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Monetary Policy, Trend Inflation and Inflation Persistence

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Abstract

This paper presents a new mechanism through which monetary policy rules affect inflation persistence. When assuming that price reset hazard functions are not constant, backward-looking dynamics emerge in the NKPC. This new mechanism makes the traditional demand channel of monetary transmission have a long-lasting effect on inflation dynamics. The Calvo model fails to convey this insight, because its constant hazard function leads those important backward-looking dynamics to be canceled out. I first analytically show how it works in a simple setup, and then solve a log-linearized model numerically around positive trend inflation. With realistic calibration of trend inflation and the monetary policy rule, the model can account for the pattern of changes in inflation persistence observed in the post-wwii U.S. data. In addition, with increasing hazard functions, the "Taylor principle" is sufficient to guarantee the determinate equilibrium even under extremely high trend inflation.

JEL classification: E31; E52

Key words: Intrinsic inflation persistence, Hazard function, Trend inflation, Monetary policy, New Keynesian Phillips curve

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1 Introduction

The current policy debate on using monetary easing to spur the U.S. economy out of the Great Recession raises a question on how stable are the inflation dynamics to warrant exceptionally easing monetary policy for an extended period. Theoretically, it is closely related to the question whether there exists a structural relationship between current inflation and its lags. If the answer is yes, monetary policy can be used to "fine-tune" the real economy, without having to worry about price stability at least in the short run.

This paper investigates the issue with primary emphasis on how the systematic part of monetary policy rules may affect inflation persistence. My analysis is based on the assumption that price reset hazard functions are history-dependent, as opposed to the common Calvo price setting. This assumption is consistent with recent empirical evidence using micro-level data (See e.g. Campbell and Eden, 2005, Alvarez, 2007 and Nakamura and Steinsson, 2008). I show analytically that, when the hazard function is non-constant with respect to the time-since-last-adjustment, there emerges a lagged-expectation channel in the generalized New Keynesian Phillips curve (GNKPC hereafter), through which the monetary policy parameter affects "intrinsic inflation persistence"(Fuhrer, 2006), and hence the backward-looking dynamics of inflation is not a structural relationship, in the sense of the Lucas (1976)'s critique.

In the standard New Keynesian monetary models, developed by Rotemberg and Woodford (1997) and Clarida, Gali, and Gertler (2000) among others, inflation is not persistent, and monetary policy, typically modeled as nominal interest rate reaction rules, has no bearing on generating inflation persistence. This prediction is, however, clearly against the broad consensus drawn from SVAR evidence that monetary policy shocks lead to a delayed and long-lasting responses of inflation (Christiano, Eichenbaum, and Evans, 1999). The theoretical response to this challenge is the hybrid NKPC, introducing some ad hoc forms of backward-looking dynamics in inflation (See e.g. Gali and Gertler, 1999 and Christiano, Eichenbaum, and Evans, 2005). According to this kind of hybrid NKPCs, even though the monetary policy shock generates persistent inflation dynamics, there is still no role played by the systematic part of monetary policy rules in generating inflation persistence, because the backward-looking dynamics in those models are invariant in monetary policy rules. Recent empirical studies\(^1\) explicitly consider the role of shifts in monetary policy rules in accounting for changes in inflation persistence. They find that the more aggressive monetary policy regime is closely related to the decline in inflation persistence after the Volcker disinflation.

Analysis in this paper is based on a generic sticky price model set forth by Wolman (1999).\(^2\) The model assumes that firms adjust their prices following a hazard function, which specifies probabilities of price adjustment conditional on the time elapsed since the price was last set. As a result, the GNKPC incorporates new dynamic components, such as lags of inflation and lagged expectations. Presenting a lagged-expectation channel makes the tradition demand channel of monetary transmission more "powerful" in the sense that monetary policy has now a "long-lasting" effect on inflation dynamics. In the macro literature, the demand channel of monetary transmission is well understood, but its effect on inflation persistence is largely unexplored, because the predominant Calvo price setting implies that only forward-looking expectations  

\(^1\)See: e.g. Davig and Doh (2008), Benati and Surico (2008) and Benati (2009).  
\(^2\)It is further studied by Mash (2003), Coenen et al. (2007), Whelan (2007) and Sheedy (2007).
matter for inflation dynamics. By contrast, non-constant hazard pricing model gives lagged expectations a role in forming inflation dynamics, so that the demand channel of monetary policy can also affect inflation persistence.

Using a simple inflation-responding Taylor rule, I show analytically that the intertemporal dependence between current inflation and its lags is a function of the interest rate responding parameter to inflation deviations and price reset hazard rates. When, for example, the inflation response parameter conforms to the "Taylor principle" and the price reset hazard function is increasing, the GNKPC exhibits a positive backward dependence. These conditions, however, are sufficient but not necessary for having intrinsic inflation persistence. In addition, the magnitude of this intertemporal dependence is positively dependent on the strength of the interest rate response to inflation deviations, implying that "intrinsic inflation dynamics" are only stable when monetary policy does not accommodate inflation.

In the numerical experiments, I solve and simulate the equilibrium dynamics of inflation using a full-fledged DSGE model featuring positive trend inflation and increasing price reset hazard functions. I find that, firstly, serial correlation of the inflation gap rises significantly with trend inflation. Secondly, how aggressively monetary policy rules react to inflation deviations has also quantitatively important impact on inflation persistence. Under the active monetary policy regime, autocorrelation coefficients are smaller at all levels of trend inflation than those under a passive monetary regime. These findings are consistent with the empirical evidence that shifts in monetary policy regimes in the early 80’s accounted for the decline in inflation persistence in the U.S. Together with my analytical result, I identify a new source of benefit associated with the active monetary policy regime in fighting inflation. When a central bank aggressively responds to inflation deviations, on the one hand, overall inflation persistence will decline, on the other hand, the intrinsic component of inflation persistence will rise. As a result, inflation persistence shifts from inherited persistence to intrinsic persistence, making intrinsic inflation dynamics more stable and less affected by the extrinsic forces.

