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The optimal Chapter 7 exemption level in a life-cycle model with asset portfolios *

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Abstract

I develop a heterogenous agent life-cycle model that examines the effects of the US personal bankruptcy law on bankruptcy filings and welfare. In addition to facing uncertainty over their labor income, agents also face wealth shocks that stem from unexpected changes in family composition or from unexpected medical expenses. I allow agents to borrow and save simultaneously. Some households borrow at high interest rates while simultaneously saving at low interest rate because of the option value of defaulting. This explains one of the puzzles in the consumption literature and is consistent with data on household asset holdings. Under chapter 7 of the US bankruptcy law, consumers can keep all wealth up to an exemption level. I show that introducing exemption levels is of particular importance in the presence of wealth shocks. My quantitative evaluations show that changes in the exemption level have an impact only for very low exemption levels. Thus, ignoring them biases welfare results. But this impact fades out rather quickly. The reason is that almost no household is affected by medium to high exemption levels because those households who might default do not have much wealth. The welfare results of changes in the exemption level are rather small, less than 0.1% of annual consumption. In contrast to the earlier literature, but consistent with the data, I do not find a strong positive relationship between the exemption level and default rates.

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1 Introduction

The steep increase in consumer bankruptcy filings in the 1990ies and early 2000s led to an increased interest in the workings of personal bankruptcy laws. On the one hand there has been a public debate leading to a reform of the US bankruptcy law. On the other hand there has been a growing interest among economists in models that are able to explain observed behaviors and that can be used to evaluate different bankruptcy policies. In this chapter, I contribute to this debate by examining the effects of different wealth exemption levels on economic outcomes and welfare in a life-cycle model.

In order to investigate the effects of changing the exemption level, I use a heterogenous agent life-cycle model. In addition to facing uncertainty over their labor income, agents also face wealth shocks that stem from unexpected changes in family composition or from unexpected medical expenses since these are the most important reasons for bankruptcies (Sullivan, Warren, and Westbrook, 2000) in the US. The model features incomplete financial markets. I allow for two assets only: unsecured debt and savings. The possibility to default then introduces some contingency and therefore moves the financial system closer to complete markets.[1] This default option gives consumer insurance against the economic consequences of the aforementioned shocks to their income or wealth. A more generous bankruptcy system, a higher exemption level to be precise, will provide more of this insurance. This comes however at the cost of worsened credit market conditions, higher interest rates and possibly complete credit rationing. In addition, having the possibility to shield some assets in the case of a default increases the incentive to save for poor agents.

The exemption level differs a lot across US states, ranging from a few thousand US dollars for example in Maryland to an unlimited amount in for example Florida. Higher exemption levels increase the incentive to default. Therefore one would expect to see a positive relationship between the exemption level and default rates, unless credit rationing becomes so sever that many households are excluded from borrowing altogether. A strong positive relationship is predicted by previous papers that investigated the optimal exemption level of Chapter 7 consumer bankruptcy code, see for example Athreya (2006).

Figure 1 however shows that there is no significant relationship between the exemption level and bankruptcy filings. [2] Even though my model, which is built on Livshits, MacGee, and Tertilt (2007), also has a positive relationship between the exemption level and bankruptcy rates, the effect is very small, in particular for exemption levels above a relatively modest value of $20,000. Thus, the model is consistent with the data. Moreover, my quantitative evaluations show that the welfare difference between different exemption levels are rather small, less than 0.1% of annual consumption. This might explain why there are differences in the exemption level across US states in the first place.

Almost all variables of interest follow a similar pattern as the bankruptcy rate. There are significant changes when the exemption is increased from zero to some small positive level. However, there are almost no further effects when the exemption level is increased further. The main reason for this is that almost no household is affected by an even higher exemption level. Almost no household holds assets close to the

[1] For a theoretical evaluation of that trade-off see Dubey, Geanakoplos, and Shubik (2005)

[2] The data are are the average per capita filing rates between 1995-2003.

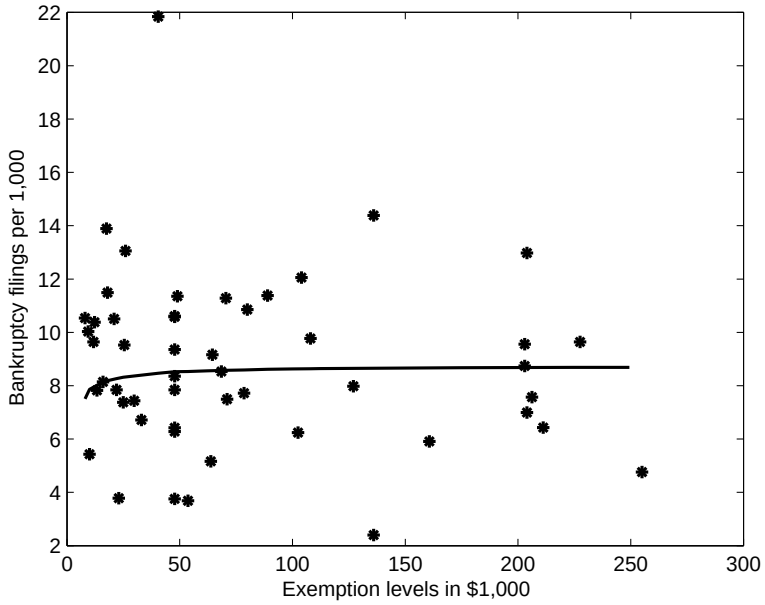

FIGURE 1: BANKRUPTCY RATES FOR DIFFERENT EXEMPTION LEVELS

exemption level even for intermediate exemption levels. If the exemption level is then increased further, asset holdings hardly increase.

The quantitative literature on consumer bankruptcy has increased a lot since Athreya's original paper in 2002. He found that eliminating the default option would be welfare increasing. Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) show that the recent tightening of the law in the US implies large welfare gains. Livshits et al. (2007) compare the US system under which future earnings are exempt after having declared bankruptcy with a European type of system under which future earnings are garnished to repay creditors. They find that the welfare differences between the systems depends on the persistence and variance of the shocks. Mateos-Planas and Seccia (2006) compare different credit market exclusion periods in a model with endogenous borrowing limits stemming from the default option with a model with exogenous borrowing limits. All of these models are one goods models. Hintermaier and Koeniger (2008) however examine the reasons for the increase in consumer bankruptcies in a model with durable and nondurable goods.

My paper is closest to Livshits et al. (2007) in that I follow their set up with labor income uncertainty and wealth shocks. Livshits et al. (2007) however have only one asset in their model and ignore the exemption level all together. This means that set it implicitly to zero. However, as my result indicate, this omission makes their welfare results slightly spurious since a positive exemption level has a particularly positive effect for very low exemption levels. My modeling of the asset market is close to Li and Sarte (2006) who also have both, unsecured debt and savings. Their model however uses an infinite horizon framework with the only uncertainty coming from changes in labor productivity. This is also the case in Athreya (2006) who investigates the optimal exemption level in a model with secured and unsecured debt. Both, life-cycle issues and wealth shocks however are important for the welfare results.

The paper is structured as follows. Section 2 explains the US bankruptcy code.

Section 3 shows how wealth shocks make a positive exemption level is particularly important. In particular, i focus on the impact on savings. Section 3 lays out the model and the computational algorithm. Section 4 shows the benchmark calibration and discusses the main mechanisms of the model. Section 5 shows the main policy experiment. I examine the impact of changing the exemption level on default rates, default reasons, borrowing and savings decisions and welfare. Section 6 concludes.

2 Consumer bankruptcy

Personal bankruptcy law in the US consist of two different procedures: Chapter 7 and Chapter 13. Under Chapter 7, all unsecured debt is discharged immediately, and future earnings cannot be garnished. This is why Chapter 7 is known as providing a "fresh start". At the same time, a person filing for bankruptcy has to surrender all wealth in excess of an exemption level. The exemption level varies across US states, ranging from $8,000 in Maryland to unlimited for housing wealth in some states, for example Florida. Therefore, we calculate the population-weighted median across states. The resulting average exemption level is $47,800 in 1993. A person can file for Chapter 7 only once every six years.

Under Chapter 13 agents can keep their wealth, debt is not discharged immediately and future earnings are garnished. A person can file for Chapter 13 every six moths. In the model, I follow Livshits et al. (2007) and let agents file first for Chapter 7 and if they follow soon afterwards they have to file for Chapter 13.

3 An illustration of the importance of wealth exemptions

In this section, I show a simple example that demonstrates that wealth exemption levels are particularly important in the presence of wealth shocks, stemming from, for example, unexpected medical expenditures. The reason for this is that the option to default can encourage savings of, particularly, the poor.

