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Trade and Imperfect Competition in General Equilibrium

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Trade and Imperfect Competition in General Equilibrium

Abstract

This paper employs a general equilibrium model of imperfect competition and trade in which capital is used to establish firms and labor is used for production. We show that two different types of equilibria may exist, one with factor price equalization and one with different factor prices. When factor prices are equalized, trade improves welfare under relatively mild conditions. However, if factor prices differ, these conditions are not sufficient for mutual gains from trade.

JEL-Code: F120, D500.

Keywords: imperfect competition, international trade, general equilibrium.

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1 Introduction

The welfare effects of trade have been a subject of examination for a long time. In the pre-1970 period, neoclassical trade theory occupied much of the literature. In neoclassical trade theory, resources are mobile within countries, but immobile across countries, and countries’ factor endowments, technologies and preferences drive production and trade. In traditional models of trade, markets are perfectly competitive, and trade unambiguously improves welfare; not necessarily for each individual, but on an aggregate level. During the 1980s, starting with the new trade theory, models have taken market imperfections into account. One strand of this literature employs general equilibrium models of monopolistic competition; such models proliferated following the seminal paper by Krugman (1980). Models of monopolistic competition assume that firms behave like monopolists and do not take strategic interactions into account when deciding on prices or outputs. These models have become very prominent in trade theory as they could explain intra-industry trade, empirically a significant share of trade between similar countries, that traditional models of trade failed to explain. These models show that economies of scale and product variety are further channels through which trade can improve welfare. Recently, such models have been extended as to incorporate firm heterogeneity; see Melitz (2003) and Melitz and Ottaviano (2008).1

The other strand of the literature is comprised of the reciprocal dumping models developed by Brander (1981), and Brander and Krugman (1983). These models are Cournot models and focus on strategic interactions which models of monopolistic competition do not take into account. These models have been extended in many ways, but their role in trade theory has remained much less influential than that of either perfectly or monopolistically competitive models of trade; see Neary (2009, 2010). The reciprocal dumping models argue that trade may occur even in the absence of either comparative advantage or product differentiation, but they fail to address many of the classic questions of trade theory (Bensel and Elmslie 1992; Neary 2009). As for the predictions about the welfare effects of trade, they are not immune against assumptions concerning the market structure.

Many of both monopolistic competition and reciprocal dumping models assume quasi-

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1In these models, firms are heterogeneous in their productivity, and each firm has to incur fixed costs, first to develop a differentiated product, then to market the product in foreign countries. Therefore only the more productive firms will be profitable and export. Such models demonstrate that trade reallocates market shares from inefficient to efficient firms which is another source of welfare gains.
linear preferences and the existence of a perfectly competitive industry that effectively buffers all factor price effects so that trade has no impact on factor markets. While this is a significant sacrifice, in particular compared to traditional models of trade in which the change of factor prices is at the heart of the analysis, its remedy is not straightforward. The dilemma is that if firms are sufficiently large, the monopolistic competition setup which assumes that firms take their industry environment as given is not appropriate anymore. Allowing for market power, however, also has implications in a general equilibrium setup which are not easy to model. For example, a very large firm may have an influence on factor prices and may take this into account. When we allow for this effect, the firm even has to take into account that it has an influence on a country’s GDP.\(^2\)

At the end of the day, the important question is at which stage firms play strategically. In monopolistic competition models, they do not play strategically at all. These models thus seem to underestimate strategic effects. In a model in which firms have a substantial influence on factor prices, they play even strategically on the GDP level, an implication with which we do not feel very comfortable because it seems to overestimate the role of strategic effects. In order to reconcile general equilibrium effects with trade under imperfect competition, Neary (2007, 2009) and Neary and Tharakan (2008) have suggested that firms are small in the large and large in the small, that is, firms take factor prices as given, but they do not take prices in their own industry as given. Thus, they exercise market power in the commodity market they are operating in, but they have no influence on factor markets. This approach allows to study factor price changes originating from imperfect competition in a general equilibrium framework. We follow this approach in the current paper as we also find it reasonable that firms compete for resources without taking account of their influence on factor prices (and thus on national income), but that they do know very well the role they play in their commodity market.

An important predecessor of our paper is the influential book on market structures and trade by Helpman and Krugman (1985). They consider trade both under oligopolistic and monopolistic competition in general equilibrium. While no general theory of imperfect competition exists, they develop results on the effects of trade that hold by and large in several models. An implicit assumption in their models is that firms do not

\(^2\)Taking this effect into account leads to severe technical difficulties; see Neary (2009) for discussions of these difficulties.
anticipate any influence on factor prices, even if only a few firms and industries exist.\textsuperscript{3} They also consider free market entry as we do, but their sufficient conditions for gains from trade focus on outcomes and not on primitives. In Peter Neary’s framework, the number of firms in each industry is given, so entry is not modeled.

In this paper, we want to make progress on two issues. First, we integrate trade and imperfect competition into a general equilibrium framework by also modeling the market for firm assets. In order to do so, we sacrifice generality and extend the famous Dornbusch-Fischer-Samuelson (DFS) model, Dornbusch \textit{et al.} (1980) to oligopolistic competition such that we do not take industry structures as given, but determine them endogenously.\textsuperscript{4} Second, we want to develop a better understanding how the effects of trade relate to the primitives. In our model, capital and labor play different roles, and there are economies of scale, and thus commodity markets are imperfectly competitive.\textsuperscript{5} The capital market is the market for firm assets, and capital is thus used to establish firms while labor is used to run the established firms. Industries differ w.r.t. their input requirements, both for capital and labor. We find that an equilibrium that equalizes factor prices may exist if countries are similar, and an equilibrium may exist in which factor prices do not equalize if countries are sufficiently different. In the first case, both countries diversify such that they produce all goods; in the second case, each country specializes and produces only a certain range of goods. Surprisingly, specialization is the potential troublemaker in our setup. We demonstrate the relatively mild conditions under which trade will unambiguously improve welfare mutually if factor prices equalize, but these are not sufficient in case of no factor price equalization. The reason is that market entry is excessive, and trade changes factor prices such that entry becomes more profitable.

The remainder of the paper is organized as follows. Section 2 introduces the model and discusses the autarky equilibrium. Section 3 discusses the implications of trade for two cases, (i) the case of factor price equalization (Subsection 3.1), and (ii) the

\textsuperscript{3}In an early paper on imperfect competition in general equilibrium, Gabszewicz and Vial (1972) assume that consumers get a share of firm outputs for their inputs, and these goods are then traded among consumers. While this avoids the modeling of factor markets, firms do not maximize profits but the real wage of their shareholders.

\textsuperscript{4}See Romalis (2004) for a model of monopolistic competition in a DFS framework. See also Dornbusch \textit{et al.} (1977) for a (one-factor) Ricardian version of their model.

\textsuperscript{5}Neary and Tharakan (2008) also have a model with two factors of production, but the factors of production in their model play a different role.
case of no factor price equalization (Subsection 3.2). Section 4 investigates how our results generalize when we allow a more general demand structure. Section 5 offers some concluding remarks. For convenience, we have relegated most of the proofs and technical details to the Appendix.

2 The model and the case of autarky

As in the DFS model, we consider a continuum of goods which are indexed by \( z \) and defined over the interval \([0, 1]\), and we assume that households are symmetric and their preferences are Cobb-Douglas. Our model wants to focus on the supply side effects, and this is the reason why we keep the demand structure of the model as simple as possible in the main body of the paper and assume that expenditure shares are identical across commodities. This allows us to specify the utility function such that 

\[
U = \int_0^1 \ln(\hat{y}(z))dz
\]

where \( \hat{y}(z) \) denotes per-capita consumption of commodity \( z \). The aggregate output of industry \( z \) is \( Y(z) = L\hat{y}(z) \) where \( L \) denotes the number of workers in the economy (which has to be replaced by \( L + L^* \) when countries trade). This preference structure leads to an inverse demand function 

\[
p(z) = I/Y(z),
\]

where \( I \) denotes income, and \( p(z) \) is the price of good \( z \). Firms in this industry use labor for producing output and capital as to establish a firm. The profit of firm \( i \) in industry \( z \) is equal to

\[
\Pi_i(z) = (p(z) - \lambda(z)w)y_i(z) - r\kappa(z).
\]

The input requirement of labor is equal to \( \lambda(z) \) in industry \( z \); \( w \) denotes the wage. Each active firm has to make an investment of size \( \kappa(z) \) as to set up a plant where \( \kappa(z) > 0 \); \( r \) denotes the rental rate. \( y_i(z) \) is the firm-level output such that \( Y(z) = \sum_i y_i(z) \). All firms within industry \( z \) are symmetric such that \( Y(z) = n(z)y_1(z) \), where \( n(z) \) denotes the number of active firms in industry \( z \).

Before we proceed we have to be more specific on the index \( z \), and for this reason we rank industries in terms of their capital input requirements without loss of generality. We assume that industries can be ranked such that the capital input requirement is a differentiable function of \( z \), and we introduce the following

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6Since \( p_Y(z) + p_Y(z)Y(z)y_i(z) = -I(n(z) - 2)/n(z)Y(z)^2 \leq 0 \), firms compete by strategic substitutes in the sense of Bulow et al. (1985).

7Assuming differentiability of the \( \kappa \)-ranking simplifies our analysis. However, all our results do also

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**Definition 1** Industries are ranked such that $z$ decreases with the capital input requirement $\kappa$: $\kappa'(z) \leq 0$.