Another striking result from the simulation exercise is that, when assuming an increasing price reset hazard function, the "Taylor principle" is sufficient to guarantee the determinate equilibrium even under extremely high trend inflation. For standard calibration values, the model is determinate even under trend inflation of 40% at the annual rate, which is much higher than the values reported in the literature under either Calvo or Taylor staggered price setting. The key mechanism at work is that the Calvo staggered price model gives rise to a large price dispersion, and the existence of trend inflation aggravates it even further. By contrast, increasing hazard functions restrict the price dispersion, and therefore help to resolve the determinacy puzzle. In the sensitivity analysis, the results discussed above are robust to a wide range of increasing hazard functions.

In the literature, many studies are closely related to this paper. Sheedy (2007) studies the generalized NKPC under a recursive formulation of the hazard function and shows that the dependence of current and lagged inflation is a function of the slope of the hazard function. In particular, increasing hazard functions result in positive backward-dependence of inflation dynamics. According to my analytical result, the increasing hazard function is only a sufficient

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but not necessary condition for having intrinsic inflation persistence. When monetary policy is comply with the "Taylor principle", intrinsic inflation persistence can be warranted even by constant or slightly decreasing hazard function. Empirical studies by Davig and Doh (2008), Benati and Surico (2008) and Benati (2009) raise similar doubts on models building intrinsic inflation persistence as a structural feature of the NKPC. For example, Benati estimates a structural backward-looking model under different monetary regimes, and finds that the backward-looking coefficients are not invariant across monetary policy regimes. In theory, alternative explanations for why monetary policy should affect intrinsic inflation persistence are provided by various authors. Cogley and Sbordone (2008) show that, when allowing for time-drifting trend inflation, the generalized NKPC also embeds a structural persistence term which is affected by trend inflation, which stems from monetary policy. Carlstrom, Fuerst, and Paustian (2009) emphasize the impact of monetary policy on the relative importance of different shocks. In particular, a more aggressive monetary policy rule will increase the relative importance of mark-up shocks relative to demand shocks.

The remainder of the paper is organized as follows: in section 2, I present the model with the generalized time-dependent pricing and derive the New Keynesian Phillips curve; section 3 shows new insights gained from relaxing the constant hazard function underlying the Calvo assumption; in section 4, I simulate the full-fledged DSGE model with positive trend inflation and alternative monetary policy regimes; section 5 contains some concluding remarks.

2 The Model

In this section, I construct a New Keynesian monetary model with general price reset hazard functions. Staggered price setting in the model allows for a general form of hazard functions introduced by Wolman (1999).

2.1 Household:

The representative household has one unit of labor endowment in all periods. She works $N_t$, consumes $C_t$, buys one-period bonds $B_{t+1}$ in period $t$, and holds at the end of the period with a quantity of money $M_t$. He maximizes the expected value of discounted future utilities with the following period utility function:

$$U_t = E_t \sum_{j=0}^{\infty} \beta^j \left[ \log C_{t+j} + \alpha \log \left( \frac{M_{t+j}}{P_{t+j}} \right) + \gamma \log (1 - N_{t+j}) \right],$$

(1)

where $\beta$ is the household’s discount factor, $\alpha$ and $\gamma$ are weights on utility of real money balance and leisure.

The household is restricted to the following budget constraint in $t$

$$C_t + \frac{B_{t+1}}{R_t} + \frac{M_t}{P_t} \leq \frac{W_t}{P_t} N_t + \frac{M_{t-1}}{P_t} + B_t + F_t,$$

(2)

where $R_t$ is the gross rate of real return on one-period bonds held from $t$ to $t+1$, and $F_t$ is profits transferred from firms to the household.
The household maximizes the expected value of his discounted utility (1), subject to the sequence of budget constraints (2). The optimality conditions for the household yield:

\[
\frac{\gamma C_t}{(1 - N_t)} = \frac{W_t}{P_t} \quad (3)
\]

\[
R_t^{-1} = \beta E_t \left[ \frac{C_{t+1}}{C_t} \right] \quad (4)
\]

\[
C_t = \frac{\kappa P_t}{M_t} + \beta E_t \left[ \frac{C_{t+1} P_t}{P_{t+1}} \right] \quad (5)
\]

In addition, two terminal conditions are needed to rule out that the household accumulates too much bonds and real money balance,

\[
\lim_{j \to \infty} E_t \left[ \frac{B_{t+j+1}}{R_{t+j}} \right] = 0, \quad (6)
\]

\[
\lim_{j \to \infty} \beta^j E_t \left[ \frac{M_{t+j}}{C_{t+j} P_{t+j}} \right] = 0. \quad (7)
\]

### 2.2 Firms

There are two kinds of firms in the economy. Perfectly competitive final good producers use the following technology to produce the final good

\[
Y_t = \left( \int_0^1 Y_{jt} \frac{\theta^{\eta-1}}{\eta} \, dj \right)^ \frac{\eta}{\theta-1}, \quad (8)
\]

where \( Y_t \) is time \( t \) production of the final good, \( Y_{jt} \) is the quantity of intermediate good \( j \) used in the production, and \( \eta \) is a parameter strictly greater than one, interpreted as the elasticity of substitution between intermediate goods. The profit maximization problem of the final good producer yields the demand for each intermediate good

\[
Y_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\eta} Y_t. \quad (9)
\]

Imposing the zero profit condition, we obtain the price index as:

\[
P_t = \left( \int_0^1 P_{jt}^{1-\eta} \, dj \right)^{\frac{1}{1-\eta}}. \quad (10)
\]

There is a continuum of monopolistically competitive firms indexed on the unit interval by \( j \in [0, 1] \). To produce intermediate goods \( Y_{jt} \), they use a Cobb-Douglas technology:

\[
Y_{jt} = Z_t L_{jt}, \quad (11)
\]

where \( L_{jt} \) is the amount of labor input demanded by the firm \( j \) in time \( t \), and \( Z_t \) is an aggregate technology shock.