Suppose an agent lives for two periods. His initial wealth is $a_0 = 1$ For simplicity, I assume that the agent has no income, that the risk free interest rate $r^f = 0.0$ and that his discount factor $\beta = 1.0$.[3].

In the second period the agent might face a health shock s that requires him to spend e in order to survive. This health shocks acts as a wealth shock and occurs with probability p. However, the agent can default on these expenses. If he defaults, he will lose all his assets up to the exemption level X. For simplicity, I set the value of the health shock $e = 1$. This ensures that the agent will always default on his expense debt.

The agent maximizes lifetime utility

$$\begin{aligned} \max_s U &= \log(c_1) + \mathbb{E}\log(c_2) \\ &= \log(a_0 - s) + (1-p)\log(s) + p\log(\min[s, X]) \end{aligned}$$

The solution to this problem will depend on the exemption level X in a non-trivial way. There are three possible cases. First, the agent might choose a level of savings that is

[3]None of the results of this section hinges on any of these assumptions. Their sole purpose is to make the analysis more transparent.

higher than the exemption level. In this case, the solution is

$$s = \frac{(1-p)\,a0}{2-p} = \bar{X}_L$$

This situation is likely for low exemption levels. Note, that this implicitly defines a value for the exemption level until this situation can occur, call this $\bar{X}_L$ If the exemption level is higher, the second case might occur and the optimal saving will be equal to the exemption level. The last case is when the exemption is so high that it becomes non-binding and the agent implements his first best level of savings

$$s = s^{FB} = \frac{1}{2}a_0 = \bar{X}_H$$

Note again, that this implicitly defines the critical exemption level $\bar{X}_H$ until this situation can occur. Therefore, we get the following result.

Result 1 *If the expense shock is so high that the agent cannot repay it, his savings are (weakly) increasing in the exemption level.*

$$s^* = \begin{cases} \frac{(1-p)a0}{2-p} & for \quad X \leq \frac{(1-p)a0}{2-p} \\ X & for \quad \frac{(1-p)a0}{2-p} < X < \frac{1}{2}a_0 \\ \frac{1}{2}a_0 & for \quad \frac{1}{2}a_0 \leq X \end{cases}$$

Formal details are relegated to the appendix. In the first and in the last case, an increase in the exemption level has no effect. In the second case, however savings increase one-for-one with the exemption level.

Of course, a higher exemption level can induce people to default even if they could afford to repay their expense debts. If they plan to do so, their savings will be lower than in a situation in which they had to self-insure. This is the well-understood negative effect of generous exemption levels on savings. It is important to note that the positive effect is particularly relevant for poor agents, whereas the negative effect is relevant for richer agents. Thus, it is important to use a heterogenous agent framework in order to investigate the aggregate effects on savings and welfare. In the next section, I will present a heterogenous agent model in which agents face uncertainty with regards to income and expense (wealth) shocks. This is not the case in Li and Sarte (2006) and Athreya (2006). These are both infinite horizon models without expense shocks. On the other hand, setting the exemption level to zero as is done by Livshits et al. (2007) will probably bias welfare results of the US bankruptcy code since positive exemptions are particularly valuable if agents face wealth shocks.

4 The model

My model framework is a partial equilibrium overlapping generations model based on the model by Livshits et al. (2007). Each household lives for J periods. Each generation consists of households of measure 1. All households are born equally without any wealth. There is no bequest motive. Therefore each household maximizes its own expected lifetime utility. There is no disutility of labor. Households face uncertainty

with respect to their future labor productivity and with respect to small and large wealth shocks, reflecting family risks and health risks.

Financial markets are incomplete, in particular there are no insurance markets in which households could ensure themselves against their labor income or their family or health risks. There are two assets in the economy. First, households can save any non-negative amount by buying a risk-free bond. This bond pays an exogenous interest rate r^f.

Second, they can also borrow non-negative amounts from financial intermediaries. This borrowing is done through notionally non-contingent debt contracts on which the household, however, can default. This default option makes these bonds partially contingent. I abstract from any issues of informational asymmetries. At the point of signing the debt contract, financial intermediaries have the same information set as the households themselves. Therefore all debt contracts are household specific in that financial intermediaries price these debts according to the characteristics of each specific household.

4.1 Households

4.1.1 Preferences

Households live for J years. For simplicity I abstract from labor-leisure choice.[4] All agents supply their unit of labor inelastically, i.e. there is no disutility of labor. Households maximize their discounted expected utility of consumption. However, in order to take varying household sizes into account, household size is expressed in terms of equivalence scale units n_j. Felicity is standard, non-decreasing and concave

$$U = \sum_{j=1}^{J} \beta^{j-1} u\left(\frac{c_j}{n_j}\right).$$

4.1.2 Productivity

Labor productivity of household i at age j is the product of three components: an age specific component e_j, a household specific persistent component z_j^i, and a household specific component transitory component η_j^i

$$y_j^i = e_j z_j^i \eta_j^i.$$

The age specific component e_j is chosen to reflect life cycle income patterns that are common across households. The household specific components reflect uncertainties over the life cycle. For example, citet{storesletten2004} estimate an AR(1) for log earnings of the following form

$$\begin{aligned} \ln\left(y_j^i\right) &= \ln\left(z_j^i\right) + \ln\left(\eta_j^i\right) + \ln g\left(x_j^i\right) \\ \ln\left(z_j^i\right) &= \rho \ln\left(z_j^i\right) + \varepsilon_j^i \end{aligned}$$

[4] I ignore labor-leisure choice mainly because the current model is already quite complicated and not because it is not interesting (see on that matter for example Li and Sarte (2006).)

where $g(\cdot)$ reflects the deterministic component of earnings. The persistent component z_j^i follows an AR(1) process with a very high autocorrelation. Storesletten, Telmer, and Yaron (2004) estimate it to be 0.99. The variance of the transitory shock is about six times as high a the variance of the persistent shock. As is standard in the literature, I will discretize the income process by using a Markov chain.

4.1.3 Wealth shocks

In addition to income uncertainty, households also face idiosyncratic wealth shocks. These wealth shocks represent expenditures that have to be incurred due to, for example, a divorce or some necessary medical treatment. It is important to note that these expenditures do not yield any utility, therefore they simply reduce the wealth of the household. If the household does not hold sufficient wealth he will have to default on these expenditures. As in Livshits et al. (2007), I assume that these shocks are i.i.d. and uncorrelated to income. It is important to note that these expenditures are due to third parties, e.g. hospitals in the case of a health shock. Thus if the agent files for bankruptcy, and he does not repay the expenditure shock, it is the hospital that loses money and not that bank. the bank only loses the amount of unsecured credit. [5]

4.2 Credit market

I assume perfect competition (free entry) in the credit market. Therefore, banks must make zero expected profit on each contract. The opportunity cost of lending is the safe rate of return on capital which is taken as exogenous. I assume that financial intermediation incurs real resource costs.

Households, who have not defaulted in the past, can hold two types of assets or any one of them: savings a and unsecured debt d. Savings earns the household rate of return r^F on his savings. Unsecured credit requires transaction costs τ^u that reflects the higher information costs that banks incur in the data when producing unsecured debt.

Furthermore, I abstract from information asymmetries in the credit market.[6] Each bank knows the borrower's age j and his the persistent component of his labor productivity z^i.[7] Therefore, by anticipating the behavior of the borrower, the banks are able to calculate the probability of default and how much they will get in the case of default. Perfect competition implies that they set the interest rate, $r(j, z, d, \gamma, X)$, such that they expect to break even. This interest rate depends on the exemption level X because it affects the incentives to default and the amount the bank recovers in this event. The banks offer a menu of one period debt contracts which consist of an amount lent d and a corresponding interest rate $r(j, z, d, \gamma, X)$ to each agent (j, z).

Households who have just defaulted are excluded from borrowing. However they can still save.

[5] While this assumption is plausible for health shocks, it is less plausible for e.g. the expenditures for a divorce since households have to pay these costs themselves. One way to model this would be to model the debt contract as a credit card contract with a pre-specified credit line (credit limit) that the household can draw on in case he faces an expenditure shock.

[6] For an analysis of bankruptcy under asymmetric information see Athreya, Tam, and Young (2007)

[7] It is immaterial whether the bank also knows how much the borrower will save. The bank can always anticipate the decision of the borrower.