Note that we do not make any assumption how labor input requirements behave. Let us now consider the domestic country which has a capital endowment of size $K$, used to establish firms, and a labor endowment of size $L$, used to produce output. All variables referring to the foreign country will be denoted by an asterisk. We are now interested in the autarky equilibrium. Even in autarky, some further assumptions have to be made to ensure that firms will be established in each industry and that industry outputs are strictly positive. For future reference, let us define

$$\Omega \equiv \int_0^1 \sqrt{\kappa(z)} dz.$$ 

We make the following

**Assumption 1** $K > 2\kappa(0) \geq 2\Omega^2$

which ensures that the capital stock is sufficiently large so that a sufficient number of firms can be established in all industries under autarky.\(^8\) Assumption 1 is not too demanding and can be understood best for constant capital input requirements, $\kappa(z) = \kappa$, across industries. In this case, $2\kappa(0) = 2\Omega^2 = 2\kappa$, and Assumption 1 requires that the capital endowment allows to establish at least two firms in each industry.\(^9\)

All entrants to an industry play a two-stage game: in the first stage, they decide on entry and carry the investment cost, in the second stage they decide on firm output.\(^10\) hold if the $\kappa$-ranking is almost everywhere differentiable.

\(^8\)Since the market for firm assets is perfectly competitive, firm ownership is diversified and capital owners take $r$ as given. They have no influence on the behavior of firms, and thus the price normalization problem cannot arise as in Dierker et al. (2003), and Dierker and Dierker (2006).

\(^9\)Note that the demand specification does not support a monopolistic outcome due to constant expenditure shares such that at least two firms are required to avoid corner solutions. See also Assumption 2.

\(^10\)Helpman and Krugman (1985, p. 85) find that the Cournot approach is useful but also lacks realism. However, Kreps and Scheinkman (1983) have shown that this approach is strategically equivalent to a game in which firms determine capacities first, and then compete by prices. This means that our results are equivalent to a game in which a firm, after having made an investment, hires labor to produce a maximum output before it will compete by prices with its rivals in its industry.
We solve the game in the usual backward induction fashion, and we follow Neary (2009) such that each active firm maximizes its profits for a given wage and rental rate, and for a given income level; this reflects the assumption that firms are large in the small, but small in the large. The first-order condition is

\[
\frac{\partial \Pi_i(z)}{\partial y_i(z)} = p(z) - \lambda(z)w + p_{Y(z)}(z)y_i(z)
\]

\[
= \frac{I}{Y(z)} - \lambda(z)w - \frac{Iy_i(z)}{Y(z)^2} = 0.
\]

Using symmetry, we find that

\[
y_i(z) = \frac{I}{\lambda(z)w} \frac{n(z) - 1}{n(z)^2}.
\]

A further innovation of our paper is that we model the market for firm assets such that capital is used to establish firms as long as entrepreneurs expect non-negative returns. This has the implication that all firms will make zero profits in equilibrium. If profits were positive (negative) in an industry, capital demand would rise (decline), reducing (increasing) industry profits due to entry (exit) to that industry. Furthermore, also the rental would increase (decline), but this effect is not taken into account by potential entrants. Ignoring the integer constraint, we now can determine market entry by the zero profit condition:

\[
\Pi_i(z) = -p_{Y(z)}(z)y_i(z)^2 - r\kappa(z) = \frac{I}{n(z)^2} - r\kappa(z) = 0
\]

which leads to

\[
n(z) = \sqrt{\frac{I}{r\kappa(z)}}.
\]

Note carefully that market entry is excessive in our setup such that firms are too small from a social perspective as shown by the industrial organization literature; see, for example, Mankiw and Whinston (1986). Of course, given economies of scale, the socially optimal solution would be to allow only one firm charging marginal costs. This firm would have to be subsidized, though. But even if a social planner could
control only market entry and not firm behavior, this literature has shown that entry is excessive when firms compete à la Cournot, and the sum of consumer surplus and profits could be increased by reducing the number of firms below the free entry level.\textsuperscript{11} The reason is that entrants steal business from their rivals, and firms become smaller, and consequently, excessive entry does not increase consumer surplus substantially but reduces profits to zero.

Our analysis will be confined to laissez-faire equilibria, and given our demand specification, we want to consider a model with competition between firms so we make

**Assumption 2** \(n(z) \geq 2\)

which implies that \(\sqrt{r\kappa(z)}/I \leq 1/2\). Assumption 2 guarantees that firm and industry outputs in equilibrium, which are given below, are positive:\textsuperscript{12}

\[
y_i(z) = \frac{r\kappa(z)}{\lambda(z)w} \left(\sqrt{\frac{I}{r\kappa(z)}} - 1\right),
\]

\[
Y(z) = \sqrt{Ir\kappa(z)} \frac{\lambda(z)}{\lambda(z)w} \left(\sqrt{\frac{I}{r\kappa(z)}} - 1\right).
\]

To close the model, we need to determine the autarky factor prices, and thus we now consider factor markets. Labor demand of industry \(z\) is equal to

\[
L(z) = \lambda(z)Y(z) = \frac{I}{w} - \frac{\sqrt{Ir\kappa(z)}}{w}
\]

and capital demand of the same industry amounts to

\[
K(z) = \kappa(z)n(z) = \sqrt{\frac{I}{r}} \sqrt{\kappa(z)}.
\]

\textsuperscript{11}We do not endogenize competition policies in this paper. Stähler and Upmann (2008) discuss the role of nationally allowing free entry or restricting local market access in a two-country model.

\textsuperscript{12}We could also accommodate a perfectly competitive industry such that \(\kappa(1) = 0\) which would lead to \(Y(1) = I/\lambda(1)w\).
Before we proceed to the determination of factor prices under autarky, we want to explore how factor demands and the capital intensity change across industries. Factor demands do not depend on $\lambda(z)$, and we find that

$$L'(z) = -\frac{\sqrt{\kappa(z)rI}}{2zw}\epsilon_\kappa(z) \geq 0, \quad K'(z) = \frac{K(z)\epsilon_\kappa(z)}{2z} \leq 0,$$

(2)

where $\epsilon_\kappa(z) \leq 0$ denotes the elasticity of $\kappa$ w.r.t. $z$. Not surprisingly, more capital-intensive industries have a larger capital demand. Furthermore, they have a lower labor demand, irrespective of the labor input requirement. The capital intensity

$$\mu(z) \equiv \frac{K(z)}{L(z)} = \frac{w}{\sqrt{\frac{r}{r\kappa(z)} - 1}}$$

(3)

is endogenous in our model, and we can now explore how it changes with $z$. We find that

$$\mu'(z) = \mu(z) \left( \frac{K'(z)}{K(z)} - \frac{L'(z)}{L(z)} \right) = \mu(z) \frac{I}{2zwL(z)} \epsilon_\kappa(z) \leq 0$$

which leads to

**Lemma 1** The capital intensity increases with the capital input requirement.

This result is also not surprising. However, the capital intensity in our model should not be confused with capital intensities in trade models of perfect competition employing linear-homogeneous production functions. Capital is used only to establish firms, and labor is used only for production. Therefore the capital intensity in our model gives the ratio of establishment inputs to operating inputs.

We are now ready to establish the equilibrium conditions for both factor markets. As for the aggregate labor market, we find that

$$L = \int_0^1 \left( \frac{I}{w} - \sqrt{Ir\kappa(z)} \right) dz = \frac{I}{w} - \sqrt{Ir} \Omega$$

must hold, and for the capital market
must hold. The model is closed by the income definition \( I = wL + rK \). Note that we have a large (infinite) number of equilibrium conditions for all commodity markets and two equilibrium conditions for the factor markets. The last condition follows from all other ones according to Walras’ Law, and hence both the labor market and the capital market equilibrium lead to

\[
\sqrt{rK} = \sqrt{wL + rK}\Omega.
\]

Without loss of generality, we can use the rental rate as a numeraire in our model such that the wage coincides with the wage-rental ratio. This yields

\[
K = \sqrt{wL + K}\Omega \iff w = \frac{K}{L} \left( \frac{K}{\Omega^2} - 1 \right).
\] (4)

The wage-rental ratio is the larger the larger (smaller) is \( K(L) \). As can be seen from eq. (4), there are now two dimensions along which countries may differ in our model. First, as in classic models, countries may differ with respect to their per-capita endowment which we define as \( k \equiv K/L \) for the domestic country (and \( k^* \equiv K^*/L^* \) for the foreign country). Second, the size of the capital stock is also crucial as a larger capital stock allows more firms to be established. In classic models, autarky wages coincide if countries’ per-capita endowments coincide. This is not true here – if relative endowments coincide, autarky wages coincide only if absolute capital endowments are also the same (which implies that labor endowments are also identical).

We can also compare the wage-rental ratio with the per-capita capital endowment. Assumption 1 implies \( K > 2\Omega^2 \) which immediately leads to \( wL > K \); see eq. (4). Thus, we observe that the wage-rental ratio is larger than per-capita capital endowment, \( w > k \), and the income share of labor is larger than the income share of capital which seems to be confirmed by empirical evidence. Finally, we can easily compute the autarky income as \( I = wL + K = K^2/\Omega^2 \) which shows that a country’s autarky income increases overproportionately with its capital stock.
3 Trade in general equilibrium

We now turn to the integrated markets in which each firm will sell in both countries and commodity prices are the same across countries. A crucial question will be whether factor prices will equalize as a result of trade, that is, whether trade can also be a perfect substitute for factor mobility in our setting. Factor price equalization is also closely related to the existence of a cone of diversification, that is, whether all goods are produced within a country. In our model, we have to explore first how factor price equalization (or non-equalization) is related to the existence of active firms in the two countries. We find that factor price equalization is crucial for diversification.