I assume that intermediate firms demand labor input on an economy-wide competitive labor market. Furthermore, because of the monopolistic power, firms can choose optimal prices for their goods to maximize real profits:

\[
\max_{\{L_{jt}\}} \Pi_{jt} = \frac{P_{jt} Y_{jt}}{P_t} - \frac{W_t}{P_t} L_{jt}, \quad (12)
\]
subject to the production technology (11) and the demand condition (9) for $Y_{jt}$. Substituting these conditions for $Y_{jt}$ and $L_{jt}$, and taking derivative w.r.t. $P_{jt}$, it yields

$$P^*_t = \frac{\eta}{\eta - 1} \frac{W_t Z_t^{-1}}{MC_t}.$$  
(13)

As seen in equation (13), the optimal price set by a monopolistic firm is equal to the product of nominal marginal cost $MC_t$ and a markup $\Omega = \frac{\eta}{\eta - 1} > 1$.

Next, integrating the production function (11) and imposing labor market clearing condition $N_t = \int_0^1 L_{jt} dj$, it yields the aggregate production function

$$Y_t = Z_t N_t.$$  
(14)

### 2.2.1 Pricing Decisions under Generalized Hazard Functions

As in Wolman (1999), I assume that the probabilities for monopolistically competitive firms to reset their prices depend on the duration of the price. This is summarized by a hazard function $h_i$, where $i \in \{0, I\}$ denotes the period-since-last-adjustment. $I$ is the maximum number of periods for which a firm’s price can be fixed. Table (1) summarizes key notations regarding the dynamics of vintages.

<table>
<thead>
<tr>
<th>Vintage</th>
<th>Hazard Rate</th>
<th>Non-adj. Rate</th>
<th>Survival Rate</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
<td>$h_i$</td>
<td>$\alpha_i = 1 - h_i$</td>
<td>$S_i = \prod_{k=0}^{i} \alpha_i$</td>
<td>$\theta(i)$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>$\theta(0)$</td>
</tr>
<tr>
<td>1</td>
<td>$h_1$</td>
<td>$\alpha_1 = 1 - h_1$</td>
<td>$S_1 = \alpha_1$</td>
<td>$\theta(1)$</td>
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<td>$\vdots$</td>
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<td>$\theta(i)$</td>
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</tr>
<tr>
<td>$I$</td>
<td>$h_I = 1$</td>
<td>$\alpha_I = 0$</td>
<td>$S_I = 0$</td>
<td>$\theta(I)$</td>
</tr>
</tbody>
</table>

Table 1: Notations of the dynamics of price-vintage-distribution.

It turns out that, as long as the hazard rates lie between zero and one, there always exists an invariant distribution of price vintages $\Theta$, obtained by solving $\theta_t(i) = \theta_{t+1}(i)$. The invariant price-duration distribution $\theta(i)$ is obtained as follows:

$$\theta(i) = \frac{S_i}{\sum_{i=0}^{I-1} S_i}, \text{ for } i = 0, 1, \cdots, I - 1.$$  
(15)

In a given period when a firm is allowed to reoptimize its price, the optimal price chosen should reflect the possibility that it will not be re-adjusted in the near future. Consequently, adjusting firms choose optimal prices that maximize the discounted sum of real profits over the time horizon during which the new price is expected to be fixed. The probability that a new price will be at least fixed for $i$ periods is given by the survival function $S_i$, defined in Table (1).

\footnote{Note that this equation is only valid under zero steady state inflation. See detailed discussion in Ascari (2004).}
The maximization problem of an average price setter is

$$\max_{P_{jt}} \Pi_t = E_t \sum_{i=0}^{l-1} S_i Q_{t,t+i} \left( \frac{P_{jt}}{P_{jt}} Y_{j,t+i} - TC_{j,t+i} Y_{j,t+i} \right),$$

where $Q_{t,t+i}$ is the real stochastic discount factor appropriate for discounting real profits from $t$ to $t+i$, given by equation (4). Substituting $Y_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\eta} Y_t$ into the profit equation, I obtain

$$P_t^* = \frac{\eta}{\eta - 1} \sum_{i=0}^{l-1} S_i E_t \left[ Q_{t,t+i} Y_{t+i} P_{jt+i} - MC_{t+i} \right],$$

where the average level of nominal marginal cost of all resetting firms is given by $MC_t = W_t Z_t^{-1}$. The optimal price is equal to the markup multiplied by a weighted sum of future marginal costs, where weights depend on the survival function. In the Calvo case, where $S_i = \theta^i$, this equation reduces to the Calvo optimal pricing condition.

Finally, given the invariant price distribution $\theta(i)$, the aggregate price can be written as a distributed sum of past reset prices. I define the aggregate optimal price which was set $k$ periods ago as $P^*_{t-k}$, then the aggregate price is obtained by

$$P_t = \left( \sum_{k=0}^{l-1} \theta(i) P^*_{t-k} \right)^{\frac{1}{\eta - 1}}. \quad (17)$$

### 2.3 Generalized New Keynesian Phillips Curve

To study implications of general hazard functions on inflation dynamics, I log-linearize the equations around the zero-inflation steady state\(^5\) and derive the generalized New Keynesian Phillips curve (GNKPC hereafter) under an arbitrary hazard function as follows:\(^6\)

$$\hat{\pi}_t = \sum_{k=0}^{l-1} \frac{\theta(k)}{1 - \theta(0)} E_{t-k} \left( \sum_{i=0}^{l-1} \beta^i S(i) \frac{\hat{mc}_{t+i-k}}{\Psi} + \sum_{i=1}^{l-1} \sum_{i=1}^{l-1} \beta^i S(i) \frac{\hat{\pi}_{t+i-k}}{\Psi} \right)$$

$$- \sum_{k=2}^{l-1} \Phi(k) \hat{\pi}_{t-k+1},$$

$$\text{where} \quad \Psi = \sum_{i=0}^{l-1} S_i \beta^i \quad \text{and} \quad \Phi(k) = \frac{\sum_{i=k}^{l-1} S(i)}{\sum_{i=0}^{l-1} S(i)}.$$

In this equation, $\hat{\pi}_t$ is defined as deviation from steady state inflation, $\hat{mc}_{t} = \frac{1}{1-N} \hat{y}_t$ is the log deviation of the average real marginal cost and $\hat{y}_t$ is the output gap between log output and the logarithm of the potential output under flexible prices.

