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## The absent-minded prisoner

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**Ernst-Moritz-Arndt-Universität Greifswald**

Rechts- und Staatswissenschaftliche Fakultät

**Wirtschaftswissenschaftliche Diskussionspapiere**

## **The Absent-Minded Prisoner**

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## **Abstract**

If one of two rational players is absent-minded for at least three rounds, cooperation in a prisoners' dilemma with a finite number of repetitions is possible. If both players are absent-minded, even two rounds of absent-mindedness can be enough for cooperation in these rounds and all rounds before. Sufficient conditions for the existence of a cooperative equilibrium are derived, a plausible interpretation of absent-mindedness in the case of many repetitions is given.

Key words: absent-mindedness, prisoners' dilemma, repeated games

JEL-Classification: C72

## **Zusammenfassung**

Wenn einer von zwei rationalen Spielern mindestens drei Runden „geistesabwesend“ ist (im unklaren darüber, in welcher Runde er sich gerade befindet), dann ist Kooperation in einem Gefangenendilemmaspiel mit endlich vielen Wiederholungen möglich. Wenn beide Spieler geistesabwesend sind, können sogar zwei Runden der Geistesabwesenheit ausreichen, damit es zum kooperativen Verhalten in diesen Runden und allen Runden davor kommt. Hinreichende Bedingungen für die Existenz kooperativer Gleichgewichtslösungen werden abgeleitet, eine plausible Interpretation der Geistesabwesenheit im Falle vieler Wiederholungen wird gegeben.

# The Absent-Minded Prisoner

## 1. Introduction

In a one-shot prisoners' dilemma rational prisoners cannot cooperate. The same is true for a finite number of repetitions of the game,<sup>1</sup> if both prisoners are rational and have perfect recall. Even if the game is played thousands of times, the possibility of a small gain in the last round makes much larger gains from cooperation over all rounds impossible. As is well known this result, which offends common sense, can be avoided by the introduction of irrationality, even if there is only a small probability of irrationality. Irrationality or fairness considerations and altruism respectively are therefore the standard explanations for the empirical finding that human beings will normally cooperate in experiments with repeated prisoners' dilemma games. However, in this paper the idea of perfect recall will be suspended. It will be shown that absent-mindedness can be sufficient for the possibility of cooperative equilibria.<sup>2</sup>

The structure of this paper is as follows. The setting of the problem mentioned above will be given in section 2. In section 3 it will be shown that the absent-mindedness has to last for at least three rounds, if only one prisoner is absent-minded. In addition the conditions for cooperation in this case will be derived. In section 4 these results will be generalized for more than three rounds and a sufficient, yet not necessary condition for the possibility of cooperation will be given. Section 5 deals with the absent-mindedness of both prisoners, in which case the sufficient condition for cooperation is weaker and two rounds of absent-mindedness can be enough for cooperating. Section 6 contains conclusions and implications.

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<sup>1</sup> There may be cooperation by an actual finite number of repetitions, if the game is probabilistic and can end every round with little probability. Nevertheless, there has to be at least the possibility of an infinite number of repetitions, otherwise backward induction applies. As life comes to an end at some point and even the whole physical world is finite and will not last forever, in reality only a finite number of rounds is possible.

<sup>2</sup> Dulleck/Oechssler (1997) show that absent-mindedness allows rational players to cooperate in the centipede game.

## 2. The problem

To simplify, the prisoners' dilemma will have the following symmetric form throughout this paper:

		Prisoner 2	
		cooperates	defects
Prisoner 1	cooperates	$x, x$	$z, w$
	defects	$w, z$	$y, y$

This means that the utility of both prisoners is  $x$ , if both of them choose to cooperate. If both defect, each of them will get  $y$  which is less than  $x$ . If only one defects, he derives an utility of  $w$  whereas his cooperating co-prisoner gets  $z$ . The fact that  $w$  is higher than  $x$  and  $z$  is lower than  $y$  leads to  $w > x > y > z$ . As is well known, rational players will always choose defection in this case because defection leads to a greater gain than cooperation regardless of the action of the other player. Nevertheless it would be better for both, if both would cooperate at the same time instead of defecting. But they will not cooperate as long as they cannot make a binding agreement.