Lemma 2 There is no coexistence of domestic and foreign firms in industry $z$ if factor prices differ.

Proof: See Appendix A.1.

The intuition for Lemma 2 is that different factor prices imply different cost structures, and thus different cost structures imply that one country will not host firms of an industry due to an inferior cost structure, and firms of that industry will be geographically concentrated in the other country. Of course, due to factor market clearance in both countries, this cannot be true for all industries so that each country will attract some industries. In what follows, we will show that two different types of equilibria, an equilibrium with factor price equalization and an equilibrium without factor price equalization, may exist. Then we will scrutinize the effects of trade for both cases, factor price equalization (FPE) and no factor price equalization (NFPE). We start with the FPE case.

3.1 Factor price equalization

In the FPE case, let the common factor prices be denoted by $w$ and $r$, respectively. Identical factor prices imply that firms are indifferent where to locate, but on aggregate

\[^{13}\text{Helpman and Krugman (1985, p. 103) discuss in Chapter 5.4 the possibility that the scale of production may be different across countries when factor prices do not equalize and the production technology is not homothetic so that firms of different scale may coexist in a free entry trade equilibrium. The specific structure of the DFS model does not include this case.}\]
their location decision has an effect on (equalized) factor prices. The aggregate number of active firms now comprises both the one located in the domestic country and the one located in the foreign country, and is given by

\[ n(z) + n^*(z) = \sqrt{\frac{I^w}{r\kappa(z)}}. \]  

(5)

where \( I^w = w(L + L^*) + r(K + K^*) \) is now world income. We let the rental rate \( r \) be the numeraire as in the case of autarky. Factor markets clear if

\[
\begin{align*}
K &= \int_0^1 n(z)\kappa(z)dz, \\
L &= \int_0^1 \frac{n(z)}{n(z) + n^*(z)} \left( \frac{I^w}{w} - \sqrt{\frac{I^w}{w}\kappa(z)} \right) dz \Rightarrow \\
wL + K &= \Upsilon \sqrt{I^w} \text{ where} \\
\Upsilon &= \int_0^1 n(z)\sqrt{\kappa(z)}dz.
\end{align*}
\]

The second-last line follows from using the factor market condition for capital and eq. (5). Balanced trade requires that production is equal to consumption, that is, \( wL + K = \Upsilon \sqrt{I^w} \), and thus leads to the same condition. Furthermore, since \( I^w = w(L + L^*) + (K + K^*) \), this condition follows also from the same exercise for the foreign country for which the trade balance condition leads to \( wL^* + K^* = I^w - \Upsilon \sqrt{I^w} \).
Hence, the four factor market conditions and the two trade balance conditions are not independent, but lead to a single condition. This condition requires that total domestic production, which is equal to domestic real income, should stand in a certain relation to world income. Note that there is only one \( \Upsilon \) which will fulfill this condition once income levels are determined. However, there is a range of different industry structures, that is, different combinations of \( n(z) \) and \( n^*(z) \), which will meet this requirement such that \( K = \int_0^1 n(z)\kappa(z)dz \) and \( K^* = \int_0^1 n^*(z)\kappa(z)dz \). Hence, we have some ambiguity in terms of industry structures, but not in terms of the relation of national incomes to world income.

Our analysis so far has assumed that an equilibrium with factor price equalization exists. We can demonstrate both the existence and the set of factor endowments under which such an equilibrium exists by considering a fully integrated economy in which factors of production are mobile across countries. In this fully integrated economy,
the wage determination is similar to the case of autarky, except that it now takes all endowments into account and factor markets should clear in aggregate. Similar to eq. (4), we find that

\[ K + K^* = \sqrt{w(L + L^*)} + K + K^*\Omega \Leftrightarrow w = \frac{K + K^*}{L + L^*} \left( \frac{K + K^*}{\Omega^2} - 1 \right). \]  (7)

Of course, this is consistent with our analysis above. We could derive eq. (7), the FPE wage, also from eq. (6) because \( K + K^* = \int_0^1 (n(z) + n^*(z)) \kappa(z) dz \); we can substitute \( n(z) + n^*(z) \) given by eq. (5) as to rewrite \( K + K^* \) which leads to the same result.

We now conduct the same thought experiment as originally developed by Dixit and Norman (1980) and ask whether we can reconstruct the same resource allocation if factors of production are immobile across countries. As it is well known, the FPE set can be constructed as a convex set of the sectoral employments of labor and capital in the fully integrated economy. Lemma 1 has demonstrated that the capital intensity increases with the capital input requirement, and thus the factor price equalization set has the standard characteristics of models with an infinite number of goods as shown by Figure 1.

![Figure 1: Factor price equalization](image-url)
$K^w$ and $L^w$ denote the world factor endowments with capital and labor, respectively. Figure 1 shows that an FPE equilibrium will always exist because part of the diagonal $00^*$ belongs to it. Compared to DFS, however, the autarky wages along this diagonal are not identical in our model except at the point of perfect symmetry, $W$. All factor allocations which guarantee equal autarky wages are given by the dotted line through $W$. Furthermore, the capital stock in both countries must be larger than $2\kappa(0)$ so as to guarantee that all goods can be produced in autarky. A common feature of models with more goods than factors is that the production pattern is indeterminate, and this is also true in our model as we have demonstrated above. Suppose that point $P$ is the allocation of internationally mobile factors across the two countries in a fully integrated economy. Since it is within the FPE set, it can easily be reconstructed without factor mobility but with trade. Drawing a line through $P$ with the slope of the wage-rental ratio $w$ yields combinations of factor allocations for which the relative income across countries stays constant. Since preferences are homothetic, point $C$ on this line, that is, the intersection with the diagonal $00^*$, gives the point of implicit consumption of factors which depends on the share of world income only. As in DFS, although we cannot pin down production patterns, we can unambiguously determine the net factor content of trade. In Figure 1, the domestic country is a net exporter of capital services and a net importer of labor services.

We now compare the wage-rental ratio before and after trade liberalization, and we find that trade increases (decreases) the domestic wage-rental ratio if

$$\frac{k}{k^*} < \frac{2K + K^* - \Omega^2}{K - \Omega^2}.$$ (8)

A similar condition holds for the foreign country. Note that the RHS of eq. (8) is unambiguously larger than unity; hence, a necessary, but not a sufficient condition for the wage-rental ratio in the domestic country to decline is that $k > k^*$. We may conclude that the wage-rental ratio may decline with trade if the domestic country is very capital-abundant. If countries are not too different in terms of their per-capita capital endowment, the wage-rental ratio will go up in both countries. Furthermore, note that both countries cannot experience a decline in the wage-rental ratio because

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14 This line can be derived from equalizing the autarky wages which leads to $K(L^w - L)(K - \Omega^2) = (K^w - K)L(K^w - K - \Omega^2)$.

15 Note carefully that $w > k^w \equiv K^w/L^w$, where $k^w$ denotes the world per-capita endowment, also holds in the integrated economy.
a decline for the domestic country requires \( k > k^* \) which would definitely lead to an increase for the foreign country. Moreover, both a positive and a negative sign in condition (8) are neither in conflict with eq. (7) nor with the construction of the FPE set. This leads to

**Proposition 1** The wage-rental ratio will increase in both countries with free trade if countries are not too different with respect to their per-capita endowments. If countries are substantially different, the country with the higher per-capita endowment may experience a decrease in the wage-rental ratio, while the other country will definitely experience an increase.

Let us put this result more in context. Since the rental is the numeraire in our model, a decline in the domestic wage-rental ratio implies that domestic income declines. World income, however, is equal to

\[
I^w = \frac{(K + K^*)^2}{\Omega^2} > \frac{K^2}{\Omega^2} + \frac{K^*^2}{\Omega^2}
\]

and larger than the sum of autarky incomes. Therefore, if a country experiences a decline in the wage-rental ratio, this country also becomes less influential in terms of its share in world income.

An increase or decrease in a country’s income, however, does not indicate welfare effects of trade as these depend also on the changes in commodity prices. The welfare effects, however, are not straightforward. On the one hand, markets become larger due to integration, and thus firms become more efficient. On the other hand, Proposition 1 has shown that the wage-rental ratio will increase in at least one country, if not in both countries, and an increase in the wage-rental ratio makes production more costly. If it increases in one country only, this country will experience a substantial increase in its share of world income and will thus be a more dominant player than before. Therefore, we have a trade-off between larger markets carrying more firms and an increase in production costs. The last effect is not present in models that allow for a perfectly competitive industry which absorbs all labor market effects.

