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# Implementation of technological breakthroughs at sector level and the technology-bias<sup>1</sup>

by

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## Abstract

Different goods are produced by different sectors in an economy. The fact that sectors use different production technologies is named technology-bias. The technology-bias is well documented and has important theoretical implications for economic growth and unemployment. We provide a theoretical model that explains the technology-bias and predicts its development. We provide empirical evidence on the development of the technology-bias and explain this development by using our model-results. Last not least, we discuss the implications of our findings for the existing growth literature and structural change literature.

**Keywords:** sector technology, implementation of technological progress, structural change, growth, multi-sector growth models

**JEL-Codes:** O14, O41

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## 1. Introduction

Different goods are produced by different sectors in an economy. In general, we can say that each sector has its own specific production technology. That is, technology differs across sectors. This fact is named “cross-sector technology-bias”. Many multi-sector models assume the existence of such a technology bias and many models imply that this bias has important impacts on the aggregate growth rate of the economy and for unemployment as well. (For some references, see section 8.)

The aim of the paper is to provide a long-run growth model, which can explain the existence of technology-bias endogenously and which predicts how this technology bias develops over time. We focus especially on the cross-sector bias regarding capital-intensities, output-elasticities of labor and factor-income-shares. That is, we analyze why sectors have different output-elasticities of labor and thus different capital-intensities and we analyze how these differences develop over time. We also provide empirical evidence on this question.

Our model is a sort of multi-sector Ramsey-Cass-Koopmans-model. General technology breakthroughs happen exogenously. They have to be implemented to improve the technology of the sectors. Implementation requires some creative ideas and, hence, depends upon the amount of labor employed in a sector. Hence, we study why some sectors implement the breakthroughs better than others.

To our knowledge, there are two papers, which have some similarities to our paper

1.) Acemoglu and Guerrieri (2006) model endogenous technological change in a two-sector framework, where endogenous growth is achieved by costly *research*. Their model yields an endogenous *cross-sector bias in TFP*.<sup>2</sup> In contrast, we focus on sector-size-dependant factors which affect implementation of general breakthroughs at sector-level (*learning-by-doing and creative-powers*); furthermore, we are interested in explaining the *cross-sector bias in output-elasticities of inputs* (not so much bias in TFP).

2.) Zuleta and Young (2007) present a two-sector model where a “backward sector” uses labor only and a “progressive sector” uses labor and capital; the output elasticity of capital in the progressive sector can be increased by investment. Due to these assumptions the backward

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<sup>2</sup> To see what the exact difference between our paper and the paper by Acemoglu and Guerrieri (2006) is, consider the following fact: The implicit sectoral production functions of Acemoglu and Guerrieri’s (2006)-model are depicted on page 15 of their paper (equation (27)). In fact these functions are of type Cobb-Douglas, like in our model. Acemoglu and Guerrieri (2006) endogenize the TFP ( $M_1$  and  $M_2$ ) of these production functions on page 36 of their paper (equations (61)). Hence, we can say that the *bias of TFP* is endogenous in their model. However, the output-elasticities of inputs ( $\alpha_1$  and  $\alpha_2$ ) are not endogenized in their model. That is, the *bias of output-elasticities of inputs* is exogenous in their model. We focus on the *bias of output-elasticities of inputs*; i.e., we try to analyse why  $\alpha_1$  and  $\alpha_2$  are different and how this difference develops over time.

sector cannot catch-up (since it cannot use capital), i.e. the technologies cannot converge. In contrast, we focus on the catching-up process of the backward sector.

Our theoretical results imply that sectoral technologies may converge or diverge, depending upon the sort of technological progress (labor-augmenting or capital-augmenting), structural change patterns (i.e. changes in factor-allocation across sectors) and distribution of capital-production. Our empirical results imply that sector-technologies converged between 1948 and 1987 in the USA, which may be explained by prevalence of capital-augmenting technological implementation in the past and/or increasing employment-shares of technologically backward sectors.

Overall, our model implies that models, which assume exogenous technology-bias, omit important dynamics of structural change and aggregate growth. The reason is, that technology-bias tends to be self-reinforcing, causes structural change (cross-technology factor-reallocation) and is affected itself by cross-technology factor reallocation. For a detailed discussion of model-implications for the existing literature see section 8.

In the next section, we present the assumptions and the equilibrium of the model. In section 3, we discuss the dynamics of the model, when no technological breakthroughs occur. In section 4, we study the impacts of a sequence of technology breakthroughs on the sectoral technology-bias (via implementation). In section 5, we study how general structural-change-patterns affect the implementation of breakthroughs at sector level. Section 6 provides a summary of the factors which have an impact on the development of the technology-bias in our model. In section 7 we provide some evidence on the development of the sectoral technology bias. In section 8 we discuss the implications of our results for the existing literature. In section 9 we provide some concluding remarks.

## 2 Model assumptions

### 2.1 Production

**Assumption 1:** Many heterogeneous goods ( $i = 1, \dots, n$ ) are produced in the economy; each good  $i$  is produced by a subsector  $i$ .

**Assumption 2:** Goods/subsectors  $i = 1, \dots, m$  are assigned to sector A; goods/subsectors  $i = m + 1, \dots, n$  are assigned to sector B ( $n > m$ ). This is only a simplifying assumption; in fact our arguments work even when there are more than two sectors.

**Assumption 3:** Capital ( $K$ ) and labor ( $L$ ) are inputs in Cobb-Douglas production functions.

**Assumption 4:** Technology is homogenous within a sector but differs across sectors. For example, all subsectors  $i = 1, \dots, m$  have the same technology, but subsectors  $i = m + 1, \dots, n$  have not the same technology as subsectors  $i = 1, \dots, m$ . Again, this is only a simplifying assumption without implications for our main results. It helps us to model the independency assumption in the next subsection.

**Assumption 5:** The degree of implementation of a general technological breakthrough depends upon the number of persons who are employed in a sector. That is, the more persons are employed in a sector, the better a technological breakthrough is implemented. This is a very important assumption; it is discussed extensively in Section 2.4.

**Assumption 6:** The implementation of a general technological breakthrough affects the TFP-growth rate and the output elasticity of capital and labor respectively. That is, implementation affects the marginal rate of technical substitution and thus the optimal capital intensity of a sector. Again, this is a very important assumption, which will be discussed extensively in Section 2.4.

These assumptions imply the following production functions of subsectors  $i$ :

$$(1a) \ Y_i = A(l_i L)^{\alpha(l_A L)} (k_i K)^{1-\alpha(l_A L)}, \quad i = 1, \dots, m$$

where

$$(1b) \ 0 < \alpha(l_A L) < 1$$

$$(1c) \ \frac{\dot{A}}{A} = g_A(l_A L)$$

$$(1d) \ l_A \equiv \sum_{i=1}^m l_i$$

$$(2a) \ Y_i = B(l_i L)^{\beta(l_B L)} (k_i K)^{1-\beta(l_B L)}, \quad i = m + 1, \dots, n$$

where

$$(2b) \ 0 < \beta(l_B L) < 1$$

$$(2c) \ \frac{\dot{B}}{B} = g_B(l_B L)$$

$$(2d) \ l_B \equiv \sum_{i=m+1}^n l_i$$

where  $Y_i$  denotes the output of good  $i$ ;  $l_i$  and  $k_i$  denote respectively the fraction of labor and capital devoted to production of good  $i$  (i.e.  $l_i$  is the employment share of subsector  $i$  and  $k_i$  is the capital share of sector  $i$ );  $K$  is the aggregate capital;  $L$  aggregate labor;  $A$  ( $B$ ) is a technology parameter of sector  $A$  ( $B$ ). Note that I omit here the time index. Furthermore, note that the index  $i$  denotes goods/subsectors.  $l_A$  and  $l_B$  denote respectively the employment shares of sector  $A$  and sector  $B$ . I will define additional sector-variables later.

The fact that the TFP-growth rates ( $g_A$  and  $g_B$ ) and the subsectoral output-elasticities of inputs depend upon the sectoral employment (equations (1b,c) and (2b,c)) comes from Assumptions 5 and 6. We discuss in Section 2.4 which assumptions are reasonable for the functional forms of  $\alpha(l_A L)$ ,  $\beta(l_B L)$ ,  $g_A(l_A L)$  and  $g_B(l_B L)$ .

**Assumption 7:** All capital and all labor are used in the production of goods  $i = 1, \dots, n$ :

$$(3) \sum_{i=1}^n l_i = 1; \quad \sum_{i=1}^n k_i = 1$$

**Assumption 8:** The amount of available labor grows at exogenous rate ( $g_L$ ):

$$(4) \frac{\dot{L}}{L} \equiv g_L$$

**Assumption 9:** All goods are consumed. Furthermore, only the good  $m$  can be used as capital.

(Note, that the model could be modified such that more than one good is used as capital e.g. in the manner of Ngai and Pissarides (2007).)

This assumption implies:

$$(5) Y_i = C_i, \quad \forall i \neq m$$

$$(6) Y_m = C_m + \dot{K} + \delta K$$

where  $C_i$  denotes consumption of good  $i$ ;  $\delta$  denotes the constant depreciation rate of capital.

**Assumption 10:** Each subsector consists of many identical, marginalistic and profit-maximizing producers; i.e. there is perfect competition in each subsector. (Eventual departures from this assumption are discussed in Section 9.)

There is perfect factor-mobility within and across sectors.

These assumptions imply that in each subsector entrepreneurs consider prices, factor-prices and technology-parameters as exogenous (i.e. determined by the market). They are price-

takers and technology-takers (i.e. they consider  $\alpha$ ,  $\beta$ ,  $g_A$  and  $g_B$  as exogenous). Hence, profit maximization and perfect factor mobility imply the following optimality conditions:

$$w = p_i \partial Y_i / \partial (l_i L), \quad \forall i$$

$$r = p_i \partial Y_i / \partial (k_i K), \quad \forall i$$

$$\frac{p_i}{p_m} = \frac{\partial Y_m / \partial (l_m L)}{\partial Y_i / \partial (l_i L)} = \frac{\partial Y_m / \partial (k_m K)}{\partial Y_i / \partial (k_i K)}, \quad \forall i$$

where  $w$  is the real wage rate,  $r$  is the real rental rate of capital and  $p_i$  is the price of good  $i$  (and correspondingly  $p_m$  is the price of good  $i = m$ ).

By using equations (1)-(3) these optimality conditions can be reformulated as follows:

$$(7a) \quad \frac{k_i}{l_i} = \frac{k_m}{l_m} \quad \text{for } i = 1, \dots, m$$

$$(7b) \quad \frac{k_i}{l_i} = \frac{\alpha(l_A L)}{1 - \alpha(l_A L)} \frac{1 - \beta(l_B L)}{\beta(l_B L)} \frac{k_m}{l_m} \quad \text{for } i = m + 1, \dots, n$$

$$(7c) \quad \frac{p_i}{p_m} = 1 \quad \text{for } i = 1, \dots, m$$

$$(7d) \quad \frac{p_i}{p_m} = \frac{\alpha(l_A L) \left( \frac{k_m K}{l_m L} \right)^{1 - \alpha(l_A L)}}{\beta(l_B L) \left( \frac{k_i K}{l_i L} \right)^{1 - \beta(l_B L)}}, \quad \text{for } i = m + 1, \dots, n$$

$$(7e) \quad r = p_m A(l_A L) [1 - \alpha(l_A L)] \left( \frac{l_m L}{k_m K} \right)^{\alpha(l_A L)}$$

$$(7f) \quad w = p_m A(l_A L) \alpha(l_A L) \left( \frac{k_m K}{l_m L} \right)^{1 - \alpha(l_A L)}$$

Note that these equations were derived under the assumption that the producers consider  $\alpha$ ,  $\beta$ ,  $g_A$  and  $g_B$  as exogenous, since the producers are marginalistic.