This result also holds in a finite number of repetitions of the game. If two players play the game again and again, they cannot build a reputation for cooperating and punishing defection in future rounds as long as there will be a last round. This is due to the fact that there will be no further game after the last round and defection therefore is the only rational strategy in the last round. So the last but one round has to be considered, yet the same applies here. Defection cannot be punished in future, because in the next round defection is certain anyway. Applying backward induction this holds true for all rounds before, until the first one is reached. Only if

there is (at least potentially) an infinite number of repetitions, cooperation becomes possible for rational players. An alternative for cooperation is the existence of irrational or bounded rational players. In this article it will be shown that cooperation in a finite prisoners' dilemma with rational players is possible if absent-mindedness exists.

Absent-mindedness means that a player at two or more decision nodes does not know at which of these nodes he is. In the repeated prisoners' dilemma this could mean, for example, that one player does not know, if he is in the 96<sup>th</sup>, 97<sup>th</sup> or 98<sup>th</sup> repetition of the game as long as he is in one of these three rounds. Over these three rounds he is absent-minded. Before, in the 95<sup>th</sup> round, he knew that he found himself in this round. Also in the 99<sup>th</sup> round he knows about the number of the round. But for the rounds in between he cannot differentiate. In the following, rounds after the (last) absent-mindedness will not be considered as it is clear that rational players will always defect in those rounds. The question is, whether rational prisoners can cooperate in the rounds where at least one of them is absent-minded. In case they can, cooperation would also be possible in the rounds before the absent-mindedness occurs because the backward induction argument against cooperation would no longer apply.

The absent-mindedness of a prisoner concerning the number of the round he finds himself in does not imply that he cannot remember the result of the last round. If the fellow prisoner has defected in the last round he is surely able to remember this. To simplify the analysis it will be taken for granted that both prisoners will choose nothing but defection in all rounds to come, if at least one of them defects in one round. To punish defection that hard is not necessary in all cases, but it will certainly yield an equilibrium as it is always an equilibrium that both prisoners do nothing but defect all the time.

### **3. One prisoner is absent-minded for three rounds**

#### **3.1. Why are at least three rounds of absent-mindedness necessary for cooperation?**

Considering that both prisoners are rational and only one of them is absent-minded for a few rounds, the question arises for how many rounds the absent-mindedness has to last at least to give cooperation a chance. By definition absent-mindedness cannot last for just one round. It



has to last for at least two rounds. However, two rounds are not enough to enable cooperation. The rational absent-minded prisoner would act on the basis of the following considerations: „I can be in the first or in the second round of my absent-mindedness. If I am in the second and therefore last round, cooperation is pointless because the other prisoner knows that this is the last round (before the end of the game or the phase of full awareness with nothing but defection) and will not cooperate. If I am in the first round of my absent-mindedness I cannot win anything by cooperation either, because in the next round the other prisoner will defect anyway. Therefore I should defect anyway, too.“<sup>3</sup> Hence, in case of absent-mindedness for only two rounds there is no cooperation, as the other prisoner will also defect in all rounds according to these considerations.

What if the absent-mindedness lasts for at least three rounds? Would not the same argumentation rule out any cooperation? This, in fact, is not the case because the prisoners could now have an incentive to cooperate in one round to get cooperation in the next one. As long as there are only two rounds of absent-mindedness this is impossible as the fully informed player will defect in the last of these rounds anyway. The non-absent-minded prisoner will also defect in the last out of three rounds for sure, but she could have an incentive to cooperate in the second one. Her incentive could be that her cooperation in the second round leads to a sufficient probability that the absent-minded prisoner will cooperate in the third round. But why should the absent-minded prisoner cooperate in the third round while the other prisoner will defect? The simple reason for this is his absent-mindedness. He does not know that he finds himself in the third round. The absent-minded prisoner will cooperate if (1) there is a sufficient probability that he is in the first round, (2) he can bring about cooperation of the fellow prisoner in the second round by cooperating himself, and (3) he can gain more by acting this way than by immediate defection.

### **3.2. A cooperative equilibrium in pure strategies**

To analyze this in more detail it will be reasonable to start with a simplification of the problem and therefore to only consider pure strategies of the absent-minded prisoner. This means that

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<sup>3</sup> This argument cannot be used if the two absent-minded rounds do not follow each other immediately. But these cases of disconnected rounds with absent-mindedness are quite implausible and will not be considered here.

the absent-minded prisoner can only choose between cooperating or defecting in all three rounds as long as he is absent-minded. He can and certainly will switch from cooperation to defection as soon as he sees that the other prisoner has defected in the last round. But assumed the other prisoner has cooperated so far, should the absent-minded one cooperate or not?

