ECONSTOR
Make Your Publications Visible.

A Service of

ZBW Leibniz-Informationszentrum Wirtschaft
Leibniz Information Centre for Economics

Sievers, Tim

Working Paper

A vector-based approach to modeling knowledge in economics

Hamburg Contemporary Economic Discussions, No. 03/2005

Provided in Cooperation with:
University of Hamburg, Chair for Economic Policy

Suggested Citation: Sievers, Tim (2005) : A vector-based approach to modeling knowledge in economics, Hamburg Contemporary Economic Discussions, No. 03/2005, ISBN 978-3-940369-05-5, University of Hamburg, Faculty of Business, Economics and Social Sciences, Chair for Economic Policy, Hamburg

This Version is available at:
https://hdl.handle.net/10419/49288

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

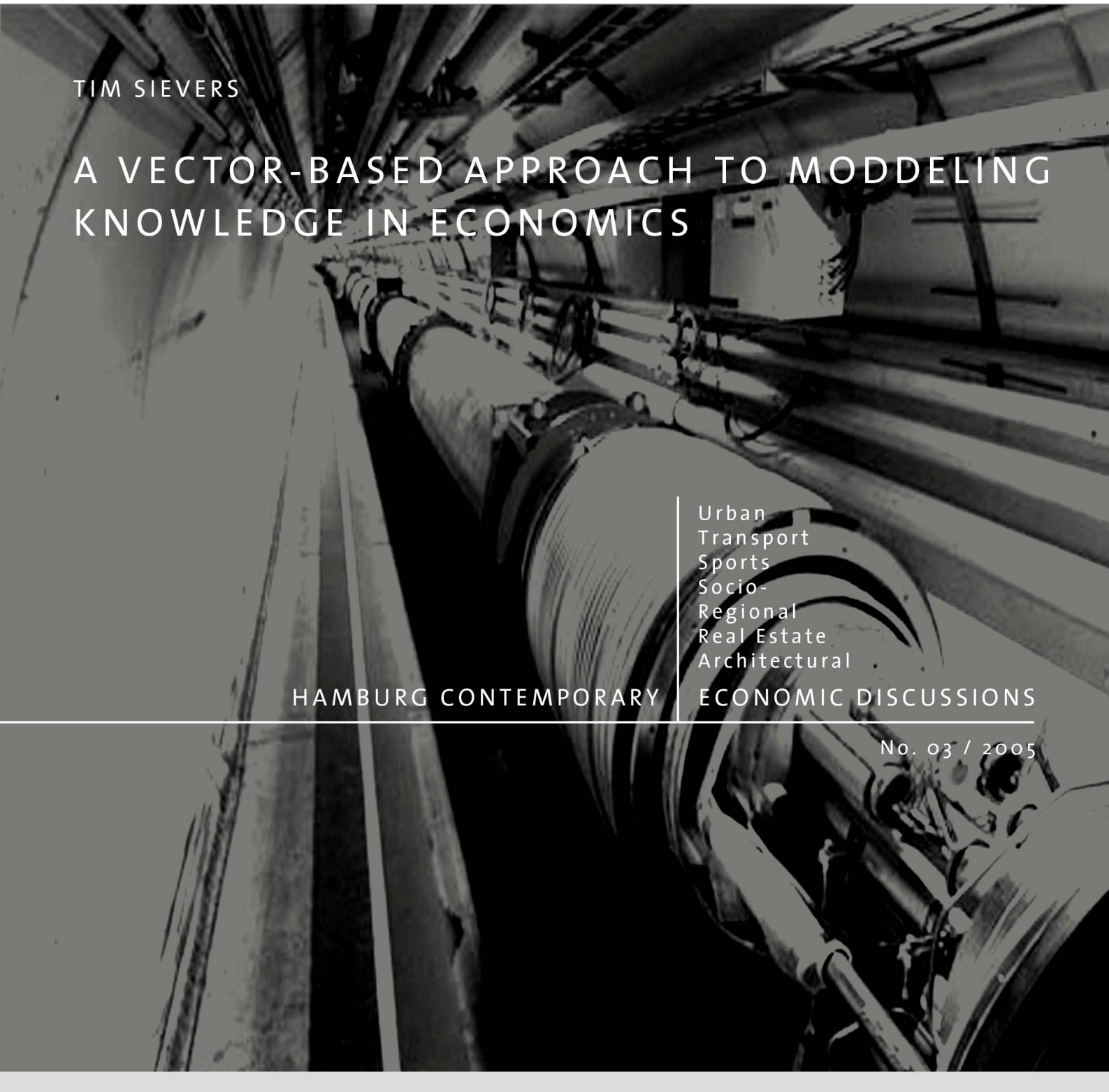
If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

