Schäfer, Andreas; Schneider, Maik T.

Working Paper
Endogenous enforcement of intellectual property, North-South trade, and growth

Working Paper // Universität Leipzig, No. 96

Provided in Cooperation with:
University of Leipzig, Faculty of Economics and Management Science

Suggested Citation: Schäfer, Andreas; Schneider, Maik T. (2011) : Endogenous enforcement of intellectual property, North-South trade, and growth, Working Paper // Universität Leipzig, No. 96, Univ., Wirtschaftswissenschaftliche Fakultät, Leipzig

This Version is available at:
http://hdl.handle.net/10419/48853

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.
You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.
If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
Endogenous Enforcement of Intellectual Property, North-South Trade, and Growth

August 2011

ISSN 1437-9384
Abstract

While most countries have harmonized intellectual property rights (IPR) legislation, the dispute about the optimal level of IPR-enforcement remains. This paper develops an endogenous growth framework with two open economies satisfying the classical North-South assumptions to study (a) IPR-enforcement in a decentralized game and (b) the desired globally-harmonized IPR-enforcement of the two regions. The results are compared to the constrained-efficient enforcement level. Our main insights are: The regions’ desired harmonized enforcement levels are higher than their equilibrium choices, however, the gap between the two shrinks with relative market size. While growth rates substantially increase when IPR-enforcement is harmonized at the North’s desired level, our numerical simulation suggests that the South may also benefit in terms of long-run welfare.

Keywords: Endogenous Growth, Intellectual Property Rights, Trade, Dynamic Game

JEL: F10, F13, O10, O30

*We would like to thank David de la Croix, Hans Gersbach, Edwin Lai, Omar Licandro, Christian Kiedaisch, Thomas Steger, Betrand Wigniolle, Ralph Winkler, seminar participants at ETH Zurich, the Universities of Leipzig and Bern, and at the EEA-ESEM 2009 (Barcelona), SURED 2010 (Ascona), DEGIT 2010 (Frankfurt), and OLG-Days 2011 (Vielsalm) for valuable comments. The usual disclaimer applies.
1 Introduction

As trade of knowledge intensive goods accelerated during the last decades, patent and copyright infringements have become a problem of highest concern. Although the Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPs) specifies a minimum set of protection standards that members of the World Trade Organization (WTO) have to assent to, the enforcement of intellectual property rights (IPRs) is still a source of great international heterogeneity and further fuels the debate about the optimal protection level of IPRs in the world.

For example, the European Commission’s IPR Enforcement Report 2009 gives account of serious problems with IPR-enforcement in a large number of mostly developing countries. Complaints include that injunctions or criminal sanctions are often difficult to obtain and civil procedures are lengthy and burdensome with high uncertainty of outcomes. Involved staff is insufficiently trained, lacks resources to effectively prosecute and convict violators, and cooperation between authorities is insufficient. For some countries the report assesses even a lack of political will indicated by their opposing in-depth enforcement discussions in international fora such as the WTO or the WIPO.¹ Studying the distributional effects of TRIPs, McCalman (2001) argues that the agreement involves transfers from developing countries to developed countries due to stronger IPR protection. These transfers are primarily determined by enforcement efforts rather than the extension of the coverage of patent protection. Thus, he reasons that the developing countries “will be more willing to extend the coverage of patent protection as required by TRIPs, but may be less willing to devote adequate resources to enforcement”. Further he predicted that “future North-South tensions over intellectual property rights are likely to be centered around enforcement issues rather than the sectoral coverage of protection offered” (McCalman, 2001, p. 181).²

In response to this heterogeneity in IPR-enforcement, efforts have been made in secret negotiations under the title Anti-Counterfeiting Trade Agreement (ACTA) with the aim to harmonize international standards of IPR-enforcement. It is reported that a preliminary agreement has been reached in October 2010 between several countries among them the

---
¹See EU Commission (2009). A similar picture is drawn in the annual Special 301 Reports by the U.S. Trade Representative, see Office of U.S. Trade Representative (2010).
²Other authors hold that even though the TRIPs-Agreement provides for mechanisms of law enforcement, these are not always implemented by the member countries (see e.g. Cychosz (2003)).
U.S. and the E.U. An ultimate objective of ACTA is that large emerging economies, “where IPR could be improved will sign up to the global pact” (EU Commission, 2008; Reuters, 2010).

Inspired by these recent developments, this paper develops an endogenous growth framework to study IPR-enforcement within the context of a classical North-South trade model. Our analysis is characterized by the following features that distinguish our paper from the previous literature. First, we assume equal treatment of all active patents in a region with respect to IPR-enforcement at any point in time. Second, a government cannot commit to IPR-enforcement for the indefinite future but after each legislative term the (new) government may adjust its enforcement efforts as it sees fit. Third, when setting its policies, the government’s planning horizon is limited.

By the first two assumptions, we intend to capture important aspects of IPR-enforcement. With regard to the first item, we argue that in reality IPR-enforcement depends on whether or not a patent is active, ruling out the possibility that IPR-enforcement distinguishes active patents by, e.g., the year of invention. Second, while formal law may be fixed for substantial time horizons, the enforcement of laws can be changed more easily, for example, by reallocating resources used for IPR-enforcement to other purposes. Our third assumption reflects an important aspect of policy making in that governments are not or not only motivated by fostering long-term welfare but are concerned with their political ends.

Incorporating these assumptions into a dynamic model with endogenous innovation arguably makes the analysis of IPR-enforcement more realistic. However, it is also particularly interesting as it adds another area of tension resulting from the different planning horizons of the governments and the innovators. At the heart of our analysis is the

3The countries are: Australia, Canada, the EU, Japan, Mexico, Morocco, New Zealand, the Republic of Korea, Singapore, Switzerland, and the U.S.

4In 2006, the European Union adopted the “IPR Enforcement Directive” to harmonize IPR-enforcement levels among its members and eschew civil procedures that are “unnecessarily complicated and costly or involve unreasonable time limits or unwarranted delays” (European Parliament, 2004).

5It might be more realistic that enforcement distinguishes between a domestic product and an invention of a foreign country. In this paper, we do not address this case and focus on national treatment only.

6For example, both, politicians’ monetary and non-monetary rewards may depend on the welfare level during their term in office. According to a large literature on the political business cycle, the welfare during the term in office also affects the incumbent politicians’ reelection probabilities (Oudiz and Sachs, 1985; Drazen, 2000; Persson and Tabellini, 2000). There is also a literature on office motivated politicians, so called populists, who pander to the public by pursuing short-term policies to maximize reelection chances. The concern of this literature is how to give incentives to implement projects that are beneficial in the long-term but come at costs in the short-run. See e.g., Müller (2007); Gersbach (2004).
governments’ classical trade-off between static efficiency and dynamic gains extended by international externalities of IPR-protection with regard to R&D incentives and profit flows.\textsuperscript{7} By choosing IPR-enforcement, the government has to trade off welfare today - by incurring deadweight losses and R&D costs - against future welfare resulting from a higher technological level. Without internalizing the full future benefits of innovations, an office-term motivated government may be more reluctant to bear the costs of great innovative activity implying a substantial burden on current welfare.

As a consequence, we find that in the decentralized equilibrium of the IPR-enforcement game, the relation between the North’s IPR-enforcement level and its own research productivity exhibits an inverted U-shaped course. When the research capacity is low, the dynamic gains of IPR-enforcement dominate and the enforcement level increases with a higher productivity of research. However, if the research capacity is very high, the current R&D costs are so large that the government reduces IPR-enforcement in response to an even higher research productivity. As the South does not engage in R&D, it neglects research expenditures but considers its influence on the R&D activity in the North. Consequently, the South’s equilibrium IPR-enforcement increases monotonically with the North’s innovative capacity. The office-motivated government in the North may, hence, possess lower incentives to enforce IPR than the one in the South when the North’s R&D productivity is very high, while the opposite is the case for low levels of R&D productivity. Further we find that a country’s relative market size positively affects its equilibrium IPR-enforcement level. The intuition is that a larger country’s impact on R&D incentives is relatively higher and therefore its incentive to freeride on the other region’s IPR-efforts are lower.

By analyzing the regions’ preferred harmonized IPR-enforcement levels, we seek to shed light on potential clashes of interest in international negotiation rounds. We compare these enforcement levels with those chosen in the decentralized equilibrium and relate both to the constrained-efficient solution reflecting the maximum welfare the two governments can achieve given they cannot escape their political-economy constraints.

Both, the North’s and the South’s desired harmonized IPR-enforcement levels are higher than their respective equilibrium choices. While the South’s preferred harmonized enforcement level is independent of relative market sizes, the one of the North typically exhibits a declining relationship with its relative market size. This contrasts with the decentralized equilibrium where the North’s equilibrium IPR-enforcement level is positively associated

\textsuperscript{7}The trade-off between static efficiency and dynamic gains was first discussed by Nordhaus (1969).
with its relative market size. This result suggests that small innovative countries show large differences between their desired harmonized levels supported in negotiation rounds concerning global IPR-enforcement and their own equilibrium choices.

Further, we find that relative to the constrained-efficient solution the regions’ IPR-enforcement levels in the decentralized equilibrium are too low. By contrast, the North’s desired harmonized enforcement level is typically higher than the constrained efficient one while that of the South is lower. As a consequence, the regions’ growth rate is highest when the harmonized IPR-enforcement level of the North is implemented. Would this rate of growth come at the expense of welfare in the South? According to a numerical exercise, our analysis suggests that the South may well gain in terms of aggregate long-run welfare by adopting the North’s desired harmonized IPR-enforcement level given a sufficiently productive R&D sector in the North. However, the opposite holds for low research capacities in the North.

The literature has approached questions regarding the international protection of IPR from two perspectives. On the one hand, from a macroeconomic, endogenous growth perspective which treats the regions’ IPR-enforcement as exogenous and examines its effects on the resulting growth rate and on welfare (Helpman, 1993; Kwan and Lai, 2003; Dinopoulos and Segerstrom, 2006; Futugami and Iwaisako, 2007). On the other hand, from a rather microeconomic, industrial organization perspective that explicitly takes IPR-enforcement as endogenous, but precludes long-run dynamics (Chin and Grossman, 1990; Deardorff, 1992; Maskus, 1990; Diwan and Rodrik, 1991; Lai and Qiu, 2003). This paper establishes a unified framework which combines these two perspectives and therefore allows to consider endogenous choices of IPRs as well as aspects of economic growth and welfare.

As we do, the seminal paper by Grossman and Lai (2004) employs a framework of variety expanding innovations, however, considers a one-shot game with respect to IPR-protection and does not allow for endogenous long-run economic growth. The one-shot game in Grossman and Lai (2004) is equivalent to a game where governments are able (1) to decide on the IPR-protection level of each vintage of inventions separately and (2) to fully commit to it in the future. This implies the theoretical possibility that at a particular point in time, all different vintages of active patents enjoy different levels of IPR-enforcement. This is precluded in our set-up. Additionally, our paper takes a complementary approach to the one by Grossman and Lai (2004) by incorporating governments’ political economy considerations. Our work is also related to Eicher and Garcia-Penalosa (2008) who present
an endogenous growth model with endogenous strength of IPR-enforcement. This paper differs from ours in that it considers a closed economy. Moreover IPR-enforcement there is not a choice variable of the government but characterized by private investments of firms to hire lawyers.

The paper is organized as follows. In Section 2, we introduce the model. We discuss the setting in which both regions choose their national IPR-enforcement decentrally in Section 3. In Section 4, we analyze the preferred harmonized enforcement levels of the North and the South. Section 5 compares the desired harmonized enforcement levels and the decentralized equilibrium with the constrained-efficient solution. We present implications for welfare in Section 6 and provide a summary and conclusions in Section 7.

2 The Model

We consider two regions, $n$ and $s$, that differ with respect to their innovative capacity. Region $n$, which we also refer to as the North, produces blueprints, that are licensed out to Region $s$, the South. For simplicity, we assume that there is no innovation activity in Region $s$. Our analysis builds on a variety-expanding-growth framework where at time $t$ a patent is enforced with probability $\omega_{j,t}$ in Region $j = s, n$. For simplicity, we assume that imitation is costless. Thus, an imitated intermediate is supplied under full competition and operating profits are zero. Both economies are populated by a measure $L_j$ of households each inelastically supplying one unit of labor in each period. There is no population growth and time moves in discrete steps $t = 0, 1, 2, ..., \infty$. In the following, we first introduce the model for given levels of IPR-enforcement in both regions and then discuss the governments’ problems concerning their IPR-enforcement choice.

---

8In a model where both regions innovate but Region $s$ possesses lower innovative capacity and without perfect knowledge spillovers between the regions, it can be shown that the ratio between the number of innovations in Region $s$ and Region $n$ tends to zero. A proof is available upon request.

9As our explicit focus is on IPR-enforcement, we assume that each innovation obtains a patent of infinite length and neglect the issue of patent breadth. Without changing our qualitative results, it would be possible to assume a finite patent length and a certain patent breadth, e.g., given exogenously via TRIPs. The regional governments then possess some leverage on determining the strength of enforcement reflected by $\omega_{j,t}$. 
2.1 Production

In Region $j$, the final good $Y_j$ is produced according to

$$Y_j = A_j L_j^{1-\alpha} \int_0^N [x_j(i)]^{\alpha} di,$$  \hspace{1cm} (1)

where $A_j$ represents a productivity measure, $L_j$ is labor input, $N$ is the measure of different intermediates invented in the North, and $x_j(i)$ stands for the amount of intermediate $i$ used in final-good production in Region $j = n, s$. The elasticity of substitution between the different intermediates is denoted by $\alpha \in (0, 1)$.

Each intermediate good $i$ is produced by a monopolist or an imitator. The production of one unit of intermediate $i$ requires one unit of the final output. We choose final output as the numeraire. Hence marginal production costs of intermediates are equal to unity. The symmetric equilibrium on the market for intermediates induces equal prices and demand for all types of intermediates, such that $p_{m,j}(i) = p_{c,j} = \frac{1}{\alpha}$, $x_{m,j}(i) = x_{c,j}(i)$ for all protected intermediates and $x_{c,j}(i) = x_{c,j}$ for all imitated intermediates.

Demand in Region $j$ for protected intermediates is

$$x_{m,j}(i) = \lambda_j \alpha^{1-\alpha},$$

with $\lambda_j = \frac{L_j A_j}{1-\alpha}$ reflecting the "effective" market size of Region $j$. Hence, a small economy in terms of its population may constitute a large effective market when its productivity level in final-good production is sufficiently large and vice versa. Patent holders located in the North can attain operating profits per period $\pi = P(\lambda_s + \lambda_n)$ with $P = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} > 0$. If an intermediate is copied and, hence, sold at the competitive price $p_{c,j} = 1$, demand increases to $x_{c,j}(i) = \lambda_j \alpha^{1-\alpha}$, and operating profits in $j$ at time $t$ are zero.

Given the enforcement level $0 \leq \omega_{j,t} \leq 1$, the number of protected intermediates at time $t$ is $\omega_{j,t} N_t$, while $[1 - \omega_{j,t}] N_t$ of the intermediates are imitated. Aggregate output in Region $j$ writes therefore as

$$Y_{j,t} = \lambda_j \left[ \int_0^{\omega_{j,t} N_t} [x_{m,j}(i)]^{\alpha} di + \int_0^{[1-\omega_{j,t}] N_t} [x_{c,j}(i)]^{\alpha} di \right].$$  \hspace{1cm} (2)

Additionally considering that $x_{m,j} = \alpha^{1-\alpha} x_{c,j}$, we obtain $Y_{j,t} = \lambda_j N_t [1+\omega_{j,t}(\alpha^{1-\alpha} - 1)] x_{c,j}^\alpha$, where $\omega_{j,t}(\alpha^{1-\alpha} - 1) < 0$ represents the deadweight loss due to monopolistic competition.\(^{10}\)

\(^{10}\) Notice that for $\omega_{j,t} = 1$, i.e., full patent protection, we obtain the standard Romer (1990) production function: $Y_{j,t} = A_j L_j^{1-\alpha} N_t x_{c,j}^\alpha$. The case without patent protection, $\omega_{j,t} = 0$, yields the highest possible output from a static perspective: $Y_{j,t} = A_j L_j^{1-\alpha} N_t x_{c,j}^\alpha$. Of course this undermines incentives to invest in R&D.
2.2 Research and development

The North performs R&D in search for new designs (blueprints) of intermediate goods. Here, we use a lab equipment specification assuming that final output (which incorporates both labor and intermediate goods) enters as the main factor of production into the R&D process. A measure \( L^e_n << L_n \) of the population in the North has the entrepreneurial skills to operate a research lab. Each research-lab operates under the cost function

\[
\zeta(\eta_t) = \frac{\delta \eta_t^2}{L^n_e}, \tag{3}
\]

where \( \eta_t \) denotes the number of new inventions at time \( t \) and \( \delta \) reflects the research productivity or the quality of the research infrastructure. Alternatively, \( \delta \) can be interpreted as a measure of the entrepreneurs’ human capital. That is, the higher the level of human capital, the lower \( \delta \) implying that lab-equipment can be used more productively. In addition, R&D is positively affected by the entrepreneurs’ average level of technological knowledge \( \frac{N_t}{L^n_e} \).