4.3 Timing

The sequence of events is shown in figure 2. A household of age j brings forward from last period: a certain credit record S, a value for the persistent component of labor productivity z_{t-1}, debt d and savings a. At the beginning of the period the expenditure shock k and his labor productivity z, η is realized. Since the household can default on the expenditure shock, this expenditure shock is simply added to the household's debt holdings. All households who carry some debt, either because they have borrowed in the previous period and (or) they have been hit by an expenditure shock then decide whether to repay or whether to default. The credit record of households who had not defaulted in the previous period and who repay also this period remain a clean credit record. The credit record of households who default this period is changes to reflect their default. A household who had already defaulted in the previous period will have had no debt $d = 0$. If this household is however hit by an expense shock, it might default again. This behavior will also be reflected by the credit status.

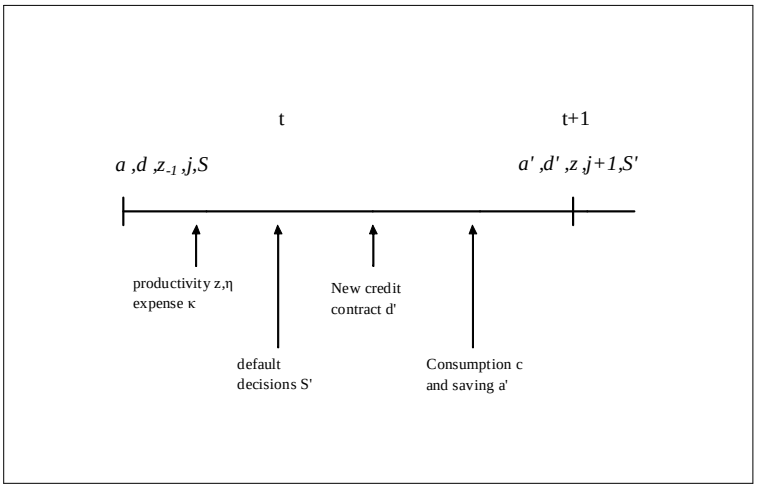

FIGURE 2: TIMING

All households with a clean credit record can borrow d in the unsecured credit market. At the end of the period, each household can decide how much to save a' and how much to consume c.

4.4 The Household's problems

As usual, the household's problem is defined recursively. In order to describe the problem three value functions are needed. V^R is the value of repaying the debt, V^D is the value of defaulting the first time under Chapter 7, and V^{DD} is the value of defaulting again after the household has already defaulted in the previous period. This last value function is needed since a household can default under Chapter 7 only once every six years.

An unconstrained agent of age j with savings a, current productivity z, η, expense shock realization κ, and debt d has to decide whether to repay or whether to default.

The value of repaying is given by

$$\begin{aligned} V_j^R(a,d,z,\eta,\kappa) &= \max_{c,a',d'} \left\{ \begin{array}{c} u\left(\frac{c}{n_j}\right) + \\ \beta \mathbb{E} \max\left[V_{j+1}^R(a',d',z',\eta',\kappa'), V_{j+1}^D(a',z',\eta')\right] \end{array} \right\} \\ \text{s.t.} \quad c + d + \frac{a'}{1+r^s} + \kappa &\leq e_j z\eta + a + \frac{d'}{1+r(j,z,d',a',X)}, \\ a' &\geq 0, \qquad d' \geq 0. \end{aligned}$$

where savings a' and new debt d' have to be non-negative. Since the agent repays, he will be unconstrained tomorrow and therefore has the option to default tomorrow, i.e. he can choose the maximum of defaulting or repaying. If the agent's debt repayment and expenditures on the expense shock exceed his income and potential new borrowing, the constraint set is empty, i.e. consumption would have to be negative. In this case the value function is set to negative infinity and the agent will have to default.

The value of defaulting is given by

$$\begin{aligned} V_j^D(a,z,\eta) &= \max_{c,a'} \left\{ \begin{array}{c} u\left(\frac{c}{n_j}\right) - \Psi + \\ \beta \mathbb{E} \max\left[V_{j+1}^R(a',0,z',\eta',\kappa'), V_{j+1}^{DD}(a,z',\eta',\kappa')\right] \end{array} \right\} \\ \text{s.t.} \quad c + \frac{a'}{1+r^s} &\leq (1-\gamma) e_j z\eta + \min[a,X] \\ a' &\geq 0. \end{aligned}$$

Since the household defaults on all unsecured debt d and all expenditures κ, their values play no role here. However if the agent defaults, he can keep assets only up to the exemption level X. In addition part of his labor income will be garnished. Event hough a household who defaults cannot borrow in the current period, the household can save. This is in contrast to Livshits et al. (2007) who do not allow the agent to save. In their paper a household is in financial autarky after a default. This assumption has been used by other authors as well because it simplifies the analysis. However, this financial autarky assumption clearly overstates the punishment from a default since there is no evidence that households who have defaulted in the past are precluded from saving. Ψ is a utility cost of defaulting and reflects both pecuniary costs and non-pecuniary costs. The pecuniary costs, for example court fees and lawyer fees, have been estimated to exceed $\$1,000$. In addition Ψ reflects the cost of the stigma of having had to declare bankruptcy. I use this parameter in the calibration to tie down the default rate. If the continuation value of defaulting exceeds the value of repaying, i.e. $V_j^D(a,z,\eta) > V_j^R(a,d,z,\eta,\kappa)$, the household will default. I denote this decision by $I_j^D(a,d,z,\eta,\kappa)$.

In the next period, the household will have no debt but he might be hit by an expense shock. If he is unable to repay the expense shock, he will have to default again. In that case, I assume that he has to surrender all his wealth and part of his income will be garnished. Therefore the value of not repaying expense debt after having already defaulted is

$$\begin{aligned} V_j^{DD}(a,z,\eta,\kappa) &= u\left(\frac{c}{n_j}\right) - \Psi + \beta \mathbb{E} \max[\left[V_{j+1}^R(0,d',z',\eta',\kappa'), V_{j+1}^D(0,z',\eta')\right] \\ \text{where} \quad c &= (1-\gamma) e_j z\eta, \qquad d' = (\kappa - a - \gamma e_j z\eta)(1+\bar{r}) \end{aligned}$$

where debt is rolled over to next period at an exogenous interest rate $\bar{r}$. This agent has no choice problem. Similarly to the agent who has not defaulted in the past, if the continuation value from defaulting again exceed the value of paying off expense debt, i.e. $V_j^{DD}(a, z, \eta, \kappa) > V_{j+1}^R(a', 0, z', \eta', \kappa')$, the agent will default a second time. I denote this decision by $I_j^{DD}(a, z, \eta, \kappa)$.

4.5 The zero profit condition of the banks

There is perfect competition (free entry) in the credit market. The banks make zero profit on each savings contract and on each unsecured loan contract. All agents, except those who default twice, can safe at the risk free interest rate r^f.

Since I abstract from asymmetric information, banks observe the household fully. This means they know the household's age j, cash on hand $e_j z\eta + a - d - \kappa$ and persistent component of productivity z. In addition they know how much the household is going to save, i.e. they know $\frac{a'}{1+r^s}$. Given a savings level $\frac{a'}{1+r^s}$ and productivity level z the bank knows in which future states of the world the household will be willing to repay and in which the household will default Therefore, for each amount of unsecured credit d', the bank can calculate the default probability $\pi(d', a', z, j, \gamma, X)$ and the amount the bank recovers in each default state. This will depend on the exemption level X , the fraction of labor income that can be garnished γ[8] and on the amount the household owes in expense debt κ'. I assume that all assets above the exemption level X and the garnished labor income are split proportionally in the repayment of the bank and expense debt. So, the bank receives a fraction $\frac{d'}{d'+\kappa'}$ of labor income $e_j z\eta$ and of the savings above the exemption level, if these savings exceed the exemption level. In addition, the credit production process incurs real costs τ^u which are assumed to be proportional to the loan size. The zero profit condition is given by

$$\begin{aligned}\left(1 + r^f + \tau^u\right) d' = & \\ & \left(1 - \pi\left(d', a', z, j, \gamma, X\right)\right)\left(1 + r\left(d', a', z, j, X\right)\right) d' \\ & + \pi\left(d', a', z, j, \gamma, X\right) \mathbb{E}\left(\frac{d'}{d' + \kappa'}\left(\gamma e_j z\eta + \max\left[a' - X, 0\right]\right)\right).\end{aligned}$$

4.6 Equilibrium

Let $=(a, d, z, j, S)$ be a state vector for an individual, where a denotes savings, d unsecured borrowing, z the persistent component of labor productivity, j, the age of the household and S the credit status. Let r^f be the exogenous interest rate, τ^u the resource costs of producing unsecured credit, γ the proportional garnishment and X the wealth exemption level. A competitive recursive equilibrium is then given by:

- value functions V_j^R, V_j^D, V_j^{DD} that solve the households problem and lead to optimal policy functions c, d', a', I^D, I^{DD},
- an interest rate function that satisfies the zero profit condition,

[8] If there was no garnishment, households would not repay any fraction of the loan. This is not according to the US bankruptcy law which requires bankruptcy filers to have acted in good faith and therefore denies filing for bankruptcy immediately after having taken out a credit.