Let prisoner 2 be the one who is not absent-minded and take the following strategy of her as fix for the moment: Prisoner 2 cooperates in the first two rounds of the absent-mindedness of prisoner 1 and defects in the third one, but if she detects that prisoner 1 has defected in the first round she will also defect in the second one. In pure strategies prisoner 1 now has to compare what he would get by cooperation versus defection in all three rounds given the strategy of prisoner 2. If prisoner 1 cooperates in all three rounds, he will get  $x$  for two rounds and only  $z$  in the last one. If he defects all the time he will win  $w$  in the first round and will then get  $y$  twice. So it is rational for him to cooperate as long as

$$(1) \quad 2x + z \geq w + 2y$$

If this condition is satisfied, the described strategy of prisoner 2 will be rational for her because she will get  $2x+w$  which is her maximal gain. If prisoner 2 started to defect earlier, she would only change one or two  $x$  into  $y$  which is less. So (1) is the condition that has to be satisfied for the existence of an cooperative equilibrium in pure strategies.

More important than the absolute amount of utility is the difference between the values. Define  $a=w-x$ ,  $b=x-y$  and  $c=y-z$ , then (1) can be rearranged to

$$(2) \quad b \geq a + c$$

This means that the advantage from cooperation instead of defection carried out by both prisoners has to be at least as high as the difference between defecting and cooperating in view of a cooperating partner plus the difference between defecting and cooperating in view of defection by the other player. The reason for this is that the absent-minded prisoner could win  $w$  instead of  $x$  in the first round and secure himself  $y$  instead of  $z$  in the third one by defecting all the time, but would lose  $x-y=b$  in the second round as a consequence.<sup>4</sup>

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<sup>4</sup> In fact he would lose a lot more as there would be no cooperation at all, but this is not relevant to his decision.

### 3.3. Mixed strategies

The absent-minded prisoner does not have to use pure strategies. Perhaps he can do better by playing a mixed strategy. The strategy of prisoner 2 shall be given as before, so that she cooperates in the first two rounds as long as prisoner 1 cooperates in the first one, otherwise she will defect in the second round as she will do in the third round anyway. Now the absent-minded prisoner can use the mixed strategy to cooperate with the probability  $p$  and to defect with the counter-probability  $(1-p)$  in every round,<sup>5</sup> if there has been unanimous cooperation before, otherwise he will defect for sure. How can the optimal probability  $p^*$  be calculated? The probabilities with which the absent-minded prisoner believes to be in one of the three rounds are crucial. It seems reasonable that these probabilities depend on  $p$ . The probability of being in the second round without having seen defection before is only  $p$  times as high as that of being in the first round, if the strategy of the other prisoner is given and he himself defects with a probability of  $(1-p)$ . The probability of being in the third round without defection before is accordingly only  $p^2$  times as high as that of being in the first round. So the expected utility of the absent-minded prisoner depending on  $p$  can be calculated:

$$(3) \quad u_1(p) = (1-p)(w+2y) + px + p(1-p)(w+y) + p^2x + p^2(1-p)y + p^3z$$

To find the optimal  $p^*$ , the utility has to be differentiated and equalized to zero:

$$(4) \quad 3p^{*2}(z-y) + 2p^*(x-w) + (x-y) = 0$$

⇒

$$(5) \quad p^* = \frac{\sqrt{(w-x)^2 + 3(x-y)(y-z)} - (w-x)}{3(y-z)} = \frac{\sqrt{a^2 + 3bc} - a}{3c}$$

---

<sup>5</sup> There is an on-going discussion, if a player actually has to behave the same way in every round he is absent-minded even if he cannot differentiate between them. Aumann/Hart/Perry (1997a) argue that this is the only rational behavior. Piccione/Rubinstein (1997a), Piccione/Rubinstein (1997b) and Lipman (1997) doubt this, but they at least agree that it is a reasonable solution, although others might exist. Also see further articles in the special issue on absent-mindedness of Games and Economic Behavior, Volume 20, Number 1, July 1997.

This solution holds as long as  $p^*$  is between 0 and 1. If it is lower,<sup>6</sup> the absent-minded prisoner should not cooperate in any case. If it is higher, he should always cooperate.