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\(^5\)Later, I relax this steady state assumption by allowing positive trend inflation.

\(^6\)A technical note is available from the author.
The GNKPC differs from the standard NKPC in two aspects. First, the GNKPC involves not only forward-looking terms, but also backward-looking dynamics, such as lagged inflation and lagged expectations. In addition, all coefficients in the GNKPC are nonlinear functions of price reset hazard rates \( \alpha_i = 1 - h_i \) and the model’s structural parameters. Thereby, short-run dynamics of the inflation gap are affected by both the shape and magnitude of hazard functions. Note that the standard NKPC emerges as a special case when assuming the hazard function to be constant over the infinite horizon. In this case, the distribution of price duration becomes \( \theta(i) = (1 - \theta)\theta^i \), and the survival function collapses to \( \theta^i \).

The economic intuition why backward-looking dynamics should emerge in the GNKPC, but are missing in the Calvo model, is the following: first, the forward-looking terms enter the Phillips curve through their influence on the current reset price. As same as in the Calvo sticky price model, the price setting in this model is forward-looking. The optimal price decision is based on the sum of current and future real marginal costs over the time span in which reset prices are fixed. Second, due to price stickiness, some fraction of past reset prices continue to affect the current aggregate price. Lagged expectational terms represent influences of past reset prices on current inflation. Last, past inflations enter the GNKPC, because they affect the lagged aggregate price \( p_{t-1} \). The higher the past inflations prevail, the higher the lagged aggregate price would be, and thereby it deters current inflation to be high. The two backward looking dynamic terms have opposing effects on inflation through \( p_t \) and \( p_{t-1} \) respectively, and the magnitudes of these counteracting effects depend on the price reset hazard function. In the general case, they should be different to each other, therefore emerge in the GNKPC, but, in the Calvo case, the constant hazard function causes lagged expectations and lagged inflation to be canceled out. This insight is also to be seen in the derivation of the Calvo NKPC:

\[
\begin{align*}
p_t &= (1 - \theta) \sum_{j=0}^{\infty} \theta^j p^*_t - j \\
&= (1 - \theta) [p^*_t + \theta p^*_t - 1 + \theta^2 p^*_t - 2 + \cdots] \\
&= (1 - \theta)p^*_t + (1 - \theta) [\theta p^*_t - 1 + \theta^2 p^*_t - 2 + \cdots] \\
&= (1 - \theta)p^*_t + \theta p_{t-1}.
\end{align*}
\]

The crucial substitution in line (3) is only possible, if the distribution of price durations takes the power function under the Calvo assumption, so that all dynamic effects of past reset prices are replaced by \( p_{t-1} \).

3 Analysis

3.1 Monetary Policy and Intrinsic Inflation Persistence

The purely forward-looking NKPC is often criticized for generating too little inflation persistence (See e.g. Fuhrer and Moore, 1995). In response to this challenge, the hybrid NKPC has been developed to capture the positive dependence of inflation on its lags (See: e.g. Gali and Gertler, 1999 and Christiano et al., 2005). According to this strand of literature, the dependency between current and lagged inflation is mechanically modeled as a fixed primitive relationship, which is
independent of changes in monetary policy. By contrast, as analyzed above, the generalized sticky price model propagates inflation persistence in a more delicate way. Unlike the hybrid NKPC, inflation persistence in this framework is the result of two counteracting channels. The first channel gives lagged inflation a direct role, which works through the past aggregate price. I call it the "front-loading channel" because it weakens inflation persistence, and its magnitude is purely determined by the price reset hazard function. By contrast, the second channel is an indirect one, where lagged inflation affects current inflation only through the expectational terms in the GNKPC, I name it the "expectation channel". In this channel, lagged inflations have positive coefficients when lagged inflations are positively correlated to other variables. Because, in the general equilibrium, the expectation formulation is determined by the whole setup of the model, the magnitude of the "expectation channel" is not only affected by hazard functions, but also by aggregate demand side of the economy and monetary policy.

To show these channels more explicitly, I assume that monetary policy follows a simple Taylor rule $i_t = \phi_n \hat{\pi}_t$, where nominal interest rate is set to respond to the deviation of inflation from its steady state, and $\phi_n$ is a non-negative responding parameter to inflation deviations. Combining with the log-linearized equation (4) and the goods market equilibrium, it yields

$$E_t[\hat{y}_{t+1}] - \hat{y}_t = i_t - E_t[\hat{\pi}_{t+1}]. \tag{19}$$

Substituting the Taylor rule into equation (19), it yields

$$E_t[\hat{\pi}_{t+1}] = \phi_n \hat{\pi}_t - E_t[\hat{y}_{t+1}] + \hat{y}_t. \tag{20}$$

Iterating this equation forward, we get higher-order expectations

$$E_t[\hat{\pi}_{t+2}] = \phi_n E_t[\hat{\pi}_{t+1}] - E_t[\hat{y}_{t+2}] + E_t[\hat{y}_{t+1}]$$

$$E_t[\hat{\pi}_{t+2}] = \phi_n^2 \hat{\pi}_t - E_t[\hat{y}_{t+2}] + (1 - \phi_n) E_t[\hat{y}_{t+1}] + \phi_n \hat{y}_t. \tag{21}$$