WWW.ECONSTOR.EU

Mitglied der Leibniz-Gemeinschaft

Universität Hamburg

Faculty Economics
and Social Sciences
Chair for Economic Policy



TIM SIEVERS

A VECTOR-BASED APPROACH TO MODDELING KNOWLEDGE IN ECONOMICS

Urban
Transport
Sports
Socio-
Regional
Real Estate
Architectural

HAMBURG CONTEMPORARY | ECONOMIC DISCUSSIONS

No. 03 / 2005

Hamburg Contemporary Economic Discussions
University of Hamburg
Faculty Economics and Social Science
Chair for Economic Policy
Von-Melle-Park 5
D-20146 Hamburg | Germany
Tel +49 40 42838 - 4622
Fax +49 40 42838 - 6251
http://www.uni-hamburg.de/economicpolicy

Editor: Wolfgang Maennig

Tim Sievers
University of Hamburg
Faculty Economics and Social Science
Chair for Economic Policy
Von-Melle-Park 5
D-20146 Hamburg | Germany
Tel +49 40 42838 - 4622
Fax +49 40 42838 - 6251
hced@econ.uni-hamburg.de

ISSN 1865 - 2441 (Print)
ISSN 1865 - 7133 (Online)

ISBN 978 - 3 - 940369 - 04 - 8 (Print)
ISBN 978 - 3 - 940369 - 05 - 5 (Online)

Tim Sievers

A Vector–based Approach to Modeling Knowledge in Economics

Abstract: This paper draws attention to two important characteristics of knowledge which so far have been left unexplored, and proposes a new method to capture them in economic modeling with the help of vectors. The direction of the vector represents the knowledge's complementarity with other knowledge and its norm represents its potential economic value.
Keywords: knowledge; innovation; innovative production; vector representation; complementarity
JEL classification: C0, O3
Version: February 2005

1 Introduction

The characteristics of knowledge in innovative activity have been discussed at length in the literature (ARROW, 1962; COHEN & LEVINTHAL, 1989; DOSI, 1988; NELSON, 1959). This note draws attention to two important characteristics which so far have been left unexplored, and proposes a new method for modeling knowledge to capture these.

(1) The same new knowledge can be of different value to different firms.

Consider for instance a company specialised in the production of musical instruments which produced new knowledge about the characteristics of certain materials. The company is likely to recognise and use only those aspects of its new knowledge which fall in the area of its existing competences to improve the quality or reduce the production costs of its instruments. Yet the same knowledge might contain valuable insights for completely different products, for instance in the area of sports equipment, where similar materials might be used. This latter economic potential, however, is likely to not be exploited by the musical instruments company.

In what follows I shall refer to the degree of fit of a firm's new knowledge with its existing knowledge (comprising competences, routines, organisational structure,

etc.) as the new knowledge's complementarity. This complementarity determines the *effective value* of knowledge for a company. I define the *potential value* of knowledge as comprising all its economically useful aspects, some of which its owner will not be able to recognise or exploit in case of less than perfect complementarity.

(2) The production of new knowledge involves uncertainty.

There are two types of uncertainty involved: firstly, about whether and how much new knowledge will be produced, and secondly, about what new knowledge exactly will be invented and how useful it will be. In the following I shall abstract from the first type of uncertainty by assuming that a certain amount of R&D expenditure will always produce an amount of knowledge of a certain potential value. However, uncertainty remains about the degree of complementarity of new knowledge and hence about its effective value for the inventor.