We can compare the change in firm size in more detail. Let the cases of autarky and trade with factor price equalization be denoted by the superscripts \( a \) and \( t \), respectively. Trade leads to a change in domestic firm-level output of
\[
y_t^i(z) = \frac{k}{k^w(\Theta(z)), \Theta(z) = \frac{K - \Omega^2}{K + K^* - \Omega^2} \frac{\Omega}{\sqrt{\kappa(z)}} (K + K^*) - \Omega^2}{\frac{\Omega}{\sqrt{\kappa(z)}} K - \Omega^2},
\]

where \( k^w \equiv (K + K^*)/(L + L^*) \) denotes the world per-capita endowment. If capital input requirements are identical across industries, that is, if \( \kappa(z) = \kappa \), then \( \Theta(z) = 1 \) and \( y_t^i(z)/y_t^a(z) = k/k^w \). Firm size in all industries will increase (decrease) in the country that has a larger (smaller) per-capita endowment. If \( \kappa' < 0 \) at least over some range, then there is one and only one \( \tilde{z} \) for which \( \Omega = \sqrt{\kappa(\tilde{z})} \) due to the Intermediate Value Theorem. The change for \( z = \tilde{z} \) is again \( y_t^i(\tilde{z})/y_t^a(\tilde{z}) = k/k^w \). Since

\[
\Theta'(z) = \frac{K - \Omega^2}{K + K^* - \Omega^2} \frac{\kappa'(z)\Omega K^*}{2 \left( K - \Omega \sqrt{\kappa(z)} \right)^2 \sqrt{\kappa(z)}} < 0,
\]

we find that the change in firm size is larger (smaller) than \( k/k^w \) for goods that are capital-intensive, \( z < \tilde{z} \), (less capital-intensive, \( z > \tilde{z} \)). We cannot even rule out that \( k/k^w \) is so large (small) that domestic firm size will increase (decrease) in all industries. Helpman and Krugman (1985) emphasize, in Chapter 5.4, the beneficial role of rationalization when market entry is free, but this rationalization effect is not guaranteed in our model due to the endogenous response of the factor markets.

The ambiguous change in firm size is an indication that we require a thorough welfare analysis. For this purpose, let us first consider a single commodity \( z \) and the change in per-capita consumption due to trade. As before, per-capita consumption is denoted by \( \hat{y}(z) \). Consider the domestic country. We already know the wage-rental ratios and income levels under autarky and trade which leads to

\[
\hat{y}^a(z) = \frac{Y^a(z)}{L} = \frac{\sqrt{\kappa(z)}}{\lambda(z)} \frac{\Omega}{K - \Omega^2} \left( \frac{K}{\Omega} \sqrt{\frac{1}{\kappa(z)}} - 1 \right), \\
\hat{y}^t(z) = \frac{Y^t(z)}{L + L^*} = \frac{\sqrt{\kappa(z)}}{\lambda(z)} \frac{\Omega}{K + K^* - \Omega^2} \left( \frac{K + K^*}{\Omega} \sqrt{\frac{1}{\kappa(z)}} - 1 \right).
\]

The difference between per-capita consumption under free trade and under autarky is therefore equal to
\[ \hat{y}'(z) - \hat{y}^a(z) = \frac{\Omega K^*(\sqrt{\kappa(z)} - \Omega)}{(K - \Omega^2)(K + K^* - \Omega^2)\lambda(z)}. \] (11)

If \( \kappa' = 0 \) for all \( z \), then \( \Omega = \sqrt{\kappa} \), and so \( \hat{y}^a(z) = \hat{y}'(z) \) for all \( z \). Hence, identical capital input requirements across all industries imply that per-capita consumption does not change, and this has a clear welfare effect:

**Lemma 3** If \( \kappa(z) = \kappa \), then trade does not change per-capita consumption of each commodity, and thus does not change welfare.

This case implies that the (positive) effects of a larger market and the (negative) effects of an increased real wage just compensate each other for each industry. Notably, the case of identical technologies has been referred to as the *featureless economy* by Peter Neary; see especially Neary (2009). In his model, there is no market entry and only one factor of production (labor). He concludes for the case of a featureless world with completely symmetric countries that trade does not change welfare because labor supply stays constant and thus wages rise. This result is independent of the number of firms. Lemma 3 shows that this is also true if (i) market entry is endogenous and (ii) the investment costs do not differ across industries. In particular, note that firms employ two factors of production in our model and that this result does not depend on the specification of labor input requirements. The crucial difference to Peter Neary’s analysis is that trade also encourages excessive market entry.

What happens if the capital input requirements strictly decrease with \( z \) over at least some range? From eq. (11), we find that \( \hat{y}^a(z) > (<)\hat{y}'(z) \) if

\[ \Omega > (<)\sqrt{\kappa(z)}, \] (12)

which suggests that per-capita-consumption will decrease for capital-intensive goods and decrease for all other goods with trade. We summarize our results in

**Proposition 2** If trade equalizes factor prices, then per-capita consumption of commodity \( z \) increases (decreases) if \( \sqrt{\kappa(z)} > (<)\Omega \).

An interesting question is now whether the change in per-capita consumption is positive for the majority or the minority of goods. To illustrate this, Figure 2 assumes that \( \kappa' < 0 \)
across the whole range, and plots three different downward sloping curves for $\sqrt{\kappa(z)}$. All plots have in common that their endpoints, $\sqrt{\kappa(0)}$ and $\sqrt{\kappa(1)}$, coincide so as to make them comparable. In the left panel, $\sqrt{\kappa(z)}$ is a linear function of $z$; thus, $\kappa(z)$ is linear-quadratic in $z$. $\Omega$ is the area under the linear graph of $\sqrt{\kappa(z)}$. Due to linearity, $\Omega$ is also equal to $\sqrt{\kappa(1)} + (\sqrt{\kappa(0)} - \sqrt{\kappa(1)})/2 = (\sqrt{\kappa(1)} + \sqrt{\kappa(0)})/2$ (note that the horizontal length is unity). Therefore, if $\sqrt{\kappa(z)}$ is linear in $z$, $\hat{z}$ is exactly equal to 0.5, so per-capita consumption of goods $z \in [0, 0.5]$ goes up, whereas per-capita consumption of goods $z \in [0.5, 1]$ goes down.

In the right panel, the linear graph is contrasted with two alternatives, a concave and a convex graph. As can be seen easily, the concave graph implies a larger $\Omega$, as the area under the graph is larger than the area in the linear case. Therefore, $\hat{z}$ must be larger than 0.5, and so the goods experiencing an increase in per-capita consumption outnumber those experiencing a decline. On the contrary, a convex graph implies a lower $\Omega$, that is, only less than one half of goods will experience an increase in per-capita consumption. This is potentially worrying, but we have to be careful here. All we have done so far is to look upon the sign of the change, but not yet on its strength. Trade will improve welfare on an aggregate level if the per-capita consumption reductions are only very moderate, while the increases in per-capita consumption are substantial.

Due to our specification of utility, trade unambiguously improves welfare if
\[
\int_0^1 \ln(\hat{y}'(z))dz - \int_0^1 \ln(\hat{y}^a(z))dz \geq 0 \iff v \equiv \int_0^1 \ln(\delta(z))dz \geq 0. \tag{13}
\]

where \(\delta(z) \equiv \hat{y}'(z)/\hat{y}^a(z)\) is the relative change of per-capita consumption from autarky to trade. Unless \(\kappa' = 0\) for all \(z\), we know from Proposition 2 that \(\delta(0) > 1 > \delta(1) > 0\). Since we use a specific utility function, we can go into the details of potential gains from trade. We find:

**Theorem 1** *In the case that trade equalizes factor prices, overall welfare unambiguously improves if*

\[
\frac{1}{2}[\ln(\delta(0)) + \ln(\delta(1))] - \text{Cov} \left[ z, \frac{\delta'(z)}{\delta(z)} \right] \geq 0.
\]

**Proof:** Integration by parts leads to

\[
v = \ln(\delta(1)) - \int_0^1 z \frac{\delta'(z)}{\delta(z)}dz
\]

and

\[
\int_0^1 z \frac{\delta'(z)}{\delta(z)}dz = \int_0^1 zdz \int_0^1 \frac{\delta'(z)}{\delta(z)}dz + \text{Cov} \left[ z, \frac{\delta'(z)}{\delta(z)} \right] \square
\]

Theorem 1 identifies two effects that contribute to the gains from trade. First, we know that (i) the increase in per-capita consumption is largest for \(z = 0\), (ii) the decrease in per-capita consumption is largest for \(z = 1\), and (iii) \(\ln(\delta(0)) > 0\) and \(\ln(\delta(1)) < 0\). Therefore, the effect \([\ln(\delta(0)) + \ln(\delta(1))]\) shows how the extreme changes contribute to gains from trade: if the increase in per-capita consumption of the most capital-intensively produced goods overcompensates the decrease in that of the least capital-intensively produced goods, then this effect contributes positively to the gains from trade such that \([\ln(\delta(0)) + \ln(\delta(1))] > 0\). Second, Theorem 1 shows that a negative covariance between \(z\) and the relative change of per-capita consumption also makes a positive contribution to the gains from trade. So, on average, the relative change in per-capita consumption should not accelerate with the change in capital intensity. This effect deals with per-capita consumption changes in between the two extremes,
and since we consider the covariance here, we just have to take the average correlation between \( z \) and the relative consumption change into account.

Note carefully that we do not require both effects to contribute positively, but at least one effect should be sufficiently strong and positive so as to overcompensate the other one if the other effect is negative. Both effects will depend on the behavior of the capital input requirements. In order to shed some more light on the role of \( \kappa(z) \), we now establish sufficient conditions for welfare improvement dependent on the behavior of capital input requirements. We summarize these findings in Corollary 1.

**Corollary 1** *In the case that trade equalizes factor prices, overall welfare unambiguously improves if the ranking of capital input requirements is not concave* \( (\kappa'' \leq 0) \).

Proof: See Appendix A.2.

