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How Trade Unions Increase Welfare

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Historically, worker movements have played a crucial role in making workplaces safer. Firms traditionally oppose better health standards. According to our interpretation, workplace safety is costly for firms but increases the average health of workers and thereby the aggregate labour supply. A laissez-faire approach in which firms set safety standards is suboptimal as workers are not fully informed of health risks associated with jobs. Safety standards set by better-informed trade unions are output and welfare increasing.

JEL codes: J 51, J 81

Keywords: occupational health and safety, trade unions, welfare

1 Introduction

The process of economic development and growth is a process of an endless introduction of new technologies. This is especially true for the early stages of the Industrial Revolution but also applies today. When new technologies are introduced, their properties are not always well understood. While a technology might promise that a certain good is provided very efficiently, the same technology could also have side effects that did not occur to the inventor. The history of the introduction of new technologies is full of countless examples.

Since as early as the Roman Empire, coal has been used as a source of energy. Systematic coal mining, however, was not carried out until the Industrial Revolution, when a massive
and steady supply of energy was required. Coal seemed like the perfect solution. Mining, however, has its side effects. In 1831, a potential causal link between working in a coal mine and black lung disease was first reported by a Scottish physician. Nowadays, black lung disease is accepted as a disease caused by the repeated and year-long inhalation of small amounts of coal dust. However, it took more than 130 years for this link to be generally accepted. Only in the 1960s, after extensive political activities by various worker groups in Pennsylvania, Ohio, and West Virginia on the Appalachian coal fields, was black lung disease recognized as an occupational disease. As a consequence, the Coal Mine Health and Safety Act was passed in 1969 which established more comprehensive rules for work conditions and also the compensation of disabled mine workers (Smith, 1987).

There is an abundance of further examples of worker movements improving health and safety conditions, including “brown lung” disease caused by exposure to cotton dust (Botsch, 1993), “white lung” disease caused, inter alia, by mining and the exposure to asbestos (Rosner and Markowitz, 1991), the health risk posed by radium (Clark, 1997), workplace exposure to dibromochloropropane, a pesticide that makes workers sterile and is linked to the risk of cancer (Robinson, 1991), the spray machine conflict in the early 1900s (Frounfelker, 2006) or conflicts in the pottery industry (Stern, 2003) and in the automobile and steel industries (Bacow, 1980, ch. 5). For an overview of the literature on the history of occupational health and safety (OHS), see Judkins (1986, p. 240). A more general history of labor standards with international comparisons is covered by Engerman (2003).

A reading of these analyses shows that the side effects caused by new ways of production reveal themselves only gradually. While there might be uncertainty about health implications of a certain job, there is initially often simply ignorance about health implications, sometimes just absence of any doubt. When workers then start sensing that “something is going wrong”, that work conditions are causing health problems, these claims are often met with doubt, not only by employers, but also by insurance companies or even the government. These analyses also clearly demonstrate that worker movements, joint collective actions by individuals, are required to raise political awareness, to lobby for changes in work conditions and to eventually bring about regulatory changes towards better OHS measures.

Similar conclusions about the importance of worker movements for triggering broader support not only for the improvement of working conditions but also for the development of the modern welfare state can be drawn when looking at Germany. During the Industrial Revolution around 1850, the issues of poverty, working and living conditions of dependent workers caused organizations to be created enabling workers to express their own interests (see e.g. Schneider, 2005, p. 15). While poverty and dependent work also existed in pre-industrial times, the contemporaneous rise of the wealthiness of some and the poverty of others was no longer accepted as “the will of God”. The first trade union in Germany, founded in June 1848 by type setters, was set up with an aim to secure the living standards of type setters, who feared competition from the steam engine and technological progress (hence, there was income orientation), but also with an aim to establish mutual health and disability insurance systems.
(Schneider, 2005, p. 27). The worker movement, represented by unions and political parties, was also incited by occupational injuries which almost caused “mass causalities” (Tennstedt et al. 1993, p. XXI), partly due to the widespread use of new technologies and fast economic growth. These movements and associated political pressure caused Bismarck, the German chancellor, to implement, inter alia, statutory accident insurance in 1884.