## 2.2 Households

Now, we assume that there are many marginalistic households. We assume here that the households are identical, although many dimensions of household-heterogeneity could be

introduced into this model without much mathematical difficulties. However, household-heterogeneity will be a topic of a separate paper.

In the following, the superfix  $\iota$  denotes the corresponding variable of the individual household. For example, while  $E$  stands for consumption expenditures of the whole economy,  $E^\iota$  stands for consumption expenditures of the household  $\iota$ . I assume that there is an arbitrary and large number of households ( $\iota=1,...,x$ ), sufficiently large to constitute marginalistic behavior of households.

We assume the following utility function, which is quite similar to the utility function used by Kongsamut et al. (1997, 2001):

$$(8a) \quad U^\iota = \int_0^\infty u(C_1^\iota, \dots, C_n^\iota) e^{-\rho t} dt, \quad \forall \iota, \quad \rho > 0$$

where

$$(8b) \quad u(C_1^\iota, \dots, C_n^\iota) = \ln \left[ \prod_{i=1}^n (C_i^\iota - \theta_i^\iota)^{\omega_i} \right], \quad \forall \iota$$

$$(8c) \quad \sum_{i=1}^m \theta_i^\iota = 0 \quad \forall \iota$$

$$(8d) \quad \sum_{i=m+1}^n \theta_i^\iota = 0 \quad \forall i$$

$$(8e) \quad \sum_i \omega_i = 1$$

where  $C_i^\iota$  denotes the consumption of good  $i$  by household  $\iota$ ;  $\rho$  is the time-preference rate.

$\theta_i^\iota$  can be interpreted as a subsistence level (if  $\theta_i^\iota$  is positive) or as a an endowment (if  $\theta_i^\iota$  is negative) of household  $\iota$  regarding good  $i$  (see also e.g. Kongsamut et al. (2001)). In fact this preference structure is non-homothetic across goods in general; hence, increasing income causes demand-shifts across goods as shown by Kongsamut et al. (2001). Furthermore, this preference-structure features a non-unitary price elasticity of demand; hence, structural change may be caused by relative-price-changes as discussed by Ngai and Pissarides (2007).

Restrictions (8c,d) are imposed here for analytical reasons. They simplify the analysis and help us to isolate the determinants of technology assimilation. They allow for the existence of a partially balanced growth path (see later), which makes our dynamic analysis traceable. Furthermore, they allow us to determine exogenously whether factor reallocation takes place across technology or not: in fact, restrictions (8c,d) prevent factor reallocation across technology A and B. Hence, later we will be able to determine whether factor reallocation



across technology takes place or not, which is helpful for isolating the determinants of technology assimilation, as we will see.

Furthermore, each household has the following dynamic constraint:

$$(9) \dot{W}^t = w\bar{L} + (r - \delta)W^t - E^t, \quad \forall t$$

where  $W^t$  is the wealth/assets of household  $t$ ,  $E^t$  are consumption expenditures of household  $t$  and  $\bar{L}$  is the (exogenous) labor-supply of household  $t$ . The latter implies that each household supplies the same amount of labor at the market.

The dynamic constraint (A.17) is standard (compare for example Barro and Sala-i-Martin (2004), p.88). It implies that the wealth of the household increases by labor-income and by (net-) interest-rate-payments and decreases by consumption expenditures.

Note that I assume that the labor supply of each household is exogenously determined.

Consumption expenditures of a household are given by

$$(10) E^t = \sum_i p_i C_i^t, \quad \forall t$$

Each household maximizes its life-time-utility (8) subject to its dynamic constraint (9)-(10). Since this optimization problem is time-separable (due to the assumption of separable time-preference and marginalistic household), it can be divided into two steps; see also, e.g., Foellmi and Zweimüller (2008), p.1320f:

- 1.) Intratemporal (static) optimization: For a given level of consumption-budget ( $E^t$ ), the household optimizes the allocation of consumption-budget across goods.
- 2.) Intertemporal (dynamic) optimization: The household determines the optimal allocation of consumption-budget across time.

### ***Intratemporal optimization:***

The household maximizes its instantaneous utility (8b-d) subject to the constraint (9)-(10), where it regards the consumption-budget ( $E^t$ ) and prices ( $p_i$ ) as exogenous. (Remember that the household is price-taker.) This yields the following optimality conditions:

$$(11a) C_i^t = \frac{\omega_i}{\omega_m} \frac{C_m^t - \theta_m^t}{p_i} + \theta_i^t, \quad \forall i, t$$

$$(11b) E^t = \frac{C_m^t - \theta_m^t}{\omega_m}, \quad \forall t$$

### ***Intertemporal optimization***

Inserting the intratemporal optimality conditions into the instantaneous utility function yields after some algebra (where we use here equations (7c,d) as well):

$$(12a) \quad u(.) = \ln E^t - \bar{\omega}_B \ln p_B + \bar{\omega}, \quad \forall t$$

where

$$(12b) \quad \bar{\omega}_B \equiv \sum_{i=m+1}^n \omega_i$$

$$(12c) \quad \bar{\omega} \equiv \sum_i \omega_i \ln(\omega_i)$$

$$(12d) \quad p_B \equiv \frac{p_i}{p_m} = \left( \frac{\alpha}{\beta} \right)^\beta \left( \frac{1-\alpha}{1-\beta} \right)^{1-\beta} B^{-1} A^{\beta/\alpha} \left( \frac{Kk_m}{Ll_m} \right)^{\beta-\alpha}, \quad i = m+1, \dots, n.$$

Now, we have determined the instantaneous utility as function of consumption-budget (and prices). (Remember that the household is price-taker, i.e. prices are exogenous from the household's point of view.) Inserting (12) into (8) yields

$$(13) \quad U^t = \int_0^\infty (\ln E^t - \bar{\omega}_B \ln p_B + \bar{\omega}) e^{-\rho t} dt, \quad \forall t$$

Thus, the intertemporal optimization problem is to optimize (13) subject to the dynamic constraint (9). This is a typical optimal control problem, which can be solved by using a Hamiltonian.  $E^t$  is control-variable and  $W^t$  is state variable. The prices ( $p_B$ ) and factor prices ( $w$  and  $r - \delta$ ) are regarded by the household as exogenous (since the household is marginalistic and thus price-taker.) Remember that  $\bar{L}$  is exogenous.

In this way we can obtain the following intertemporal optimality condition after some algebra:

$$(14) \quad \frac{\dot{E}^t}{E^t} = r - \delta - \rho, \quad \forall t$$

Note that here and in the following we use use(d) good  $i = m$  as numeraire; thus

$$(15) \quad p_m = 1$$

## **2.3 Aggregates, equilibrium and market clearing**

We define now aggregate output ( $Y$ ) as follows:

$$(16a) \quad Y \equiv \sum_{i=1}^n p_i Y_i$$

Aggregate consumption expenditures ( $E$ ), aggregate consumption of good  $i$  ( $C_i$ ) and aggregate subsistence needs/endowments ( $\theta_i$ ) are given by the sum of their individual counterparts respectively, i.e.

$$(16b) \ E \equiv \sum_t E^t = \sum_{i=1}^n p_i C_i$$

$$(16c) \ C_i = \sum_t C_i^t, \quad \forall i$$

$$(16d) \ \theta_i = \sum_t \theta_i^t, \quad \forall i$$

We assume that all markets are in equilibrium and there is market clearing. That is equations for the goods-market-clearing (5) and (6) hold. There is no unemployment, i.e. labor-market-clearing requires

$$(17) \ L = \sum_t \bar{L}$$

Last not least, since the wealth/assets can only be invested in production-capital ( $K$ ), the following relation must be true if there is capital-market-clearing

$$(18) \ K = \sum_t W^t$$

(see also, e.g. Barro and Sala-i-Martin (2004), p.97). That is, all assets are invested in capital (capital-market-clearing).

By using these aggregate definitions, market clearing conditions and the optimality conditions from the previous sections, we can obtain the following equations describing the development of aggregates, sectors and subsectors after some algebra:

*Subsectors:*

$$(19) \ C_i = \frac{\omega_i}{\omega_m} \frac{C_m - \theta_m}{p_i} + \theta_i, \quad \forall i$$

*Sectors:*

$$(20) \ l_B = \frac{\bar{\omega}_B E}{Y} \left[ \alpha + (1 - \alpha) \frac{l_m}{k_m} \right] \frac{\beta}{\alpha}$$

$$(21) \ l_A = 1 - l_B$$

*Aggregates:*

$$(22) \quad \frac{\dot{E}}{E} = r - \delta - \rho = \left( \frac{l_m}{k_m} \right)^\alpha (1 - \alpha) AL^\alpha K^{-\alpha} - \delta - \rho$$

$$(23) \quad \dot{K} = Y - \delta K - E$$

$$(24) \quad Y = \left[ \alpha + (1 - \alpha) \frac{l_m}{k_m} \right] \left( \frac{k_m}{l_m} \right)^{1-\alpha} AL^\alpha K^{1-\alpha}$$

$$(24) \quad \frac{l_m}{k_m} = 1 - \frac{\beta - \alpha}{\alpha(1 - \alpha)} \bar{\omega}_n \frac{E}{\left( \frac{k_m}{l_m} \right)^{1-\alpha} AL^\alpha K^{1-\alpha}}$$

Note that in these equations  $\alpha$  and  $\beta$  are still functions of sectoral employment, i.e.  $\alpha = \alpha(l_A L)$  and  $\beta = \beta(l_B L)$ . However, we omitted the functional arguments for the sake of clarity of the formulas.

Now the remaining task is to define the functions  $\alpha = \alpha(l_A L)$  and  $\beta = \beta(l_B L)$ . This is done in the following section.

## 2.4 Technological progress and its implementation

### 2.4.1 General breakthroughs and their implementation

We assume that some general technological breakthroughs occur during our observation period. Such breakthroughs may be rare “big breakthroughs”, which have fundamental and long-lasting impacts on the production-structure of the economy. Examples of such breakthroughs may be inventions which lead to Kondratjew-waves (e.g. steam engine, microchip). On the other hand the breakthroughs in our model may be as well some “smaller inventions” which occur more frequently.

We do not model the emergence of such breakthroughs endogenously. In fact, this has been done in endogenous growth theory (in research and development models). Why such breakthroughs occur and at which rate they occur is not in focus of our model anyway. We are rather interested in the pattern of their implementation across sectors. Studying this pattern does not necessarily require endogenous modeling of technological breakthroughs. Therefore, we keep them exogenous. However, we may imagine that such breakthroughs come from basic research.

