} \mid \{r_t\}_{t=0}^{\infty}) = \kappa_V + \]

\[ + \sum_{t=0}^{\infty} \beta^t \left\{ \ln (1 - \tau_t) + \theta_G \ln [B_{t+1} - (1 + r_t)B_t + \tau_t(1 + \gamma)^t L] \right\}, \quad (8) \]

in which \( \kappa_V \) is a constant that does not affect optimization regarding the determination of the optimal sequence \( \{\{\tau_t, B_{t+1}\}\}_{t=0}^{\infty} \). In Appendix A we show that first-order conditions \( \partial V / \partial \tau_t = 0 \) and \( \partial V / \partial B_{t+1} = 0 \), lead to the price-dependent \( \{\{r_t\}_{t=0}^{\infty}\} \) solutions,

\[ G_t = \theta_G (1 - \tau_t) Y_t, \quad (9) \]

\[ B_1 - (1 + r_0) B_0 = \phi(\tau_0) - L, \quad (10) \]

and

\[ B_t = B_0 \prod_{j=0}^{t-1} (1 + r_j) + \phi(\tau_0) \frac{1 - \beta^t}{1 - \beta} \prod_{j=1}^{t-1} (1 + r_j) - \]

\[ - \frac{L}{1 + \gamma} \left[ (1 + \gamma)^t + \sum_{s=1}^{t-1} (1 + \gamma)^s \prod_{i=s}^{t-1} (1 + r_i) \right], \quad \text{for all } t \geq 2, \quad (11) \]
in which
\[ \phi (\tau_0) \equiv (1 + \theta_G) (1 - \tau_0) L. \]

Equations (11) and (10) are only the general solution to the system of optimality conditions (shown in Appendix A), since the coefficient \( \phi (\tau_0) \) is still undetermined. By specifying the level of \( \tau_0 \), the level of government in period 0 and the level of outstanding fiscal debt in period 1 \( (B_1) \) are fully determined (see equations (9) and (10)). In the following section we analyze how the government specifies \( \tau_0 \) for a given stream of interest rates.

### 2.1.3 Fiscal solvency considerations and optimal government size

Given a fully pre-specified stream of future interest rates, \( \{r_t\}_{t=0}^{\infty} \), the government must make sure that \( \tau_0 \) is not too low, so that it can avoid fiscal insolvency. In addition, as implied by equation (9), the government must ensure that the current tax rate is not too high, so as to avoid entering a socially undesirable suboptimal path of under-providing the public good in which fiscal resources saved by low interest rates are wasted. Such considerations are resolved by meeting the fiscal-debt transversality condition,

\[
\lim_{t \to \infty} \frac{B_t}{\prod_{j=0}^{t-1} (1 + r_j)} = 0. \tag{12}
\]

Combining the transversality condition given by equation (12) with equation (11) determines coefficient \( \phi (\tau_0) \), namely,

\[
\phi (\tau_0) = (1 - \beta) [W_1 - (1 + r_0) B_0], \tag{13}
\]

in which \( W_1 \) is the present value of all future GDP levels, conditional on a given interest-rate stream \( \{r_t\}_{t=1}^{\infty} \). In other words, \( W_1 \) is the total worth of the private sector of the economy,
with\textsuperscript{12}
\[ W_1 = \lim_{t \to \infty} \frac{L}{1+\gamma} \left[ (1+\gamma)^t + \sum_{s=1}^{t-1} (1+\gamma)^s \prod_{i=s}^{t-1} (1+r_i) \right] . \tag{15} \]

After combining equations (9) and (13) we derive the optimal government size of fiscal spending as a percentage of GDP in period 0,
\[ \frac{G_0}{Y_0} = \frac{\theta_G}{1 + \theta_G} (1 - \beta) \left[ \frac{W_1}{Y_0} - (1 + r_0) \frac{B_0}{Y_0} \right] , \]
or, more generally, for all \( t = 0, 1, \ldots, \)
\[ \frac{G_t}{Y_t} = \frac{\theta_G}{1 + \theta_G} (1 - \beta) \left( \begin{array}{c}
\frac{z_t W_{t+1}}{Y_t} \\
\frac{B_t}{Y_t} \\
\frac{r_t B_t}{Y_t}
\end{array} \right) . \tag{16} \]

Equation (16) is intuitive. The term \( r_t B_t/Y_t \) shows how the current cost of servicing the debt decreases \( G_t/Y_t \). The term \( B_t/Y_t \) reveals that future taxes must pay back the outstanding sovereign debt-GDP ratio, which also contributes to reducing \( G_t/Y_t \) in the current period. Finally, the term \( z_t W_{t+1}/Y_t \) contains all future interest rates, \( \{r_s\}_{s=t+1}^{\infty} \). Equation (15) reveals that higher future interest rates reduce current economy’s worth (\( z_t W_{t+1}/Y_t \)) through increasing the future interest burden for servicing sovereign debt, decreasing \( G_t/Y_t \) as well.

It remains to determine the level of interest rates in any future period. In order to do so, we model external creditors in a way that continues to offer analytical tractability to international-equilibrium setting, and to further applications. The next two sections present the external-creditors model and interest-rate determination.

\textsuperscript{12}The general form of (15) is,
\[ W_{t+1} = \lim_{k \to \infty} \frac{L}{1+\gamma} \left[ (1+\gamma)^k + \sum_{s=t+1}^{k-1} (1+\gamma)^s \prod_{i=s}^{k-1} (1+r_i) \right] . \tag{14} \]
2.2 The external creditors

We denote all external-creditor variables using a star. For simplicity, assume that external creditors only hold bonds, and maximize their total life-time utility given derived from consumption,

\[ \sum_{t=0}^{\infty} \beta^t \ln (c_t^*) \]  

subject to the budget constraint,

\[ B_{t+1}^* = (1 + r_t) B_t^* - c_t^* . \]

Notice that the rate of time preference, \((1 - \beta) / \beta\), in the utility function of creditors, \((17)\), is equal to the rate of time preference of domestic households.

The solution to the problem of maximizing \((17)\) subject to \((18)\) is,

\[ c_t^* = (1 - \beta) \beta^t \prod_{i=0}^{t} (1 + r_i) B_0^* , \]

which implies that

\[ B_1^* = \beta (1 + r_0) B_0^* . \]

Equation \((19)\) determines the demand for bonds by external creditors in period 1. Logarithmic preferences are responsible for this compact algebraic solution given by \((19)\), which implies that demand for external debt in period 1 depends only on the return of bonds in period 0. In the following section we combine \((19)\) with domestic supply of bonds in order to determine interest rates.

2.3 Determining interest-rate levels

Interest-rate levels are determined by equalizing demand and supply of government bonds. In particular, demand for bonds one period ahead, \(B_1^*\), is given by equation \((19)\). Supply
of bonds is given by combining the optimal level of government spending with the fiscal-budget constraint. To determine supply of bonds in period 0 we combine equation (10) with equation (13) to obtain,

\[ B_1 = \beta (1 + r_0) B_0 + (1 - \beta) W_1 - L. \]  

(20)

After applying the equilibrium condition \( B_1 = B_1^* \), and given that \( B_0 = B_0^* \), equations (20) and (19) imply,

\[ W_1 = \frac{L}{1 - \beta}. \]  

(21)

Since our analysis can be generalized to all \( t \in \{0, 1, \ldots\} \), equation (21) can also be generalized to,

\[ W_{t+1} = \frac{L}{1 - \beta}, \quad t = 0, 1, \ldots. \]  

(22)

It is straightforward to see from equation (15) that the sequence \( \{W_{t+1}\}_{t=0}^{\infty} \) satisfies the recursion,

\[ W_{t+1} = \frac{1 + \gamma}{1 + r_{t+1}} W_{t+2} + L, \quad t = 0, 1, \ldots. \]  

(23)

After substituting (22) into (23) we obtain,

\[ r_{t+1} = r^{ss} = \frac{1 + \gamma}{\beta} - 1, \quad t = 0, 1, \ldots, \]  

(24)

in which the symbol “\( r^{ss} \)” denotes a steady-state level of international interest rates. As we show in the next section, since \( \beta (1 + r^{ss}) / (1 + \gamma) = 1 \), having \( r_t = r^{ss} \) for all \( t \in \{0, 1, \ldots\} \), implies a balanced-growth steady state in the domestic economy, characterized by a constant tax rate, debt-GDP ratio, and public-consumption-GDP ratio throughout the whole horizon.

