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Endogenous Intellectual Property Rights and North-South Trade*

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Abstract

Even though most countries have agreed to a harmonization of intellectual property rights by signing the Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS), there is still much dispute about the optimal level of protection of intellectual property rights in the world. Particularly some developing countries argue that the high protection standards in TRIPS benefit the North at the expense of their own welfare. On the other hand, many developed countries, mostly located in the northern hemisphere, have the impression that the legal practice in the South leaves much to be desired. In this paper, we provide a framework unifying micro- and macroeconomic perspectives which is capable to analyze the North's and the South's incentives for providing IPR protection. This research suggests that current IPR policies are conducive for economic growth. Moreover, the South may experience welfare gains if the research productivity of the North is not too low.

Keywords: Endogenous Growth, Intellectual Property Rights, Trade, Dynamic Game

JEL: F10, F13, O10, O30

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1 Introduction

As trade of knowledge intensive goods accelerated during the last decades, patent and copyright infringements have become a problem of highest concern which resulted in the Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS) to secure minimum standards of protection of intellectual property rights (IPR).¹ Even though the TRIPS agreement lead to an international harmonization of IPR legislation, the enforcement of which is still a source of great international heterogeneity in IPR-protection.² This heterogeneity further fuels the ongoing debate about the optimal protection level of IPRs in the world. Particularly some developing countries argue that the high protection standards in TRIPS benefit the North at the expense of their own welfare. On the other hand, many developed countries, mostly located in the northern hemisphere, have the impression that the legal practice in the South leaves much to be desired.³ For example, the European Commission's IPR Enforcement Report 2009 gives account of serious problems with IPR-enforcement in a large number of mostly developing countries. Complaints include that injunctions and criminal sanctions are often difficult to obtain, civil procedures are lengthy and burdensome with high uncertainty of outcomes. Even if the police conducts raids, a lot of times there is no proper follow-up with seized pirate products finding their way back to the market. Involved staff is insufficiently trained, lacks resources to effectively prosecute and convict violators, and cooperation between authorities is insufficient. Oftentimes the report assesses a lack of political will with countries opposing in-depth enforcement discussions in international fora such as the WTO or the WIPO. A similar picture is drawn in the annual Special 301 Reports by the U.S. Trade Representative. In response to heterogeneity in IPR-enforcement, the European Union adopted the "IPR Enforcement Directive" to harmonize IPR-enforcement levels and eschew civil procedures that are "unnecessarily complicated and costly or involve unreasonable time limits or un-

¹TRIPS specifies a minimum set of protection that each member state has to guarantee to foreign investors. Prior to TRIPS, the protection of intellectual property was subject to the principle of nationality, i.e. national IPRs must be extended to foreign investors but without prescribing specific levels of IPR-protection. Moreover IPRs fell under the jurisdiction of the World Intellectual Property Organization (WIPO), a U.N.-agency and were largely beyond the scope of the GATT.

²The TRIPS-Agreement provides for mechanisms of law enforcement. These, however, are not always implemented by the member countries. (see e.g. Cychosz, 2003)

³Empirical assessments of the arguments brought forward by the South and the North are still in its infancy. Chauduri, Goldberg and Jia (2006), estimate the effects of global patent protection on the pharmaceutical market of India and conclude: welfare losses in the pharmaceutical market of India amount to \$ 305 million in 2000, if the US legislation would have been applied. In contrast profit gains of foreign producers amount to \$ 19,6 million.

warranted delays”. Further, efforts are being made in secret negotiations under the title Anti-Counterfeiting Trade Agreement (ACTA) to establish international standards on IPR-enforcement. It has been reported that a preliminary agreement has been reached in October 2010 between Australia, Canada, the EU, Japan, Mexico, Morocco, New Zealand, the Republic of Korea, Singapore, Switzerland, and the U.S. An ultimate objective of ACTA is that large emerging economies, “where IPR could be improved will sign up to the global pact” (European Commission, 2007).

Inspired by these recent developments, we develop a dynamic IPR-enforcement game between two regions to re-examine the following questions.

- What are the implications, if governments can only commit themselves credibly to an enforcement level of IPR-protection for the near future, for example one decade?
- Given the existence of this commitment problem, what are the determinants of different enforcement levels in the North and the South, if both regions act decentrally and in a non-cooperative fashion?
- How large is the difference between the preferred harmonized enforcement levels of IPR protection between the North and the South?
- What are the implications for economic growth and economic welfare in the North and the South, if their respective preferred harmonized protection levels were implemented?

Our focus on IPR-enforcement leads us to make two important assumptions that distinguishes our paper from the previous literature. First, while formal law may provide governments with a commitment device over long time horizons, the enforcement of laws can be changed more easily for example by reallocating resources used for IPR-enforcement to other purposes. In principle, after each legislative term the (new) government may adapt its enforcement efforts. Second, we assume that all active patents at a certain point in time receive equal treatment with respect to IPR-enforcement. In particular, we argue that it is not very realistic that the enforcement of IPR distinguishes active patents by the year of invention.⁴

⁴It might be more realistic that enforcement distinguishes between a domestic product and an invention of a foreign country. In this paper, we do not address this case.

Governments may commit to certain laws but not necessarily to their enforcement for all times. As governments and economic conditions change over the lifespan of a patent, administrations may reallocate financial resources used for IPR protection which could hamper prosecution for contributory infringement or imply longer trials in court for example. Further, the level of enforcement also reflects the availability of injunctions and the burden of proof procedures.

In order to address the questions at hand, we present a time discrete endogenous growth model of the Romer (1990)-type with international trade including the following features: First, R&D firms make profits in expectation. Second, governments commit themselves to the level of IPR enforcement for the next period only. A scenario which we label as the imperfect-commitment case as opposed to the full-commitment case which refers to a scenario in which the government could commit itself to the entire path of enforcement levels from time zero to infinity. Moreover, the level of current IPR enforcement affects both the new developed and all existing technology vintages. Imperfect commitment induces lower enforcement levels as compared to the full-commitment case and leads to a different qualitative behavior of the enforcement level with respect to changes in research productivity. In autarky, each country faces the well known trade off between static welfare losses due to monopoly power of intermediate producers and the dynamic gains from new innovations. In open economies this trade off becomes richer, as each country has an incentive to freeride on the protection levels of its trading partners which depends inversely on national market sizes. As a natural outcome of a decentralized Nash-equilibrium, governments would provide underprotection of IPRs on a global level. As regards the respective preferred harmonized protection levels it can be expected that the clash of interests between the North and the South amplifies as compared to the decentralized Nash-equilibrium, since both regions translate their own interest with a higher multiplier - the world market size and not their own market size - into the world trading system. Moreover, the relative market size of the North is now inversely and not positively related to its enforcement level, since a low relative world market size of the North allows for an "export" of dead-weight losses to the South. Scotchmer (p.336) argues in the same line of direction: "*In the TRIPS negotiation, the rich industrialized nations such as the United States pushed hard for strengthening intellectual property rights. The rich industrialized nations have both high innovative capacity and large markets. ... it is the innovative capacity, not the size of the market, that caused the large developed countries to be strong advocates of intellectual property. The size of the market cuts the other way. ..., small, very*

innovative countries should be most enthusiastic about strengthening global intellectual property rights. Switzerland, for example, was a strong advocate.” As regards the implications for economic growth and welfare, we find that the enforcement level of the North exceeds the one of the South, if the research productivity of the North is sufficiently low. Under this circumstances, we find that the implementation of the preferred harmonized protection level of the North is conducive for global economic growth, and may be welfare improving for the South, given that the research productivity of the North is not too low. The implementation of the preferred harmonized protection level of the South is harmful for welfare in the North.

In the academic literature, these questions have been approached from two perspectives.⁵ On the one hand, from a macroeconomic, growth theoretical perspective which treats the countries’ IPR protection to be exogenous and conducts a comparative statics analysis of the resulting growth rate with respect to the protection levels. On the other hand, there is a more micro-economic, industrial organization type of literature that explicitly takes IPR-protection as endogenous, but is static in nature. Until now, a unified framework which combines these two perspectives and therefore allows to consider endogenous choices of IPRs and aspects of economic growth and welfare is still missing. This research provides such a framework.

Diwan and Rodrik (1991) assess global welfare effects of harmonization, but they are maximizing aggregate welfare and this is different from implementing the preferred harmonized levels of IPR- protection of the North or the South into the global economy. Chin and Grossman (1990), and Deardorff (1992), analyze welfare effects of harmonization in a setup where the South does not innovate and the governments either provide full or zero protection. All of these seminal papers are static (partial) equilibrium models. Helpman (1993), is the only dynamic model in this time which analyzes the interaction between innovation, imitation, and IPRs, where IPRs are captured by an exogenous parameter altering the capacity of the South to imitate innovations of the North. Lai and Qiu (2003), propose a static multi-sectoral North-South trade model, where national governments set a certain patent length concluding that both regions can gain through implementing the IPR-level of the North in the world trading system. Eicher and Garcia-Penalosa (2008), examine private investments to enforce intellectual property rights protection in a closed economy taking formal institutions such as the existence of a patent office as given. In contrast, we focus on a country’s government

⁵We discuss related literature further below more in detail.

to provide intellectual property right enforcement. Although we do not neglect that private investments might be necessary for effective IPR-enforcement, we argue that the central institution-builder is the national authority. In a sense, our approach is complementary to that of Eicher (2008) in that the state's provision of IPR-protection shapes the productivity of private investment to enforce protection. In a seminal work Grossman and Lai (2004) analyze an open economy under simultaneous choices of IPR protection. There, the strength of IPRs depends on the innovative capacity and the market size of the North. In our model, the strength of IPR protection is also driven by the North's innovative capacity and its market size. But in our setting the market size of the North has as argued above a negative effect on the preferred harmonized level of IPR-protection in the world as a higher amount of deadweight losses accrues in the North.

The remainder of paper is organized as follows. In Section A we introduce the general framework. We shortly illustrate the classical trade-off in a closed economy. In Section 3, we introduce our North-South trade model which can be considered as a limiting case of a world with incomplete knowledge spillover. In this regime both regions implement their IPR-enforcement levels in a noncooperative fashion. In Section 4, we turn to the analysis of the preferred harmonized enforcement levels in the North and the South. In Section 5 we present the optimal solution of a global government to which we related the solutions of the preceding sections with respect to their implications for economic growth and welfare - Section 6. Finally, Section 7 provides a summary and concludes.

2 The Model

We consider a two-country world in which both countries or regions, n and s , differ with respect to their innovative capacities expressed, i.e. Region n is more productive in research than Region s , such that this world corresponds in the limit to a classical North-South trade model with R&D taking place in Region n . Thus, the developed Region n that may be labeled as the North produces blueprints, which are licensed out to Region s , the South. Our analysis builds on a variety-expanding-growth framework of the Romer (1990)-type, such that economic growth is driven by purposeful investments in R&D. In period t , production takes place given the state of technological knowledge and the state - i.e. the probability - of IPR enforcement denoted by $\omega_{j,t}$, $j = s, n$. That is, in period t the patent is enforced with probability $\omega_{j,t}$ in Region s or n , respectively,

and the patent holder receives operating profits of the current period. To the contrary, with probability $1 - \omega_{j,t}$ the patent is not enforced and the intermediate is imitated. In period t the government of Region j commits itself credibly to an enforcement level of IPR protection for the subsequent period $t + 1$. Hence, R&D takes place in period t given the current stock of technological knowledge and the commitment of governments to the enforcement level of IPR protection for the subsequent period, $\omega_{j,t+1}$. Simplifying matters without losing economic insights, we assume that imitation is costless. Thus, an imitated intermediate is supplied under full competition and operating profits are zero which could be interpreted as a normalization.⁶

Both economies are populated by a measure L_j , $j = s, n$ of identical households that inelastically supplies one unit of labor in each period. There is no population growth and time moves in discrete steps $t = 0, 1, 2, \dots, \infty$.

2.1 Final Good Production

In Region $j = s, n$, the final good Y_j is produced by using labor L_j and a range of intermediates indexed by i . The production function takes the following form

$$Y_j = A_j L_j^{1-\alpha} \int_0^{N_n} [x_j(i)]^\alpha di, \quad (1)$$

where A_j represents a productivity measure, N_n is the measure of different intermediates invented in the North, and $x_j(i)$ stands for the amount of intermediate i used in final good production. The elasticity of substitution between the different intermediates is denoted by $\alpha \in (0, 1)$.

2.2 Intermediate Goods Production

Each intermediate good i is produced by a monopolist or an imitator and requires only final output Y_j as a factor of production. Moreover, production of all types of intermediates i is subject to the same technology in the sense that production of one unit x_j requires one unit final output Y_j . We choose final output Y_j as the numeraire such that $p_{Y_j} = 1$ and marginal production costs of intermediates are equal to unity. The implied symmetric equilibrium on the market for intermediates induces equal

⁶In contrast to Grossman and Lai (2004) our focus is explicitly on intellectual property rights *enforcement*. For simplicity we thus assume an infinite patent length and neglect the issue of patent breadth.

prices and demand for all types of intermediates, such that $p_{mj}(i) = p_{mj} = 1/\alpha$, $x_{m,j}(i) = x_{m,j}$ for all protected intermediates and $p_{c,j}(i) = p_{c,j} = 1$, $x_{c,j}(i) = x_{c,j}$ for all imitated intermediates. Demand in Region j for protected intermediates reads therefore as

$$x_{m,j} = \lambda_j \alpha^{\frac{2}{1-\alpha}}, \quad (2)$$

with $\lambda_j = L_j A_j^{\frac{1}{1-\alpha}}$ reflecting the "effective" market size of Region $j = s, n$. Obviously, effective market size is not a matter of being rich or poor. A small economy in terms of its population may exhibit a large effective market when its productivity level in final good production is sufficiently large and vice versa. Technology owners located in the North facing (2) yield operating profits in period t

$$\pi_{m,n} = \bar{P}(\lambda_s + \lambda_n), \quad (3)$$

with $\bar{P} = \left(\frac{1-\alpha}{\alpha}\right) \alpha^{\frac{2}{1-\alpha}} > 0$.

If an intermediate is copied and, hence, sold at the competitive price $p_{c,j} = 1$, demand increases up to

$$x_{c,j} = \lambda_j \alpha^{\frac{1}{1-\alpha}}, \quad (4)$$

and operating profits in j at t are zero. Denoting the number of protected intermediates at time t by $N_{m,j,t} = \omega_{j,t} * N_{n,t}$, and the ones that are imitated by $N_{c,j,t} = [1 - \omega_{j,t}] * N_{n,t}$ aggregate output in Region j writes therefore as

$$Y_{j,t} = \lambda_j \left[\int_0^{N_{m,j,t}} [x_{m,j}(i)]^\alpha di + \int_0^{N_{c,j,t}} [x_{c,j}(i)]^\alpha di \right]. \quad (5)$$

As additionally $x_{m,j} = \alpha^{\frac{1}{1-\alpha}} x_{c,j}$, we obtain for $Y_{j,t}$:

$$Y_{j,t} = \lambda_j N_{n,t} [1 + \omega_{j,t} (\alpha^{\frac{\alpha}{1-\alpha}} - 1)] x_{c,j}^\alpha, \quad (6)$$

where $\omega_{j,t} (\alpha^{\frac{\alpha}{1-\alpha}} - 1) < 0$ represents deadweight losses due to monopolistic competition.⁷

2.3 Research and Development

R&D constitutes the search for new designs (blueprints) of machines which requires resources. To this end research firms rent labor services and capital inputs. It simplifies the analysis considerably though if we assume that final output which incorporates

⁷Notice that for $\omega_{j,t} = 1$, i.e., full patent protection, we obtain the standard Romer (1990) production function: $Y_{j,t} = A_j L_j^{1-\alpha} N_{n,t} \underbrace{(\alpha^{\frac{1}{1-\alpha}} x_{c,j})^\alpha}_{x_{m,j}}$. The case without patent protection, $\omega_{j,t} = 0$, yields the

highest possible output from a static perspective: $Y_{j,t} = A_j L_j^{1-\alpha} N_{n,t} x_{c,j}^\alpha$. Of course this undermines incentives to invest in R&D.

both labor and machines enters as the only factor of production into the R&D process. In addition we assume, that R&D output is positively affected by a research lab's knowledge capital which is reflected in its simplest form by the average number of innovations or the economy's average level of technological knowledge $\frac{N_{n,t}}{L_n}$. In particular, we assume the following cost function on the research-lab level:

$$\zeta_n(\eta_{n,t}) = \frac{\delta_n \eta_{n,t}^2}{\frac{N_{n,t}}{L_n}}, \quad (7)$$

where η_t denotes the number of new inventions at time t and δ_n reflects the research productivity or the quality of research infrastructure which could alternatively be interpreted as a measure of a research-lab's human capital. That is, the higher the level of human capital, the lower δ_n which implies that lab-equipment can be used more productively.⁸ The assumption of decreasing returns on the firm and industry level with respect to R&D expenditures has been supported empirically, f.e. by Pakes and Griliches (1984) and Hall et al. (1988). On the macro level, the probably most important source of decreasing returns in R&D can be seen in an increased probability of duplicative research through an increasing number of both rivals and expenditures, even though the R&D process as such may be driven by large spillovers (Amir, 2000; Kortum, 1993; Klette and Kortum, 2004). In a related line of argumentation it is also possible to think of plausible limits in transforming an ever increasing stock of new ideas into usable knowledge for production (Weitzman, 1998).⁹ We would also like to stress that we are not challenging approaches employing constant returns to scale production functions in R&D, but that we consider the assumption of decreasing returns to scale as very convenient for our purposes - we come back to this point further below. In our setting a new blueprint invented in period t becoming productive in $t + 1$ receives a patent of infinite length that is enforced with probability $\omega_{j,t+1}$ in period $t + 1$. Accordingly, the expected value of an invention i at time t reads as

$$E_{o,t} = E_{o,t}[V(i)] = \sum_{\tau=t+1}^{\infty} \beta^{\tau} \bar{P}(\lambda_n \omega_{n,\tau} + \lambda_s \omega_{s,\tau}). \quad (8)$$

⁸The assumption that both research productivity or human capital as well as the current technology stock play a positive role for innovative output and are complementary to a certain extent is standard in the literature. For example, in Romer (1990), p. 86, the aggregate stock of designs evolves according to $\dot{A} = \delta_a H_A A$, where A is the stock of designs, H_A is human capital and δ_a is a productivity parameter.

⁹From an aggregate perspective, decreasing returns may also reflect heterogeneity in the cost of research projects. A similar argument can be found in Scotchmer (2004) (ch. 11).

Optimality requires that marginal costs for an additional invention must equal its expected value. Consequently, inventions per research lab are determined by

$$\eta_{n,t} = E_{o,t} \frac{N_{n,t}}{2\delta_n L_n}, \quad (9)$$

such that aggregate technological knowledge evolves according to

$$N_{n,t+1} - N_{n,t} = \eta_{n,t} L_n = E_{o,t} \frac{N_{n,t}}{2\delta_n}. \quad (10)$$

2.3.1 The Household's and the Government's Problem

Given optimal decisions of households in Region j , the government of Region j maximizes aggregate welfare by implementing an optimal enforcement level of IPR protection, for the subsequent period. In this setting the government cannot commit itself credibly to an enforcement level of IPRs for all times $t \rightarrow \infty$ but only for the next period $t + 1$ which seems to be very realistic whenever one period encompasses more than one decade.

Each of the households seeks to maximize

$$U_{j,t} = \sum_{t=0}^{\infty} \beta^t c_{j,t}, \quad (11)$$

where $0 < \beta < 1$ is a discount factor and $\frac{1-\beta}{\beta}$ is the rate of time preference, such that the government of Region j solves¹⁰

$$\max_{\{w_{j,t+1}\}_{t=0}^{\infty}} W_j = \sum_{t=0}^{\infty} \beta^t c_{j,t} L_j, \quad (12)$$

subject to

$$N_{n,t+1} = N_{n,t} \left[1 + \frac{E_{o,t}}{2\delta_n} \right], \quad (13)$$

where

$$c_s L_s = N_{n,t} \lambda_s [\bar{Y} + \omega_{s,t} (\bar{D} - \bar{P})], \quad (14)$$

$$c_n L_n = N_{n,t} \left[\lambda_n (\bar{Y} + \omega_{n,t} \bar{D}) + \lambda_s \omega_{s,t} \bar{P} - \frac{E_{o,t}^2}{4\delta_n} \right], \quad (15)$$

¹⁰The linear instantaneous utility function induces a knife-edge condition $1+r = \frac{1}{\beta}$ and the objective function converges to an upper bound as t approaches to ∞ whenever the economy's long-run growth rate does not exceed the rate of time preference, i.e. $g_{BGP} < \frac{1-\beta}{\beta}$.

with $\bar{Y} = \alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}} > 0$ reflecting the contribution of an intermediate to final production net of production costs for intermediates and $\bar{D} = \alpha^{\frac{2\alpha}{1-\alpha}} - \alpha^{\frac{\alpha}{1-\alpha}} + \alpha^{\frac{1}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}} < 0$ representing the deadweight-loss factor net of production costs for intermediates. We will describe the features of the welfare maximization problem more in detail in the next section.

3 Non-cooperative Enforcement of IPRs

In this section, we introduce strategic interaction between governments in terms of national levels of IPR-enforcement. Both regions set their respective levels of IPR-protection simultaneously and in a non-cooperative fashion. At this point the fundamental conflict of interests between the two regions becomes already visible. The discounted profit flow per innovation in the North depends on the IPR-policies of both regions, while deadweight losses accrue on the respective national markets only. We will come back to this problem further below. Prior to that we introduce the two regions' welfare maximization problems and derive the steady-state levels of IPR-protection in a decentralized Nash-equilibrium.

3.1 The Problem of the South

The objective function of the South's government at time t is specified as

$$U_{s,t} = \sum_{t=0}^{\infty} \beta^t N_{n,t} \lambda_s [\bar{Y} + \omega_{s,t}(\bar{D} - \bar{P})]. \quad (16)$$

Again \bar{Y} reflects the contribution of an additional intermediate to total production if IPRs were not enforced, \bar{D} captures deadweight losses due to IPR-protection and \bar{P} represents profits of technology owners in the North. As the dynamics of the technology stock is given by (10), we yield via dynamic programming the reaction function of the South from the first-order condition along the balanced growth path - i.e. imposing $\omega_{j,t+1} = \omega_{j,t} = \omega_j$ with $j = n, s$ - as:¹¹

$$\omega_s^s(\omega_n) = - \left(\frac{1-\beta}{2-\beta} \right) \left[\frac{\bar{Y}}{\bar{D}-\bar{P}} + \frac{2\Delta_o \lambda_o}{\beta \bar{P} \lambda_s} \right] - \frac{1}{2-\beta} \frac{\lambda_n}{\lambda_s} \omega_n, \quad (17)$$

¹¹The first-order condition reads as: $R^s(\omega_n, \omega_s) = \left(1 + \frac{E_{o,t}}{2\delta_n}\right) (\bar{D}-\bar{P}) + \frac{\beta \lambda_s \bar{P}}{2\delta} [\bar{Y} + \omega_{s,t+1}(\bar{D}-\bar{P})] = 0$.

where $\Delta_o = \frac{\delta_n}{\lambda_o}$ and $\lambda_o = \lambda_n + \lambda_s$.¹²

In light of (17), we are thus able to establish the following proposition:

Proposition 1

- (i) *The steady state level of intellectual property rights protection in the South is a strategic substitute of intellectual property rights protection of the North.*
- (ii) *For ω_n given, the South's IPR-enforcement increases with the effective market size of the South, λ_s , and with the research productivity of the North – i.e., is decreasing in δ_n .*

Item(ii) reveals that the level of IPR-enforcement in the South depends on its own market size. Whenever the South exhibits a large effective market size its impact on E_o becomes large such that its incentive to freeride on the North's protection levels shrinks and vice versa. Moreover it is also worth to stress that the South's level of IPR-protection may even be complete, that is $\omega_s = 1$, for any given level of IPR-enforcement in the North, if Δ_o is sufficiently low, e.g., due to a high research productivity in the North. In the limit $\Delta_o \rightarrow 0$, the level of IPR-protection in the South will thus reach its upper boundary $w_s = 1$ for sufficiently small Δ_o and ω_n .¹³ Further below we will argue that this is not a remote possibility but under certain circumstances a possible outcome in equilibrium although the South's research sector is inactive.