- and correct default probabilities $\pi\left(d', a', z, j, \gamma, X\right) = E\left(I_j^D\left(a', d', z', \eta', \kappa'\right)\right)$

4.7 Computational algorithm

In this section, I present an overview of the computational algorithm. As is standard in the literature, I solve this program by backward induction. I assume that the household faces no uncertainty in the last period of his life and cannot default. I discretize the asset space, the persistent state of productivity, the temporary productivity and the expense shock.

Algorithm 2 *1. Solve the value functions for the last period T.*

2. *Given the values in T, I solve the households problems in $T-1$. Since agents are not allowed to default in period T, this is simple.*
3. *In period $T-2$, I set up a grid of savings values a' and borrowing values d'. These two grids form a matrix.*
   (a) *Then, I calculate the default probabilities and associated recoveries for each entry of this matrix and each level or persistent productivity by using the continuation values in period $T-1$ by looping over persistent and transitory productivity and the expense shock. These default probabilities imply an interest rate for each pair of values a', d', z.*
   (b) *Given this array, I solve for the optimal decision of the households. Since households who have defaulted cannot borrow, calculating their value functions is standard.*
4. *I continue to do that until the first period*
5. *I simulate the model for 10 million households.*

5 Calibration

In this section, I first show the parametrization. Afterwards, I describe the results and compare the model's implications to the data.

5.1 Parametrization

5.1.1 Fixed parameters

Since I want to compare my results to the results obtained by Livshits et al. (2007), I follow their parametrization in all respects with one addition. Their model does not have the utility cost Ψ which I need to calibrate the default rate. In essence, all of the parameter values are based on empirical studies. The only exceptions being the garnishment rate Γ and the utility cost Ψ which are calibrated to match the average debt to income ratio and the observed default rate.

In order to simplify computations, each model period corresponds to three years. Households are born at age 20, retire with 65 and die at age 74. This implies that a

life in the model has 18 periods, where the last three periods are spent in retirement. As already mentioned, households face no uncertainty in retirement.[9] In the following, I report only annual values since these are more familiar than triennial values.

The felicity function features constant relative risk aversion

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

where σ is the coefficient of relative risk aversion and set to 2.0. The discount factor is equal to 0.94. The family size life-cycle comes from Fernandez-Villaverde and Krueger (2007) which in turn is based on the US Census data for 1990.

The interest rate on saving r^f is set to 4.0 percent. This is in line with estimates of the average return to capital in the US. The transaction cost for unsecured credit τ^u is based on the costs of producing credit card debt and is also set to 4.0 percent Evans and Schmalensee (1999).

The AR(1) income process

$$\begin{aligned} \ln\left(y_j^i\right) &= \ln\left(z_j^i\right) + \ln\left(\eta_j^i\right) + \ln g\left(x_j^i\right) \\ \ln\left(z_j^i\right) &= \rho \ln\left(z_j^i\right) + \varepsilon_j^i \end{aligned}$$

is parameterized by the following values which are based on citet{storesletten2004}: The autocorrelation coefficient ρ is set to 0.99. Its innovation is assumed to be normal, $\varepsilon_j^i \sim N(0, \sigma_\varepsilon)$, with variance $\sigma_\varepsilon^2 = 0.007$. That transitory shock η is also assumed to be normal $\eta_j^i \sim N(0, \sigma_\eta)$ with variance $\sigma_\eta^2 = 0.043$. All these annual values are mapped into triennial values and then discretized into a Markov process with five states . The transition matrix $\Pi(z'|z)$ is assumed to be age-independent. The transitory shock is discretized using three states where ten percent of the population receive a positive and ten a negative shock.

Upon entering retirement, there are no further shocks. In order to make retirement income (social security) dependent on earnings, it has two components: first a lump-sum component equal to 35% of average earnings and then an individual specific component consisting of 30 percent of the last earning.

The expense shock κ can take three values $\{0, \kappa_1, \kappa_2\}$. The first value means no shock. The small shock, κ_1, is set to $\$10,973$ annually and has probability $\pi_1 = 2.368$ percent. Livshits et al. (2007) aggregate three different shocks of similar size. First, a divorce shock which has probability 1.244 percept which leads to expenditures on the divorce and a loss in economies of scale. Second, an unwanted pregnancy which occurs with 0.5 percent. Lastly, medical shocks that are not too big and which affect 0.625 percent of households each year. The big shock κ_2 is purely a large medical expense shock. This is set to $\$34,154$ annually and has probability $\pi_2 = 0.153$ percent. This means that a small fraction of households are hit by very large medical expenditure shocks.[10]

[9] This assumption is innocuous for income uncertainty since retirees receive social security benefits. It is less plausible for expense shocks. But I maintain it for computational simplicity. Since old people do not borrow much and hardly ever default in the data, this assumption is unlikely to bias the results.

[10] For details on these data see Livshits et al. (2007), and also the working paper version.

Table 1: The fixed parameters

Parmeter	**Symbol**	**Value**
CRRA	σ	2
Risk free rate	r^f	4%
trasaction cost	τ^u	4%
expense shocks	κ_1, κ_2	$\$1,097, \$34,154$
probability of expense shocks	π_1, π_2	$2.369\%, 0.153\%$
Transitory states	η_1, η_2, η_3	$[0.6151, 0.9785, 1.5568]$
Transitory probabilties	$p_{\eta_1}, p_{\eta_2}, p_{\eta_3}$	$[0.1, 0.8, 0.1]$
Persistent states	z_1, z_2, z_3, z_4, z_5	$\begin{bmatrix} 0.3799 & 0.6311 & 0.8613 & 1.1754 & 1.9523 \end{bmatrix}$
Transition matrix	$\Gamma(z'\|z)$	$\begin{bmatrix} 0.8638 & 0.1351 & 0.0011 & 0.0 & 0.0 \\ 0.1351 & 0.6778 & 0.1838 & 0.0034 & 0.0 \\ 0.0011 & 0.1838 & 0.6302 & 0.1838 & 0.0011 \\ 0.0 & 0.0034 & 0.1838 & 0.6778 & 0.1351 \\ 0.0 & 0.0 & 0.0011 & 0.1351 & 0.8638 \end{bmatrix}$
Exemption level	X	$\$47,800$

As shown in fig 1, the wealth exemption level X. varies tremendously across states. It ranges from a few thousand dollars to an unlimited amount in some states. I set it to the population weighted median value of $\$47,800$. This value is a higher than what is mostly used in the literature (see for example Athreya (2006)). One reason for the difference is that I include all exemption levels and not only the homestead exemption. Thus while it is true that the homestead exemption in Maryland is zero, there is an exemption on personal belongings of $\$2,500$ and one on tools of trade of $\$5,000$. Moreover, since it is easy to hide some assets, I think that very low exemption levels are not plausible. A second reason is that I use a higher value for top-coding of the exemption levels in states that have an unlimited homestead exemption. However as an additional robustness check, I recalibrated the model to a low exemption level and the results do not change much. Table 1 summarizes all the parameters.

5.1.2 Calibrated parameters

The remaining parameters to be set are the utility cost of bankruptcy Ψ and the garnishment parameter γ. Livshits et al. (2007) do not include the utility cost in their model. They calibrate γ in order to match the observed debt to earnings ratio of 8.4 percent. The default rate is then endogenously determined in their model. Their model does very well and manages to explain 85 percent of observed chapter 7 defaults. However, this result is not robust. My model nests their model as a special case. In particular if I set the exemption level to zero, I get exactly their results. But if I set the exemption level to a plausible one, I get way too many defaults. Therefore, I calibrate Ψ to match the observed default rate of 0.84 percent. The calibrated parameters are shown in table 2.

As can be seen in table 3, the model matches the targeted moments very well. The bankruptcy target are all non-business related Chapter 7 bankruptcies, averaged between 1995-1999.

Table 2: The calibrated parameters

Parmeter	Symbol	Value
Garnishment	γ	34.6%
Utility cost	Ψ	0.1

Table 3: The targets

Target	Data	Model
Debt to earnings ratio	8.4%	8.39%
Chapter 7 bankruptcies	0.84%	0.85%

5.2 Benchmark model

In this subsection, I first present some implications of the model. Then, I will discuss some policy functions. Lastly, I will show some life-cycle implications of the model.