Equation (24) determines the entire sequence of interest rates except for this in period 0, which is indeterminate. By substituting equation (22) into (16), the optimal government size of fiscal spending as a percentage of GDP in period 0 is,

\[ \frac{G_0}{Y_0} = \frac{\theta_G}{1 + \theta_G} \left( \frac{1}{1 - \beta} - \frac{B_0}{Y_0} - \frac{B_0}{Y_0} \right). \]  

(25)
2.3.1 Debt-GDP-ratio dynamics in the benchmark model

As explained in the previous section, international capital-markets equilibrium in our benchmark model implies that the interest in period 0, \( r_0 \), is indeterminate, while \( r_t = r^{ss} \) for all \( t \in \{1, 2, \ldots\} \). So, let the interest rate \( r_0 \) take any value \( r_0 > -1 \). For notational simplicity, let the debt-GDP ratio and the public-consumption-GDP ratio be denoted by,

\[
b_t \equiv \frac{B_t}{Y_t} \quad \text{and} \quad g_t \equiv \frac{G_t}{Y_t}, \quad t = 0, 1, \ldots.
\]

After combining equations (20) and (21),

\[
b_1 = \frac{\beta (1 + r_0)}{1 + \gamma} b_0 . \tag{26}
\]

Considering the generalization of the fiscal budget constraint in domestic equilibrium, given by equation (20) for \( t \in \{1, 2, \ldots\} \), i.e. \( B_{t+1} = \beta (1 + r_t) B_t + (1 - \beta) z_t W_{t+1} - Y_t \), and substituting the international-equilibrium interest rates, \( r_t = r^{ss} \) for all \( t \in \{1, 2, \ldots\} \), we obtain,

\[
b_t = b_1 , \quad t = 1, 2, \ldots . \tag{27}
\]

Equations (26) and (27) imply that the debt-GDP ratio stays at the same level from period 1 and on, while its level in period 1 depends on \( r_0 \). In Appendix A we show that one optimality condition is \( \tau_t = 1 - [\beta / (1 + \gamma)]^t \Pi_{j=1}^t (1 + r_j) \cdot (1 - \tau_0) \), which implies that, since \( r_t = r^{ss} \) for all \( t \in \{1, 2, \ldots\} \),

\[
\tau_t = \tau_0 , \quad t = 0, 1, \ldots , \tag{28}
\]

no matter if \( r_0 \neq r^{ss} \) or not. In addition, (9) and (28) imply that

\[
g_t = g_0 , \quad t = 0, 1, \ldots , \tag{29}
\]

for any \( r_0 \) as well, no matter if \( r_0 \neq r^{ss} \) or not. So, equations (28) and (29) imply that, in international equilibrium, there is a smoothing of taxes and public-consumption-GDP
ratios. Yet, as $r_0$ increases, equation (25) implies that $g_0$ goes down, and, as a consequence, equation (9) implies that $\tau_0$ increases. Finally, equation (26) shows that an increase in $r_0$ causes $b_1$ to increase as well, which adds to the debt burden of the domestic economy and enhances the need for a permanent increase in tax rates and for a permanent drop in public-consumption-GDP ratios.

A remarkable benchmark is to set,

$$r_0 = r^{ss}.$$ \hfill (30)

Under (30), equation (26) implies that,

$$b_t = b_0, \quad t = 0, 1, \ldots,$$ \hfill (31)

while (25) and (9) imply,

$$g_0 = \frac{\theta_G}{1 + \theta_G} \left[ 1 - \frac{1 - \beta}{\beta} (1 + \gamma) b_0 \right] \quad \text{and} \quad \tau_0 = 1 - \frac{g_0}{\theta_G}. \hfill (32)$$

The benchmark given by (31) determines the level of current international interest rates (with $r_0$ being given by (30)) which guarantees that the domestic country’s debt-to-GDP ratio remains constant throughout the infinite horizon.

3. Corruption: rent-seeking groups, the voracity effect and external sovereign debt

Here we extend our setup by including rent-seeking groups which have influence on fiscal-budget setting and manage to consume resources for themselves. We focus on deriving the Markov-Nash equilibrium of our setup with rent-seeking groups and we study the long-run impact of these groups on fiscal policy. We demonstrate that the ability of rent-seeking groups to consume resources leads to a form of a “voracity effect” on public spending which
implies higher tax rates, lower public-good levels available to society, and excess sovereign-debt issuing in the long run.\textsuperscript{13}

3.1 Model with rent-seeking groups

We begin with the model developed in Section 2 and introduce $N$ rent-seeking groups in the domestic economy, which have the power to expropriate resources through paying lower taxes. In each period $t \in \{0, 1, \ldots\}$, a rent-seeking group $j \in \{1, \ldots, N\}$ manages to extract a total rent of size $C_{j,t}^R$. Let the population mass of each rent-seeking group be $\mu_j$ with $\sum_{j=1}^{N} \mu_j < 1$, and let all households participating in a rent-seeking group $j$ be identical within the group for all $j \in \{1, \ldots, N\}$, and having equal shares of private individual rents, $c_{j,t}^R$. So, the total rent expropriated by group $j$ in the form of private consumption is,

$$C_{j,t}^R = \mu_j \cdot c_{j,t}^R ,$$

and this rent is distributed equally among the rent-seeking group’s individual members in the form of tax benefits. So, the government’s fiscal-budget constraint is,

$$B_{t+1} = (1 + r_t) B_t + G_t + \sum_{j=1}^{N} C_{j,t}^R - \tau_t Y_t ,$$

For simplicity we assume that participation in a rent-seeking group is exogenous and costless (e.g., inherited participation). Yet, we assume that private consumption of a rent-seeking individual household, $c_t$, and the rent $c_{j,t}^R$ are considered to be separate goods. The reason behind this distinction is that individual rents of a member of group $j \in \{1, \ldots, N\}$, $c_{j,t}^R$, appear in the form of private benefits that differ from the typical consumer basket. Yet, we assume that the price per unit of $c_{j,t}^R$ is equal to the cost of the consumer basket a-priori, for all $j \in \{1, \ldots, N\}$ and for all $t \in \{0, 1, \ldots\}$.

\textsuperscript{13}The term “voracity effect” has been introduced byLane and Tornell (1996) and Tornell and Lane (1999). Tornell and Lane (1999) have argued that, when some economies lack strong legal political institutions, they may experience a more than proportional redistribution of fiscal resources after a positive shock to their income, decreasing growth. Their mechanism relies upon rent-seeking groups stealing public resources.
An individual rent-seeker who is a member of \( j \in \{1, ..., N\} \) does not control the level of consumption \( c_{j,t}^R \) that she obtains from the group through any form of private effort or cost.\(^{14}\) So, the utility function of this individual rent-seeker of group \( j \) is,

\[
\sum_{t=0}^{\infty} \beta^t \left[ \ln(c_{j,t}) + \theta_l \ln(1 - l_{j,t}) + \theta_G \ln(G_t) + \omega_j \ln(c_{j,t}^R) \right],
\]

with \( \omega_j > 0 \), and her economic problem is maximizing (35) subject to the budget constraint

\[
c_{j,t} = (1 - \tau_t) z_t l_{j,t}.
\]

The problem of any non-rent-seeker is this of maximizing (3) subject to (5), exactly as in the benchmark model of Section 2. Optimal choices are given by,

\[
l_{j,t} = l_t = \frac{\theta_l}{1 + \theta_l} = L, \quad t = 0, 1, ..., \]

while

\[
c_{j,t} = c_t = (1 - \tau_t) z_t L.
\]

Since labor supply is identical across rent seekers and non rent seekers, \( Y_t = z_t \cdot L \), as it has also been in the benchmark model presented in Section 2.