3.2 The Problem of the North

Since the research sector of Region n is active, its objective function incorporates R&D expenditures and profit flows from the South to the North, $N_{n,t} \lambda_s \bar{P} \omega_{s,t}$, which are subject to the IPR-protection level in the South. The discounted flow of expected profits and the law of motion of the technology stock are captured by (8) and (10). Hence the optimization problem of the North writes as

$$\begin{aligned} \max_{\{\omega_{n,t+1}\}_{t \in \mathbb{N}_+}} U_n &= \sum_{t=0}^{\infty} \beta^t N_{n,t} \left[\lambda_n (\bar{Y} + \omega_{n,t} \bar{D}) + \lambda_s \omega_{s,t} \bar{P} - \frac{E_{o,t}^2}{4\delta_n} \right], \\ \text{s.t.} & \quad (10). \end{aligned}$$

¹²We indicate, the reaction function of a region n or s by respective superscripts.

¹³Notice that positive consumption levels at any feasible level of IPR-protection require $\bar{Y} > \bar{P} - \bar{D}$. Consequently, the first term in brackets of (17) is greater than 1 (i.e. $\frac{\bar{Y}}{\bar{D}-\bar{P}} < -1$).

The reaction function of the North in steady state $\omega_n^n(\omega_s)$ is given by:¹⁴

$$\begin{aligned} R^n(\omega_n, \omega_s) &= \frac{\bar{D} - \bar{P}}{2} \frac{\beta}{1 - \beta} \bar{P} \left(\frac{\lambda_n}{\lambda_o} \omega_n + \frac{\lambda_s}{\lambda_o} \omega_s \right) + \bar{D} \Delta_o \\ &+ \frac{\beta \bar{P}}{2} \left[\frac{\lambda_n}{\lambda_o} (\bar{Y} + \omega_n \bar{D}) + \frac{\lambda_s}{\lambda_o} \bar{P} \omega_s - \left(\frac{\beta}{1 - \beta} \right)^2 \frac{\bar{P}^2}{4 \Delta_o} \left(\frac{\lambda_n}{\lambda_o} \omega_n + \frac{\lambda_s}{\lambda_o} \omega_s \right)^2 \right] \\ &= 0. \end{aligned}$$

We therefore obtain:

Lemma 1

- (i) *There exists a unique economically sensible solution $\omega_n^n(\omega_s)$ to $R_n(\omega_n, \omega_s) = 0$.*
- (ii) *The North's reaction function $\omega_n^n(\omega_s)$ is strictly decreasing and strictly concave on \mathbb{R}_+ .*

The proof can be found in the appendix.

Before we turn to the details of the equilibrium analysis it is worth noting that national levels of IPR-protection are strategic substitutes to foreign protection levels. The intuition behind this result is simple. Any region which increases its level of IPR-protection carries automatically accrued deadweight losses per innovation. The increased protection level however spills over into the discounted flow of profits per innovation (see Eq. (8)). Consequently, governments freeride on each other's efforts of IPR-protection and the dynamic game with open economies exhibits the characteristics of a classical public good game with the corresponding inefficient outcomes in the Nash-solution.

3.3 Equilibrium

In this section we analyze the equilibrium properties of the dynamic IPR-protection game between the North and the South. To this end we portray economically reasonable equilibrium outcomes of this game and perform an inspection of its comparative-static behavior in the next subsection, with respect to changes in research costs relative to a given world market size per innovation, i.e. $\Delta_o = \frac{\delta_n}{\lambda_o}$ and the relative market size $\frac{\lambda_n}{\lambda_o}$. For this purpose, it is convenient to rewrite the reaction function of the South (17) as¹⁵

$$\omega_n^s(\omega_s) = (1 - \beta) \left[\frac{\bar{Y}}{\bar{D} - \bar{P}} \frac{\lambda_s}{\lambda_n} + \frac{2 \Delta_o}{\beta \bar{P}} \frac{\lambda_o}{\lambda_n} \right] - (2 - \beta) \frac{\lambda_s}{\lambda_n} \omega_s. \quad (18)$$

¹⁴The first-order condition reads as:

$$R^n(\omega_{n,t+1}, \omega_{s,t+1}) = \bar{D} + \bar{P} \left(1 - \frac{E_{ot}}{\delta_n} \right) + \beta \frac{\bar{P} \lambda_n}{2 \delta_n} \left[\bar{Y} + \omega_{n,t+1} \bar{D} + \bar{P} \left(\omega_{n,t+1} + \omega_{s,t+1} \frac{\lambda_s}{\lambda_n} \right) - \frac{E_{o,t+1}^2}{2 \delta_n \lambda_n} \right] = 0.$$

¹⁵Note that this is possible as $\omega_s(\omega_n)$ is a bijection.

Now it is possible to show that the reaction function of the North $\omega_n^n(\omega_s)$ and the one of the South $\omega_n^s(\omega_s)$ possess only one economically meaningful intersection which we denote by $(\omega_n^e; \omega_s^e)$ - an example is presented in Figure 2.¹⁶ Again, we indicate the resulting equilibrium levels of IPR-enforcement by a bar: $(\bar{\omega}_n^e; \bar{\omega}_s^e)$. Further, let us define $\hat{y} \equiv \max\{\min\{y, 1\}, 0\}$ and $\hat{y}(x) \equiv \max\{\min\{y(x), 1\}, 0\}$ for a constant y and a function $y(x)$, respectively. The following proposition characterizes the levels of IPR-protection in a steady-state equilibrium, $(\bar{\omega}_n^e; \bar{\omega}_s^e)$

Proposition 2 (Steady state equilibrium)

In steady state, there exists a unique Nash-equilibrium of the IPR-protection game in the North-South model. The unique enforcement levels in equilibrium are characterized by

$$\bar{\omega}_n^e = \begin{cases} \hat{\omega}_n^n(0), & \text{if } \omega_s^e \leq 0, \\ \hat{\omega}_n^e, & \text{if } \omega_s^e \in (0, 1), \\ \hat{\omega}_n^n(1), & \text{if } \omega_s^e \geq 1, \end{cases}$$

$$\bar{\omega}_s^e = \begin{cases} \hat{\omega}_s^s(0), & \text{if } \omega_n^e \leq 0, \\ \hat{\omega}_s^e, & \text{if } \omega_n^e \in (0, 1), \\ \hat{\omega}_s^s(1), & \text{if } \omega_n^e \geq 1. \end{cases}$$

The proof can be found in Appendix C.5.

At this point several remarks are at order. First, we cannot rule out the possibility of zero IPR-protection in the South, i.e. $\bar{\omega}_s^e = 0$. In this case, the model switches structurally to the closed-economy setting. Since we know, moreover, that respective levels of IPR-enforcement are considered as strategic substitutes, trade opening between North and South lowers the enforcement level of the North provided that $\bar{\omega}_s^e > 0$. Second, trade opening in the South enhances the level of IPR-protection compared to autarky since the South – even though it does not conduct research – internalizes the effect of its IPR-protection level on R&D-incentives in the North.

3.4 Comparative Statics

We are now in the position to perform a comparative-static analysis. As a change in regional market sizes, λ_j , $j = n, s$, can affect the total world market size, respective relative market sizes of the North and South, or both, we analyze either a change in total world market size, keeping the respective relative market sizes constant or a

¹⁶At one of the intersections ω_n is always negative, whereas at the other intersection it can be both positive and negative. A formal proof can be found in Appendix C.5.

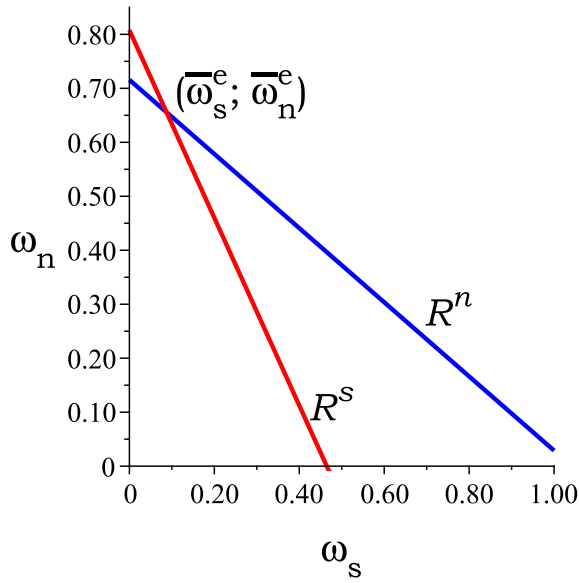


Figure 1: Nash-equilibrium of the North-South policy game

change in the relative market sizes, leaving the total market size unaffected.¹⁷ Thus there are two parameter shifts in the center of our interest:

1. Changes in research productivity and/or total effective market size are captured by changes in $\Delta_o = \frac{\lambda_o}{\delta_n}$, with $\lambda_o = \lambda_n + \lambda_s$. Reasonably, changes in Δ_o do not reflect changes in relative market sizes which have different implications.
2. We therefore analyze changes in effective market sizes $\frac{\lambda_n}{\lambda_o}$ separately, for given Δ_o , i.e. total market size λ_o and research productivity of the North δ_n .

We begin our analysis with Δ_o and ask in which direction the intersection of the reaction functions $(\bar{\omega}_n^e; \bar{\omega}_s^e)$ moves in response to changes in Δ_o . The equilibrium represented by the pair $(\bar{\omega}_n^e; \bar{\omega}_s^e)$ can be derived in the following way: inserting $\omega_s^s(\omega_n)$ into $R^n(\omega_n, \omega_s)$ yields a quadratic function $R_n^e(\omega_n)$. The function $R_n^e(\omega_n)$ possesses two real roots where only one is economically sensible and corresponds to ω_n^e . Inserting ω_n^e into $\omega_s^s(\omega_n)$, we obtain ω_s^e . Perceiving now ω_n^e and ω_s^e as functions in Δ_o , we obtain

Lemma 2

- (i) ω_n^e is strictly concave in Δ_o .

¹⁷For example an increase in the North's market size leaving that of the South unaffected would increase both, the world market size and the relative market size of North. Consequently, the effect on the IPR-protection level would be a combination of the two effects. For this reason, it seems natural to isolate the resulting effects from each other.

(ii) ω_s^e is strictly convex in Δ_o .

The proof can be found in the appendix.

As far as IPR-enforcement levels in the South are concerned, ω_s^e is a declining line in Δ_o if there is no IPR-protection in the North. For positive protection levels in the North, the South's enforcement level must be strictly below this line (see expression (17)) as the protection level of the North acts as a strategic substitute from the perspective of the South. Consequently, the protection level of the South becomes convex as IPR-protection in the North is concave. To characterize the comparative-static behavior of our IPR-protection game, we have to account for corner solutions, explicitly (see Proposition 2). Given Lemma 2, there exists a critical level $\Delta_o^{0,j}$, $j = n, s$, for both countries individually, such that for any $\Delta_o > \Delta_o^{0,j}$ the respective country j does not establish positive levels of IPR-enforcement.¹⁸ This implies for the situation $\Delta_o^{0,s} < \Delta_o^{0,n}$ – i.e., the South's critical threshold level is smaller than that of the North – that for all $\Delta_o^{0,n} > \Delta_o > \Delta_o^{0,s}$ the South does not offer protection in equilibrium whereas the North acts as in autarky. The opposite holds true in the situation where $\Delta_o^{0,n} < \Delta_o^{0,s}$. In the following we focus on the case $\Delta_o^{0,s} < \Delta_o^{0,n}$ and define $\Delta_o^0 \equiv \Delta_o^{0,s}$ as the smallest threshold corresponding to the South. This condition seems to be more reasonable compared to the opposite case as it implies a minimum effective market size of the North relative to the South

$$\frac{\lambda_n}{\lambda_s} > \frac{\bar{D}}{\bar{D} - \bar{P}}. \quad (19)$$

In the next proposition, we characterize the comparative statics of equilibrium IPR-protection levels with respect to changes in Δ_o given that condition (19) holds - see also Figure 3.

Proposition 3 (Effect of Δ_o on IPR-enforcement)

If $\frac{\lambda_n}{\lambda_s} > \frac{\bar{D}}{\bar{D} - \bar{P}}$ then

- (i) $\bar{\omega}_s^e$ is positive and strictly decreasing with Δ_o for all $\Delta_o < \Delta_o^0$. For all $\Delta_o \geq \Delta_o^0$, $\bar{\omega}_s^e = 0$.
- (ii) $\bar{\omega}_n^e$ is identical to its value in autarky for all $\Delta_o > \Delta_o^0$.
- (iii) There exists a unique value $\Delta_o^{eq} < \Delta_o^0$ where $\bar{\omega}_n^e = \bar{\omega}_s^e$. For all $\Delta_o < \Delta_o^{eq}$, $\bar{\omega}_n^e < \bar{\omega}_s^e$ and for all $\Delta_o^0 > \Delta_o > \Delta_o^{eq}$, $\bar{\omega}_n^e > \bar{\omega}_s^e > 0$.

¹⁸This claim is also verified in the proof of Proposition 3.

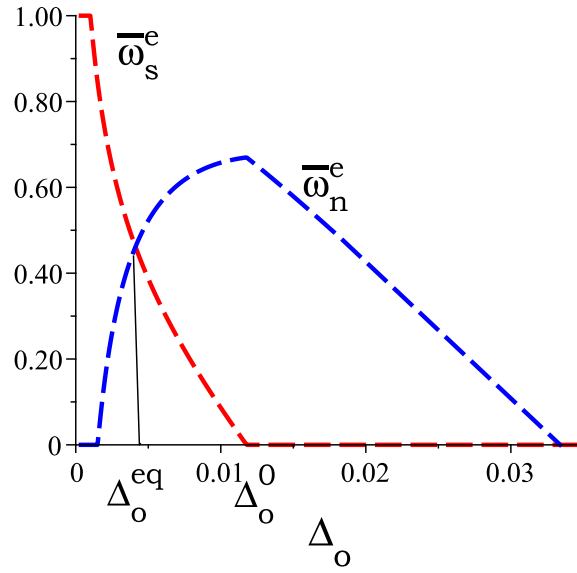


Figure 2: Change in the equilibrium pair $(\bar{\omega}_n^e; \bar{\omega}_s^e)$ in response to variations in Δ_o .

(iv) If additionally $\frac{\lambda_s}{\lambda_n} > \tilde{\lambda}_o$, then there exists a Δ_o^{crit} with

$$\frac{\partial \bar{\omega}_n^e}{\partial \Delta_o} \begin{cases} > 0, & \text{if } \Delta_o < \Delta_o^{crit} \\ = 0, & \text{if } \Delta_o = \Delta_o^{crit} \\ < 0, & \text{if } \Delta_o^0 \geq \Delta_o > \Delta_o^{crit} \end{cases} .$$

(v) If $\frac{\lambda_s}{\lambda_n} \leq \tilde{\lambda}_o$, then the North's equilibrium level of IPR-protection is increasing in Δ_o over the entire interval $\Delta_o < \Delta_o^a$.

The threshold $\tilde{\lambda}_o$ is defined by

$$\tilde{\lambda}_o = \frac{(2 - \beta)(\bar{D} - \bar{P})(-(1 - \beta)\beta\bar{P} - \sqrt{\beta\bar{P}B})}{-\beta\bar{P}B + \sqrt{\beta\bar{P}B}[(3 - \beta)(2 - \beta)\bar{D} - (4 - 3\beta)\bar{P}]},$$

where $B = (-2(2 - \beta)(1 - \beta)\bar{D} - \beta(3 - 2\beta)\bar{P})$.

The proof can be found in the appendix.

While Items (i) and (ii) of Proposition 3 summarize the preceding discussion with respect to corner or interior solutions induced by $\Delta_o \leq \Delta_o^0$, the results in Items (iii)-(v) apply to the intersection of the countries' reaction functions. If the relative effective market size of the North is comparatively low, in the sense that $\frac{\lambda_s}{\lambda_n} > \tilde{\lambda}_o$ (Item (iv)), the IPR-enforcement level of the North follows in equilibrium an inverted u-shaped pattern with respect to increases in Δ_o . To the contrary, if $\frac{\lambda_s}{\lambda_n} < \tilde{\lambda}_o$ (Item (v)), the protection

level of the North is monotonically increasing in Δ_o . In this case, the incentive to lower the enforcement level of IPRs is counteracted by a comparatively high relative effective world market size of the North. Moreover, it is important to stress that the model may also include the case of full IPR-protection on a global scale, i.e. $\bar{\omega}_n^e = \bar{\omega}_s^e = 1$, in some small environment around Δ_o^{eq} , as well as the case of very low protection levels, whenever Δ_o is very large. Item (iii) suggests that higher enforcement levels of IPR-protection in the North compared to the South result for example from a sufficiently low research productivity in the North, such that $\Delta_o > \Delta_o^{eq}$.

Before turning our attention to the comparative statics with respect to changes in relative market sizes, we should clarify the effect of Δ_o on the BGP-growth rate which is driven by the evolution of the world technology stock through R&D efforts undertaken in the North. Noting that the optimal degree of IPR-enforcement depends on the parameters α, β, λ_n and λ_s , we denote the subset of the parameter space for which $(\omega_n^e, \omega_s^e) \in (0, 1)^2$ – i.e., for which we obtain interior solutions – by $\Gamma_I \subset (0, 1)^2 \times \mathbb{R}_{++}^4$. We are now able to formulate the next proposition which verifies the same relationship between Δ_o and economic growth.

Proposition 4 (Effect of Δ_o on steady state growth)

The global rate of growth in steady state is strictly decreasing with Δ_o on Γ_I .

The proof can be found in the appendix.

Thus, even though the North may reduce the level of IPR-protection for low values of Δ_o , the steady state-growth rate is strictly decreasing with Δ_o .

Finally we examine the role of relative market sizes for IPR-protection and growth.

We focus again on interior equilibria, i.e., on economies with parameter values in Γ_I .

We obtain

Proposition 5 (Effect of relative effective market size)

Both countries' IPR protection levels in equilibrium increase with their relative effective market sizes. The global rate of growth is unaffected by the relative effective market sizes.

The proof can be found in the appendix.

Governments react with tighter IPR-enforcement in response to an increase in their national relative market share, since their relative weights incorporated in the first-order conditions increases as well. Symmetrically, the region with a lower world market share reduces its IPR-enforcement level since its effect on discounted profits per innovation is

4 Harmonization of IPR-enforcement

We now explore the preferred harmonized protection levels of the North and South by conducting the following thought experiment: which level of IPR protection would the North and South perceive as optimal for the world economy, i.e. strategic interaction is absent and the North and South optimize over the total effective market size of the world economy. Since we already have demonstrated in the previous section that decentrally chosen levels of IPR enforcement coincide rather by accident, we should expect that the North and South would also propose different harmonized levels of IPR-protection in general. Harmonization means in our context that $\omega_j^h = \omega_n = \omega_s$, with $j = n, s$. Thus, the incentive to freeride on the other region's level of IPR-enforcement is absent. The focus of this section is to examine the magnitude of the difference between ω_s^h and ω_n^h given that each region could propose a global level of IPR protection.

4.1 The Problem of the South and Comparative Statics

We begin with the optimization problem of the government located in the South. By choosing an optimal harmonized level of IPR-protection, ω_s^h , the South also determines the level of IPR-protection in the North, such that the South's optimization problem reads as follows

$$\begin{aligned} \max_{\{\omega_{s,t+1}^h\}_{t=0}^{\infty}} U_s^h &= \sum_{t=0}^{\infty} \beta^t N_{n,t} \lambda_s [\bar{Y} + \omega_{s,t}^h (\bar{D} - \bar{P})] \\ \text{s.t. } N_{n,t+1} &= N_{n,t} \left(1 + \frac{E_{h,t}}{2\delta_n} \right), \end{aligned}$$

where discounted profits per invention are specified as

$$E_{h,t} = \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \omega_{s,\tau}^h \bar{P} \lambda_o. \quad (20)$$

¹⁹The BGP-growth rate remains unchanged because the incentive to innovate depends on total world market size given national IPR-policies.

Hence, we obtain along the balanced growth path, the preferred harmonized enforcement level of the South as²⁰

$$\omega_s^h = -\frac{1-\beta}{2-\beta} \left(\frac{\bar{Y}}{\bar{D}-\bar{P}} + \frac{2\Delta_o}{\beta\bar{P}} \right). \quad (21)$$

The desired harmonized protection level of the South differs from the one in the decentralized solution in two respects: First, the government of the South internalizes the impact of its policy on the overall effective world market size λ_o rather than λ_s only, since $\Delta_o = \frac{\delta_n}{\lambda_o}$. Thus the induced effect on innovative activities in the North is higher compared to the non-cooperative scenario. Second, also as a consequence of harmonization, the optimal level of the South is independent of ω_n . Consequently, we obtain for the comparative statics:

Proposition 6 (Desired harmonized IPR-enforcement of the South)

The preferred harmonized level of intellectual property rights protection of the South

- (i) *increases with effective market sizes of the South and the North and with the research productivity of the North,*
- (ii) *is complete – i.e., $\omega_s^h = 1$ – if the research productivity of the North is large enough (δ_n small enough).*

4.2 The Problem of the North and Comparative Statics

The optimization problem of the North incorporates now profit inflows from the South which are contrary to the decentralized protection game subject to the proposed harmonized protection level ω_n^h of the North

$$\max_{\{\omega_{n,t+1}^h\}_{t=0}^{\infty}} U_n^h = \sum_{t=0}^{\infty} \beta^t N_{n,t} \lambda_n \left[\bar{Y} + \omega_{n,t}^h \left(\bar{D} + \bar{P} \frac{\lambda_s}{\lambda_n} \right) - \frac{E_{h,t}^2}{4\delta_n \lambda_n} \right], \quad (22)$$

$$\text{s.t. } N_{n,t+1} = N_{n,t} \left(1 + \frac{E_{h,t}}{2\delta_n} \right). \quad (23)$$

Before proceeding, we should exclude corner solutions which are economically of little interest.²¹ Intuitively, violations of the second-order condition to problem (22) result from a comparatively large effective world market size of the South, such that the gains

²⁰The first-order condition reads as: $\left(1 + \frac{E_{h,t}}{2\delta_n} \right) (\bar{D} - \bar{P}) = -\beta \frac{\lambda_o \bar{P}}{2\delta_n} [\bar{Y} + \omega_{s,t+1}^h (\bar{D} - \bar{P})]$.

²¹In the case of corner solutions $\omega_n^h \in \{0, 1\}$, the desired level of harmonized IPR-enforcement may be constant in Δ_o respectively in λ_n/λ_s .

(profit flows to the North) necessarily overcompensate the costs (aggregate deadweight losses in the North) due to a marginal increase in ω_n^h , such that the North opts always for complete protection, i.e. $\omega_n^h = 1$. The next lemma identifies an upper limit for $\frac{\lambda_s}{\lambda_n}$ such that the second-order condition is satisfied:

Lemma 3 (Second-order condition)

The North's optimization problem satisfies the second-order condition for a maximum if and only if

$$\frac{\lambda_s}{\lambda_n} < 1 - \frac{2\bar{D}}{\bar{P}}. \quad (24)$$

In the following we restrict our analysis to the case where (24) holds.²² In steady state, the optimal level of global IPR-protection from the perspective of the North satisfies²³

$$\begin{aligned} R_n^h(\omega_n^h) &= \Delta_o(\bar{D} + \frac{\lambda_s}{\lambda_n}\bar{P}) + \frac{E_h}{2\lambda_o}(\bar{D} - \bar{P}) \\ &+ \frac{\beta\bar{P}}{2} \left[\bar{Y} + \omega_n^h \left(\bar{D} + \bar{P} \frac{\lambda_s}{\lambda_n} \right) - \frac{E_h^2}{4\Delta_o\lambda_o\lambda_n} \right] = 0. \end{aligned} \quad (25)$$

The next proposition systemizes the effects of a change in Δ_o on the preferred harmonized protection level of the North, given the relative world market size $\frac{\lambda_n}{\lambda_s}$ (Items (i) and (ii)). The effects of a change in relative world market size, given Δ_o are presented in Item (iii).