The outcome of this discussion about historical episodes in advanced OECD countries is threefold: (i) A safe workplace, in short OHS, does not come for free: Achievements of the modern welfare state, which today are taken for granted, were hotly disputed in the past. (ii) There is a conflict of interest between unions and firms, which goes beyond pure wage bill issues. In many cases, industry, insurance companies and often also the government initially object to any demands for compensation or changes in health standards simply because there is no clear scientific medical evidence for the claimed nexus between certain symptoms and the professional activity. (iii) Unions\(^2\) played a crucial role in pushing for OHS standards and prepared and fought for what is (almost generally) accepted today as a positive aspect of modern welfare states (see e.g. Brugiavini et al. (2001, ch. II.2.1), Agell (1999, p. F144) and the discussion below). Only once workers succeed in forming large groups and in lobbying for their joint interests is there enough political visibility in order for changes in OHS regulations to take place. To put it briefly, in the spirit of Freeman and Medoff’s (1984) “collective voice”: Trade unions have a “good face” as well.

The purpose of this paper is to understand why it took worker movements (rather than the government or employers) to start the development of insurance mechanisms. Why did worker movements eventually lead to the creation of government agencies which regulate OHS nowadays and what are the determinants of endogenous OHS standards?

We shall construct a model which highlights the key ingredients for understanding the importance of worker movements in the past. Jobs have two effects on workers - they provide income and they affect health. In order to keep the analysis as simple as possible, we will assume that workers are entirely ignorant about the health implications of jobs: job choice is based purely on the wage paid by the employer. Returning to the coal miner example from above, workers were simply not aware of the potential risk of black lung disease.\(^3\) We consider an economy with one homogenous good and assume perfect competition on goods and labour markets, implying, inter alia, full employment. Unions do not cause unemployment in our setup. Given the absence of any information on the health risk of working, the production process exerts a negative externality on workers’ health. OHS standards can in principle reduce this negative externality but they also reduce the total factor productivity (TFP) of

\(^{2}\)We will often use ‘union’ as name for more informal worker groups, worker movements or worker associations. Union, as used here, does not necessarily describe a well-organized and at times bureaucratic institution as is nowadays the case in some OECD countries.

\(^{3}\)We see this complete ignorance as a short-cut to a Bayesian learning setup where workers form a prior about health implications and it takes time to learn the true health consequences of a job. See Viscusi (1979, 1980) for various applications of Bayesian learning to uncertainty about health implications of jobs.
firms, reflecting the fact that OHS is costly. As long as health effects of working are disputed, no employer or government would concede better working conditions.

The role of worker movements is to provide and confirm information about the health effects of working. An individual worker does not have enough time and makes too few observations to discern job-related health effects from other health effects. A group of workers, a union, has many members and thereby more observations. Learning is much faster and unions can thereby help internalize the externality.

In standard trade union models, the objective of trade unions is to maximize the wage income of their members. We extend this arguably narrow perspective and portray trade unions as having both high wage income and good health standards as their objective. We then find determinants of OHS standards by letting unions set OHS standards. This monopoly view of OHS-setting unions and employment-setting firms is - as in wage-setting models of unions - a short-cut to a more complete setup with endogenous union membership where workers form groups to increase the speed of learning.

Some of our findings are as follows: Each firm individually is opposed to higher OHS standards as they reduce TFP and thereby profits. Unlike compensating differentials setups with complete information, competitive markets here are unable to take health effects caused by technologies into account: individuals can not judge with sufficiently high precision to what extent a certain job will affect the health. The laissez-faire factor allocation is characterized by inefficiently high sickness leaves. If better-informed firm-level trade unions set OHS standards, the positive effect on the improved health of their members balances the negative effect of lower employment due to lower TFP. If there are economy-wide or occupational unions, OHS standards are more comprehensive as unions also take the negative health effect on overall labour supply into account. If unions are not too extreme in their health preferences, higher OHS standards than those favoured by firms increase economy-wide output and increase welfare. The presence of unions is welfare-increasing.