### 3.2 Influence of rent-seeking groups in politics: symmetric Markov-Nash equilibrium

In the benchmark model, the idea of maximizing social welfare in an economy with identical households reflects the need of political support for any proposed policies. In the presence of rent-seeking groups, policy is set through rent-seeking groups. In particular, rent-seeking groups compete noncooperatively with other groups for rents, while ensuring that they have the support of the broader public. In order to gain the support of the broader public,
each rent-seeking group \( j \in \{1, \ldots, N\} \) maximizes a convex combination of, (i) the sum of individual utilities of non-rent seekers and, (ii) the group’s utility derived by the stream of the group’s consumption \( C_{j,t}^R \). So, rent-seeking group \( j \in \{1, \ldots, N\} \) picks the stream \( \{C_{j,t}, \tau_t, B_{t+1}\}_{t=0}^\infty \) that maximizes the utility function

\[
\hat{V}_j \left( \left\{ C_{j,t}^R, \tau_t, B_t \right\}_{t=0}^\infty \mid \left\{ C_{i,t}^R \right\}_{i \neq j}^N, \{r_t\}_{t=0}^\infty \right) = \kappa_{V_j} + \\
+ \sum_{t=0}^\infty \beta^t \left[ \ln (1 - \tau_t) + \theta_G \ln (G_t) + \theta_{R,j} \ln (C_{j,t}^R) \right]
\]

subject to,

\[
G_t = B_{t+1} - (1 + r_t)B_t - \sum_{i=1 \atop i \neq j}^N C_{i,t}^R (B_t, Y_t \mid \{r_s\}_{s=t}^\infty) - C_{j,t}^R + \tau_t Y_t ,
\]

and

\[
Y_t = z_t L = (1 + \gamma)^t L ,
\]

in which \( \kappa_{V_j} \) is a constant which does not affect optimization, the term, \( \ln (1 - \tau_t) + \theta_G \ln (G_t) \), is the policy-dependent variable component of non-rent-seeker momentary indirect utility, while \( \theta_{R,j} > 0 \) is the weight placed on utility derived from the group’s extracted rent. The set \( \{C_{i,t}^R\}_{i \neq j} \) is the Markov-Nash strategies of type \( C_{i,t}^R = C_{i,t}^R (B_t, z_t \mid \{r_s\}_{s=t}^\infty) \) by all other rent-seeking groups. We also assume symmetry in political influence and size of groups, namely:

\[
\theta_{R,j} = \theta_R \quad \text{and} \quad \mu_j = \mu \quad \text{for all} \ j \in \{1, \ldots, N\} .
\]

Definition 1 gives the concept of domestic political equilibrium with rent-seeking groups that we employ.

**Definition 1**  Given a stream of interest rates, \( \{r_t\}_{t=0}^\infty \), a Markov-Perfect Nash Political Equilibrium (MPNPE) is a set of strategies, \( \{C_{i,t}^R\}_{i=1}^N \) of the form \( C_{i,t}^R = \)
and a set of policy decision rules \( \{ T, B \} \) of the form \( \tau_t = T(B_t, z_t \mid \{ r_s \}_{s=t}^{\infty}) \) and \( B_{t+1} = B(B_t, z_t \mid \{ r_s \}_{s=t}^{\infty}) \), such that each and every rent seeking group \( j \in \{1, \ldots, N\} \) maximizes (39) subject to (34), (41), \( \{ T, B \} \), and \( \{ C^{i,R} \}_{i \neq j} \).

Markov-perfect games can be expressed in a recursive format through the use of Bellman equations.\(^{15}\) Proposition 1 provides a MPNPE in linear strategies.

**Proposition 1** Given a stream of interest rates, \( \{ r_t \}_{t=0}^{\infty} \), there is a symmetric MPNPE equilibrium given by,

\[
C^{i,R}(B_t, z_t \mid \{ r_s \}_{s=t}^{\infty}) = \xi_R \cdot [z_t W_{t+1} - (1 + r_t) B_t]
\]

for all \( i \in \{1, \ldots, N\} \), in which \( W_{t+1} \) is given by (14).

\[
\xi_R = \frac{(1 - \beta) \theta_R}{1 + \theta_G + \theta_R + (N - 1)(1 - \beta) \theta_R},
\]

and also

\[
B_{t+1} = B(B_t, z_t \mid \{ r_s \}_{s=t}^{\infty}) = \beta_N (1 + r_t) B_t + (1 - \beta_N) z_t W_{t+1} - Y_t,
\]

in which

\[
\beta_N = \frac{1 + \theta_G + \theta_R}{1 + \theta_G + \theta_R + (N - 1)(1 - \beta) \theta_R} \beta,
\]

while

\[
\tau_t = T(B_t, z_t \mid \{ r_s \}_{s=t}^{\infty}) = 1 - \frac{1}{\theta_G Y_t} G_t,
\]

in which

\[
\frac{G_t}{Y_t} = \frac{\theta_G}{1 + \theta_G + N \theta_R} (1 - \beta_N) \left[ z_t W_{t+1} - (1 + r_t) \frac{B_t}{Y_t} \right].
\]

\(^{15}\)In order to formulate Bellman equations, the value function of each player would be of the form \( V^j(B_t, z_t \mid \{ C^{i,R} \}_{i \neq j}, T, B, \{ r_s \}_{s=t}^{\infty}) \). In the Proof of Proposition 1, which appears in Appendix B, we use Lagrangians and not Bellman equations.
In the following section we use the results of Proposition 1 in order to examine the possibility of sovereign traps by paying special attention to the formation of international interest rates.

### 3.3 Determination of international interest rates and the possibility of sovereign traps

The key insight offered by Proposition 1 is the introduction of a new discount factor, $\beta_N$, which is related to $\beta$ through equation (46). Equation (46) reveals that if $N \geq 2$, then $\beta_N < \beta$, i.e. the domestic economy becomes collectively more impatient due to the strategic voracity effect of interest groups. Since $\beta$ is the discount factor of external creditors, this voracity effect on collective patience reflected on $\beta_N$ creates a tendency to oversupply bonds compared to the case of $N = 1$, or compared to the case of no corruption. The following section distinguishes the case of $N = 1$ from this of $N \geq 2$, and demonstrates that the voracity effect on bond supply leads to high interest rates and explosive debt-GDP ratios.

#### 3.3.1 One big “mob”

In the case of $N = 1$, the discount factor $\beta_N = \beta$. Equating external-creditor demand for bonds, which is given by (19), with domestic supply of bonds, given by (45) with $\beta_N$ replaced by $\beta$, leads to equation (22) and consequently to (24), i.e. $r_{t+1} = r^{ss}$ for all $t \in \{0, 1, \ldots\}$. Setting also $r_0 = r^{ss}$ gives equation (31) of Section 2, namely that, $b_t = b_0$ for all $t \in \{0, 1, \ldots\}$. Yet, unlike equation (32) in Section 2, equations (48) and (47) imply

$$g_0 = \frac{\theta_G}{1 + \theta_G + \theta_R} \left[ 1 - \frac{1 - \beta}{\beta} (1 + \gamma) b_0 \right] \quad \text{and} \quad \tau_0 = 1 - \frac{g_0}{\theta_G} , \quad (49)$$
while \( g_t = g_0 \) and \( \tau_t = \tau_0 \) for all for all \( t \in \{0,1,...\} \). Equation (49) implies that, in the presence of one rent-seeking group, steady-state public consumption as a share of GDP is lower compared to this of the benchmark model of Section 2. According to (49), the steady state tax rate is also higher.

### 3.3.2 More than one noncooperative rent-seeking groups

The case with \( N \geq 2 \) rent-seeking groups is more complicated. Proposition 2 summarizes the dynamics of interest rates.