A new blueprint invented in period \( t \) can be employed in final-good production from \( t + 1 \) on and it receives a patent of infinite length. Accordingly, the expected value of an invention \( i \) at time \( t \) reads as

\[
E_t[V(i)] = \sum_{\tau = t+1}^{\infty} \beta^\tau P\left(\lambda_n + \lambda_s\right).
\]

As \( E_t[V(i)] \) is the same for all \( i \), we will use the abbreviation \( E_t \) in the following. Optimality requires that marginal costs for an additional invention must equal its expected value. Consequently, inventions per research lab are given by

\[
\eta_t = \frac{E_t \frac{N_t}{2L^n_e}}{\delta \eta^2}, \tag{5}
\]

The assumption that both, research productivity (or human capital) as well as the current technology stock play a positive role for innovative output and are complementary to a certain extent is standard in the literature. For example, in Romer (1990, p. 86), the aggregate stock of designs evolves according to \( \dot{A} = \delta_a H A \), where \( A \) is the stock of designs, \( H A \) is human capital and \( \delta_a \) is a productivity parameter. The assumption of decreasing returns on the firm and industry level with respect to R&D expenditures has been supported empirically, e.g., by Pakes and Griliches (1984) and Hall et al. (1988). On the macro level, the probably most important source of decreasing returns in R&D can be seen in an increased probability of duplicative research through an increasing number of both rivals and expenditures, even though the R&D process as such may be driven by large spillovers (Amir, 2000; Kortum, 1993; Klette and Kortum, 2004). In a related line of argument, it is possible to think of plausible limits in transforming an ever increasing stock of new ideas into usable knowledge for production (Weitzman, 1998). From an aggregate perspective, decreasing returns may also reflect heterogeneity in the cost of research projects. A similar argument can be found in Scotchmer (2004, ch. 11). Convex costs of R&D are also widely used in the industrial organization type literature on IPR-protection (see e.g., Chin and Grossman, 1990; McCalman, 2002; Lai and Qiu, 2003).
and the aggregate stock of technological knowledge evolves according to

\[ N_{t+1} - N_t = \eta_t L_n = E_t \frac{N_t}{2\delta}, \tag{6} \]

2.3 The household’s and the government’s problem

Concentrating on the governments’ IPR-enforcement decisions, we keep the individual household’s problem deliberately simple. The households in Region \( j \) maximize

\[ U_{j,t} = \sum_{\tau=t}^{\infty} \beta^{\tau} c_{j,\tau}, \tag{7} \]

where \( 0 < \beta < 1 \) is a discount factor.\(^{12}\) For the entrepreneurs in the North, the maximization problem reduces to the decision of how much of their income (labor income plus the profit flows from their active patents) to invest in R&D and how much to consume in each period. This problem is solved by (5). The households in the North without entrepreneurial skills as well as the households in the South consume their labor income in each period.

As motivated in the introduction, we intend to examine the effects of politically motivated short-sighted governments that do not fully take into account the long-run consequences of their actions. The simplest way to incorporate this aspect into our model is to assume that at any time \( t \) the governments in both regions choose an optimal enforcement level of IPR so as to maximize\(^{13}\)

\[ W_{j,t} = \sum_{\tau=t}^{t+1} \beta^{\tau} C_{j,\tau}, \tag{8} \]

subject to (6). \( C_{j,t} \) stands for aggregate consumption in country \( j \) at time \( t \). As mentioned in the introduction, we make two additional assumptions concerning the governments’ IPR-enforcement choices. First, governments can only commit to a level of

---

\(^{12}\) Note that this implies that \( \frac{1-\beta}{\beta} \) is the rate of time preference which, in equilibrium, must be equal to the interest rate.

\(^{13}\) Oudiz and Sachs (1985) argue that restricting the planning horizon of the government as we do it here is a natural way to incorporate short-sightedness of governments into dynamic macroeconomic models. Our particular modelling choice regarding the planning horizon of the government could be motivated via short-lived households (with two-period lives). A minority of the households is altruistic and entertain research labs. At the cost of further complexity, we could interpret output \( Y \) as sophisticated machinery that can be used either in research or to produce the consumption good via technology \( F(L^u, Y) \), where \( L^u \) denotes unskilled labor. Under the assumption that unskilled workers constitute the non-altruistic (short-sighted) majority and \( L^u \) and \( Y \) are complements, there exists a conflict between R&D expenditures and machinery for the production of the consumption good. Concerning IPR-policy, a re-election motivated government would then adopt a the short-sighted view of the majority of unskilled workers.
IPR-enforcement for the subsequent period, i.e. $\omega_{j,t+1}$, but not for the indefinite future. For example, while in office at time $t$, the government can increase training efforts of staff responsible for the prosecution and conviction of imitators of protected intermediates. A larger number of trained officials will then be available in $t+1$ to enforce the laws on IPR. Similar arguments apply with respect to other resources or capacity building necessary for effective enforcement. Second, we assume that the enforcement level chosen by the government in Region $j$ applies to all active patents in the same way.

In a typical period $t$, the sequence of events can be summarized as follows. First, intermediate-good production and final-good production take place given the technology stock $N_t$ and IPR-enforcement level $\omega_{j,t}$. Then the government announces the level of IPR-enforcement $\omega_{j,t+1}$ and thereafter the entrepreneurs decide how much to invest in R&D. Finally, the households consume.

At any time $t$ aggregate consumption in the North as well as the dynamics of the technology stock (6) depend on the R&D expenditures in $t$ which reflect the entrepreneurs’ expectations about future IPR-enforcement beyond $t+1$. Let us denote these expectations at time $t$ by $\Omega'_{t+2} \equiv \{\omega'_{n,t}, \omega'_{s,t}\}_{t+2}^\infty$ and the vector of IPR-enforcement that will finally realize by $\Omega_{t+2}$. When deciding on IPR-enforcement, $\omega_{j,t+1}$, the governments have expectations about the entrepreneurs’ expectations $\Omega'_{t+2}$, which we refer to by $\Omega'_{g,t+2}$, and on how the entrepreneurs adapt their expectations in response to the governments’ enforcement choices for period $t+1$, $\omega_{j,t+1}$. Even under the assumption of rational expectations, this structure allows for a plenitude of subgame-perfect equilibria. Here, we intend to minimize complexity by focussing on equilibria that satisfy the following assumption.

**Assumption 1**

(i) At any time $t$, the entrepreneurs’ expectations about future IPR-enforcement $\Omega'_{t+2}$ do not depend on $\omega_{j,t+1}$.

(ii) Each government $j$ takes $N_t$, $\omega_{n,t}, \omega_{s,t}$, $(\omega_{k,t+1}, k \neq j)$ and item (i) as given and maximizes (8) subject to (6) according to its expectations $\Omega'_{t+2}$. Governments do not condition their choices on the history of play before time $t$.

(iii) Given (i) and (ii), expectations are rational, i.e. $\Omega'_{t+2} = \Omega_{t+2}$.

Two remarks are in order. First, in Item (ii) we have used parenthesis for the other region’s IPR-enforcement choice at time $t$, because this is taken as given by each government in the game where IPR-enforcement is chosen decentrally. Later we consider regimes
where a government is able to determine both regions’ enforcement levels in which, of course, the other region’s IPR-enforcement is not taken as given. Second, given Item (i) of Assumption 1, the entrepreneurs’ expectations can only be rational if the future governments’ optimal enforcement choices do not depend on the technology stock. This is the case as we will see below.

3 Decentralized Enforcement of IPRs

In this section, we examine the strategic interaction between governments with respect to their national levels of IPR-enforcement. We focus on unique subgame-perfect equilibria (SPE) in steady state satisfying Assumption 1. In the next two subsections, we study the South’s and the North’s maximization problems and describe the SPE in steady state thereafter.

3.1 The problem of the South

The objective function of the government in the South at time $t$ can be written as

$$W_{s,t} = \sum_{\tau=t}^{t+1} \beta^\tau N_{s,\tau} [Y + \omega_{s,\tau} (D - P)],$$

(9)

where $Y \equiv \alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}} > 0$ reflects the contribution of an intermediate to final output net of production costs for intermediates and $D \equiv \alpha^{\frac{2\alpha}{1-\alpha}} - \alpha^{\frac{\alpha}{1-\alpha}} + \alpha^{\frac{1}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}} < 0$ represents the deadweight-loss factor net of production costs for intermediates. The expression $\omega_{s,\tau} N_{s} \lambda_{s} P$ indicates the profits accruing to the technology owners in the North. The South’s objective (9) and the constraint (6) reveal the government’s trade-off between static efficiency and dynamic gains: Stronger IPR-enforcement involves higher deadweight losses and profit flows to the North while it increases the incentives to innovate in the North (via $E_t$) and thereby leads to higher productivity of domestic final-good production in the South. Solving the South’s optimization problem, the reaction function along the balanced growth path with $\omega_{j,t+1} = \omega_{j,t} = \omega_j$, writes as

$$\omega_s(\omega_n) = - \frac{1 - \beta}{2 - \beta} \left[ \frac{Y}{D - P} + \frac{2 \Delta}{\beta P} \left( 1 + \frac{\lambda_n}{\lambda_s} \right) \right] - \frac{1}{2 - \beta} \frac{\lambda_n}{\lambda_s} \omega_n,$$

(10)

We suppress time indices for steady-state variables. The first-order condition reads as: $R^s(\omega_n, \omega_s) = (1 + \frac{E_t}{28}) (D - P) + \frac{\beta \lambda_n}{28} [Y + \omega_{s,t+1} (D - P)] = 0.$
where $\lambda \equiv \lambda_n + \lambda_s$ denotes the effective size of the world market and $\Delta \equiv \frac{\delta}{\lambda}$ represents the North’s research capacity relative to the aggregate effective market size. This notation turns out to be very convenient for separating the effects of the aggregate world market size, $\lambda$, from those of the relative effective market sizes, $\frac{\lambda_n}{\lambda_s}$. In light of (10), we establish the following proposition:

**Proposition 1 (IPR-enforcement in South)**

(i) The steady-state level of IPR-enforcement in the South is a strategic substitute to IPR-enforcement in the North.

(ii) For $\omega_n$ given, the South’s IPR-enforcement increases with the effective market size of the South, $\lambda_s$, and with the research productivity of the North – i.e., it is decreasing in $\Delta$.

The result in Item (i) originates from the fact that IPR-enforcement constitutes a global public good as far as R&D incentives are concerned. With respect to Item (ii), the South’s impact on the value of a patent becomes larger when it exhibits a larger effective market size, thereby reducing its incentive to free-ride on the North’s protection levels.\(^{15}\)

### 3.2 The problem of the North

In contrast to the government’s objective in the South, the government in the North additionally accounts for R&D expenditures, $E^2_t/4\delta$, and profit flows from the South to the North, $\omega_{s,t}N_t\lambda_sP$, which are subject to IPR-enforcement in the South. Hence, the North’s government maximizes

$$W_{n,t} = \sum_{\tau=t}^{t+1} \beta^\tau N_\tau \left[ \lambda_n(Y + \omega_{n,\tau}D) + \lambda_s\omega_{s,\tau}P - \frac{E^2_\tau}{4\delta} \right],$$

subject to (6). We obtain the first-order condition

$$-N_t\frac{E^2_t}{2\delta}\lambda_nP + N_{t+1}\lambda_nD + N_t\frac{\beta\lambda_nP}{2\delta} \left[ \lambda_n(Y + \omega_{n,t+1}D) + \lambda_s\omega_{s,t+1}P - \frac{E^2_{t+1}}{4\delta} \right] = 0.$$  

(12)

A marginal increase in $\omega_{n,t+1}$ involves higher R&D costs in period $t$ lowering current consumption. This is reflected by the first term in (12). The second term represents the

\(^{15}\)Note that the South’s level of IPR-enforcement may be perfect, that is $\omega_s = 1$. This can be the case if either $\Delta$ is sufficiently low, i.e., the research productivity in the North relative to the effective world market is large or the relative size of the effective market in the South is very large implying a small value of $\frac{\lambda_n}{\lambda_s}$. Further notice that positive consumption levels at any feasible level of IPR-enforcement require $Y > P - D$. Consequently, the first term in brackets of (10) is greater than 1 (i.e. $\frac{Y}{D-P} < -1$).
marginal increase in the deadweight loss in period $t + 1$. Finally, the marginal benefits are captured by the last summand of (12) which multiplies the additional number of innovations, $N_{t+1} - N_t = N_t \frac{\beta \lambda_n P}{2 \delta}$, induced by the marginal increase in IPR-enforcement, with the future welfare gains per innovation as expressed by the term in brackets.

From (12), we obtain in steady state

$$R_n(\omega_n, \omega_s) \equiv \frac{\bar{E}}{2\Delta} (D - P) + D + \frac{\beta P}{2\Delta} \left[ \frac{\lambda_n}{\lambda} (Y + \omega_n D) + \frac{\lambda_s}{\lambda} P \omega_s - \frac{\bar{E}^2}{4\Delta} \right] = 0,$$

where $\bar{E} = E/\lambda$. Note that $\bar{E}$ only depends on the relative effective market sizes, $\lambda_n/\lambda_s$, but not on $\lambda$. Equation (13) implicitly defines the reaction function of the North, $\omega_n^*(\omega_s)$.

In the first term of (13), we combined the R&D costs and the deadweight losses of the innovations created in period $t$, while the second term represents the deadweight losses resulting from enforcing the patents created before time $t$. The government’s future welfare gains induced by a marginal increase in the North’s level of IPR-enforcement are still captured in the third term. In the appendix, we show:

**Lemma 1**

(i) There exists a unique economically sensible solution $\omega_n^*(\omega_s)$ to $R_n(\omega_n, \omega_s) = 0$.

(ii) The North’s reaction function $\omega_n^*(\omega_s)$ is strictly decreasing and strictly concave on the relevant interval $[0, 1]$.

Lemma 1’s implication of strategic substitutability between $\omega_n$ and $\omega_s$ from the perspective of the North is not obvious. A higher $\omega_s$ implies higher profit inflows from the South to the North for all active patents and for those intermediates that are developed in $t$. On the one hand, this increases the North’s incentives to tighten its level of IPR-enforcement. On the other hand, the global public good problem with respect to R&D-incentives acts to reduce IPR-enforcement in the North when the South increases its enforcement level. As verified in the proof of Lemma 1 the public good aspect dominates. Hence, national levels of IPR-enforcement are strategic substitutes to foreign enforcement levels.

---

16Note that, by assumption, the marginally higher IPR-enforcement applies to all active patents in $t + 1$, $N_{t+1}$. This includes all innovations created before time $t$, $N_t$, as well as those invented in period $t$, $N_t \frac{\Delta}{2\delta}$. 
3.3 Equilibrium

The reaction functions of the North, $\omega_n^r(\omega_s)$, and the South, $\omega_s^r(\omega_n)$, possess only one potentially economically meaningful intersection which we denote by $(\omega_n^e, \omega_s^e)$.\textsuperscript{17} However, the intersection may lie outside of the feasible set $[0, 1]^2$. To account for corner solutions, let us introduce the notation $\hat{z} \equiv \max\{\min\{z, 1\}, 0\}$ and $\hat{z}(x) \equiv \max\{\min\{z(x), 1\}, 0\}$ for a constant $z$ and a function $z(x)$, respectively. Now we are able to characterize the levels of IPR-enforcement in a steady-state SPE, $(\omega_n^e, \omega_s^e)$.

**Proposition 2 (Steady-State SPE)**

In steady state, there exists a unique subgame-perfect equilibrium of the IPR-enforcement game satisfying Assumption 1. The unique enforcement levels in equilibrium are characterized by

$$
\omega_n^e = \begin{cases} 
\hat{\omega}_n^r(0), & \text{if } \omega_s^e \leq 0, \\
\hat{\omega}_n^r, & \text{if } \omega_s^e \in (0, 1), \\
\hat{\omega}_n^r(1), & \text{if } \omega_s^e \geq 1,
\end{cases}
$$

$$
\omega_s^e = \begin{cases} 
\hat{\omega}_s^r(0), & \text{if } \omega_n^e \leq 0, \\
\hat{\omega}_s^r, & \text{if } \omega_n^e \in (0, 1), \\
\hat{\omega}_s^r(1), & \text{if } \omega_n^e \geq 1.
\end{cases}
$$

The proof can be found in Appendix A.2. At this point two remarks are interesting. First, there is the possibility of zero IPR-enforcement in the South, i.e., $\omega_s^e = 0$. In this case, the model switches structurally to a closed-economy setting. As the regions’ IPR-enforcement levels are strategic substitutes, trade opening between North and South lowers the enforcement level of the North provided that $\omega_s^e > 0$. Second, trade opening in the South enhances the level of IPR-enforcement compared to autarky since the South – even though it does not conduct research – internalizes the effect of its IPR-enforcement level on R&D-incentives in the North.

3.4 The roles of research capacity and market sizes

In our model, the enforcement levels in the steady-state SPE are entirely determined by the ‘primitives’ $\alpha, \beta, \Delta,$ and $\lambda_n/\lambda_s$. Our interest centers on how the decentralized steady-state equilibrium is affected (1) by the research capacity of the North and the global effective market size captured by the parameter $\Delta$ and (2) by the relative effective market

\textsuperscript{17}A formal proof can be found in Appendix A.2.
size of the North and the South, $\lambda_n/\lambda_s$, for a given aggregate market size, i.e. for a given $\Delta$.\textsuperscript{18}

We begin with $\Delta$. Perceiving $\omega^n_e$ and $\omega^s_e$ as functions in $\Delta$, we obtain

**Lemma 2**

*In an interior equilibrium where $(\omega^n_e, \omega^s_e) \in (0, 1)^2$,*

(i) $\omega^n_e$ is strictly concave in $\Delta$.

(ii) $\omega^s_e$ is strictly convex in $\Delta$.

In the proof given in the appendix, we first show that $\omega^n_e$ is strictly concave in $\Delta$ in an interior equilibrium. As far as IPR-enforcement levels in the South are concerned, $\omega^s_e$ is a declining line in $\Delta$ if there is no IPR-enforcement in the North. For positive protection levels in the North, the South’s enforcement level must be strictly below this line as the protection level of the North acts as a strategic substitute. Consequently, the protection level of the South becomes convex since IPR-enforcement in the North is concave.