5.2.1 Model implications

In order to assess the model, I present some further comparisons between the model and data in table 4. The average interest rate is similar to the average interest rate the Federal Reserve Board reports on two-year personal loans. The fraction of households with negative net wealth is too high in the model. Wolff (2007) reports that 18 percent of households have negative net wealth. However, he also shows that 27 of households have wealth of less than $5,000.

Table 4: Model implications

Variable	Data	Model
Average borrowing rate	11.2%	10.3%
Households with negative net worth	18%	27.5%
Relative earning of defaulters	49.1%	52.9%
Average debt to income ratio of defaulters	187%	232.5%
Recovery probability	$< 5\%$	1.8%

The earnings of defaulters are about one half in the model and in the data. The amount of debt households hold at the time of filing for bankruptcy is a bit too high in the model compared to the data. However, the numbers from Sullivan et al. (2000) are based on a relatively small sample of bankruptcy cases. In the model, the lender almost never collects anything in the case the household defaults. This is because households who default do not have savings in excess of the exemption level. This is consistent with the data because there are very few cases in which the lenders recover anything. Thus, overall the model fits the data rather well.

5.2.2 Policy functions

In this subsection, I first show how the possibility to save affects the price and availability of credit. Then I show examples of policy functions of the household.

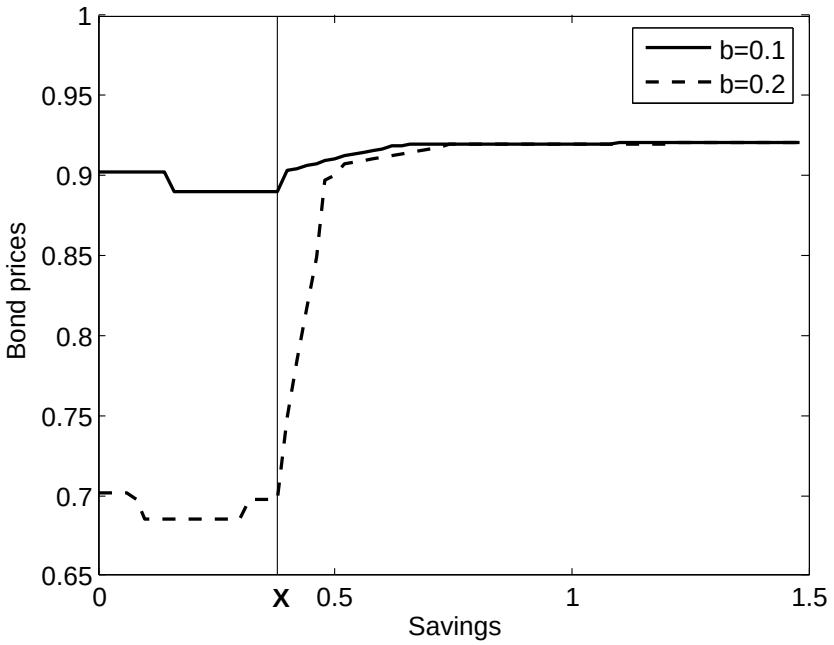

FIGURE 3: BOND PRICES FOR $b = 0.1$ AND $b = 0.2$ FOR DIFFERENT LEVELS OF SAVINGS

The prices at which households can borrow depends on their default incentives. As usual, the more households borrow the higher is the incentive to default and therefore the lower will the price of a bond that is a promise to repay a fixed amount be. In addition, the incentive to default also depends on the exemption level. There are two opposing effects at play. On the one hand, the more a household has saved, to better it can afford to repay its debt. On the other hand, for savings up to the exemption level, the more the household has saved, keeping the repayment requirement constant, the better off the household will be in default.

Figure 3 shows the price of bonds with repayment requirement $b = 0.1$ and $b = 0.2$ respectively, for different levels of savings of agents with low labor productivity. If the household has no savings, it will default in some states. If savings are positive but below the exemption level, the household is better off defaulting, therefore the bond price falls. In this case banks will recover nothing in case of a default. Once savings exceed the exemption level, banks recover part of the loan because assets in excess off the exemption level go to the bank. This is reflected in an increase in the price of the bond. Since the household loses all its assets above the exemption level, default becomes costlier. This lowers the incentive to default. If the household has saved a lot, it will actually never default and therefore it will be able to borrow at the risk free rate.

Figure 3 also shows that these incentives depend on the amount borrowed. If repayment requirements are high, as is the case when the household borrows $b = 0.2$, then the default incentive is already high without savings. An increase in savings increases the default incentive further. And, similar to the case of borrowing only $b = 0.1$, once the household saves more than the exemption level, the recovery rate of the bank increases and therefore the loan price increases. It is important to note

that these pricing functions pertain to particular bonds and agents might or might not choose an element of this particular pricing function. Equilibrium choices are shown in figure 4.

Figure 4 shows several policy functions of households aged 26, have a low persistent labor productivity and that have not defaulted in the previous period. All panels have the cash on hand of the agent after the current shocks were realized, production has taken place and previous debt (or assets) has been repayed on the x-axis. Panel A shows the amount they borrow. Panel B shows the (annual) interest rate they have to pay on these loans. Panel C shows their savings decision. Recall that they save at the constant risk-free interest rate. Panel D shows their consumption decision.

Poor agents, those with cash on hand less than -0.075 cannot borrow more than 0.18. Since they will default when they receive a bad shock, they have to pay a higher interest rate. These agents prefer not to save since their marginal utility of consuming immediately is so high. Agents with cash on hand of more than -0.075 start borrowing less. Therefore the interest rate declines.

The most interesting agents are those with cash on hand between 0.08 and 0.27. These agents borrow at an (annual) interest rate around 10.5% and save at the lower rate of 4% simultaneously. They do this because they can default on the unsecured debt in bad states. Agents with cash on hand around 0.25 are actually net savers. Nevertheless, they are willing to pay this high interest rate on their debt because of the insurance offered by its partial contingency. If they default, they can keep all of their savings since these are less than the exemption level. And, conversely, the banks will recover nothing in case of a default.

Agents with cash on hand above 0.27 do not borrow. Therefore, the interest rate in panel B is not shown. These agents only save at the risk-free rate. However, those who have cash on hand less than the worst expenditure shock, which has a value around 0.82, might have negative cash on hand next period. In this case, they might default on this expense debt. After defaulting they can keep their savings up to the exemption level.

5.2.3 Life-cycle implications

Income and consumption of the model have the observed hump over the life-cycle. In this subsection, I show the fraction of borrowers, the amount borrowed and the interest rates over the life-cycle. Since the key contribution of paper is to introduce two assets into a life-cycle model, I show how these variables differ across agents who either only borrow or who borrow and save simultaneously. The left column of figure 5 shows households who only borrow. The right column shows households who borrow and save.

The first row of figure 5 shows that the fraction of agents who only borrow is higher across all age groups. The second row shows that these household borrow substantially more than households who borrow and save. Both these quantities show a hump-shape with an increase at the end of the working lives.

In figure 4, we saw that it is the very poor agents who only borrow. They do this because their current marginal utility of consumption is so high that they prefer not to save at all. Since these agents are also likely to default, they have to pay higher interest rates, as can be seen in the last row of 5. The incentive to default of these agents rises