**Proposition 2** If \( N \geq 2 \), then

\[
b_{t+1} = \frac{\beta (1 + r_t)}{1 + \gamma} b_t , \quad \text{for all } t \in \{0,1,...\} ,
\]

(50)

\[
r_t > r^{ss} \text{ and } b_{t+1} > b_t , \quad \text{for all } t \in \{1,2,...\} ,
\]

(51)

with

\[
\lim_{t \to \infty} r_t = r^{ss} \text{ and } \lim_{t \to \infty} b_t = \frac{1}{(1 - \beta) (1 + r^{ss})} = \frac{\beta}{(1 - \beta) (1 + \gamma)} ,
\]

(52)

and

\[
\frac{1 - \beta}{\beta} (\beta - \beta_N) b_t + \frac{\beta_N}{1 + \gamma} b_t = \frac{1}{1 + r_t} , \quad \text{for all } t \in \{1,2,...\} ,
\]

(53)

while \((b_0, r_0)\) satisfy,

\[
(1 + r_0) b_0 = \frac{1 + \gamma}{\beta} b_1 < \frac{1}{1 - \beta} .
\]

**Proof** See Appendix B. \( \square \)

In order to calculate the interest rates in this case, we can apply backward induction using equations (53) and (50). One way is to start with \( b_t = b^{ss} \), and then to use \( b_{t-1} = b^{ss} - \varepsilon \) for
some $\varepsilon > 0$ arbitrarily small, and substituting this value for $b_{t-1}$ in equation (53). Then, we can continue backwards using (53) and (50).\textsuperscript{16}

The key message given by Proposition 2 is that if the domestic economy remains in free markets forever, then the rent-seeking groups increase the debt-GDP perpetually. The limiting level of the debt-GDP ratio corresponds to exhausting the domestic economy’s total wealth-to-GDP ratio, when the latter is evaluated at the asymptotic level of interest rates, $r^{ss}$, i.e. $b_t \to 1/ [(1 - \beta) (1 + r^{ss})]$. This result holds for all initial debt-GDP levels $b_0$ with $b_0 < 1/ [(1 - \beta) (1 + r^{ss})]$.

A commons problem which arises through rent-seeking in politics is the key source leading to our results. With more than one non-cooperating rent-seeking groups, there is over-exploitation of public resources. This over-exploitation tendency leads to a form of collective impatience, which is reflected in the domestic government’s discount factor $\beta_N$. With $\beta_N < \beta$ whenever $N \geq 2$, the free-capital-markets outcome under prefect foresight is an infinitely-long stream of high interest rates ($r_t > r^{ss}$ for all $t \in \{1, 2, \ldots\}$). These high interest rates lead to a perpetual increase in the debt-GDP ratio. We return to these dynamics in our discussion of sovereign bailouts in Section 4.

4. Debt-GDP thresholds and sovereign bailouts

Our framework is capable of isolating how the recent international-market pressures on the cost of financing the newly issued sovereign debt affect optimal sovereign debt policy. These international interest-rate pressures have been particularly strong for the newly issued debt of Greece, Ireland, and other countries of the EU periphery such as Portugal and Spain (bond spreads in relation to equilibrium interest rates of German government bonds), after the recent financial crisis of 2008.

\textsuperscript{16}In the proof of Proposition 2, we provide an explicit solution to this backward induction procedure.
Although a formal theory that explains bond spreads would require the explicit modeling of default risk, default expectations, and speculator informational asymmetries, such an analysis is beyond the scope of our study.\footnote{For a formal theory of expectations about disaster risk and rational learning about the likelihood of rare disasters, see, for example, Koulovatianos and Wieland (2011).} Here, we are interested in delivering a simple framework that enables us to evaluate bailout plans through paper-and-pencil methods. The focus of our analysis is to emphasize the fiscal pressure caused by the level of internationally formed interest rates. To this end, a deterministic environment allows us to avoid complexities such as the determination of the optimal maturity structure of bonds, or the incorporation of expectations asymmetries in a dynamic game between the domestic economy and external creditors.\footnote{A study presenting an analytically solved example that demonstrates the complexity of dynamic games under imperfect information is Koulovatianos (2010).}

4.1 A trap of high international sovereign interest rates

Throughout this section we assume that the underlying model is that of optimal-fiscal policy, presented in Section 2. In other words, we abstract from rent-seeking groups in our main discussion of traps and bailouts and we return to this extension in a later section which examines bailouts under corruption. We focus on a scenario in which a country, perhaps after a disaster in its immediately recent past, finds itself with a high outstanding external-sovereign-debt-GDP ratio in period 0. If such a debt-GDP ratio, $b_0$, is higher than a certain threshold, $\kappa$, then we assume that external creditors (asymmetrically) think of this country as being constrained.\footnote{Policy makers and analysts continue to use simple rules of thumb to judge risks and to assess fiscal sustainability (see Mody and Saravia, 2003). Reinhart and Rogoff (2010) and Manasse and Roubini (2008) have identified explicit external debt to GDP thresholds using empirical data for emerging economies that are associated with default. More specifically, using a novel methodology, Manasse and Roubini (2009) identify several important indicators that predict future sovereign defaults. Most notably their analysis identifies external debt to GDP ratio (exceeding 50%) as one of the most important predictors of the majority of sovereign defaults episodes in their sample.} In particular, external creditors think that, even if interest rates
from period 1 onwards are $r^{ss}$, there is high disutility due to the discrepancy between taxes and spending ($t_t - g_t = r^{ss}b_t = r^{ss}b_1$ for all $t \in \{1, 2, \ldots\}$, and both private and public consumption are low). External creditors believe that this disutility creates social pressure for partial default, i.e., a haircut of magnitude $b_0 - \kappa$, so as to return to the socially bearable debt-GDP ratio, $\kappa$. So, conditional on any anticipated stream of interest rates from period 1 and on, $\{r_t\}_{t=1}^{\infty}$, equation (19) implies that the external-creditors demand of domestic bonds is,

$$ \hat{B}_1^* = \beta (1 + r_0) \lambda_0 B_0^* , $$

in which $\lambda_0$ is the post-haircut fraction of outstanding debt, given by,

$$ \lambda_0 = \begin{cases} 
1 - (b_0 - \kappa) & \text{if } b_0 \geq \kappa \\
1 & \text{else} 
\end{cases} , $$

and $\hat{B}_1^*$ denotes external-creditor demand that incorporates the anticipation of a haircut, and $\hat{\kappa} \geq \kappa$.  

Domestic supply for bonds is still given by equation (20), i.e., $B_1 = \beta (1 + r_0) B_0 + (1 - \beta) W_1 - L$. Even if the domestic government does not intend to make a haircut of magnitude $1 - \lambda_0$, the interest rate in period 0 must increase. If the previously anticipated interest rate was $r_0 = r^{ss}$, the price of 1-period zero-coupon bonds, $P^{ss}$, must change according to $\hat{P} / P^{ss} = \lambda_0$, in which $\hat{P}$ is the price that incorporates the anticipation of a haircut, while $P^{ss}$ does not incorporate such an expectation and corresponds to intrinsic bond return equal to $r^{ss}$. With $P^{ss} = 1 / (1 + r^{ss})$, and $\hat{P} = 1 / (1 + \hat{r}_0)$, the new gross-effective interest

\[^{20}\text{Note that equation (16) implies lower public good provision with increases in the debt to GDP ratio.}\]

\[^{21}\text{We also assume that } b_0 - \hat{\kappa} < 1. \text{ The reason why } \hat{\kappa} \text{ may be higher than } \kappa \text{ is that external creditors may still require a premium for some period after a partial default. Alternatively, if } \hat{\kappa} = \kappa, \text{ external creditors may continue to buy bonds. This is a simplification. One may think of such an assumption in two ways: (a) the default risk premium is proportional to the haircut although in principle it can be higher to the point where the bond price is substantially lower, or (b) the country will honor future debt obligations in the future despite defaulting today. Our key goal here is to demonstrate the sovereign trap, and we leave the explicit pricing of bonds to models with uncertainty and information asymmetries.}\]
rate is,
\[ 1 + \hat{r}_0 = \frac{1 + r_{ss}}{\lambda_0} . \] (56)