To fully characterize the comparative-statics, we have to account for corner solutions. There exists a critical level $\Delta^0_j$, for both regions individually, such that for any $\Delta > \Delta^0_j$, country $j$ is not willing to enforce IPRs.\textsuperscript{19} This implies for the situation $\Delta^0_s < \Delta^0_n$ – i.e., the South’s critical threshold level is smaller than the one of the North – that for all $\Delta^0_s > \Delta > \Delta^0_n$ the South does not offer protection in equilibrium while the North acts as in autarky. The opposite holds true in the situation where $\Delta^0_n < \Delta^0_s$. In the following, we focus on the case $\Delta^0_s < \Delta^0_n$ and define $\Delta^0 \equiv \Delta^0_s$ as the smallest threshold corresponding to the South. This condition seems to match reality more closely compared to the opposite case, as it implies a minimum effective market size of the North relative to the South

$$\frac{\lambda_n}{\lambda_s} > \frac{D}{D - P}. \quad (14)$$

Note that the right-hand side of (14) is smaller than one. Hence the inequality is always satisfied if $\lambda_n > \lambda_s$, but it also holds if $\lambda_n$ is not too much smaller than $\lambda_s$. In the next proposition, we characterize the comparative statics of equilibrium IPR-enforcement levels with respect to changes in $\Delta$ given that condition (14) holds.

\textsuperscript{18}For example, an increase in the North’s market size leaving that of the South unaffected would increase both, the world market size and the relative market size of North. Consequently, the effect on the IPR-enforcement level would be a combination of the two effects. For this reason, it seems natural to isolate the resulting effects from each other.

\textsuperscript{19}This claim is verified analytically in the proof of Proposition 3.
Figure 1: Steady state equilibrium pairs \((\omega^n_e, \omega^e_s)\) dependent on \(\Delta\), with \(\alpha = 0.3; \beta = 0.3; \lambda = 1; \lambda_n = 0.445\).

**Proposition 3 (Effect of \(\Delta\) on IPR-enforcement)**

If \(\frac{\lambda_n}{\lambda_s} > \frac{D}{D-F}\), then

(i) \(\omega^e_s\) is positive and strictly decreasing with \(\Delta\) for all \(\Delta < \Delta^0\), and \(\omega^e_s = 0\) for all \(\Delta \geq \Delta^0\).

(ii) For interior values, \(\omega^n_e\) exhibits an inverted U-shaped relationship with \(\Delta\). \(\omega^n_e\) is identical to its value in autarky for \(\Delta > \Delta^0\).

(iii) There exists a unique value \(\Delta^x < \Delta^0\) where \(\omega^n_e = \omega^e_s\). For all interior equilibria, \(\omega^n_e < \omega^e_s\) if \(\Delta < \Delta^x\), and \(\omega^n_e > \omega^e_s > 0\) if \(\Delta > \Delta^x\).

The proof can be found in the appendix. Proposition 3 is illustrated in Figure 1. Intuitively, \(\omega^e_s\) declines with \(\Delta\) because a larger value of \(\Delta = \frac{\delta}{\lambda}\) (i.e. declining research capacity \((\delta \uparrow)\) or declining effective world market size \((\lambda \downarrow)\)) implies a lower lever exercised by the South’s IPR-enforcement on innovation incentives in the North. The convex shape for interior values of \(\omega^e_s\) arises, as discussed earlier, from the public-good aspect of IPR-enforcement on R&D-incentives. In contrast to the literature, our model predicts an inverted U-shaped relation between the North’s level of IPR-enforcement and \(\Delta\). An intuition for this result can be gained from scrutiny of the North’s reaction function (13) for a given \(\omega_s\).\(^{20}\) Using the implicit-function theorem, the partial derivative of \(\omega^n_s(\omega_s)\)

\(^{20}\)The effect of \(\Delta\) via \(\omega_s\) changes \(\omega^n_s\) quantitatively but does not affect the inverted-U shape of \(\omega^n_s\) in
with respect to $\Delta$ can be written as $\frac{\partial \omega_n^*}{\partial \Delta} = -\frac{\partial R(\omega_n, \omega_s)}{\partial \Delta} \frac{\partial R(\omega_n, \omega_s)}{\partial \omega_n}$. As we show in the appendix the denominator is negative, implying that the sign of $\frac{\partial \omega_n^*}{\partial \Delta}$ is identical to the one of $\frac{\partial R(\omega_n, \omega_s)}{\partial \Delta}$, which may be positive or negative. On the one hand, a decline in $\Delta$ involves an increase in the number of innovations, $(\beta P/2\Delta)$. On the other hand, it increases current R&D expenditures and future deadweight losses (first term in (13)). Additionally, welfare per innovation (term in brackets in (13)) declines when $\Delta$ becomes smaller because next period’s R&D expenditures increase, as well. The benefits of a marginal increase in $\omega$ (the higher number of innovations) are not increasing as strongly when $\Delta$ becomes smaller as the the marginal costs (additional R&D-costs and deadweight losses), implying an inverted U-shaped relation between $\omega_n^*$ and $\Delta$.

It is important to emphasize that this result is not an implication of convex R&D-costs at the research lab level. The central assumptions behind this result are that the government does not take full account of the future benefits of R&D and enforces all active patents at the same strength. It is straightforward to show that in the case of a far-sighted government which could commit to a particular enforcement level for each vintage over the entire lifetime of its patent (such as in Grossman and Lai (2004)), a monotonically declining relationship between $\omega_n$ and $\Delta$ would result.\(^{21}\) Similarly, in a one shot game where the government determines the level of IPR-enforcement once and for all. The governments’ limited time horizons and the necessity to enforce all active patents at the same strength result in different weights between the marginal benefits and the marginal costs of IPR-enforcement in the government’s first-order condition. While the government can only influence the profit flows and deadweight losses in the next period, the induced additional costs for R&D that accrue in the current period account for the entire net present value of future profits. The latter cost term takes the dominant role for small values of $\Delta$ leading to an increasing relationship between IPR-enforcement and $\Delta$ in the North. To the contrary, the South’s decision problem is independent from R&D-expenditures, such that the dynamic gains from the perspective of the South are monotonically increasing with the research productivity of the North. As a consequence of this result, we may find lower IPR-enforcement levels in the North than in the South for sufficiently small $\Delta$, and vice versa if $\Delta$ is sufficiently large.

Before turning our attention to the comparative statics with respect to relative market sizes, $\frac{\lambda_n}{\lambda_s}$, we verify that in interior equilibria, the global rate of growth on the balanced

\(^{21}\) A proof is provided upon request.
growth path increases when the research capacity becomes larger, even though the North’s level of IPR-enforcement may be declining at low values of $\Delta$.

**Proposition 4 (Effect of $\Delta$ on steady-state growth)**

In interior equilibria $(\omega_e^n, \omega_e^s) \in (0, 1)^2$, the global steady-state growth rate strictly decreases with $\Delta$.

The proof can be found in Appendix A.5. Finally we turn to the role of relative market sizes for IPR-enforcement and economic growth. We focus again on interior equilibria.

**Proposition 5 (Effect of relative effective market size)**

In interior equilibria $(\omega_e^n, \omega_e^s) \in (0, 1)^2$, both countries’ IPR-enforcement levels increase with their relative effective market sizes. The steady-state growth rate is unaffected by the relative effective market sizes.

The proof is provided in Appendix A.5. Governments tighten IPR-enforcement in response to an increase in their relative market share, since their relative levers in inducing innovations increase. The region becoming relatively smaller by a marginal change in the relative effective market sizes reduces its IPR-enforcement level in a symmetric way such that the global discounted profits to be earned in expectation by an entrepreneur in the North remain unchanged. As a consequence, the steady-state growth rate remains unaffected. In sum, a change in a country’s effective market size will affect the growth rate only through its effect on the total world market size but not via a change in the its relative market size.

## 4 Harmonization of IPR-enforcement

As discussed in the introduction, some countries make an effort to harmonize IPR-enforcement globally, e.g., via ACTA. In this respect, it is interesting to explore which harmonized IPR-enforcement level the governments of Regions $n$ and $s$ would like to implement given it had the power to do so. These enforcement levels may shed light on the differences that need to be bridged in international negotiation rounds.\footnote{In the formal bargaining problem, the governments’ most preferred IPR-enforcement levels are the points on the boundary of the feasible set which will realize if the respective regional government possesses all the bargaining power. The threat point of the problem is the decentralized equilibrium as described in the previous section. How close to governments’ ideal enforcement levels the bargaining outcome will be depends on the relative bargaining power, of course.}
In our context harmonization means that both regions are subject to the same level of IPR-enforcement. Hence, expected discounted profits per invention are specified as

\[ E_t^h = \sum_{\tau=1}^{\infty} \beta^{\tau-t} \omega_{j,\tau}^h P \lambda, \]  

(15)

where \( \omega_{j,\tau}^h \) represents the harmonized IPR-enforcement level preferred by Region \( j \). The evolution of the technology stock is again captured by (6), where discounted profits are now determined by (15), such that

\[ N_{n,t+1} = N_t \left( 1 + \frac{E_t^h}{2\Delta} \right). \]  

(16)

With respect to the governments’ decision problems, we keep with the two major assumptions that there is only commitment on IPR-enforcement for one period and all active patents have to be enforced at the same strength. One may argue that an agreement in the international arena could serve as a commitment device, partially at least. However, particularly where IPR-enforcement is concerned rather than formal laws, there is also the possibility of renegotiations after each period. Here, we stress the latter point.\(^{23}\) This also allows us to directly compare the results to the ones in the decentralized setting.

4.1 Desired harmonized enforcement level of the South

We begin with the optimization problem of the government located in the South which chooses a single level of IPR-enforcement that applies to both regions. The South maximizes

\[ W_{s,t}^h = \sum_{\tau=t}^{t+1} \beta^{\tau-t} N_{\tau} \lambda_s \left[ Y + \omega_{s,\tau}^h (D - P) \right], \]  

(17)

subject to (16). Along the balanced growth path, we obtain as the preferred harmonized enforcement level of the South\(^{24}\)

\[ \omega_s^h = \frac{1 - \beta}{2 - \beta} \left( \frac{Y}{D - P} + \frac{2\Delta}{\beta P} \right). \]  

(18)

\(^{23}\)Allowing for commitment over a longer finite time horizon would increase the desired levels of IPR-protection but would not change the characteristics of the problem qualitatively.

\(^{24}\)Note that we still assume that the government is able to adjust IPR-enforcement after each period. Consequently, in period \( t \) the South determines the optimal harmonized enforcement level \( \omega_{s,t+1}^h \) taking as given the rational beliefs of the entrepreneurs about future governments’ optimal decisions (see Assumption 1). The first-order condition reads as \( \left( 1 + \frac{E_t^h}{2\Delta} \right) (D - P) = -\beta \frac{P}{2\Delta} [Y + \omega_{s,t+1}^h (D - P)] \).
Compared to the decentralized protection game (see Equation (10)), the desired harmonized enforcement level of the South is larger, since the marginal benefits in terms of R&D incentives increase due to the larger market size in the optimization problem ($\lambda$ versus $\lambda_s$) for which enforcement is determined. At the same time, the marginal costs in terms of deadweight losses in the South and profit outflows to the North remain as in the decentralized setting. In addition, $\omega_h^s$ is independent from relative market sizes. Equation (18) reveals that $\omega_h^s$ increases with the North’s research capacity but is independent of the relative effective market sizes. We summarize these observations in the next proposition.

**Proposition 6 (Desired harmonized IPR-enforcement of the South)**
The preferred harmonized level of IPR-enforcement of the South increases with the North’s research productivity and the global effective market size but is independent of the relative market sizes.

### 4.2 Desired harmonized enforcement level of the North

The objective of the government in the North includes profit inflows from the South which are – contrary to the decentralized IPR-enforcement game – subject to the harmonized enforcement level of the North

$$W_{n,t}^h = \sum_{\tau=t}^{t+\tau} \beta^\tau N_\tau \left[ \lambda_n (Y + \omega_{n,\tau}^h D) + \lambda_s P \omega_{n,\tau}^h - \frac{(E_{n,\tau}^h)^2}{4\delta} \right].$$

In steady state, the North’s optimal level of global IPR-enforcement, $\omega_n^h$, satisfies

$$R_n^h(\omega_n^h) \equiv \frac{\bar{E}_n^h}{2}(D - P) + \Delta(D + \frac{\lambda_s}{\lambda_n} P) + \frac{\beta P}{2} \left[ Y + \omega_n^{h}(D + P\frac{\lambda_s}{\lambda_n}) - \frac{(E_n^h)^2}{4\Delta} \left(1 + \frac{\lambda_s}{\lambda_n}\right) \right] = 0.$$  (20)

where $\bar{E}_n^h = \frac{\beta}{\beta - P} P \omega_n^{h}$. The next proposition verifies that (20) possess a unique economically sensible solution and describes the effects of changes in $\Delta$ and the relative market sizes, $\frac{\lambda_s}{\lambda_n}$, on the preferred harmonized enforcement level of the North.

---

25The North’s first-order condition reads $R_n^h(\omega_n^{h,t+1}) = \frac{\bar{E}_n^h}{2X}(D - P) + \Delta(D + \frac{\lambda_s}{\lambda_n} P) + \frac{\beta P}{2} \left[ Y + \omega_n^{h,t+1}(D + P\frac{\lambda_s}{\lambda_n}) - \frac{(E_n^{h,t+1})^2}{4\Delta}\lambda_n \right] = 0$. As the North controls profit inflows from the South, the second-order condition for the problem described above may be violated, if $\frac{\lambda_s}{\lambda_n}$ is large enough, such that the marginal gains from profit inflows to the North always overcompensate the marginal R&D costs and deadweight losses in the North. Then the North opts for complete protection $\omega_n^{h,\tau} = 1$, $\forall \tau$. In the following, we consider the more interesting case where the second-order condition for a maximum is satisfied, such that $\frac{\lambda_s}{\lambda_n} > \frac{P - 2D}{2P} > 1$, since $D < 0$. 

---
Proposition 7 (Desired harmonized IPR-enforcement of the North)

There exists a unique economically sensible solution to the North’s optimization problem. The North’s desired harmonized level of global IPR-enforcement depends on its research productivity and its relative effective market size as follows:

(i) If \( \frac{\lambda_n}{\lambda_s} < -\frac{P}{\Delta} \), then the North’s desired level of global IPR-enforcement, \( \omega_{hn} \), increases with \( \Delta \).

(ii) If \( \frac{\lambda_n}{\lambda_s} > -\frac{P}{\Delta} \), then there exists a unique value \( \Delta^m > 0 \) where for all \( \Delta > (\Delta^m) \), the North’s desired level of IPR-enforcement, \( \omega_{hn} \), decreases (increases) with \( \Delta \).

(iii) There exists a unique value \( \bar{\Delta} > 0 \), where for all \( \Delta > (\bar{\Delta}) \), the North’s desired level of IPR-enforcement, \( \omega_{hn} \), decreases (increases) with \( \frac{\lambda_n}{\lambda_s} \).

The proof can be found in Appendix A.7. Concerning the effects of research capacity and global effective market size (\( \Delta \)), Proposition 7 distinguishes two cases. In the first, (i), an increase of global IPR-enforcement involves less additional deadweight losses in the North (\( -\lambda_nD \)) than additional profit inflows from the South (\( \lambda_sP \)). Thus, the only costs associated with IPR-enforcement are the research costs, and the North’s main objective in enforcing global IPRs is to reap profits from the South. The latter is cheaper when \( \Delta \) increases as this implies lower aggregate R&D expenditures. As a consequence, there is a positive relation between \( \omega_{hn} \) and \( \Delta \). In the second case, (ii), the profit inflows from the South are lower than the deadweight losses in the North incurred by an increase in global IPR-enforcement. In this scenario, the North’s first-order condition with respect to its most preferred harmonized enforcement level shows a similar structure as the one in the decentralized game with the difference that a part of the North’s deadweight losses are compensated for by higher profit inflows from the South. As a consequence, we also obtain an inverted U-shaped relationship between \( \omega_{hn} \) and \( \Delta \) for which the same intuition as provided in the discussion of the decentralized setting can be applied.

Contrary to the decentralized enforcement game, the relative effective market size exhibits a non-monotonic effect on the North’s desired level of IPR-enforcement as indicated by Item (iii) of Proposition 7. The reason is that changes in the relative effective market sizes change the weights attached to the different components in the North’s objective function. As an illustration consider the effect of an increase of \( \lambda_n/\lambda_s \) given \( \lambda \) on the government’s welfare objective in period \( t \). The latter writes as

\[
N_t \left[ \lambda_n Y + \omega_{hn} \left( \lambda_n D + \lambda_s P \right) - \frac{(E_{h}^2)^2}{4\Delta} \right].
\]

(21)
Figure 2: Desired harmonized IPR-enforcement of the North (solid lines) in response to an increase in $\frac{\Delta}{\Delta}$ (gray versus black solid line). Dashed line: desired harmonized IPR-enforcement level of the South. Parameters: $\alpha = 0.3; \beta = 0.35; \lambda = 1; \lambda_n = 0.78$.

Substituting $\lambda_s$ by $\lambda - \lambda_n$ and taking the derivative with respect to $\lambda_n$ given $\lambda$ yields

$$N_t \lambda_n \left[ Y + \omega_n^h \left(D - P\right)\right].$$

(22)

Apparently, the marginal change of the North’s periodic welfare with respect to changes in its own relative market size (22) is structurally equivalent to the periodic welfare of the South and is independent of research expenditures since $E_t^h$ depends only on the effective world market size $\lambda$, which remains unchanged. Intuitively, a larger effective market size of the North gives higher weight to final-good production and deadweight losses in the North and lower weight to the profit inflow from the South. That is, an increase in the effective market size of the North gives higher weight to those components of the North’s periodic welfare that are also present in the South’s. Hence, the desired IPR-enforcement level of the Northern government approaches the one of the South when $\lambda_n/\lambda_s$ increases. However, it will never coincide with $\omega_n^h$, since $\omega_n^h$ represents the solution under autarky for $\lambda_s = 0$ with R&D expenditures still being positive. A graphical illustration is presented in Figure 2, where the solid gray line reflects the desired harmonized enforcement level of the North for a lower relative market size $\lambda_n/\lambda_s$ compared to the dark solid line. The dark solid curve is closer to the dashed line which represents the desired harmonized enforcement level of the South.\footnote{Formally this can be seen as follows. As a direct consequence of the arguments above, it follows that the derivative of $R_t^h(\omega_n^h)$ with respect to $\lambda_n/\lambda_s$ is equivalent to the South’s first-order condition in steady...}
According to this intuition and Proposition 7 (iii), we infer:

**Proposition 8 (North’s and South’s desired harmonized IPR-enforcement)**

For interior values of $\omega^h_n$ and $\omega^h_s$, $\omega^h_n < \omega^h_s$ if $\Delta < \bar{\Delta}$, and $\omega^h_n > \omega^h_s$ if $\Delta > \bar{\Delta}$.