There are two important aspects regarding the effects of such breakthroughs on production.

- 1.) General technological breakthroughs, such as electricity or microchip, do not directly improve the technologies of sectors and industries. Rather a lot of research, ideas

and/or inventions are necessary to implement a general breakthrough at sector level. For example, improvements in productivity of services, which come from the invention of the microchip, required a lot of ideas and research in software programming and hardware before they increased the productivity of services. That is, general breakthroughs require further breakthroughs to be implemented. Hence, we can distinguish between general breakthroughs and “implementation breakthroughs”.

- 2.) Depending upon the industry or sector, different sorts of ideas are necessary to improve the production technology. The improvement of the productivity of a banker by using a microchip requires different sorts of knowledge/ideas in comparison to the improvement of the productivity of a car-producer by a microchip. In other words, each industry/sector requires its own sector-specific ideas/knowledge/inventions to implement a general breakthrough. There are no or few spill-overs between sectors. (On the other hand, within a sector or industry strong spill-overs may exist.)

These two points are essential for the cross-sector technology-bias, since the need for sector specific ideas to implement breakthroughs constitutes a basis for cross-sector technology bias.

The following assumption is aimed to keep our analysis traceable:

**Assumption 11:** The implementation of a general breakthrough occurs within a finite period of time after the happening of the breakthrough.

Although we may also think of the implementation process as lasting forever, we have to restrict the period of time in order to keep our model solvable. In some sense, we may imagine that the relevant/important part of the implementation occurs in some finite period; the impacts of implementations, which happen long time after the breakthrough has happened, may be less relevant (in comparison to the impacts of the implementations of newer breakthroughs).

#### ***2.4.2 The relation between sector-size and degree of implementation***

We assume that the degree of implementation of a general breakthrough depends upon the size of a sector. We can think of two aspects which relate sector size to degree of implementation:

- 1.) The more labor is employed in a sector, the more creative power is concentrated on the production processes of a sector. Hence, the probability that an implementation-idea arises during the production process is higher. This aspect is closely related to

learning-by-doing-models of endogenous-growth. Overall, we can assume that the more labor is employed in a sector the better the technology improvement through implementation of general breakthroughs. The ideas on implementation of a breakthrough occur accidentally.

- 2.) Implementation research may require the overcoming of large fix costs and sunk-cost and may be associated with high risk. Hence, large sectors may have more financial power to overcome such costs.
- 3.) Of course, there are further aspects, which may relate sector-size (market-size) to degree of implementation, for example the relationship between market-size and competition and incentives to be competitive/ innovative. However, I have no clear ideas on straightforward/unambiguous chains of arguments regarding this. For some related discussion, see e.g. Klevorick et al. (1995) and Pavitt (1985).

Overall, we can summarize this discussion in the following assumption:

**Assumption 12:** There is a positive relationship between sector-size and degree of implementation of breakthroughs. That is, the bigger the sector, the stronger the technology-improvement through implementation of a breakthrough.

Note that this assumption is reflected by the fact that we have expressed our production function parameters as functions of sectoral employment, i.e. we assumed  $\alpha = \alpha(l_A L)$ ,  $\beta = \beta(l_B L)$ ,  $g_A = g_A(l_A L)$  and  $g_B = g_B(l_B L)$ . That is, we measure sector-size by its employment share. However, we could also use capital-input as indicator of sector-size; our key-results would not change (see Theorem 3).

### ***2.4.3 The impacts of implementation on sector technology***

We assume that the implementation of a breakthrough does not only affect the TFP-growth rate but also the output-elasticity of inputs. That is, when a breakthrough is implemented the marginal rate of technological substitution is affected. Thus, the optimal capital intensity of a sector changes through implementation.

Acemoglu (2002) provides a discussion and “microfoundation” of this fact. In a model, where technological progress may be labor-augmenting and capital-augmenting, he shows that technological progress leads to changes of factor-income-shares and marginal rate of technical substitution. If technological progress is labor-augmenting (capital-augmenting), the labor-income-share (capital-income-share) increases.

Since this form of microfoundation is quite complex, we omit it here in order to keep our model traceable. Instead we simply assume that implementation affects the parameter of our Cobb-Douglas production function and we distinguish two cases:

***Assumption 13:***

- 1.) If the implementation is *capital-augmenting*, the output-elasticity (and thus the income-share) of capital increases and the TFP-growth rate increases. That is,  $\alpha(l_A L)$  and/or  $\beta(l_B L)$  decreases and  $g_A$  and/or  $g_B$  increases.
- 2.) On the other hand if the implementation is *labor-augmenting*, the labor-income-share increases and the TFP-growth rate increases. That is,  $\alpha(l_A L)$  and/or  $\beta(l_B L)$  increases and  $g_A$  and/or  $g_B$  increases. We know from standard growth theory (e.g. Ramsey-Cass-Koopmans model) that the equilibrium growth rate of output is among others given by the TFP-growth rate divided by the output-elasticity of labor, e.g.  $g_A / \alpha$ . We will see later that in our model the PBGP-growth rate is given by  $g_A / \alpha$ . Hence, an increase in  $\alpha$  reduces the equilibrium growth rate. In general, technological progress is not associated with a decrease in the equilibrium growth rate. Therefore, in the case of labor-augmenting technological progress we may assume that the increase in  $g_A$  is stronger than the decrease in  $\alpha$ ; hence the equilibrium growth rate  $g_A / \alpha$  increases. (The same arguments apply to the sector B; i.e. we assume that labor-augmenting technological progress increases  $g_B / \beta$ .)

#### ***2.4.4 Summary of the impacts of a technological breakthrough***

Now we can sum up the discussion of section 2.4 as follows:

We assume that several technological breakthroughs occur over time. Each breakthrough is implemented over a finite period of time. The implementation of a breakthrough in a sector improves the technology of all subsectors, which belong to this sector (within-sector spill-overs). However, the subsectors that belong to the other sector do not profit from this implementation (no cross-sector spill-overs). If the breakthrough is implemented in sector A (B),  $g_A$  ( $g_B$ ) increases, where the increase in  $g_A$  ( $g_B$ ) is the stronger, the higher  $l_A L$  ( $l_B L$ ) is. Regarding the impacts on output-elasticities of inputs we have to consider two cases:

- 1.) Implementation is labor-augmenting: Implementation in sector A (B) increases  $\alpha$  ( $\beta$ ), where the increase in  $\alpha$  ( $\beta$ ) is the stronger, the higher  $l_A L$  ( $l_B L$ ) is. The overall-change in  $g_A / \alpha$  ( $g_B / \beta$ ) is positive.
- 2.) Implementation is capital-augmenting: Implementation in sector A (B) decreases  $\alpha$  ( $\beta$ ), where the decrease in  $\alpha$  ( $\beta$ ) is the stronger, the higher  $l_A L$  ( $l_B L$ ) is.

In fact, the model is now fully specified and we can now analyze the development of technology-bias over time. In the next section we analyze the model-dynamics when there are no breakthroughs. Then, in section 4, we analyze how a technology break-through affects this technology-bias. In section 5, we study the impact of structural change on this relationship. Afterward there are several sections, in which we will discuss and summarize the results.

### 3 Dynamics in the absence of technology breakthroughs

In this section we assume that there are no technology-breakthroughs; hence,  $\alpha$ ,  $\beta$ ,  $g_A$  and  $g_B$  are exogenous and constant.

**Definition 1:** A partially balanced growth path (PBGP) is an equilibrium growth path where aggregates ( $Y$ ,  $K$  and  $E$ ) grow at a constant rate.

Note that this definition does require balanced growth for aggregate variables. However, it does not require balanced growth for subsectoral variables (e.g. for subsectoral outputs).

**Lemma 1:** When  $\alpha$ ,  $\beta$ ,  $g_A$  and  $g_B$  are assumed to be exogenous and constant, equations (22) to (24) imply that there exists a unique PBGP, where aggregates ( $Y$ ,  $K$ , and  $E$ ) grow at constant rate  $g^*$  and where  $l_m / k_m$  is constant. The PBGP-growth rate is given by

$$g^* = \frac{g_A}{\alpha} + g_L.$$

**Proof:** is self-evident.

**Lemma 2a:** A saddle-path, along which the economy converges to the PBGP, exists in the neighborhood of the PBGP.

**Lemma 2b:** The PBGP is locally stable.

**Proof:** See APPENDIX.



Lemma 2 ensures that, if a technological breakthrough shifts the economy away from the PBGP, the economy will converge to the PBGP again, provided that the departure from the PBGP is not too strong.

**Lemma 3:** *Along the PBGP, labor is not reallocated between sectors A and B. That is, labor is not reallocated across technology, i.e.  $l_A$  and  $l_B$  are constant.*

**Proof:** This lemma is implied by equations (20) and (21) and by Lemma 1. **Q.E.D.**

Note, however, that it can be easily shown that labor is reallocated within each of the sectors A and B. Furthermore, note that in section 5 we will allow for labor reallocation between sectors A and B.

#### **4 Dynamics in the absence of cross-technology structural change: self-reinforcing technology-convergence/divergence**

Now we study how a technology breakthrough affects the situation described in the previous section. Furthermore, we assume an initial situation where the technologies differ across sectors.

As discussed in section 2.4, a technology breakthrough induces a finite period of time where the breakthrough is implemented. During this period of implementation, the sectoral technologies change. When the implementation-period is finished, the economy converges to the new PBGP.<sup>3</sup>

As we will show now, the change in sectoral technologies during the implementation-period leads to a change of cross-technology factor-allocation ( $l_A, l_B$ ). The change in this allocation affects the ability of the sectors to implement the next breakthrough (see e.g. section 2.4.4). Hence, over time (and over a sequence of breakthroughs) the sectoral technologies may converge or diverge over time.

We will elaborate now the factors that determine whether the technologies converge or diverge. To do so we calculate the employment shares along the PBGP. By using equations (20)/(21) and some of the equations from APPENDIX we obtain the following employment shares along the PBGP:

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<sup>3</sup> Note that the discussion in APPENDIX implies that the PBGP-values of the economy depend upon the technology parameters of the production functions. Hence, we know that technology-implementation leads to a departure from the old PBGP and convergence to the new PBGP (after the period of implementation is finished).

$$(25) \quad l_B^* = \beta \frac{1 - \alpha + \alpha \frac{\rho + \delta + g_L + \frac{g_A}{\alpha}}{\rho}}{(\beta - \alpha)\bar{\omega}_B + \alpha \frac{\rho + \delta + g_L + \frac{g_A}{\alpha}}{\rho}}$$

$$(26) \quad l_A^* = 1 - l_B^*$$

An asterisk denotes here the PBGP-value.