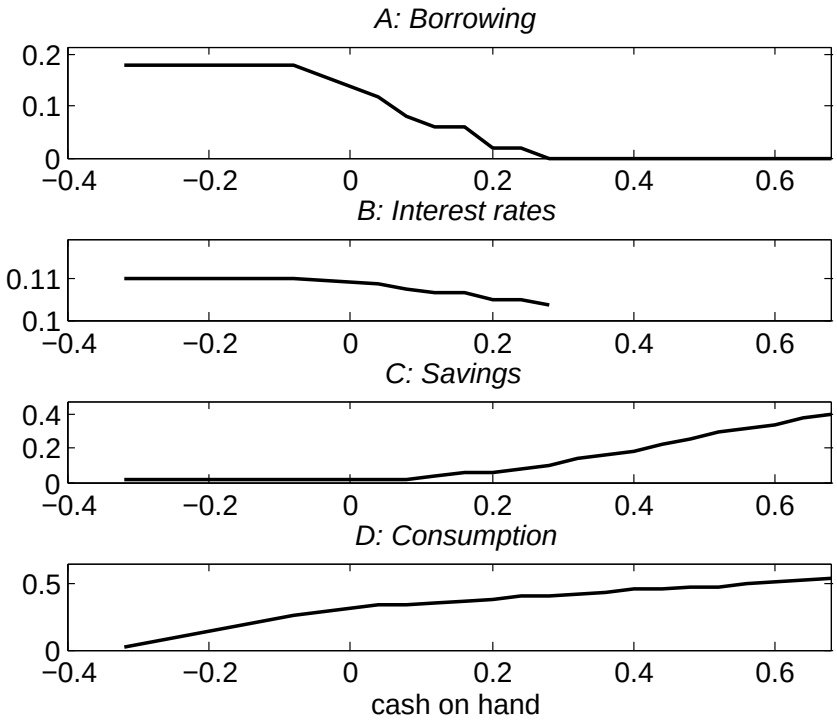

FIGURE 4: POLICY FUNCTIONS OF AN AGENT AGED 26 AND WITH LOW PERSISTENT LABOR PRODUCTIVITY

almost monotonically over their life cycle. The main reason for this is that there is less time left and therefore the punishment of being excluded from the unsecured credit market has a declining impact. This is particularly apparent in the last period in which agents can default. The second row shows that an increase in borrowers at age 62, and in particular an increase in the amount they borrow. This higher level of borrowing combined with no further concern for the future, because there is no uncertainty after 65, explains the sharp increase in interest rates in the last periods of life.

The interest rates of borrowers who also save is lower, even though conditional on the loan size, they have a higher incentive to default. But as we have seen in 4, agents who borrow and save are relatively richer than those who only borrow. Their default incentives are relatively constant over their life-cycle, therefore the interest rate they have to pay do not change much.

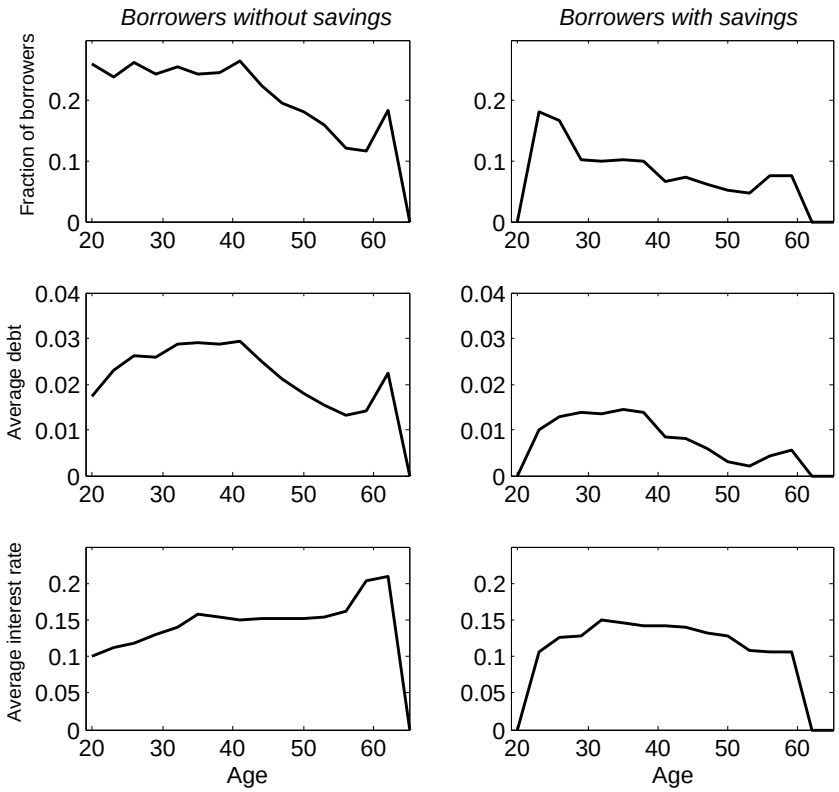

FIGURE 5: FRACTION OF BORROWERS, AVERAGE DEBT AND INTEREST RATES OF BORROWERS WHO DO NOT SAVE AND OF BORROWERS WHO SAVE

6 Policy experiment

In this section, I present the implications of varying the exemption level from zero to high levels. In particular, I will investigate the following exemption levels: The lowest level is $X = 0$, the case implicitly analyzed by Livshits et al. (2007) because they do not have an exemption level in their model. A low levels $X = 0.063$ which corresponds to the observed minimum of \$8,000 found in Maryland. Then the benchmark value of $X = 0.38$ corresponding to \$47,800 which is the population weighted median level. And a higher level $X = 1$ which corresponds to about \$124,700. I will report the maximum of almost \$250,000 found in Kansas only occasionally since these results are usually the same as the once obtained for $X = 1$.

6.1 Default rates

Since the incentive to default increases with the exemption level, default should be positively correlated with the exemption level.[11] However, as shown in figure 1 there is no positive relationship between the exemption level and the occurrence of default. In fact, the correlation is slightly negative. My model does not produce a negative relationship. It produces a small positive one with non-linear effects. However, a regression on the data shows that a positive coefficient cannot be excluded. In particular, the correlation my model produces is within the 95 percent confidence interval.

Previous models, for example Athreya (2008) for consumers or Mankart and Rodano (2007) for entrepreneurs, find a strong positive relationship between the exemption level and default rates. The reason my model predicts only modest increases in the default rate when the exemption is very low and almost no increase once it exceeds \$30,000 is that almost no household is affected by such high exemption levels. Households that might default have assets that are below the exemption level. This is explained in more detail later on.

The non-linear effect at low exemption levels indicates that not including the exemption level might lead to spurious results. My model nests the model by Livshits et al. (2007). In particular, setting the exemption level to zero makes my model equivalent to their model. Therefore, I also contrast their results with mine.[12]

In addition to looking at average default rates, it is instructive to look at default rates over the life-cycle. Figure 6 shows default rates for three different exemption levels and the default rates observed in the data.[13] While the benchmark model gets the hump-shape over the life-cycle right, the peak in the default rate occurs too early compared to the data. In addition, defaults pick up at the end of the life in the model.[14]

The case of $X = 0$ is the case analyzed by citet{Livshits2007}. In this case the

[11] This is unless credit rationing is so severe that the most risky borrrowers are excluded from the market completely. In this case the selection effect might overturn the positive relationship between the exemption level and the default rate.

[12] Since I do not recalibrate the model for each exemption level, the results that I report for $X = 0$ and their results differ. However, these differences are very small since the value of my calibrated variables are very close to the values in their calibration.

[13] For observed bankruptcies, I used the data from Sullivan et al. (2000) and adjusted the mean.

[14] The model produces this marked increase because everyone retires for sure at 65 and I assume that there is no further uncertainty. If the model included heterogeneity with respect to retirement age and additional uncertainty this peak would flatten out.

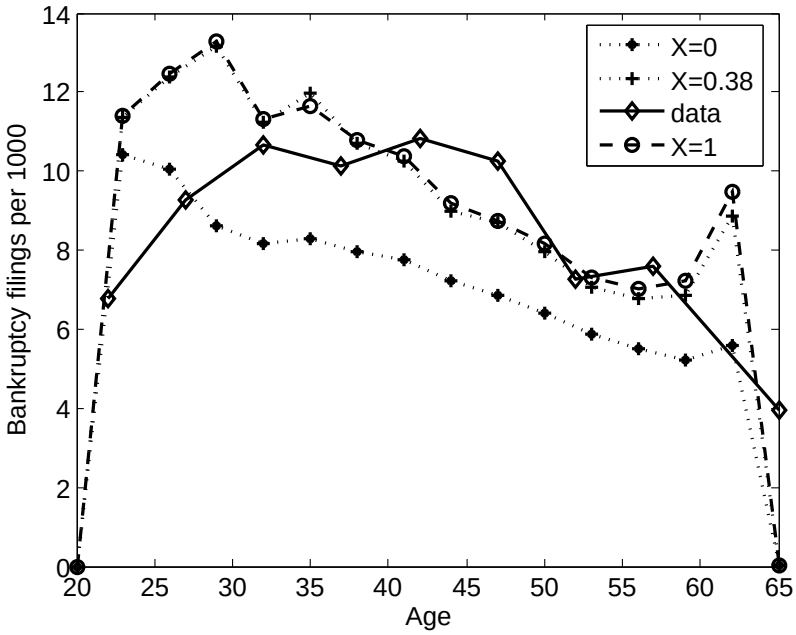

FIGURE 6: DEFAULTS OVER THE LIFE CYCLE: DATA, BENCHMARK MODEL ($X = 0.38$), LOW EXEMPTION ($X = 0$) AND HIGH EXEMPTION ($X = 1$)

peak occurs in the first period in which default is possible in the model. [15] Thus, this version of the model does worse in describing default rates over the life-cycle. Even a very small exemption level 0.06 (not shown) already implies a peak in third model period, i.e. at age 29.