The proposition is illustrated in Figure 2 and a proof is provided in the appendix. Proposition 8 implies that when the research productivity in the North is very large ($\Delta$ sufficiently small), the South may even desire a higher harmonized enforcement level than the North. Likely, reality is described by $\Delta > \bar{\Delta}$ and $\Delta > \Delta^x$ implying that $\omega^h_n > \omega^h_s$ and $\omega^e_n > \omega^e_s$. Then the North’s desired harmonized enforcement level increases with the relative market size of the South while its equilibrium enforcement level in the decentralized game declines. Consequently, a relatively larger Southern market widens the gap between $\omega^h_n$ and $\omega^e_n$. The opposite is true for the South: Its desired harmonized level is independent from the relative market sizes, while the equilibrium level $\omega^e_s$ increases with the South’s relative market size. This implies that the difference between $\omega^e_s$ and $\omega^h_s$ becomes smaller, since $\omega^e_s < \omega^h_s$ as argued in Section 4.1.

In particular, with regard to the ACTA-negotiations our results suggest that small countries located in the North strongly favor tighter IPR-enforcement as they benefit most from higher profit inflows from the South, with the latter incurring the correspondingly large amount of deadweight losses.

$$\frac{\partial R^h_n(\omega^h_n)}{\partial \lambda^h_n} = R^h_s(\omega^h_s) \equiv \left(1 + \frac{\tilde{E}^h}{2\Delta}\right)(D - P) + \beta \frac{P}{2\Delta}(Y + \omega^h_j(D - P)), $$

with $j = n, s$. The optimal level of IPR-enforcement of the South is given by $\frac{\partial R^h_n(\omega^h_n)}{\partial x^h_s} = R^h_s(\omega^h_s) = 0$. As (23) is decreasing with $\omega^h_j$, and given that $\Delta$ is such that the desired value of IPR-enforcement of the North is higher than that of the South – i.e., $\omega^h_n > \omega^h_s$ – an increase of the North’s relative market size would imply $\frac{\partial R^h_n(\omega^h_n)}{\partial \lambda^h_n} < 0$ and thus $\frac{d\omega^h_n}{d\lambda^h_n} < 0$. This argument follows directly from the implicit-function theorem. The opposite holds true if $\omega^h_n < \omega^h_s$ (see Figure 2).

Recall that $\omega^h_n \equiv \max[\min\{\omega^h_n, 1\}, 0]$. Moreover, we changed the set of parameters for illustrative purposes of interior solutions without altering the qualitative results. The set of parameters employed in the previous section violates the second order condition of the North, such that the North would choose full protection, i.e. $\omega^h_n = 1$. The parameters used in this section imply a corner solution in the decentralized enforcement game, such that the South opts very fast for zero protection and the North behaves as in autarky.

Scotchmer (2004, p. 336 and 346) notes that during the TRIPS negotiations countries with smaller markets were in favor of stronger protection.
5 Constrained Efficiency and Comparison of Regimes

What would be the maximum welfare that governments could achieve by coordinating their respective levels of IPR-enforcement, but given their inability to escape their political economy constraints? We have in mind a global government choosing pairs of $(\omega_{n,t+1}, \omega_{s,t+1})$ so as to maximize the sum of the regional governments’ welfare. Since expected profits depend only on the path of $\Phi_t = \lambda_n \omega_{n,t} + \lambda_s \omega_{s,t}$ and not on particular values of $\omega_{n,t}$ and $\omega_{s,t}$, we can rewrite the maximization problem of a global government in terms of $\Phi_t$.\(^{29}\) Hence the constrained efficient pairs of IPR-enforcement, $(\omega_{n,t+1}, \omega_{s,t+1})$, are obtained by solving

$$\max_{\Phi_{t+1}} W = \sum_{t=1}^{t+1} \beta^t N_t \left[ Y + D (\lambda_n \omega_{n,t} + \lambda_s \omega_{s,t}) - \frac{E_t}{4\Delta} - \frac{E^2_t}{4\Delta} \right],$$

subject to (6). The necessary condition for a constrained welfare maximum in steady state reads as

$$D + \frac{\tilde{E}}{2\Delta} (D - P) + \frac{\beta P}{2\Delta} \left( Y + D\Phi/\lambda - \frac{\tilde{E}^2}{4\Delta} \right) = 0. \quad (24)$$

Sidestepping the multiplicity of optimal solutions to the global government’s problem, we focus on the (unique) constrained efficient harmonized solution where the optimal enforcement level $\omega^p$ is implemented in both regions and solves (24). In this case, we obtain $\Phi = \lambda \omega^p$ and (24) coincides with the first-order condition of a closed economy with effective market size $\lambda$.

The constrained efficient solution serves as a theoretical point of reference to which we relate the enforcement levels obtained from the previous sections. The different levels of IPR-enforcement are depicted in Figure 3. The regions’ preferred harmonized and the constrained-efficient enforcement levels intersect at $\bar{\Delta}$ such that $\omega^h_n > \omega^p > \omega^h_s$, if $\Delta > \bar{\Delta}$, while $\omega^h_n < \omega^p < \omega^h_s$, if $\Delta < \bar{\Delta}$.

The intuition behind this result can be described as follows: for $\lambda_s \to 0$ and $\lambda_n \to \lambda$ the preferred harmonized enforcement level of the North must equal the constrained-efficient solution $(\omega^h_n = \omega^p)$ since the world economy consists of the North only. According to Item (iii) of Proposition 7, $\omega^h_n$ increases with $\lambda_s/\lambda_n$ if $\Delta > \bar{\Delta}$, but declines if $\Delta < \bar{\Delta}$.\(^{30}\) Hence, starting from the situation where $\lambda_s = 0$ and $\lambda_n = \lambda$, an increase in $\lambda_s/\lambda_n$ turns

\(^{29}\)An equivalent result is obtained by Grossman and Lai (2004).

\(^{30}\)Intuitively, in the former case the motive of gaining profit flows from the South dominates while in the latter $\Delta$ is so small that increased R&D expenditures are of greater concern.
ωₙ counterclockwise around ∆ implying ωₙ > (<) ωₚ if ∆ > (<) ∆. As discussed in the previous section, for a declining ratio λₛ/λₙ, ωₙ approaches ωₙ but will not coincide with it in the limit λₛ/λₙ → 0. Accordingly, Proposition 8 implies ωₙ < (>) ωₚ for ∆ > (<) ∆. As a consequence, the constrained-efficient IPR-enforcement level is in between the desired harmonized enforcement levels of the North and the South for all ∆ ≠ ∆.

Concerning the decentralized enforcement level in the North, we know from Proposition 5 that \( \frac{\partial \omega^n}{\partial \lambda_n} > 0 \). Moreover, in the situation where λₙ = λ, ωₚ coincides with the constrained-efficient enforcement level \( \omega_p \). An increase in \( \lambda_s/\lambda_n \) thus implies \( \omega^n < \omega_p \). According to our previous discussion, it further involves \( \omega^n < \omega_h \) for ∆ ≥ ∆. However, this relation may not be satisfied for all ∆ < ∆. We summarize our observations in the following proposition.

**Proposition 9 (Comparison of IPR-enforcement regimes)**

(i) At ∆ = ∆ the regions’ preferred harmonized enforcement levels correspond to the constrained efficient harmonized IPR-enforcement, i.e., \( \omega^h = \omega^n = \omega_p \).

(ii) For ∆ < ∆, \( \omega^h \) is above and \( \omega^n \) below the constrained-efficient level of IPR-

---

The reason is that if \( \lambda_n = \lambda \), the decision problem of the North is entirely described by the constrained efficient problem.

In this case both, \( \omega^n \) and \( \omega^h \) decline with \( \lambda_s/\lambda_n \). Conditions under which \( \omega^n < \omega^h \) for all ∆ ≥ ∆ will be provided upon request.
enforcement. For all $\Delta > \bar{\Delta}$, $\omega_n^h$ is below and $\omega_n^h$ above the constrained-efficient level of IPR-enforcement.

(iii) The decentralized equilibrium level of IPR-enforcement in the North, $\omega_n^e$, is always below the constrained-efficient level and lower than the North’s desired harmonized enforcement level if $\Delta \geq \bar{\Delta}$.

The world economy is arguably best described by $\Delta > \bar{\Delta}$. As stated in Proposition 9 and depicted in Figure 3, the preferred harmonized enforcement level of the North exceeds the constrained-efficient level which in turn is higher than the preferred harmonized enforcement level of the South. Since the steady-state growth rate is a linear function of IPR-enforcement, the implementation of $\omega_n^h$ would be most conducive for economic growth.\textsuperscript{33} On the other hand, for small values of $\Delta$, the decentralized game yields the lowest aggregate incentives for R&D and consequently the lowest steady-state growth rate. Interestingly, the latter may even fall below the resulting growth rate if the South’s desired harmonized level of IPR-enforcement were adopted globally.\textsuperscript{34}

6 Welfare

Whether the South should adopt the IPR-standards of the North is one of the most debated questions in the political arena.\textsuperscript{35} However, it is not clear to which IPR-standards of the North the discussion refers to: the equilibrium choice of the North or its desired harmonized enforcement level. Figure 3 in the previous section suggests that even though the difference between the North’s and the South’s equilibrium choices can be substantial, the South’s desired harmonized IPR-enforcement level can be quite close to the North’s equilibrium choice. Hence, a binding adoption of the North’s equilibrium enforcement level might not be such a contentious issue as opposed to the implementation of the North’s most preferred harmonized protection level of IPRs. We therefore explore the welfare effects in the South resulting from the implementation of $\omega_n^h$ along the balanced growth path as compared to the implementation of $\omega_n^h$.\textsuperscript{36} Aggregate welfare in the South

\textsuperscript{33} As can be inferred directly from (6), the steady-state growth rate can be written as $g(\omega_n^h) = \frac{\tilde{E}_2}{\Delta}$.\textsuperscript{34} This can be the case when $\lambda_s/\lambda_n$ is large (however still satisfying (14)). Using the set of parameters employed in this section, we obtain $\omega_n^e = 0$ (even for $\Delta < \bar{\Delta}$). The North behaves as in autarky where $\omega_n^e < \omega^p < \omega_n^h$ for $\Delta > \bar{\Delta} = 0.005$. The resulting growth rate per year for $\Delta = 0.009$ implying $\omega_n^e \approx 0.85$ and $\omega_n^h \approx 0.75$ equals $g^e \approx 3.6\%$ and $g(\omega_n^h) \approx 3.8\%$.\textsuperscript{35} See e.g., Lai and Qiu (2003).\textsuperscript{36} We do not consider welfare effects in the North, which are very intuitive: The implementation of $\omega_n^h$ in the North causes welfare losses there relative to the implementation of $\omega_n^h$, since the South neglects
$f \rightarrow \omega f_s = 0$

$\Delta \rightarrow \omega f_s = 0$

$\bar{W}_{nh}s$ corresponds to $\omega h_s$; $\bar{W}_{nh}s$ corresponds to $\omega h_n$; $\bar{W}_{sf}s$ corresponds to $\omega f_s$.

The results are depicted in Figure 4.

The gray solid line reflects the South’s overall welfare, $W_s(\omega h_s)$, obtained from its government’s preferred harmonized enforcement level, while the dashed line represents the long-term welfare level, $W_s(\omega h_n)$, realized by accepting the Northern government’s desired harmonized enforcement level. Figure 4 indicates that for a relatively high research capacity in the North ($\Delta f < \Delta f_0$), the implementation of $\omega h_n$ would induce welfare gains in the South compared to the implementation of its own preferred harmonized enforcement level, $\omega h_s$. For large values of $\Delta$, however, the South suffers welfare losses by implementing $\omega h_n$ rather than $\omega h_s$.

The result that the South gains in welfare from implementing the desired harmonized IPR-enforcement level of the North can be explained by the Southern government’s limited time horizon. To illustrate this, we calculate the simple one-shot solution to maximizing welfare in the South given by (25). That is, the government in the South selects the impact of $\omega h$ on R&D expenditures and profit inflows to the North.

We use the same set of parameters as before: $\alpha = 0.3; \beta = 0.4; \lambda = 1; \lambda_n = 0.78$. Details on the calculations can be obtained upon request.
global level of IPR-enforcement at $t = 0$ which is then fixed for all times. The welfare level realized from the implementation of this enforcement level, which we denote by $\omega_f$, is indicated by the dashed dark line in Figure 4.\textsuperscript{38} Here we directly observe that the South’s welfare level obtained from the implementation of the North’s desired harmonized enforcement level approximates the one realized in the full commitment case for $\Delta < \Delta_0_f$. To the contrary, the implementation of $\omega_h$ causes welfare losses for $\Delta > \Delta_0_f$, since the South would set even $\omega_f$ equal to zero. Clearly the intuition is that the government in the South would enforce IPR stronger if its planning horizon accounted for the entire future welfare associated with innovations.\textsuperscript{39} Hence, if the research capacity of the North is large, accepting the North’s desired level of IPR-enforcement in international negotiation rounds such as ACTA, would foster long-term welfare in the South. However, the opposite is true when the research capacity is low, such that $\Delta > \Delta_0_f$.

\section{Summary and Conclusions}

Even though most countries have agreed to harmonize intellectual property rights by signing TRIPs, there is much dispute about the \textit{enforcement} of IPR in the world. This paper examines IPR-enforcement in an endogenous growth framework with two open economies. We incorporate three assumptions that distinguish our paper from the previous literature and add realistic features to the model. These are that in each economy all active patents are enforced at the same (endogenously chosen) strength, the governments cannot fully commit to IPR-enforcement for the indefinite future and have limited planning horizons, e.g. due to re-election concerns.

While the governments in the decentralized game provide too little IPR-enforcement relative to the constrained efficient solution that maximizes the governments’ aggregate welfare under the previous assumptions, both regions, the North and the South, desire higher IPR-protection relative to the equilibrium enforcement levels if they were able to

\textsuperscript{38}Note that $\omega_f$ must satisfy the first-order condition

$$
\frac{2\delta}{\bar{E}}(1 + \frac{\bar{E}}{2\Delta})(D - P) + \sum_{\tau=t+1}^{\infty} \beta^{\tau-t}(1 + g)^{\tau-t-1}\lambda_s [Y + \omega_f(D - P)] = 0.
$$

The first-order condition concerning the (one-shot) full-commitment problem differs from the one with limited commitment (18) with respect to the second summand which represents the discounted benefit of a change in IPR-enforcement for all future periods. It follows that the South would prefer a higher harmonized enforcement level when full commitment were available – i.e., $W_s(\omega_f) > W_s(\omega_h)$.

\textsuperscript{39}Note that $\omega_h = 0$ for $\Delta > \Delta_0_h$. 

28
select a harmonized world enforcement level. Typically, the North’s desired harmonized enforcement level is larger than the constrained efficient one while that of the South is lower. The difference between the North’s and the South’s desired harmonized enforcement levels increases with the relative market size of the South, thus amplifying the clash of interests in international negotiations. Moreover, we find that the smaller a region’s relative market size, the larger is the difference between its equilibrium choice and the ideal harmonized enforcement desired on the international level.

Concerning the discussion whether the South suffers welfare losses from adopting the desired IPR-enforcement levels of the North, our numerical welfare example suggests that as long as the North’s research capacity is not too low, the South may well benefit in terms of overall long-term welfare. However, when the research capacity is low, the dynamic gains realized would not justify the large profit outflows even from a long-term welfare perspective.

It is frequently assumed in the political economy literature as well as in parts of the dynamic macroeconomic literature that governments act in a short-sighted way. Our paper highlights that such an assumption can change the above results in counterintuitive ways for very high levels of the North’s research capabilities. In particular the North’s short-sighted government’s IPR-enforcement level in equilibrium and also the desired harmonized level may decline with its research productivity. As a consequence, the short-sighted government in the South may choose a higher equilibrium and desired harmonized enforcement level than the North.

Our paper opens up several avenues for future research. It would be interesting to extend the enforcement game to one where both regions are active in research and to consider more than two countries. Further, the framework developed can be used to study several important aspects of IPR-protection such as blocking patents, differences in preferences between the countries or principal-agent problems in R&D joint ventures and their consequences for long-run development.
A Appendix

A.1 Proof of Lemma 1

(i) Solving \( R_n(\omega_n, \omega_s) = 0 \) for \( \omega_n \) yields

\[
\omega_{n1}(\omega_s) = \frac{1}{\beta^2 P^2 \lambda_n} \left[ G(\omega_s) + 2\sqrt{\Delta \lambda H(\omega_s)} \right]
\]

\[
\omega_{n2}(\omega_s) = \frac{1}{\beta^2 P^2 \lambda_n} \left[ G(\omega_s) - 2\sqrt{\Delta \lambda H(\omega_s)} \right]
\]

where

\[
G(\omega_s) = -\beta^2 \lambda_s P^2 \omega_s - 2(1 - \beta) \Delta \lambda (-2 - \beta) D + P < 0,
\]

\[
H(\omega_s) = \Delta \lambda [(2 - 3\beta + \beta^2)D^2 + (1 - \beta)^2 (P^2 - 4(1 - \beta)DP)]
\]

\[
+ (1 - \beta)^2 \beta^2 P^2 (\lambda_n Y - \lambda_s (D - P)\omega_s) > 0.
\]

The signs of \( G(\omega_s) \) and \( H(\omega_s) \) imply that \( \omega_{n2}(\omega_s) \) is negative for all values \( \omega_s \geq 0 \). In contrast, \( \omega_{n1}(\omega_s) \) can be positive. Hence the latter is the only economically sensible solution and we define \( \omega^*_n(\omega_s) \equiv \omega_{n1}(\omega_s) \).