These equations imply that:

$$(27) \quad \frac{\partial l_B^*}{\partial \beta} > 0 \quad \frac{\partial l_A^*}{\partial \beta} < 0$$

$$(28) \quad \frac{\partial l_B^*}{\partial \alpha} < 0 \quad \frac{\partial l_A^*}{\partial \alpha} > 0$$

Note that in the calculation of (28), we assumed that an increase in  $\alpha$  is associated with an increase in  $g_A$ , such that  $g_A / \alpha$  increases if  $\alpha$  increases, according to our discussion in point 2) of Assumption 13.

Now we can postulate the following:

**Theorem 1:** *Implementation of breakthroughs leads to a change in the PBGP-values of the employment-shares of sectors A and B ( $l_A$  and  $l_B$ ). That is, implementation of breakthroughs leads to cross-technology labor-reallocation.*

**Proof:** This theorem is implied by equations (27)/(28) and Assumption 13. **Q.E.D.**

Note that this theorem holds even if implementation is such that  $\alpha$  and  $\beta$  increase by the same amount. The reason is the following: a change in  $\alpha$  and/or  $\beta$  leads to a change in average economy-wide output-elasticity of labor; i.e. the average capital-intensity of the economy ( $K/L$ ) changes (as implied by Lemma 1). Since sector A (and especially subsector m) produces capital, a change in average capital intensity affects the employment-share of sector A, ceteris paribus. (See also equations (20) and (21) and note that aggregate investment is equal to  $Y - E$ ).

Of course the change in  $l_A$  and  $l_B$  comes not only from this change in aggregate capital demand. Additionally, the following fact determines the allocation  $(l_A, l_B)$ : Optimal factor-

allocation across sectors is determined by the profitability of factor-inputs in sectors, since we assume perfect factor mobility across sectors. That is, sectors, which have relatively high output-elasticity of labor, employ more labor in comparison to sectors, which have relatively low output-elasticity of labor. Therefore, a change in cross-sector technology-bias ( $\frac{\alpha}{\beta}$ ) induces an adjustment of factor-input-shares, such that optimal factor-input-shares are achieved. (To understand this fact, note that equations (20) and (21) imply that a change in  $\frac{\alpha}{\beta}$  induces a change in  $(l_A, l_B)$  even if we keep  $l_m/k_m, E$  and  $Y$  constant.)

Hence, we can postulate the following corollary:

**Corollary from Theorem 1:** *The change in cross-technology labor reallocation  $(l_A, l_B)$  postulated in Theorem 1 comes from two forces:*

- 1.) *A change in sectoral output-elasticities of inputs affects the aggregate capital demand; since only sector A produces capital, the changes in aggregate capital demand induce factor-reallocation across sectors A and B.*
- 2.) *A change in cross-sector technology-bias  $(\alpha/\beta)$  changes the optimal factor-allocation across sectors. Thus, when there is factor-mobility across sectors, cross-sector factor reallocations are induced in order to achieve the new optimal cross-sector factor allocation.*

Theorem 1 is the basis for technology convergence and divergence, since the cross-technology labor-reallocation changes the sector's relative potential to implement future breakthroughs (according to Assumption 13).

According to the discussion in section 2.4.4 we distinguish between two cases.

1.) *Implementation is labor-augmenting*

According to our discussion in section 2.4.4 and equations (27)/(28) we can postulate the following causal chain: A breakthrough leads to implementation in sector A (B), which leads to increase in  $\alpha$  ( $\beta$ ), which leads to increase in  $l_A L$  ( $l_B L$ ), which leads to better implementation of the next breakthrough.

2.) *Implementation is capital-augmenting*

According to our discussion in section 2.4.4 and equations (27)/(28) we can postulate the following causal chain: A breakthrough leads to implementation in sector A (B), which leads to decrease in  $\alpha$  ( $\beta$ ), which leads to decrease in  $l_A L$  ( $l_B L$ ), which leads to weaker implementation of the next breakthrough.

Hence, we can postulate the following theorem:

**Theorem 2:** *a) If implementation is labor-augmenting, the implementation of a breakthrough in a sector increases the sector's employment-share. If implementation is capital-augmenting, the implementation of breakthrough in a sector reduces the sector's employment-share.*

*b) If implementation is labor-augmenting, the implementation of a breakthrough in a sector increases the sector's ability to implement the next breakthrough improves. If implementation is capital-augmenting, the implementation of breakthrough in a sector reduces the sector's ability to implement the next breakthrough.*

**Proof:** This theorem is implied by equations (27)/(28) and Assumption 13. **Q.E.D.**

**Corollary from Theorem 2:** *a) Labor-augmenting technological progress tends to increase the technology-bias. b) Capital-augmenting technological progress tends to reduce the technology-bias.*

**Proof:** Now assume that we have a small sector (few labor is employed in this sector), which has not implemented much breakthroughs in the past; thus it has a backward technology. Furthermore, assume that we have a big sector (much labor is employed in this sector), which implemented a lot of technology in the past. Hence, the big sector is much technologically advanced. If breakthroughs tend to be implemented in labor-augmenting fashion (in the whole economy), the technological bias is increased: Due to its size, the big sector is able to implement the next breakthrough very well; hence; the output-elasticity of labor will increase strongly in the big sector; therefore, the employment of the big sector will increase strongly as well; thus the next breakthrough will be implemented much better in comparison to the last breakthrough. In contrast, the small sector's employment will increase only by a small amount and the next breakthrough will be implemented only little better in comparison to the last breakthrough. Hence, overall, the ability to implement the breakthroughs increases strongly in the big sector and increases weakly in the small sector over time. Hence, technology-bias increases over time. Of course there is a limit to this process, since the employment-share and the output-elasticity of labor cannot exceed 1. Hence, it may be possible, that the implementation process stops (or slows down increasingly) after some time in the big sector, and the small-sector starts to catch-up. Analogous arguments can be used to show that part b) of the corollary is true. **Q.E.D.**

**Theorem 3:** *Theorem 2 and its Corollary hold, even if we assume that the ability to implement breakthroughs is positively correlated with the amount of capital employed in the sector. That*

is, Theorem 2 and its Corollary hold, even if we assume that instead of relations (1b,c) and (2b,c), the following relations were true:

$$\alpha = \alpha(k_A K), \quad g_A = g_A(k_A K), \quad \beta = \beta(k_B K) \quad \text{and} \quad g_B = g_B(k_B K)$$

where the capital-level of the sector is positively correlated with the ability to implement

breakthroughs and where  $k_A \equiv \sum_{i=1}^m k_i$  and  $k_B \equiv \sum_{i=m+1}^n k_i$ .

**Proof:** By using equations (7b), (C.6) and (25), it can be shown that  $k_B$  is given by

$$k_B = (1 - \beta) \frac{\alpha + (1 - \alpha) \frac{\rho}{\rho + \delta + g_L + g_A / \alpha}}{1 - \alpha - (\beta - \alpha) \bar{\omega}_B}. \quad \text{Which implies, for example, that } \frac{\partial k_B}{\partial \beta} > 0. \text{ Hence,}$$

for example, by using the arguments from above it can be shown that it is still true that labor-augmenting progress increases sector's ability to implement future breakthroughs and capital-augmenting progress reduces sector's ability to implement future breakthroughs. The rest of the proof is trivial. **Q.E.D.**

Hence, this theorem implies that our results are not sensitive to the measure of sector-size. It does not matter, whether we measure sector-size by labor-employment or by capital-employment.

## 5 The impacts of cross-technology structural change

We define here structural change as changes in employment shares of (sub)sectors. That is, we say that structural change takes place if (sub)sectoral employment shares change. It can be easily shown that structural change takes place even along the PBGP in our model.

Cross-technology structural change means here that labor is reallocated across technology. Hence, in our model, changes in  $(l_A, l_B)$  indicate that cross-technology structural change takes place.

In this section we show that cross-technology structural change can affect the technology-bias; i.e. structural change can lead to technology convergence/divergence.

Structural change is a well known fact and has been modeled by, e.g., Baumol (1967), Kongamut et al. (2001), Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008) and Foellmi and Zweimueller (2008).

The literature on structural change implies that structural change (across technology) is caused by several determinants; especially, non-homothetic preferences (as modeled by Kongsamut et al. 2001), cross-sector-differences in TFP-growth (as modeled by Ngai and

Pissarides 2007) and cross-sector-differences in capital-intensities (as modeled by Acemoglu and Guerrieri 2008). Although all these determinants are included in our model, our parameter-restriction (8c,d) prevent them from taking effect across technology; hence,  $l_A$  and  $l_B$  are constant along the PBGP. Instead of deviating from parameter-restrictions (8c,d), which would make our analysis quite complicated, we model structural change as follows: We assume that the utility-weights of technology change exogenously; i.e.  $\bar{\omega}_B$  changes. This leads to changes in demand and to structural change across technology. Hence, we do not model the structural change (mentioned above) explicitly. Note that our way of modeling has, in fact, very similar dynamic implications in comparison to the implications of structural change models, which model the structural change determinants explicitly. In some sense, our way of modeling only omits the microfoundations, which are anyway provided by the previous literature.

**Lemma 4:** *A onetime change in  $\bar{\omega}_B$  causes a deviation from the old PBGP and convergence to the new PBGP.*

**Proof:** This lemma is implied by equations (C.5) and Lemma 2. **Q.E.D.**

**Theorem 4:** *a) A onetime increase in  $\bar{\omega}_B$  induces labor-reallocation from sector A to sector B, provided that  $\alpha > \beta$ ; i.e.  $l_A$  decreases and  $l_B$  increases. b) A onetime increase in  $\bar{\omega}_B$  induces labor-reallocation from sector B to sector A, provided that  $\alpha < \beta$ ; i.e.  $l_A$  increases and  $l_B$  decreases. c) The stronger the difference between  $\alpha$  and  $\beta$ , the stronger the change in  $l_A$  and  $l_B$ . d) Analogous results are obtained for the case that there is a onetime decrease in  $\bar{\omega}_B$ .*

**Proof:** This lemma is implied by equation (25).

**Corollary from Theorems 2 and 4:** *Depending on the structural change pattern (i.e. whether  $\bar{\omega}_B$  increases or decreases), structural change may lead to technology convergence or divergence between sectors A and B.*

This corollary will be discussed further during the discussion of the relevance of our results for existing literature in section 8.

## **6 Summary of the factors which determine technology-convergence/divergence**

In our model we have discussed three main factors which determine the technology bias: 1.) the sort of implementation (labor-augmenting vs. capital-augmenting), 2.) capital-production and 3.) structural change:

1.) Our model implies that sectoral technologies diverge, if breakthroughs are such that in most sectors/industries they are implemented in the labor-augmenting way. Labor-augmenting technological progress reduces the optimal capital-intensity. The productivity boost in the big sectors, which implement the breakthroughs, is only attainable if their labor-input is increased. However, in contrast to capital, labor cannot be accumulated; hence, the big sectors pull labor from small sectors. This fact reduces the ability of small sectors to implement future breakthroughs (less creative potential). In contrast, capital-augmenting implementation increases the capital intensity. Hence, big sectors can release some part of their labor-force, since it can be substituted by capital (which can be accumulated endogenously). This labor-force is reallocated to small sectors, which increases their creative potential and thus ability to implement future breakthroughs. (See also the Corollaries from Theorems 1 and 2.)