Proposition 7 (Desired harmonized IPR-enforcement of the North)

There exists a unique economically sensible solution to the North's optimization problem. The North's desired harmonized level of global IPR-protection depends on its research productivity and its relative effective market size as follows:

- (i) *If $\frac{\lambda_n}{\lambda_s} < -\frac{\bar{P}}{\bar{D}}$, then the North's desired level of global IPR-protection, ω_n^h , increases with Δ_o .*
- (ii) *If $\frac{\lambda_n}{\lambda_s} > -\frac{\bar{P}}{\bar{D}}$, then there exists a unique value $\tilde{\Delta}_o > 0$ where*
 - (a) *for all $\Delta_o < \tilde{\Delta}_o$, the North's desired level of IPR-protection, ω_n^h , increases with Δ_o .*
 - (b) *for all $\Delta_o > \tilde{\Delta}_o$, the North's desired level of IPR-protection, ω_n^h , decreases with Δ_o .*

²²Recall that \bar{D} is negative and, hence, the right-hand side of (24) is greater than 1.

²³In general: $R_n^H(\omega_{n,t}^h, \omega_{n,t+1}^h) = \Delta_o(\bar{D} + \frac{\lambda_s}{\lambda_n}\bar{P}) + \frac{E_{h,t}}{2\lambda_o}(\bar{D} - \bar{P}) + \frac{\beta\bar{P}}{2} \left[\bar{Y} + \omega_{n,t+1}^h \left(\bar{D} + \bar{P} \frac{\lambda_s}{\lambda_n} \right) - \frac{E_{h,t+1}^2}{4\Delta_o\lambda_o\lambda_n} \right] = 0.$

(iii) There exists a unique value $\bar{\Delta}_o > 0$, where

- (a) for all $\Delta_o < \bar{\Delta}_o$, the North's desired level of IPR-protection, ω_n^h , increases with $\frac{\lambda_n}{\lambda_s}$.
- (b) for all $\Delta_o > \bar{\Delta}_o$, the North's desired level of IPR-protection, ω_n^h , decreases with $\frac{\lambda_n}{\lambda_s}$.

The proof can be found in the appendix.

Items (i) and (ii) reveal some interesting economic insights with respect to changes in ω_n^h in response to variations in Δ_o . For the subsequent argumentation, it is important to note that the incentive to freeride on the South's IPR-policy is absent and possibly substituted by the incentive to export deadweight losses to the South. The possibility of the latter in turn depends on relative market sizes of the North and the South. Whenever the effective market size in the North is relatively low compared to the South - see Item (i), aggregate deadweight losses in the North ($-\lambda_n \bar{D}$) are lower than aggregate profit inflows from the South ($\lambda_s \bar{P}$). In this case the North calls for a higher protection level in response to a lower research productivity relative to the world market size per innovation, i.e., higher Δ_o . In the opposite case (Item (ii)) the effective market size of the North is relatively high such that aggregate deadweight losses in the North exceed profit inflows from the South. Hence there exists a critical $\Delta_o = \tilde{\Delta}_o$ in the sense that the North reduces ω_n^h for $\Delta_o > \tilde{\Delta}_o$.²⁴ In this context, it is also important to emphasize the difference between the results we just discussed and the behavior of the North in the non-cooperative enforcement game, especially the results contained in Proposition 3. In the non-cooperative protection game each country takes its impact on research incentives into account. Whenever this impact is low, due to a comparatively low market share, the incentive to free-ride increases due to strategic substitutability between national enforcement levels. Hence, in this scenario a high relative effective market share of the North, in the sense that $\frac{\lambda_s}{\lambda_n} \leq \tilde{\lambda}$, induces a monotone increase of ω_n^e in Δ_o and a reduction after some threshold Δ_o^{crit} , whenever the world market share of the South is sufficiently high, i.e. $\frac{\lambda_s}{\lambda_n} > \tilde{\lambda}$. The insight of Proposition 7 in turn points in the opposite direction. Given a total effective world market size, the North proposes its preferred harmonized protection level under absence of strategic interaction, but taking account for the effect of its proposed IPR-policy on profit inflows from the

²⁴The proof follows directly from the Implicit function theorem. The sign of $\frac{\partial \omega_n^h}{\partial \Delta_o}$ is determined by $\frac{\partial R_n^h(\omega_n^h)}{\partial \Delta_o}$, where the latter is always positive for $-(\lambda_n \bar{D} + \lambda_s \bar{P}) < 0$.

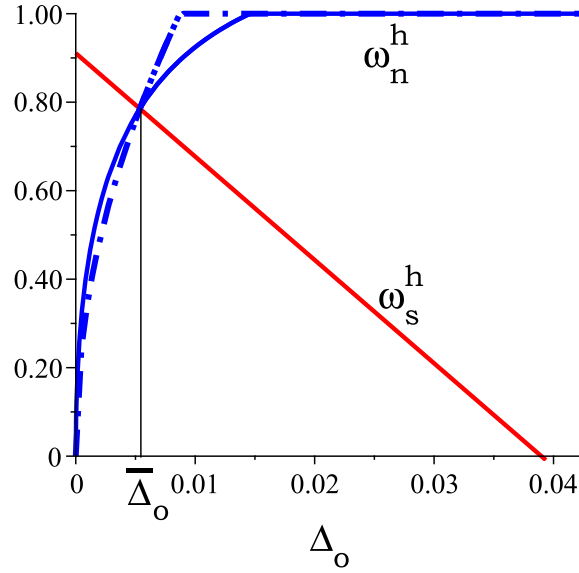


Figure 3: Reaction of the preferred harmonized IPR enforcement levels in the North and the South (solid lines) in response to changes in Δ_o . Dashed line: reduction of the relative world market share of the North, $\frac{\lambda_n}{\lambda_s}$.

South. Hence, the North is able to increase IPRs at the expense of the South in terms of deadweight losses. Whenever the relative world market share of the South is big enough, the North is able to increase IPRs and to benefit from profit inflows from the South which carries deadweight losses. Hence, whenever the relative world market share of the South is sufficiently high ($\frac{\lambda_n}{\lambda_s} < -\frac{\bar{P}}{\bar{D}}$), ω_n^h is monotonically increasing in Δ_o .

Item (iii) of Proposition 7 implies that $\omega_n^h(\Delta_o)$ and $\omega_s^h(\Delta_o)$ intersect at $\bar{\Delta}_o$ and that this intersection remains at $\bar{\Delta}_o$ for different relative effective market sizes (see Figure 4). Further ω_n^h increases (decreases) in $\frac{\lambda_n}{\lambda_s}$ for all $\Delta_o < (>) \bar{\Delta}_o$. In the limit $\frac{\lambda_s}{\lambda_n} \rightarrow 0$ (or $\lambda_n \rightarrow \lambda_o$) the North's desired enforcement level converges to the one it would choose in autarky. To gain an intuition consider first the effect of an increase of λ_n/λ_s given λ_o on the North's aggregate utility in period t . Aggregate utility of the North at time t writes as

$$N_{n,t} \left[\lambda_n \bar{Y} + \omega_{n,t}^h (\lambda_n \bar{D} + \lambda_s \bar{P}) - \frac{E_{nt}^2}{4\Delta_n} \right]. \quad (26)$$

Substituting λ_s by $\lambda_o - \lambda_n$ and taking the derivative with respect to λ_n given λ_o yields

$$N_{n,t} \lambda_n \left[\bar{Y} + \omega_{n,t}^h (\bar{D} - \bar{P}) \right]. \quad (27)$$

Interestingly, the last expression is independent of research incentives since the effective world market size λ_o remains unchanged. Therefore, the marginal change of the North's utility with respect to changes in its own relative market size is structurally equivalent to aggregate utility of the South. Intuitively, a larger effective market size of the North induces higher total production, higher deadweight losses in the North and a lower profit flow from the South to the North. The derivative of $R_n^h(\omega_n^h)$ with respect to λ_n corresponds to the South's first-order condition in steady state:

$$\begin{aligned} \frac{\partial R_n^h(\omega_n^h)}{\partial \lambda_n} &= R_s^h(\omega_s^h) \\ &= \left(1 + \frac{\beta \bar{P} \omega_j^h}{2(1-\beta)\Delta_o}\right) (\bar{D} - \bar{P}) + \beta \frac{\bar{P}}{2\Delta_o} (\bar{Y} + \omega_j^h (\bar{D} - \bar{P})), \end{aligned} \quad (28)$$

with $j = n, s$.

The optimal level of IPR-protection of the South is given by $\frac{\partial R_n^h(\omega_n^h)}{\partial \lambda_n} = R_s^h(\omega_s^h) = 0$. As (28) is decreasing in ω_j^h , $j = n, s$, and given that Δ_o is such that the desired value of IPR-protection of the North is higher than that of the South – i.e., $\omega_n^h > \omega_s^h$ – an increase of the North's market size would imply $\frac{\partial R_n^h(\omega_n^h)}{\partial \lambda_n} < 0$ and thus $\frac{d\omega_n^h}{d\lambda_n} < 0$.²⁵ The opposite holds true if Δ_o is such that $\omega_n^h < \omega_s^h$. Intuitively, according to our previous arguments, an increase in the effective market size of the North gives higher weight to those components of the North's aggregate utility that are also present in the South's utility. Hence, the optimal IPR-protection level of the North moves towards the one of the South.

Against this background, the fundamental question centers around conditions which induce a higher protection level of intellectual property in the North compared to the South:

Proposition 8

For all $0 < \Delta_o < \bar{\Delta}_o$, $\hat{\omega}_n^h \leq \hat{\omega}_s^h$ and for all $\Delta_o > \bar{\Delta}_o$, $\hat{\omega}_n^h \geq \hat{\omega}_s^h$.

The proof is given in the appendix.²⁶

Proposition 8 is a direct consequence of the insights gained from Proposition 7 and expression (28). The former implies for $\Delta_o < \bar{\Delta}_o$ that the level of IPR-protection of the North is increasing in its own relative effective market size. The intuition from (28) is that the optimal level of IPRs in the North moves in the direction of that of the South. Hence, for all $\Delta_o < \bar{\Delta}_o$ the South's optimal level of IPR-protection must be

²⁵The argument follows directly from the implicit function theorem.

²⁶Recall that $\hat{\omega}_n^h \equiv \max\{\min\{\omega_n^h, 1\}, 0\}$.

strictly higher than that of the North, i.e., $\bar{\omega}_s^h > \bar{\omega}_n^h$ (or both countries choose the corner solution $\bar{\omega}_n^h = \bar{\omega}_s^h = 1$). The line of argument applies vice versa for the case $\Delta_o > \bar{\Delta}_o$. Proposition 8 implies that there is an interval of some sufficiently small Δ_o which induce a lower desired harmonized protection level of the North compared to the South and vice versa for sufficiently high Δ_o . As regards the conflict of interests between the North and the South with respect to the as optimal perceived protection levels in the world trading system our model is able to shed light into the fundamentals behind this conflict. Assuming that reality is described by $\Delta_o > \bar{\Delta}_o$, the North perceives a substantially stronger level of IPR-protection ω_n^h as optimal compared to the closed-economy scenario. In addition our model is able to explain why small countries located in the North should be in favor of a tighter IPR-protection in the TRIPS negotiations as they are able to export a comparatively large amount of deadweight losses to the South and benefit from higher profit inflows from the South.

5 Optimal Enforcement Level for the World Economy

This section constitutes the theoretical counterpart to the previous sections in the sense that we characterize levels of IPR-protection which maximize aggregate welfare of the two regions under consideration.²⁷ Since expected profits however depend on the path of $\Omega_t^p = \lambda_n \omega_{n,t} + \lambda_s \omega_{s,t}$ only and not on particular values $\omega_{n,t}$ and $\omega_{s,t}$, we can rewrite the maximization problem of a global planning unit in terms of Ω_t^p . In order to assemble overall efficiency, national levels of IPR-protection are irrelevant. What matters with regard to overall efficiency is the implementation of an optimal path Ω_t^p . Efficient pairs of IPR-protection, (ω_n, ω_s) are thus obtained by solving the following dynamic program

$$\begin{aligned} \max_{\Omega_{t+1}^p} W &= \sum_{t=0}^{\infty} \beta^t N_{n,t} \left[\bar{Y} \lambda_o + \bar{D} \underbrace{(\lambda_n \omega_{n,t} + \lambda_s \omega_{s,t})}_{\Omega_t^p} - \frac{E_{o,t}^2}{4\delta_n} \right], \\ \text{s.t. } N_{n,t+1} &= N_{n,t} + \frac{E_{o,t} N_{n,t}}{2\delta_n}. \end{aligned} \quad (29)$$

Again, using standard dynamic programming methods, the necessary condition for an overall welfare maximum reads in steady state as

$$\bar{D} + \frac{E_o}{2\delta_n} (\bar{D} - \bar{P}) + \frac{\beta \bar{P}}{2\delta_n} \left(\bar{Y} \lambda_o + \bar{D} \Omega^p - \frac{E_o^2}{4\delta_n} \right) = 0, \quad (30)$$

²⁷Note that this is different from maximizing the sum of the utilities of the representative consumers.

which coincides to the first-order condition of a closed economy with effective market size λ_o .

6 Implications for Economic Growth and Welfare

In this section, we compare the comparative-static results of the previous sections, i.e. non-cooperative enforcement of IPRs, harmonization and efficient solution, with regard to their respective levels of IPR enforcement and their implications for economic growth and welfare. To begin with we start with the enforcement levels of IPRs.

6.1 IPR-Enforcement and Economic Growth

As we discussed earlier, the reaction function of the South in the non-cooperative protection game is given by

$$\omega_s^s(\omega_n) = -\frac{1-\beta}{2-\beta} \left[\frac{\bar{Y}}{\bar{D}-\bar{P}} + \frac{2\Delta_o \lambda_o}{\beta \bar{P} \lambda_s} \right] - \frac{1}{2-\beta} \frac{\lambda_n}{\lambda_s} \omega_n. \quad (31)$$

Intuitively for the South, the preferred harmonized level of IPR-protection is determined by the last expression for $\lambda_s = \lambda_o$ ($\rightarrow \lambda_n = 0$) and $\omega_n = 0$, such that

$$\omega_s^h = -\frac{1-\beta}{2-\beta} \left[\frac{\bar{Y}}{\bar{D}-\bar{P}} + \frac{2\Delta_o}{\beta \bar{P}} \right]. \quad (32)$$

For $\Delta_o = 0$, the South's enforcement level of the decentralized protection game equals its preferred harmonized protection level, i.e. $\bar{\omega}_s^e = \omega_s^h = -\frac{1-\beta}{2-\beta} \frac{\bar{Y}}{\bar{D}-\bar{P}} > 0$, since $\bar{D} < 0$ and $\bar{\omega}_n^e = 0$, or $\bar{\omega}_s^e = \omega_s^h = 1$, if $-\frac{1-\beta}{2-\beta} \frac{\bar{Y}}{\bar{D}-\bar{P}} \geq 1$. Since $\lim_{\Delta_o \rightarrow 0} \omega_n^e = \frac{\bar{Y}}{\bar{D}-\bar{P}} \frac{\lambda_s}{\lambda_n} < 0$ it follows that $\bar{\omega}_n^e = 0$, if $\Delta_o \in [0; \underline{\Delta}_o]$, where $\omega_n^e(\underline{\Delta}_o) = 0$. Consequently, $\bar{\omega}_s^e$ declines in a linear fashion in $\Delta_o \in [0; \underline{\Delta}_o]$ and with a higher slope than ω_s^h , since $\frac{\lambda_o}{\lambda_s} > 1$. For $\Delta_o > \underline{\Delta}_o^s$, $\omega_n^e > 0$ and ω_s^e becomes (strictly) convex and declines even faster in Δ_o until $\Delta_o = \bar{\Delta}_o^s$. Hence, $\bar{\Delta}_o^s < \bar{\Delta}_o^{sh}$, where $\Delta_o = \bar{\Delta}_o^{sh} \rightarrow \omega_s^h = 0$. Consequently, the preferred harmonized enforcement level of the South, ω_s^h , is always higher than $\bar{\omega}_s^e$ for $\Delta_o \in (0, \bar{\Delta}_o^{sh})$.

We already clarified that ω_s^h and ω_n^h intersect at $\Delta_o = \bar{\Delta}_o$. It is precisely this $\bar{\Delta}_o$ which turns out to be very useful in order to identify the relationship between the three different regimes under consideration such that we can establish the following proposition

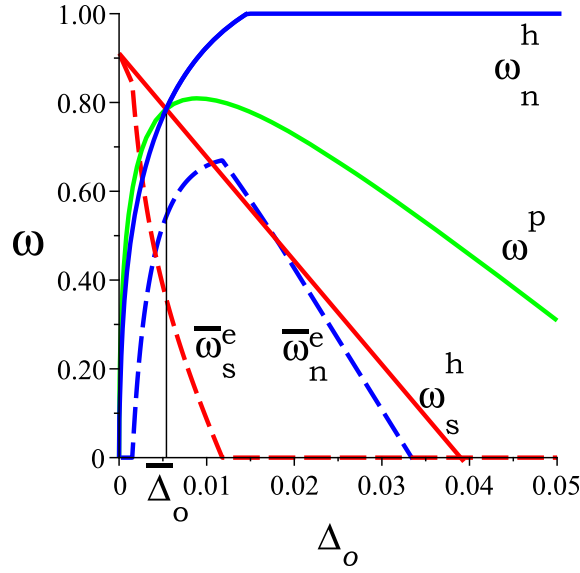


Figure 4: Comparison between the regimes: non-cooperative enforcement (dashed lines), efficient solution (clear solid line) and preferred harmonized enforcement levels (dark solid lines).

Proposition 9

- (i) At $\Delta_o = \bar{\Delta}_o$ the harmonized levels of IPR correspond to the efficient level, i.e. $\omega_s^h = \omega_n^h = \omega^p$.
- (ii) For $\Delta_o < \bar{\Delta}_o$, ω_s^h is above and ω_n^h below the efficient level of IPR-enforcement. For all $\Delta_o > \bar{\Delta}_o$, ω_s^h is below and ω_n^h above the efficient level of IPR-enforcement.
- (iii) The non-cooperative enforcement level of IPR protection in the North, ω_n^n , is always below the efficient level and the preferred harmonized level, i.e. $\omega_n^n < \omega_s^h < \omega^p$ for $\Delta_o < \bar{\Delta}_o$ and $\omega_n^n < \omega^p < \omega_n^h$ for $\Delta_o > \bar{\Delta}_o$.

Figure 5 visualizes Proposition 9. The harmonized solutions ω_s^h and ω_n^h are depicted by dark solid lines and the efficient solution by the clear solid line. All the three of them intersect at $\bar{\Delta}_o$. Hence, our model suggests that a world economy characterized by $\Delta_o > \bar{\Delta}_o$ in the sense that either the effective world market size per innovation is low and/or the research productivity of the North is comparatively low, the preferred harmonized enforcement rate of the North exceeds the efficient level which in turn is higher than the preferred harmonized enforcement rate of the South, i.e. $\omega_n^h > \omega^p > \omega_s^h$. Consequently, the adoption of ω_n^h would be conducive for economic growth, since $g_i^h = \frac{\beta}{1-\beta} \frac{\bar{P}}{2\Delta_o} \omega_j^h, j = n, s$, and $\omega_n^h > \omega_s^h$.

6.2 Welfare

We now turn to the implications of different enforcement levels of IPR protection for economic welfare in the North and the South along the balanced growth path. To this end, we implement in both regions the preferred harmonized enforcement levels of the North and the South, respectively. Moreover, we relate this scenarios to the case where the government of the South would be able to solve its commitment problem in the sense that this region could commit credibly to enforce a path of its preferred harmonized protection level for all times. Since the observable clash of interests between the North and the South is characterized by $\omega_n^h > \omega_s^h$, we restrict our analysis to $\Delta_o > \bar{\Delta}_o$. Welfare along the BGP is denoted by \bar{U}_s^h, \bar{U}_n^h such that

$$\bar{U}_s^h(\omega_j^h) = \frac{1}{1 - \beta(1 + g_j^h)} \lambda_s \left[\bar{Y} + \omega_j^h (\bar{D} - \bar{P}) \right], \quad (33)$$

and

$$\bar{U}_n^h(\omega_j^h) = \frac{1}{1 - \beta(1 + g_j^h)} \lambda_n \left[\bar{Y} + \omega_j^h (\bar{D} + \bar{P} \frac{\lambda_s}{\lambda_n}) - \frac{\left(\frac{\beta}{1-\beta} \bar{P} \omega_j^h \right)^2}{4\Delta_o} \right], \quad (34)$$

with $g_j^h = \frac{\bar{P}\omega_j^h}{2\Delta_o} \frac{\beta}{1-\beta} = \text{const.}$, since $\omega_j^h = \text{const.}$, and $j = n, s$.

Since $\frac{\partial g}{\partial \Delta_o} < 0$, there exists a Δ_o^{\min} , such that $\frac{1}{1-\beta(1+g)} \rightarrow +\infty$ for $g \rightarrow \frac{1-\beta}{\beta}$, such that $\bar{U}_j^h(\omega_j^h)$, $j = n, s$ approaches infinity, when Δ_o declines to Δ_o^{\min} . Moreover, ω_s^h is declining in Δ_o , whereas ω_n^h may be increasing or may follow an inverted u-shaped course in Δ_o . In the latter $\omega_n^h = 0$, if $\Delta_o \geq \Delta_o^{0nh} = \frac{1}{2} \frac{\bar{Y}\bar{P}\beta}{-\bar{P}\frac{\lambda_s}{\lambda_n} - \bar{D}}$. ω_s^h equals zero, in turn, for $\Delta_o \geq \Delta_o^{0sh}$, with $\Delta_o^{0sh} = \frac{1}{2} \frac{\bar{Y}\bar{P}\beta}{\bar{P} - \bar{D}}$, where $\Delta_o^{0nh} > \Delta_o^{0sh}$, since $\bar{P}(1 + \frac{\lambda_s}{\lambda_n}) > 0$. Hence, the implementation of ω_s^h in the North implies $\omega_s^h = 0$ and thus $g = 0$ for $\Delta_o \geq \Delta_o^{0sh}$, with $\bar{U}_n^h(\omega_s^h = 0) = \frac{\lambda_n \bar{Y}}{1-\beta}$. The implementation of ω_n^h in the North instead implies $\omega_n^h = 0$ and thus $\bar{U}_n^h(\omega_n^h = 0) = \frac{\lambda_n \bar{Y}}{1-\beta}$ at $\Delta_o^{0nh} > \Delta_o^{0sh}$ only. Hence, utility is declining in Δ_o and remains constant for any Δ_o implying zero IPR-protection. It follows for $\Delta_o \in (\bar{\Delta}_o; \bar{\Delta}_o^{0nh})$: $\bar{U}_n^h(\omega_n^h) > \bar{U}_n^h(\omega_s^h)$, i.e. the implementation of the South's preferred protection level in the North would cause welfare losses in the North - see Figure 6(a).²⁸

With regard to the welfare effects of the South, we take account for three different policies: first, the implementation of ω_s^h or ω_n^h and the case where the South is able to

²⁸Conjecture: welfare losses in the North would increase for $\frac{\lambda_n}{\lambda_s} < -\frac{\bar{P}}{\bar{D}}$, since $\omega_n^h \rightarrow 1$ as Δ_o declines.