So far studies of innovative production have failed to take these knowledge-specific properties seriously. For instance, KLETTE & KORTUM (2004) explicitly take into account the importance of uncertainty, but their model fails to capture complementarity. In their model an invention adds the same amount to any firm's "knowledge capital" regardless of whether it is close to the firm's current activities or not.[1]

To capture the special properties of innovative production I propose to model knowledge as a vector characterised by a length, i.e. a scalar which reflects its potential value, and a relative direction, i.e. an angle which reflects its complementarity.

This type of representation also allows to deal with a conceptual problem which results from the importance of cumulativeness of knowledge: New knowledge builds on existing knowledge and often only becomes "inventable" on the basis

[1] Cobb-Douglas type innovative production functions suffer from the same limitation (e.g., KLETTE, 1996).

of what was known before. This entails for instance the "standing on the shoulders of giants" effect but works in reverse as well. New knowledge might allow to put previously existing knowledge to other uses and thereby lead to further commercial innovation. To capture the cumulativeness of knowledge requires the ability to model the outcome of the combination of two sets of knowledge. With a one-dimensional variable, however, this is difficult to handle because the importance of complementarity means that additivity does not hold for knowledge in a one-dimensional sense. The increase in effective knowledge due to any new knowledge is dependent on the degree of complementarity of the new knowledge.

2 Vector Representation of Knowledge

Let $f_i^t: \mathbb{R} \to \mathbb{R} \times (0°, 360°)$ be the knowledge production technology for *i* in period *t* such that

$$f_t^i: e_t^i \to \left\|\vec{n}_t^i\right\|, \beta\left(\vec{n}_t^i\right) \tag{1}$$

where e_t^i is a scalar denoting *i*'s R&D expenditure in period *t*. $\vec{n}_t^i$ is the vector of new knowledge. $\beta\left(\vec{n}_t^i\right) \in (0°, 360°)$ is a random variable following a known probability distribution denoting the direction of $\vec{n}_t^i$. $\left\|\vec{n}_t^i\right\|$ is the Euclidean norm of $\vec{n}_t^i$ and a known function $g_t^i: \mathbb{R} \to \mathbb{R}$ of e_t^i,

$$\left\|\vec{n}_t^i\right\| = g_t^i\left(e_t^i\right) \tag{2}$$

with $g_t^{i\prime} > 0$, $g_t^{i\prime\prime} > 0$. The interpretation of the scalar $\left\|\vec{n}_t^i\right\|$ is that it denotes the length and hence the potential value of $\vec{n}_t^i$, while $\beta\left(\vec{n}_t^i\right)$ is the basis for evaluating the relative direction and hence complementarity of $\vec{n}_t^i$. This allows to operationalise the notion of uncertainty in innovative activity.

Knowledge accumulation in this setting is captured with the help of simple vector addition

$$\vec{k}_t^i = \vec{k}_{t-1}^i + \vec{n}_i^i \tag{3}$$

where $\vec{k}_t^i$ is the total knowledge vector of firm *i* in period *t* which results from adding period *t*'s new knowledge, $\vec{n}_t^i$, to the inherited knowledge vector from the previous period, $\vec{k}_{t-1}^i$ (see Figure 1).

As the defining asset of an innovative firm one can think of $\vec{k}_{t-1}^i$ as a proxy for its inherited competences and organisation. The knowledge accumulation process above then reflects the evolution of the firm over time.

To the extent that it is exclusive (for instance, due to effective patent protection) new knowledge can be valuable for its owner because it affords a protected market position. Assume that the innovative firm's profit is increasing in the effective value of its new knowledge. Consider the following simple payoff function

$$\Pi_t^i = h\left[\alpha\left(\vec{n}_t^i, \vec{k}_{t-1}^i\right)\right] \cdot \left\|\vec{n}_t^i\right\| \tag{4}$$

where $\alpha\left(\vec{n}_t^i, \vec{k}_{t-1}^i\right) \in (0°, 180°)$ denotes the (smallest) difference between $\beta\left(\vec{n}_t^i\right)$ and $\beta\left(\vec{k}_{t-1}^i\right)$ and h is a continuously differentiable function monotonously decreasing in $\alpha\left(\vec{n}_t^i, \vec{k}_{t-1}^i\right)$, with $h(0) = 1$.[2] As an inverse index of complementarity $\alpha\left(\vec{n}_t^i, \vec{k}_{t-1}^i\right)$ is an important determinant of the effective value of new knowledge, i.e. of how much of the knowledge's potential value a particular firm can realise.