For analytical simplicity, I use a simple version of the GNKPC with a maximum price duration of $J = 3$ periods. I also assume $\beta = 1$, then the GNKPC can be expressed as follows

$$\hat{\pi}_t = \frac{1}{\Psi} E_t \left( (\alpha_1 + \alpha_1 \alpha_2) \hat{\pi}_{t-1} + \alpha_1 \alpha_2 \hat{\pi}_{t+2} + \omega \hat{y}_t + \alpha_1 \omega \hat{y}_{t+1} + \alpha_1 \alpha_2 \omega \hat{y}_{t+2} \right)$$

$$+ \frac{\alpha_1}{\Psi} E_{t-1} \left( (\alpha_1 + \alpha_1 \alpha_2) \hat{\pi}_t + \alpha_1 \alpha_2 \hat{\pi}_{t+1} + \omega \hat{y}_{t-1} + \alpha_1 \omega \hat{y}_t + \alpha_1 \alpha_2 \omega \hat{y}_{t+1} \right)$$

$$+ \frac{\alpha_1 \alpha_2}{\Psi} E_{t-2} \left( (\alpha_1 + \alpha_1 \alpha_2) \hat{\pi}_{t-1} + \alpha_1 \alpha_2 \hat{\pi}_{t-2} + \omega \hat{y}_{t-2} + \alpha_1 \omega \hat{y}_{t-1} + \alpha_1 \alpha_2 \omega \hat{y}_t \right)$$

$$- \frac{\alpha_2}{1 + \alpha_2} \hat{\pi}_{t-1},$$

where $\Psi = (\alpha_1 + \alpha_1 \alpha_2) (1 + \alpha_1 + \alpha_1 \alpha_2), \quad \omega = \frac{1}{1 - N}. \tag{22}$

Substituting expectation equations (20) and (21) into GNKPC (22), we obtain
\[ \pi_t = \Psi_1 E_t [\pi_{t+1}] + \Psi_2 E_t [\pi_{t+2}] + \Psi_3 \pi_{t-1} + \Psi_4 \pi_{t-2} + AR(\hat{y}_t), \]  \tag{23}

where:

\[ \Psi_1 = \frac{1}{1 + \alpha_1 + \alpha_1 \alpha_2}, \]
\[ \Psi_2 = \frac{\alpha_1 \alpha_2 (\phi - 1) + \alpha_1 \alpha_2 (\phi_{\pi}^2 - \alpha_2) + (\phi_{\pi} \alpha_1 - \alpha_2)}{(1 + \alpha_2) (1 + \alpha_1 + \alpha_1 \alpha_2)}, \]
\[ \Psi_3 = \frac{\phi_{\pi} \alpha_1 \alpha_2 (2 + \phi_{\pi} \alpha_2)}{(1 + \alpha_2) (1 + \alpha_1 + \alpha_1 \alpha_2)}. \]

Inflation persistence in this equilibrium Phillips curve is driven by both intrinsic and extrinsic sources. \( AR(\hat{y}_t) \) represents a complex autoregressive process of the extrinsic driving force, while the "intrinsic" inflation dynamic is driven by the lags of inflation and inflation expectations \( (\Psi_1 E_t [\pi_{t+1}] + \Psi_2 E_t [\pi_{t+2}] + \Psi_3 \pi_{t-1} + \Psi_4 \pi_{t-2}) \). It is worthy to note that the backward-looking part \( (\Psi_3 \pi_{t-1} + \Psi_4 \pi_{t-2}) \) is totally absent in the standard Calvo NKPC, while, in the hybrid NKPC, its coefficients are invariant to monetary policy. By contrast, intrinsic persistent coefficients \( (\Psi_3, \Psi_4) \) in the GNKPC are functions of policy parameter \( \phi_{\pi} \), as well as hazard rates \( \alpha_1, \alpha_2 \). In the following propositions, I formally show how those parameters affect intrinsic inflation persistence in the GNKPC.

**Proposition 1:** The GNKPC (23) exhibits positive intrinsic persistent coefficients

\[ \frac{\alpha_1 \alpha_2 (\phi_{\pi} - 1) + \alpha_1 \alpha_2 (\phi_{\pi}^2 - \alpha_2) + (\phi_{\pi} \alpha_1 - \alpha_2)}{(1 + \alpha_2) (1 + \alpha_1 + \alpha_1 \alpha_2)} > 0 \]  \tag{24}
\[ \frac{\phi_{\pi} \alpha_1 \alpha_2 (2 + \phi_{\pi} \alpha_2)}{(1 + \alpha_2) (1 + \alpha_1 + \alpha_1 \alpha_2)} > 0, \]  \tag{25}

when the following condition is fulfilled

\[ \alpha_1 \alpha_2 (\phi_{\pi} - 1) + \alpha_1 \alpha_2 (\phi_{\pi}^2 - \alpha_2) + (\phi_{\pi} \alpha_1 - \alpha_2) > 0. \]  \tag{26}

**Proof:** It is obvious to see that, inequality (25) holds for all reasonable values of parameters I consider, namely \( \phi_{\pi} > 0 \) and \( 0 < \alpha_1, \alpha_2 < 1 \). and inequality (24) holds, when \( \alpha_1 \alpha_2 (\phi_{\pi} - 1) + \alpha_1 \alpha_2 (\phi_{\pi}^2 - \alpha_2) + (\phi_{\pi} \alpha_1 - \alpha_2) > 0 \).

To draw some instructive economic intuitions from the condition (26), I study inequality by parts. First, when \( \phi_{\pi} - 1 > 0 \), it implies that monetary policy conforms to the "Taylor principle", i.e. monetary policy should react strongly to inflation deviations by adjusting nominal interest rate more than one for one. The "Taylor principle" is one of the most important guidelines for the practice of modern monetary policy. In addition, it is also the necessary condition for a DSGE model with the Taylor rule to have a determinate equilibrium. I view, therefore, that this condition is fulfilled under a normal circumstance. Given \( \phi_{\pi} > 1, \phi_{\pi}^2 - \alpha_2 > 0 \) holds too.