(ii) Taking the second derivative of \( \omega^*_n(\omega_s) \) with respect to \( \omega_s \) gives

\[
\frac{d^2 \omega^*_n(\omega_s)}{d \omega_s^2} = \frac{(1 - \beta)^4 \beta^2 \Delta^2 \lambda^2 \lambda_s^2 (D - P)^2 P^2}{2 \lambda_n [H(\omega_s)]^{\frac{1}{2}}} < 0, \ \forall \omega_s \geq 0.
\]

Note that \( H(\omega_s) > 0 \) for all \( \omega_s \geq 0 \). Thus \( \omega^*_n(\omega_s) \) is strictly concave.

To show that \( \omega^*_n(\omega_s) \) is strictly decreasing on \([0, 1]\), we use the implicit-function theorem. The partial derivative of \( R_n(\omega_n, \omega_s) \) with respect to \( \omega_n \) reads

\[
\frac{\partial R_n(\omega_n, \omega_s)}{\partial \omega_n} = \frac{1}{2 \partial \omega_n} \frac{\partial \tilde{E}}{\partial \omega_n} (D - P) + \frac{\beta P}{2} \left( \frac{D - \tilde{E}}{2 \Delta} \frac{\partial \tilde{E}}{\partial \omega_n} \right) < 0.
\]

As the monopoly distortion \( D \) is negative, the derivative is smaller than zero for all \( (\omega_n, \omega_s) \in \mathbb{R}_+^2 \). The derivative of \( R_n(\omega_n, \omega_s) \) with respect to \( \omega_s \) can be written as

\[
\frac{\partial R_n(\omega_n, \omega_s)}{\partial \omega_s} = -\frac{\beta^2}{1 - \beta} \frac{P^2 \lambda_s}{2 \lambda} + \frac{1}{2 \partial \omega_s} \frac{\partial \tilde{E}}{\partial \omega_s} \left( D - \frac{\beta P \tilde{E}}{2 \Delta} \right) < 0.
\]

The implicit-function theorem then implies

\[
\frac{d \omega_n}{d \omega_s} = -\frac{\partial R_n(\omega_n, \omega_s)}{\partial \omega_n} \frac{\partial R_n(\omega_n, \omega_s)}{\partial \omega_s} < 0.
\]
A.2 Proof of Proposition 2

The intuition of the proof can be summarized as follows. First, we show that there is a unique economically sensible intersection of the reaction functions of the North and the South. Then we verify that the reaction function of the North intersects the one of the South from below. This implies that there exists a stable “Cobb-web” mechanism towards the intersection of the reaction functions. This mechanism leads to a unique equilibrium which is the intersection of the reaction function itself if the intersection is in the feasible set. Otherwise it determines a unique equilibrium on the boundary of the feasible set.

(1)

We show that there is a unique economically sensible intersection of the reaction functions of the North and the South.

Let us define \( \omega_s \) as the solution to \( H(\omega_s) = 0 \), where \( H(\omega_s) \) is given in the proof of Lemma 1. Since \( H(\omega_s) > 0, \forall \omega_s \geq 0 \), we obtain \( \omega_s < 0 \) and that \( \omega^*_n(\omega_s) \) is a real number for all \( \omega_s > \omega_s \). Further, \( \omega^*_n(\omega_s) \) is strictly concave on \((\omega_s, \infty)\) according to the proof of Lemma 1. Inserting \( \omega^*_s(\omega_n) \) given by (10) into \( R_n(\omega_n, \omega_s) \) yields \( R^*_n(\omega_n) \). Solving \( R^*_n(\omega_n) = 0 \) for \( \omega_n \) gives

\[
\omega^*_n = \frac{1}{\beta^2 P^2 \lambda_n} \left[ Q_1(\Delta) + 2\sqrt{(2 - \beta)^2 \lambda^2 Q_2(\Delta)} \right],
\]

(\text{A.2})

\[
\omega^*_{n2} = \frac{1}{\beta^2 P^2 \lambda_n} \left[ Q_1(\Delta) - 2\sqrt{(2 - \beta)^2 \lambda^2 Q_2(\Delta)} \right],
\]

where

\[
Q_1(\Delta) = \frac{\beta^2 \lambda_n Y P^2}{D - P} + 2\Delta \lambda((3 - \beta)(2 - \beta)D - (4 - 3\beta)P) < 0,
\]

\[
Q_2(\Delta) = (3 - \beta)^2 D^2 - 4(3 - 2\beta)D P + P^2 (2(2 - \beta)\Delta + \beta^2 Y) > 0.
\]

Since \( Q_2(\Delta) > 0 \), \( R^*_n(\omega_n) \) possesses two real roots – i.e., the reaction functions \( \omega^*_n(\omega_s) \) and \( \omega^*_s(\omega_n) \) possess two intersections on the real plane. As \( Q_1(\Delta) < 0, \omega^*_{n2} \) is strictly negative for all relevant parameter values and only \( \omega^*_{n1} \) possesses economical relevance. Hence, we have \( \omega^*_N = \omega^*_n \).

Given a unique \( \omega^*_n \), we can immediately infer from (10) that \( \omega^*_s = \omega^*_s(\omega^*_n) \) is also unique.

(2)

Now, we show that the reaction of the North intersects the one of the South from below.
We define \( \bar{\omega}_s \equiv \omega_s^r(\omega_{n2}) \) and the inverse of the South’s reaction function
\[
\omega_n^r(\omega_s) = (1 - \beta) \left[ \frac{Y}{D - P \lambda_n} + \frac{2 \Delta \lambda}{\beta P \lambda_n} \right] - (2 - \beta) \frac{\lambda_s}{\lambda_n} \omega_s.
\] (A.3)

Part (1) of the proof together with strict concavity of \( \omega_n^r(\omega_s) \) on \((\omega_s, \infty)\) and \( \omega_n^s(\omega_s) \) being a strictly decreasing linear function yields the following lemma.

**Lemma 3**

On the interval \((\omega_s, \bar{\omega}_s)\), \( \omega_n^r(\omega_s) \) intersects \( \omega_n^s(\omega_s) \) from below.

\[
\omega_n^r(\omega_s) < \omega_n^s(\omega_s), \quad \text{if} \quad \omega_s < \omega_s < \omega_s^x
\]
\[
\omega_n^r(\omega_s) > \omega_n^s(\omega_s), \quad \text{if} \quad \omega_s^x < \omega_s < \bar{\omega}_s.
\]

(3)

We have to show that

(i) if \( \omega_s^x \leq 0 \) and \( \omega_s^x \leq 0 \), the unique equilibrium is \((\omega_n, \omega_s) = (\hat{\omega}_n^r(0), 0)\).

(ii) if \( \omega_n^x \leq 0 \) and \( \omega_n^x \in (0, 1) \), the unique equilibrium is \((\omega_n, \omega_s) = (0, \hat{\omega}_s^r(0))\).

(iii) if \( \omega_n^x \leq 0 \) and \( \omega_n^x \geq 1 \), the unique equilibrium is \((\omega_n, \omega_s) = (\hat{\omega}_n^r(1), \hat{\omega}_n^r(0))\).

(iv) if \( \omega_n^x \in (0, 1) \) and \( \omega_n^x \leq 0 \), the unique equilibrium is \((\omega_n, \omega_s) = (\omega_n^r(0), 0)\).

(v) if \( \omega_n^x \in (0, 1) \) and \( \omega_n^x \in (0, 1) \), the unique equilibrium is \((\omega_n, \omega_s) = (\omega_n^r, \omega_n^x)\).

(vi) if \( \omega_n^x \in (0, 1) \) and \( \omega_n^x \geq 1 \), the unique equilibrium is \((\omega_n, \omega_s) = (\hat{\omega}_n^r(1), 1)\).

(vii) if \( \omega_n^x \geq 1 \) and \( \omega_n^x \leq 0 \), the unique equilibrium is \((\omega_n, \omega_s) = (\hat{\omega}_n^r(0), \hat{\omega}_s^r(1))\).

(viii) if \( \omega_n^x \geq 1 \) and \( \omega_n^x \in (0, 1) \), the unique equilibrium is \((\omega_n, \omega_s) = (1, \hat{\omega}_s^r(1))\).

(ix) if \( \omega_n^x \geq 1 \) and \( \omega_n^x \geq 1 \), the unique equilibrium is \((\omega_n, \omega_s) = (1, 1)\).

The existence of the equilibrium is established as follows.

(i) Suppose that \( \omega_n^x \leq 0 \) and \( \omega_n^x \leq 0 \). Given \( \omega_s = 0 \), the best response of North is \( \hat{\omega}_n^r(0) \).

Given \( \omega_n = \hat{\omega}_n^r(0) \), we obtain \( \omega_s^r(\omega_n^r(0)) \leq 0 \) by using Lemma 3 and the fact that \( \omega_n^r(\omega_n) \) is strictly declining. Consequently, the South’s best response to \( \omega_n = \hat{\omega}_n^r(0) \) is \( \omega_s = 0 \).

\(^{40}\)Note that this is possible as \( \omega_s(\omega_n) \) is a bijection.
Suppose that $\omega^r_n \leq 0$ and $\omega^s_n \in (0, 1)$. Given $\omega_s = \hat{\omega}^r_n(0)$, then best response of North is $0$ because $\omega^r_n(\hat{\omega}^r_n(0)) \leq 0$ due to $\omega^r_s(\omega_n)$ being a strictly declining function and Lemma 3. Given $\omega_n = 0$, the South’s best response is $\hat{\omega}^s_n(0)$.

Suppose that $\omega^r_n \leq 0$ and $\omega^s_n \geq 1$. We distinguish the cases where $\omega^r_n(1) \leq 0$ and $\omega^r_n(1) > 0$. If $\omega^r_n(1) \leq 0$, the equilibrium can be written as $(\omega_n, \omega_s) = (0, \hat{\omega}^r_n(0))$. $\omega^r_n(1) \leq 0$ together with Lemma 3 and $\omega^r_s(\omega_n)$ strictly declining implies that $\omega^r_n(\hat{\omega}^r_n(0)) \leq 0$. Hence, the best response of the North is $\omega_n = 0$. Further, given $\omega_n = 0$, $\hat{\omega}^r_n(0)$ is the best response of the South.

If $\omega^r_n(1) > 0$, the equilibrium can be written as $(\omega_n, \omega_s) = (\hat{\omega}^r_n(1), 1)$. Given $\omega_s = 1$, $\hat{\omega}^r_n(1)$ is best response of North. $\omega^r_n(1) > 0$ together with Lemma 3 and $\omega^r_s(\omega_n)$ strictly declining imply $\omega^r_s(\omega^r_n(1)) \geq 1$. Consequently, the South’s best response is $\omega_s = 1$.

Suppose that $\omega^r_n \in (0, 1)$ and $\omega^s_n \leq 0$. Given $\omega_s = 0$, the best response of North is $\hat{\omega}^r_n(0)$. Given $\omega_n = \hat{\omega}^r_n(0)$, $\omega^r_s(\omega^r_n(0)) \leq 0$ follows from Lemma 3 and $\omega^r_s(\omega_n)$ being a strictly declining function. Hence, the South’s best response is $\omega_s = 0$.

Let $\omega^r_n \in (0, 1)$ and $\omega^s_n \in (0, 1)$. Then $(\omega_n, \omega_s) = (\omega^r_n, \omega^s_n)$ is an equilibrium by the definition of the reaction functions.

Let $\omega^r_n \in (0, 1)$ and $\omega^s_n \geq 1$. Given $\omega_s = 1$, the best response of North is $\hat{\omega}^r_n(1)$. Given $\omega_n = \hat{\omega}^r_n(1)$, the South’s best response is $\omega_s = 1$ as $\omega^r_s(\omega^r_n(1)) \geq 1$ due to Lemma 3 and $\omega^r_s(\omega_n)$ being a strictly declining function.

Suppose that $\omega^r_n \geq 1$ and $\omega^s_n \leq 0$. We distinguish the cases where $\omega^r_n(0) \leq 1$ and $\omega^r_n(0) > 1$. If $\omega^r_n(0) \leq 1$, the equilibrium can be written as $(\omega_n, \omega_s) = (\hat{\omega}^r_n(0), 0)$. Given $\omega_s = 0$, $\hat{\omega}^r_n(0)$ is best response of the North. Due to Lemma 3 and $\omega^r_s(\omega_n)$ being a strictly declining function, $\omega^r_s(\hat{\omega}^r_n(0)) \leq 0$. Consequently, South’s best response is $\omega_s = 0$.

If $\omega^r_n(0) > 1$, the equilibrium can be written as $(\omega_n, \omega_s) = (1, \hat{\omega}^r_n(1))$. $\omega^r_n(0) > 1$ together with Lemma 3 and $\omega^r_s(\omega_n)$ being a strictly declining function implies that $\omega^r_s(\hat{\omega}^r_n(1)) \geq 1$. Hence, the best response of the North is $\omega_n = 1$. Further, given $\omega_n = 1$, $\hat{\omega}^r_n(1)$ is the best response of the South.

Suppose that $\omega^r_n \geq 1$ and $\omega^s_n \in (0, 1)$. Given $\omega_s = \hat{\omega}^r_n(1)$, Lemma 3 and $\omega^r_s(\omega_n)$ strictly declining imply that $\omega^r_s(\hat{\omega}^r_n(1)) \geq 1$. Consequently, the North’s best response is $\omega_n = 1$. Given $\omega_n = 1$, the South’s best response is $\omega_s = \hat{\omega}^r_n(1)$.
Let \( \omega_n^x \geq 1 \) and \( \omega_s^x \geq 1 \). Since both functions, \( \omega_n^r(\omega_s) \) and \( \omega_s^r(\omega_s) \) are declining on \( \mathbb{R}_+ \), \( \omega_n^x \geq 1 \) and \( \omega_s^x \geq 1 \) implies that \( \omega_n^r(\omega_s) \), \( \omega_s^r(\omega_s) \) \( \geq 1 \) for all \( \omega_s \in [0, 1] \) and \( \omega_n \in [0, 1] \). Consequently, given \( \omega_s = 1 \), \( \omega_n^r(1) \geq 1 \) leading to \( \omega_n = 1 \) as the best response of the North. Given \( \omega_n = 1 \), the best response of the South is \( \omega_s = 1 \) as \( \omega_s^r \geq 1 \).

Concerning uniqueness, Lemma 3 and the fact that \( \omega_n^r(\omega_s) \) and \( \omega_s^r(\omega_s) \) are strictly declining functions imply that \( \forall \omega_s \in [0, 1] \) and \( \omega_s \neq \omega_s^e \), we have \( \omega_n^r(\omega_s) \neq \omega_s^e \). Further \( \forall \omega_n \in [0, 1] \) and \( \omega_n \neq \omega_n^e \), we obtain \( \omega_n^r(\omega_n) \neq \omega_n^e \). As a consequence, the equilibrium \( (\omega_n, \omega_s) = (\bar{\omega}_n^x, \omega_s^e) \) as given in Proposition 2 is unique.

### A.3 Proof of Lemma 2

From the proof of Proposition 2, we know that

\[
\omega_n^x = \frac{1}{\beta^2 P^2 \lambda_n} \left[ Q_1(\Delta) + 2 \sqrt{(2 - \beta)^2 \Delta \lambda^2 Q_2(\Delta)} \right],
\]

where

\[
Q_1(\Delta) = \frac{\beta^2 \lambda_s Y P^2}{D - P} + 2 \Delta \lambda (3 - \beta)(2 - \beta)D - (4 - 3\beta)P < 0,
\]

\[
Q_2(\Delta) = (3 - \beta)^2 \Delta D^2 - 4(3 - 2\beta)\Delta DP + P^2(2(2 - \beta)\Delta + \beta^2 Y) > 0.
\]

The second derivative of \( \omega_n^x \) with respect to \( \Delta \) reads

\[
\frac{d^2 \omega_n^x}{d \Delta^2} = -\frac{(2 - \beta)\beta^2 P^2 \lambda Y}{2\lambda_n Q_2(\Delta)^{\frac{3}{2}}} < 0.
\]

Concerning the convexity of the South’s IPR-level in \( \Delta \), we use equation (10) and take the second derivative with respect to \( \Delta \) to obtain

\[
\frac{d^2 \omega_s^x}{d \Delta^2} = -\frac{1}{2 - \beta} \frac{\lambda_n d^2 \omega_n^x}{d \Delta^2} < 0. \tag{A.4}
\]

Since \( \frac{d^2 \omega_s^x}{d \Delta^2} < 0 \), \( \frac{d^2 \omega_s^x}{d \Delta^2} \) must be positive and hence, \( \omega_s^x \) is strictly convex in \( \Delta \).

### A.4 Proof of Proposition 3

To verify the three items of Proposition 3, it is necessary to show that \( \omega_s^x \) is strictly convex and declining with \( \Delta \), while \( \omega_n^x \) is strictly concave and exhibits an inverted U-shaped relation with \( \Delta \). Then determining the roots of \( \omega_s^x \) and \( \omega_n^x \) in \( \Delta \) identifies \( \Delta_s^0 \) and \( \Delta_n^0 \). Comparing \( \Delta_s^0 \) and \( \Delta_n^0 \) yields condition (14).
Item (i) of Proposition 3 follows from the properties of $\omega^r_s$ mentioned above. For Item (ii) it is necessary to additionally show that $\omega^r_n(0)$ (i.e., the North’s IPR-enforcement level in autarky) is strictly concave in $\Delta$ and intersects with $\omega^r_n$ from above at $\Delta^0(\equiv \Delta^0_n)$. Since $\omega^r_n$ is identical to $\omega^r_s$ for all $\Delta < \Delta^0$ and identical to $\omega^r_n(0)$ for all $\Delta \geq \Delta^0$, this implies that $\omega^r_n$ is strictly concave and shows an inverted U-shape over the entire relevant interval, but is – of course – not differentiable at $\Delta^0$. Item (iii) follows from the properties of $\omega^r_o$ and $\omega^r_s$ given that condition (14) is satisfied.