2.) Changes in capital-intensity of sectors affect the output of the capital-producing sector. In general, we can say that the manufacturing sector produces the largest part of capital; see e.g. Kongamut et al. (1997). Hence, implementation of breakthroughs in any sector affect the employment-share of the capital-producing sector and hence its ability to implement future technological breakthroughs. (See also the Corollary from Theorem 1.)

3.) As mentioned in the previous sectors, structural change is caused by different determinants. Hence, all these determinants affect the cross-sector-factor allocation and thus relative abilities to implement technology. (See also the Corollary from Theorems 2 and 4).

Last not least, there are several further factors, which are not explicitly modeled here, but which affect the model outcome. These factors rather depend on the nature of the breakthrough and/or on the nature the sectoral output. Some discussion on the nature of sectors and its implications for the scope of technological progress can be found, e.g. in the essays by Wolfe (1955), Baumol (1967) and Klevorick et al. (1995).

1.) It may happen that some breakthroughs are rather implementable in capital-augmenting or labor-augmenting way. That is, it may depend on the nature of the breakthrough whether it is primarily labor-augmentingly or capital-augmentingly implemented.

2.) Some sectors (e.g. services) may implement a breakthrough (e.g. electricity) in a primarily labor-augmenting way and other sector (e.g. agriculture) may implement the same breakthrough (electricity) in a primarily capital-augmenting way. This would yield technology-divergence. That is, in which way the breakthroughs are primarily implemented may depend on the nature of the output of a sector.

3.) Simply speaking, it may also be accidental that over some period of time breakthroughs occur, which are implementable in only one sector; which would yield technology convergence. As well technical feasibility may dictate the order of breakthroughs and thus development of the technology-bias.

Overall, there seems to be some accidental or technical component which determines the timely order of breakthroughs and their implementability in a sector.

## 7 Empirical evidence on technology convergence

In this section we construct a simple measure of cross-sector technology-bias and analyze its development over time. Since quite easy measurable, we use sectoral employment shares as indices of sectoral technology (output-elasticity of labor).

The measure of technology-bias, which we suggest is the following:

$$(29) \text{Var}_1 = \sum_i l_i^0 \left( \frac{1}{\alpha_i^t} - \frac{\bar{1}}{\alpha_i} \right)^2$$

where

$$(30) \frac{\bar{1}}{\alpha_i} \equiv \sum_i l_i^0 \frac{1}{\alpha_i^t}$$

or alternatively

$$(31) \text{Var}_2 = \sum_i l_i^T \left( \frac{1}{\alpha_i^t} - \frac{\bar{1}}{\alpha_i} \right)^2$$

where

$$(32) \frac{\bar{1}}{\alpha_i} \equiv \sum_i l_i^T \frac{1}{\alpha_i^t}$$

where  $i$  denotes sector;  $\alpha_i^t$  is the labor-income share of sector  $i$  at time  $t$  (or approximately output-elasticity of labor);  $l_i^t$  is the employment-share of sector  $i$  at time  $t$ ;  $l_i^0$  is the employment-share of sector  $i$  at the beginning of the observation period;  $l_i^T$  is the employment-share of sector  $i$  at the end of the observation period.



That is, our measure of cross-sector technology variation is simply a variance. Note that it makes sense to use the employment-shares as weighting-factors in variance-calculation, since

it is theoretically reasonable:  $\sum_i l_i \frac{1}{\alpha_i} = \sum_i l_i \frac{p_i Y_i}{w l_i L} = \frac{\sum_i p_i Y_i}{w L} = \frac{Y}{w L}$ . Hence, this term is equal

to the reciprocal of the aggregate labor-income-share.

Note that we do not use the actual employment-shares to calculate the technology-variance, but the employment-shares from the beginning of the observation period. (For control we also calculate the variance by using employment-shares at the end of the period.) The reason for this is that the employment shares change over time and they would eventually bias the technology development. Hence, with actual employment shares we could not say whether the technology-variance decreased or whether some changes in employment-shares led to the result.

For the calculations in this section I use the data for the U.S.A., which is available at the website of the U.S. Department of Commerce (Bureau of Economic Analysis). I use the U.S.-Gross-Domestic-Product-(GDP)-by-Industry-Data, which is based on the sector-definition from the “Standard Industrial Classification System”, which defines the following sectors:

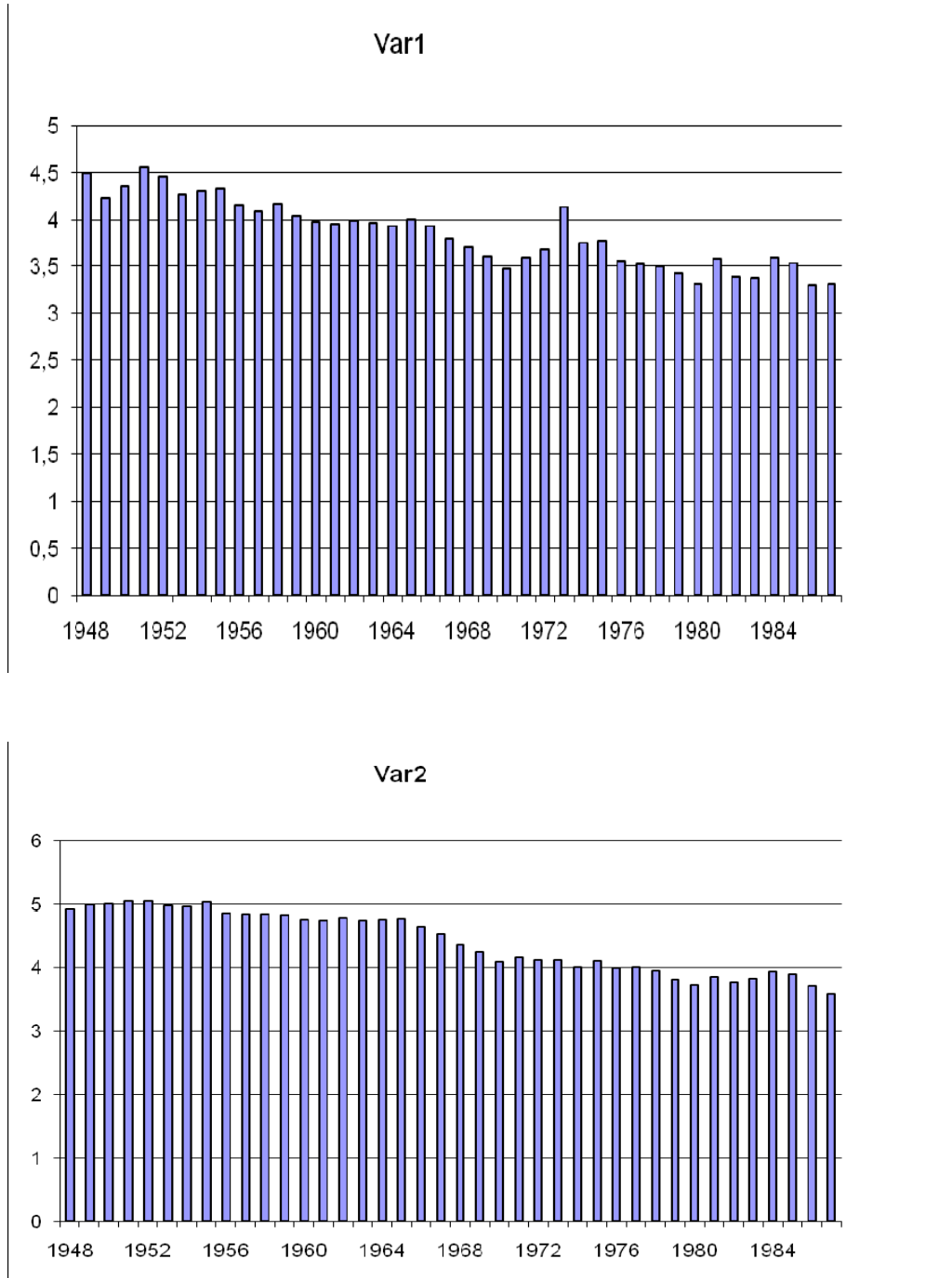
- (1) Agriculture, forestry, and fishing
- (2) Mining
- (3) Construction
- (4) Manufacturing
- (5) Transportation and public utilities
- (6) Wholesale trade
- (7) Retail trade
- (8) Finance, insurance, and real estate
- (9) Services

My calculations are based on the data for the period 1948-1987. Uniform data for longer time-periods is not available, since the “Standard Industrial Classification System” has been modified over time (hence, the sector definition after 1987 is not the same as the sector definition before 1987).

To calculate the sectoral labor-income-shares ( $\alpha_i$ ) I divide “(Nominal) Compensation of Employees” by “(Nominal) Value Added by Industry” in each sector. The sectoral

employment shares ( $l_i$ ) are calculated by using the sectoral data on “Full-time Equivalent Employees”. (This approach is similar to that used by Acemogu and Guerrieri (2008)).

The following two figures depict the development of the two measures over time



Both measures imply that the cross-sector technology-variance is decreasing. Hence, it seems that technologies converged during the observation period.

In fact, our model-explanations of this finding are:

- Over the long-run in the past, technological progress has been capital-augmenting; hence technologies are converging now.
- Structural change has increased the employment in technologically backward sectors much. Therefore, they are able to implement new breakthroughs and are catching-up.

These explanations are discussed in the next chapter.

## 8 Relevance of the results for existing literature

The results of this paper are closely related to the theoretical and empirical literature about Baumol's cost disease. Examples of this literature are the papers by Baumol et al. (1967), Ngai and Pissarides (2007), Nordhaus (2008) and Acemoglu and Guerrieri (2008). In fact, this literature implies that factors are reallocated to technologically backward sectors. In part, this literature implies that therefore the real GDP-growth rate decreases. Our model and our empirical evidence imply that this reallocation process leads to technological catching-up of backward sectors. Hence, the technologies may converge over the very long-run. Therefore, the slow-down of the real GDP-growth rate may cease and structural change may slow-down (since in this literature the technology-bias is a key determinant of the direction and strength of structural change).

There is some literature, which models technologically heterogeneous sectors; especially, this literature assumes that output-elasticities of inputs (i.e. the  $\alpha_i$ 's) differ across sectors, but assumes that this technology-bias is exogenous. Examples are the models by Kongsamut et al. (1997), Echevarria (1997, 2000), Golin et al (2002), Meckl (2002), Jensen and Larsen (2004), Greenwood and Uysal (2005), Bah (2007), Golin et al. (2007), Zuleta and Young (2007), Acemoglu and Guerrieri (2008) and Buera and Kaboski (2009). Our model adds a micro-foundation of the technology-bias of these models in part. Especially, our model and our empirical evidence imply that eventually this technology-bias may vanish in the future and complicated modeling of this technology-bias may not be necessary anymore. Furthermore, our model implies that the predictions of these models regarding the structural change dynamics may be biased. Our model results imply that when the technology-bias is endogenized several relationships between structural change and technology arise, which are omitted in the models with exogenous technology-bias:

- 1.) Technology implementation causes by itself structural change (Theorem 2)
- 2.) Technology-change slows-down/accelerates structural change patterns (Theorem 2 and 4).
- 3.) Structural change affects the technology bias (Corollary from Theorems 2 and 4).