If the domestic government starts with \( b_0 > \kappa \), and (erroneously) anticipates that \( \{r_t = r_{ss}\}_{t=1}^{\infty} \), then equation (21) implies that \( W_1 = L/(1 - \beta) \).\(^{22}\) In turn, equation (20) implies that \( B_1 = \beta (1 + \hat{r}_0) B_0 \). So, \( b_1 = \beta (1 + \hat{r}_0) b_0/(1 + \gamma) \), and after this equation is combined with (56) and (24), it is,
\[ b_1 = \frac{1}{\lambda_0} b_0 > b_0 > \kappa . \] (57)

Under this scenario, equation (57) shows that the domestic government intensifies its problem in period 1, and the interest rate in period 1, \( \hat{r}_1 \) is expected to rise to a higher level than \( \hat{r}_0 \), since external creditors anticipate an even bigger haircut of size \( b_1 - \kappa > b_0 - \kappa \). This is a simple demonstration of a trap of high international sovereign interest rates which may, indeed, force the domestic government to default within a short period of time.\(^{23}\)

Countries participating in a monetary union such as the Euro zone have the special feature that they cannot smooth out the effects of such a trap through loose monetary policy. For countries outside a monetary union, one traditional way of achieving this monetary smoothing is through the domestic government selling bonds to domestic commercial banks

\(^{22}\)Such an error may be due to an asymmetry in disaster-risk expectations between the domestic government and external creditors. One example of a study which shows how disaster-risk expectations under informational imperfections can be formulated in continuous time is Koulovatianos and Wieland (2011). Concerning the direction of change of interest rates, in a deterministic environment the term structure is flat, so arbitrage would lead to this new intrinsic annual return, \( \hat{r}_0 \), for bonds of any maturity. In a model with uncertainty and imperfect information the term structure need not be flat, but the interest-rate burden should change towards the same direction for bonds of all maturities. For the purposes of our analysis here, we can rationalize the outcome of \( \hat{r}_0 \) through an implicit assumption that there is a momentum of having different expectations between external creditors and the domestic economy. An example of an early study that models informational asymmetries between governments and external creditors explicitly is this of Cole et al. (1995).

\(^{23}\)Cole and Kehoe (2000) formally illustrate how self-fulfilling default expectations can lead to sovereign debt crises. The crisis ensues when creditors lose confidence in the government’s ability to roll over debt. In their model, market beliefs can trigger a default when a country’s fundamentals such as the level, maturity structure of government debt and capital stock are within a crisis zone. One of the important insights generated by their model is the role played by the maturity structure of debt, with longer maturities shrinking the region of self-fulfilling defaults.
which are partly owned by the state. A high supply of bonds that carry high default risk to the domestic banking system obliges the domestic central bank to buy these bonds through the issuing of more domestic currency to the system, an inflationary policy. Following such a strategy is not possible for countries within a monetary union, even partially.\footnote{Even for a country outside a monetary union with high outstanding external sovereign debt, such a strategy of selling bonds to its domestic state-owned commercial banks has its limits. This strategy leads to high inflation, which can cause the domestic currency to depreciate. Domestic currency depreciation would increase the monetary value of external debt in foreign currencies which become more and more expensive.} For this reason, sovereign-debt bailouts arise as natural solutions for such entrapped countries within a monetary union, in order to avoid risks of bank runs. The following section assesses the success potential of such plans subject to the level of initial outstanding external sovereign debt, and subject to bailout-plan duration and to the level of commonly agreed interest rates.

### 4.2 Sovereign bailout plans

In case countries enter a high-interest-rate trap as described in the previous section, sovereign bailout plans offer financing with controlled interest rates over a certain period of time. The goal of a bailout plan is to assist a country to reach a certain sovereign debt-GDP ratio, say, below a threshold level $\kappa$, which allows the country to return to free international capital markets in order to borrow.\footnote{We assume that, due to the risk of banking-risk contagion within a monetary union, bailout plans do not allow for sovereign-debt haircuts. So, we focus on bailout plans in the form of the rescue package for Greece in May 2010.} Since sovereign states are independent democracies, policy setting is endogenous. Policy endogeneity is the most demanding aspect of evaluating the success prospects of bailout plans. Our model fills this gap and can serve as a simple vehicle in order to carry out an analysis of the success prospects of a bailout plan.\footnote{Our model abstracts from the endogeneity of growth rates, and especially from the connection between tax burdens and growth prospects. This connection, in turn, can affect default risk, and the success prospects of a bailout plan. These elements may not be the most crucial in evaluating the success potential of bailouts. Nevertheless, the explicit modeling of growth rates and default risk are useful extensions for future work.}
Let the duration of a bailout plan be $T$ years with a fixed and controlled interest rate, $r_b$. In such a setting, starting from period 0, the bailout plan is effective until period $T - 1$, and its goal is to achieve

$$b_T \leq \kappa,$$

which can allow the country to return to free capital markets from period $T$ and on. So, the bailout plan must specify parameters $T$ and $r_b$.\footnote{In practice, the IMF-EU plan for Euro-zone countries specifies a more explicit plan of monitoring the progress of policy-making in the bailed-out country. Yet, each democratic country is free to opt out from the plan, which makes the analysis of optimal policy the most crucial constraint captured by our model.} In Appendix C we show that for all bailout periods, $t \in \{0, \ldots, T - 1\}$, the law of motion for the debt-GDP ratio is

$$b_{t+1} = \alpha b_t + (1 - \alpha) \left[ 1 - \left( \frac{1 + \gamma}{1 + r_b} \right)^{T-t} \right] \frac{1}{r_b - \gamma},$$

in which,

$$\alpha \equiv \frac{\beta (1 + r_b)}{1 + \gamma}.$$

Equation (59) implies that, for each $t \in \{0, \ldots, T - 1\}$, the intersection of the line implied by (59) with the 45-degree line in the $(b_t, b_{t+1})$ space is,

$$\bar{b}_t = \left[ 1 - \left( \frac{1 + \gamma}{1 + r_b} \right)^{T-t} \right] \frac{1}{r_b - \gamma}.$$

Given equation (60), a distinction is crucial: whether $r_b$ is greater than $r^{ss}$ or not. If $r_b > r^{ss}$, then the bailout plan must secure that $b_t < \bar{b}_t$ for all $t \in \{0, \ldots, T - 1\}$ in order to guarantee that the domestic government’s optimal debt-GDP ratio, $b_t$, decreases over time. A geometric analysis demonstrates that, in case $r_b > r^{ss}$, the requirement for a decreasing optimal debt-GDP ratio over time is having

$$b_t < \bar{b}_t,$$

for all $t \in \{0, \ldots, T - 1\}$, since allowing $b_t > \bar{b}_t$ for some $t \in \{0, \ldots, T - 1\}$ will lead to an increasing debt-GDP ratio after that ($b_{t+1} > b_t$ for all $t \geq \bar{t}$).
On the contrary, providing a low interest rate, $\gamma < r_b < r^{ss}$, will always lead to eventual success, as long as bailout duration, $T$, is not too long to generate incentives for issuing more debt, due to the low cost of sovereign borrowing. In Appendix A we demonstrate that the explicit dynamics of $b_t$ under the bailout plan are given by,

$$b_t = \frac{1}{r_b - \gamma} - \alpha^t \left( \frac{1}{r_b - \gamma} - b_0 \right) - \frac{1 - \alpha}{r_b - \gamma} \left( 1 + \gamma \right) \frac{T-t+1}{1-\beta} \frac{1 - \beta^t}{1-\beta},$$

for all $t \in \{0, ..., T-1\}$, which is the solution of equation (59). To see that a bailout plan with $\gamma < r_b < r^{ss}$ should not last for too long, notice that equation (62) can be re-written as,

$$b_t = \frac{1}{r_b - \gamma} - \frac{(1-\alpha)}{(r_b - \gamma) (1-\beta)} \left( 1 + r_b \right)^t \left\{ b_0 - \frac{1+\gamma}{1+\gamma} - \frac{\beta}{1-r_b} \left( 1 + \gamma \right)^T \right\} \alpha^t.$$  

If $T$ is too high, then the factor multiplying $\alpha^t$ in the last term of equation (63) may be negative, and since $\alpha < 1$ when $\gamma < r_b < r^{ss}$, $b_t$ may start increasing after some time. Nevertheless, if the design of a bailout plan avoids this possibility, then low interest rates with $\gamma < r_b < r^{ss}$ typically lead to a faster transition of $b_t$ to a level below $\kappa$, compared to the case of $r_b > r^{ss}$.