The strategy of prisoner 2 was treated as given. But what would be the condition for prisoner 2 to hold to this strategy? Clearly she will play this strategy, if  $p^*$  is 1 or higher and prisoner 1 cooperates in all three rounds for sure. Then the result is the same as in pure strategies, but the conditions on the parameters are stronger:

$$(6) \quad p^* \geq 1 \quad \Leftrightarrow \quad \frac{\sqrt{a^2 + 3bc} - a}{3c} \geq 1$$

$\Rightarrow$

$$(7) \quad b \geq 2a + 3c$$

As long as (7) holds, an equilibrium exists where both prisoners cooperate in the first two rounds of absent-mindedness, whereas in the third round only the absent-minded one cooperates and the other prisoner defects. However, it is not these three rounds that are really important but the possibility of rational cooperation in all the rounds before the absent-mindedness.

There are even more cases where the second prisoner holds to the strategy to cooperate in the first two rounds and to defect in the third. She will do this as long as it will bring her a greater utility than the action of defection before. The expected utility of starting to defect in the first, second or third round depending on  $p^*$  has to be compared. Prisoner 2 gets the following expected utilities:

$$(8) \quad u_2^1 = p^*(w + 2y) + (1 - p^*)3y$$

$$(9) \quad u_2^2 = p^*x + (1 - p^*)(z + 2y) + p^{*2}(w + y) + p^*(1 - p^*)2y$$

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<sup>6</sup> There are two solutions to equation (4), but the other one (it is (5) with a negative sign before the root) is always negative and makes the second derivation of (3) positive, so that it would be a minimum instead of a maximum, whereas putting  $p^*$  in the second derivation of (3) is always negative.

$$(10) \quad u_2^3 = p^* x + (1 - p^*)(z + 2y) + p^{*2} x + p^*(1 - p^*)(z + y) + p^{*3} w + p^{*2} (1 - p^*)y$$

For prisoner 2 to cooperate in the first two rounds it has to hold

$$(11) \quad u_2^1 \leq u_2^3 \geq u_2^2$$

To check first the case  $u_2^1 \leq u_2^3$ , it will be found by rearranging (8) and (10)

$$(12) \quad u_2^1 \leq u_2^3 \Leftrightarrow 3y + p^* w - p^* y \leq 2y + z + p^* x - p^* y + p^{*2} x - p^{*2} z + p^{*3} w - p^{*3} y$$

$\Leftrightarrow$

$$(13) \quad 0 \leq p^{*3} (a + b) + p^{*2} (b + c) - p^* a - c$$

If (13) is equal to 0, it has exactly one positive solution<sup>7</sup>  $p^M$ :

$$(14) \quad p^M = \frac{a - c + \sqrt{a^2 + 2ac + 4bc + c^2}}{2(a + b)}$$

For (12) and (13) to hold  $p^*$  has to be higher than  $p^M$ . To prevent defection in the second round the following inequalities have to hold:

$$(15) \quad u_2^3 \geq u_2^2 \quad \Leftrightarrow \quad 2y + z + p^* x - p^* y + p^{*2} x - p^{*2} z + p^{*3} w - p^{*3} y \\ \geq 2y + z + p^* x - p^* z + p^{*2} w - p^{*2} y$$

$\Leftrightarrow$

$$(16) \quad p^{*3} (a + b) + p^{*2} (c - a) - p^* c \geq 0$$

---

<sup>7</sup> There are also two negative solutions, one is -1 and the other one (14) with a negative sign before the root.

If (16) is equal to 0, it also has exactly one positive solution<sup>8</sup> and is equal to (14). This means that cooperating for two rounds is either better than cooperating for only one as it is better than not cooperating at all or that it is worse than both alternatives. It is better as long as<sup>9</sup>

$$(17) \quad p^* = \frac{\sqrt{a^2 + 3bc} - a}{3c} > p^M = \frac{a - c + \sqrt{a^2 + 2ac + 4bc + c^2}}{2(a + b)}$$

Unfortunately the author is not able to simplify (17). So for any given parameters (17) has to be checked separately. If at least one of the inequalities (7) or (17) is fulfilled, there is a cooperative equilibrium, otherwise there is not.

#### 4. One prisoner is absent-minded for more than three rounds

So far it has been shown under which conditions cooperation is possible, if one of the players is absent-minded for three rounds. What will happen, if the absent-mindedness lasts for longer than three rounds? Will the conditions for cooperation be relaxed or tightened?

##### 4.1. Four rounds absent-mindedness

What will happen, if prisoner 1 is absent-minded for four rounds? In pure strategies prisoner 2 could be expected to cooperate the first three rounds of absent-mindedness and to defect in the fourth one, if prisoner 1 cooperates in all four rounds. Prisoner 1 could rationally choose cooperation instead of defection as a pure strategy as long as

$$(18) \quad 3x + z \geq w + 3y$$

⇒

$$(19) \quad 2b \geq a + c$$

---

<sup>8</sup> One of the other solutions is 0 instead of -1, the third solution is also the same as follows from (13).