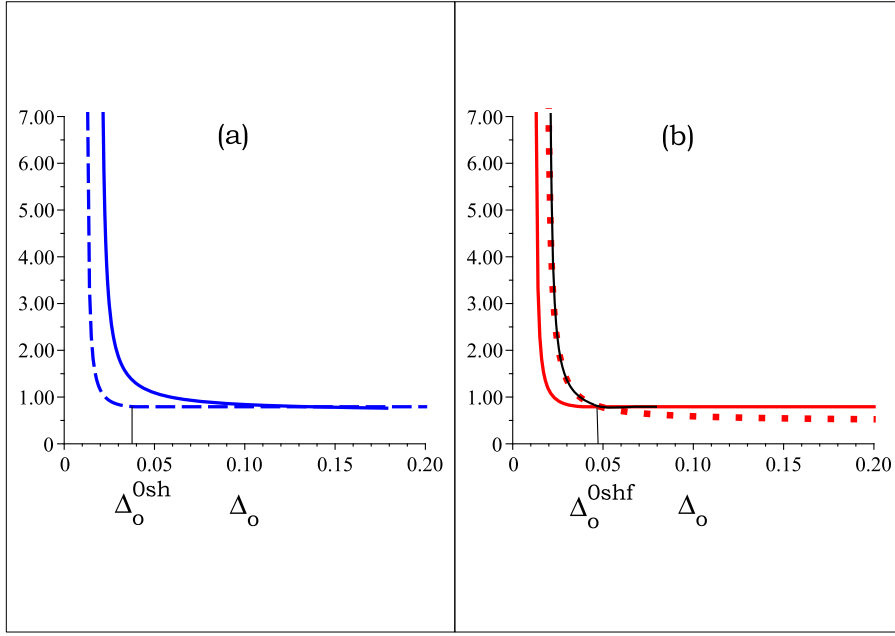


Figure 5: (a): $\bar{U}_n^h(\omega_n^h)$ - solid line; $\bar{U}_n^h(\omega_s^h)$ - dashed line.
(b): $\bar{U}_s^h(\omega_s^h)$ - clear solid line; $\bar{U}_s^h(\omega_s^{hf})$ - dark solid line; $\bar{U}_s^h(\omega_n^h)$ - dotted line.

solve its commitment problem, such that $\omega_s^{h,f}$ would satisfy

$$\frac{2\delta}{\bar{P}}(1+g)(\bar{D}-\bar{P}) + \sum_{\tau=t+1}^{\infty} \beta^{\tau-t}(1+g)^{\tau-t-1} \lambda_s \left[\bar{Y} + \omega_s^{h,f}(\bar{D}-\bar{P}) \right] = 0.$$

Again the full-commitment solution differs compared to (21) with respect to the second summand which represents the discounted benefit of a change in IPRs for all future periods such that the South would prefer a higher harmonized enforcement level if it could solve its commitment problem. Hence, $\bar{U}_s^h(\omega_s^{hf}) > \bar{U}_s^h(\omega_s^h)$.

Proposition 10

In the full-commitment case, the preferred harmonized enforcement level of IPRs of the South, ω_s^{hf} , is declining in Δ_o and approaches zero at $\Delta_o^{0shf} = \frac{1}{2} \frac{\beta}{1-\beta} \frac{\bar{Y}\bar{P}}{\bar{P}-\bar{D}}$, where $\Delta_o^{shf} > \Delta_o^{sh}$ and $\Delta_o^{shf} < \Delta_o^{nh}$, if $(1-\beta) + \frac{\bar{D}}{\bar{P}} > \frac{\lambda_n}{\lambda_s}$.

Hence, ω_s^{hf} as well as ω_s^h are declining in Δ_o , where $\omega_s^{hf} = 0$ for $\Delta_o \geq \Delta_o^{shf}$ and $\Delta_o^{shf} > \Delta_o^{sh}$. Hence, $\bar{U}_s^h(\omega_s^h) = \bar{U}_s^h(\omega_s^{hf}) = \frac{\lambda_s \bar{Y}}{1-\beta}$, if $\Delta_o \geq \Delta_o^{shf}$. The implementation of ω_n^h in the South causes welfare losses in the South, if $\omega_s^{hf} = 0$ and $\omega_n^h > 0$. Hence, $\bar{U}_s^h(\omega_n^h) < \bar{U}_s^h(\omega_s^h = 0) = \bar{U}_s^h(\omega_s^{hf} = 0)$, if $\Delta_o \in (\Delta_o^{shf}; \Delta_o^{nh})$. Since the South considers profit outflows to the North as costs, it neglects an important contribution to the benefits of higher enforcement levels from the perspective of the North. In fact the

South could increase welfare if it adopted ω_n^h (dotted line in Figure 6(b)), as long as the research productivity of the North is not too low, i.e. $\Delta_o < \Delta_o^{shf}$.

7 Summary and Conclusions

Even though most countries have agreed to harmonize intellectual property rights by signing the Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS), there is still much dispute about the optimal level of protection of intellectual property rights in the world. This paper unifies micro- and macroeconomic perspectives from the academic and political debate and addresses the following questions

- What are the implications, if governments can only commit themselves credibly to enforce a certain level of IPR-protection for the near future, only, for example one decade?
- Given the existence of this commitment problem, what are the determinants of different enforcement levels in the North and the South, if both regions act decentrally and in a non-cooperative fashion?
- How large is the difference between the preferred harmonized enforcement levels of IPR protection between the North and the South?
- What are the implications for economic growth and economic welfare in the North and the South, if their respective preferred harmonized protection levels were implemented?

With regard to these questions our research suggests the following results

- Imperfect commitment of governments with respect to the enforcement level of IPR protection induces lower enforcement levels compared to the full-commitment case, such that economic growth is reduced as well. Moreover in the former, the IPR-enforcement level follows an inverted u-shaped pattern with respect to declining research productivities or declining market sizes per innovation, whereas the enforcement level is monotonously declining in the latter.
- Since IPR-enforcement exhibits the characteristics of a public good, the North and the South have an incentive to freeride on each others enforcement levels.

The incentive to freeride, i.e. to reduce its own enforcement level shrinks however with an increase in the own market size, since the impact on R&D incentives is growing as well. The lower the research productivity of the North or the lower the world market size per innovation the lower the enforcement level of IPR protection in the South. Conversely, the enforcement level of the North is increasing with a decline in research productivity or a decline in the market size per innovation. Hence, for sufficiently low research productivities or world market sizes per innovation, the enforcement level of the North exceeds the one of the South. Given a relatively low relative market share of the North, the North starts to reduce its own enforcement level, whenever research productivity or relative market size are sufficiently low. To the contrary, if the North exhibits a relatively high market share, its enforcement level is monotonously increasing with a declining research productivity or effective market size.

- When the North and the South decide about their optimal harmonized protection levels for the world economy, strategic interaction between enforcement levels is absent, since either the one or the other is implemented. Compared to the decentralized solution both countries take their decisions now based on the world market size. The South's enforcement is therefore increasing in the total world market size and the research productivity of the North. Relative world market sizes in turn play a crucial role in the decision problem of the North. Given a sufficiently low relative market share of the North, the North is in the position to export the costs of its policies (deadweight losses) to the South and benefits from high profit inflows from the South. Thus, the preferred harmonized enforcement level of the North is monotonously increasing for declining research productivities or world market sizes. Conversely, for a sufficiently high relative market share of the North, the North must carry a substantial part of the costs in terms of deadweight losses, such that the preferred harmonized protection level of the North follows an inverted u-shaped pattern with respect to a decline in research productivity or total world market size. An increase (decline) in the relative market size of the North reduces its preferred enforcement level, if research productivity or effective market size are sufficiently low (high). Hence small and innovative countries located in the North push stronger for tighter IPR-enforcement than big ones given that the research productivity and the world market size are not to high.

- Given a sufficiently low research productivity or world market size per innovation, the preferred harmonized enforcement level of IPR-protection in the North exceeds the efficient enforcement level for the North and the South which in turn is higher than the preferred enforcement level of the South. Under these circumstances, the implementation of the preferred harmonized enforcement level of the North is conducive for economic growth. The implementation of the preferred harmonized enforcement level of the South in the world trading system would cause welfare losses in the North. The adoption of the preferred harmonized protection level of the North in turn causes welfare gains for the South given that research productivity of the North and world market size per innovation are not too low.

Appendix

A Closed Economy

Our analysis builds on a variety-expanding-growth framework of the Romer (1990)-type, such that economic growth is driven by purposeful investments in R&D. In period t , production takes place given the state of technological knowledge and the state - i.e. the probability - of IPR enforcement denoted by $\omega_{a,t}$. That is, in period t the patent is enforced with probability $\omega_{a,t}$ and the patent holder receives operating profits of the current period. To the contrary, with probability $1 - \omega_{a,t}$ the patent is not enforced and the intermediate is imitated. In period t the government commits itself credibly to an enforcement level of IPR protection for the subsequent period $t + 1$. Hence, R&D takes place in period t given the current stock of technological knowledge and the commitment of the government to the enforcement level of IPR protection for the subsequent period, $\omega_{a,t+1}$. Simplifying matters without losing economic insights, we assume that imitation is costless. Thus, an imitated intermediate is supplied under full competition and operating profits are zero which could be interpreted as a normalization.²⁹

A.1 The Model

We consider an economy populated by a measure L of identical households that inelastically supplies one unit of labor in each period. There is no population growth and time moves in discrete steps $t = 0, 1, 2, \dots, \infty$.

A.1.1 Final Good Production

The final good Y is produced by using labor L and a range of intermediates indexed by i . The production function takes the following form

$$Y = AL^{1-\alpha} \int_0^N [x(i)]^\alpha di, \quad (35)$$

where A represents a productivity measure, N is the measure of different intermediates, and $x(i)$ stands for the amount of intermediate i used in final good production. The elasticity of substitution between the different intermediates is denoted by $\alpha \in (0, 1)$.

²⁹In contrast to Grossman and Lai (2004) our focus is explicitly on intellectual property rights *enforcement*. For simplicity we thus assume an infinite patent length and neglect the issue of patent breadth.

A.1.2 Intermediate Goods Production

Each intermediate good i is produced by a monopolist or an imitator and requires only final output Y as a factor of production. Moreover, production of all types of intermediates i is subject to the same technology in the sense that production of one unit x requires one unit final output Y . We choose final output Y as the numeraire such that $p_Y = 1$ and marginal production costs of intermediates are equal to unity. The implied symmetric equilibrium on the market for intermediates induces equal prices and demand for all types of intermediates, such that $p_m(i) = p_m = 1/\alpha$, $x_m(i) = x_m$ for all protected intermediates and $p_c(i) = p_c = 1$, $x_c(i) = x_c$ for all imitated intermediates. Demand for protected intermediates reads therefore as

$$x_m = LA^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} \quad (36)$$

generating profits of technology owners in period t as

$$\pi_m = \left(\frac{1-\alpha}{\alpha} \right) \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L. \quad (37)$$

If an intermediate is copied and, hence, sold at the competitive price $p_c = 1$, demand increases up to

$$x_c = LA^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}}, \quad (38)$$

and profits in t are zero. Denoting the number of protected intermediates at time t by $N_{m,t} = \omega_{a,t} * N_t$, and the ones that are imitated by $N_{c,t} = [1 - \omega_{a,t}] * N_t$ aggregate output writes as

$$Y_t = AL^{1-\alpha} \left[\int_0^{N_{m,t}} [x_m(i)]^\alpha di + \int_0^{N_{c,t}} [x_c(i)]^\alpha di \right]. \quad (39)$$

As additionally $x_m = \alpha^{\frac{1}{1-\alpha}} x_c$, we obtain for Y_t :

$$Y_t = AL^{1-\alpha} N_t [1 + \omega_{a,t} (\alpha^{\frac{\alpha}{1-\alpha}} - 1)] x_c^\alpha, \quad (40)$$

where $\omega_{a,t} (\alpha^{\frac{\alpha}{1-\alpha}} - 1) < 0$ represents deadweight losses due to monopolistic competition.³⁰

³⁰Notice that for $\omega_{a,t} = 1$, i.e., full patent protection, we obtain the standard Romer (1990) production function: $Y_t = AL^{1-\alpha} N_t (\underbrace{\alpha^{\frac{1}{1-\alpha}} x_c}_{x_m})^\alpha$. The case without patent protection, $\omega_{a,t} = 0$, yields the highest possible output from a static perspective: $Y_t = AL^{1-\alpha} N_t x_c^\alpha$. Of course this undermines incentives to invest in R&D.

A.1.3 Research and Development

R&D constitutes the search for new designs (blueprints) of machines which requires resources. To this end research firms rent labor services and capital inputs. It simplifies the analysis considerably though if we assume that final output which incorporates both labor and machines enters as the only factor of production into the R&D process. In addition we assume, that R&D output is positively affected by a research lab's knowledge capital which is reflected in its simplest form by the average number of innovations or the economy's average level of technological knowledge $\frac{N_t}{L}$. In particular, we assume the following cost function on the research-lab level:

$$\zeta(\eta_t) = \frac{\delta_a \eta_t^2}{\frac{N_t}{L}}, \quad (41)$$

where η_t denotes the number of new inventions at time t and δ_a reflects the research productivity or the quality of research infrastructure which could alternatively be interpreted as a measure of a research-lab's human capital. That is, the higher the level of human capital, the lower δ_a which implies that lab-equipment can be used more productively.³¹ The assumption of decreasing returns on the firm and industry level with respect to R&D expenditures has been supported empirically, f.e. by Pakes and Griliches (1984) and Hall et al. (1988). On the macro level, the probably most important source of decreasing returns in R&D can be seen in an increased probability of duplicative research through an increasing number of both rivals and expenditures, even though the R&D process as such may be driven by large spillovers (Amir, 2000; Kortum, 1993; Klette and Kortum, 2004). In a related line of argumentation it is also possible to think of plausible limits in transforming an ever increasing stock of new ideas into usable knowledge for production (Weitzman, 1998).³² We would also like to stress that we are not challenging approaches employing constant returns to scale production functions in R&D, but that we consider the assumption of decreasing returns to scale as very convenient for our purposes - we come back to this point further below. In our setting a new blueprint invented in period t becoming productive in $t + 1$ receives a patent of infinite length that is enforced with probability $\omega_{a,t+1}$ in period $t + 1$.

³¹The assumption that both research productivity or human capital as well as the current technology stock play a positive role for innovative output and are complementary to a certain extent is standard in the literature. For example, in Romer (1990), p. 86, the aggregate stock of designs evolves according to $\dot{A} = \delta_a H_A A$, where A is the stock of designs, H_A is human capital and δ_a is a productivity parameter.

³²From an aggregate perspective, decreasing returns may also reflect heterogeneity in the cost of research projects. A similar argument can be found in Scotchmer (2004) (ch. 11).

Accordingly, the expected value of an invention i at time t reads as

$$E_{a,t} = E_{a,t}[V(i)] = \sum_{\tau=t+1}^{\infty} \beta^{\tau} \pi \omega_{a,\tau} = \sum_{\tau=t+1}^{\infty} \beta^{\tau} \bar{P} \lambda_a \omega_{a,\tau} \quad (42)$$

with $\bar{P} = \left(\frac{1-\alpha}{\alpha}\right) \alpha^{\frac{2}{1-\alpha}} > 0$ and $\lambda_a = A^{\frac{1}{1-\alpha}} L$ reflecting "effective" market size for intermediates. Obviously, effective market size is not a matter of being rich or poor. A small economy, in terms of its population may exhibit a large effective market when its productivity level in final good production is sufficiently large and vice versa.

Optimality requires that marginal costs for an additional invention must equal its expected value. Consequently, inventions per research lab are determined by

$$\eta_t = E_{a,t} \frac{N_t}{2\delta_a L}, \quad (43)$$

such that aggregate technological knowledge evolves according to

$$N_{t+1} - N_t = \eta_t L = E_{a,t} \frac{N_t}{2\delta_a}. \quad (44)$$

A.1.4 The Household's and the Government's Problem

Given optimal decisions of households, the government maximizes aggregate welfare by implementing an optimal enforcement level of IPR protection for the subsequent period. In this setting the government cannot commit itself credibly to an enforcement level of intellectual property for all times $t \rightarrow \infty$ but only for the next period $t+1$ which seems to be very realistic whenever one period encompasses more than one decade.

Each of the households seeks to maximize

$$U_t = \sum_{t=0}^{\infty} \beta^t c_t, \quad (45)$$

where $0 < \beta < 1$ is a discount factor and $\frac{1-\beta}{\beta}$ is the rate of time preference.³³ Apparently the household's optimization problem is already solved for by the optimal decision on R&D-investment for a given path of IPR-enforcement, such that the government solves

$$\max_{\{w_{a,t+1}\}_{t=0}^{\infty}} W = \sum_{t=0}^{\infty} \beta^t c_t L, \quad (46)$$

³³The linear instantaneous utility function induces a knife-edge condition $1+r = \frac{1}{\beta}$ and the objective function converges to an upper bound as t approaches to ∞ whenever the economy's long-run growth rate does not exceed the rate of time preference, i.e. $g_{BGP} < \frac{1-\beta}{\beta}$.

subject to

$$N_{t+1} = N_t \left[1 + \frac{E_{a,t}}{2\delta_a} \right], \quad (47)$$

where $c_t L = N_t [\lambda_a (\bar{Y} + \omega_t \bar{D}) - \frac{E_{a,t}^2}{4\delta_a}]$ and $E_{a,t} = \sum_{\tau=t+1}^{\infty} \beta^\tau \lambda_a \bar{P} \omega_{a,\tau}$. The parameter $\bar{Y} = \alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}} > 0$ reflects the contribution of an intermediate to final production net of production costs for intermediates, if there were no IPR-enforcement and $\bar{D} = \alpha^{\frac{2\alpha}{1-\alpha}} - \alpha^{\frac{\alpha}{1-\alpha}} + \alpha^{\frac{1}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}} < 0$ represents the deadweight-loss factor net of production costs for intermediates.

A.2 Optimal IPR-Protection in Autarky

The government commits itself to an optimal level of IPR-enforcement $\omega_{a,t+1}$ given $\omega_{a,t}$ and the current state of technology N_t . The corresponding Bellman equation reads as

$$V(N_t) = \max_{\omega_{a,t+1}} \left\{ N_t \lambda_a \left[\bar{Y} + \omega_{a,t} \bar{D} - \frac{E_{a,t}^2}{4\delta_a \lambda_a} \right] + \beta V(N_{t+1}) \right\} \quad (48)$$

implying the following first order condition with respect to $\omega_{a,t+1}$

$$-N_t \frac{E_{a,t} \frac{\partial E_{a,t}}{\partial \omega_{a,t+1}}}{2\delta_a} + \beta \left(\frac{\partial V(N_{t+1})}{\partial N_{t+1}} + \frac{\partial V(N_{t+1})}{\partial N_{t+1}} \frac{\partial N_{t+1}}{\partial \omega_{a,t+1}} \right) = 0, \quad (49)$$

such that we obtain in light of the Envelope theorem³⁴

$$-\frac{E_{a,t} \bar{P}}{2\delta_a} + \left(1 + \frac{E_{a,t}}{2\delta_a} \right) \bar{D} + \frac{\beta \bar{P}}{2\delta_a} \left[\lambda_a (\bar{Y} + \omega_{a,t+1} \bar{D}) - \frac{E_{a,t+1}^2}{2\delta_a} \right] = 0. \quad (50)$$

The first term in the first-order condition represents additional R&D expenditures in the current period induced by a marginally higher $\omega_{a,t+1}$. Additionally, an increase in $\omega_{a,t+1}$ procures higher deadweight losses measured by the second term which can also be expressed as $\frac{N_{t+1}}{N_t} \bar{D}$. The third term reflects the effect of a change in $\omega_{a,t+1}$ on N_{t+1} and the thereby induced impact of the latter on future consumption possibilities. The focus of our analysis builds on the steady state growth path of the economy which

³⁴More in detail, we yield: $\frac{\partial V(N_{t+1})}{\partial \omega_{a,t+1}} = N_{t+1} \lambda_a \bar{D}$; $\frac{\partial V(N_{t+1})}{\partial N_{t+1}} = \lambda_a (\bar{Y} \omega_{a,t+1} \bar{D}) - \frac{E_{a,t+1}^2}{4\delta_a}$; $\frac{\partial N_{t+1}}{\partial \omega_{a,t+1}} = N_t \frac{\partial E_{a,t}}{\partial \omega_{a,t+1}}$.

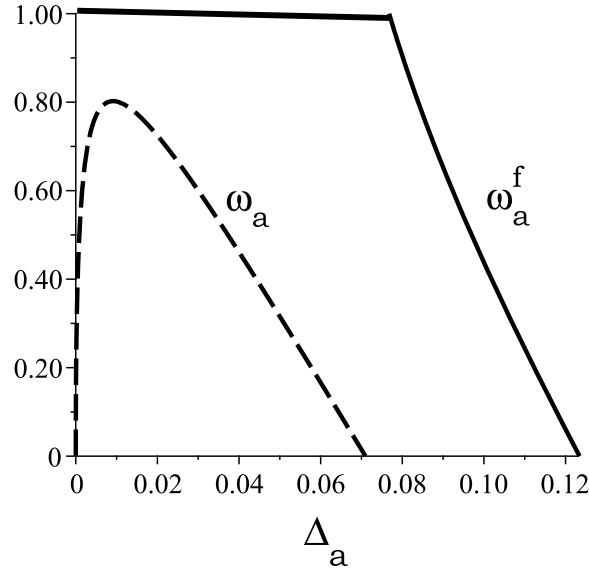


Figure 6: Behavior of the IPR-enforcement level in the full-commitment case (solid line) and imperfect-commitment case (dashed line) dependent on $\Delta_a = \frac{\delta_a}{\lambda_a}$.

implies $\omega_{a,t+1} = \omega_{a,t} = \omega_a$. In this case the first order condition writes as

$$\begin{aligned}
R(\omega_a) &= \underbrace{\bar{D}\Delta_a + \frac{\beta}{1-\beta} \frac{\bar{P}\omega_a}{2} (\bar{D} - \bar{P}) + \frac{\beta\bar{P}}{2} \left[\omega_a\bar{D} - \left(\frac{\beta}{1-\beta} \right)^2 \frac{\bar{P}^2\omega_a^2}{4\Delta_a} \right]}_{<0, \text{ marg. cost of IPR}} \\
&+ \underbrace{\frac{\beta\bar{P}}{2} \bar{Y}}_{>0, \text{ marg. benefit of IPR}} \\
&= 0.
\end{aligned} \tag{51}$$

In (51), we used Δ_a to denote research productivity relative to effective market size, i.e., $\Delta_a \equiv \frac{\delta_a}{\lambda_a}$ which proves to be very convenient for the subsequent comparative statics. At this place there are two points worth being noticed: first, the only benefit term of increased IPRs is captured by the increase in future output. This term must dominate the cost terms consisting of current and future R&D expenditures and future deadweight losses. Second, the first-order condition is quadratic in ω_a . Solving the first order condition in steady state for ω_a and examining the comparative statics with respect to Δ_a yields:

Proposition 11

- (i) There exists a unique (economically sensible) ω_a that satisfies the first-order condition in steady state (51).
- (ii) The optimal value ω_a is strictly concave with respect to Δ_a . I.e. there exists a critical level $\Delta_{a,crit} > 0$ with the property that

$$\frac{\partial \omega_a}{\partial \Delta_a} \begin{cases} > 0, & \text{if } \Delta_a < \Delta_{a,crit}, \\ = 0, & \text{if } \Delta_a = \Delta_{a,crit}, \\ < 0, & \text{if } \Delta_a > \Delta_{a,crit}. \end{cases}$$

The proof is given in the appendix.³⁵

Since we defined the level of IPR-enforcement ω_a as a probability, the optimal level of IPR enforcement is given by

$$\bar{\omega}_a = \begin{cases} 1 & \text{if } \omega_a > 1, \\ \omega_a & \text{if } \omega_a \in [0, 1], \\ 0 & \text{if } \omega_a < 0. \end{cases}$$

Proposition 11 implies an inverted-u-shape relation between the enforcement level ω_a of IPRs and $\Delta_a = \frac{\delta_a}{\lambda_a}$, i.e. the protection of IPRs is first increasing ($\Delta_a < \Delta_{a,crit}$) in research costs relative to effective market size per innovation and declining for $\Delta_a > \Delta_{a,crit}$ (see Figure 1). The result that a country which exhibits a high research productivity aims to reduce IPR-protection if either research productivity improves even further or the market becomes larger may seem counterintuitive and stands in contrast to the results in the previous literature. In order to give an economic intuition, we start from a different point of origin. In light of the Implicit function theorem we yield from the first-order condition

$$\frac{\partial \omega_a}{\partial \Delta_a} = \frac{\bar{D} + \beta \left(\frac{\beta}{1-\beta}\right)^2 \frac{\bar{P}^3 \omega_a^2}{8\Delta_a^2}}{- \left[\frac{\beta}{1-\beta} \frac{\bar{P}}{2} (\bar{D} - \bar{P}) + \frac{\beta}{2} \bar{P} \bar{D} - \beta \bar{P} \left(\frac{\beta}{1-\beta}\right)^2 \frac{\bar{P}^2 \omega_a}{4\Delta_a} \right]}. \quad (52)$$

Apparently, the denominator is unambiguously positive, since $\bar{D} < 0$. The numerator in turn may be positive, negative or equal to zero at $\Delta_a = \Delta_{a,crit}$. Finally both the numerator and the denominator are declining in Δ_a . Consequently, the non-monotonous behavior of ω_a with respect to an increase in Δ_a depends on whether

³⁵As the first order condition is a quadratic function of ω , it possesses two roots. In the appendix we also show that one is always negative, whereas the other can be both positive and negative. Hence the latter is the only one that is economically meaningful. ω_a satisfies the first-order condition in a strictly mathematical sense.