Capital owners favour higher OHS standards than individual firms. Capital owners see that an economy-wide improvement in health increases labour supply and thereby returns to capital owners - as long as the positive health effect is not overcompensated by the negative TFP effect. Capital owners might even favour higher OHS standards than firm-level unions. However, capital owners could never be at the origin of improving work standards as they simply do not feel (in the literal sense of the word) health effects. They have no incentive to form “capitalists movements” as bad working conditions do not affect them. When we compare capital owners to economy-wide unions, unions desire higher OHS standards as they value health per se (capital does not become sick but workers do). Hence, both at the firm level and at the economy-wide level, there is conflict of interest between unions on the one hand and firms and capital owners, respectively, on the other. But for a certain range of OHS,

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4Capital owners here and in what follows denote a federation which represents the joint interest of capital owners in an economy. Individuals looking only at capital income in one specific firm would never agree on higher OHS standards.
unions and capital owners agree on increasing OHS standards. This explains why after some initial historical dispute and controversies over OHS standards - most OHS standards in OECD countries are no longer hotly disputed today.

2 Related literature

Our paper is related to various strands of literature. First, there is obviously a huge literature on trade unions, and it would be impossible to provide a summary here which does any justice to the various substrands. While it seems fair to argue that most contributions attribute a distorting (efficiency-reducing) role to unions\(^5\), there are also some economists that find positive aspects in union behaviour: Brugiavini et al. (2001, ch. II.2.1) see unions as the precursor to the modern welfare state. They write on p. 163 that “unions developed mutual insurance as part of associational self-help to compensate for the lack of private insurance or public social protection. At the same time, they mobilized [...] for the expansion of social rights. Increasingly, many of the protective functions that unions provided [...] came to be taken over by the state”.\(^6\) A by now well-accepted argument was made by Freeman and Medoff (1984): By providing a “collective voice”, unions provide information which otherwise would not be available. Malcomson (1983) argues that unions increase efficiency as they improve the allocation of risk-bearing between firms and workers. Acemoglu et al. (2001) argue that unions induce training and provide insurance and Boeri and Burda (2009) show that workers prefer collective bargaining in the presence of market imperfections. Booth and Chatterji (1998) and Viscusi (1979, ch. 11) show how trade union bargaining with monopsonistic firms increases social welfare and Agell (1999, p. F144), more generally, argues that “certain institutions may serve quite useful purposes” in the labour market. We put forward OHS standards as an example of such a useful institution. We believe that this beneficial historical aspect of worker movements for what are now modern societies and the role unions can play in developing countries today has not received sufficient credit so far. Our contribution lies in the emphasis and analysis, in the framework of a very simple model, of the informational and learning advantage of a union in a world with incomplete information and side effects caused by new technologies.

Second, and maybe most importantly, our view of multi-feature workplaces is related to but differs starkly from the equalizing differences approach of Rosen (1974, 1986). Equalizing differences are traditionally derived in setups with perfect information. When workers know

\(^5\)Distortions can have their positive sides in second-best worlds or when it comes to collecting rents. See Mezzetti and Dinopoulos (1991) for an example with an employment-oriented union in an international trade setup with imperfect competition.

\(^6\)Historical evidence linking union growth to their provision of insurance (strikes, unemployment, sickness, burial cost) for the Netherlands and Britain is provided by van Leeuwen (1997). Quantitative evidence for the United States for union decline due to an expanding welfare state is provided by Neumann and Rissman (1984).
about all job characteristics and all markets are competitive, factor allocation is efficient and any institution would be distorting. Given the historical situation and technological examples we have in mind, workers having perfect information does not appear to be a realistic assumption. We therefore choose the other extreme and assume that workers are unable to learn anything about work-related health implications. While the reality certainly lies somewhere inbetween, the justification for our assumption is simple: When new technologies become available, workers and often society as a whole do not know a lot about potential side effects. Health implications may only become apparent over the long-term and workers might simply not have the time to learn about these implications. Hence, with regard to learning processes which take a very long time, we assume right away that it is impossible for the individual workers to learn of health effects. As a consequence, a decentralized factor allocation is inefficient. In contrast, trade unions consisting of a large number of workers have access to many observations about jobs, can collect this information and can therefore learn more easily. In fact, we assume that unions have perfect information and can therefore internalize externalities, increase efficiency, output and welfare.