In fact point 1) implies that technology implementation is itself a structural change determinant. Hence, it should be added to the traditional structural change determinants, which are studied in the literature (non-homothetic preferences, exogenous TFP-bias and exogenous bias in output-elasticities of inputs).

The theoretical results by Acemoglu and Guerrieri (2008) imply that cross-sector-differences in factor-income-shares are not compatible with the relatively stable development of the aggregate economy, which is described by “Kaldor’s stylized facts of economic growth”. (This result is supported by Kongsamut et al. (1997)). Nevertheless, in the simulation of their model they find that the model satisfies the Kaldor-facts approximately. Our model and our empirical evidence imply the following explanation of this fact: It is possible that the sectoral technologies have already converged significantly. Hence, the cross technology-bias is sufficiently small and has therefore relatively weak impacts on the aggregate economy.

Valentinyi and Herrendorf (2008) show that capital-intensities are relatively similar at low degree of disaggregation (i.e. when comparing agriculture, manufacturing and services). Our model and our empirical evidence imply that this finding may have come from the technology-convergence-tendencies described in our paper.

Last not least: neoclassical endogenous growth theory studies the explanation of aggregate production functions (or: aggregate technological progress). However, aggregate production functions are some weighted averages of sectoral production functions. Hence, an endogenous growth theory without the study of the question how technologies are implemented at sector level seems to be incomplete. Since our model is neoclassical in many aspects, our model may be regarded as a contribution to the neoclassical endogenous growth theory.

## 9 Concluding remarks

We have presented a model of technology-implementation at sector level. We have elaborated several aspects of this process, which can explain and predict the development of the technology-bias at sector level. The focus of our model has been on the technology-bias regarding capital-intensities, output-elasticities of inputs and factor-income-shares. That is, we have analyzed why not all sectors have the same capital-intensity (output-elasticity of capital) and how this technology-bias develops over time. These questions are especially important, since many models assume the existence of such a technology-bias and since endogenous technology-bias would affect their results. These aspects among others have been discussed in section 8.

As discussed in section 6, our model provides three main factors which determine whether technologies converge/diverge at sector level: 1.) sort of implementation of general technological breakthroughs (labor-augmenting vs. capital-augmenting implementation), 2.) changes in capital-production due to changes in average capital intensity and 3.) structural change.

Our empirical evidence (in section 7) implies that technologies have converged in the USA between 1948 and 1987. Our model explanation of this fact is that technological breakthroughs have been implemented in capital-augmenting manner in the past and that structural change (increasing factor-shares of technologically-backward sectors) has increased the ability of backward-sectors to implement general technological breakthroughs.

Note that our model may be regarded as a baseline model only; it shows how technology implementation and cross-sector technology-convergence can be modeled. It could be used to study several further questions, which are important for cross-sector technology convergence. Especially, we think that it could be interesting to study the question of how do patent rights (monopolistic competition) and implementation-research-costs affect technological convergence.

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## APPENDIX

First, I show by using linear approximation that the saddle-path-feature of the PBGP is given (Lemma 2a). Then I prove local stability by using a phase diagram (Lemma 2b).

### Existence of a saddle-path (Lemma 2a)

First I rearrange the aggregate equation system (22)-(24) as follows:

$$\dot{\hat{K}} = \left( \frac{l_m}{k_m} \right)^{\alpha-1} (\alpha + (1-\alpha) \frac{l_m}{k_m}) \hat{K}^{1-\alpha} - \hat{E} - (\delta + g_L + \frac{g_A}{\alpha}) \hat{K} \quad (C.1)$$

$$\frac{\dot{\hat{E}}}{\hat{E}} = (1-\alpha) \left( \frac{l_m}{k_m} \right)^{\alpha} \hat{K}^{-\alpha} - \delta - \rho - g_L - \frac{g_A}{\alpha} \quad (C.2)$$

$$\frac{l_m}{k_m} = 1 - \frac{\beta - \alpha}{\alpha(1-\alpha)} \bar{\omega}_B \frac{\hat{E}}{\hat{K}^{1-\alpha} \left( \frac{l_m}{k_m} \right)^{\alpha-1}} \quad (C.3)$$

where aggregate variables are expressed in “labor-efficiency units”, i.e. they are divided by

$$LA^{\frac{1}{\alpha}}; \text{ hence } \hat{K} \equiv \frac{K}{LA^{\frac{1}{\alpha}}} \text{ and } \hat{E} \equiv \frac{E}{LA^{\frac{1}{\alpha}}}.$$

These equations imply that  $\hat{K}$ ,  $\hat{E}$  and  $l_m/k_m$  have the following values along the PBGP

$$\hat{K}^* = \sigma^{\frac{1}{\alpha}} \left( \frac{l_m}{k_m} \right)^* \quad (C.4)$$

$$\hat{E}^* = \alpha \sigma^{\frac{1-\alpha}{\alpha}} + \rho \sigma^{\frac{1}{\alpha}} \left( \frac{l_m}{k_m} \right)^* \quad (C.5)$$

$$\left( \frac{l_m}{k_m} \right)^* = \frac{\alpha}{1-\alpha} \frac{1-\alpha - (\beta-\alpha) \bar{\omega}_B}{\alpha + (\beta-\alpha) \bar{\omega}_B} \frac{\rho}{1-\alpha} \sigma \quad (C.6)$$

$$\text{where } \sigma \equiv \frac{1-\alpha}{\delta + \rho + g_L + \frac{g_A}{\alpha}}$$

where an asterisk denotes the PBGP-value of the corresponding variable.

The proof of local saddle-path-stability of the PBGP is analogous to the proof by Acemoglu and Guerrieri (2008) (see there for details and see also Acemoglu (2009), pp. 269-273, 926).

First, I have to show that the determinant of the Jacobian of the differential equation system (C.1)-(C.2) (where  $l_m/k_m$  is given by equation (C.3)) is different from zero when evaluated at

the PBGP (i.e. for  $\hat{K}^*, \hat{E}^*, \left(\frac{l_m}{k_m}\right)^*$  from equations (C.4)-(C.6)). This implies that this differential equation system is hyperbolic and can be linearly approximated around  $\hat{K}^*, \hat{E}^*, \left(\frac{l_m}{k_m}\right)^*$  (Grobman-Hartman-Theorem; see as well Acemoglu (2009), p. 926, and Acemoglu and Guerrieri (2008)). The determinant of the Jacobian is given by:

$$|J| = \begin{vmatrix} \frac{\partial \dot{\hat{K}}}{\partial \hat{K}} & \frac{\partial \dot{\hat{K}}}{\partial \hat{E}} \\ \frac{\partial \dot{\hat{E}}}{\partial \hat{K}} & \frac{\partial \dot{\hat{E}}}{\partial \hat{E}} \end{vmatrix} = \frac{\partial \dot{\hat{K}}}{\partial \hat{K}} \frac{\partial \dot{\hat{E}}}{\partial \hat{E}} - \frac{\partial \dot{\hat{E}}}{\partial \hat{K}} \frac{\partial \dot{\hat{K}}}{\partial \hat{E}} \quad (\text{C.7})$$

The derivatives of equations (C.1)-(C.2) are given by:

$$\begin{aligned} \frac{\partial \dot{\hat{K}}}{\partial \hat{K}} &= (1-\alpha) \hat{K}^{-\alpha} \left( \alpha \left( \frac{l_m}{k_m} \right)^{\alpha-1} + (1-\alpha) \left( \frac{l_m}{k_m} \right)^{\alpha} \right) \\ &\quad + \hat{K}^{1-\alpha} \left( -(1-\alpha) \alpha \left( \frac{l_m}{k_m} \right)^{\alpha-2} + \alpha (1-\alpha) \left( \frac{l_m}{k_m} \right)^{\alpha-1} \right) \frac{\partial \left( \frac{l_m}{k_m} \right)}{\partial \hat{K}} - \left( \delta + g_L + \frac{g_A}{\alpha} \right) \\ \frac{\partial \dot{\hat{K}}}{\partial \hat{E}} &= \hat{K}^{1-\alpha} \left( -(1-\alpha) \alpha \left( \frac{l_m}{k_m} \right)^{\alpha-2} + \alpha (1-\alpha) \left( \frac{l_m}{k_m} \right)^{\alpha-1} \right) \frac{\partial \left( \frac{l_m}{k_m} \right)}{\partial \hat{E}} - 1 \\ \frac{\partial \dot{\hat{E}}}{\partial \hat{K}} &= (1-\alpha) \hat{E} \left( -\alpha \hat{K}^{-\alpha-1} \left( \frac{l_m}{k_m} \right)^{\alpha} + \alpha \hat{K}^{-\alpha} \left( \frac{l_m}{k_m} \right)^{\alpha-1} \frac{\partial \left( \frac{l_m}{k_m} \right)}{\partial \hat{K}} \right) \\ \frac{\partial \dot{\hat{E}}}{\partial \hat{E}} &= \left( (1-\alpha) \left( \frac{l_m}{k_m} \right)^{\alpha} \hat{K}^{-\alpha} - \delta - \rho - g_L - \frac{g_A}{\alpha} \right) \\ &\quad + (1-\alpha) \hat{E} \hat{K}^{-\alpha} \alpha \left( \frac{l_m}{k_m} \right)^{\alpha-1} \frac{\partial \left( \frac{l_m}{k_m} \right)}{\partial \hat{E}} \end{aligned} \quad (\text{C.8})$$

where the derivatives of equation (C.3) are given by

$$\begin{aligned}
\frac{\partial \left( \frac{l_m}{k_m} \right)}{\partial \hat{E}} &= - \frac{\frac{\beta - \alpha}{\alpha(1 - \alpha)} \bar{\omega}_B \left( \frac{l_m}{k_m} \right)^{1 - \alpha}}{\left( 1 + \frac{\beta - \alpha}{\alpha} \bar{\omega}_B \frac{\hat{E}}{\hat{K}^{1 - \alpha} \left( \frac{l_m}{k_m} \right)^\alpha} \right)} \\
\frac{\partial \left( \frac{l_m}{k_m} \right)}{\partial \hat{K}} &= \frac{\frac{\beta - \alpha}{\alpha} \bar{\omega}_B \frac{\hat{E}}{\hat{K}^{2 - \alpha} \left( \frac{l_m}{k_m} \right)^{\alpha - 1}}}{\left( 1 + \frac{\beta - \alpha}{\alpha} \bar{\omega}_B \frac{\hat{E}}{\hat{K}^{1 - \alpha} \left( \frac{l_m}{k_m} \right)^\alpha} \right)} \quad (C.9)
\end{aligned}$$

Inserting the derivatives (C.8) and (C.9) into (C.7) and inserting the PBGP-values from equations (C.4)-(C.6) yields after some algebra the following value of the determinant of the Jacobian evaluated at the PBGP:

$$|J|^* = \frac{-\alpha(\beta - \alpha)\bar{\omega}_B \frac{\hat{E}^*}{\hat{K}^*} \left[ \rho + \frac{\alpha}{(\beta - \alpha)\bar{\omega}_B} \frac{1 - \alpha}{\sigma} \right]}{\alpha + (\beta - \alpha)\bar{\omega}_B \frac{\hat{E}^*}{(\hat{K}^*)^{1 - \alpha} \left[ \left( \frac{l_m}{k_m} \right)^* \right]^\alpha}} \quad (C.10)$$

This equation can be transformed further by using equations (C.4)-(C.6):

$$|J|^* = \frac{-\frac{\hat{E}^*}{\hat{K}^*} \frac{\alpha}{\sigma} [1 - \alpha - (\beta - \alpha)\bar{\omega}_B]}{\alpha \left( \frac{l_m}{k_m} \right)^* + 1 - \alpha} \quad (C.11)$$

Note that  $\frac{\hat{E}^*}{\hat{K}^*}$  and  $\left( \frac{l_m}{k_m} \right)^*$  are positive and are given by equations (C.4)-(C.6).