<sup>9</sup> It is not possible that both values are equal.

This is less demanding than (2). What happens in mixed strategies? If prisoner 2 cooperates in the first three rounds and defects only in the last one or if she has observed defection before, then prisoner 1 will get by cooperating with the probability of  $p$ , if there has been no defection before, and defecting otherwise

$$(20) \quad u_1(p) = (1-p)(w+3y) + px + p(1-p)(w+2y) + p^2x + p^2(1-p)(w+y) \\ + p^3x + p^3(1-p)y + p^4z$$

By rearranging, differentiating in  $p$ , and setting equal to 0, the following condition for the optimal  $p^*$  can be found:

$$(21) \quad x - y + 2p^*(x - y) - 3p^{*2}(w - x) - 4p^{*3}(y - z) = b + 2p^*b - 3p^{*2}a - 4p^{*3}c = 0$$

Instead of searching the (quite complicated) condition under which prisoner 2 will hold to the given strategy depending on  $p^*$ , the analysis will be restricted to the cases where prisoner 1 cooperates in all rounds, that is  $p^* \geq 1$ . For these cases it follows from (21) that the following inequality has to hold:

$$(22) \quad 3b \geq 3a + 4c$$

By comparing (22) and (7) it can be seen that the conditions on the parameters are far less strict in four rounds of absent-mindedness than in three. Cooperation is easier to achieve, if the absent-mindedness lasts longer.

## 4.2. The general case

What happens generally, if the absent-mindedness lasts for  $n$  rounds following one after the other? Prisoner 2 should cooperate  $n-1$  rounds and defect in the  $n$ th one or as soon as she detects prisoner 1 has defected. Taking this strategy as given and  $i$  standing for the number of the round, prisoner 1 gets  $w + (n-i)y$  in every round but the last one for the rest of all absent-minded rounds by defecting for the first time, which he does with probability  $(1-p)$ . Without defection he gets  $x$  in every round, but the probability of cooperation is repeated  $p$  in every round and has to be multiplied for all rounds  $i$  so far. If the  $n$ th round is reached in

cooperation, prisoner 1 gets  $y$  or  $z$  depending on defecting or cooperating in this round. Therefore, the general form of his utility is

$$(23) \quad u_1(p) = \sum_{i=1}^{n-1} p^{i-1} (1-p)(w + (n-i)y) + p^i x + p^{n-1} (1-p)y + p^n z$$

By rearranging this yields

$$(24) \quad u_1(p) = w + (n-1)y + \sum_{i=1}^{n-2} p^i (x-y) + p^{n-1} (x-w) + p^n (z-y)$$

To maximize utility this has to be differentiated in  $p$  and set equal to 0:

$$(25) \quad \sum_{i=1}^{n-2} i p^{*i-1} (x-y) - (n-1) p^{*n-2} (w-x) - n p^{*n-1} (y-z) \\ = \sum_{i=1}^{n-2} i p^{*i-1} b - (n-1) p^{*n-2} a - n p^{*n-1} c = 0$$

By setting  $p^* = 1$  the following condition can be derived from (25):

$$(26) \quad \frac{(n-2)(n-1)}{2} b \geq (n-1)a + nc$$

As long as (26) is satisfied<sup>10</sup> cooperation is possible in a repeated prisoners' dilemma where one prisoner is absent-minded for  $n$  rounds. This condition is less and less demanding the higher  $n$  is.

## 5. Both prisoners are absent-minded

To give a complete analysis of both prisoners being absent-minded it would be necessary to define not only a probability  $p$  with which prisoner 1 cooperates but also a probability  $q$  of prisoner's 2 cooperation. The utility of each prisoner depending on these probabilities would

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<sup>10</sup> There are more cases because the conditions for cooperation with  $p^*$  less than one have not been analyzed. (26) is sufficient for cooperation, but not necessary.



have to be maximized and insoluble equations would follow. Therefore the analysis will be restricted to the case where both prisoners cooperate in all rounds of absent-mindedness with probability 1. The conditions for this case are certainly stricter than necessary, but this is an advantage. If the following conditions are fulfilled, cooperation will surely be possible, even though more possibilities of cooperation exist.