$\bar{D} \leq \beta \left(\frac{\beta}{1-\beta} \right)^2 \frac{\bar{P}^3 \omega_a^2}{8\Delta_a^2}$. Whenever Δ_a is low, the productivity of R&D is high compared to the effective market size per innovation which induces high R&D expenditures in equilibrium possibly exceeding the constant deadweight loss term \bar{D} ($\Delta_a < \Delta_{a,crit}$). In this case private R&D expenditures overcompensate the deadweight loss term per innovation and the government reacts towards a reduction in R&D productivity (increase in δ_a) by an increase in the protection level of IPRs, ω_a . Whenever $\Delta_a > \Delta_{a,crit}$, the situation reverses. R&D becomes more and more unproductive and private R&D expenditures in equilibrium are lower than the deadweight loss per innovation, such that the government reduces the enforcement level of IPRs in response to further increases in Δ_a . It is also important to stress that this result does not hinge on the quadratic form of our cost function. The quadratic cost function induces a first-order condition which is quadratic in ω_a , see Eq.(50). The upper result depends on the fact that R&D expenditures are inversely related to Δ_a , see Eq.(43), whereas the deadweight loss per innovation remains constant.³⁶ The latter result in turn is driven by the way in which the government commits itself to its IPR-policy in the subsequent period. By doing so, the government focuses on costs and benefits in the current and subsequent period. It therefore neglects the impact of ω_a on benefits beyond period $t + 1$. Assuming for the moment that the government could commit itself to a path of enforcement levels $\omega_{a,t}^f$ for all periods, the corresponding Lagrangian reads as

$$\begin{aligned} \mathcal{L} = & \sum_{t=0}^{\infty} \beta^t \prod_{i=0}^t (1 + g_{a,i}) \lambda_a N_0 \left[\bar{Y} + \omega_{a,t}^f \bar{D} - \frac{E_{a,t}^2}{4\delta_a \lambda_a} \right] \\ & + \mu_t \left[1 + g_{t+1} - \left(1 + \frac{E_{a,t}}{2\delta_a} \right) \right], \end{aligned} \quad (53)$$

where the first-order conditions are identified through $\mathcal{L}_{\omega_{a,t+1}^f} = \mathcal{L}_{g_{t+1}} = 0$,³⁷ such that we obtain along the BGP

$$\begin{aligned} & \frac{2\delta_a}{\bar{P}} \left[(1 + g) \bar{D} - \frac{\beta}{1-\beta} \frac{\bar{P}^2 \lambda_a \omega_a^f}{2\Delta_a} \right] \\ & + \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} (1 + g)^{\tau-t-1} \lambda_a \left[\bar{Y} + \omega_a^f \bar{D} - \left(\frac{\beta}{1-\beta} \right)^2 \frac{\bar{P}^2 (\omega_a^f)^2}{4\Delta_a} \right] = 0. \end{aligned} \quad (54)$$

³⁶Obviously, constant marginal costs would only dampen the negative effect of Δ_a on equilibrium R&D-expenditures.

³⁷The first-order conditions read as: $\mathcal{L}_{\omega_{a,t+1}^f} = -\frac{E_{a,t} \frac{\partial E_{a,t}}{\partial \omega_{a,t+1}^f}}{2\delta_a} + \beta(1 + g_{t+1}) \lambda_a \bar{D} - \mu_t \frac{E_{a,t} \frac{\partial E_{a,t}}{\partial \omega_{a,t+1}^f}}{2\delta_a} = 0$; $\mathcal{L}_{g_{a,t+1}} = \mu_t + \sum_{t=0}^{\infty} \beta^t \prod_{i=2}^t (1 + g_{a,i}) \lambda_a N_0 \left[\bar{Y} + \omega_{a,t}^f \bar{D} - \frac{E_{a,t}^2}{4\delta_a \lambda_a} \right]$, with $N_0 = 1, g_0 = 0$ and $E_{\omega_{a,t+1}^f} = \beta \lambda_a \bar{P}$.

In light of (54) we are able to establish the following proposition

Proposition 12

- (i) *In the full-commitment scenario exists a unique economically feasible solution ω_a^f satisfying (54).*
- (ii) *The optimal enforcement level ω_a^f is monotonically declining in Δ_a .*

Compared to the non-full commitment case (see Figure 1) there are several points worth being noticed: Both solutions differ only with respect to the second summand which represents discounted net-benefits of IPRs, i.e. (51) and (54) coincide for $\tau = t + 1$. While the government in the full-commitment case accounts for benefits in all future periods, the benefit term is higher compared to the imperfect-commitment solution. The cost terms captured by the first summands respectively however are identical. Consequently, the levels of IPRs in the full commitment case exceed the levels resulting from the imperfect commitment scenario, i.e. $\bar{\omega}_a^f > \bar{\omega}_a$, as long as $\bar{\omega}_a, \bar{\omega}_a^f \in (0; 1)$. Moreover in the full commitment case, the level of IPRs is monotonically declining in Δ_a . Hence, the above mentioned non-monotonous behavior of ω_a with respect to changes in Δ_a is owed to the commitment problem of the government and not to the quadratic cost function. Finally for the imperfect commitment case, the economy's growth rate along the balanced growth path reads as

$$g_a = \frac{\beta}{1 - \beta} \frac{\bar{P}\bar{\omega}_a}{2\Delta_a}. \tag{55}$$

Given Δ_a , imperfect commitment of the government with respect to future IPR-levels induces necessarily a lower growth rate, i.e. $g_a < g_a^f$ as $\bar{\omega}_a^f > \bar{\omega}_a$ for interior solutions. In light of equation (55) it is obvious that the growth rate is declining in Δ_a as long as the optimal level of IPR-protection is declining in Δ_a as well, i.e. $\Delta_a > \Delta_{a,crit}$. We show in the appendix that g_a is also declining in Δ_a , for $\Delta_a < \Delta_{a,crit}$. With respect to changes in Δ_a , we therefore obtain:

Proposition 13

The growth rate of the closed economy is strictly decreasing with Δ_a .

It is also worth to notice that the effect of L on bgp-growth stems from the market size effect and is not induced by the empirically non-valid scale effect.³⁸

³⁸Proposition 13 implies a unique lower bound for Δ_a such that the government's utility maximization problem is well defined. More precisely, for the government's objective function to converge to a finite value we require $g_a < \frac{1-\beta}{\beta}$. Accordingly, the lower boundary $\Delta_{a,l}$ is defined by $g_a = \frac{1-\beta}{\beta}$. Consequently, whether increasing IPR-protection with Δ_a can occur depends on whether $\Delta_{a,crit} > \Delta_{a,l}$.

B The Generalized Model

B.1 Open Economy

We now turn to the analysis of an open economy setting, where both countries are investing in $R\&D$. We assume that there are full knowledge spillovers in the sense that the costs of invention do not only decrease with the number of a country's own measure of intermediates but also with the number of imported foreign intermediates. More formally,

$$\zeta_j(\eta_{jt}) = \delta_N \eta_{jt}^2 \frac{L_j}{N_t}, \quad (56)$$

where $j \in \{N, S\}$ and $N_t = N_{it} + N_{jt}$. The stream of (expected) discounted profits stemming from an invention in the open economy accrues to

$$E_{Ot} = \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} Z_3 (\omega_{N\tau} \lambda_N + \omega_{S\tau} \lambda_S). \quad (57)$$

Profit maximization by the researchers results in a number $\eta_{jt} = \frac{E_{Ot} N_t}{2\delta_j L_j}$ of new blueprints for intermediates in period t per researcher. This leads to the following dynamics of the countries' technology stocks:

$$N_{Nt+1} = N_{Nt} + \frac{E_{Ot} N_t}{2\delta_N}, \quad (58a)$$

$$N_{St+1} = N_{St} + \frac{E_{Ot} N_t}{2\delta_S}. \quad (58b)$$

Inspection of the equations of motion yields the following result.

Lemma 4

There exists a unique balanced growth path (BGP) where the technology stocks of both economies grow at the same rate. On the BGP the relation of the countries' technology stocks at each point in time is proportional to the countries' research productivities, i.e., $\frac{N_{Nt}}{N_{St}} = \frac{\delta_S}{\delta_N}$.

Both countries are symmetric with respect to their objective. Consider the utility maximization problem of country j .

$$\begin{aligned} \max_{\{\omega_{jt+1}\}_{t \in \mathbb{N}_+}} U_j &= \sum_{t=0}^{\infty} \beta^t \left[N_t \lambda_j (Z_1 + \omega_{jt} Z_2) - N_{kt} \omega_{jt} Z_3 \lambda_j + N_{jt} \omega_{kt} Z_3 \lambda_k - \frac{E_{Ot}^2}{4\delta_j} \right] \\ \text{s.t.} & \quad (58a), (58b), \end{aligned}$$

where $j, k \in \{N, S\}$, $j \neq k$. The first term of the objective function comprises total production and the deadweight loss incurred by intellectual property rights. The second term indicates how much country j pays for licenses of the blueprints from country k . Additionally, there are profits of country j 's intermediate firms from offering their intermediates in the foreign country k . With some mathematical transformations, the first-order condition of country j can be written as

$$\begin{aligned} R_j(\omega_{jt+1}, \omega_{kt+1}) &= Z_2 + \frac{1}{2}E_{Ot}(Z_2 - Z_3)\left(\frac{1}{\delta_N} + \frac{1}{\delta_S}\right) - \frac{N_{kt}}{N_t}Z_3 \\ &\quad + \frac{\beta Z_3}{2\delta_j} \left[\lambda_j(Z_1 + \omega_{jt+1}Z_2) + \omega_{kt+1}\lambda_k Z_3 - \frac{E_{Ot+1}^2}{4\delta_j} \right] \\ &\quad + \frac{\beta Z_3}{2\delta_k} \left[\lambda_j(Z_1 + \omega_{jt+1}Z_2) - \omega_{jt+1}\lambda_j Z_3 - \frac{E_{Ot+1}^2}{4\delta_j} \right] = 0 \end{aligned}$$

The sufficient condition is satisfied as $\frac{\partial R_j(\omega_{jt+1}, \omega_{kt+1})}{\partial \omega_{jt+1}} < 0$.³⁹ In steady state, we have $\omega_{jt+1} = \omega_{jt} = \omega_j$. Further, from Lemma 4, we know that the relation of the technology stocks will converge to the ratio of the R&D costs – i.e., $\frac{N_{Nt}}{N_{St}} = \frac{\delta_S}{\delta_N}$. Consequently, in steady state the reaction function of country j , $R_j(\omega_{jt+1}, \omega_{kt+1})$, transforms to

$$\begin{aligned} R_j^s(\omega_j, \omega_k) &= Z_2 + \frac{1}{2}E_O(Z_2 - Z_3)\left(\frac{1}{\delta_N} + \frac{1}{\delta_S}\right) - \frac{\delta_j}{\delta_N + \delta_S}Z_3 \quad (59) \\ &\quad + \frac{\beta Z_3}{2\delta_j} \left[\lambda_j(Z_1 + \omega_j Z_2) + \omega_k \lambda_k Z_3 - \frac{E_O^2}{4\delta_j} \right] \\ &\quad + \frac{\beta Z_3}{2\delta_k} \left[\lambda_j(Z_1 + \omega_j Z_2) - \omega_j \lambda_j Z_3 - \frac{E_O^2}{4\delta_j} \right] = 0 \end{aligned}$$

It is now interesting to see whether the countries' levels of IPR-protection are strategic substitutes or strategic complements. The following proposition gives a clear answer.

Proposition 14

Country j 's level of intellectual property rights protection is a strategic substitute to that of country k and vice versa.

The proof is given in the appendix.

³⁹More precisely, we obtain

$$\frac{\partial R_j(\omega_{jt+1}, \omega_{kt+1})}{\partial \omega_{jt+1}} = \frac{\beta Z_3 \lambda_j}{2\delta_j} \left[(Z_2 - Z_3)\left(1 + 2\frac{\delta_j}{\delta_k}\right) + Z_2 \right] < 0.$$

B.2 Open Economies without Knowledge Spillovers

We will now consider the case where the countries trade intermediates but cannot benefit from knowledge spillovers from the technologies of the other country. This means that the cost function of R&D changes to

$$\zeta_j^{nS}(\eta_{jt}) = \delta_j \eta_{jt}^2 \frac{L_j}{N_{jt}}.$$

Consequently, the dynamics of the national technology stocks write

$$N_{jt+1} = N_{jt} \left(1 + \frac{E_{Ot}}{2\delta_j} \right).$$

The growth rate in country j is given by

$$\frac{N_{jt+1} - N_{jt}}{N_{jt}} = \frac{E_{Ot}}{2\delta_j}.$$

Evidently, a balanced growth path along which both countries grow at the same rate is only possible if $\delta_N = \delta_S$. In this case, the initial ratio of technological knowledge, $n_t = \frac{N_{St}}{N_{Nt}}$ is preserved over time. So we have $n_t = n$, $\forall t$. Suppose that $\delta_N < \delta_S$. This implies that country N 's technology stock grows faster than that of country S for all times. Accordingly, n_t converges to zero in the long run, whereas it would grow to infinity if $\delta_S < \delta_N$. We summarize this observation in the following lemma:

Lemma 5

If $\delta_N = \delta_S$ there exists a balanced growth path where the ratio of the technology stocks is identical to the ratio of the technology stocks before the countries opened up for trade. Otherwise the technology stocks diverge over time, where the technology stock of the country with the higher research productivity is growing faster.

The utility of country N in the case without knowledge spillovers can be written as

$$U_N = \sum_{t=0}^{\infty} \beta^t N_{Nt} \lambda_N \left[(1 + n_t)(Z_1 + \omega_{Nt} Z_2) - n_t \omega_{Nt} Z_3 + Z_3 \omega_{St} \frac{\lambda_S}{\lambda_N} - \frac{E_{Ot}^2}{2\delta_N \lambda_N} \right],$$

and that of country S as

$$U_S = \sum_{t=0}^{\infty} \beta^t N_{Nt} \lambda_S \left[(1 + n_t)(Z_1 + \omega_{St} Z_2) - \omega_{St} Z_3 + Z_3 n_t \omega_{Nt} \frac{\lambda_N}{\lambda_S} - \frac{E_{Ot}^2}{2\delta_S \lambda_S} \right].$$

We can see that n_t reflects the importance of the intermediates from country S in the countries' final good production and for the profit streams between the countries. That

is, if n_t is low the intermediates from country N play the most important role in final good production and country S is a net-importer of technology. The opposite holds true if n_t is high.

The first-order condition of the countries in the dynamic game without spillovers read:

$$\begin{aligned}
R_N(\omega_{Nt+1}, \omega_{St+1}) &= (1 + n_t)Z_2 + Z_3 \left(1 - n_t - \frac{E_{Ot}}{\delta_N}\right) & (60) \\
&+ \beta \frac{Z_3 \lambda_N}{2\delta_N} \left[Z_1 + \omega_{Nt+1}Z_2 + Z_3 \left(\omega_{Nt+1} + \omega_{St+1} \frac{\lambda_S}{\lambda_N}\right) - \frac{E_{Ot+1}^2}{2\delta_N \lambda_N} \right] \\
&+ \beta n_t \frac{Z_3 \lambda_N}{2\delta_S} [Z_1 + \omega_{Nt+1}(Z_2 - Z_3)] = 0,
\end{aligned}$$

$$\begin{aligned}
R_S(\omega_{Nt+1}, \omega_{St+1}) &= (1 + n_t)Z_2 + Z_3 \left(n_t - 1 - n_t \frac{E_{Ot}}{\delta_S}\right) & (61) \\
&+ \beta n_t \frac{Z_3 \lambda_S}{2\delta_S} \left[Z_1 + \omega_{St+1}Z_2 + Z_3 \left(\omega_{St+1} + \omega_{Nt+1} \frac{\lambda_N}{\lambda_S}\right) - \frac{E_{Ot+1}^2}{2\delta_S \lambda_S} \right] \\
&+ \beta \frac{Z_3 \lambda_S}{2\delta_N} [Z_1 + \omega_{St+1}(Z_2 - Z_3)] = 0.
\end{aligned}$$

In the first-order conditions, the terms in the first line reflect the static loss of IPR-protection. The second line represents the marginal utility gains from additional intermediates produced 'at home' and the third line the marginal utility gain from those produced abroad. As the costs of R&D are decreasing with technological knowledge, if n_t is increasing, the marginal utility gain from additional intermediates of country N is increasing in importance whereas that of intermediates of country S is decreasing. In the limit, $n_t \rightarrow 0$, the first order conditions write:

$$\begin{aligned}
\lim_{n_t \rightarrow 0} R_N(\omega_{Nt+1}, \omega_{St+1}) &= Z_2 + Z_3 \left(1 - \frac{E_{Ot}}{\delta_N}\right) & (62) \\
&+ \beta \frac{Z_3 \lambda_N}{2\delta_N} \left[Z_1 + \omega_{Nt+1}Z_2 + Z_3 \left(\omega_{Nt+1} + \omega_{St+1} \frac{\lambda_S}{\lambda_N}\right) - \frac{E_{Ot+1}^2}{2\delta_N \lambda_N} \right],
\end{aligned}$$

$$\lim_{n_t \rightarrow 0} R_S(\omega_{Nt}, \omega_{St}) = Z_2 - Z_3 + \beta \frac{Z_3 \lambda_S}{2\delta_N} [Z_1 + \omega_{St+1}(Z_2 - Z_3)]. \quad (63)$$

A comparison with the first-order conditions in the classical North-South model reveals that (62) is identical to the North's first-order condition (??) and (63) is identical to the first-order condition of the South as given by (??). Hence we obtain the following result:

Proposition 15

The dynamic system of two open economies without knowledge spillovers converges to the dynamics of the classical North-South model if $\delta_N \neq \delta_S$.

The proposition implies that the North-South model studied before is not only relevant for cases when there is only one country that conducts research, but also for settings where the economies' technology stocks grow at different rates in the long run.

We now examine the balanced growth path that the economies will converge to in case they possess the same level of human capital, i.e. $\delta_N = \delta_S$. We can again state:

Proposition 16

In the model without knowledge spillovers, country N's level of intellectual property rights protection is a strategic substitute to that of country S and vice versa.

The proof proceeds along the same lines as that in the full spillover model. It is available upon request.

C Proofs

C.1 Proof of Proposition 11

(i) Solving the first-order condition for ω yields

$$\begin{aligned}\omega_{a1} &= \frac{2(1-\beta)}{\beta^2 Z_3^2} \left[((2-\beta)Z_2 - Z_3)\Delta + \sqrt{\Delta X(\Delta)} \right], \\ \omega_{a2} &= \frac{2(1-\beta)}{\beta^2 Z_3^2} \left[((2-\beta)Z_2 - Z_3)\Delta - \sqrt{\Delta X(\Delta)} \right],\end{aligned}$$

where $X(\Delta) = 4\Delta(1-\beta)Z_2(Z_2-Z_3) + Z_3^2\Delta + \beta^2(Z_1Z_3^2 + Z_2^2\Delta) > 0$. As $(2-\beta)Z_2 - Z_3 < 0$ only the solution where the square-root is added contains economic meaning. Hence, the optimal level is uniquely given by

$$\omega_a = \omega_{a1} = \frac{2(1-\beta)}{\beta^2 Z_3^2} \left[((2-\beta)Z_2 - Z_3)\Delta + \sqrt{\Delta X(\Delta)} \right].$$

(ii) We now show that ω_a possesses a unique maximum at Δ_{crit} . To this end, we first observe from the second derivative of ω_a with respect to Δ that ω_a is strictly concave in Δ .

$$\frac{d^2\omega_a}{d\Delta^2} = -\frac{(1-\beta)\beta^2 Z_1^2 Z_3^2}{2[\Delta X(\Delta)]^{\frac{3}{2}}} < 0.$$

Further, taking the limit $\Delta \rightarrow 0$ of ω_a and $\frac{d\omega_a}{d\Delta}$ yields:

$$\lim_{\Delta \rightarrow 0} \omega_a = 0, \tag{64a}$$

$$\lim_{\Delta \rightarrow 0} \frac{d\omega_a}{d\Delta} = +\infty. \tag{64b}$$

For ω_a to possess a unique maximum, it remains to be shown that there exists a Δ_{crit} such that $\frac{d\omega_a}{d\Delta} < 0$ for all $\Delta > \Delta_{crit}$. Using the implicit function theorem, we obtain

$$\frac{d\omega_a}{d\Delta} = - \frac{\left. \frac{\partial R(\omega)}{\partial \Delta} \right|_{\omega=\omega_a}}{\left. \frac{\partial R(\omega)}{\partial \omega} \right|_{\omega=\omega_a}}$$

Since the second derivative of $R(\omega)$ with respect to ω reads

$$\frac{d^2 R(\omega)}{d\omega^2} = - \left(\frac{\beta}{1-\beta} \right)^2 \frac{Z_3^2}{4\Delta} < 0, \quad (65)$$

$R(\omega)$ is strictly concave in ω_N . Further we know from part (i) of this proof that $R(\omega)$ possesses two roots, the larger of which is ω_a . It follows that $\left. \frac{\partial R(\omega)}{\partial \omega} \right|_{\omega=\omega_a} < 0$ and the sign of $\frac{d\omega_a}{d\Delta}$ is identical to the sign of $\left. \frac{\partial R(\omega)}{\partial \Delta} \right|_{\omega=\omega_a}$. The derivative of $R(\omega)$ with respect to Δ writes

$$\frac{\partial R(\omega)}{\partial \Delta} = Z_2 + \frac{\beta^3 Z_3^3 \omega^2}{8(1-\beta)^2 \Delta^2}, \quad (66)$$

and is strictly increasing with ω . Since $Z_2 < 0$, there exists a unique ω_{crit} for which $\left. \frac{\partial R(\omega)}{\partial \Delta} \right|_{\omega=\omega_{crit}} = 0$:

$$\omega_{crit} = \frac{2(1-\beta)\Delta}{\beta Z_3} \sqrt{-\frac{2Z_2}{\beta Z_3}}. \quad (67)$$

This implies that $\left. \frac{\partial R(\omega)}{\partial \Delta} \right|_{\omega} < (>)0$ for all $\omega > (<)\omega_{crit}$. Equation (67) reveals that ω_{crit} is linearly increasing with Δ and $\omega_{crit} = 0$ at $\Delta = 0$. This together with the strict concavity of ω_a and the properties given in (64a) and (64b) imply that there will be a unique intersection of ω_a and ω_{crit} in Δ . From the definition of ω_{crit} , this intersection must be at the maximum of ω_a . We denote the maximizer by Δ_{crit} . Item (ii) of Proposition 11 follows directly. Note that (64a) and (64b) also imply a positive maximum level of IPR-protection – i.e., the autark economy will not be in the corner solution of zero protection for all values of Δ . \square

C.2 Proof of Proposition 12

The only economic meaningful solution to (54) is given by

$$\omega_a^f = \frac{2(\beta-1)}{\beta^2(Z_3-2Z_2)} \left[\Delta X_1 + \sqrt{\Delta^2 X_2 + \Delta X_3} \right], \quad (68)$$

with $X_1 = Z_2(2-3\beta) + Z_3(1-\beta)$; $X_2 = Z_2^2(5\beta^2 - 8\beta + 4) - 4Z_2Z_3(1-\beta)^2 + Z_3^2(1-\beta)^2$; $X_3 = Z_1Z_3\beta^2(1-2Z_2)$.