The proof is organized as follows. First, we derive $\omega^r_s(0)$ and $\omega^r_n(0)$ as well as some notation and lemmata that will be used throughout the proof. Then, we show the existence of $\Delta^0_n$ and that condition (14) is necessary and sufficient for $\Delta^0_o < \Delta^0_n$. In the remainder of the proof, we verify items (i)-(iii) of the proposition.

![Figure 5: Illustration of the proof of Proposition 3](image)

From the South’s reaction function (10), we obtain the values of IPR-protection in the South given that $\omega_n = 0$ as

$$\omega_s^r(0) = -\left(\frac{1 - \beta}{2 - \beta}\right) \left[\frac{Y}{D - P} + \frac{2\Delta}{\beta P \lambda_s}\right]. \tag{A.5}$$

$\omega_s^r(0)$ is zero at the value

$$\Delta^s_m = -\frac{\beta \lambda_s Y P}{2\lambda(D - P)}. \tag{A.6}$$

Now, consider the level of IPR-protection of the North such that the South would just choose a zero level of protection. This corresponds to the inverse of $\omega_s^r(\omega_n)$ at the point $\omega_s = 0$ – i.e.,

$$\omega_n^s(0) = -\left(1 - \beta\right) \left[\frac{\lambda_s}{\lambda_n} \frac{Y}{D - P} + \frac{2\Delta}{\beta P \lambda_n}\right]. \tag{A.7}$$
\( \omega^s(0) \) defines a line in the \( \omega - \Delta \) coordinate plane that intersects with \( \omega^r(0) \) at \( \Delta^s \).

Let us now consider \( R^e_n(\omega_n) \), which is derived by inserting \( \omega^r(\omega_n) \) as given by (10) into the first-order condition of the North (13). From the first part of the proof of Proposition 2, we know that \( R^e_n(\omega_n) \) possesses two real roots, \( \omega^x_n_1 \) and \( \omega^x_n_2 \). The economically sensible one is the larger root \( \omega^x_n_1 \) implying \( \omega^x_n \equiv \omega^x_n_1 \). By showing that \( R^e_n(\omega_n) \) is strictly concave, we establish

**Lemma 4**

(i) \( \forall \omega_n > \omega^x_n_2, R^e_n(\omega_n) > (\omega^x_n, \omega_n) \), \( \omega_n < (\omega^x_n, \omega_n) \).

(ii) \( \frac{dR^e_n(\omega_n)}{d\omega_n} \bigg|_{\omega_n = \omega^x_n} < 0 \).

**Proof.** \( R^e_n(\omega_n) \) can be written as

\[
R^e_n(\omega_n) = \frac{D - P}{2} \frac{\beta}{2 - \beta}P \left( \frac{\lambda_n}{\lambda} \omega_n - A_1 \right) + \Delta D + \frac{\beta P}{2} \left[ \frac{\lambda_n}{\lambda}(Y + \omega_n D) - P \frac{1 - \beta}{2 - \beta}A_1 - P \frac{\lambda_n}{\lambda} \frac{\omega_n}{2 - \beta} - \left( \frac{\beta}{2 - \beta} \right)^2 \frac{P^2}{4\Delta} \left( \frac{\lambda_n}{\lambda} \omega_n - A_1 \right)^2 \right].
\]

where \( A_1 = \frac{Y}{D - P} \frac{\lambda_n}{\lambda} + \frac{2\Delta}{\beta P} \). Taking the second derivative with respect to \( \omega_n \), we obtain

\[
\frac{\partial^2 R^e_n(\omega_n)}{\partial \omega_n^2} = -\left( \frac{\beta}{2 - \beta} \right)^2 \frac{P^2}{2\Delta} \left( \frac{\lambda_n}{\lambda} \right)^2 < 0.
\]

This verifies Lemma 4. \( \square \)

The level of IPR-protection of the North when the South chooses \( \omega_s = 0 \) is given by \( R_n(\omega_n, \omega_s = 0) = 0 \). Since the second derivative of \( R(\omega) \) with respect to \( \omega \) reads

\[
\frac{d^2 R_n(\omega_n, 0)}{d\omega_n^2} = -\left( \frac{\beta}{1 - \beta} \right)^2 \frac{P^2}{4\Delta} < 0,
\]

\( R_n(\omega_n, 0) \) is strictly concave in \( \omega_n \). It also possesses two roots

\[
\omega^a_n_1 = \frac{2(1 - \beta)}{\beta^2 P^2} \left[ ((2 - \beta)D - P) \Delta \frac{\lambda}{\lambda_n} + \sqrt{\Delta \frac{\lambda}{\lambda_n} X(\Delta \frac{\lambda}{\lambda_n})} \right],
\]

\[
\omega^a_n_2 = \frac{2(1 - \beta)}{\beta^2 P^2} \left[ ((2 - \beta)D - P) \Delta \frac{\lambda}{\lambda_n} - \sqrt{\Delta \frac{\lambda}{\lambda_n} X(\Delta \frac{\lambda}{\lambda_n})} \right],
\]

where \( X(\Delta \frac{\lambda}{\lambda_n}) = 4\Delta \frac{\lambda}{\lambda_n} (1 - \beta) D (D - P) + P^2 \Delta \frac{\lambda}{\lambda_n} + \beta^2 (YP + D^2 \Delta \frac{\lambda}{\lambda_n}) > 0 \). Only \( \omega^a_n_1 \) is economically sensible. Hence we define the level of IPR-protection of the North when the South provides no IPR-protection by \( \omega^a_n \equiv \omega^a_n_1 \). Using the same line of argument as with regard to Lemma 4, we are now able to formulate:
Lemma 5

(i) \( \forall \omega > \omega_n^a, R_n(\omega_n, 0) > (>) 0 \Leftrightarrow \omega_n < (>) \omega_n^a \).

(ii) \( \frac{dR_n(\omega_n, 0)}{d\omega_n} \bigg|_{\omega_n=\omega_n^a} < 0 \).

Further, we show

Lemma 6

\( \omega_n^a > 0 \) and \( \omega_n^x > 0 \) at \( \Delta_m^s \) if and only if \( \frac{\lambda_n}{\lambda_s} > \frac{D}{D-P} \).

Proof. The condition that \( \omega_n^x > 0 \) and \( \omega_n^a > 0 \) at \( \Delta_m^s \) is equivalent to \( R_n^e(0) > 0 \) and \( R_n(0,0) > 0 \) at \( \Delta_m^s \) according to Lemmata 4 and 5. Inserting \( \Delta_m^s \) given in equation (A.6) into \( R_n^e(0) > 0 \) and \( R_n(0,0) > 0 \) yields

\[ R_n^e(0) > 0 \Leftrightarrow R_n(0,0) > 0 \Leftrightarrow -\frac{\beta Y P (\lambda_s D - \lambda_n (D - P))}{2 \lambda (D - P)} > 0 \]
\[ \Leftrightarrow \frac{\lambda_n}{\lambda_s} > \frac{D}{D - P}. \]

Let us now establish

Lemma 7

\( \omega_n^a \) possesses a unique maximum at \( \Delta_c \).

Proof. First we obtain from (A.2) that

\[ \lim_{\Delta \to 0} \omega_n^x = \frac{\lambda_s Y}{\lambda_n D - P} < 0 \quad \text{(A.9a)} \]
\[ \lim_{\Delta \to 0} \frac{\partial \omega_n^x}{\partial \Delta} = +\infty. \quad \text{(A.9b)} \]

Using the implicit function theorem, the sign of

\[ \frac{d\omega_n^x}{d\Delta} = -\frac{\partial R_n^e(\omega_n)}{\partial \Delta} \bigg|_{\omega_n=\omega_n^x}, \Delta > 0, \]

is identical to that of \( \frac{\partial R_n^e(\omega_n)}{\partial \Delta} \) because \( \frac{dR_n^e(\omega_n)}{d\omega_n} < 0 \) at \( \omega_n = \omega_n^x \) due to Lemma 4 (ii).

\[ \frac{\partial R_n^e(\omega_n)}{\partial \Delta} = -\frac{\beta P B}{2(2-\beta)^2} - \sqrt{\beta P} \frac{\beta P (\lambda_s Y - \lambda_n \omega_n^0(D - P))}{8(2-\beta)^2 \Delta \lambda (D - P)}. \quad \text{(A.10)} \]

According to (A.10), \( \frac{\partial R_n^e(\omega_n)}{\partial \Delta} \) is strictly increasing with \( \omega_n \). Hence there exists a \( \omega_n, \text{crit} \), defined by \( \frac{\partial R_n^e(\omega_n)}{\partial \Delta} = 0 \), for which \( \frac{\partial R_n^e(\omega_n)}{\partial \Delta} \) is (>) 0 if and only if \( \omega_n > (>) \omega_n, \text{crit} \). \( \omega_n, \text{crit} \) can be written as

\[ \omega_n, \text{crit} = \frac{\lambda_s Y}{\lambda_n D - P} + \frac{\lambda}{\lambda_n \beta P} \frac{2 \Delta}{\beta PB}. \]
This reveals that $\omega_{n,crit}$ is increasing linearly with $\Delta$ and that

$$\lim_{\Delta \to 0} \omega_{n,crit} = \lim_{\Delta \to 0} \omega_n^x = \frac{\lambda_s}{\lambda_n} \frac{Y}{D - P} < 0.$$ 

Considering (A.9b) and the strict concavity of $\omega_n^x$, we can directly infer that there will be a unique intersection of $\omega_n^x$ and $\omega_{n,crit}$ at a $\Delta > 0$ which we call $\Delta_c$. Due to the definition of $\omega_{n,crit}$, this intersection is at the maximum of $\omega_n^x$ in $\Delta$. □

Using the same line of argument as in Lemma 7, it can be shown that

**Lemma 8**

$\omega_n^a$ is strictly concave in $\Delta$ and there exists a $\Delta_{crit} > 0$ where it possesses a unique maximum.

The proof will be provided upon request. Now we are able to show the existence of $\Delta_n^0$.

**Lemma 9**

There exists a $\Delta_n^0$ such that $\omega_n^e = 0$ for all $\Delta \geq \Delta_n^0$.

To establish the existence of $\Delta_n^0$, we have to distinguish between the cases where $\Delta_n^0 > \Delta_s^0$ and $\Delta_n^0 \leq \Delta_s^0$. In the first case, existence of a $\Delta_n^0 > \Delta_s^0$ requires that $\omega_n^a \leq 0$ for all $\Delta$ larger than a certain threshold value. Consider first the IPR-level in autarky $\omega_n^a$. Due to the strong concavity of $\omega_n^a$ and since $\frac{d\omega_n^a}{d\Delta} < 0$ for all $\Delta > \Delta_{crit}$ (see Lemma 8, there exists a threshold of $\Delta$ where $\omega_n^a \leq 0$ for all $\Delta$ larger than this threshold.

A threshold level $\Delta_n^0 \leq \Delta_s^0$ requires that $\omega_n^x \leq 0$ for all $\Delta$ larger than a certain threshold value. Such a threshold value of $\Delta$ exists since $\omega_n^x$ is strictly concave in $\Delta$ and $\frac{\partial \omega_n^x}{\partial \Delta} < 0$ for all $\Delta < \Delta_c$ according to Lemma 7.

Finally we establish

**Lemma 10**

$\Delta_n^0 > \Delta_s^0$ if and only if $\frac{\lambda_n}{\lambda_s} > \frac{D}{D - P}$.

A necessary and sufficient condition for $\Delta_n^0 > \Delta_s^0$ is that $\omega_n^a > 0$ at $\Delta_n^0$. This condition is sufficient as $\Delta_s^0$ is smaller than or equal to $\Delta_m^s$. The condition is necessary because if $\omega_n^a < 0$ at $\Delta_m^s$ then $\omega_n^a < 0$ for all $\Delta \geq \Delta_m^s$. Further we know from Lemma 6, that $\omega_n^a < 0$ at $\Delta_m^s$ if and only if $\omega_n^a < 0$ at $\Delta_n^0$. This implies that there exists a $\Delta < \Delta_m^s$ for

\[\Delta_n^0 \leq \Delta_m^s\] follows from $\omega_m^a \leq \omega_m^a(0)$ because $\omega_m^s$ is a strategic substitute to $\omega_n$ and $\omega_n \geq 0$ in equilibrium.
which \( \omega_s^x > 0 \) and that \( \Delta^0 = \Delta_m^x \). Consequently, \( \omega_n^a > 0 \) at \( \Delta_m^x \) is necessary for \( \Delta_n^0 > \Delta^0 \). According to Lemma 6, \( \omega_n^a > 0 \) at \( \Delta_m^x \) if and only if \( \frac{\lambda_n}{\lambda_s} > \frac{D_{n-p}}{P} \).

Next we verify items (i)-(iii) of Proposition 3

(i): \( \omega_s^x \) is strictly declining for the following reason. Since \( \frac{\lambda_n}{\lambda_s} > \frac{D_{n-p}}{P} \) holds by assumption, \( \omega_n^a > 0 \) at the point \( \Delta_m^x \). This implies that \( \omega_s^x = \omega_s^x(\omega_n^a) \) is negative at \( \Delta_m^x \). Further, \( \omega_s^x \) must be smaller than or equal to \( \omega_s^x(0) \) if \( \omega_s^x \geq 0 \). According to Lemma 2, \( \omega_s^x \) is strictly convex. It follows from (A.9a) that \( \omega_s^x > 0 \) for some \( \Delta < \Delta_m^x \). This, together with \( \omega_s^x < 0 \) at \( \Delta_m^x \) implies that \( \omega_s^x \) possesses a unique root \( \Delta^0 \) in the relevant interval \([0, \Delta_m^x]\) and is strictly decreasing for all \( \Delta < \Delta^0 \). Since

\[
\omega_s^x = \begin{cases} 
\omega_s^x(0), & \text{if } \omega_n^a \leq 0, \\
\omega_s^x, & \text{if } \omega_n^a \in (0, 1), \\
\omega_s^0(1), & \text{if } \omega_n^a \geq 1,
\end{cases}
\]

\( \omega_s^x \) is (weakly) decreasing and positive for all \( \Delta < \Delta^0 \).

To verify that the equilibrium enforcement of the South takes the corner solution \( \omega_s^x = 0 \) for all \( \Delta \) larger then \( \Delta^0 \), we have to consider \( \omega_n^a \) which indicates the North’s best response to \( \omega_s = 0 \). Only if \( \omega_n^a > \omega_s^x(0) \) for all \( \Delta \in (\Delta^0, \Delta_m^x) \), will the South choose \( \omega_s^x = 0 \) for all \( \Delta \geq \Delta^0 \). Since \( \omega_n^a = \omega_s^x = \omega_s^x(0) \) at \( \Delta^0 \), \( \omega_n^a \) is strictly concave in \( \Delta \), and \( \omega_n^a > 0 \) at \( \Delta_m^x \), \( \omega_n^a \) does not intersect \( \omega_s^x(0) \) in the interval \( \Delta \in (\Delta^0, \Delta_m^x) \).

(ii): As \( \omega_s^x = 0 \) for all \( \Delta > \Delta^0 \), the North acts as if in autarky because in this case there are no profit inflows from the South and, hence, neither are there additional incentives to conduct R&D in the North. Accordingly the government in the North acts as if \( \lambda_s = 0 \) and \( \lambda_n = \lambda \).

The inverted U-shape of \( \omega_s^x \) follows from the following line of argument. According to the previously established lemmata, both \( \omega_n^x \) and \( \omega_n^a \) are strictly concave and follow an inverted U-shape in \( \Delta \). \( \omega_n^a \) combines \( \omega_n^x \) for \( \Delta < \Delta^0 \) and \( \omega_n^a \) for \( \Delta > \Delta^0 \). \( \omega_n^x \) and \( \omega_n^a \) intersect at \( \Delta^0 \) and \( \omega_n^x > \omega_n^a \) for \( \Delta > \Delta^0 \) while \( \omega_n^x < \omega_n^a \) for \( \Delta < \Delta^0 \). The strict concavity of both \( \omega_n^x \) and \( \omega_n^a \) then implies that at the intersection \( \frac{\partial \omega_n^x}{\partial \Delta} < \frac{\partial \omega_n^a}{\partial \Delta} \). Now it follows directly that \( \omega_n^x \) is concave and exhibits an inverted U-shaped form on the interval of \( \Delta \) where \( \omega_n^x > 0 \).