As we have demonstrated in Section 2, if the bailout plan meets the goal of $b_T \leq \kappa$, then, from period $T$ and on, the country returns to international capital markets with interest rate $r^{ss}$, and $b_t = b_T \leq \kappa$ for all $t \geq T$. Proposition 1 describes the features of successful bailout plans.

**Proposition 3** Given an initial sovereign-debt-GDP ratio $b_0 > \kappa$, a bailout plan characterized by a combination $(r_b, T)$ with $r_b > \gamma$, is successful if $T$ is the smallest integer such that

$$\frac{1}{r_b - \gamma} - \alpha^T \left( \frac{1}{r_b - \gamma} - b_0 \right) - \frac{1 - \alpha}{r_b - \gamma} \left( 1 + \gamma \right) \frac{1 - \beta^T}{1-\beta} \leq \kappa,$$  

29
and in the special case of $r_b > r^{ss}$, the tuple $(r_b, T)$ should also satisfy

$$\frac{1}{r_b - \gamma} - b_0 > \left(\frac{1 + \gamma}{1 + r_b}\right)^T \frac{1}{(1 - \beta)(1 + r_b)} \left[\frac{1}{\beta^T} - \frac{\beta(1 + r_b) - (1 + \gamma)}{r_b - \gamma}\right].$$ (65)

**Proof** See Appendix C. □

Proposition 3 shows that in the process of designing a bailout plan, the choice of $(r_b, T)$ with $r_b > r^{ss}$ needs to take into account particular constraints. The most important constraint we have emphasized through our model is the political feasibility of a bailout plan. Since EU-type monetary-union countries are democracies, optimality of domestic policy is a key concern. In the following section we study bailout plans in the presence of political pathologies such as corruption through rent-seeking groups.

### 4.3 Corruption and bailouts in the Euro zone

It seems that corruption in the south-periphery Euro-zone countries is a major concern of the public and also of country analysts. A glance at the “IMF country reports” International Monetary Fund (2011a,b,d) for Greece, Italy, and Portugal, make explicit reference to the need for reducing rent-seeking activities in the first three countries, while the country report for Ireland (International Monetary Fund (2011c)) focuses on Ireland’s banking problems. Corruption indicators constructed from various institutions (World Bank, OECD and the Transparency International Organization) uniformly rank the corruption level in these countries highest among all Euro area countries. For example, in Arghyrou (2010), the three-year average (2007, 2008 and 2009) of the perception corruption index from Transparency International ranks Italy as 45th and Greece as 50th in a sample of 115 countries (with lower ranking equivalent to higher corruption), surprisingly even lower than many developing countries with weak political and social institutions. Spain and Portugal rank
closely behind.\textsuperscript{28}

In our model interest rates are the cornerstone of diagnosing sovereign-debt pathologies. So, in Figure 2, for a period before the introduction of the Euro, and up to recently, we plot the 10-year-maturity yield in the countries of the south periphery of the Eurozone, in Ireland, and we also use Germany as a yardstick.\textsuperscript{29} Ireland has the lowest corruption index, and its yield in the 1990s is close to that of Germany. The problems of Ireland start after the fall of 2008, with its spreads following the same path as Portuguese spreads in the post-credit-crunch era. Table 1 shows that Ireland never exhibited high corruption, even back in year 1995. But Ireland’s sovereign debt is exploding due to its problems in the banking sector.\textsuperscript{30} According to our model, Ireland falls in the theoretical category of our analysis in which rent-seeking groups are not a burden, but its trapped due to its debt burden. Based on our analysis above, Ireland’s bailout plan seems most likely to succeed. For Greece and Portugal, we refer to the sections specializing in bailouts and corruption.

4.3.1 One big “mob” and bailouts

After substituting (49) into the value function of a non-rent seeker, the distortion caused by the resources spent by the rent-seeking group (driven by the presence of parameter $\theta_R$ in (49)) leads to lower welfare of non-rent seekers. On the contrary, rent-seeking households

\textsuperscript{28}All three indices of corruption correlate very strongly with each other and aim at measuring the overall extent of corruption in the public sector (e.g. bribes, kickbacks, public funds appropriation, etc.). It is also worth noting that credit ratings agencies (e.g. Standard and Poor’s), which have also played a role in the development of the EU sovereign crisis by influencing creditors default expectations, take into account corruption as a factor in the political dimension that determines sovereign ratings and consequently affect country specific interest rates.

\textsuperscript{29}The 10-year-maturity yields in Figure 1 are nominal. For the pre-Euro period in the 1990s, even if we had controlled for inflation expectations, (a) the picture would have been similar, and (b) in that case we would have lost the inflation premium, which still reflects a credibility concern. One reason for that credibility concern is corruption, but other concerns must be at play as well (such as growth prospects of particular economies, etc.).

\textsuperscript{30}In International Monetary Fund (2011c, p. 21), it is mentioned: “Debt (in Ireland) is projected to peak at 120 percent of GDP in 2013, and to then decline gradually.”
have higher well being compared to non-rent seekers, due to the additional consumption of the rent. In the case that a country’s outstanding external debt is high enough to make external creditors concerned about a haircut in the fashion described by equation (55), then a bailout plan as discussed in Section 3 may be implemented. In this case of one big “mob” \(N = 1\), a bailout-plan analysis is the same as in Section 3. However, it may be that social dissatisfaction of non rent seekers may justify setting the threshold \(\kappa\) of (55) to lower levels compared to this of the benchmark model.

In International Monetary Fund (2011b, p. 28, footnote 5), it is mentioned: “Italy ranks 67th on the Transparency International Corruption Perception Index 2010, among the lowest in the EU. The Council of Europe’s Group of States against Corruption in its last evaluation of Italy (2008) noted that "...corruption in public administration is widely diffused" and recognized the "connections (that) exist between corruption and organized crime." ” According to our analysis, if Italy enters a bailout plan, the possibility that its rent-seeking groups play a cooperative strategy may give to Italy an advantage. For Portugal, in International Monetary Fund (2011d, p. 66), among the stated objectives is mentioned: “Ensure a level playing field and minimize rent-seeking behavior by strengthening competition and sectoral regulators; eliminate special rights of the state in private companies (golden shares); [...]” Perhaps, at a first stage, the request by the European Commission for consensus among political parties in Portugal aims at placing Portugal in the category of one rent-seeking group in order to avoid the commons problem explained above.
4.3.2 More than one rent-seeking groups and bailouts

In Section 3 we have shown that when rent-seeking groups are more than one, the debt-GDP ratio always increases over time.\textsuperscript{31} As a result, any bailout plan would never lead to a desired debt-GDP ratio target asymptotically, unless the bailout plan controlled interest rates forever. In light of this observation, the main target of a bailout plan should be this of eliminating rent-seeking groups in the corrupt economy.

In the case of corruption with $N \geq 2$, the problem of a bailed-out country is structural. If, for example, rent-seeking groups are tied to a strong bipartisan system, then the monetary union should aim at receiving guarantees of transparency by the two dominant political parties of a bailed-out country. The requirement of transparency towards the rest of the lenders of the monetary union may eliminate rent-seeking groups. This requirement should complement any other features of the bailout plan (such as the tuple $(r_b, T)$ discussed in Proposition 3).

In Figure 2, the nominal yield for Greece was very close to its German counterpart between years 2001-2008. Perhaps, during that period, external creditors had formed the belief that Greek rent-seeking groups would decrease their fiscal stealing due to Euro-zone monitoring. Yet, post-credit crunch facts revealed that Greece’s fiscal voracity problem remains to date. The persistent failure of political parties in Greece to cooperate and the increased polarization of its bipartisan system makes the goal of fighting corruption in Greece a top priority.