3 Applications

The vector methodology offers promising ways in which to revisit some existing debates in knowledge economics. Even the simple payoff structure assumed above provides a theoretical basis to look at the case for efficiency enhancing reallocation of knowledge as a driver for knowledge transfers. The importance of complementarity and uncertainty in innovative activity can lead to a gap between the potential and the effective value of new knowledge for a specific agent. Reallocation of new knowledge to a firm with more complementary existing knowledge might become a profitable opportunity. This provides a market-based framework for the analysis of channels for knowledge transfers, e.g.

[2] An example for h is $h(x) = \cos(x)$.

through the M&A or the labour market. SIEVERS (2005) for instance uses the vector methodology to develop a model of clustering which does not rely on knowledge spillovers but derives from the importance of uncertainty and complementarity inherent in innovative activity and the cluster's role of alleviating informational imperfections.

Another possible extension applies to the "Schumpeterian" debate on the relationship between firm size and innovative success. This strand of the literature acknowledges the importance of uncertainty and complementarity and introduces the latter with a distinction between "drastic" and "incremental" innovation.[3] The restriction to a binary, discrete approach to complementarity and the lack of a framework for knowledge accumulation are major shortcomings. With vector representation the role of firm size for innovative success follows naturally from an alteration of the payoff structure, for instance to $\Pi_t^i = \left\|\vec{k}_t^i\right\| - \left\|\vec{k}_{t-1}^i\right\|$, without having to sacrifice the special characteristics of innovative production (Figure 2).

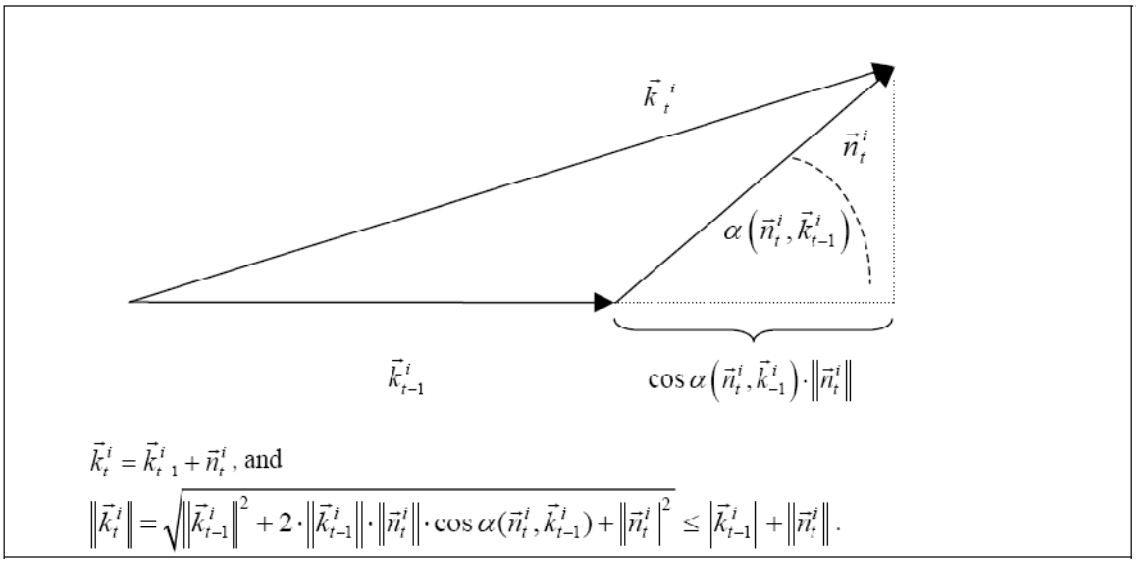
4 Conclusion

I propose to use vector representation as a simple yet powerful analytical tool to model knowledge more accurately and consistently, which has so far not been explored in the literature. This is useful for at least two reasons: The first is that the inherent properties of knowledge might be drivers of dynamics which – for lack of alternative explanations – have been interpreted as caused by other forces. An example that springs to mind is the role of knowledge spillovers as an explanation for clustering of innovative activity. The second reason is that inferences from any model involving knowledge as input or output and not taking into account its special characteristics emphasised in this paper must be interpreted with caution. The potential sensitivity of results with regard to the left out properties needs to be tested.