In order to verify the effect of changes in Δ_a on ω_a^f we choose an indirect strategy, since $\frac{\partial \omega_a^w}{\partial \Delta_a}$ turns out to be quite dizzying. For this reason notice that

1. $\omega_a^f = 0$, if $\Delta_a = 0$ or $\Delta_a = -\frac{1}{2} \frac{\beta Z_3 Z_1}{(1-\beta) Z_2} > 0$.
2. ω_a^f approaches $-\infty$ as Δ_a approaches $+\infty$.

Remark:

$$\begin{aligned} \lim_{\Delta \rightarrow \infty} \omega_a^f &= \text{sign} \left\{ \overbrace{\frac{1}{\beta^2 Z_3 (2Z_2 - Z_3)}}^{<0} \right. \\ &\quad \left. \overbrace{\left[Z_2(\beta(3\beta - 5) + 2) - Z_2(1 - \beta)^2 + (1 - \beta)\sqrt{\Phi} \right]}^{>0} \right\} \infty \\ &= -\infty, \end{aligned} \quad (69)$$

with $\Phi = 5Z_2^2\beta^2 - 4Z_2\beta^2 Z_3 - 8Z_2^2\beta + 8Z_2\beta Z_3 + Z_3^2\beta^2 - 2\beta Z_3^2 + 4Z_2^2 - 4Z_2 Z_3 + Z_3^2$.

3. As moreover

$$\frac{\partial^2 \omega_a^f}{\partial \Delta^2} = -\frac{2(\beta - 1)}{\beta^2(Z_3 - 2Z_2)} \left[\frac{1}{4} \frac{X_3^2}{(\Delta(\Delta X_2 + X_3)\sqrt{\Delta(\Delta X_2 + X_3)})} \right] > 0, \quad (70)$$

for $\Delta_a > 0$, since $\Delta(\Delta X_2 + X_3) > 0$ for economic meaningful solutions. In addition we yield: $\lim_{\Delta \rightarrow 0} \frac{\partial^2 \omega}{\partial \Delta^2} = +\infty$ and $\lim_{\Delta \rightarrow \infty} \frac{\partial^2 \omega}{\partial \Delta^2} = 0$, such that $\frac{\partial^2 \omega}{\partial \Delta^2} \geq 0 \forall \Delta \geq 0$.

From 3. we can exclude an inverted u-shaped relation between ω_a^f and Δ_a . Since, ω_a^f becomes negative for $\Delta_a > -\frac{1}{2} \frac{\beta Z_3 Z_1}{(1-\beta) Z_2}$, we can also exclude an u-shaped relation over a range of Δ_a , since this would require an inverted-u-shaped relation afterwards, such that ω_a^f turns negative. It follows that $\omega_a^f = 0$ for $\Delta_a^f = 0$ and $\omega_a^f > 0$ for $\Delta \in (0, -\frac{1}{2} \frac{\beta Z_3 Z_1}{(1-\beta) Z_2})$, with $\frac{\partial \omega}{\partial \Delta} < 0 \forall \Delta > 0$.

C.3 Proof of Proposition 13

Inserting ω_a into (55) and taking the derivative with respect to Δ yields

$$\frac{dg}{d\Delta} = -\frac{\beta Z_1 Z_3}{2\Delta \sqrt{\Delta} X(\Delta)} < 0.$$

□

C.4 Proof of Lemma 1

(i) Solving $R_N(\omega_N, \omega_S) = 0$ for ω_N yields

$$\begin{aligned}\omega_{N1}^N(\omega_S) &= \frac{1}{\beta^2 Z_3^2 \lambda_N} \left[G(\omega_S) + 2\sqrt{\Delta\lambda H(\omega_S)} \right] \\ \omega_{N2}^N(\omega_S) &= \frac{1}{\beta^2 Z_3^2 \lambda_N} \left[G(\omega_S) - 2\sqrt{\Delta\lambda H(\omega_S)} \right]\end{aligned}\tag{71}$$

where

$$\begin{aligned}G(\omega_S) &= -\beta^2 \lambda_S Z_3^2 \omega_S - 2(1 - \beta)\Delta\lambda(-(2 - \beta)Z_2 + Z_3) < 0, \\ H(\omega_S) &= \Delta\lambda[(2 - 3\beta + \beta^2)^2 Z_2^2 + (1 - \beta)^2(Z_3^2 - 4(1 - \beta)Z_2 Z_3)] \\ &\quad + (1 - \beta)^2 \beta^2 Z_3^2 (\lambda_N Z_1 - \lambda_S(Z_2 - Z_3)\omega_S) > 0.\end{aligned}$$

The signs of $G(\omega_S)$ and $H(\omega_S)$ imply that $\omega_{N2}^N(\omega_S)$ is negative for all values $\omega_S \geq 0$. In contrast, $\omega_{N1}^N(\omega_S)$ can be positive. Hence it is the only economically sensible solution and we define $\omega_N^N(\omega_S) \equiv \omega_{N1}^N(\omega_S)$.

(ii) Taking the second derivative of $\omega_N^N(\omega_S)$ with respect to ω_S gives

$$\frac{d^2 \omega_N^N(\omega_S)}{d\omega_S^2} = -\frac{(1 - \beta)^4 \beta^2 \Delta^2 \lambda^2 \lambda_S^2 (Z_2 - Z_3)^2 Z_3^2}{2\lambda_N [H(\omega_S)]^{\frac{3}{2}}} < 0, \quad \forall \omega_S \geq 0.$$

The negative sign for all $\omega_S \geq 0$ follows from $H(\omega_S) > 0$ for all $\omega_S \geq 0$.

To show that $\omega_N^N(\omega_S)$ is strictly decreasing on \mathbb{R}_+ , we use the implicit function theorem.

The partial derivative of $R_N(\omega_N, \omega_S)$ with respect to ω_N reads

$$\frac{\partial R_N(\omega_N, \omega_S)}{\partial \omega_N} = -\left(\frac{1}{1 - \beta}\right) \frac{\partial E_O}{\partial \omega_N} \frac{Z_3 L_N}{2\delta_N} + \frac{\beta Z_3 L_N}{2\delta_N} \left[A_N^{\frac{1}{1-\alpha}} Z_2 - \frac{E_O \frac{\partial E_O}{\partial \omega_N}}{2\delta_N} \right],$$

where E_O denotes the discounted profits of an innovation steady state. As the monopoly distortion Z_2 is negative, the entire derivative is smaller than zero for all $(\omega_N, \omega_S) \in [0, 1]^2$. With respect to ω_S , the derivative of $R_N(\omega_N, \omega_S)$ can be written as

$$\frac{\partial R_N(\omega_N, \omega_S)}{\partial \omega_S} = \left(\frac{1}{1 - \beta} - \frac{\beta}{2}\right) \frac{Z_3^2 \lambda_S}{\delta_N} - \frac{\beta}{1 - \beta} \frac{Z_3^2 \lambda_S E_O}{4\delta_N^2}.$$

This derivative is negative for all $(\omega_N, \omega_S) \in [0, 1]^2$, since β is smaller than one and positive.

According to the implicit function theorem, we can deduce that $\omega_N^N(\omega_S)$ is strictly decreasing on the interval $[0, 1]$. As $\omega_N^N(\omega_S)$ is also strictly concave on \mathbb{R}_+ , it follows that it must be strictly decreasing on \mathbb{R}_+ .

This implies that the inverse of $\omega_N^N(\omega_S)$ which we denote by $\omega_S^N(\omega_N)$ exists at least for all $\omega_N \leq \omega_N^N(0)$. \square

C.5 Proof of Proposition 2

We have to show that

- (i) if $\omega_N^e \leq 0$ and $\omega_S^e \leq 0$, the unique equilibrium is $(\omega_N, \omega_S) = (\hat{\omega}_N^N(0), 0)$.
- (ii) if $\omega_N^e \leq 0$ and $\omega_S^e \in (0, 1)$, the unique equilibrium is $(\omega_N, \omega_S) = (0, \hat{\omega}_S^S(0))$.
- (iii) if $\omega_N^e \leq 0$ and $\omega_S^e \geq 1$, the unique equilibrium is $(\omega_N, \omega_S) = (\hat{\omega}_N^N(1), \hat{\omega}_S^S(0))$.
- (iv) if $\omega_N^e \in (0, 1)$ and $\omega_S^e \leq 0$, the unique equilibrium is $(\omega_N, \omega_S) = (\hat{\omega}_N^N(0), 0)$.
- (v) if $\omega_N^e \in (0, 1)$ and $\omega_S^e \in (0, 1)$, the unique equilibrium is $(\omega_N, \omega_S) = (\omega_N^e, \omega_S^e)$.
- (vi) if $\omega_N^e \in (0, 1)$ and $\omega_S^e \geq 1$, the unique equilibrium is $(\omega_N, \omega_S) = (\hat{\omega}_N^N(1), 1)$.
- (vii) if $\omega_N^e \geq 1$ and $\omega_S^e \leq 0$, the unique equilibrium is $(\omega_N, \omega_S) = (\hat{\omega}_N^N(0), \hat{\omega}_S^S(1))$.
- (viii) if $\omega_N^e \geq 1$ and $\omega_S^e \in (0, 1)$, the unique equilibrium is $(\omega_N, \omega_S) = (1, \hat{\omega}_S^S(1))$.
- (ix) if $\omega_N^e \geq 1$ and $\omega_S^e \geq 1$, the unique equilibrium is $(\omega_N, \omega_S) = (1, 1)$.

The proof proceeds as follows. First, we show that there is a unique economically sensible intersection of the reaction functions of the North and the South – i.e., of $\omega_N^N(\omega_S)$ and $\omega_N^S(\omega_S)$. Second, we characterize the relative positions of the reaction functions $\omega_N^N(\omega_S)$ and $\omega_N^S(\omega_S)$ around this intersection. Third, we show existence of the equilibrium. Fourth, we establish the uniqueness of the equilibrium.

(1)

Let us define $\underline{\omega}_S$ as the solution to $H(\omega_S) = 0$. It follows from the proof of Lemma 1 that $\underline{\omega}_S < 0$ and that $\omega_N^N(\omega_S)$ is a real number for all $\omega_S > \underline{\omega}_S$. Further, $\omega_N^N(\omega_S)$ is strictly concave on $(\underline{\omega}_S, \infty)$ according to the proof of Lemma 1. Inserting $\omega_S^S(\omega_N)$ given by (17) into $R_N(\omega_N, \omega_S)$ yields $R_N^e(\omega_N)$. Solving $R_N^e(\omega_N) = 0$ for ω_N gives

$$\begin{aligned}\omega_{N1}^e &= \frac{1}{\beta^2 Z_3^2 \lambda_N} \left[Q_1(\Delta_o) + 2\sqrt{(2-\beta)^2 \Delta_o \lambda^2 Q_2(\Delta_o)} \right], \\ \omega_{N2}^e &= \frac{1}{\beta^2 Z_3^2 \lambda_N} \left[Q_1(\Delta_o) - 2\sqrt{(2-\beta)^2 \Delta_o \lambda^2 Q_2(\Delta_o)} \right],\end{aligned}\tag{72}$$

where

$$Q_1(\Delta_o) = \frac{\beta^2 \lambda_S Z_1 Z_3^2}{Z_2 - Z_3} + 2\Delta_o \lambda((3 - \beta)(2 - \beta)Z_2 - (4 - 3\beta)Z_3) < 0,$$

$$Q_2(\Delta_o) = (3 - \beta)^2 \Delta_o Z_2^2 - 4(3 - 2\beta)\Delta_o Z_2 Z_3 + Z_3^2(2(2 - \beta)\Delta_o + \beta^2 Z_1) > 0.$$

Since $Q_2(\Delta_o) > 0$, $R_N^e(\omega_N)$ possesses two real roots – i.e., the functions $\omega_N^N(\omega_S)$ and $\omega_N^S(\omega_S)$ possess two intersections on the real plane. As $Q_1(\Delta_o) < 0$, ω_{N2}^e is strictly negative for all relevant parameter values and only ω_{N1}^e possesses economical relevance. Hence, we have $\omega_N^e = \omega_{N1}^e$.

Given a unique ω_N^e , we can immediately infer from (17) that $\omega_S^e = \omega_S^S(\omega_N^e)$ is also unique.

(2)

First, we define $\bar{\omega}_S \equiv \omega_S^S(\omega_{N2}^e)$. Part (1) of the proof together with strict concavity of $\omega_N^N(\omega_S)$ on $(\underline{\omega}_S, \infty)$ and $\omega_N^S(\omega_S)$ being a strictly decreasing linear function yields the following lemma.

Lemma 6

$$\begin{aligned} \omega_N^N(\omega_S) < \omega_N^S(\omega_S), & \quad \text{if } \underline{\omega}_S < \omega_S < \omega_S^e \\ \omega_N^N(\omega_S) > \omega_N^S(\omega_S), & \quad \text{if } \omega_S^e < \omega_S < \bar{\omega}_S. \end{aligned}$$

For example, given that (ω_N^e, ω_S^e) is in $(0, 1)^2$, we obtain for the abscissa and ordinate intercepts that $\omega_N^N(0) < \omega_N^S(0)$ and $\omega_S^S(0) > \omega_S^e(0)$.

(3)

- (i) Suppose that $\omega_N^e \leq 0$ and $\omega_S^e \leq 0$. Given $\omega_S = 0$, the best response of North is $\hat{\omega}_N^N(0)$. Given $\omega_N = \hat{\omega}_N^N(0)$, we obtain $\omega_S^S(\omega_N^N(0)) \leq 0$ by using Lemma 6 and the fact that $\omega_S^S(\omega_N)$ is strictly declining. Consequently, the South's best response to $\omega_N = \hat{\omega}_N^N(0)$ is $\omega_S = 0$.
- (ii) Suppose that $\omega_N^e \leq 0$ and $\omega_S^e \in (0, 1)$. Given $\omega_S = \hat{\omega}_S^S(0)$, then best response of North is 0 because $\omega_N^N(\hat{\omega}_S^S(0)) \leq 0$ due to $\omega_S^S(\omega_N)$ being a strictly declining function and Lemma 6. Given $\omega_N = 0$, the South's best response is $\hat{\omega}_S^S(0)$.
- (iii) Suppose that $\omega_N^e \leq 0$ and $\omega_S^e \geq 1$. We distinguish the cases where $\omega_N^N(1) \leq 0$ and $\omega_N^N(1) > 0$. If $\omega_N^N(1) \leq 0$, the equilibrium can be written as $(\omega_N, \omega_S) = (0, \hat{\omega}_S^S(0))$.

$\omega_N^N(1) \leq 0$ together with Lemma 6 and $\omega_S^S(\omega_N)$ strictly declining implies that $\omega_N^N(\hat{\omega}_S^S(0)) \leq 0$. Hence, the best response of the North is $\omega_N = 0$. Further, given $\omega_N = 0$, $\hat{\omega}_S^S(0)$ is the best response of the South.

If $\omega_N^N(1) > 0$, the equilibrium can be written as $(\omega_N, \omega_S) = (\hat{\omega}_N^N(1), 1)$. Given $\omega_S = 1$, $\hat{\omega}_N^N(1)$ is best response of North. $\omega_N^N(1) > 0$ together with Lemma 6 and $\omega_S^S(\omega_N)$ strictly declining imply $\omega_S^S(\omega_N^N(1)) \geq 1$. Consequently, South's best response is $\omega_S = 1$.

(iv) Suppose that $\omega_N^e \in (0, 1)$ and $\omega_S^e \leq 0$. Given $\omega_S = 0$, the best response of North is $\hat{\omega}_N^N(0)$. Given $\omega_N = \hat{\omega}_N^N(0)$, $\omega_S^S(\omega_N^N(0)) \leq 0$ follows from Lemma 6 and $\omega_S^S(\omega_N)$ being a strictly declining function. Hence, the South's best response is $\omega_S = 0$.

(v) Let $\omega_N^e \in (0, 1)$ and $\omega_S^e \in (0, 1)$. Then $(\omega_N, \omega_S) = (\omega_N^e, \omega_S^e)$ is an equilibrium by the definition of the reaction functions.

(vi) Let $\omega_N^e \in (0, 1)$ and $\omega_S^e \geq 1$. Given $\omega_S = 1$, the best response of North is $\hat{\omega}_N^N(1)$. Given $\omega_N = \hat{\omega}_N^N(1)$, the South's best response is $\omega_S = 1$ as $\omega_S^S(\omega_N^N(1)) \geq 1$ due to Lemma 6 and $\omega_S^S(\omega_N)$ being a strictly declining function.

(vii) Suppose that $\omega_N^e \geq 1$ and $\omega_S^e \leq 0$. We distinguish the cases where $\omega_N^N(0) \leq 1$ and $\omega_N^N(0) > 1$. If $\omega_N^N(0) \leq 1$, the equilibrium can be written as $(\omega_N, \omega_S) = (\hat{\omega}_N^N(0), 0)$. Given $\omega_S = 0$, $\hat{\omega}_N^N(0)$ is best response of the North. Due to Lemma 6 and $\omega_S^S(\omega_N)$ being a strictly declining function, $\omega_S^S(\omega_N^N(0)) \leq 0$. Consequently, South's best response is $\omega_S = 0$.

If $\omega_N^N(0) > 1$, the equilibrium can be written as $(\omega_N, \omega_S) = (1, \hat{\omega}_S^S(1))$. $\omega_N^N(0) > 1$ together with Lemma 6 and $\omega_S^S(\omega_N)$ being a strictly declining function implies that $\omega_N^N(\hat{\omega}_S^S(1)) \geq 1$. Hence, the best response of the North is $\omega_N = 1$. Further, given $\omega_N = 1$, $\hat{\omega}_S^S(1)$ is the best response of the South.

(viii) Suppose that $\omega_N^e \geq 1$ and $\omega_S^e \in (0, 1)$. Given $\omega_S = \hat{\omega}_S^S(1)$, Lemma 6 and $\omega_S^S(\omega_N)$ strictly declining imply that $\omega_N^N(\hat{\omega}_S^S(1)) \geq 1$. Consequently, the North's best response is $\omega_N = 1$. Given $\omega_N = 1$, the South's best response is $\omega_S = \hat{\omega}_S^S(1)$.

(ix) Let $\omega_N^e \geq 1$ and $\omega_S^e \geq 1$. Since both functions, $\omega_N^N(\omega_S)$ and $\omega_S^S(\omega_S)$ are declining on \mathbb{R}_+ , $\omega_N^e \geq 1$ and $\omega_S^e \geq 1$ implies that $\omega_N^N(\omega_S), \omega_S^S(\omega_S) \geq 1$ for all $\omega_S \in [0, 1]$ and $\omega_S^S(\omega_N) \geq 1$ for all $\omega_N \in [0, 1]$. Consequently, given $\omega_S = 1$, $\omega_N^N(1) \geq 1$ leading to $\omega_N = 1$ as the best response of the North. Given $\omega_N = 1$, the best response of the South is $\omega_S = 1$ as $\omega_S^S(1) \geq 1$.

(4)

Lemma 6 and the fact that $\omega_S^S(\omega_N)$ and $\omega_N^S(\omega_S)$ are strictly declining functions imply that $\forall \omega_S \in [0, 1]$, $\omega_S \neq \bar{\omega}_S^e$, we have $\hat{\omega}_S^S(\hat{\omega}_N^N(\omega_S)) \neq \omega_S$. Further $\forall \omega_N \in [0, 1]$, $\omega_N \neq \bar{\omega}_N^e$, we obtain $\hat{\omega}_N^N(\hat{\omega}_S^S(\omega_N)) \neq \omega_N$. As a consequence, the equilibrium $(\omega_N, \omega_S) = (\bar{\omega}_N^e, \bar{\omega}_S^e)$ as given in Proposition 2 is unique. □

C.6 Proof of Lemma 2

From the proof of Proposition 2, we know that

$$\omega_N^e = \frac{1}{\beta^2 Z_3^2 \lambda_N} \left[Q_1(\Delta_o) + 2\sqrt{(2-\beta)^2 \Delta_o \lambda_o^2 Q_2(\Delta_o)} \right],$$

where

$$\begin{aligned} Q_1(\Delta_o) &= \frac{\beta^2 \lambda_S Z_1 Z_3^2}{Z_2 - Z_3} + 2\Delta_o \lambda_o ((3-\beta)(2-\beta)Z_2 - (4-3\beta)Z_3) < 0, \\ Q_2(\Delta_o) &= (3-\beta)^2 \Delta_o Z_2^2 - 4(3-2\beta)\Delta_o Z_2 Z_3 + Z_3^2 (2(2-\beta)\Delta_o + \beta^2 Z_1) > 0. \end{aligned}$$

The second derivative of ω_N^e with respect to Δ_o reads

$$\frac{d^2 \omega_N^e}{d\Delta_o^2} = -\frac{(2-\beta)\beta^2 Z_3^2 \lambda_o Z_1}{2\lambda_N Q_2(\Delta_o)^{\frac{3}{2}}}. \quad (73)$$

From (73) we directly observe that the second derivative of ω_N^e with respect to Δ_o is strictly negative, and consequently, ω_N^e is strictly concave in Δ_o .

Concerning the convexity of the South's IPR-level in Δ_o we use equation (17) and take the second derivative with respect to Δ_o to obtain

$$\frac{d^2 \omega_S^e}{d\Delta_o^2} = -\frac{1}{2-\beta} \frac{\lambda_N}{\lambda_S} \frac{d^2 \omega_N^e}{d\Delta_o^2}, \quad (74)$$

Since $\frac{d^2 \omega_N^e}{d\Delta_o^2} < 0$, $\frac{d^2 \omega_S^e}{d\Delta_o^2}$ must be positive and hence, ω_S^e is strictly convex in Δ_o . (Note that the strict convexity comes from the South's IPR-protection being a strategic substitute to that of the North.) □

C.7 Proof of Proposition 3

The proof is organized as follows. First, we derive $\omega_S^S(0)$ and $\omega_N^S(0)$ as well as some notation and lemmata that will be used throughout the proof. Then, we show the

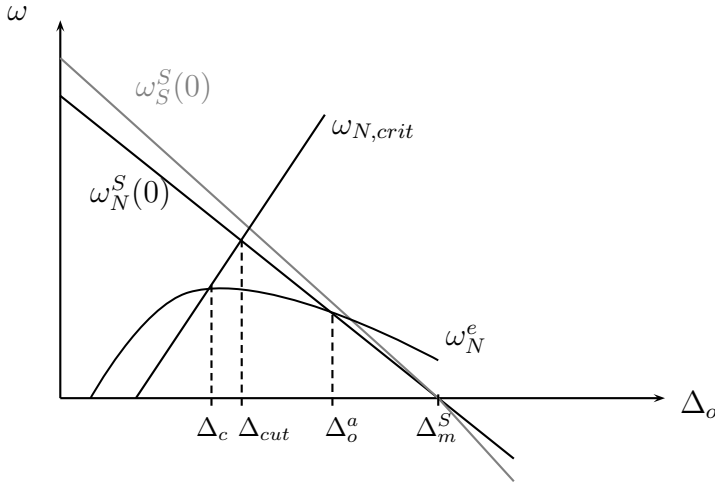


Figure 7: Illustration of the proof of Proposition 3

existence of $\Delta_o^{a,N}$ and that condition (19) is necessary and sufficient for $\Delta_o^{a,S} < \Delta_o^{a,N}$. In the remainder of the proof, we verify items (i)-(v) of the proposition.