Finally, the rapidly growing literature on child labour touches upon some aspects covered here. For example, Doepke and Zilibotti (2005) analyse how attitudes towards child labour regulation can change over time. Baland and Robinson (2000) derive determinants of child labour and generally find that child labour is inefficient. In contrast, Krueger and Donohue (2005) find that a child-labour ban is not necessarily welfare-increasing. To the extent that child labour is bad for the health and safety of children, our analysis implicitly studies the effects of trade unions on child labour. In fact, Doepke and Zilibotti (2005, p. 1494) mention that the “trade union movement played a key role in lobbying for the introduction of child labour regulation”. Baland and Robinson (2000, footnote 17) make a similar point. This literature, however, does not focus on unions as an institution as we do here and does not attempt to work out the potentially beneficial effects unions and their use of their market power can have. A companion paper (Donado and Wälde, 2010) qualitatively and quantitatively studies the effect of globalization for labour standards in the North and in the South in the presence of unions as portrayed here.

3 The model

Our economy produces a homogenous good. Aggregate output amounts to $Y$. A typical firm produces the quantity $y$ by employing capital $k$ and labour $l$, the latter of which is measured in working hours. All firms use the same technology with TFP $A(s)$,

$$ y = A(s) f(k, l), $$

where capital and labour inputs have the usual neoclassical effects on output. Given our historical perspective on what are now OECD countries or our focus on developing countries
today, we assume that firms can hire from a spot market. There are no hiring or firing costs and it does not take any time to find a worker.

The central focus of this paper is OHS. This aspect is reflected in the production process in the TFP component $A(s)$. TFP in a firm or in a country is influenced by many factors, starting from very technology-specific aspects (like the age distribution of the capital stock or the management and communication skills of staff) and ranging to more economy-wide influences (like the institutional stability, the political regime, or the education level of workers). The more important factor influencing TFP for our arguments is OHS $s$. A job is safe(r) if a worker is (more) certain to return home in good health after 8 (or more) hours of work. We reflect safer jobs by a higher $s > 0$.

Safe workplaces are clearly in the interest of the worker, and in many cases, OHS is also a central concern for employers. If safety measures increase the smoothness of a production process, employers should be in favour of high safety standards. An accident in a coal mine, costing not only lives of workers but also letting the production process break down for weeks, is clearly not in the interest of the firm. In many cases, however, there is a fundamental conflict of interest. In the case of low-skill workers or workers needing only general (i.e. not firm-specific) human capital to perform their job and in countries where firms do not (have to) pay sickness-leave (i.e. whenever firms can easily replace their workers), firms have no economic interest in the state of health of their workers. Quite to the contrary, OHS measures are costly. A workplace where coal miners are well protected against black lung disease or ore miners against silicosis is more costly than one without protection measures like ventilation systems. A worker who spends half an hour dressing and undressing (helmets, safety glasses, gloves, entire suits etc.) is less productive than a worker who starts doing his job right away.

What matters for our results is that workers value safety more than firms. For modelling purposes, we go to the extreme and exclude firms from any benefits from higher safety. We capture safety costs by letting OHS measures reduce TFP, $A_s < 0$. Given the spot market assumption, a sick worker would simply be replaced by a new healthy worker.

An individual values consumption $c$ and health $z$ and both are determined by the job an individual chooses. A job is therefore a differentiated good as in Rosen (1974). Let $z(s, m)$ denote the share of potential working hours that an individual is healthy and can work. Currie and Madrian (1999) summarize the literature on health and labour markets. They document a positive relationship between health and income with health having a larger effect on hours than on wages. While it is true that the link between health and labour market participation is less clear-cut (Currie and Madrian stress that this could be due to an abundance of methodological problems), in the following we feel safe to assume that longer working hours $m$ under bad OHS standards are bad for health, $z_m < 0$, but safety measures

\footnote{This is the standard assumption in the literature on compensating differentials, see e.g. Rosen (1986). If $A$ increased in $s$, no uncertain jobs would ever be observed. One can always imagine that $A$ initially increases in $s$ but decreases above some threshold level. It could be that low $s$ reduces labour productivity rather than TFP. For simplicity, we will continue to use the term TFP.}
s improve health, $z_s > 0$ (subscripts denote partial derivatives). Utility of workers increases in consumption $c$ and health $z(s, m)$ but with a decreasing slope, $u_c > 0$, $u_{cc} < 0$ and $u_z > 0$, $u_{zz} < 0$. Letting all individuals work the same number of hours $m$, we can suppress $m$ and use

$$u = u(c, z(s))$$

as utility function. Health is important for two reasons: It matters per se and consumption rises due to longer hours worked. All workers are identical in their preferences.