[3] See HENDERSON (1993) for an overview and extension.

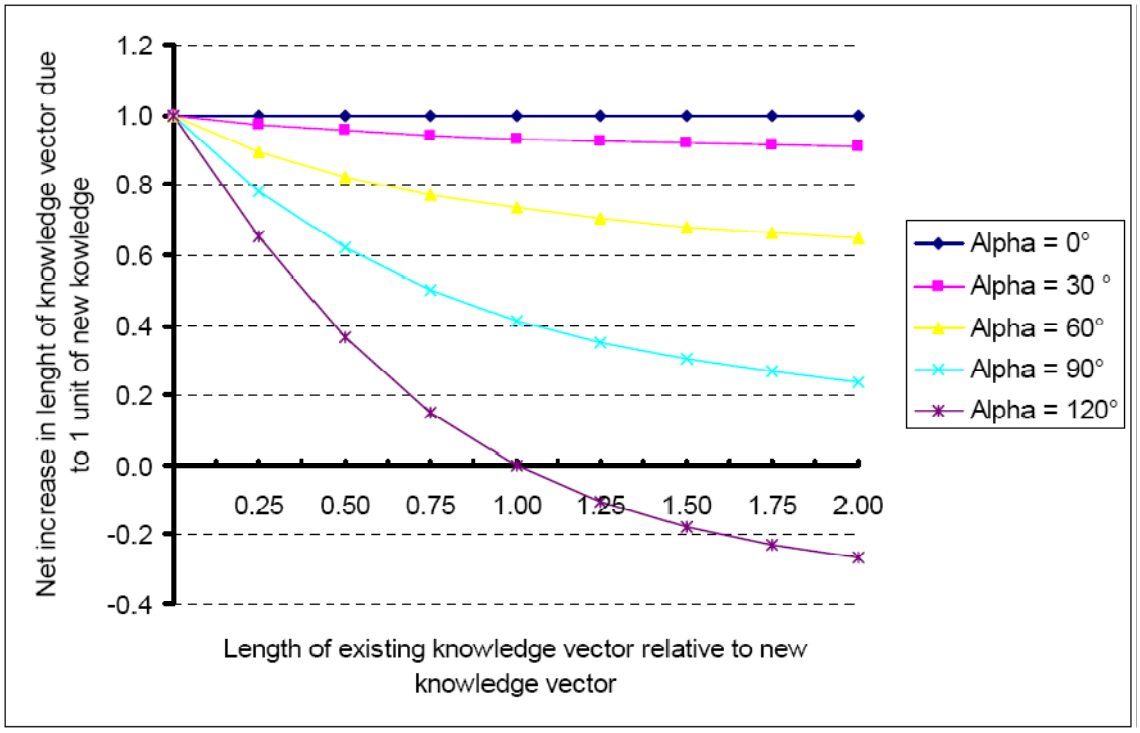
5 Appendix

Fig. 1 Cumulativeness and Complementarity with Vector Representation



Source: Own illustration.

Fig. 2 Impact of New Knowledge as a Function of Non-complementarity (Alpha) and Existing Knowledge



Source: Own illustration.

Literature

ARROW, K. J. (1962). The Economic Implications of Learning by Doing. *Review of Economic Studies, 29*(3), 155-173.

COHEN, W. M., & LEVINTHAL, D. A. (1989). Innovation and Learning: The Two Faces of R & D. *Economic Journal, 99*(397), 569-596.

DOSI, G. (1988). The Nature of the Innovative Process. In G. DOSI, C. FREEMAN, R. NELSON, G. SILVERBERG & L. SOETE (Eds.), *Technical Change and Economic Theory*. London & New York: Pinter Publishers.

HENDERSON, R. (1993). Underinvestment and Incompetence as Responses to Radical Innovation: Evidence from the Photolithographic Alignment Equipment Industry. *RAND Journal of Economics, 24*(2), 248-270.