From the South's reaction function (17) we obtain the values of IPR-protection in the South given that $\omega_N = 0$ as

$$\omega_S^S(0) = - \left(\frac{1-\beta}{2-\beta} \right) \left[\frac{Z_1}{Z_2 - Z_3} + \frac{2\Delta_o \lambda_o}{\beta Z_3 \lambda_S} \right]. \quad (75)$$

$\omega_S^S(0)$ is zero at the value

$$\Delta_m^S = \frac{\beta \lambda_S Z_1 Z_3}{2\lambda_o (Z_2 - Z_3)}. \quad (76)$$

Now, consider the level of IPR-protection of the North such that the South would just choose a zero level of protection. This corresponds to the inverse of $\omega_S^S(\omega_N)$ at the point $\omega_S = 0$ – i.e.,

$$\omega_N^S(0) = -(1-\beta) \left[\frac{\lambda_S}{\lambda_N} \frac{Z_1}{Z_2 - Z_3} + \frac{2\Delta \lambda_o}{\beta Z_3 \lambda_N} \right]. \quad (77)$$

$\omega_N^S(0)$ defines a line in the $\omega - \Delta$ coordinate plane that intersects with $\omega_S^S(0)$ at Δ_m^S .

Let us now consider $R_N^e(\omega_N)$, which is derived by inserting $\omega_S^S(\omega_N)$ as given by (17) into the first-order condition of the North (18). From the first part of the proof of Proposition 2 we know that $R_N^e(\omega_N)$ possesses two real roots, ω_{N1}^e and ω_{N2}^e . The economically sensible one is the larger root ω_{N1}^e implying $\omega_N^e \equiv \omega_{N1}^e$. By showing that $R_N^e(\omega_N)$ is strictly concave, we establish

Lemma 7

(i) $\forall \omega_N > \omega_{N2}^e, R_N^e(\omega_N) > (<) 0 \Leftrightarrow \omega_N < (>) \omega_N^e.$

(ii) $\left. \frac{dR_N^e(\omega_N)}{d\omega_N} \right|_{\omega_N=\omega_N^e} < 0.$

Proof. $R_N^e(\omega_N)$ can be written as

$$R_N^e(\omega_N) = \frac{Z_2 - Z_3}{2} \frac{\beta}{2 - \beta} Z_3 \left(\frac{\lambda_N}{\lambda_o} \omega_N - A_1 \right) + \Delta_o Z_2 + \frac{\beta Z_3}{2} \left[\frac{\lambda_N}{\lambda_o} (Z_1 + \omega_N Z_2) - Z_3 \frac{1 - \beta}{2 - \beta} A_1 - Z_3 \frac{\lambda_N}{\lambda_o} \frac{\omega_N}{2 - \beta} - \left(\frac{\beta}{2 - \beta} \right)^2 \frac{Z_3^2}{4\Delta_o} \left(\frac{\lambda_N}{\lambda_o} \omega_N - A_1 \right)^2 \right].$$

where $A_1 = \frac{Z_1}{Z_2 - Z_3} \frac{\lambda_S}{\lambda_o} + \frac{2\Delta_o}{\beta Z_3}$. Taking the second derivative with respect to ω_N , we obtain

$$\frac{\partial^2 R_N^e(\omega_N)}{\partial \omega_N^2} = - \left(\frac{\beta}{2 - \beta} \right)^2 \frac{Z_3^2}{2\Delta_o} \left(\frac{\lambda_N}{\lambda_o} \right)^2 < 0.$$

This verifies Lemma 7. □

The level of IPR-protection of the North when the South chooses $\omega_S = 0$ is given by $R^N(\omega_N, \omega_S = 0) = 0$. Interpreting the parameter λ_o as λ_N in $R(\omega)$ as given by equation (51), we obtain

$$R^N(\omega_N, 0) = \frac{\lambda_N}{\lambda_o} R(\omega_N). \tag{78}$$

Since the second derivative of $R(\omega)$ with respect to ω reads

$$\frac{d^2 R(\omega)}{d\omega^2} = - \left(\frac{\beta}{1 - \beta} \right)^2 \frac{Z_3^2}{4\Delta} < 0, \tag{79}$$

$R(\omega)$ is strictly concave in ω_N . From the proof of Proposition 11 we know that $R(\omega)$ possesses two roots. As a consequence of (78), we infer that $R^N(\omega_N, 0)$ is strictly concave in ω_N and also possesses two roots. In fact, the roots of $R^N(\omega_N, 0)$ correspond to ω_{a1} and ω_{a2} given in the proof of Proposition 11 when substituting Δ by $\Delta_o \frac{\lambda_o}{\lambda_N}$. Let us denote the resulting solutions by ω_{N1}^a and ω_{N2}^a , respectively.⁴⁰ Only ω_{N1}^a is economically sensible. Hence we define the level of IPR-protection of the North when

⁴⁰The solutions can be written as

$$\omega_{N1}^a = \frac{2(1 - \beta)}{\beta^2 Z_3^2} \left[((2 - \beta)Z_2 - Z_3)\Delta_o \frac{\lambda_o}{\lambda_N} + \sqrt{\Delta_o \frac{\lambda_o}{\lambda_N} X(\Delta_o \frac{\lambda_o}{\lambda_N})} \right],$$

$$\omega_{N2}^a = \frac{2(1 - \beta)}{\beta^2 Z_3^2} \left[((2 - \beta)Z_2 - Z_3)\Delta_o \frac{\lambda_o}{\lambda_N} - \sqrt{\Delta_o \frac{\lambda_o}{\lambda_N} X(\Delta_o \frac{\lambda_o}{\lambda_N})} \right],$$

where $X(\Delta) = 4\Delta_o \frac{\lambda_o}{\lambda_N} (1 - \beta) Z_2 (Z_2 - Z_3) + Z_3^2 \Delta_o \frac{\lambda_o}{\lambda_N} + \beta^2 (Z_1 Z_3^2 + Z_2^2 \Delta_o \frac{\lambda_o}{\lambda_N}) > 0$.

the South provides no IPR-protection by $\omega_N^a \equiv \omega_{N1}^a$. Using the same line of argument as with regard to Lemma 7, we are now able to formulate:

Lemma 8

- (i) $\forall \omega > \omega_{N2}^a, R^N(\omega_N, 0) > (<) 0 \Leftrightarrow \omega_N < (>) \omega_N^a.$
- (ii) $\left. \frac{dR^N(\omega_N, 0)}{d\omega_N} \right|_{\omega_N = \omega_N^a} < 0.$

Further, we show

Lemma 9

$\omega_N^a > 0$ and $\omega_N^e > 0$ at Δ_m^S if and only if $\frac{\lambda_N}{\lambda_S} > \frac{Z_2}{Z_2 - Z_3}.$

Proof. Since $\omega_{N2}, \omega_{N2}^a < 0, \omega_N^e > 0$ and $\omega_N^a > 0$ at Δ_m^S is equivalent to $R_N^e(0) > 0$ and $R^N(0, 0) > 0$ at Δ_m^S according to Lemmata 7 and 8. Inserting Δ_m^S given in equation (76) into $R_N^e(0) > 0$ and $R^N(0, 0) > 0$ yields

$$\begin{aligned} R_N^e(0) > 0 \Leftrightarrow R^N(0, 0) > 0 \Leftrightarrow -\frac{\beta Z_1 Z_3 (\lambda_S Z_2 - \lambda_N (Z_2 - Z_3))}{2\lambda_o (Z_2 - Z_3)} > 0 \\ \Leftrightarrow \frac{\lambda_N}{\lambda_S} > \frac{Z_2}{Z_2 - Z_3}. \end{aligned}$$

□

Let us now establish

Lemma 10

ω_N^e possesses a unique maximum at $\Delta_c.$

Proof. By a similar line of argument as in the proof of Proposition 11, we show Lemma 10. First we obtain from (72) that

$$\lim_{\Delta_o \rightarrow 0} \omega_N^e = \frac{\lambda_S}{\lambda_N} \frac{Z_1}{Z_2 - Z_3} < 0 \quad (80a)$$

$$\lim_{\Delta_o \rightarrow 0} \frac{\partial \omega_N^e}{\partial \Delta_o} = +\infty. \quad (80b)$$

Using the implicit function theorem, the sign of

$$\frac{d\omega_N^e}{d\Delta_o} = -\frac{\frac{\partial R_N^e(\omega_N)}{\partial \Delta_o}}{\frac{dR_N^e(\omega_N)}{d\omega_N}}, \quad \Delta_o > 0,$$

is identical to that of $\frac{\partial R_N^e(\omega_N)}{\partial \Delta_o}$ because $\frac{dR_N^e(\omega_N)}{d\omega_N} < 0$ at $\omega_N = \omega_N^e$ due to Lemma 7 (ii).

$$\frac{\partial R_N^e(\omega_N)}{\partial \Delta_o} = -\frac{\beta Z_3 B}{2(2 - \beta)^2} - \sqrt{\beta Z_3} \frac{\beta Z_3 (\lambda_S Z_1 - \lambda_N \omega_N (Z_2 - Z_3))}{8(2 - \beta)^2 \Delta_o \lambda_o (Z_2 - Z_3)}. \quad (81)$$

According to (81), $\frac{\partial R_N^e(\omega_N)}{\partial \Delta_o}$ is strictly increasing with ω_N . Hence there exists a $\omega_{N,crit}$, defined by $\frac{\partial R_N^e(\omega_N)}{\partial \Delta_o} = 0$, for which $\frac{\partial R_N^e(\omega_N)}{\partial \Delta_o} > (<) 0$ if and only if $\omega_N > (<) \omega_{N,crit}$. $\omega_{N,crit}$ can be written as

$$\omega_{N,crit} = \frac{\lambda_S}{\lambda_N} \frac{Z_1}{Z_2 - Z_3} + \frac{\lambda_o}{\lambda_N} \frac{2\Delta_o}{\beta Z_3} \sqrt{\beta Z_3 B}.$$

This reveals that $\omega_{N,crit}$ is increasing linearly with Δ_o and that

$$\lim_{\Delta_o \rightarrow 0} \omega_{N,crit} = \lim_{\Delta_o \rightarrow 0} \omega_N^e = \frac{\lambda_S}{\lambda_N} \frac{Z_1}{Z_2 - Z_3} < 0.$$

Considering (80b) and the strict concavity of ω_N^e , we can directly infer that there will be a unique intersection of ω_N^e and $\omega_{N,crit}$ at a $\Delta_o > 0$ which we call Δ_c . Due to the definition of $\omega_{N,crit}$, this intersection is at the maximum of ω_N^e in Δ_o . \square

Now we are able to show the existence of $\Delta_o^{a,N}$.

Lemma 11

There exists a $\Delta_o^{a,N}$ such that $\bar{\omega}_N^e = 0$ for all $\Delta_o \geq \Delta_o^{a,N}$.

To establish the existence of $\Delta_o^{a,N}$, we have to distinguish between the cases where $\Delta_o^{a,N} > \Delta_o^{a,S}$ and $\Delta_o^{a,N} \leq \Delta_o^{a,S}$. In the first case, existence of a $\Delta_o^{a,N} > \Delta_o^{a,S}$ requires that $\omega_N^a \leq 0$ for all Δ_o larger than a certain threshold value. Consider first the IPR-level in autarky ω_a . Due to the strong concavity of ω_a and since $\frac{d\omega_a}{d\Delta} < 0$ for all $\Delta > \Delta_{crit}$, there exists a threshold of Δ where $\omega_a \leq 0$ for all Δ larger than this threshold. It follows further from equation (78) that such a threshold also exists for ω_N^a in Δ_o .

A threshold level $\Delta_o^{a,N} \leq \Delta_o^{a,S}$ requires that $\omega_N^e \leq 0$ for all Δ_o larger than a certain threshold value. Such a threshold value of Δ_o exists since ω_N^e is strictly concave in Δ_o and $\frac{\partial \omega_N^e}{\partial \Delta_o} < 0$ for all $\Delta_o < \Delta_c$ according to Lemma 10. \square

Finally we establish

Lemma 12

$\Delta_o^{a,N} > \Delta_o^{a,S}$ if and only if $\frac{\lambda_N}{\lambda_S} > \frac{Z_2}{Z_2 - Z_3}$.

A necessary and sufficient condition for $\Delta_o^{a,N} > \Delta_o^{a,S}$ is that $\omega_N^a > 0$ at Δ_m^S . This condition is sufficient as $\Delta_o^{a,S}$ is smaller than or equal to Δ_m^S .⁴¹ The condition is necessary because if $\omega_N^a < 0$ at Δ_m^S then $\omega_N^a < 0$ for all $\Delta_o \geq \Delta_m^S$. Further we know from Lemma 9, that $\omega_N^e < 0$ at Δ_m^S if and only if $\omega_N^a < 0$ at Δ_m^S . This implies that

⁴¹ $\Delta_o^{a,S} \leq \Delta_m^S$ follows from $\bar{\omega}_S^e \leq \omega_S^S(0)$ because ω_S is a strategic substitute to ω_N and $\omega_N \geq 0$ in equilibrium.

there exists a $\Delta_o < \Delta_m^S$ for which $\omega_S^e > 0$ and that $\Delta_o^{a,S} = \Delta_m^S$. Consequently, $\omega_N^a > 0$ at Δ_m^S is necessary for $\Delta_o^{a,N} > \Delta_o^{a,S}$. According to Lemma 11, $\omega_N^a > 0$ at Δ_m^S if and only if $\frac{\lambda_N}{\lambda_S} > \frac{Z_2}{Z_2 - Z_3}$. \square

Next we verify items (i)-(v) of Proposition 3

(i): $\bar{\omega}_S^e$ is strictly declining for the following reason. Since $\frac{\lambda_N}{\lambda_S} > \frac{Z_2}{Z_2 - Z_3}$ holds by assumption, $\omega_N^e > 0$ at the point Δ_m^S . This implies that $\omega_S^e = \omega_S^S(\omega_N^e)$ is negative at Δ_m^S . Further, ω_S^e must be smaller than or equal to $\omega_S^S(0)$ if $\omega_N^e \geq 0$. According to Lemma 2, ω_S^e is strictly convex. It follows from (80a) that $\omega_S^e > 0$ for some $\Delta_o < \Delta_m^S$. This, together with $\omega_S^e < 0$ at Δ_m^S implies that ω_S^e possesses a unique root Δ_a in the relevant interval $[0, \Delta_m^S]$ and is strictly decreasing for all $\Delta_o < \Delta_a$. Since

$$\bar{\omega}_S^e = \begin{cases} \hat{\omega}_S^S(0), & \text{if } \omega_N^e \leq 0, \\ \hat{\omega}_S^e, & \text{if } \omega_N^e \in (0, 1), \\ \hat{\omega}_S^S(1), & \text{if } \omega_N^e \geq 1, \end{cases}$$

$\bar{\omega}_S^e$ is (weakly) decreasing and positive for all $\Delta_o < \Delta_a$.

To verify that the equilibrium enforcement of the South takes the corner solution $\bar{\omega}_S^e = 0$ for all Δ_o larger than Δ_a , we have to consider ω_N^a which indicates the North's best response to $\omega_S = 0$. Only if $\omega_N^a > \omega_N^S(0)$ for all $\Delta_o \in (\Delta_a, \Delta_m^S)$, will the South choose $\bar{\omega}_S^e = 0$ for all $\Delta_o \geq \Delta_a$. It follows from (78) that $\omega_N^a = \omega_N^e = \omega_N^S(0)$ at Δ_a . Since ω_N^a is strictly concave in Δ_o and $\omega_N^a > 0$ at Δ_m^S , ω_N^a does not intersect $\omega_N^S(0)$ in the interval $\Delta_o \in (\Delta_a, \Delta_m^S)$.

(ii): As $\bar{\omega}_S^e = 0$ for all $\Delta_o > \Delta_a$, the claim that the North acts as if in autarky follows directly from the equivalence of the first-order conditions given by (78). Another way to see the equivalence is by setting $\omega_{S,t} = 0 \forall t$ in the North's maximization problem in Section 3.2 and comparing it to the government's maximization problem in autarky given in Section A.1.4.

(iii): According to Lemma 2, ω_N^e is strictly concave in Δ_o . Further, we have $\lim_{\Delta_o \rightarrow 0} \omega_N^e < 0$ according to (80a). From $\omega_N^e > 0$ at Δ_m^S (Lemma 9), we infer that $\omega_N^e > 0$ at Δ_o^a . ω_S^e is strictly convex and strictly declining on $[0, \Delta_o^a]$. Further, $\omega_S^e = 0$ at Δ_o^a . As a consequence, there is exactly one intersection ω_N^e and ω_S^e on $[0, \Delta_o^a]$. Denoting the value of Δ_o at the intersection by Δ_{eq} , we obtain directly that for all $\Delta_o < \Delta_{eq}$, $\omega_S^e > \omega_N^e$ and for all $\Delta_{eq} < \Delta_o \leq \Delta_o^a$, $\omega_S^e < \omega_N^e$.

(iv) + (v): According to Lemma 10, ω_N^e possesses a unique maximum at Δ_c . It follows directly that ω_N^e increases (decreases) with Δ_o for all $\Delta_o < (>) \Delta_c$.

We know that for all $\Delta_o > \Delta_o^a$, the North's IPR-levels correspond to its autarky levels since the South does not provide any protection. Hence, whether there exists an interval where the North's IPR-protection is decreasing with Δ_o while the South still offers positive enforcement, depends on whether $\Delta_c < \Delta_o^a$. Otherwise ω_N^e is strictly increasing with Δ_o for all $\Delta_o < \Delta_o^a$ as described in item (iv) of the proposition. To verify items (iv) and (v) of the Proposition, we need to show that $\Delta_c < \Delta_o^a$ if and only if $\frac{\lambda_S}{\lambda_o} > \tilde{\lambda}$.

To this end, we consider the two lines $\omega_N^S(0)$ and $\omega_{N,crit}$, defined by $\frac{\partial R_N^e(\omega_N)}{\partial \Delta_o} = 0$. $\omega_{N,crit}$ intersects with ω_N^e at the maximum of ω_N^e at Δ_c (Lemma 10) and $\omega_N^S(0)$ intersects with ω_N^e at the point Δ_o^a where ω_S^e becomes zero.⁴² This is illustrated in Figure 7. Further $\omega_{N,crit}$ is increasing with Δ_o and $\omega_N^S(0)$ is decreasing with Δ_o . Consequently, the value Δ_{cut} where both lines intersect will be larger than Δ_c if and only if $\Delta_c < \Delta_o^a$ and smaller than Δ_c if and only if $\Delta_c > \Delta_o^a$.

Hence the condition $\Delta_c > \Delta_{cut}$ is equivalent to $\Delta_c < \Delta_o^a$. We now complete the proof by showing that the latter condition is equivalent to $\frac{\lambda_S}{\lambda_o} > \tilde{\lambda}$.

Equating $\omega_N^S(0)$ and $\omega_{N,crit}$, we obtain Δ_{cut} as

$$\Delta_{cut} = \frac{\lambda_S}{\lambda_o} \frac{(2 - \beta)\beta^2 Z_3^2 Z_1}{2(Z_2 - Z_3)[(1 - \beta)\beta Z_3 - \sqrt{\beta Z_3 B}]}.$$

For Δ_c , we obtain

$$\Delta_c = \frac{-(2 - \beta)^2 \beta^2 Z_3^2 Z_1}{\beta Z_3 B + \sqrt{\beta Z_3 B}[(3 - \beta)(2 - \beta)Z_2 - (4 - 3\beta)Z_3]}.$$

With few mathematical transformations, one can arrive at $\Delta_c > \Delta_{cut} \Leftrightarrow \frac{\lambda_S}{\lambda_o} > \tilde{\lambda}$. \square

C.8 Proof of Proposition 4

The growth rate of the economy writes

$$g = \frac{\beta Z_3 \left(\frac{\lambda_N}{\lambda_o} \omega_N^e + \frac{\lambda_S}{\lambda_o} \omega_S^e \right)}{2(1 - \beta)\Delta_o}.$$

Inserting ω_N^e and ω_S^e as given by (72) and (17) and differentiating with respect to Δ_o yields

$$\frac{dg}{d\Delta_o} = \frac{-\beta Z_1}{2\Delta_o \sqrt{\Delta_o(\beta^2 Z_1 Z_3^2 + \Delta_o((3 - \beta)^2 Z_2^2 - 4(3 - 2\beta)Z_2 Z_3 + 2(2 - \beta)Z_3^2))}} < 0.$$

⁴²Note $\omega_N^S(0)$ is linear and declines with Δ_o as can be observed in equation (77). Together with the properties of ω_N^e as described in item (iii) of this proof, this implies that there is exactly one intersection of $\omega_N^S(0)$ with ω_N^e .

C.9 Proof of Proposition 5

First we show that ω_N^e increases with $\frac{\lambda_N}{\lambda_S}$ and, thereafter, that ω_S^e decreases with $\frac{\lambda_N}{\lambda_S}$. Finally, we verify that the growth rate is invariant with $\frac{\lambda_N}{\lambda_S}$ given Δ_o .

$$1. \left. \frac{d\omega_N^e}{d\frac{\lambda_N}{\lambda_S}} \right|_{\lambda_o} > 0:$$

Using the implicit function theorem, we obtain

$$\left. \frac{d\omega_N^e}{d\frac{\lambda_N}{\lambda_S}} \right|_{\lambda_o} = \left. \frac{d\omega_N^e}{d\lambda_N} \right|_{\lambda_o} = - \frac{\left. \frac{\partial R_N^e(\omega_N)}{\partial \lambda_N} \right|_{\lambda_o}}{\left. \frac{\partial R_N^e(\omega_N)}{\partial \omega_N} \right|_{\lambda_o}}.$$

We know from Lemma 7 (ii) in the proof of Proposition 7 that $\left. \frac{\partial R_N^e(\omega_N)}{\partial \omega_N} \right|_{\omega_N=\omega_N^e} < 0$. Consequently, the sign of $\left. \frac{d\omega_N^e}{d\lambda_N} \right|_{\lambda_o}$ is identical to that of $\left. \frac{dR_N^e(\omega_N)}{d\lambda_N} \right|_{\lambda_o}$. For the derivative of $R_N^e(\omega_N)$ with respect to λ_N given the total market size λ_o , we can write

$$\left. \frac{\partial R_N^e(\omega_N)}{\partial \lambda_N} \right|_{\lambda_o} = \frac{\beta Z_3 (Z_1 + \omega_N (Z_2 - Z_3))}{4 \Delta_o (2 - \beta)^2 \lambda_o^2 (Z_2 - Z_3)^2} [2 \Delta_o \lambda_o (Z_2 - Z_3) ((3 - \beta)(2 - \beta) Z_2 - (4 - 3\beta) Z_3) + \beta^2 Z_3^2 (\lambda_S Z_1 - \lambda_N \omega_N (Z_2 - Z_3))] \quad (82)$$

Since $Z_1 + Z_2 - Z_3 > 0$, it can be readily observed from (82) that $\left. \frac{dR_N^e(\omega_N)}{d\lambda_N} \right|_{\lambda_o} > 0$. Hence, if we have an interior solution where $\bar{\omega}_N^e \in (0, 1)$, the North's IPR-level strictly increases with its relative effective market size.

$$2. \left. \frac{d\omega_S^e}{d\frac{\lambda_N}{\lambda_S}} \right|_{\lambda_o} > 0:$$

We insert $\lambda_S = \lambda_o - \lambda_N$ into (17) and take the derivative with respect to λ_N given the total market size λ_o . We obtain

$$\left. \frac{d\omega_S^e}{d\lambda_N} \right|_{\lambda_o} = - \frac{1 - \beta}{2 - \beta} \frac{2 \Delta_o \lambda_o}{\lambda_S^2} - \frac{\omega_N}{2 - \beta} \frac{\lambda_o}{\lambda_S^2} - \frac{\lambda_N}{\lambda_S} \frac{1}{2 - \beta} \left. \frac{d\omega_N^e}{d\lambda_N} \right|_{\lambda_o} < 0 \quad (83)$$

As we know from the first part of the proof that $\left. \frac{d\omega_N^e}{d\lambda_N} \right|_{\lambda_o} > 0$, it follows that $\left. \frac{d\omega_S^e}{d\lambda_N} \right|_{\lambda_o} < 0$. This verifies that the South (at an interior solution) also increases IPRs if its relative market size increases.