\textsuperscript{31}In particular we have shown that $b_t \to 1/[(1 - \beta)(1 + r^{as})]$, i.e. $b_t$ asymptotically converges to exhausting the domestic economy’s total wealth-to-GDP ratio, with the latter being evaluated at the asymptotic level of interest rates, $r^{as}$.
5. Conclusion

The recent sovereign-debt crisis in the Euro zone has led to a vivid political debate concerning the future effectiveness of implemented sovereign bailout plans. A key aspect to be understood in this debate is the determinants of policy setting in bailed-out governments. Any EU country which may be candidate for entering the EU sovereign bailout plan is an independent democracy which has full responsibility (and almost full discretion) for setting its own fiscal policy. So, bailout-plan effectiveness involves a political-economy analysis.

In our analysis of endogenous policy setting we have emphasized the role of international-market interest rates. We have studied forward-looking optimal policy which takes into account that debt-GDP ratios should not be raised too much due to the high future cost of debt servicing. Yet, as we have demonstrated, there is a threshold level of interest rates above which a government will still optimally increase its external-sovereign-debt-GDP-ratio over time. It is this interest-rate impact on optimal policy that can lead countries with high outstanding external-sovereign-debt-GDP-ratio to a trap. If external creditors anticipate sovereign default, then they lend at high interest rates. These high interest rates can, in turn, lead to a self-fulfilling default outcome as for example in Cole and Kehoe (2000).

EU-type sovereign bailout plans aim at aiding countries into such traps to return to free capital markets, mainly through putting them on a track of reducing their outstanding debt-GDP-ratio. The key to such a quick recovery is having a bailout plan with low interest rates. Yet, if bailed-out countries suffer from corruption in the form of rent-seeking groups stealing public resources, then a bailout plan may be hopeless unless rent-seeking groups are eliminated. The reason is, rent-seeking groups lead to a collective impatience for public goods and rents, which leads to oversupply of bonds. International markets that anticipate such an oversupply of bonds in the future, demand high interest rates, which, in turn, lead
to an explosive debt-GDP ratio over time. Our analysis of rent-seeking groups may also explain some resistance from the side of the broader public to debt austerity plans. Even non-rent seekers support a policy of excessive sovereign-debt-GDP ratios, as a response to the fact that rent seekers expropriate public resources. In brief, if bailed-out countries suffer from corruption, then the goal of the bailout plan should be this of eliminating rent-seeking groups.

We believe that our model provides a useful starting point for several extensions to models with endogenous international interest rates. A first natural extension is taxation that does not only distort welfare (as is the case in our model), but also GDP-level or GDP-growth performance. Another natural extension is the inclusion of uncertainty, informational imperfections or asymmetries, and explicit credibility concerns. Finally, a deeper understanding of bailout plans would consider the political economy of voters and governments of other monetary-union countries which finance bailouts. Such analyses can shed light on structural reasons behind imbalances or misalignment of fiscal policies within the EU, and point at ways of solving such problems.
6. Appendix A

Optimal policy-setting in the benchmark model

First-order conditions $\partial V/\partial \tau_t = 0$ and $\partial V/\partial B_{t+1} = 0$, imply equation (9) and also,

$$\frac{G_{t+1}}{G_t} = \beta(1 + r_{t+1})$$ \hspace{1cm} (66)

and

$$G_t = B_{t+1} - (1 + r_t)B_t + \tau_t Y_t$$ \hspace{1cm} (67)

Combining equations (9) and (67) leads to,

$$B_{t+1} - (1 + r_t)B_t = \left[ \theta_G - (1 + \theta_G) \tau_t \right] Y_t$$ \hspace{1cm} (68)

Moreover, solving (66) forward gives $G_t = \beta^t \prod_{j=1}^t (1 + r_j) \cdot G_0$, which can be combined with (9) to obtain,

$$\tau_t = 1 - \left( \frac{\beta}{1 + \gamma} \right)^t \cdot \prod_{j=1}^t (1 + r_j) \cdot (1 - \tau_0)$$ \hspace{1cm} (69)

Equation (68) together with (69) and (7) yield the recursion,

$$B_{t+1} - (1 + r_t)B_t = \phi(\tau_0) \cdot \beta^t \prod_{j=1}^t (1 + r_j) - (1 + \gamma)^t L, \hspace{0.5cm} \text{for all } t \geq 1$$ \hspace{1cm} (70)

together with (10). Solving equation (70) forward, leads to equation (11).

7. Appendix B

Proof of Proposition 1

We take a guess on the functional form of strategies,

$$C^{i,R}(B_t, z_t \mid \{r_s\}_{s=t}^{\infty}) = \xi_i \cdot (1 + r_t) \left( z_t \frac{W_{t+1}}{1 + r_t} - B_t \right) \hspace{0.5cm} \text{for all } i \in \{1, ..., N\}$$ \hspace{1cm} (71)
in which \( \{\xi_i\}_{i=1}^{N} \) is a set of undetermined coefficients. The Lagrangian of group \( j \)'s problem, this of maximizing (39) subject to (34), (41) given the guess given by (71) for all \( i \neq j \) is,

\[
\mathcal{L}_j = \kappa_{V_j} + \sum_{t=0}^{\infty} \beta^t \left[ \ln (1 - \tau_t) + \theta_G \ln (G_t) + \theta_{R,j} \ln (C_{j,t}^R) \right] + \\
+ \lambda_{j,t} \left[ B_{t+1} - (1 + r_t) B_t - G_t - \sum_{i=1 \atop i \neq j}^{N} \xi_i (1 + r_t) \left( \frac{z_t W_{t+1}}{1 + r_t} - B_t \right) - C_{j,t}^R + \tau_t Y_t \right]
\]

First-order conditions lead to,

\[
G_t = \theta_G (1 - \tau_t) Y_t , \tag{72}
\]

\[
C_{j,t}^R = \frac{\theta_{R,j}}{\theta_G} G_t = \frac{\theta_R}{\theta_G} G_t = C_t^R , \tag{73}
\]

due to that \( \theta_{R,j} = \theta_R \) for all \( j \in \{1, ..., N\} \), and

\[
\frac{G_{t+1}}{G_t} = \beta \left( 1 - \sum_{i=1 \atop i \neq j}^{N} \xi_i \right) (1 + r_{t+1}) = \beta \left[ 1 - (N - 1) \xi_R \right] (1 + r_{t+1}) , \tag{74}
\]

due to the symmetry of the problem, which allows us to consider that \( \xi_i = \xi_R \) for all \( i \in \{1, ..., N\} \). Combining (72) and (73), and substituting them into (34) we obtain,

\[
B_{t+1} = (1 + r_t) B_t + \frac{1 + \theta_G + N \theta_R}{\theta_G} G_t - Y_t . \tag{75}
\]

Using equations (72) through (75), the rest of the analysis follows this in Section 2, in which, after imposing the transversality condition \( \lim_{t \to \infty} B_t / \prod_{j=0}^{t-1} (1 + r_j) = 0 \), we arrive at,

\[
G_t = \frac{\theta_G}{1 + \theta_H} (1 - \beta_N) \left[ z_t W_{t+1} - (1 + r_t) B_t \right] . \tag{76}
\]

From (76) and (74) it is,

\[
C_t^R = \frac{\theta_R}{1 + \theta_H} (1 - \beta_N) \left[ z_t W_{t+1} - (1 + r_t) B_t \right] . \tag{77}
\]
Equations (71) and (77) imply that

\[ \xi_R = \frac{\theta_R}{1 + \theta_H} (1 - \beta_N) , \]

and since \( \beta_N = \beta [1 - (N - 1) \xi_R] \), we can prove (44) and (46). Proving (45) and (47) follows from direct substitution, completing the proof of the proposition. □

**Proof of Proposition 2**

Equating demand for bonds (equation (19)) and supply of bonds (equation (45)) leads to,

\[ (\beta - \beta_N) (1 + r_t) b_t = (1 - \beta_N) \left[ \frac{(1 + \gamma)^t W_{t+1}}{Y_t} - \frac{1}{1 - \beta_N} \right] . \] (78)

Substituting (78) into (45) leads to equation (50).