KLETTE, T. J. (1996). R&D, Scope Economies, and Plant Performance. *RAND Journal of Economics, 27*(3), 502-522.

KLETTE, T. J., & KORTUM, S. (2004). Innovating Firms and Aggregate Innovation. *Journal of Political Economy, 112*(5), 986-1018.

NELSON, R. R. (1959). The Simple Economics of Basic Scientific Research. *Journal of Political Economy, 67*(3), 297-306.

SIEVERS, T. (2005). Information-Driven Clustering – an Alternative to the Knowledge Spillover Story. *Hamburg Contemporary Economic Discussions, No. 02/2005*.

Hamburg Contemporary Economic Discussions

(Download: http://www.uni-hamburg.de/economicpolicy/discussions.html)

01/2005 FEDDERSEN, A. / MAENNIG, W.: Trends in Competitive Balance: Is there Evidence for Growing Imbalance in Professional Sport Leagues?, January 2005.

02/2005 SIEVERS, T.: Information-driven Clustering – An Alternative to the Knowledge Spillover Story, February 2005.

03/2005 SIEVERS, T.: A Vector-based Approach to Modeling Knowledge in Economics, February 2005.

04/2005 BUETTNER, N. / MAENNIG, W. / MENSSEN, M.: Zur Ableitung einfacher Multiplikatoren für die Planung von Infrastrukturkosten anhand der Aufwendungen für Sportstätten – eine Untersuchung anhand der Fußball-WM 2006, May 2005.

01/2006 FEDDERSEN, A.: Economic Consequences of the UEFA Champions League for National Championships – The Case of Germany, May 2006.

02/2006 FEDDERSEN, A.: Measuring Between-season Competitive Balance with Markov Chains, July 2006.

03/2006 FEDDERSEN, A. / VOEPEL, H.: Staatliche Hilfen für Profifußballclubs in finanziellen Notlagen? – Die Kommunen im Konflikt zwischen Imageeffekten und Moral-Hazard-Problemen, September 2006.

04/2006 MAENNIG, W. / SCHWARTHOFF, F.: Stadium Architecture and Regional Economic Development: International Experience and the Plans of Durban, October 2006.

Hamburg Contemporary Economic Discussions

(Download: http://www.uni-hamburg.de/economicpolicy/discussions.html)

01 AHLFELDT, G. / MAENNIG, W.: The Role of Architecture on Urban Revitalization: The Case of "Olympic Arenas" in Berlin-Prenzlauer Berg, 2007.

02 FEDDERSEN, A. / MAENNIG, W. / ZIMMERMANN, P.: How to Win the Olympic Games – The Empirics of Key Success Factors of Olympic Bids, 2007.

03 AHLFELDT, G. / MAENNIG, W.: The Impact of Sports Arenas on Locational Attractivity: Evidence from Berlin, 2007.

04 DU PLESSIS, S. / MAENNIG, W.: World Cup 2010: South African Economic Perspectives and Policy Challenges Informed by the Experience of Germany 2006, 2007.

05 HEYNE, M. / MAENNIG, W. / SUESSMUTH, B.: Mega-sporting Events as Experience Goods, 2007.

06 DUST, L. / MAENNIG, W.: Shrinking and Growing Metropolitan Areas – Asymmetric Real Estate Price Reactions? The Case of German Single-family Houses, 2007.

07 JASMAND, S. / MAENNIG, W.: Regional Income and Employment Effects of the 1972 Munich Olympic Summer Games, 2007.

08 HAGN, F. / MAENNIG W.: Labour Market Effects of the 2006 Soccer World Cup in Germany, 2007.

09 HAGN, F. / MAENNIG, W.: Employment Effects of the World Cup 1974 in Germany.

10 MAENNIG, W.: One Year Later: A Re-appraisal of the Economics of the 2006 Soccer World Cup, 2007.

11 AHLFELDT, G., MAENNIG, W.: Assessing External Effects of City Airports: Land Values in Berlin, 2007.

Hamburg

Contemporary Economic Discussions

ISSN 1865-2441 (PRINT)
ISSN 1865-7133 (ONLINE)

ISBN 978-3-940369-04-8 (PRINT)
ISBN 978-3-940369-05-5 (ONLINE)