3. Consider now the steady state growth rate in equilibrium:

$$g = \frac{\beta Z_3 (\lambda_N \omega_N^e + \lambda_S \omega_S^e)}{2(1-\beta)\lambda_o \Delta_o}.$$

Inserting ω_N^e as given by (72), ω_S^e and substituting λ_S by $\lambda_o - \lambda_N$, we obtain for the derivative with respect to λ_N given λ_o :

$$\left. \frac{dg}{d\lambda_N} \right|_{\lambda_o} = 0.$$

□

C.10 Proof of Proposition 7

We start with the items (i) and (ii). The proof proceeds as follows.

We first show that there is a unique solution to the North's optimization problem. In this first step, we also establish some lemmata that characterize the properties of $R_N^H(\omega_N^H)$ and the optimal solution ω_N^H that will be used throughout the proof. Then we verify items (i) – (iii) of Proposition 7.

(1)

To prove the uniqueness of the North's optimal IPR-protection level, we start by establishing the following lemma.

Lemma 13

$R_N^H(\omega_N^H)$ is a strictly concave function and strictly declining on \mathbb{R}_+ .

Proof. Consider the function $R_N^H(\omega^H)$ as given by (25). $R_N^H(\omega^H)$ is strictly concave because the second derivative can be written as

$$\frac{\partial^2 R_N^H(\omega^H)}{\partial(\omega^H)^2} = -\frac{\beta Z_3 \lambda_o}{4(1-\beta)^2 \Delta_o \lambda_N} < 0. \quad (84)$$

The first derivative, $\frac{\partial R_N^H(\omega^H)}{\partial \omega^H}$, reads

$$\frac{\partial R_N^H(\omega_N^H)}{\partial \omega_N^H} = \frac{\beta Z_3}{2(1-\beta)} (Z_2 - Z_3) + \frac{\beta Z_3}{2} \left(Z_2 + \frac{\lambda_S}{\lambda_N} Z_3 \right) - \frac{E_H \beta Z_3}{2 \Delta_o \lambda_N (1-\beta)}.$$

As the last fraction is positive for all $\omega^H \in [0, 1]$, $\frac{\partial R_N^H(\omega^H)}{\partial \omega^H} < 0$ on the interval $[0, 1]$ if the first two summands together are negative. This is the case if

$$\frac{\lambda_S}{\lambda_N} < -\frac{Z_2}{Z_3} \frac{2-\beta}{1-\beta} + \frac{1}{1-\beta}. \quad (85)$$

Recall from Lemma 3 that the sufficient condition for a maximum of the government's problem is

$$\frac{\lambda_S}{\lambda_N} < -\frac{2Z_2}{Z_3} + 1.$$

Since $\frac{2-\beta}{1-\beta} > 2$ and $\frac{1}{1-\beta} > 1$ for all $\beta \in (0, 1)$, the second-order condition of the government's problem is stronger than condition (85). That is, all values of $\frac{\lambda_S}{\lambda_N}$ that satisfy the second-order condition (24) will also satisfy (85). This verifies that $R_N^H(\omega^H)$ is strictly declining on the interval $[0, 1]$. As $R_N^H(\omega^H)$ is strictly concave, it must be declining on \mathbb{R}_+ . \square

$R_N^H(\omega^H)$ possesses the following roots.

$$\begin{aligned}\omega_{N1}^H &= \frac{2(1-\beta)}{\beta^2 \lambda_N Z_3^2} \left(\Delta_o \left((1-\beta) \left(Z_2 + \frac{\lambda_S}{\lambda_N} Z_3 \right) + Z_2 - Z_3 \right) + \sqrt{\Delta_o Q_3(\Delta_o)} \right), \\ \omega_{N2}^H &= \frac{2(1-\beta)}{\beta^2 \lambda_N Z_3^2} \left(\Delta_o \left((1-\beta) \left(Z_2 + \frac{\lambda_S}{\lambda_N} Z_3 \right) + Z_2 - Z_3 \right) - \sqrt{\Delta_o Q_3(\Delta_o)} \right),\end{aligned}\quad (86)$$

where $Q_3(\Delta_o) = \beta^2 \frac{\lambda_o}{\lambda_N} Z_1 Z_3^2 + \Delta_o (2\beta \frac{\lambda_o}{\lambda_N} (Z_2 + \frac{\lambda_S}{\lambda_N} Z_3) Z_3 + ((1-\beta)(Z_2 + \frac{\lambda_S}{\lambda_N} Z_3) + Z_2 - Z_3)^2)$. Since only real roots may possess economic meaning in our context, we restrict ourselves to the case where $Q_3(\Delta_o) > 0$. Then it follows that only ω_{N1}^H may assume positive values while ω_{N2}^H is always negative. Consequently, there is a unique economically sensible solution $\omega_N^H \equiv \omega_{N1}^H$.

The North's desired harmonized IPR-level ω_N^H possesses the following properties.

Lemma 14

- (a) ω_N^H is a strictly concave function in Δ_o .
- (b) $\lim_{\Delta_o \rightarrow 0} \omega_N^H = 0$.
- (c) $\lim_{\Delta_o \rightarrow 0} \frac{\partial \omega_N^H}{\partial \Delta_o} = 0$.

Proof. (a) The second derivative of ω_N^H with respect to Δ_o writes

$$\frac{\partial^2 \omega_N^H}{\partial \Delta_o^2} = -\frac{(1-\beta)\beta^2 \frac{\lambda_o}{\lambda_N} Z_1^2 Z_3^2}{2(\Delta_o Q_3(\Delta_o))^{\frac{3}{2}}} < 0.$$

This verifies the concavity of ω_N^H .

Item (b) can be observed directly in equation (86).

(c) The derivative of ω_N^H with respect to Δ_o can be written as

$$\frac{\partial \omega_N^H}{\partial \Delta_o} = \frac{2(1-\beta)}{\beta^2 \frac{\lambda_o}{\lambda_N} Z_3^2} \left[(1-\beta) \left(Z_2 + \frac{\lambda_S}{\lambda_N} Z_3 \right) + Z_2 - Z_3 + \frac{Q_4(\Delta_o)}{\sqrt{\Delta_o Q_3(\Delta_o)}} \right],$$

where $Q_4(\Delta_o) = \beta^2 \frac{\lambda_o}{\lambda_N} Z_1 Z_3^2 - 2\Delta_o(1 - \beta - \beta \frac{\lambda_o}{\lambda_N})(Z_2 + \frac{\lambda_S}{\lambda_N} Z_3) + Z_2 Z_3 + Z_3^2$. Since $\lim_{\Delta_o \rightarrow 0} \Delta_o Q_3(\Delta_o) = 0$, it depends on the sign of $\lim_{\Delta_o \rightarrow 0} Q_4(\Delta_o)$ whether the limit of $\frac{\partial \omega_N^H}{\partial \Delta_o}$ at $\Delta_o = 0$ will be plus or minus infinity. We obtain (iii) as $\lim_{\Delta_o \rightarrow 0} Q_4(\Delta_o) > 0$. \square

(2)

Now we can show items (i) and (ii) of Proposition 7. Using the implicit function theorem, we have

$$\frac{d\omega_N^H}{d\Delta_o} = - \frac{\frac{\partial R_N^H(\omega_N^H)}{\partial \Delta_o}}{\frac{\partial R_N^H(\omega_N^H)}{\partial \omega_N^H}}.$$

$\frac{\partial R_N^H(\omega_N^H)}{\partial \omega_N^H}$ is negative according to Lemma 13. Consequently, the sign of $\frac{d\omega_N^H}{d\Delta_o}$ is equal to that of $\frac{\partial R_N^H(\omega_N^H)}{\partial \Delta_o}$. We obtain

$$\frac{\partial R_N^H(\omega_N^H)}{\partial \Delta_o} = Z_2 + \frac{\lambda_S}{\lambda_N} Z_3 + \frac{\beta^3 Z_3^3 (\omega_N^H)^2 \lambda_o}{8(1 - \beta)^2 \Delta_o^2 \lambda_N}.$$

Hence, $\frac{d\omega_N^H}{d\Delta} < 0$ if and only if

$$Z_2 \lambda_N + Z_3 \lambda_S < - \frac{\beta^3 Z_3^3 (\omega_N^H)^2 \lambda_o}{8(1 - \beta)^2 \Delta_o^2 \lambda_N}. \quad (87)$$

The right hand side of (87) is clearly negative. Thus if $Z_2 \lambda_N + Z_3 \lambda_S > 0$, which is equivalent to $\frac{\lambda_S}{\lambda_N} > -\frac{Z_2}{Z_3}$, condition (87) is not satisfied. In this case, we obtain $\frac{d\omega_N^H}{d\Delta_o} > 0$. This proves (i).

With respect to (ii), suppose that $\frac{\lambda_S}{\lambda_N} < -\frac{Z_2}{Z_3}$. Then (87) defines a critical value of IPR-protection ω_N^c , for which $\frac{d\omega_N^H}{d\Delta_o} > (<) 0$ if and only if $\omega_N^H > (<) \omega_N^c$. The critical value is

$$\omega_N^c = \frac{2\Delta_o(1 - \beta)}{\beta Z_3} \sqrt{-\frac{2(Z_2 + \frac{\lambda_S}{\lambda_N} Z_3) \lambda_o}{\beta Z_3 \lambda_N}} \quad (88)$$

Equation (88) reveals that ω_N^c is a linear function of Δ_o with a positive finite slope and $\lim_{\Delta_o \rightarrow 0} \omega_N^c = 0$. Together with the properties of ω_N^H as given in Lemma 14, we can conclude that there exists a unique $\tilde{\Delta}_o > 0$ such that $\frac{d\omega_N^H}{d\Delta_o} > 0$ for all $\Delta_o \in (0, \tilde{\Delta}_o)$ and $\frac{d\omega_N^H}{d\Delta_o} < 0$ for all $\Delta_o \in (\tilde{\Delta}_o, \infty)$. This verifies claim (ii).

Now we turn to (iii). According to the implicit function theorem, we can write

$$\frac{d\omega_N^H}{d\frac{\lambda_N}{\lambda_S}} = - \frac{\frac{\partial R_N^H(\omega_N^H)}{\partial \lambda_N}}{\frac{\partial R_N^H(\omega_N^H)}{\partial \omega_N^H}}.$$

Since $\frac{\partial R_N^H(\omega_N^H)}{\partial \omega_N^H} < 0$ (Lemma 13), the sign of $\frac{d\omega_N^H}{d\lambda_N}$ is equal to the sign of $\frac{\partial R_N^H(\omega_N^H)}{\partial \lambda_N}$ for given λ_o . The latter derivative can be written as

$$\frac{\partial R_N^H(\omega_N^H)}{\partial \lambda_N} = \left(1 + \frac{\beta Z_3 \omega_N^H}{2(1-\beta)\Delta_o}\right) (Z_2 - Z_3) + \beta \frac{Z_3}{2\Delta_o} (Z_1 + \omega_N^H(Z_2 - Z_3)). \quad (89)$$

Again, $\frac{\partial R_N^H(\omega_N^H)}{\partial \lambda_N} = 0$ defines a critical value of IPR-protection, ω_N^{\prime} , such that $\frac{\partial R_N^H(\omega_N^H)}{\partial \lambda_N} > (<) 0$ if and only if $\omega_N^H < (>) \omega_N^{\prime}$. Note that (89) is identical to the first-order condition to the South's maximization problem. Hence, $\bar{\Delta}_o \omega_N^{\prime}$ is identical to ω_S^H . Further, we define $\underline{\Delta}_o$ as the level of research productivity relative to total effective market size where $\omega_S^H = 0$. $\underline{\Delta}_o$ can be expressed as

$$\underline{\Delta}_o = -\frac{\beta Z_3 Z_1}{2(Z_2 - Z_3)} > 0.$$

Since ω_S^H is declining with Δ_o (see (21)), $\underline{\Delta}_o > 0$ implies that $\omega_S^H > 0$ at $\Delta_o = 0$. It follows that $\omega_N^H < \omega_S^H$ for small values of Δ_o according to the properties of ω_N^H as described in Lemma 14. Since ω_N^H is strictly concave in Δ_o , a necessary and sufficient condition for the existence of a $\bar{\Delta}_o$ such that $\omega_N^H < (>) \omega_S^H$ for all $\Delta_o < (>) \bar{\Delta}_o$ is that $\omega_N^H > \omega_S^H (= 0)$ at $\underline{\Delta}_o$.

By a similar line of argument as in the proof of Lemma 7, we can infer from Lemma 13 that $\omega_N^H > 0$ at $\underline{\Delta}_o$ if and only if $R_N^H(0) > 0$ given $\Delta_o = \underline{\Delta}_o$. The latter can be written as

$$R_N^H(0)|_{\Delta_o=\underline{\Delta}_o} = -\frac{\beta Z_1 Z_3^2}{2(Z_2 - Z_3)} \frac{\lambda_o}{\lambda_N} > 0.$$

This verifies that ω_N^H and ω_S^H possess exactly one intersection where $\omega_N^H, \omega_S^H > 0$. We denote the value of Δ_o at this intersection by $\bar{\Delta}_o$. It now follows directly that $\frac{d\omega_N^H}{d\lambda_N} < (>) 0$ for all $\Delta_o > (<) \bar{\Delta}_o$. \square

C.11 Proof of Proposition 8

The proof of Proposition 8 follows directly from the last part of the proof of Proposition 7, where we have shown that ω_N^H and ω_S^H possess a unique intersection where both ω_N^H and ω_S^H are greater than zero. $\bar{\Delta}_o$ is the level of Δ_o at this intersection. It follows further from the proof of Proposition 7 that $\omega_N^H < \omega_S^H$ if $\Delta_o < \bar{\Delta}_o$ and $\omega_N^H > \omega_S^H$ if $\Delta_o > \bar{\Delta}_o$. \square

C.12 Proof of Proposition 9

In order to verify the existence of a unique intersection between ω^p, ω_n^h and ω_s^h , we set $\omega^p = \omega_s^h$, with ω_s^h given from (21), and obtain from the first-order condition of the planer solution one economically meaningful $\bar{\Delta}_o$, such that

$$\bar{\Delta}_o = \frac{1}{2} \frac{\beta \bar{Y} \bar{P} [\beta(2 - \beta) - 2 + \sqrt{\Phi}]}{(3\beta - 4)(\bar{D} - \bar{P})}, \quad (90)$$

with $\Phi = \beta[\beta(5 - 4\beta + \beta^2) - 4] + 4$.

Setting now $\omega_n^h = \omega_s^h$ and $\Delta_o = \bar{\Delta}_o$, the first-order condition of the harmonization scenario in the North is given by

$$\begin{aligned} R_n^h(\omega_n^h = \omega_s^h)|_{\bar{\Delta}_o} &= \frac{1}{2} \beta^2 \bar{Y} \bar{P}^2 \left[-(\lambda_n + \lambda_s) \beta^3 + \beta^3 \lambda_o + (\lambda_n + \lambda_s) 5\beta^2 - 5\beta^2 \lambda_o \right. \\ &\quad + (\lambda_n + \lambda_s) \sqrt{\Phi_2} - \beta \lambda_o \sqrt{\Phi_2} - (\lambda_n + \lambda_s) 11\beta + \lambda_o 11\beta \\ &\quad \left. + 10(\lambda_n + \lambda_s) - 10\lambda_o - (\lambda_n + \lambda_s) 3\sqrt{\Phi_2} + 3\lambda_o \sqrt{\Phi_2} \right] \\ &\quad \times \left[\lambda_n \left[\beta^2 + (2 - \beta) - \sqrt{\Phi_2} \right] (\beta - 2)(3\beta - 4)(\bar{D} - \bar{Y}) \right]^{-1}, \quad (91) \end{aligned}$$

with $\Phi_2 = \sqrt{(\beta^2 + 1)(\beta - 2)^2}$.

Obviously $R_n^h(\omega_n^h = \omega_s^h)|_{\bar{\Delta}_o} = 0$, since $\lambda_n + \lambda_s = \lambda_o$ which verifies Item (i) of the proposition.

Item (ii) follows from the fact that the first-order condition of the North in the harmonization scenario corresponds to the planer solution if the effective market size of the North converges to the total effective world market size, i.e. $\lambda_n \rightarrow \lambda_o$

$$\begin{aligned} \lim_{\lambda_n \rightarrow \lambda_o} R_n^h &= \bar{D} \Delta_o + \frac{\beta}{1 - \beta} \frac{\bar{P} \omega_n^h}{2} (\bar{D} - \bar{P}) \\ &\quad + \frac{\beta \bar{P}}{2} \left[\bar{Y} + \omega_n^h \bar{D} - \left(\frac{\beta}{1 - \beta} \right)^2 \frac{\bar{P}^2 (\omega_n^h)^2}{4 \Delta_o} \right] = R^p \quad (92) \end{aligned}$$

and Item (iii) of Proposition 7, i.e. $\frac{\partial \omega_n^h}{\partial \lambda_s} \geq 0$ for $\Delta_o \leq \bar{\Delta}_o$ which implies for $\lambda_n < \lambda$ and $\lambda_s > 0$ that $\omega_n^h \leq \omega^p$, if $\Delta_o \leq \bar{\Delta}_o$. The remainder follows from Proposition 8.

For the proof of Item (iii) notice the following facts

1. $\lambda_s \rightarrow 0$ and $\lambda_n \rightarrow \lambda$ imply that the first-order condition of the North in the decentralized protection game, R_n^n , converges to the efficient solution R^p , i.e. $R_n^n = R^p$. Since moreover, $\frac{\partial \omega_n^n}{\partial \lambda_s} > 0$, it follows that $\omega_n^n < \omega^p$.
2. As $\Delta_o > \bar{\Delta}_o$ implies $\omega_n^h > \omega^p$, we know that: $\omega_n^h > \omega^p > \omega_n^n$ for $\Delta_o > \bar{\Delta}_o$.

3. Similarly, $\Delta_o = \bar{\Delta}_o$ implies $\omega_n^h = \omega^p$. Hence, $\omega_n^h > \omega_n^n$ for $\Delta_o = \bar{\Delta}_o$.
4. $\Delta_o \rightarrow 0$ implies $\omega_n^h = \omega^p = 0$ and $\omega_n^e = \frac{\bar{Y}\lambda_s}{\lambda_n(D-P)} < 0$. As $\frac{\partial \omega_n^n}{\partial \Delta_o} > 0$, at least for $\Delta_o \in [0, \Delta_o^{crit})$, it follows that $\bar{\omega}_n^e = 0$ for $\Delta_o \in [0, \underline{\Delta}_o^s]$. Consequently, $\omega_n^n < \omega_n^h < \omega^p$ for $\Delta_o \in (0, \hat{\Delta}_o]$.
5. Since ω_n^n is strictly concave in Δ_o , it follows by continuity that $\omega_n^n < \omega_n^h < \omega^p$ for $\Delta_o \in (\underline{\Delta}_o^s, \bar{\Delta}_o)$.

C.13 Proof of Proposition 14

In order to verify that $\frac{d\omega_j}{d\omega_k} < 0$, we will use the implicit function theorem. First we show that $\frac{\partial R_j^s(\omega_j, \omega_k)}{\partial \omega_j} < 0$ and thereafter that $\frac{\partial R_j(\omega_j, \omega_k)}{\partial \omega_k} < 0$.

$$(1) \frac{\partial R_j^s(\omega_j, \omega_k)}{\partial \omega_j} < 0.$$

The partial derivative of $R_j^s(\omega_j, \omega_k)$ with respect to ω_j is

$$\begin{aligned} \frac{\partial R_j^s(\omega_j, \omega_k)}{\partial \omega_j} &= \frac{Z_2 - Z_3}{2} \left(\frac{1}{\delta_N} + \frac{1}{\delta_S} \right) \frac{\beta}{1 - \beta} \lambda_j Z_3 \\ &\quad + \frac{\beta Z_3}{2\delta_j} \left[\lambda_j Z_2 - \frac{E_O}{2\delta_j} \frac{\beta}{1 - \beta} \lambda_j Z_3 \right] \\ &\quad + \frac{\beta Z_3}{2\delta_k} \left[\lambda_j (Z_2 - Z_3) - \frac{E_O}{2\delta_j} \frac{\beta}{1 - \beta} \lambda_j Z_3 \right] < 0. \end{aligned}$$

Since $Z_2 < 0$ we obtain $\frac{\partial R_j(\omega_j, \omega_k)}{\partial \omega_j} < 0$.

$$(2) \frac{\partial R_j(\omega_j, \omega_k)}{\partial \omega_k} < 0.$$

Taking the partial derivative of $R_j(\omega_j, \omega_k)$ with respect to the level of IPR-protection in country k yields

$$\begin{aligned} \frac{\partial R_j^s(\omega_j, \omega_k)}{\partial \omega_k} &= \frac{Z_2 - Z_3}{2} \left(\frac{1}{\delta_N} + \frac{1}{\delta_S} \right) \frac{\beta}{1 - \beta} \lambda_k Z_3 \\ &\quad + \frac{\beta Z_3}{2\delta_j} \left(\lambda_k Z_3 - \frac{E_O}{2\delta_j} \frac{\beta}{1 - \beta} \lambda_k Z_3 \right) - \frac{\beta Z_3^2 \lambda_k}{2\delta_k} \frac{\beta}{1 - \beta} \frac{E_O}{2\delta_j} \end{aligned}$$

This derivative can be rewritten in the following way:

$$\begin{aligned} \frac{\partial R_j^s(\omega_j, \omega_k)}{\partial \omega_k} &= \frac{Z_2}{2} \left(\frac{1}{\delta_N} + \frac{1}{\delta_S} \right) \frac{\beta}{1 - \beta} \lambda_k Z_3 + \frac{\beta Z_3^2 \lambda_k}{2\delta_j} \left(1 - \frac{1}{1 - \beta} \right) - \frac{\beta Z_3^2 \lambda_k}{2\delta_k (1 - \beta)} \\ &\quad - \frac{\beta Z_3^2 \lambda_k E_O}{4\delta_j} \frac{\beta}{1 - \beta} \left(\frac{1}{\delta_N} + \frac{1}{\delta_S} \right) < 0 \end{aligned}$$

As $\frac{1}{1-\beta} > 1$, we obtain $\frac{\partial R_j^s(\omega_j, \omega_k)}{\partial \omega_k} < 0$. Hence, IPR-protection in country j is a strategic substitute to IPR-enforcement in country k . As this result is general – i.e., does not depend on any of the parameters that may differ in the two countries – by symmetry, IPR-protection in country k is also a strategic substitute to that of country j . \square

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