We can see that the determinant evaluated at PBGP is different from zero. Hence, the PBGP is hyperbolic. Furthermore, equations (C.10) and (C.11) imply that  $|J|^* < 0$ . (Equation (C.10) implies that  $|J|^* < 0$ , if  $\beta - \alpha > 0$ ; equation (C.11) implies that  $|J|^* < 0$ , if  $\beta - \alpha < 0$  as well.) Our differential equation system consists of two differential equations ((C.1) and (C.2)) and of two variables ( $\hat{E}$  and  $\hat{K}$ ), where we have one state and one control-variable. Hence, saddle-

path-stability of the PBGP requires that there exist one negative (and one positive) eigenvalue of the differential equation system when evaluated at PBGP (see also Acemoglu and Guerrieri (2008) and Acemoglu (2009), pp. 269-273). Since  $|J|^* < 0$  we can be sure that this is the case. ( $|J|^* < 0$  can exist only if one eigenvalue is positive and the other eigenvalue is negative. If both eigenvalues were negative or if both eigenvalues were positive, the determinant  $|J|^*$  would be positive.) Therefore, the PBGP is locally saddle-path-stable, i.e. Lemma 2a is proved. *Q.E.D.*

### Local stability (Lemma 2b)

I study here only the case where output-elasticity of capital in investment goods industries ( $i=m$ ) is relatively low in comparison to the output-elasticity of capital in the consumption goods industries ( $\forall i \neq m$ ), i.e. I assume  $\beta < \alpha$ . This is consistent with the empirical evidence presented and discussed in Valentinyi and Herrendorf (2008) (see there especially p.826). Note, however, that the qualitative stability results for the other case (i.e.  $\beta > \alpha$ ) are the same.

To show the stability-features of the PBGP, the three-dimensional system (C.1)-(C.3) has to be transformed into a two dimensional system, in order to allow me using a phase-diagram.

By defining the variable  $\kappa \equiv \frac{\hat{K}k_m}{l_m}$ , the system (C.1)-(C.3) can be reformulated as follows

(after some algebra):

$$(C.12) \quad \frac{\dot{\hat{E}}}{\hat{E}} = (1 - \alpha)\kappa^{-\alpha} - \left( \delta + \rho + g_L + \frac{g_A}{\alpha} \right)$$

$$(C.13) \quad \frac{\dot{\kappa}}{\kappa} = \frac{\kappa^{-\alpha} - (\delta + g_L + \frac{g_A}{\alpha}) - \frac{\hat{E}}{\kappa} \left( 1 - \frac{\alpha - \beta}{\alpha(1 - \alpha)} \bar{\omega}_B \rho \kappa^\alpha \right)}{1 + \frac{\alpha - \beta}{1 - \alpha} \bar{\omega}_B \frac{\hat{E}}{\kappa^{1 - \alpha}}}$$

I can focus attention on showing that the stationary point of this differential equation system is stable:  $\kappa$  and  $\hat{E}$  are jointly in steady state only if  $\hat{K}$ ,  $\hat{E}$  and  $k_m/l_m$  are jointly in steady state; furthermore,  $\hat{K}$ ,  $\hat{E}$  and  $k_m/l_m$  are jointly in steady state only if  $\kappa$  and  $\hat{E}$  are jointly in steady state. Therefore, the proof of stability of the stationary point of system (C.12)-(C.13)

implies stability of the stationary point of system (C.1)-(C.3). Hence, in the following I will prove stability of the stationary point of system (C.12)-(C.13).

It follows from equations (C.12) and (C.13) that the steady-state-loci of the two variables are given by

$$(C.12a) \quad \frac{\dot{\hat{E}}}{\hat{E}} = 0: \quad \kappa^* = \left( \frac{1-\alpha}{\delta + \rho + g_L + \frac{g_A}{\alpha}} \right)^{\frac{1}{\alpha}}$$

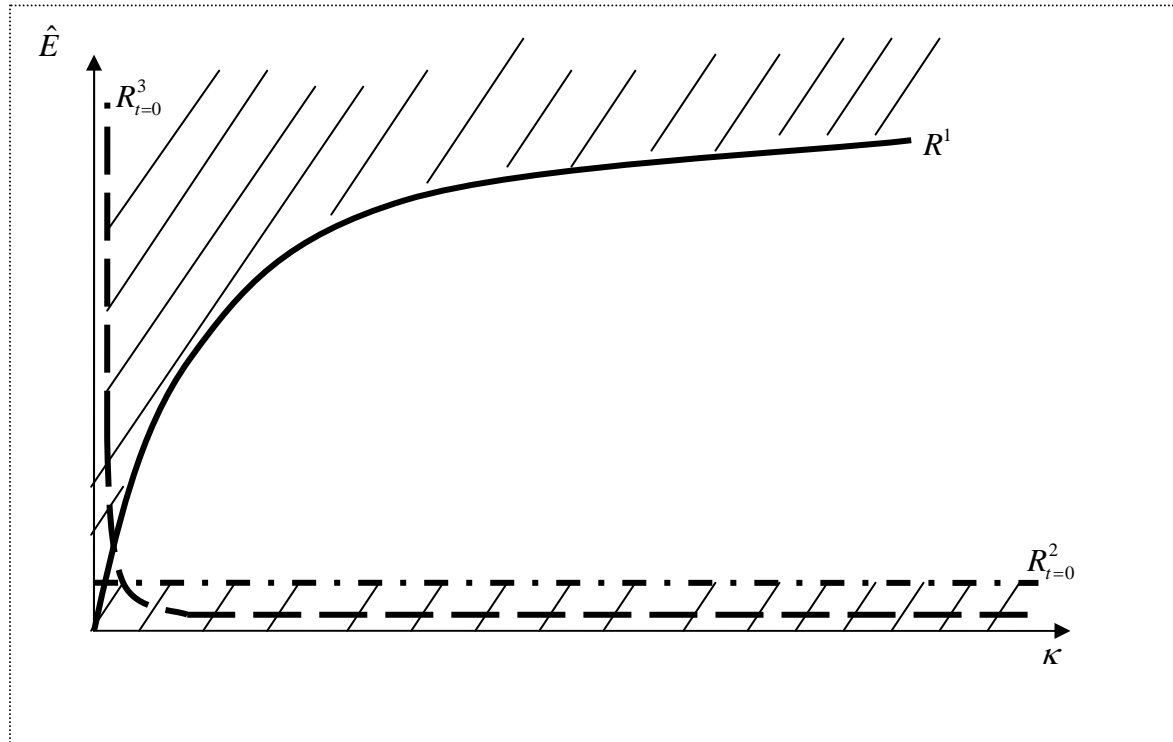
$$(C.13a) \quad \frac{\dot{\kappa}}{\kappa} = 0: \quad \hat{E}_{\kappa=0} = \frac{\kappa^{-\alpha} - (\delta + g_L + \frac{g_A}{\alpha})}{1 - \rho \frac{\alpha - \beta}{\alpha(1-\alpha)} \bar{\omega}_B \kappa^\alpha} \kappa$$

Now, I could depict the differential equation system (C.12)-(C.13) in the phase space  $(\hat{E}, \kappa)$ .

Before doing so, I show that not the whole phase space  $(\hat{E}, \kappa)$  is economically meaningful.

The economically meaningful phase-space is restricted by three curves  $(R^1, R_t^2, R_t^3)$ , as shown in the following figure and as derived below:

**Figure C.1: Relevant space of the phase diagram**



Only the space below the  $R^1$ -line is economically meaningful, since the employment-share of at least one sub-sector  $i$  is negative in the space above the  $R^1$ -line. This can be seen from the following fact:

It follows from equations (3) and (7a,b) after some algebra that

$$(C.14) \quad \frac{l_m}{k_m} = 1 - \left( \frac{\beta - \alpha}{\beta(1 - \alpha)} \right) \sum_{i=m+1}^n l_i$$

Since,  $l_i$  cannot be negative (hence,  $0 \leq \sum_{i=m+1}^n l_i \leq 1$ ) this equation implies that

$$(C.15) \quad \frac{l_m}{k_m} < \frac{\alpha(1 - \beta)}{\beta(1 - \alpha)}$$

Inserting equation (C.3) into this relation yields

$$(C.16) \quad R^1 : \hat{E} < \frac{\alpha}{\beta} \frac{1}{\omega_B} \kappa^{1-\alpha}$$

Hence, the space above  $R^1$  is not feasible. When the economy reaches a point on  $R^1$ , no labor is used in sub-sectors  $i=1, \dots, m$ . If I impose Inada-conditions on the production functions, as usual, this means that the output of sub-sectors  $i=1, \dots, m$  is equal to zero, which means that the consumption of these sectors is equal to zero. Our utility function implies that life-time utility is infinitely negative in this case. Hence, the household prefers not to be at the  $R^1$ -curve. Note that actually the  $R^1$ -curve is only an outer limit: Since we have existence-minima in our utility function, the utility function becomes infinitely negative when the consumption of one of these goods falls below its subsistence level. Hence, even when the consumption of all goods is positive, it may be the case that the utility function is infinitely negative due to violation of some existence minima. Therefore, the actual constraint is somewhere below the  $R^1$ -curve. However, this fact does not change the qualitative results of the stability analysis.