On the aggregate level, consumption equals output $C = Y$ and labour demand $L$ equals labour supply,

$$L = z(s) N.$$  \hspace{1cm} (3)

The latter is given by potential employment $N$ (also measured in hours and assumed to be fixed) multiplied by the share $z(s)$ of time workers are healthy and can actually work. Improved safety, implying improved health, implies higher labour supply.

We finally turn to trade unions. Depending on the degree of centralization of negotiations and wage setting, literature usually classifies countries in three groups (see e.g. Calmfors and Drifill, 1988): (1) highly decentralized systems with wage setting at the firm level (i.e. USA and Canada), (2) intermediate degree of centralization (most continental European countries), and (3) highly centralized systems with wage setting at the national level (i.e. Nordic countries and Austria). We will also consider different degrees of centralization and model the two polar cases of highly decentralized and highly centralized systems.

In a decentralized setup, unions operate at the firm level and are therefore small in comparison to the economy as a whole. As we view spot markets as the best description of labour markets for activities as described in the introduction, there is no attachment of workers to the firm. Hence, membership of firm-level unions is just as volatile as employment at the firm. As a consequence, the union only cares about the overall well-being of the $l$ workers in this particular firm. As households value consumption and health, we let unions value these quantities as well. Consumption depends on capital and labour income and union members might also have some capital income. Observing union activities, however, we find it more appropriate to model unions as institutions which focus on labour income or the employment situation in general. Unions neglect the capital market position of their members and focus on the wage sum of their members. Given historical examples about union behaviour in now OECD countries and preferences of households in (2), unions also care about a worker’s health and a union’s utility function reads

$$v = v(wl, z(s)),$$  \hspace{1cm} (4)

Labour income $wl$ of union members depends on the market wage $w$ and on labour demand $l$ as chosen by the firm. Depending on the importance attached to each of these two objectives, the union might be called income-oriented or health-oriented.\textsuperscript{8}

\textsuperscript{8}For an introduction to the discussion on the appropriate specification of union preferences, see Oswald
In some countries, unions are large or form a confederation. Their basic objectives are the same but they now represent not only the workers of a particular firm but the whole labour force,

\[ V = V(wL, z(s)), \quad V_{wL} > 0, \quad V_z > 0. \tag{5} \]

The main difference compared to the firm-level union is that health now has two positive channels, as in individual preferences (2): health matters per se and through higher labour supply visible here through \( L \). An alternative to economy-wide unions, also captured by (5), are occupation-specific unions. As long as a union takes the effect of standards on all workers into account (e.g. because a union represents all coal miners and not just those currently employed in one particular firm), beneficial labour supply effects as a result of higher standards are internalized by the union.

4 Centralized and decentralized OHS setting

This section explores the behaviour of a planner and OHS levels in a decentralized economy. This allows us to understand the basic mechanism of why trade unions in principle can have positive welfare and output effects.

4.1 The planner

As all firms use the same technologies, we can simply insert aggregate capital endowment \( K \) into (1). After also inserting the labour-market equilibrium condition (3), total output is given by

\[ Y(s) = A(s) f(K, z(s)N). \tag{6} \]

Welfare comparisons require a social welfare function. With identical preferences and homogeneous firms, all workers will be equally healthy. The only source of heterogeneity of households could be wealth holdings. However, as our static framework is agnostic about wealth distributions, we will work with the assumption of a representative consumer. We can therefore use the individual utility function (2) and obtain a social welfare function by inserting aggregate consumption,

\[ U(s) = U(C(s), z(s)) = U(Y(s), z(s)). \tag{7} \]

A social planner maximizing social welfare (7) chooses a safety level \( s_U \) that satisfies (see app. A.1)

\[ \varepsilon_{UYA} \varepsilon_{AS} = [\varepsilon_{UY} \varepsilon_{YL} + \varepsilon_{Uz}] \varepsilon_{z}, \tag{8} \]

\[ (1982) \text{ and Booth (1995, ch. 4). Note that even for modern Britain, there is evidence that physical working conditions is one important issue over which trade unions and management bargain (Millward et al., 1992, pp. 249-254).} \]
where, for readability, all elasticities throughout this paper are defined as positive quantities. Hence, the OHS elasticity of TFP and the inverse wage elasticity of labour demand require a minus sign in their definition,

\[ \varepsilon_{xg} \equiv -\frac{\partial x}{\partial g} \frac{g}{x}, \text{ for } xg \in \{As, wL\} \quad \text{and} \quad \varepsilon_{ad} \equiv \frac{\partial a}{\partial d} \frac{d}{a}, \text{ for } ad \notin \{As, wL\}. \tag{9} \]