Besides the monetary policy parameter, the shape of hazard functions is also an important factor in determining intrinsic inflation persistence in the condition (26). Sheedy (2007) studies the generalized NKPC under a recursive formulation of the hazard function and concludes that
increasing hazard functions give rise to intrinsic inflation persistence. According to the condition (26), his conclusion is only a sufficient, but not necessary condition for generating intrinsic inflation persistence. The increasing hazard function implies $\alpha_1 > \alpha_2$. This condition together with the "Taylor principle" will guarantee that the condition (26) is satisfied $(\phi_n \alpha_1 - \alpha_2 > 0)$, but it is not necessary condition, because Equation (24) requires the sum to be greater than zero, but not by parts.

**Proposition 2:** The intrinsic persistent coefficients are increasing in inflation response parameter $\phi_n$.

Proof:

\[
\begin{align*}
\frac{\partial \Psi_3}{\partial \phi_n} &= \frac{\alpha_1 + \alpha_1 \alpha_2 + 2\alpha_1 \alpha_2 \phi_n}{(1 + \alpha_2)(1 + \alpha_1 + \alpha_1 \alpha_2)} > 0 \\
\frac{\partial \Psi_4}{\partial \phi_n} &= \frac{2\alpha_1 \alpha_2 (1 + \phi_n \alpha_2)}{(1 + \alpha_2)(1 + \alpha_1 + \alpha_1 \alpha_2)} > 0,
\end{align*}
\]

This result has important implications for monetary policy. First of all, since the coefficient on lagged inflation depends on the parameter in the monetary policy rule, the "intrinsic inflation persistence" in the reduced-form Phillips curve is not a structural relationship in the sense of the Lucas critique (Lucas, 1976). Secondly, the monetary policy stance can strengthen or weaken the "intrinsic anchor" of inflation dynamics through the traditional demand channel of monetary transmission. When monetary policy reacts aggressively to inflation deviations, implying that the real interest rate will always rise to counter whatever reasons that cause increase in inflation. In this circumstance, agents in the economy will form expectations betting that inflation will not deviate by a large amount from its past values, so that, at the aggregate level, inflation dynamics are well anchored on its own history. This belief is stronger, when monetary policy is more aggressive on inflation deviations. Effects of expectation channel on inflation is well understood in macroeconomics, but what is new in the model is that the demand channel of monetary transmission has a long-lasting effect on inflation through the presence of lagged expectations in the GNKPC, so that monetary policy affects not only the current inflation, but also inflation persistence in the future.

Furthermore, one can see that monetary policy parameter even affects intrinsic inflation persistence in an accelerating way, as the second derivatives with respect to $\phi_n$ are also positive.

### 4 Monetary Policy, Trend Inflation and Inflation Persistence - A Numerical Assessment

To study the equilibrium dynamics of inflation, I solve and simulate the log-linearized DSGE model around non-zero steady state inflation. Recent monetary economic literature emphasizes the role played by positive trend inflation on inflation persistence. Seminal work by Cogley and Sbordone (2008) show that, when allowing for time-drifting trend inflation, the generalized NKPC also embeds a structural persistence term which is affected by the drifting trend inflation, which results from monetary policy. More importantly, many authors also find that positive trend inflation requires stronger responses by the central bank to achieve stabilization.
in a determinate equilibrium than under zero trend inflation. For example Coibion and Gorodnichenko (2008) show that the Taylor principle breaks down when trend inflation exceeds 1.2% per year. With trend inflation around 7 percent per year, the response to inflation deviation needed to guarantee the determinacy is 10 times higher than that with zero trend inflation. Motivated by these results, I log-linearized the equilibrium equations around non-zero steady state inflation. As seen in the following log-linearized equations, trend inflation \( \pi_t \) affects inflation dynamics in a very complex manner. In the next section, I conduct a numerical assessment by solving this log-linearized DSGE model

\[
\begin{align*}
\hat{y}_t &= \hat{n}_t + \eta(\hat{p}_t - \hat{p}_t), \\
\sum_{k=0}^{i-1} \theta(k) \Pi^{-k} \hat{p}^*_t &= \sum_{k=0}^{i-1} \theta(k) \Pi^{-k} \hat{p}^*_{t-k}, \\
\hat{c}_t &= \hat{y}_t, \\
\hat{p}_t &= \sum_{k=0}^{i-1} \chi(k) \hat{p}^*_{t-k}, \\
\hat{p}^*_{t} &= \sum_{i=0}^{i-1} \Psi(i) E_t[\hat{mc}^r_{t+i} + \hat{p}_{t+i}], \\
\hat{mc}^r_t &= \frac{N}{1+N} \hat{n}_t + \hat{y}_t, \\
E_t[\hat{c}_{t+1}] - \hat{c}_t &= \iota_t - E_t[\hat{\pi}_{t+1}], \\
\hat{\pi}_t &= \hat{p}_t - \hat{p}_{t-1}, \\
\iota_t &= \phi_t \hat{\pi}_t + \epsilon_t, \text{ where } \epsilon_t \sim i.i.d.(0, \sigma^2)
\end{align*}
\]

All real variables \( (\hat{y}_t, \hat{n}_t, \hat{c}_t, \hat{mc}^r_t) \) are expressed in terms of log deviations from the non-stochastic steady state. Prices \( (\hat{p}_t, \hat{p}^*_t, \hat{p}^*_{t}) \) are first detrended and log-linearized around the common trend. The inflation gap \( \hat{\pi}_t \) is the deviation of inflation from its constant trend and \( \iota_t \) denotes the net rate of nominal interest.