The main result that increases in the exemption level beyond an intermediate level do not lead to an increase in bankruptcies can also be seen in figure 6. The case of a high exemption level $X = 1$ is almost indistinguishable from the benchmark case $X = 0.38$. Default rates are only marginally higher during the last periods of life.

6.2 Default reasons

Households in the model are exposed to three types of uncertainty: expense (wealth) shocks, changes in persistent labor productivity and transitory income shocks. In this section, I compare the default reasons across two exemption levels, $X = 0$ and $X = .38$. The former is the case implicitly analyzed in Livshits et al. (2007), the latter is the benchmark case.

The biggest difference between the two panels in table 5 is that the fraction of defaulters who have experienced no wealth shock more than doubles from 6.6% to 16.2% when the exemption level is increased from $X = 0$ to $X = 0.38$. In particular, households who have experienced no wealth shock but whose persistent productivity has dropped compared to the previous period increases from 4.9% to 11.7%. In the data [16], the fraction of defaulters who report job reasons is at least as high as those

[15] See also figure 1 in Livshits et al. (2007).

[16] See figure 1.2 on page 16 in Livshits et al. (2007).

Table 5: Default by reason for X=0 and X=0.38

A: Exemption X=0	Expense shock			
	Low	High	None	Total
No income fall	63.8%	10.2%	1.0%	75.1%
Fall in persistent income only	8.1%	1.6%	4.9%	14.6%
Fall in transitory income only	7.3%	1.2%	0.2%	8.7%
Both fall	0.9%	0.2%	0.6%	1.7%
Total	80.2%	13.2%	6.6%	100.0%
B: Exemption X=0.38	Expense shock			
	Low	High	None	Total
No income fall	55.1%	8.8%	2.3%	66.1%
Fall in persistent income only	8.6%	1.4%	11.7%	21.7%
Fall in transitory income only	7.4%	1.1%	0.9%	9.4%
Both fall	1.3%	0.2%	1.3%	2.8%
Total	72.4%	11.4%	16.2%	100.0%

reporting expense shocks. Thus, the benchmark model with a positive exemption level probably still overstates the role of expense shocks. But it is a significant improvement over a model without any exemption. Further increases in the exemption level (not shown) do not lead to any significant changes in the default reasons.

6.3 Debt and savings

Figure 7 shows the average debt over the life-cycle. Panel A shows debt of agents who do not save. Panel B shows debt of agents who save. Panel C shows total debt.

First, if the exemption is 0, no borrower will ever save. This is because, in case of a default, he has to surrender all assets above the exemption level which in this case simple means all assets. Therefore, in panel B the line for $X = 0$ corresponds with the x-axis. If the exemption is positive, for example $X = 0.38$ as in the benchmark case, some agents will borrow and save simultaneously (see panel B). But since this is mainly a substitution, this lowers the borrowing amount in panel A of figure 7.

The net effect can be seen in panel C. A positive exemption leads to slightly more borrowing in the first half of the life-cycle and to slightly less in the second half. Figure 7 shows again that a further increase in the exemption level from $X = 0.38$ to $X = 1$ leads to almost no change. The two lines are indistinguishable in all three panels. This can also be seen in figure 9.

Figure 8 shows the assets held by households at the moment of filing for bankruptcy. If the exemption level is zero, less than six percent of households have positive savings at the tome of default. If the exemption level is increased to a still very low level of 0.06, the distribution shifts outwards. This means agents hold more wealth at the moment of default. The reason for this is that being able to keep some wealth leads agents who might default to increase their savings since they can keep it now. Further increasing the exemption level to $X = 0.38$ alters the distribution somewhat. But an increase to $X = 1$ has again almost no additional effect.

Figure 9 shows aggregate savings and aggregate borrowing for exemption levels,

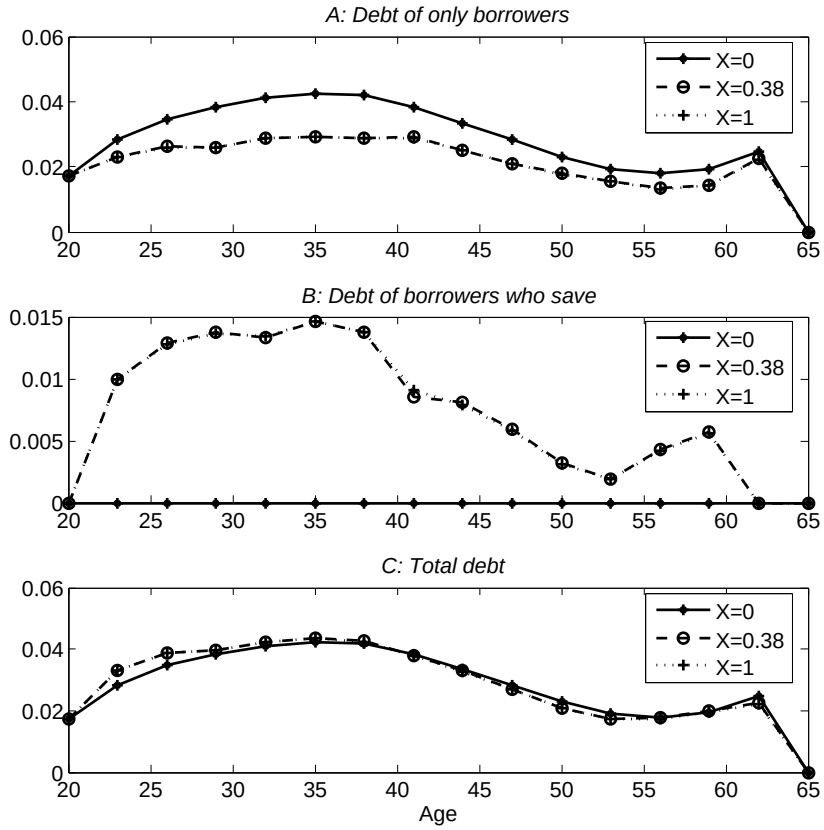

FIGURE 7: DEBTS OVER THE LIFE CYCLE: BENCHMARK MODEL ($X = 0.38$), LOW EXEMPTION ($X = 0$) AND HIGH EXEMPTION ($X = 1$)

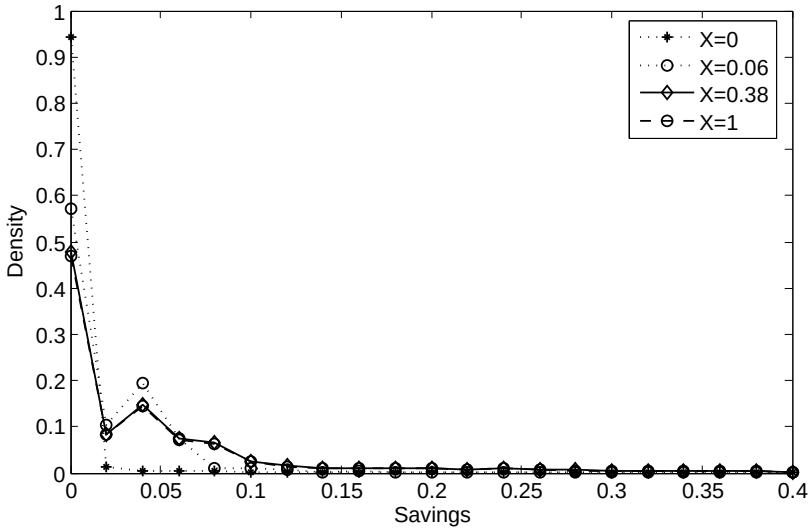

FIGURE 8: DISTRIBUTION OF ASSETS OF DEFAULTERS FOR DIFFERENT EXEMPTION LEVELS

ranging from \$0 to \$250,000. Borrowing increases rapidly for low levels of savings before it falls back to a smaller level. And then, it remains unchanged for exemption levels higher than $X = 0.2$.

Savings however keep on increasing for all levels of the wealth exemption, even though the increases get smaller. Nevertheless, it is almost the only variable that keeps changing even for high exemption levels. This confirms the discussion in section 3 that savings can increase when the exemption level increases. The reason is that the insurance through a high exemption level now is available also for relatively richer households. This leads these households to increase their savings supply.