The intuition for Corollary 1 is as follows. We observe a structural change due to trade. Proposition 2 shows that per-capita consumption declines for goods which are not produced capital-intensively; these have become overproportionately expensive, and this effect is not overcompensated by the larger market, because the investment costs are not substantial. Conversely, per-capita consumption of capital-intensively produced goods increases, because a larger market can accommodate more firms. Consequently, \( \kappa'' \leq 0 \) guarantees that capital input requirements do not fall underproportionately with \( z \) so that a sufficiently large number of commodities experiences a sufficiently large increase in per-capita consumption.\(^{16}\)

Note carefully that even if these welfare conditions are felt as not too demanding, we would like to make clear that gains from trade do not come natural in our setup. From a social perspective, there is excessive firm entry, and this distortion is the larger the larger is the capital input requirement \( \kappa(z) \). Since the wage-rental ratio will increase, at least for one (dominant) country, this effect is even emphasized as it becomes relatively less costly to establish a firm. Theorem 1 demonstrates that this effect is likely to be overcompensated by the increase in the size of the market.

We can also compare our welfare result to the reciprocal dumping model of Brander and Krugman (1983). They have a model of symmetric countries, one factor of production,

\(^{16}\)Note that the sufficient condition \( \kappa'' \leq 0 \) also implies the convexity of \( \sqrt{\kappa(z)} \) in Figure 2 because

\[
d^2 \sqrt{\kappa(z)}/dz^2 = -\kappa'(z)^2/4\kappa(z)^{3/2} + \kappa''(z)/2\sqrt{\kappa(z)} \leq 0.
\]
and households have quasi-linear preferences. Since there is perfect competition in one sector, trade has no effect on factor prices. They show that trade is always welfare-improving if market entry is free because firms unambiguously become larger and climb down the average cost curve.\footnote{Brander and Krugman (1983) also consider restricted trade by using trade costs. For the case of free trade, that is, zero trade costs, trade is also welfare improving when market structures are exogenous. This is not true for high trade costs. When market structures are endogenous, trade is always welfare improving, irrespective of the size of trade costs.} Our model takes factor price changes into account, and it shows that the overall welfare effects of trade can still be positive.

### 3.2 Different factor prices

In the last section, we have characterized the trade equilibrium which equalizes factor prices. In this section we scrutinize an equilibrium in which factor prices continue to differ after trade liberalization. The analysis of trade without factor price equalization is usually not on the agenda if an FPE equilibrium exists. However, we will show in this subsection that trade may not lead to mutual welfare gains, so this case deserves a thorough analysis.

We know from Lemma 2 that different factor prices imply geographical concentration of industries. Thus, we are now interested in the possible specialization patterns for the equilibrium factor prices. Given Lemma 2, an industry will be hosted by the country having the lowest price for the equilibrium factor prices. Let \( p(z) (p^*(z)) \) denote the price if the domestic (foreign) country hosts industry \( z \). Given \( Y(z) \), prices are given by

\[
p(z) = \frac{\lambda(z) \omega}{\Psi(z)} \quad \text{where} \quad \Psi(z) = 1 - \sqrt{\frac{r \kappa(z)}{I \omega}},
\]

if production takes place in the domestic country; and a similar expression can be written for the foreign country. Since all factor markets have to be cleared, there must be at least one indifferent industry \( \hat{z} \) for which \( p(\hat{z}) = p^*(\hat{z}) \). Thus, we find that

\[
\frac{\lambda(\hat{z}) \omega}{\Psi(\hat{z})} = \frac{\lambda(\hat{z}) \omega^*}{\Psi^*(\hat{z})} \iff \frac{\omega}{\Psi(\hat{z})} = \frac{\omega^*}{\Psi^*(\hat{z})}.
\]

\[
(14)
\]
The pattern of specialization is then straightforward, but we need a slightly stronger assumption on the behavior of the capital input requirements, that is,

**Assumption 3** $\kappa' < 0$ across the whole range for the NFPE case.

We find:

**Proposition 3** In an NFPE equilibrium, one country has the higher equilibrium wage and the lower equilibrium rental compared to the other country. This country hosts industries in the range $z \in [0, \hat{z}]$, and the other country hosts industries in the range $z \in [\hat{z}, 1]$.

Proof: See Appendix A.3.

We can also identify which country is more capital-abundant such that these patterns may emerge. We know from Lemma 1 that the capital intensity will increase with the capital input requirement, which is negatively related to $z$, that is, the capital intensity will decrease with $z$. This will also be true along the range of commodities produced within each country. Assume that the domestic country produces in the range $z \in [0, \hat{z}]$; in this case, $\mu'(z) < 0$ for $z \in [0, \hat{z}]$ and $\mu^*(z) < 0$ for $z \in [\hat{z}, 1]$. Using the definition of $\mu$, given by eq. (3), and the definition of $\hat{z}$, given by eq. (14), we find that

$$\mu(\hat{z}) = \sqrt{\frac{r^*}{r}} \mu^*(\hat{z}) > \mu^*(\hat{z}).$$

Proposition 3 has shown that the domestic country must have a lower rental than the foreign country if it produces in the capital-intensive range, and this is the reason why $\mu(\hat{z}) > \mu^*(\hat{z})$. Therefore, the capital intensity in both countries behaves like in Figure 3.

Factor market clearance implies that relative factor demand for capital is a weighted average of capital intensities. Thus, it follows immediately from the Intermediate Value Theorem that the weighted average will be higher (lower) in the country hosting the capital-(labor-)intensive industries, and this must be matched by the per-capita capital endowment. In our case, it follows that $k > k^*$. Additionally, there are clearly lower bounds on the difference in relative factor endowments. In order to make Figure 3 consistent with the relative factor endowments, a sufficient (but not yet necessary) requirement is that $k \geq \mu(\hat{z})$ and $k^* \leq \mu^*(\hat{z})$. Therefore we conclude:
**Proposition 4** An NFPE equilibrium may exist only if countries are sufficiently different with respect to their relative capital endowments. If it exists, then the capital-abundant country produces the capital-intensive commodities.

In the NFPE case, we find that imperfect competition cannot reverse the specialization patterns as they are well known from classic trade models. Furthermore, a direct implication of Proposition 4 is that the capital-abundant country will be a net exporter of the capital services and a net importer of labor services as embodied in trade.\(^{18}\)

If we maintain our assumption that the domestic country produces goods in the range \([0, \hat{z}]\), then domestic labor and capital markets clear if

\[
L = \int_0^{\hat{z}} \left( \frac{I^w}{w} - \frac{\sqrt{I^w r K(z)}}{w} \right) dz,
\]

\[
K = \frac{I^w}{r} \int_0^{\hat{z}} \sqrt{K(z)} dz,
\]

where world income is now \(I^w = wL + rK + w^*L^* + r^*K^*\). Rearranging the capital

\(^{18}\)See Brecher and Choudhri (1982) for this result in a Heckscher-Ohlin model without factor price equalization.
market condition and putting it into the labor market condition yields

\[ L = \frac{I^w \hat{z} - rK}{w}. \]  

(15)

A similar exercise for the foreign country gives

\[ L^* = \frac{I^w (1 - \hat{z}) - r^* K^*}{w^*}. \]  

(16)

Rearranging these equations yields the trade balance condition, that is, the domestic country’s income should be equal to its production such that

\[ wL + rK = I^w \hat{z}. \]

So we have three equations, the two equations clearing the capital markets – one for each country – and the trade balance. Furthermore, eq. (14) determines \( \hat{z} \). In summary, we have four independent equations, and, after using the domestic rental rate as a numeraire again, that is, \( r = 1 \), four unknowns, \( w, w^*, r^* \) and \( \hat{z} \), and we can show:

**Proposition 5** If an NFPE equilibrium exists, it is unique.

Proof: See Appendix A.4.

What are the welfare effects of trade in the case of NFPE? Similar effects are at work as in the case of FPE, that is, the effect of a larger market and the effect of a potential increase in production costs. Additionally, countries now specialize completely in a certain range of goods as indicated by Proposition 4. As to analyze the welfare effects, we have simulated the model, assuming labor input requirements are constant across industries, \( \forall z \in [0, 1], \lambda(z) = 1 \). Also we assign a specific function to \( \kappa(z) \), that is, \( \kappa(z) = 2 - z \). We know from Corollary 1 that trade would improve welfare in this setup if factor prices could equalize. The reason is that \( \kappa'' = 0 \) guarantees positive gains from trade in the FPE case. Table 1 summarizes the results of our simulation.

Table 1 shows the factor endowments, the relative factor prices, income levels and the welfare change indices \( v \) and \( v^* \) for both countries. In all ten simulations, the factor
endowments are outside the FPE set. The simulations differ such that we increase the capital stock of the capital-abundant, domestic country by 10. We see that this increase in the domestic capital stock increases both the wage-rental in the domestic country under autarky and trade. However, the increase under trade is much larger. Both countries gain from trade in the first three simulations, but once $K = 60$ is reached, it is only the foreign country that gains, and these gains increase when the domestic capital stock is increased further. For the domestic country, the gains from trade become smaller over the whole range and turn negative after simulation # 3 which shows that trade may reduce welfare and proves

\textbf{Theorem 2} In the case that trade does not equalize factor prices, the welfare effects can be negative even if parameters are such that they are unambiguously positive in an FPE equilibrium.