(iii): According to Lemma 2, \( \omega_n^x \) is strictly concave in \( \Delta \). Further, we have \( \lim_{\Delta \to 0} \frac{\partial \omega_n^x}{\partial \omega_n^a} < 0 \) according to (A.9a). From \( \omega_n^x > 0 \) at \( \Delta_m^s \) (Lemma 6), we infer that \( \omega_n^x > 0 \) at \( \Delta^0 \). \( \omega_s^x \) is strictly convex and strictly declining on \([0, \Delta^0]\). Further, \( \omega_s^x = 0 \) at \( \Delta^0 \). As a consequence, there is exactly one intersection \( \omega_n^x \) and \( \omega_s^x \) on \([0, \Delta^0]\). Denoting the value of \( \Delta \) at the
intersection by $\Delta^x$, we obtain directly that for all $\Delta < \Delta^x$, $\omega^x_s > \omega^x_n$ and for all $\Delta^x < \Delta \leq \Delta^0$, $\omega^x_s < \omega^x_n$.

\[\text{A.5 Proof of Proposition 4}\]

The equilibrium growth rate of both economies writes

\[g = \frac{\beta P(\frac{\lambda_n}{\lambda_s} \omega^x_n + \frac{\lambda_s}{\lambda_n} \omega^x_s)}{2(1 - \beta)\Delta}.\]

Inserting\(^\text{42}\)

\[\omega^x_n = \frac{1}{\beta^2 P^2 \lambda_n} \left[ Q_1(\Delta) + 2\sqrt{(2 - \beta)\Delta^2} Q_2(\Delta) \right] \]

and $\omega^x_s$ as given by (10) and differentiating with respect to $\Delta$ yields

\[\frac{dg}{d\Delta} = -\beta Y < 0.\]

\[\text{A.6 Proof of Proposition 5}\]

First we show that $\omega^x_n$ increases with $\frac{\lambda_n}{\lambda_s}$ and, thereafter, that $\omega^x_s$ decreases with $\frac{\lambda_n}{\lambda_s}$.

Finally, we verify that the growth rate is invariant with $\frac{\lambda_n}{\lambda_s}$ given $\Delta$.

1. Let us consider $R^x_n(\omega_n)$, which is derived by inserting $\omega^x_s(\omega_n)$ as given by (10) into the first-order condition of the North (13). As shown in the first part of the proof of Proposition 2, $R^x_n(\omega_n)$ possesses two real roots, of which only the larger one is economically sensible and is denoted by $\omega^x_n$. Using the implicit-function theorem, we obtain

\[\frac{d\omega^x_n}{d\frac{\lambda_n}{\lambda_s}} \bigg|_{\lambda} = \frac{d\omega^x_n}{d\lambda_n} \bigg|_{\lambda} = -\frac{\partial R^x_n(\omega_n)}{\partial \omega_n} \bigg|_{\omega_n} \frac{\partial R^x_n(\omega_n)}{\partial \lambda_n} \bigg|_{\omega_n}.\]

Concerning the sign of the denominator, we can show that $\frac{\partial R^x_n(\omega_n)}{\partial \omega_n} \bigg|_{\omega_n} \omega_n \omega^x_n < 0$ by

\(^{42}\)Details on how $\omega^x_n$ is derived can be found in the extended appendix of the (Schäfer and Schneider, 2011).
verifying that \( R^e_n(\omega_n) \) is strictly concave.\(^{43} \) \( R^e_n(\omega_n) \) can be written as
\[
R^e_n(\omega_n) = \frac{D - P}{2} \frac{\beta}{2 - \beta} P \left( \frac{\lambda_n}{\lambda} \omega_n - A_1 \right) + \Delta D + \]
\[
\frac{\beta P}{2} \left[ \frac{\lambda_n}{\lambda} (Y + \omega_n D) - P \frac{1 - \beta}{2 - \beta} A_1 - P \frac{\lambda_n}{\lambda} \frac{\omega_n}{2 - \beta} - \left( \frac{\beta}{2 - \beta} \right)^2 \frac{P^2}{4\Delta} \left( \frac{\lambda_n}{\lambda} \omega_n - A_1 \right)^2 \right].
\]
where \( A_1 = \frac{Y}{D - P} \frac{\lambda}{\lambda} + \frac{\omega_n}{\beta P} \). Taking the second derivative with respect to \( \omega_n \), we obtain
\[
\frac{\partial^2 R^e_n(\omega_n)}{\partial \omega_n^2} = -\left( \frac{\beta}{2 - \beta} \right)^2 \frac{P^2}{2\Delta} \left( \frac{\lambda_n}{\lambda} \right)^2 < 0.
\]
As a consequence of the denominator being negative, the sign of \( \frac{dR^e_n(\omega_n)}{d\lambda_n} \) is identical to that of \( \frac{dR^e_n(\omega_n)}{d\lambda_n} \). For the derivative of \( R^e_n(\omega_n) \) with respect to \( \lambda_n \) given the total market size \( \lambda \), we can write
\[
\frac{\partial R^e_n(\omega_n)}{\partial \lambda_n} \bigg|_{\lambda} = \frac{\beta P(Y + \omega_n (D - P))}{4\Delta (2 - \beta)^2 \lambda^2 (D - P)^2} \left[ 2\Delta \lambda (D - P)((3 - \beta)(2 - \beta)D - (4 - 3\beta)P) + \beta^2 P^2 (\lambda Y - \lambda_n \omega_n (D - P)) \right]. \tag{A.12}
\]
Since \( Y + D - P > 0 \), it can be readily observed from (A.12) that \( \frac{dR^e_n(\omega_n)}{d\lambda_n} \bigg|_{\lambda} > 0 \). Hence, if we have an interior solution where \( \omega_n^c \in (0, 1) \), the North’s IPR-enforcement level strictly increases with its relative effective market size.

2. We insert \( \lambda_s = \lambda - \lambda_n \) into (10) and take the derivative with respect to \( \lambda_n \) given the total market size \( \lambda \). We obtain
\[
\frac{d\omega^x_s}{d\lambda_n} \bigg|_{\lambda} = -\frac{1 - \beta}{2 - \beta} \frac{2\Delta \lambda}{\lambda^2} - \frac{\omega_n}{2 - \beta} \frac{\lambda_n}{\lambda} \frac{1 - \beta}{2 - \beta} \frac{d\omega^x_s}{d\lambda_n} \bigg|_{\lambda} < 0 \tag{A.13}
\]
As we know from the first part of the proof that \( \frac{d\omega^x_s}{d\lambda_n} \bigg|_{\lambda} > 0 \), it follows that \( \frac{d\omega^x_s}{d\lambda_n} \bigg|_{\lambda} < 0 \). This verifies that the South (at an interior solution) also increases IPR-enforcement if its relative market size increases.

3. Consider now the steady-state growth rate in equilibrium:
\[
g = \frac{\beta P(\lambda_n \omega^x_n + \lambda_s \omega^x_s)}{2(1 - \beta)\lambda \Delta}.
\]
Inserting \( \omega^x_n \) as given by (A.11), \( \omega^x_s \) and substituting \( \lambda_s \) by \( \lambda - \lambda_n \), we obtain for the derivative with respect to \( \lambda_n \) given \( \lambda \): \( \frac{dg}{d\lambda_n} \bigg|_{\lambda} = 0 \).

\(^{43}\)This follows from the facts mentioned above: that \( R^e_n(\omega_n) \) possesses two real roots and \( \omega^x_n \) is the larger one of the two.
A.7 Proof of Proposition 7

The proof first shows that there is a unique solution to the North’s optimization problem. Then we show that the desired harmonized enforcement level $\omega_n^h$ is strictly concave in $\Delta$ by verifying that the second derivative is negative. The derivative of $\omega_n^h$ with respect to $\Delta$ is always positive if the condition given in Item (i) of Proposition 7 is satisfied. Otherwise the derivative will change its sign for larger values of $\Delta$ implying an inverted U-shaped relation between $\omega_n^h$ and $\Delta$. This verifies Item (ii) of Proposition 7. With respect to Item (iii), we first show that $\omega_n^h$ increases (decreases) with its relative effective market size if $\omega_n^h < (>) \omega_n^s$. Using the properties of $\omega_n^h$ and $\omega_n^s$ on the relevant interval of $\Delta$, we show that there exists a unique $\Delta$ such that $\omega_n^h > (\omega_n^h$ if and only if $\Delta > (\Delta).$ This proves Item (iii) of Proposition 7.

(1) We show that there is a unique solution to the North’s optimization problem. In this first step, we also establish some lemmata that characterize the properties of $R_n^h(\omega_n^h)$ and the optimal solution $\omega_n^h$. These will be useful to verify Items (i) – (iii) of Proposition 7.

We start by establishing the following lemma.

**Lemma 11**

$R_n^h(\omega_n^h)$ is a strictly concave function and strictly declining on $\mathbb{R}_+$. 

**Proof.** Consider the function $R_n^h(\omega_n^h)$ as given by (20). $R_n^h(\omega_n^h)$ is strictly concave because the second derivative can be written as

$$\frac{\partial^2 R_n^h(\omega_n^h)}{\partial (\omega_n^h)^2} = -\frac{\beta P \lambda}{4(1-\beta)^2 \Delta \lambda_n} < 0. \quad (A.14)$$

The first derivative, $\frac{\partial R_n^h(\omega_n^h)}{\partial \omega_n^h}$, reads

$$\frac{\partial R_n^h(\omega_n^h)}{\partial \omega_n^h} = \frac{\beta P}{2(1-\beta)}(D - P) + \frac{\beta P}{2}(D + \frac{\lambda_s}{\lambda_n} P) - \frac{E^h \beta P}{2 \Delta \lambda_n(1-\beta)}. \quad (A.15)$$

As the last term is positive, $\frac{\partial R_n^h(\omega_n^h)}{\partial \omega_n^h} < 0$ if the first two summands together are negative. This is the case if

$$\frac{\lambda_s}{\lambda_n} < \frac{D}{P} \frac{2 - \beta}{1 \beta} + \frac{1}{1 \beta}. \quad (A.16)$$

As stated in the main text, the sufficient condition for a maximum of the government’s problem is

$$\frac{\lambda_s}{\lambda_n} < -\frac{2D}{P} + 1. \quad (A.16)$$
Since $\frac{2-\beta}{1-\beta} > 2$ and $\frac{1}{1-\beta} > 1$ for all $\beta \in (0,1)$, the second-order condition of the government’s problem, (A.16), is stronger than condition (A.15). That is, all values of $\frac{\lambda_n}{\lambda_m}$ that satisfy the second-order condition (A.16) will also satisfy (A.15). Hence, for the relevant parameter set satisfying condition (A.16), $R_n^h(\omega_n^h)$ is strictly concave, it must be declining with $\omega_n^h$ on $\mathbb{R}_+$. 

$R_n^h(\omega_n^h)$ possesses the following roots.

\[
\omega_{n1}^h = \frac{2(1-\beta)}{\beta^2 \lambda_n P^2} \left( \Delta ((1-\beta)(D + \frac{\lambda_n}{\lambda_m}P) + D - P) + \Delta Q_3(\Delta) \right), \quad (A.17)
\]

\[
\omega_{n2}^h = \frac{2(1-\beta)}{\beta^2 \lambda_n P^2} \left( \Delta ((1-\beta)(D + \frac{\lambda_n}{\lambda_m}P) + D - P) - \sqrt{\Delta Q_3(\Delta)} \right),
\]

where $Q_3(\Delta) = \beta^2 \lambda_n Y P^2 + \Delta \left[ 2\beta \frac{\lambda}{\lambda_m} (D + \frac{\lambda_n}{\lambda_m}P) + ((1-\beta)(D + \frac{\lambda_n}{\lambda_m}P) + D - P)^2 \right]$. Since only real roots may possess economic meaning in our context, we restrict ourselves to the case where $Q_3(\Delta) > 0$. Then it follows that only $\omega_{n1}^h$ may assume positive values while $\omega_{n2}^h$ is always negative. Consequently, there is a unique economically sensible solution $\omega_n^h \equiv \omega_{n1}^h$.

The North’s desired harmonized IPR-level $\omega_n^h$ possesses the following properties.

**Lemma 12**

(a) $\omega_n^h$ is a strictly concave function in $\Delta$.

(b) $\lim_{\Delta \to 0} \omega_n^h = 0$.

(c) $\lim_{\Delta \to 0} \frac{\partial \omega_n^h}{\partial \Delta} = \infty$.

**Proof.** (a) The second derivative of $\omega_n^h$ with respect to $\Delta$ writes

\[
\frac{\partial^2 \omega_n^h}{\partial \Delta^2} = \frac{(1-\beta)\beta^2 \frac{\lambda}{\lambda_m} Y^2 P^2}{2(\Delta Q_3(\Delta))^{\frac{3}{2}}} < 0.
\]

This verifies the concavity of $\omega_n^h$.

Item (b) can be observed directly in equation (A.17).

(c) The derivative of $\omega_n^h$ with respect to $\Delta$ can be written as

\[
\frac{\partial \omega_n^h}{\partial \Delta} = \frac{2(1-\beta)}{\beta^2 \lambda_n P^2} \left[ (1-\beta)(D + \frac{\lambda_n}{\lambda_m}P) + D - P + \frac{Q_4(\Delta)}{\sqrt{\Delta Q_3(\Delta)}} \right],
\]

where $Q_4(\Delta) = \beta^2 \lambda_n Y P^2 - 2\Delta (1-\beta-\beta \frac{\lambda}{\lambda_m})(D + \frac{\lambda_n}{\lambda_m}P + D)P + P^2$. Since $\lim_{\Delta \to 0} \Delta Q_3(\Delta) = 0$, it depends on the sign of $\lim_{\Delta \to 0} Q_4(\Delta)$ whether the limit of $\frac{\partial \omega_n^h}{\partial \Delta}$ at $\Delta = 0$ will be plus or minus infinity. We obtain c) as $\lim_{\Delta \to 0} Q_4(\Delta) > 0$. 

43
Now we can show items (i) and (ii) of Proposition 7. Using the implicit function theorem, we have
\[
\frac{d\omega_n^h}{d\Delta} = -\frac{\partial R_n^h(\omega_n^h)}{\partial \lambda_n^h}.
\]
\[
\frac{\partial R_n^h(\omega_n^h)}{\partial \lambda_n^h}
\]
is negative according to Lemma 11. Consequently, the sign of \(\frac{d\omega_n^h}{d\Delta}\) is equal to that of \(\frac{\partial R_n^h(\omega_n^h)}{\partial \lambda_n^h}\). We obtain
\[
\frac{\partial R_n^h(\omega_n^h)}{\partial \Delta} = D + \frac{\lambda_s}{\lambda_n} P + \frac{\beta^3 P^3 (\omega_n^h)^2}{8(1 - \beta)^2 \Delta^2} \lambda.
\]
Hence, \(\frac{d\omega_n^h}{d\Delta} < 0\) if and only if
\[
D\lambda_n + P\lambda_s < -\frac{\beta^3 P^3 (\omega_n^h)^2}{8(1 - \beta)^2 \Delta^2} \lambda_n. \tag{A.18}
\]
The right hand side of (A.18) is clearly negative. Thus if \(D\lambda_n + P\lambda_s > 0\), which is equivalent to \(\frac{\lambda_s}{\lambda_n} > -\frac{P}{D}\), condition (A.18) is not satisfied and we obtain \(\frac{d\omega_n^h}{d\Delta} > 0\). This proves (i).

With respect to (ii), suppose that \(\frac{\lambda_s}{\lambda_n} > -\frac{P}{D}\). Then (A.18) defines a critical value of IPR-enforcement \(\omega_n^c\), for which \(\frac{d\omega_n^h}{d\Delta} > (\prec) 0\) if and only if \(\omega_n^h > (\prec) \omega_n^c\). The critical value is
\[
\omega_n^c = \frac{2\Delta(1 - \beta)}{\beta P} \sqrt{-\frac{2(D + \frac{\lambda_s}{\lambda_n} P)}{\beta P} \lambda_{\lambda_n}}. \tag{A.19}
\]
Equation (A.19) reveals that \(\omega_n^c\) is a linear function of \(\Delta\) with a positive finite slope and \(\lim_{\Delta \to 0} \omega_n^c = 0\). Together with the properties of \(\omega_n^h\) as given in Lemma 12, we can conclude that there exists a unique \(\Delta_m > 0\) such that \(\frac{d\omega_n^h}{d\Delta} > 0\) for all \(\Delta \in (0, \Delta_m)\) and \(\frac{d\omega_n^h}{d\Delta} < 0\) for all \(\Delta > \Delta_m\). This verifies claim (ii).

Now we turn to (iii). According to the implicit function theorem, we can write
\[
\frac{d\omega_n^h}{d\lambda_n^s} = -\frac{\partial R_n^h(\omega_n^h)}{\partial \lambda_n^s}. \tag{A.20}
\]
Since \(\frac{\partial R_n^h(\omega_n^h)}{\partial \lambda_n^s} < 0\) (Lemma 11), the sign of \(\frac{d\omega_n^h}{d\lambda_n^s}\) is equal to the sign of \(\frac{\partial R_n^h(\omega_n^h)}{\partial \lambda_n^s}\) as given by
\[
\frac{\partial R_n^h(\omega_n^h)}{\partial \lambda_n^s} = R_n^h(\omega_n^h) \equiv \left(1 + \frac{\tilde{E}^n}{2\Delta}\right)(D - P) + \beta\frac{P}{2\Delta}(Y + \omega_j^h(D - P)),
\]
44
which is equivalent to the South’s first-order condition in steady state (cf. footnote 21).
Again, \( \frac{\partial R_n^h}{\partial \lambda_n} = 0 \) defines a critical value of IPR-protection, \( \omega_n^c' \), such that \( \frac{\partial R_n^h(\omega_n^h)}{\partial \lambda_n} > ( < ) 0 \) if and only if \( \omega_n^h < ( > ) \omega_n^c' \). Since (A.20) is identical to the first-order condition to the South’s maximization problem, \( \omega_n^c' \) is identical to \( \omega_n^h \). Further, we define \( \Delta \) as the level of research productivity relative to total effective market size where \( \omega_n^h = 0 \). \( \Delta \) can be expressed as
\[
\Delta = -\frac{\beta PY}{2(D - P)} > 0.
\]
Since \( \omega_n^h \) is declining with \( \Delta \) (see (18)), \( \Delta > 0 \) implies that \( \omega_n^h > 0 \) at \( \Delta = 0 \). It follows that \( \omega_n^h < \omega_s^h \) for small values of \( \Delta \) according to the properties of \( \omega_n^h \) as described in Lemma 12. Since \( \omega_n^h \) is strictly concave in \( \Delta \), a necessary and sufficient condition for the existence of a \( \Delta \) such that \( \omega_n^h < ( > ) \omega_s^h \) for all \( \Delta < ( > ) \Delta \) is that \( \omega_n^h > \omega_s^h (= 0) \) at \( \Delta \).