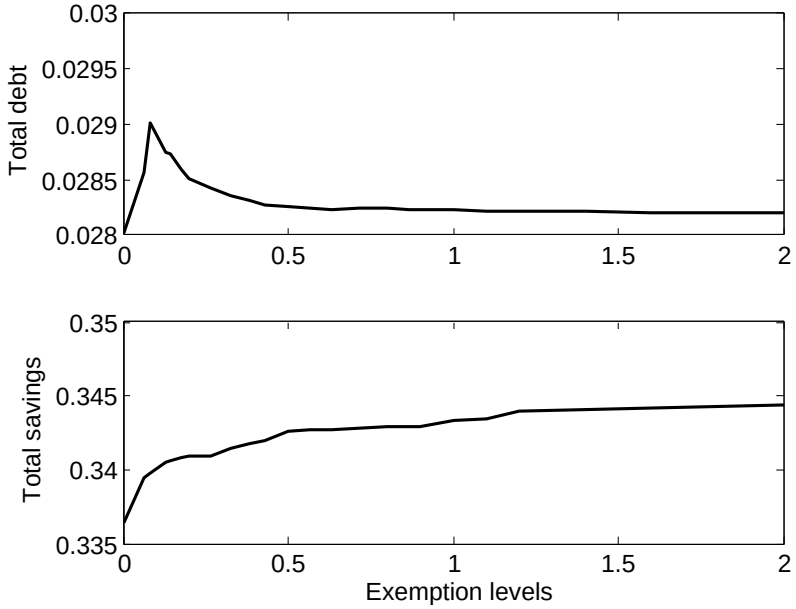

FIGURE 9: DEBTS AND SAVINGS FOR DIFFERENT EXEMPTION LEVELS.

6.4 Welfare

The top panel in figure 10 shows the welfare impacts of changing the exemption level. As utilitarian welfare measure, I use the percent increase in lifetime consumption necessary to make the households equally well off under both regimes (ECV). $X = 0.38$ is the benchmark case. A negative number here means that this particular exemption level is worse than the benchmark and vice versa. The bottom panel shows the variance of log consumption. A lower variance means that consumption is more equally distributed which, from an ex ante perspective, makes (ceteris paribus) households better off.

Figure 10 shows that there are welfare gains from moving from a very low level of the exemption ($X = 0$) to an intermediate level ($X = 0.3$). The bottom panel shows that these welfare gains are obtained by decreasing the variance of log consumption, i.e. by distributing consumption more equally. Thus, a positive exemption level allows for more risk-sharing in this economy. The welfare gains from further increases are extremely small.

Livshits et al. (2007) compare a US style system in which debt is wiped out upon

default (fresh start) with a European style system where this is not the case. They find that the fresh start system is better by about 0.06% in terms of ECV. Incorporating the second important feature of the US system, a positive exemption level, doubles this welfare differences.

The net supply of savings, aggregate savings minus aggregate debt in figure 9, is increasing in this model. While the model is a partial equilibrium model, this, at least, suggests that general equilibrium effects are unlikely to overturn the case for high exemption levels.

In figure 1, I have shown that the model implies only small differences in bankruptcy rates for different exemption levels. I have also shown that this is consistent with the data. In addition, figure 10 shows that the welfare differences between positive exemption levels are very small. The very fact that there are huge differences in exemption levels across US states suggests that the welfare implications are probably not that big. Otherwise, at least if the political process were efficient, a convergence of exemption levels should have occurred over the last decades.

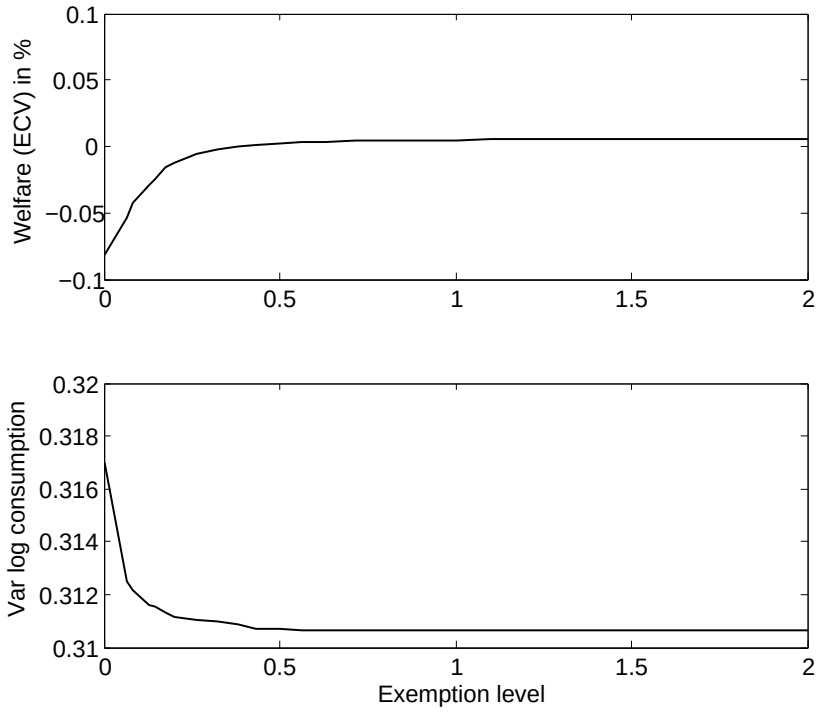

Figure 10: Welfare and variance of log consumption for different exemption levels.

7 Conclusion

In this paper, I develop a heterogenous agent life-cycle model in which agents are subject to three types of shocks: persistent and transitory labor productivity shocks and expense shocks. Financial markets in the model are incomplete but agents can insure themselves against the risk by holding a portfolio of unsecured debt and savings.

I show that including the possibility to keep some of the assets, i.e. a positive wealth exemption level, as is the case in all US states, is important.

A positive exemption level increases aggregate savings and welfare. However, I also show that increases in the exemption level beyond $25,000-$30,000 have hardly any effect. First, the default rate does not increase any further. This is consistent with the data which show no positive correlation between the exemption level and default rates. Additionally, the wide variation in exemption levels across US states is consistent with a rational political process if welfare differences are small. This is indeed the case in the model.

One limitation of the model is that it assumes that both type of expenditure shocks are born by someone else in case of default. This is plausible for high medical expense debts where hospitals often do not get paid. However for divorces, unwanted pregnancies etc. it is more reasonable to assume that households have to bear these costs themselves. They could do this if they had access to a credit line on which they could draw in these situations. In that case the loan pricing would be more difficult since the loan would actually resemble a credit card with a credit card limit. Incorporating this into a life-cycle model with expenditure shocks is left for future research.

8 Appendix

8.1 Derivations of result 1 in section 3

This is a partial proof of result 1 in the section 3. The utility function is

$$\begin{aligned} \max_s U &= \log(c_1) + \mathbb{E}\log(c_2) \\ &= \log(a_0 - s) + (1-p)\log(s) + p\log(\min[s, X]) \end{aligned}$$

Proof. There are three possible cases

$$s^* = \begin{cases} \frac{(1-p)a0}{2-p} & \text{for} \quad X \leq \frac{(1-p)a0}{2-p} \\ X & \text{for} \quad \frac{(1-p)a0}{2-p} < X < \frac{1}{2}a_0 \\ \frac{1}{2}a_0 & \text{for} \quad \frac{1}{2}a_0 \leq X \end{cases}$$

If $\min[s, X] = X$, then the last term in (1) is independent of s, therefore the first order condition with respect to s is

$$\frac{1}{a_0 - s}(-1) + \frac{(1-p)}{s} = 0$$

solving for s yields

$$s^* = \frac{(1-p)\,a0}{2-p}$$

Now, note that this was obtained under the assumption that $\min[s, X] = X$. Therefore, this results holds only for $\frac{(1-p)a0}{2-p} \geq X$

If $\min[s, X] = s$, then the problem is actually a standard problem

$$\max_s U = \log(a_0 - s) + \log(s)$$

with first order condition

$$\frac{-1}{a_0 - s} + \frac{1}{s} = 0$$

and therefore

$$s^* = \frac{1}{2}a_0$$

Note, that this case was obtained under the assumption that $\min[s, X] = s$. Therefore, it holds only $\frac{1}{2}a_0 \leq X$.

Lastly, note that for any $p \in (0, 1)$

$$\frac{1-p}{2-p} < \frac{1}{2},$$

therefore, as long as there is a positive probability for an expense shock, there will be an exemption level such that $\frac{(1-p)a0}{2-p} < X < \frac{1}{2}a_0$. In that case $s^* = X$. ■

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