Theorem 2 is remarkable because it is in sharp contrast to the common view that specialization is the key for gains from trade. In our setup, this is not true. So why does trade make the domestic country even suffer if it does not equalize factor prices at the same time? Even if the wage-rental ratios were similar across the two countries, the distortions due to excessive entry would be more substantial in the domestic country because this country hosts the capital-intensive industries. Furthermore, the increase in the wage-rental ratio even exacerbates these distortive effects because it becomes relatively less expensive to establish a firm and more expensive to run it which leads to a reduction in firm size. This effect will be pronounced when the capital stock is relatively large compared to the foreign country. In the FPE case, the distortive effect is symmetric, but in the NFPE case, it is the domestic country which creates more distortive effects than the foreign country. For a large enough asymmetry, this effect is able to eat up the domestic gains from specialization.

4 Extension to general demand structures

In the last sections, we have assumed that each commodity has the same income share. This assumption has allowed us to focus on the role capital input requirements play for welfare with and without factor price equalization. In this section, we discuss whether

\[19\text{The computations of these simulations are available upon request.}\]
and how our results extend to the case of general demand structures. We keep the Cobb-Douglas assumption as in DFS, but now we allow income shares, denoted by $\alpha(z)$, to differ across goods. More formally, utility is now given by $U = \int_0^1 \alpha(z) \ln(\hat{y}(z)) dz$ with $\int_0^1 \alpha(z) dz = 1$. This generalization has straightforward implications for the number of firms entering industry $z$, firm-level outputs and labor and capital demands of industry $z$, respectively, which we summarize here:

$$n(z) = \sqrt{\frac{\alpha(z)I}{r\kappa(z)}},$$

$$y_i(z) = \frac{r\kappa(z)}{\lambda(z)w} \left( \sqrt{\frac{\alpha(z)I}{r\kappa(z)}} - 1 \right),$$

$$L(z) = \frac{\alpha(z)I}{w} - \frac{\sqrt{\alpha(z)Ir\kappa(z)}}{w},$$

$$K(z) = \sqrt{\frac{\alpha(z)I}{r}} \sqrt{\kappa(z)}.$$

What do different income shares change from the viewpoint of firms and potential entrants to an industry? With $\alpha(z) = 1$ as in the previous sections, the ease of entry into an industry was determined only by the capital investment. With different income shares, also the size of $\alpha(z)$ plays a role. It determines the market potential of an industry, and thus a larger $\alpha$ makes investment easier. It is thus now the ratio of investment costs to the market potential which determines the ease of entry to an industry. Consequently, let us define $\beta(z) \equiv \kappa(z)/\alpha(z)$ as the effective investment cost, that is, the investment cost $\kappa$ corrected by market size. We now rank industries such that $\beta(z)$ is non-increasing. Let $\Gamma \equiv \int_0^1 \sqrt{\alpha(z)\kappa(z)} dz = \int_0^1 \alpha(z)\sqrt{\beta(z)} dz$, and we find similarly that income levels are equal to $I = wL + K = K^2/\Gamma^2$ under autarky and equal to $I^w = w(L + L^*) + (K + K^*) = (K + K^*)^2/\Gamma^2$ under trade and factor price equalization.\(^{20}\) We make similar assumption for $\beta(z)$ and $\Gamma$ as we did for $\kappa(z)$ and $\Omega$, respectively, such that at least two firms are active in each industry.

Let us now explore whether and how our former results on labor and capital demand

\(^{20}\)Note that we still assume that the ranking of effective investment costs leads to a differentiable function which declines with $z$. The corresponding $\alpha(z)$ is a function, but neither continuous nor differentiable; it is a cloud in the $\alpha(z) - z$—space. Therefore, any integral involving $\alpha(z)$ is not necessarily Riemann-integrable, but since (i) utility and technology are bounded from above and (ii) $\int_0^1 \alpha(z) dz = 1$, these integrals are Lebesgue-integrable. Lebesgue-integrability guarantees that the basic theorems on expectations and covariances as we will use them below hold.
and capital intensities carry over to the case of general demand structures. As for factor demands, it is now not clear that labor (capital) demand increases (decreases) with $z$ as before. However, the capital intensity is given by

$$
\mu(z) = \frac{w}{r} \frac{\sqrt{I}}{\sqrt{r\beta(z)} - 1}
$$

and has thus the same properties as before (see eq. (3) with $\beta(z)$ replacing $\kappa(z)$). Therefore, our results in Subsection 3.2 carry over with $\beta(z)$ as the effective investment cost.

We have already demonstrated in this subsection that welfare results can be negative, so there is no need to go any further with this using general demand structures.

Instead, we now want to focus on the welfare results with factor price equalization when demand structures exhibit different income shares. As before, let us first consider how per-capita consumption changes with trade.\footnote{We let the rental rate $r$ be the numeraire as in the previous cases.} We find that the difference between per-capita consumption under free trade and under autarky is now equal to

$$
\hat{y}^t(z) - \hat{y}^a(z) = \frac{\Gamma K^* (\sqrt{\beta(z)} - \Gamma)}{(K - \Gamma^2)(K + K^* - \Gamma^2)\lambda(z)},
$$

so that $\hat{y}^a(z) > (<) \hat{y}^t(z)$ if

$$
\Gamma > (<) \sqrt{\beta(z)}.
$$

Note carefully that $\Gamma = \int_0^1 \alpha(z) \sqrt{\beta(z)} dz$ is now the \textit{weighted} average of the square root of the effective investment costs, while $\Omega$ was the unweighted average of the square root of investment costs. However, due to the Intermediate Value Theorem, it must still be true that per-capita consumption increases for low $z$’s and decreases for high $z$’s. Since $\Gamma$ is now a weighted average, the weights also play a role for the strength of the effects on per-capita consumption. This can also be seen by the generalized condition for welfare improvement which includes the income shares:

$$
v = \int_0^1 \alpha(z) \ln (\delta(z)) dz \geq 0.
$$

\footnote{We let the rental rate $r$ be the numeraire as in the previous cases.}
Using (20), we can now turn to the welfare effects with general demand structures.

**Theorem 3** In the case that trade equalizes factor prices, overall welfare unambiguously improves if

\[
\frac{1}{2} \left[ \ln(\delta(0)) + \ln(\delta(1)) \right] - \text{Cov} \left[ z, \frac{\delta'(z)}{\delta(z)} \right] + \text{Cov} \left[ \alpha(z), \ln(\delta(z)) \right] \geq 0.
\]

Proof:

\[
\int_0^1 \alpha(z) \ln(\delta(z)) \, dz = \int_0^1 \alpha(z) \, dz \int_0^1 \ln(\delta(z)) \, dz + \text{Cov} \left[ \alpha(z), \ln(\delta(z)) \right].
\]

\[
\int_0^1 \alpha(z) \, dz = 1,
\]

and the rest of the proof for \( \int_0^1 \ln(\delta(z)) \, dz \) follows from Theorem 1. □

Compared to Theorem 1, there is a third term which contributes to gains from trade. Trade improves welfare also if the increase in per-capita consumptions occurs in those industries in which the income share is large. This does not have to be true across all industries but should hold on average as indicated by the respective covariance. Since \( \kappa \) still plays a role, a positive correlation between income shares and increases in per-capita consumption is clearly welfare-improving.

We now turn to sufficient conditions which trace back gains from trade to the behavior of the effective investment costs and the income shares. For this purpose, let us define a measure of demand changes

\[
g(z) \equiv \frac{\sqrt{\beta(z)} - \Gamma}{\sqrt{\Gamma w} - \sqrt{\beta(z)}}.
\]

which measures how demand changes for commodity \( z \) due to trade. In particular, we know that trade will be welfare improving if \( \int_0^1 \alpha(z)g(z) \, dz \geq 0 \), because this condition implies that the autarky consumption bundle is also feasible under trade. We can now develop two different sets of sufficient conditions which have in common that they require a positive correlation between income shares and the change in consumption patterns.

**Corollary 2** In the case that trade equalizes factor prices, overall welfare unambiguously improves if
(i) the income shares and the measure of demand changes are positively correlated \( \text{Cov}[\alpha(z), g(z)] \geq 0 \) and the ranking of effective capital input requirements is not concave \( (\beta'' \leq 0) \)

or

(ii) \( \text{Cov}[\alpha(z) \ln(\delta(z))] \geq 0 \) and the ranking of effective capital input requirements is sufficiently convex such that \( \beta''(z) \leq -\beta'(z)^2/2\beta(z) \).

Proof: See Appendix A.5.

Both conditions now refer to the ranking of effective capital input requirements. If this is sufficiently convex, the number of commodities experiencing a substantial increase in per-capita consumption is sufficiently large. Furthermore, these changes should apply to those commodities which, on average, play an important role as measured by their income shares. Thus, all results from the previous sections carry over to a large extent to the case of general demand structures. However, they require the qualification that it should be more the important commodities that benefit from positive changes. This does not have to be true for each commodity with a high income share, but it should hold on average as measured by the covariance.

5 Concluding remarks

This paper has shown that the general equilibrium effects of factor price changes can play a crucial role for the welfare effects of trade. We have developed a model in which the industry structure is endogenenous and depends on factor endowments. Hence, also the market for firm assets is endogenous in our model. In an equilibrium in which factor prices do not equalize, the general equilibrium effects may even make a country worse off. However, gains from trade are guaranteed under relatively mild conditions if trade equalizes factor prices, but this does not mean that per-capita consumption will increase across the board.