Now I turn to the  $R_t^2$  and  $R_t^3$ -curves. I have to take account of the non-negativity-constraints on consumption ( $C_i > 0 \forall i$ ), since our Stone-Geary-type utility function can give rise to negative consumption. By using equations (7c), (11b), (12d), (15), (16b) and (19) the non-negativity-constraints ( $C_i > 0 \forall i$ ) can be transformed as follows:

$$(C.17) \quad \hat{E} > \frac{-\theta_i}{\omega_i} \frac{1}{LA^{\frac{1}{\alpha}}} \quad i = 1, \dots, m$$

$$(C.18) \quad \hat{E} > \frac{-\theta_i}{\omega_i} \left( \frac{\alpha(1 - \beta)}{\beta(1 - \alpha)} \right)^\beta \frac{1 - \alpha}{1 - \beta} \frac{1}{LBA^{\frac{2(1 - \beta) - (1 - \alpha)}{\alpha}}} \frac{1}{\kappa^{\alpha - \beta}} \quad i = m + 1, \dots, n$$

This set of constraints implies that at any point of time only two constraints are binding, namely those with respectively the largest  $\frac{-\theta_i}{\omega_i}$ . Hence, the set (C.17), (C.18) can be reduced

to the following set:

$$(C.19) \ R_t^2: \quad \hat{E} > \frac{-\theta_j}{\omega_j} \frac{1}{LA^{\frac{1}{\alpha}}}$$

$$\text{where } \frac{-\theta_j}{\omega_j} > \frac{-\theta_i}{\omega_i} \quad i = 1, \dots, m$$

and  $1 \leq j \leq m$ .

$$(C.20) \ R_t^3: \quad \hat{E} > \frac{-\theta_x}{\omega_x} \left( \frac{\alpha(1-\beta)}{\beta(1-\alpha)} \right)^\beta \frac{1-\alpha}{1-\beta} \frac{1}{LBA^{\frac{2(1-\beta)-(1-\alpha)}{\alpha}}} \frac{1}{\kappa^{\alpha-\beta}}$$

$$\text{where } \frac{-\theta_x}{\omega_x} > \frac{-\theta_i}{\omega_i} \quad i = m+1, \dots, n$$

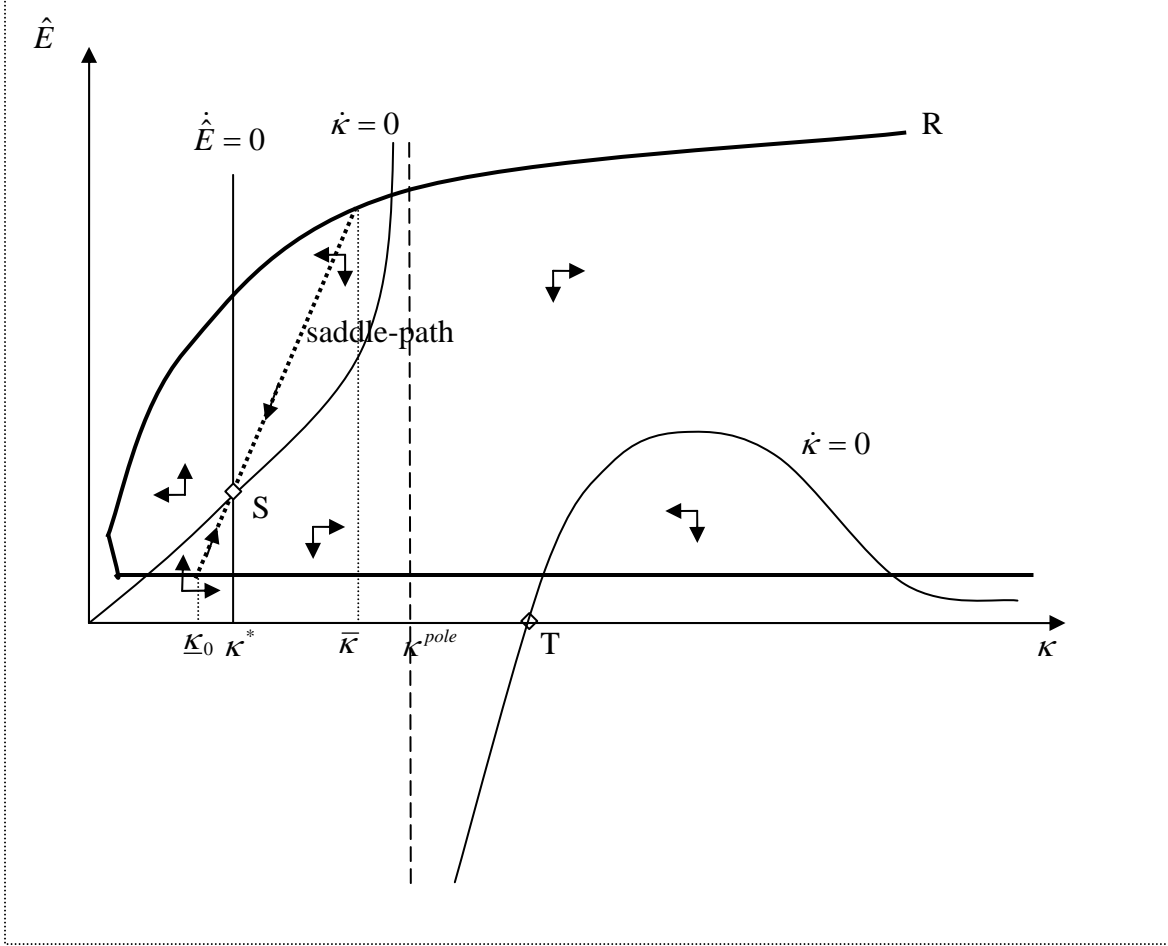
and  $m+1 \leq x \leq n$

These constraints are time-dependent. It depends upon the parameter setting whether  $R_t^2$  or whether  $R_t^3$  is binding at a point of time. In Figure C.1 I have depicted examples for these constraints for the initial state of the system. Only the space above the constraints is economically meaningful, since below the constraints the consumption of at least one good is negative. Last not least, note that equations (C.19)/(C.20) imply that the  $R_t^2$ -curve and the  $R_t^3$ -curve converge to the axes of the phase-diagram as time approaches infinity.

Now, I depict the differential equation system (C.12)-(C.13) in the phase space  $(\hat{E}, \kappa)$ .

**Figure C.2:** The differential equation system (C.12)-(C.13) in the phase-space for

$$\frac{\alpha(1-\alpha)}{\bar{\omega}_B \rho(\alpha-\beta)} < \frac{1}{\delta + g_L + \frac{g_A}{\alpha}}$$



Note that I have depicted here only the relevant (or: binding) parts of the restriction-set of Figure C.1 as a bold line  $R$ .

As we can see, the  $\dot{\kappa}=0$ -locus has a pole at  $\kappa^{pole} = \left( \frac{\alpha(1-\alpha)}{\bar{\omega}_B \rho(\alpha-\beta)} \right)^{\frac{1}{\alpha}}$ .

The phase diagram implies that there must be a saddle-path along which the system converges to the stationary point  $S$  (where  $S$  is actually the PBGP). The length of the saddle-path is restricted by the restrictions of the meaningful space  $R^1, R_t^2, R_t^3$  (bold line). In other words, only if the initial  $\kappa$  ( $\kappa_0$ ) is somewhere between  $\underline{\kappa}_0$  and  $\bar{\kappa}$ <sup>4</sup>, the economy can be on the saddle-path. Therefore, the system can be only *locally* saddle-path stable. Now, I have to

<sup>4</sup> Note that  $\bar{\kappa}$  must be somewhat smaller than depicted in this diagram, since, as discussed above,  $R^1$ -curve is only an „outer limit“.



show that the system will be on the saddle-path if  $\underline{\kappa}_0 < \kappa_0 < \bar{\kappa}$ . Furthermore, I have to discuss what happens if  $\kappa_0$  is not within this range.

Every trajectory, which starts *above the saddle-path or left from  $\underline{\kappa}_0$* , reaches the  $R^1$ -curve in finite time. As discussed above, the life-time utility becomes infinitely negative if the household reaches the  $R^1$ -curve. These arguments imply that the representative household will never choose to start above the saddle path if  $\underline{\kappa}_0 < \kappa_0 < \bar{\kappa}$ , since all the trajectories above the saddle-path lead to a state where life-time-utility is infinitely negative.

Furthermore, all initial points that are situated *below the saddle-path or right from  $\bar{\kappa}$*  converge to the point T. If the system reaches one of the constraints  $(R_t^2, R_t^3)$  during this convergence process, it moves along the binding constraint towards T. However, the transversality condition is violated in T. Therefore, T is not an equilibrium. To see that the transversality condition is violated in T consider the following facts: The transversality condition is given by  $\lim_{t \rightarrow \infty} \psi K e^{-\rho t} > 0$ , where  $\psi$  is the costate variable in the Hamiltonian function (shadow-price of capital). This transversality condition can be reformulated such that

we obtain:  $\lim_{t \rightarrow \infty} (1 - \alpha) \kappa^{-\alpha} - \delta - g_L - \frac{g_A}{\alpha} > 0$ , which is equivalent to:  $\lim_{t \rightarrow \infty} \kappa < \left( \frac{1 - \alpha}{\delta + g_L + \frac{g_A}{\alpha}} \right)^{\frac{1}{\alpha}}$ .

However, equation (C.13a) implies that in point T in Figure C.2  $\kappa = \left( \frac{1}{\delta + g_L + \frac{g_A}{\alpha}} \right)^{\frac{1}{\alpha}}$ . Hence,

the transversality condition is violated if the system converges to point T.

Overall, we know that, if  $\underline{\kappa}_0 < \kappa_0 < \bar{\kappa}$ , the household always decides to be on the saddle-path. Hence, we know that for  $\underline{\kappa}_0 < \kappa_0 < \bar{\kappa}$  the economy converges to the PBGP. In this sense, **the PBGP is locally stable (within the range  $\underline{\kappa}_0 < \kappa_0 < \bar{\kappa}$ )**.

**If the initial capital is too small** ( $\kappa_0 < \underline{\kappa}_0$ ), the economy converges to a state where some existence minima are not satisfied (curve  $R^1$ ) and thus utility becomes infinitely negative. This may be interpreted as a **development trap**. For example, Malthusian theories imply that in this case some part of the population dies, which would yield an increase in per-capita-capital (and hence an increase in  $\kappa_0$ ).

On the other hand, **if initial capital-level is too large** ( $\kappa_0 > \bar{\kappa}$ ), all trajectories violate the transversality condition. Therefore, in this case, **the representative household must waste a part of its initial capital** to come into the feasible area ( $\underline{\kappa}_0 < \kappa_0 < \bar{\kappa}$ ).

Furthermore, note that there are always some happenings that reduce the capital stock, e.g. wars (like the Second World War) or natural catastrophes. These happenings could shift the economy into feasible space ( $\underline{\kappa}_0 < \kappa_0 < \bar{\kappa}$ ).

Note that Figure C.2 depicts the phase diagram for parameter constellations, which satisfy the

condition  $\frac{\alpha(1-\alpha)}{\bar{\omega}_B \rho(\alpha-\beta)} < \frac{1}{\delta + g_L + \frac{g_A}{\alpha}}$ . For parameter constellations, which satisfy the

condition  $\frac{\alpha(1-\alpha)}{\bar{\omega}_B \rho(\alpha-\beta)} > \frac{1}{\delta + g_L + \frac{g_A}{\alpha}}$ , the discussion and the qualitative results are nearly the

same. The only difference is that the  $\dot{\kappa} = 0$ -locus is humpshaped (concave) for  $\kappa < \kappa^{pole}$ . However, all the qualitative results remain the same (local stability of PBGP for some range  $\underline{\kappa}_0 < \kappa_0 < \bar{\kappa}$  and “infeasibility” for  $\kappa_0 < \underline{\kappa}_0$  and  $\kappa_0 > \bar{\kappa}$ ). **Q.E.D.**