From (23) it is,

\[ \frac{W_{t+2}}{L} = \frac{1 + r_{t+1}}{1 + \gamma} \left( \frac{W_{t+1}}{L} - 1 \right) . \] (79)

After considering equation (78) one period ahead and substituting (79) into it, we combine the result with (78) referring to period \( t \), and, after some algebra, the result is

\[ \left[ (1 - \beta) (1 - \beta_N) \frac{W_{t+1}}{L} - (1 - \beta - \beta_N) \right] \frac{1 + r_{t+1}}{1 + \gamma} = 1 . \] (80)

After rearranging terms in (78) it is,

\[ (1 - \beta_N) \frac{W_{t+1}}{L} = 1 + (\beta - \beta_N) (1 + r_t) b_t , \] (81)

and after substituting (81) into (80) we arrive at

\[ [(1 - \beta) (\beta - \beta_N) (1 + r_t) b_t + \beta_N] \frac{1 + r_{t+1}}{1 + \gamma} = 1 . \] (82)
Combining (82) with (50) proves (53).

For proving the dynamics given by (51), equation (82) implies that
\[(1 - \beta) (\beta - \beta_N) (1 + r_t) b_t + \beta_N < \beta \iff \frac{\beta (1 + r_{t+1})}{1 + \gamma} > 1 \iff r_{t+1} > r^{ss},\]
or
\[b_t < \frac{1}{(1 - \beta) (1 + r_t)} \iff r_{t+1} > r^{ss} \quad \text{for all } t \in \{0, 1, \ldots\} . \quad \text{(83)}\]
Optimization requires that \(G_t > 0\) for all \(t \in \{0, 1, \ldots\}\), and so equation (48) implies
\[G_t > 0 \iff \frac{W_{t+1}}{L} > (1 + r_t) b_t , \quad \text{for all } t \in \{0, 1, \ldots\} . \quad \text{(84)}\]
Combining (81) with (84) leads to
\[G_t > 0 \iff b_t < \frac{1}{(1 - \beta) (1 + r_t)} \quad \text{for all } t \in \{0, 1, \ldots\} , \quad \text{(85)}\]
and given (83) proves that \(r_{t+1} > r^{ss} \quad \text{for all } t \in \{0, 1, \ldots\}\) in (51).

Combining (53) and (50) directly and using the recursion
\[\frac{1 - \beta}{\beta} (\beta - \beta_N) b_t + \frac{\beta_N}{1 + \gamma} = \frac{\beta}{1 + \gamma} b_t b_{t+1},\]
or,
\[\frac{1}{b_{t+1}} = \frac{\beta_N}{\beta} \cdot \frac{1}{b_t} + \frac{1 - \beta}{\beta} (1 + \gamma) \left(1 - \frac{\beta_N}{\beta}\right),\]
with solution,
\[\frac{1}{b_t} - \frac{1 - \beta}{\beta} (1 + \gamma) = \left(\frac{\beta_N}{\beta}\right)^{t-1} \left[\frac{1}{b_1} - \frac{1 - \beta}{\beta} (1 + \gamma)\right], \quad \text{(86)}\]
since (53) holds for \(t \in \{1, 2, \ldots\}\). since \(1/b_1 - (1 + \gamma) (1 - \beta) / \beta\) in (86) is equivalent to \(b_1 < 1/[(1 - \beta) (1 + r^{ss})]\), (85) implies that \(b_1 < 1/[(1 - \beta) (1 + r^{ss})]\) holds as well. In addition, since for \(N \geq 2\) equation (46) implies that \(\beta_N < \beta\), after taking the limit \(t \to \infty\), equation (86) proves that \(b_t \to 1/[(1 - \beta) (1 + r^{ss})]\) in (52), and substituting \(1/[(1 - \beta) (1 + r^{ss})]\) for
\( b_t \) in (53) proves that \( r_t \to r^{ss} \). Finally, for all \( b_0 \), there exists an \( r_0 \) such that, according to (53),

\[
(1 + r_0) b_0 = \frac{1 + \gamma}{\beta} b_1 < \frac{1}{1 - \beta},
\]

proving the proposition. \( \square \)

8. Appendix C

To calculate the law of motion given by (59), notice that, under a proposed bailout plan \((r_b, T)\), equation (15) implies,

\[
W_1 = \left\{ \left[ 1 - \left( \frac{1 + \gamma}{1 + r_b} \right)^T \right] \frac{1 + r_b}{r_b - \gamma} + \left( \frac{1 + \gamma}{1 + r_b} \right)^T \frac{1}{1 - \beta} \right\} L. \tag{87}
\]

Combining (87) with (20) leads to (59), which is the generalized version for all \( t \in \{0, ..., T - 1\} \).

For the solution of (59) given by (62), express (59) as,

\[
b_{t+1} = \alpha b_t + \zeta - \eta \omega^t, \tag{88}
\]

in which \( \zeta \equiv (1 - \alpha) / (r_b - \gamma) \), \( \eta \equiv [(1 + \gamma) / (1 + r_b)]^T \), and \( \omega \equiv (1 + r_b) / (1 + \gamma) \). Solving (88) forward leads to,

\[
b_t = \alpha^t b_0 + \zeta \frac{1 - \alpha^t}{1 - \alpha} - \eta \omega^{t-1} \sum_{j=0}^{t-1} \left( \frac{\alpha}{\omega} \right)^j,
\]

which is equivalent to (62).

Proof of Proposition 3

Inequality (64) reflects the condition \( b_T \leq \kappa \) and it is derived directly from (62) after setting \( t = T \). Regarding inequality (65), in the case of \( r_b > r^{ss} \), we need to guarantee that
$b_t < \bar{b}_t$, for all $t \in \{0, ..., T - 1\}$, as required by (61) for stability. Fix any $t \in \{0, ..., T - 2\}$ and observe that (60) implies,

$$
\bar{b}_{t+1} = \left[ 1 - \left( \frac{1 + \gamma}{1 + r_b} \right)^{T-t-1} \right] \frac{1}{r_b - \gamma}.
$$

In order to meet the requirement $b_{t+1} < \bar{b}_{t+1}$, combine (59) with (89) to obtain, after some algebra,

$$
\bar{b}_t - b_t > \left( \frac{1 + \gamma}{1 + r_b} \right)^{T-t} \frac{1}{\beta (1 + r_b)},
$$

which can be expressed as,

$$
\frac{1}{r_b - \gamma} - b_t > \left( \frac{1 + \gamma}{1 + r_b} \right)^{T-t} \left[ \frac{1}{\beta (1 + r_b)} + \frac{1}{r_b - \gamma} \right].
$$

After substituting (62) into (90) and after some algebra we arrive at,

$$
\frac{1}{r_b - \gamma} - b_0 > \left( \frac{1 + \gamma}{1 + r_b} \right)^T \frac{1}{(1 + r_b)(1 - \beta)} \left[ \frac{1}{\beta^{t+1}} - \frac{\beta (1 + r_b) - (1 + \gamma)}{r_b - \gamma} \right].
$$

Inequality (91) must hold for all $t \in \{0, ..., T - 1\}$. Since $1/\beta < 1/\beta^T$, substituting $t = T - 1$ into (91) guarantees that (91) holds for all $t \in \{0, ..., T - 1\}$, and doing so leads to inequality (65). □
REFERENCES


Figure 1  Source: European Central Bank, and International Monetary Fund.
Figure 2 -- Source: ECB, Secondary market yields of government bonds with maturities of close to 10 years, Ranking according to the Corruption Perception Index 2010 from Transparency International (lower ranking means lower corruption).
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Source: Transparency International
Note: Higher score means lower corruption and numbers appearing in parentheses next to each score is the country’s world corruption ranking based on the score in each particular year.