Overall, our paper has led to new insight emphasizing the important role feedback effects from factor markets can play for trade liberalization in the presence of economies of scale. In particular, free entry is excessive in our setup and is the potential troublemaker. Our paper thus offers a framework for analyzing trade under imperfect competition when (i) strategic interactions between firms are regarded as important while
product differentiation is regarded as less important. In this sense, our model is complementary to the well-known monopolistic competition models. In these models, firms do not act strategically, and hence entry is not subject to the business stealing effect, and an increase in product variety is an important source for gains from trade. The simplicity of the DFS framework does not allow us to endogenize the product space, and we know from the industrial organization literature that the effect of free entry is ambiguous for welfare if firms compete with differentiated products. However, there is no reason why the business stealing effect should completely disappear in a more general model with strategic interactions on an industry level.

Of course, whether strategic interactions matter or not is, in the end, an empirical question. In models of monopolistic competition, however, there is no room for a decrease in per-capita consumption. This is also true for heterogeneous firm models because all surviving firms will have become more efficient. A decline in consumption of some commodities cannot be explained by trade in this setup, and if it happens, different technological progress across industries and/or changes in tastes are the usual suspects. Our model can show that trade changes factor prices and also makes production in some industries overproportionately costly. This effect is even pronounced when trade does not lead to factor price equalization.

Our model is static in nature, so we do not consider how capital formation and technological progress will affect trade patterns in the long run. On the one hand, an increase of the wage-rental ratio is potentially bad news for capital formation. On the other hand, research and development (R&D) are a fixed cost as well, so our assumption of constant capital input requirements cannot hold in the long run. If R&D is considered to be capital-intensive (or skilled-labor-intensive), then a decrease in the wage-rental ratio makes R&D investment less expensive, so we should expect more innovation. Long et al. (2011) consider the role innovation and trade play in an oligopolistic model with strategic interactions of potentially heterogeneous firms for output, entry and R&D. However, they do not consider general equilibrium effects from factor markets. We leave it to future research to explore how the effects of R&D can be incorporated into an oligopolistic general equilibrium model of trade.
Appendix

A.1 Proof of Lemma 2

In an integrated market: \( Y(z) = \sum_i y_i(z) + \sum_i y_i^*(z) \). The first-order conditions are

\[
\frac{\partial \Pi_i(z)}{\partial y_i(z)} = p(z) - \lambda(z)w + p_{Y(z)}(z)y_i(z) = \frac{I^w}{Y(z)} - \lambda(z)w - \frac{I^w y_i(z)}{Y(z)^2} = 0,
\]

\[
\frac{\partial \Pi_i^*(z)}{\partial y^*_i(z)} = p(z) - \lambda(z)w^* + p_{Y(z)}(z)y^*_i(z) = \frac{I^w}{Y(z)} - \lambda(z)w^* - \frac{I^w y_i^*(z)}{Y(z)^2} = 0.
\]

All firms within industry \( z \) are symmetric such that \( Y(z) = n(z)y_i(z) + n^*(z)y^*_i(z) \). Using symmetry, we find that

\[
y_i(z) = \frac{I^w(n(z) + n^*(z) - 1)(n^*(z)w^* - (n^*(z) - 1)w)}{\lambda(z)(n(z)w + n^*(z)w^*)^2},
\]

\[
y^*_i(z) = \frac{I^w(n(z) + n^*(z) - 1)(n(z)w - (n(z) - 1)w)}{\lambda(z)(n(z)w + n^*(z)w^*)^2},
\]

leading to maximized profits

\[
\Pi_i(z) = \frac{I^w(n^*(z)w^* - (n^*(z) - 1)w)^2}{(n(z)w + n^*(z)w^*)^2} - r^*\kappa(z) = 0,
\]

\[
\Pi^*_i(z) = \frac{I^w(n(z)w - (n(z) - 1)w^*)^2}{(n(z)w + n^*(z)w^*)^2} - r^*\kappa(z) = 0,
\]

which are equal to zero in equilibrium. Differentiation of maximized profits w.r.t. \( n(z) \) and \( n^*(z) \) leads to

\[
|J| = \frac{\partial \Pi_i(z)}{\partial n(z)} \frac{\partial \Pi^*_i(z)}{\partial n^*(z)} - \frac{\partial \Pi_i(z)}{\partial n^*(z)} \frac{\partial \Pi^*_i(z)}{\partial n(z)} = 0,
\]

which proves that coexistence is impossible for different factor prices.
A.2 Proof of Corollary 1

Welfare unambiguously improves if the consumption bundle under autarky is also feasible under trade, that is

$$\int_0^1 p(z)(\hat{y}'(z) - \hat{y}''(z))dz \geq 0,$$

the sign of which is equivalent to

$$\int_0^1 f(z)dz \geq 0, f(z) \equiv \frac{\sqrt{\kappa(z)} - \Omega}{\sqrt{I^w} - \sqrt{\kappa(z)}}.$$

We are interested in the behavior of $f(z)$. The first-order derivative is equal to

$$f'(z) = \frac{\kappa'(z) \left(\sqrt{I^w} - \Omega\right)}{2\sqrt{\kappa(z)} \left(\sqrt{I^w} - \sqrt{\kappa(z)}\right)^2} < 0,$$

and the second-order derivative is equal to

$$f''(z) = -\frac{\left(\sqrt{I^w} - \Omega\right) \left(\left(\sqrt{I^w} - 3\sqrt{\kappa(z)}\right) \kappa'(z)^2 - 2 \left(\sqrt{I^w} - \sqrt{\kappa(z)}\right) \kappa(z) \kappa''(z)\right)}{4 \left(\sqrt{I^w} - \sqrt{\kappa(z)}\right)^3 \kappa(z)^{3/2}}$$

and unambiguously negative if $\kappa'' \leq 0$ because

$$\sqrt{I^w} = \frac{K + K^*}{\Omega} > 4\frac{\kappa(0)}{\Omega} = 4\frac{\sqrt{\kappa(0)} \sqrt{\kappa(0)}}{\int_0^1 \sqrt{\kappa(z)}dz} > 4\sqrt{\kappa(0)} \geq 4\Omega.$$

Therefore, $f$ is a convex function implying

$$\int_0^1 \frac{\sqrt{\kappa(z)} - \Omega}{\sqrt{I^w} - \sqrt{\kappa(z)}}dz \geq \frac{\int_0^1 \sqrt{\kappa(z)}dz - \Omega}{\sqrt{I^w} - \int_0^1 \sqrt{\kappa(z)}dz} = 0.$$
A.3 Proof of Proposition 3

Define the following functions

\[
\theta(z) = w \left( 1 - \sqrt{\frac{r^* \kappa(z)}{I^w}} \right), \phi(z) = w^* \left( 1 - \sqrt{\frac{\kappa(z)}{I^w}} \right). 
\]

Using the domestic rental as a numeraire, that is \( r = 1 \), at \( \hat{z} \) :

\[
\theta'(\hat{z}) = \phi'(\hat{z}).
\]

Differentiation yields

\[
\theta'(z) = -\frac{w}{2} \sqrt{\frac{r^*}{I^w \kappa(z)}} \kappa'(z) > 0, \quad \phi'(z) = -\frac{w^*}{2} \sqrt{\frac{1}{I^w \kappa(z)}} \kappa'(z) > 0.
\]

At any intersection of \( \theta(z) \) and \( \phi(z) \) in the \( \theta/\phi - z \)-space, either \( \theta'(z) > \phi'(z) \) or \( \theta'(z) < \phi'(z) \). If \( \theta'(z) > \phi'(z) \), prices are lower for all \( z < (>) \hat{z} \) in the domestic (foreign) country and it must be that \( \sqrt{r^*} w > w^* \). On the contrary, if \( \theta'(z) < \phi'(z) \), prices are lower for all \( z < (>) \hat{z} \) in the foreign (domestic) country and it must be that \( \sqrt{r^*} w < w^* \). Hence, there can be only one \( \hat{z} \) as two intersections would warrant \( \sqrt{r^*} w > w^* \) and \( \sqrt{r^*} w < w^* \) at the same time.

A.4 Proof of Proposition 5

A direct implication of Proposition 3 is that a certain vector of factor prices implies a unique \( \hat{z} \). Suppose that \( \hat{z} \) is given. Differentiating the equilibrium conditions

\[
\begin{align*}
 wL + K - \hat{z}I^w &= 0, \\
 K - \sqrt{I^w} \int_0^{\hat{z}} \sqrt{\kappa(z)} dz &= 0, \\
 K^* - \sqrt{I^w} \int_{\hat{z}}^1 \sqrt{\kappa(z)} dz &= 0, \\
 I^w - wL - K - w^* L^* - r^* K^* &= 0
\end{align*}
\]

yields

\[
\begin{bmatrix}
 L & 0 & 0 & \hat{z} \\
 0 & 0 & 0 & \xi_{24} \\
 0 & 0 & \xi_{33} & \xi_{34} \\
 -L & -L^* & -K^* & 1
\end{bmatrix}
\begin{bmatrix}
 dw \\
 dw^* \\
 dr^* \\
 dI^w
\end{bmatrix} = 0,
\]

(A.1)
where

\[
\begin{align*}
\xi_{24} &= -\frac{1}{2} \sqrt{\frac{1}{I^w}} \int_0^z \sqrt{\kappa(z)} dz < 0 \\
\xi_{33} &= \frac{1}{2r^*} \sqrt{I^w r^*} \int_{\hat{z}}^1 \sqrt{\kappa(z)} dz > 0 \\
\xi_{34} &= -\frac{1}{2} \sqrt{\frac{1}{I^w r^*}} \int_{\hat{z}}^1 \sqrt{\kappa(z)} dz < 0.
\end{align*}
\]

The Jacobian determinant of the matrix in (A.1) is equal to \( \xi_{24} \xi_{33} LL^* \) and is thus unambiguously negative. The unambiguous sign implies that there is one and only one vector of factor prices for a given \( \hat{z} \) and, due to Proposition 3, one and only one \( \hat{z} \) for any vector of factor prices, and the equilibrium is unique.