### 5.1. Cooperation with two rounds of absent-mindedness

Under 2.1. it has been shown that there has to be a minimum of three rounds of absent-mindedness for rational cooperation to become possible if only one of the prisoners is absent-minded. If both prisoners are absent-minded, this does not hold. Even two rounds of absent-mindedness will do. This is possible because none of the prisoners knows for sure when the last round of absent-mindedness will be. So each prisoner has to compare the advantage of defection with the disadvantage of destroying the willingness to cooperate of the other prisoner in the next round, if there is one.

It certainly is assumed here that both prisoners are absent-minded in the same rounds and that these follow each other. Both prisoners are symmetric, therefore the examination can be restricted to prisoner 1. The strategy of prisoner 2 shall be given as follows: she cooperates in both rounds of absent-mindedness, unless she sees that the other prisoner defected (in the first round) which will be replied by defection. Now it has to be investigated under which conditions it is rational for player 1 to use the same strategy, so that there is a cooperative equilibrium.

If prisoner 1 uses the mixed strategy to cooperate with probability  $p$  and to defect with the counter-probability  $(1-p)$ , he gets the utility

$$(27) \quad u_1(p) = (1-p)(w+y) + px + p(1-p)w + p^2x$$

To maximize the utility this has to be differentiated in  $p$  and to be set equal to 0:

$$(28) \quad x - y - 2p^*(w-x) = b - 2p^*a = 0$$

For sure cooperation of prisoner 1 ( $p^* \geq 1$ ) the following has to hold:

$$(29) \quad b \geq 2a$$

By comparing (29) with (7) it can be seen that cooperation between two absent-minded prisoners is more likely even in two rounds than cooperation with only one prisoner being absent-minded over three rounds.

## 5.2. Cooperation in $n$ rounds

The result of two rounds of joint absent-mindedness can be generalized for  $n$  rounds. To determine the utility of prisoner 1, given a cooperative strategy of prisoner 2, the same reasoning and formula can be applied as for (23) with only one difference. In the last round prisoner 2 will cooperate instead of defecting, so that prisoner 1 can get  $w$  or  $x$  instead of  $y$  and  $z$  respectively.

$$(30) \quad u_1(p) = \sum_{i=1}^{n-1} p^{i-1} (1-p)(w + (n-i)y) + p^i x + p^{n-1} (1-p)w + p^n x$$

$$= w + (n-1)y + \sum_{i=1}^{n-1} p^i (x-y) + p^n (x-w)$$

The condition for utility-maximization is:

$$(31) \quad \sum_{i=1}^{n-1} i p^{*i-1} (x-y) - n p^{*n-1} (w-x) = \sum_{i=1}^{n-1} i p^{*i-1} b - n p^{*n-1} a = 0$$

For  $p^*$  not being less than 1 and therefore unanimous cooperation the following condition can be derived:

$$(32) \quad \frac{n-1}{2} b \geq a$$

If (32) is met, two rational prisoners who are absent-minded for the same  $n$  rounds can definitely cooperate in these rounds of absent-mindedness and in all rounds before.

## 6. Conclusions

In this paper it has been shown that cooperation between rational players in a finitely often repeated prisoners' dilemma is possible, if one of the players is absent-minded for at least three rounds. A sufficient condition for the possibility of cooperation has been derived, too. If both players are absent-minded, this condition is weaker and even two rounds of absent-mindedness will do.

This result is not simply an unimportant application of a remote part of game theory. The possibility of cooperation in the prisoners' dilemma is quite important in economics and politics and there is a straight interpretation of absent-mindedness in this context. The behavior of the absent-minded driver<sup>11</sup> is rather pathological. If someone forgets at one exit where he is, this may be quite normal. That he will forget at the second exit again and even forget that he had the same problem in connection with the last exit seems rather fictitious.<sup>12</sup>

In case of a prisoners' dilemma repeated many times the interpretation of absent-mindedness is more straightforward. If one player does not count the rounds of the game he may know that he is approaching the end of the game, but he does not know in which of the last rounds he finds himself in. In the next rounds he will not know more, until the last round is over.

This also opens the possibility of an empirical test concerning the question, if absent-mindedness adds to the explanation of cooperative behavior of human beings. An often repeated prisoners' dilemma, say one hundred or thousand repetitions, is once played displaying the number of each round and once without displaying it. In the former case there should be less cooperation than in the latter. Absent-mindedness could even be endogenous in a model by introducing some small costs for counting the rounds.

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<sup>11</sup> See Aumann/Hart/Perry (1997a) and Piccione/Rubinstein (1997a).

<sup>12</sup> For the point that there is a difference between absent-mindedness and forgetfulness see Aumann/Hart/Perry (1997b).

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