Condition (8) balances the welfare-increasing and welfare-decreasing effects of increased safety. The left-hand side captures the cost of increased safety caused by a lower TFP: A one-percent increase in the safety level reduces the TFP and thereby output by \( \varepsilon_{Y, A} \varepsilon_{As} \) percent. Multiplying this with the output elasticity of welfare, \( \varepsilon_{UY} \), yields the percentage reduction in welfare. For maximum welfare, this negative effect of increased safety has to be equal to the positive effect on the right-hand side. A one-percent increase in safety increases the share of time working by \( \varepsilon_{zs} \) percent. This gives, multiplied by \( \varepsilon_{Uz} \) and by \( \varepsilon_{UY} \varepsilon_{YL} \) respectively, the percentage increase in utility caused by better health and higher income.

If the planner focused only on output maximization (that is, if \( \varepsilon_{Uz} = 0 \)), the optimality condition giving the output-maximizing safety level \( s^Y \) would read

\[ \varepsilon_{Y, A} \varepsilon_{As} = \varepsilon_{Y, L} \varepsilon_{zs}. \tag{10} \]

This condition balances the output-decreasing effect on the left-hand side with the output-increasing effect on the right-hand side. Interestingly, one can prove that for the general production function in (6) the welfare-maximizing safety level is always higher than the output-maximizing safety level, \( s^U > s^Y \).

4.2 The decentralized economy

The standard view to a setup with multiple job characteristics is Rosen’s (1974, 1986) equalizing-differences approach. According to this approach, workers enjoy (or dislike) job characteristics in addition to the wage and a worker’s utility function would look like the one we use in (2). The difference to our approach consists in the criteria for choosing a job. In the equalizing-differences approach, workers have full information about job characteristics and the choice of jobs would depend both on health implications \( z(s) \) and on income leading to a consumption level \( c \). Firms can therefore choose wage-safety pairs on a worker’s indifference curve. The resulting market equilibrium would be efficient.

The crucial difference from our approach lies in our historical perspective of unions in contemporary OECD countries and the conclusions we draw about information. Workers

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9Intuitively, the proof (see app. C.1) runs as follows: Let \( s \) maximize output in (6). Now add health to this objective function and obtain (7). As the health term monotonically increases in \( s \), a somewhat higher health level is better as a marginal increase in health does not reduce output at \( s = s^Y \) but does increase the health term. Hence, \( s^U > s^Y \). Clearly, how much \( s^U \) exceeds \( s^Y \) depends on how strongly health is valued, how strongly health increases and how fast output drops when \( s \) increases.
do not have sufficient information (neither would society as a whole) to perfectly evaluate the impact of work, a certain job or a specific technology on health. Workers could form expectations but their expectations need to be - in the absence of perfect information - based on a prior in a Bayesian learning sense. Perfectly competitive firms taking a safety-wage trade-off into account would then set an inefficient safety level if the prior is not identical to the true distribution of the health impact of a job. When on the job, workers would of course gradually learn about health implications of work, but each single worker makes just a few observations, especially when health also depends on other factors than just work and certain health impacts come with a long delay or can not easily be observed (as the examples in the introduction have shown). There is simply not enough variation; econometrically speaking, there is not a sufficient number of observations to draw firm conclusions and learning can take more than a lifetime. To capture this idea in the simplest way possible, we assume here that workers choose employment based only on the wage and firms choose employment taking the wage rate as given. This will qualitatively imply the same type of inefficiency one would observe in a Bayesian setup (as employed e.g. by Viscusi, 1979, 1980). The advantage of this shortcut is clearly the much simpler analytical tractability.