A.5 Proof of Corollary 2

For part (i), we use the approach that welfare unambiguously improves if the consumption bundle under autarky is also feasible under trade which leads to

\[
\int_0^1 \alpha(z)g(z)dz \geq 0, g(z) = \frac{\sqrt{\beta(z)} - \Gamma}{\sqrt{I^w} - \sqrt{\beta(z)}},
\]

which can be rewritten as

\[
\int_0^1 \alpha(z)g(z)dz = \int_0^1 g(z)dz + \text{Cov}[\alpha(z), g(z)].
\]

If \( \text{Cov}[\alpha(z), g(z)] \geq 0 \) and \( \beta'' \leq 0 \), then

\[
\int_0^1 \alpha(z)g(z)dz \geq \int_0^1 g(z)dz.
\]

Now consider function \( \tilde{g}(z) \) which is equivalent to \( g(z) \):

\[
\tilde{g}(z) = \frac{\alpha(z) \sqrt{\beta(z)} - \alpha(z) \Gamma}{\alpha(z) \sqrt{I^w} - \alpha(z) \sqrt{\beta(z)}} = g(z).
\]

Of course, \( \tilde{g}'(z) = g'(z) \) and \( \tilde{g}''(z) = g''(z) \). The proof for convexity of \( g(z) \) is similar to the one for convexity of \( f(z) \) in the proof of Theorem 1, see Appendix A.2. Convexity of \( g(z) \) also implies convexity of \( \tilde{g}(z) \), and thus
\[
\int_{0}^{1} g(z)dz = \int_{0}^{1} \tilde{g}(z)dz \geq \int_{0}^{1} \alpha(z)\sqrt{\beta(z)}dz - \Gamma = 0
\]
due to Jensen’s Inequality Theorem.

The proof for part (ii) proceeds in several steps and also uses \( v \) and Jensen’s Inequality Theorem. Consider

\[
\int_{0}^{1} h(z)dz, h(z) = \ln \delta(z), \delta(z) = \frac{\tilde{y}'(z)}{\tilde{y}^a(z)} = \frac{K - \Gamma^2}{K + K^* - \Gamma^2} \frac{K + K^* - \Gamma \sqrt{\beta(z)}}{K - \Gamma \sqrt{\beta(z)}},
\]

which differs from \( v \) that does not involve the income shares. We are interested in the behavior of \( h(z) \). The first-order derivative is equal to

\[
h'(z) = \frac{\delta'(z)}{\delta(z)} = \frac{K^* \Gamma \beta'(z)}{2\sqrt{\beta(z)} \left( K - \sqrt{\beta(z)} \Gamma \right) \left( K + K^* - \sqrt{\beta(z)} \Gamma \right)} < 0,
\]

where

\[
\delta'(z) = \frac{K^* \Gamma \beta'(z) (K - \Gamma^2)}{2\sqrt{\beta(z)} \left( K - \sqrt{\beta(z)} \Gamma \right)^2 (K + K^* - \Gamma^2)} < 0.
\]

The second-order derivative is equal to

\[
\delta''(z) = -\frac{K^* \Gamma (K - \Gamma^2) \left( (K - 3\Gamma \sqrt{\beta(z)}) \beta'(z)^2 - 2 \left( K - \Gamma \sqrt{\beta(z)} \right) \beta(z) \beta''(z) \right)}{4(K + K^* - \Gamma^2) \left( K - \Gamma \sqrt{\beta(z)} \right) \beta(z)^{3/2}}.
\]

\( \delta''(z) < 0 \) if

\[
\rho(\cdot) \equiv \left( K - 3\Gamma \sqrt{\beta(z)} \right) \beta'(z)^2 - 2 \left( K - \Gamma \sqrt{\beta(z)} \right) \beta(z) \beta''(z) > 0.
\]

Note that

\[
\begin{align*}
\rho_K &= \beta'(z)^2 - 2\beta(z)\beta''(z) > 0 \text{ if } \beta''(z) < 0, \\
\rho(\cdot) = 0 &\Rightarrow K = \Gamma \sqrt{\beta(z)} \left( 1 + \frac{2\beta'(z)^2}{\beta'(z)^2 - 2\beta(z)\beta''(z)} \right).
\end{align*}
\]

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The restriction on the minimum number of firms implies that \( K > 2\Gamma \sqrt{\beta(z)} \), so we find that \( \delta \) is convex if

\[
\frac{2\beta'(z)^2}{\beta'(z)^2 - 2\beta(z)\beta''(z)} \leq 1 \iff \beta''(z) \leq \frac{\beta'(z)^2}{2\beta(z)},
\]

because this implies that (i) \( K \) is always large enough, and (ii) \( \beta''(z) < 0 \). Now consider function \( \tilde{\delta}(z) \) which is equivalent to \( \delta(z) \):

\[
\tilde{\delta}(z) = \frac{K - \Gamma^2}{K + K^* - \Gamma^2} \frac{\alpha(z)(K + K^*) - \Gamma \alpha(z) \sqrt{\beta(z)}}{\alpha(z)K - \Gamma \alpha(z) \sqrt{\beta(z)}} = \delta(z).
\]

Of course, \( \tilde{\delta}'(z) = \delta'(z) \) and \( \tilde{\delta}''(z) = \delta''(z) \). Convexity of \( \delta(z) \) implies also convexity of \( \tilde{\delta}(z) \), and thus

\[
\int_{0}^{1} \delta(z) dz = \int_{0}^{1} \tilde{\delta}(z) dz \geq \frac{K - \Gamma^2}{K + K^* - \Gamma^2} \frac{K + K^* - \Gamma^2}{K - \Gamma^2} = 1
\]

due to Jensen’s Inequality Theorem. The second-order derivative of \( h(z) \) is equal to

\[
h''(z) = -\frac{K^* \Gamma (K - \Gamma^2)}{4(K + K^* - \Gamma^2) \delta(z)^2 \left(K - \Gamma \sqrt{\beta(z)}\right)^3 \beta(z)^{3/2}} \sigma(\cdot)
\]

where \( \sigma(\cdot) \equiv \beta'(z) \left(2 \left(K - \Gamma \sqrt{\beta(z)}\right) \beta(z) \delta'(z) + \delta(z) \left(K - 3\Gamma \sqrt{\beta(z)}\right) \beta'(z)\right) - 2\delta(z) \left(K - \Gamma \sqrt{\beta(z)}\right) \beta(z) \beta''(z) \). If \( \sigma(\cdot) > 0 \), then \( h''(z) < 0 \).

Note that

\[
\sigma_K = \delta(z) \left(\beta'(z)^2 - 2\beta(z) \beta''(z)\right) + 2\beta(z) \delta'(z) \beta'(z),
\]

\[
\sigma(\cdot) = 0 \Rightarrow K = \Gamma \sqrt{\beta(z)} \left(1 + \frac{2\beta'(z)^2}{\Lambda + \beta'(z)^2 - 2\beta(z) \beta''(z)}\right)
\]

where

\[
\Lambda = \frac{2\beta(z) \delta'(z) \beta'(z) \delta(z)}{\delta(z)} > 0.
\]
First, we observe that $\sigma_K$ is positive because $\beta'(z)^2 - 2\beta(z)\beta''(z) > 0$ and $\delta'(z)\beta'(z) > 0$, given our assumption about the behavior of the $\beta(z)$-function. Second, the restriction on the minimum number of firms implies again that $K > 2\Gamma\sqrt{\beta(z)}$, and we find that

$$1 + \frac{2\beta'(z)^2}{\Lambda + \beta'(z)^2 - 2\beta(z)\beta''(z)} < 1 + \frac{2\beta'(z)^2}{\beta'(z)^2 - 2\beta(z)\beta''(z)} \leq 2,$$

proving that $h$ is convex, so that

$$\int_0^1 h(\delta(z))dz \geq h\left(\int_0^1 \delta(z)dz\right) = \ln\left(\int_0^1 \delta(z)dz\right) \geq 0.$$ 

Furthermore,

$$v = \int_0^1 \alpha(z)\ln(\delta(z))dz = \int_0^1 \ln(\delta(z))dz + \text{Cov}[\alpha(z),\ln(\delta(z))]$$

which is clearly positive if $\text{Cov}[\alpha(z),\ln(\delta(z))] > 0$. 

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Table 1: Simulation Results

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<th>Sim#</th>
<th>K</th>
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<th>L</th>
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<th>( w^{*a} )</th>
<th>( w^* )</th>
<th>( r^* )</th>
<th>( I^a )</th>
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Notes: The superscript \( a \) denotes the case of autarky. The numeraire for relative factor prices is the rental, that is, \( r^a = 1 \) for \( w^a, r^{*a} = 1 \) for \( w^{*a} \) and \( r = 1 \) for \( w, w^* \) and \( r^* \), respectively.
References