By a similar line of argument as in the proof of Lemma 4, we can infer from Lemma 11 that \( \omega_n^h > 0 \) at \( \Delta \) if and only if \( R_n^h(0) > 0 \) given \( \Delta = \Delta \). The latter can be written as
\[
R_n^h(0)|_{\Delta=\Delta} = -\frac{\beta Y P^2}{2(D - P)} \frac{\lambda}{\lambda_n} > 0.
\]
This verifies that \( \omega_n^h \) and \( \omega_s^h \) possess exactly one intersection where \( \omega_n^h, \omega_s^h > 0 \). We denote the value of \( \Delta \) at this intersection by \( \tilde{\Delta} \). It now follows directly that \( \frac{d\omega_n^h}{d\Delta_n} < ( > ) 0 \) for all \( \Delta > ( < ) \tilde{\Delta} \).

A.8 Proof of Proposition 8

The proof of Proposition 8 follows directly from the last part of the proof of Proposition 7, where we have shown that \( \omega_n^h \) and \( \omega_s^h \) possess a unique intersection where both \( \omega_n^h \) and \( \omega_s^h \) are greater than zero. \( \tilde{\Delta} \) is the level of \( \Delta \) at this intersection. It follows further from the proof of Proposition 7 that \( \omega_n^h < \omega_s^h \) if \( \Delta < \tilde{\Delta} \) and \( \omega_n^h > \omega_s^h \) if \( \Delta > \tilde{\Delta} \).
References


<table>
<thead>
<tr>
<th>Nr.</th>
<th>Autor/Innen</th>
<th>Titel</th>
<th>Jahr</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Wolfgang Bernhardt</td>
<td>Stock Options wegen oder gegen Shareholder Value? Vergütungsmodelle für Vorstände und Führungskräfte</td>
<td>04/98</td>
</tr>
<tr>
<td>2</td>
<td>Thomas Lenk / Volkmar Teichmann</td>
<td>Bei der Reform der Finanzverfassung die neuen Bundesländer nicht vergessen!</td>
<td>10/98</td>
</tr>
<tr>
<td>4</td>
<td>Kristin Wallner</td>
<td>Möglichkeiten und Grenzen kooperativer Standortgestaltung zur Revitalisierung von Innenstädten</td>
<td>12/98</td>
</tr>
<tr>
<td>5</td>
<td>Gerhardt Wolff</td>
<td>Brauchen wir eine weitere Internationalisierung der Betriebswirtschaftslehre?</td>
<td>01/99</td>
</tr>
<tr>
<td>6</td>
<td>Thomas Lenk / Friedrich Schneider</td>
<td>Zurück zu mehr Föderalismus: Ein Vorschlag zur Neujustierung des Finanzausgleichs in der Bundesrepublik Deutschland unter besonderer Berücksichtigung der neuen Bundesländer</td>
<td>12/98</td>
</tr>
<tr>
<td>7</td>
<td>Thomas Lenk</td>
<td>Kooperativer Föderalismus – Wettbewerbsorientierter Föderalismus</td>
<td>03/99</td>
</tr>
<tr>
<td>8</td>
<td>Thomas Lenk / Andreas Mathes</td>
<td>EU – Osterweiterung – Finanzierbar?</td>
<td>03/99</td>
</tr>
<tr>
<td>9</td>
<td>Thomas Lenk / Volkmar Teichmann</td>
<td>Die ökonomischen Wirkungen verschiedener Forderungen zur Neujustierung des Länderfinanzausgleichs in der Bundesrepublik Deutschland. Eine empirische Analyse unter Einbeziehung der Normenkontrollanträge der Länder Baden-Württemberg, Bayern und Hessen sowie der Stellungnahmen verschiedener Bundesländer</td>
<td>09/99</td>
</tr>
<tr>
<td>10</td>
<td>Kai-Uwe Graw</td>
<td>Gedanken zur Entwicklung der Strukturen im Bereich der Wasserversorgung unter besonderer Berücksichtigung kleiner und mittlerer Unternehmen</td>
<td>10/99</td>
</tr>
<tr>
<td>11</td>
<td>Adolf Wagner</td>
<td>Materialien zur Konjunkturforschung</td>
<td>12/99</td>
</tr>
<tr>
<td>12</td>
<td>Anja Birke</td>
<td>Die Übertragung westdeutscher Institutionen auf die ostdeutsche Wirklichkeit – ein erfolgversprechendes Zusammenspiel oder Aufdeckung systematischer Mängel? Ein empirischer Beitrag für den kommunalen Finanzausgleich am Beispiel Sachsen</td>
<td>02/00</td>
</tr>
<tr>
<td>13</td>
<td>Rolf H. Hasse</td>
<td>Internationaler Kapitalverkehr in den letzten 40 Jahren – Wahlstandsmotor oder Krisenursache?</td>
<td>03/00</td>
</tr>
<tr>
<td>14</td>
<td>Wolfgang Bernhardt</td>
<td>Unternehmensführung (Corporate Governance) und Hauptversammlung</td>
<td>04/00</td>
</tr>
<tr>
<td>15</td>
<td>Adolf Wagner</td>
<td>Materialien zur Wachstumsforschung</td>
<td>03/00</td>
</tr>
<tr>
<td>16</td>
<td>Thomas Lenk / Anja Birke</td>
<td>Determinanten des kommunalen Gebührenerentkommens unter besonderer Berücksichtigung der neuen Bundesländer</td>
<td>04/00</td>
</tr>
<tr>
<td>17</td>
<td>Thomas Lenk</td>
<td>Finanzwirtschaftliche Auswirkungen des Bundesverfassungsgerichtsurteils zum Länderfinanzausgleich vom 11.11.1999</td>
<td>04/00</td>
</tr>
<tr>
<td>18</td>
<td>Dirk Büttel</td>
<td>Continuous linear utility for preferences on convex sets in normal vector spaces</td>
<td>05/00</td>
</tr>
<tr>
<td>19</td>
<td>Stefan Dierkes / Stephanie Hannrath</td>
<td>Steuerung dezentraler Investitionsentscheidungen bei nutzungabhängigem und nutzungsunabhängigem Verschleiß des Anlagenvermögens</td>
<td>06/00</td>
</tr>
<tr>
<td>20</td>
<td>Thomas Lenk / Andreas Mathes / Olaf Hirschfeld</td>
<td>Zur Trennung von Bundes- und Landeskompetenzen in der Finanzverfassung Deutschlands</td>
<td>07/00</td>
</tr>
<tr>
<td>21</td>
<td>Stefan Dierkes</td>
<td>Marktwerte, Kapitalkosten und Betafaktoren bei wertabhängiger Finanzierung</td>
<td>10/00</td>
</tr>
<tr>
<td>22</td>
<td>Thomas Lenk</td>
<td>Intergovernmental Fiscal Relationships in Germany: Requirement for New Regulations?</td>
<td>03/00</td>
</tr>
<tr>
<td>23</td>
<td>Wolfgang Bernhardt</td>
<td>Stock Options – Aktuelle Fragen Besteuerung, Bewertung, Offenlegung</td>
<td>03/01</td>
</tr>
<tr>
<td>Nr.</td>
<td>Autor/innen</td>
<td>Titel</td>
<td>Abdruckjahr</td>
</tr>
<tr>
<td>-----</td>
<td>--------------------------</td>
<td>-------------------------------------------------------------------------------------------</td>
<td>-------------</td>
</tr>
<tr>
<td>24</td>
<td>Thomas Lenk</td>
<td>Die „kleine Reform“ des Länderfinanzausgleichs als Nukleus für die „große Finanzverfassungsreform“? 10/2001</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>Sören Bär</td>
<td>Grundzüge eines Tourismusmarkting, untersucht für den Südraum Leipzig 05/2002</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>Wolfgang Bernhardt</td>
<td>Der Deutsche Corporate Governance Kodex: Zuwahl (comply) oder Abwahl (explain)? 06/2002</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>Adolf Wagner</td>
<td>Konjunkturtheorie, Globalisierung und Evolutionsökonomik 08/2002</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>Adolf Wagner</td>
<td>Zur Profilbildung der Universitäten 08/2002</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>Thomas Lenk / Anja Birke</td>
<td>The Measurement of Expenditure Needs in the Fiscal Equalization at the Local Level Empirical Evidence from German Municipalities 10/2002</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>Wolfgang Bernhardt</td>
<td>Die Lust am Fliegen Eine Parabel auf viel Corporate Governance und wenig Unternehmensführung 11/2002</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>Uwe Haubold / Michael Nowak</td>
<td>Risikooanalyse für Langfrist-Investments Eine simulationsbasierte Studie 12/2002</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>Thomas Lenk</td>
<td>Die Neuregelung des bundesstaatlichen Finanzausgleichs auf Basis der Steuerschätzung Mai 2002 und einer aktualisierten Bevölkerungsstatistik 12/2002</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>Uwe Haubold / Michael Nowak</td>
<td>Auswirkungen der Renditeverteilungsannahme auf Anlageentscheidungen Eine simulationsbasierte Studie 02/2003</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>Wolfgang Bernhard</td>
<td>Corporate Governance Kodex für den Mittel-Stand? 06/2003</td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>Hermut Kormann</td>
<td>Familienunternehmen: Grundfragen mit finanzwirtschaftlichen Bezug 10/2003</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>Matthias Folk</td>
<td>Launhardtsche Trichter 11/2003</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>Wolfgang Bernhardt</td>
<td>Corporate Governance statt Unternehmensführung 11/2003</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>Thomas Lenk / Karolina Kaiser</td>
<td>Das Prämienmodell im Länderfinanzausgleich – Anreiz- und Verteilungsmittelwirkungen 11/2003</td>
<td></td>
</tr>
<tr>
<td>43</td>
<td>Sabine Klinger</td>
<td>Die Volkswirtschaftliche Gesamtrechnung des Haushaltsektors in einer Matrix 03/2004</td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>Thomas Lenk / Heide Köpping</td>
<td>Strategien zur Armutsbekämpfung und -vermeidung in Ostdeutschland: 05/2004</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>Wolfgang Bernhardt</td>
<td>Sommernachtsfantasien Corporate Governance im Land der Träume. 07/2004</td>
<td></td>
</tr>
<tr>
<td>46</td>
<td>Thomas Lenk / Karolina Kaiser</td>
<td>The Premium Model in the German Fiscal Equalization System 12/2004</td>
<td></td>
</tr>
<tr>
<td>47</td>
<td>Thomas Lenk / Christine Falken</td>
<td>Komparative Analyse ausgewählter Indikatoren des Kommunalwirtschaftlichen Gesamtergebnisses 05/2005</td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>Michael Nowak / Stephan Barth</td>
<td>Immobilienanlagen im Portfolio institutioneller Investoren am Beispiel von Versicherungsunternehmen Auswirkungen auf die Risikosituation 08/2005</td>
<td></td>
</tr>
<tr>
<td>Nr.</td>
<td>Autor/In</td>
<td>Titel</td>
<td></td>
</tr>
<tr>
<td>-----</td>
<td>--------------------------</td>
<td>----------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>Anja Eichhorst / Karolina Kaiser</td>
<td>The Institutional Design of Bailouts and Its Role in Hardening Budget Constraints in Federations 03/2006</td>
<td></td>
</tr>
<tr>
<td>53</td>
<td>Gunther Schnabl</td>
<td>Die Grenzen der monetären Integration in Europa 08/2006</td>
<td></td>
</tr>
<tr>
<td>56</td>
<td>Ullrich Heilemann / Aninka Blaschzik</td>
<td>Indicators and the German Business Cycle A Multivariate Perspective on Indicators of Ifo, OECD, and ZEW 01/2007</td>
<td></td>
</tr>
<tr>
<td>59</td>
<td>Christine Falken / Mario Schmidt</td>
<td>Kameralistik versus Doppik Zur Informationsfunktion des alten und neuen Rechnungswesens der Kommunen Teil I: Einführende und Erläuternde Betrachtungen zum Systemwechsel im kommunalen Rechnungswesen 01/2007</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>Christine Falken / Mario Schmidt</td>
<td>Kameralistik versus Doppik Zur Informationsfunktion des alten und neuen Rechnungswesens der Kommunen Teil II Bewertung der Informationsfunktion im Vergleich 01/2007</td>
<td></td>
</tr>
<tr>
<td>61</td>
<td>Udo Hielscher</td>
<td>Mont dalla citta di fiarane Innovative Finanzierungen im Zeitalter Der Medici. Wurzeln der modernen Finanzmärkte 03/2007</td>
<td></td>
</tr>
<tr>
<td>63</td>
<td>Adolff Wagner</td>
<td>Regionalökonomik Konvergierende oder divergierende Regionalentwicklungen 08/2007</td>
<td></td>
</tr>
<tr>
<td>67</td>
<td>Wolfgang Bernhardt</td>
<td>5 Jahre Deutscher Corporate Governance Kondex Eine Erfolgsgeschichte? 01/2008</td>
<td></td>
</tr>
<tr>
<td>69</td>
<td>Christian Groth / Karl-Josef Koch / Thomas M. Steger</td>
<td>When economic growth is less than exponential 02/2008</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>Andreas Bohne / Linda Kochmann</td>
<td>Ökonomische Umweltbewertung und endogene Entwicklung peripherer Regionen Synthese einer Methodik und einer Theorie 02/2008</td>
<td></td>
</tr>
<tr>
<td>71</td>
<td>Andreas Bohne / Linda Kochmann / Jan Slavik / Lenka Slaviková</td>
<td>Deutsch-tschechische Bibliographie Studien der kontingenten Bewertung in Mittel- und Osteuropa 06/2008</td>
<td></td>
</tr>
</tbody>
</table>
Nr. 72 Paul Lehmann / Christoph Schröter-Schlaack  
Regulating Land Development with Tradable Permits:  
What Can We Learn from Air Pollution Control?  
08/2008

Nr. 73 Ronald McKinon / Gunther Schnabl  
China’s Exchange Rate Impasse and the Weak U.S. Dollar  
10/2008

Nr. 74 Wolfgang Bernhardt  
Managervergütungen in der Finanz- und Wirtschaftskrise  
Rückkehr zu (guter) Ordnung, [klugem] Maß und [vernünftigem] Ziel?  
12/2008

Nr. 75 Moritz Schularick / Thomas M. Steger  
Financial Integration, Investment, and Economic Growth:  
Evidence From Two Eras of Financial Globalization  
12/2008

Nr. 76 Gunther Schnabl / Stephan Freitag  
An Asymmetry Matrix in Global Current Accounts  
01/2009

Nr. 77 Christina Ziegler  
Testing Predictive Ability of Business Cycle Indicators for the Euro Area  
01/2009

Nr. 78 Thomas Lenk / Oliver Rottmann / Florian F. Wottek  
Public Corporate Governance in Public Enterprises  
Transparency in the Face of Divergent Positions of Interest  
02/2009

Nr. 79 Thomas Steger / Lucas Bratschger  
Globalization, the Volatility of Intermediate Goods Prices, and Economic Growth  
02/2009

Nr. 80 Marcelo Munoz Escobar / Robert Hohler  
Institutional Sustainability of Payment for Watershed Ecosystem Services.  
Enabling conditions of institutional arrangement in watersheds  
04/2009

Nr. 81 Robert Hohler / WU Chanyou / DUAN Ning  
Sustainable Development of Industrial Parks  
07/2009

Nr. 82 Georg Quaas  
Realgrößen und Preisindizes im alten und im neuen VGR-System  
10/2009

Nr. 83 Ullrich Heilemann / Hagen Findke  
Empirical Determination of Aggregate Demand and Supply Curves:  
The Example of the RWI Business Cycle Model  
12/2009

Nr. 84 Gunther Schnabl / Andreas Hoffmann  
The Theory of Optimum Currency Areas and Growth in Emerging Markets  
03/2010

Nr. 85 Georg Quaas  
Does the macroeconomic policy of the global economy’s leader cause the worldwide asymmetry in current accounts?  
03/2010

Nr. 86 Volker Grossmann / Thomas M. Steger / Timo Trimborn  
Quantifying Optimal Growth Policy  
06/2010

Nr. 87 Wolfgang Bernhardt  
Corporate Governance Kodex für Familienunternehmen?  
Eine Widerrede  
06/2010

Nr. 88 Philipp Mandel / Bernd Süssmuth  
A Re-Examination of the Role of Gender in Determining Digital Piracy Behavior  
07/2010

Nr. 89 Philipp Mandel / Bernd Süssmuth  
Size Matters.  
The Relevance and Hicksian Surplus of Agreeable College Class Size  
07/2010

Nr. 90 Thomas Kohstall / Bernd Süssmuth  
Cyclical Dynamics of Prevention Spending and Occupational Injuries in Germany: 1886-2009  
07/2010

Nr. 91 Martina Padmanabhan  
Gender and Institutional Analysis.  
A Feminist Approach to Economic and Social Norms  
08/2010

Nr. 92 Gunther Schnabl / Anagar Belke  
Finanzkrise, globale Liquidität und makroökonomischer Exit  
09/2010

Nr. 93 Ullrich Heilemann / Roland Schuhr / Heinz Josef Münch  
A “perfect storm”? The present crisis and German crisis patterns  
12/2010

Nr. 94 Gunther Schnabl / Holger Ziemann  
Die Deutsche Wiedervereinigung und die europäische Schuldenkrise im Lichte der Theorie optimaler  
Währungsräume  
06/2011

Nr. 95 Andreas Hoffmann / Gunther Schnabl  
Symmetrische Regeln und asymmetrisches Handeln in der Geld- und Finanzpolitik  
07/2011

Nr. 96 Andreas Schöfer / Mark T. Schneider  
Endogenous Enforcement of Intellectual Property, North-South Trade, and Growth  
08/2011