Given this focus of workers on wages (and capital owners on returns), optimal firm behaviour yields the familiar equality between marginal productivities and factor rewards,

$$w = A(s) f_l(k, l), \quad r = A(s) f_k(k, l).$$  \hfill (11)

In a laissez-faire economy, a firm fixes, in addition to the stock of labour and capital, the safety level $s$. The derivative of profits with respect to the safety level is $d\pi/ds = A_s$, i.e. it is negative. Firms only see the TFP-reducing impact of increased safety. As a consequence, firms would like OHS standards to be as low as possible. The comparison point to the central planner solution $s^U$ or $s^Y$ is a laissez-faire safety level of $s^\pi$. Given that we exclude negative safety levels, we can set $s^\pi$ to zero (or to the level where $A(s)$ starts to fall, see fn. 7). The resulting equilibrium is clearly inefficient.

### 4.3 Capital owners

Given the assumption of a representative consumer discussed before (7), one could wonder why there should ever be a conflict of interest in this economy. We see the representative consumer assumption as a convenient shortcut which allows us to work with a social welfare function (7) that abstracts from the distribution of wealth. We nevertheless look at two types of institutions: trade unions and a federation of capital owners. These institutions represent interests as if their members received only labour income or only capital income.

\footnote{The same would be true for small “entrepreneurs” who invest in their own firm. Someone owning $k$ in a firm and calculating the safety level which maximizes $rk$ would also find that it is optimal to reduce $s$ as much as possible.}
A more “realistic” model would include a distribution of wealth and would thereby justify endogenously conflicting interests. The conclusion one would draw concerning optimal safety levels for capital and labour would be identical, as we now see.

Let us compare the firm safety level to one which would be set by a federation uniting all capital owners in an economy. At the country level, the safety level $s^R$ that maximizes total capital income $r(s)K$ is described by (see app. A.5)

$$\varepsilon_{rA}\varepsilon_{As} = \varepsilon_{rL}\varepsilon_{zs},$$

where again the elasticities are defined as in (9). Here, capital holders do not only consider the TFP-reducing impact (on the left) but also the health-increasing impact (on the right) of increased safety. The reason for this is that interest rates depend on output, and, as we have already seen, output can be increased by improving the workers’ health in a country.

The safety differences between the planner, the firms and capital owners highlights the externality caused by the production process. If the planner focused on TFP only, as does each firm, OHS $s$ would be as low as possible since this increases output (6). A low safety level, however, decreases the share $z(s)$ of time a worker is healthy and can work. This reduces aggregate labour supply $z(s)N$ and therefore output (6). Hence, the starting point of our analysis of the effects of union activity is a second-best world where production exerts a negative externality on health. Output in a laissez-faire economy is inefficiently low and adding an institution - in our case a union - that sets OHS standards can improve efficiency.

## 5 Endogenous OHS with trade unions

The previous section explored the effects of the negative production externality. We will now show that if trade unions are introduced, the distorting effect can be reduced or even eliminated. Why does the union have the knowledge and means required to do so? There are two reasons: First, unions have many members and the more members there are, the easier it is to learn about a job situation. Due to its size, the union can collect information more easily than individuals. Second, in contrast to a loose group of workers that have no institutional connection, unions have the means to “prove” the link between bad work conditions and health. They can monitor the credibility of individual claims about work conditions more easily\(^{11}\) and they also have the power to impose better working conditions. Unions are a means to overcome the information and credibility problem of individual workers (see, for example, Fenn and Ashby (2004, p. 46) and Robinson (1991, pp. 41-7)).\(^{12}\)

\(^{11}\)The importance of unions in alleviating moral hazard problems has already been stressed by Beveridge in 1909 (quote taken from van Leeuwen, 1997, p. 786). Beveridge claims that unions of his time were in the best position to monitor the appropriate use of unemployment benefit payments.

\(^{12}\)Firms can also learn faster than individual workers as a firm hires many workers. Once the firm has learned about negative health effects of a certain technology, however, it might not be in the firm’s interest to reveal this information as workers with health problems that were incurred in the past could then file claims.
We will first analyse the principles of optimal union behaviour in a general setup. We compare the implied safety levels with those optimal for capital owners. This allows us to see under which conditions and to which extent there is a conflict of interest between unions and capital owners. We will then look at various examples (with Cobb-Douglas (CD) and CES production and utility functions) to reveal the precise determinants of welfare gains and potential conflicts of interests. This will show the potential but also the limits of union activity on social welfare. We will consider a decentralized system (firm-level unions) and a centralized system (trade union confederation).