**4.1 Calibration**

In the calibration, I parameterize the hazard function in a parsimonious way. In particular, the functional form I apply is the hazard function of the Weibull distribution with two parameters:

\[
h(j) = \begin{cases} 
\frac{\lambda^j}{\lambda - 1} \left( \frac{j}{\lambda} \right)^{\gamma - 1}, & \text{when } h(j) < 1 \\
1, & \text{when } h(j) > 1
\end{cases}
\]

\(1 \) I exclude hazard rate greater than one, because, in my theoretical setup, firms are not allowed to adjust their prices more than once per period.
λ is the scale parameter, which controls the average duration of price adjustment, while τ is the shape parameter to determine the monotonic property of the hazard function. The shape parameter enables the incorporation of a wide range of hazard functions by using various values. In fact, any value of the shape parameter that is greater than one corresponds to an increasing hazard function, while values ranging between zero and one lead to a decreasing hazard function. By setting the shape parameter to one, we can retrieve a Poisson process. In figure (1), I plot the Weibull distributions and corresponding hazard functions with different values of the shape parameter.

I choose $\lambda = 3$ based on empirical evidence from micro-level data. It implies an average price duration of three quarters, which is largely consistent with the median price durations of 7 - 9 months documented by Nakamura and Steinsson (2008). The shape parameter is allowed to vary in the interval between one and three, which covers a wide range of increasing hazard functions. As for the rest of the structural parameters, I follow values commonly used in the literature. I assume $\beta = 0.99$, which implies a steady state real return on financial assets of about four percent per annum. I choose the steady state market labor share $N = 1/3$. The elasticity of substitution between intermediate goods $\eta = 10$, which implies the desired markup over marginal cost should be about 11%. Finally, I set the standard deviation of the shock to the Taylor rule to be 1%.

4.2 Simulation Results

In the numerical experiments, I study how the serial correlation in inflation reacts to the monetary policy rule and trend inflation. Since this DSGE model is only driven by a transitory shock to the Taylor rule, it is not intended to capture the realistic level of inflation persistence in the data, instead, the focus is the pattern of changes in inflation persistence.

---

8Microeconometric studies typically find that aggregate hazard function is downward sloping, however, my rationale for focusing only on increasing hazard functions is two-fold: first, increasing hazard functions are theoretically consistent with the micro-founded state-dependent pricing models (Dotsey et al., 1999). In addition, Alvarez et al. (2005) show that decreasing hazards could simply result from the aggregation mechanism over individual hazard functions with different durations.
Table 2: Second moments of the simulated data (\tau=2)

<table>
<thead>
<tr>
<th>\tau = 2</th>
<th>AR(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.005</td>
</tr>
<tr>
<td>$\varphi_\pi = 1.5$</td>
<td>0.2059</td>
</tr>
<tr>
<td>$\varphi_\pi = 1.01$</td>
<td>0.2530</td>
</tr>
</tbody>
</table>

In table (2), I report the first-order autocorrelation coefficient of the inflation gap under different values of the gross quarterly rate of trend inflation $\Pi$ and the response parameter to inflation deviations in the Taylor rule $\varphi_\pi$. In this simulation exercise, I use the shape parameter of $\tau = 2$ in the Weibull distribution.

The first noteworthy result from the table is that inflation persistence increases with trend inflation, but decreases in the inflation response parameter. Under two monetary policy regimes, serial correlation of the inflation gap rises significantly with trend inflation. This finding is consistent with empirical evidence,\(^9\) showing that various measures of inflation persistence declined significantly since early 80’s, which coincides with the drop of average inflation level during the same period. In addition, the monetary policy regime has also a quantitatively important impact on inflation persistence. Under the active monetary policy regime ($\varphi_\pi = 1.5$), autocorrelation coefficients are smaller at all levels of trend inflation than those under a passive monetary regime ($\varphi_\pi = 1.01$). This result is also consistent with empirical evidence, see e.g. Davig and Doh (2008) and Benati and Surico (2008), that shifts in monetary policy regimes in the early 80’s accounted for changes in inflation persistence in the U.S. data. Along with my analytical result in Proposition 2, we can conclude an additional benefit of active monetary policy in fighting inflation. When a central bank aggressively adjusts the nominal interest rate in response to the inflation deviation, on the one hand, overall inflation persistence will decline, on the other hand, the intrinsic component of inflation persistence will rise. As a result, inflation persistence shifts from inherited persistence to intrinsic persistence, making inflation dynamics more stable and less affected by the extrinsic forces. Put into another words, when the central bank is "hawkish", inflation is more firmly anchored on its own past.

The second striking result from this simulation exercise is that, when the price reset hazard function is increasing, the Taylor principle is sufficient to guarantee a determinate equilibrium even when facing extremely high trend inflation. Under my baseline calibration, the model is determinate even under trend inflation as high as annual rate of 40% ($\Pi = 1.1$), which is much higher than values reported in the literature under either Calvo or Taylor staggered price setting.\(^10\) For example, with standard calibration of Calvo sticky price models, Ascarì and Ropele (2009) report that for levels of trend inflation greater than the annual rate of 2.42%, the simple Taylor principle breaks down. It seems, however, that the Taylor (1980) staggered price model is more robust to positive trend inflation. Kiley (2007) and Hornstein and Wolman (2005) study this case, and find that equilibrium determinacy is more difficult to achieve through reasonable specifications of the Taylor rules at levels around 4 percent per year. The key mechanism at

\(^9\)See: e.g. Levin and Piger (2003), Stock and Watson (2007), and Cogley, Primiceri, and Sargent (2010).
work is that the Calvo staggered price model gives rise to a large price dispersion and existence of trend inflation aggravates the price dispersion even further. The Taylor staggered pricing model, however, limits it by setting a maximum price duration, which reins the price dispersion. In addition, the Taylor model also implies a special form of increasing hazard function, which takes values of either zero during the price contract and one at the end of the price contract. This indicates that changing the shape of hazard functions from the constant one in the Calvo model to a more general form is the key to resolve this determinacy puzzle. In the following, I conduct a robustness check of results discussed above by simulating the GNKPC model under some alternative shapes of hazard functions.

$$AR(1)$$

| $\phi_n = 1.5$ | 0.3031 | 0.3273 | 0.3539 | 0.4116 | 0.5359 | 0.5586 |
| $\phi_n = 1.01$ | 0.3541 | 0.3768 | 0.4018 | 0.4554 | 0.5600 | 0.5882 |

Table 3: Second moments of the simulated data (tau=3)

In Table (3), I report the same measure of inflation persistence using the shape parameter of $\tau = 3$, indicating a more increasing hazard function. The results repeat the pattern from the table (2), except that the inflation gap is more persistent under each pair of $(\phi_n, II)$. This is consistent with the analytical result of Proposition 1, predicting that, ceteris paribus, intrinsic inflation persistence, and hence the overall level of inflation persistence, will rise, when the hazard function is more increasing.

$$AR(1)$$

| $\phi_n = 1.5$ | 0.0749 | 0.0475 | 0.0295 | 0.0182 | 0.1798 | indeterminate |
| $\phi_n = 1.01$ | 0.0973 | 0.0579 | 0.0356 | 0.0238 | 0.1959 | indeterminate |

Table 4: Second moments of the simulated data (tau=1.3)

In Table (4), I report the simulation result with a mild increasing hazard function ($\tau = 1.3$), which is closer to that in the Calvo model. In this case, I get very little inflation persistence, which reminds us of the famous critique on the standard Calvo sticky price model for generating too little inflation persistence (See: e.g. Fuhrer and Moore, 1995). Moreover, when the hazard function becomes flatter, the Taylor rule ceases to warrant for determinacy under a high trend inflation. One can show that, when flattening the hazard function further, indeterminacy will occur under lower and lower levels of trend inflation.

4.3 Empirical Relevance of the Model

In this section, I show that the GNKPC model can account for the pattern of changes in inflation persistence in the U.S. post-WWII monetary history. New consensus emerges in monetary economics, that monetary policy regime, trend inflation and inflation persistence are closely related phenomena. Empirical work, e.g. Cogley, Primiceri, and Sargent (2010) and Fuhrer
(2009) among others, documents a "low-high-low" pattern of changes in inflation persistence since the 60’s in the U.S. This change is argued to be closely related to the shift in monetary policy regime in the early 80’s during Volcker disinflation.\textsuperscript{11} To see if the GNKPC model can capture this historical "low-high-low" pattern of inflation persistence, I calibrate the model using realistic settings of trend inflation and the Taylor rule parameter in four sub-periods in the U.S. post-WWII monetary history.

<table>
<thead>
<tr>
<th></th>
<th>60’s (II =1.005)</th>
<th>70’s (II =1.02)</th>
<th>early 80’s (II =1.02)</th>
<th>90’s (II =1.005)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Trend Inflation</strong></td>
<td>2%</td>
<td>7%</td>
<td>7%</td>
<td>2%</td>
</tr>
<tr>
<td><strong>Persistence (Data)</strong></td>
<td>1.5</td>
<td>1.01</td>
<td>2</td>
<td>1.5</td>
</tr>
<tr>
<td><strong>Persistence (Model)</strong></td>
<td>0.327</td>
<td>0.455</td>
<td>0.376</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 5: Second moments of the simulated data (tau=1.3)

In Table (5), I simulate the course of inflation persistence under the reference values taken from the study by Cogley, Primiceri, and Sargent (2010). I consider four sub-periods. First, the 60’s were a period with both low trend inflation and an active monetary regime. I choose trend inflation to be 2% at the annual rate during this period and $\phi_p = 1.5$. Second, the 70’s were characterized by high trend inflation and passive monetary policy (Clarida, Gali, and Gertler, 2000). I set trend inflation to be 7%, which is consistent with most estimates in the literature. The Taylor rule parameter is set to be a little bit above one to reflect weak responses of monetary policy to inflation deviations and guarantee determinacy. Third, the early 80’s were marked as a transitional time from high inflation to a low inflation regime. I use a relative high value of the inflation response parameter in the Taylor rule ($\phi_p = 2$) to mirror the period of Volcker disinflation. Finally, the 90’s were well known as a period of "Great Moderation" and the U.S. monetary policy under chairman Greenspan was conducted complying with the "Taylor Principle". Using these values, I simulate the model with an increasing hazard function ($\tau = 3$). The final row of the table shows, that the simulated inflation persistence is well in line with the reference values observed in the data. Inflation persistence was low during the 60’s, rose during the 70’s, and dropped under Volcker disinflation and finally settled on 0.3 during the 90’s. With only i.i.d. monetary policy shocks, my simulations reflect quite well this "low-high-low" pattern, despite that it fails to generate the high magnitude of inflation persistence in the 70’s. This result suggests that the inner propagation mechanism of the GNKPC model is capable to capture, to a large extent, the complex interactions among monetary policy rule, trend inflation and inflation persistence in the data.

\textsuperscript{11}See e.g. Davig and Doh, 2008, Benati and Surico, 2008 and Carlstrom et al., 2009
5 Conclusion

In this paper, I present a model in which the monetary policy rule affects inflation persistence. The key assumption on which the analysis are based is that price reset hazard functions are history-dependent. When probability of price resetting depends on the duration of prices, lagged expectations of inflation emerge in the generalized NKPC. As a result, policy-induced interest rate changes have a long-lasting effect on inflation dynamics.

The implications of this model for the current policy debate are as follows: first, the reduced-form positive dependence of inflation and its lags is not the "structural" relationship, upon which monetary policy operates. In the model, the magnitude of the intertemporal dependence is positively dependent on the strength of the interest rate response to inflation deviations. Thus, intertemporal inflation dynamics are only stable when monetary policy does not accommodate inflation. Second, this model reveals a new source of benefit associated with the active monetary policy in fighting inflation. When a central bank aggressively responds to inflation deviations, inflation persistence shifts from inherited persistence to intrinsic persistence, making inflation more stably anchored on its own history, so that it is less affected by the extrinsic driving forces. On the other hand, when the central bank holds an easing policy stance on inflation, they would face a capricious inflation dynamic even in the short run.
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