5.1 The general case

- Firm-level unions

In basically all OECD countries, today and in the past, unionised and non-unionised sectors coexist. Union densities change over time and sometimes unionized firms compete with non-unionized firms. Various explanations can be offered for both the coexistence and varying union densities. In a competitive setup à la Rosen with heterogenous firms, one can imagine that firms offering the more dangerous jobs are unionised while others are not. In the theoretical literature on “deunionisation”, Acemoglu et al. (2001) show how biased technological change can be the reason for both deunionisation and an increase in wage inequality. In their setup, workers have an explicit choice whether to unionize or not.

We abstract from these important issues as we want to compare our approach to the canonical model of trade unions. In the traditional monopoly union model (see Dunlop, 1944, Oswald, 1982), unions set the wage, firms choose employment and unemployment is the inefficient equilibrium outcome. We give unions market power as well, assuming that it is beneficial for workers to join a union and that unions succeed in learning about the work-health link better than workers and unions succeed in solving the monitoring problem. This is our highly condensed version of historical processes: Historically, worker movements do not have any market power when they start. Political parties are often the vehicle through which public attention and support increase. If new regulations then improve OHS standards, they are put into force by the government. Indirectly, however, these new regulations are set by worker movements and this is what we capture here. Unions use their market power not to set wages - as in the traditional model - but to set the safety level \( s \). While unions in the real world are concerned with several issues of which wage negotiation is an important one, we focus here entirely on union activities related to improving work conditions as described in the introduction. Wages are perfectly flexible in our setup and there is no unemployment.

\[13\] Giving unions market power allows us to use the elegant monopoly union setup. This should not suggest, however, that we make a second-best world argument where one distortion (the market power of unions) corrects for another distortion (imperfect knowledge). Unions are beneficial even without (or despite) market power as they provide a superior (collective) learning technology in comparison to individualistic learning. Future work could use a Bayesian learning setup where collective information collection alone improves welfare.
At the firm level, employment \( l \) in the union’s objective function (4) is given by the firm’s labour demand from (11) which, through TFP, is a function of the safety level, \( l = l(A(s)) \). The wage rate \( w \) and the firm’s capital stock \( k \) in the labour demand function \( l(\cdot) \) are taken as parametric by the union. The choice of the safety level \( s^v \) is perceived by the union to affect labour demand through TFP and health \( z(s) \). Assuming an interior solution, the first-order condition of maximizing (4) subject to \( l = l(A(s)) \) is given with (9) by (see app. A.3)

\[
\varepsilon_{vwl}\varepsilon_{IA}\varepsilon_{As} = \varepsilon_{wz}\varepsilon_{zs}.
\]

As in the planner’s trade-off, safety has a positive as well as a negative effect here. The negative effect on the left-hand side comes through the reduction in labour demand by the firm as a result of the cost associated with a higher level of safety: A one-percent increase of safety decreases TFP by \( \varepsilon_{As} \) percent and the labour demand by \( \varepsilon_{lA}\varepsilon_{As} \) percent. Multiplying this with \( \varepsilon_{vwl} \) gives the percentage reduction in utility. The positive effect on the right-hand side is the direct effect of improved health on utility: A one-percent increase in the safety level increases health by \( \varepsilon_{zs} \) percent which multiplied by \( \varepsilon_{wz} \) gives the percentage increase in utility.

The differences between the union’s optimal \( s^v \) from (13) and the planner’s \( s^U \) from (8) stem from three sources: First, the union might value health differently than the central planner, i.e. \( v(\cdot) \) might differ from \( U(\cdot) \). In fact, the union might value health more (i.e. \( \varepsilon_{wz} \) might be greater than \( \varepsilon_{Us} \)) since all workers are affected by workplace conditions while not all consumers are, as some consumers might live on capital income only. Second, the union cares about labour income \( wl \) only and not about total consumption \( C \). In other words, capital income of capital owners is not taken into account. Third, maybe most surprisingly, firm-level unions without fixed membership do not take into account the positive effect of an increased level of health on the labour supply and thereby on output, the \( \varepsilon_{UY}\varepsilon_{YL}\varepsilon_{zs} \) term in (8).

- The trade union confederation

The union confederation has the same objectives as the firm-level union even though it represents, not only the workers from a particular firm, but the whole labour force. Consequently, employment in the union confederation’s objective function (5) is economy-wide labour supply \( L = z(s^V)N \) and the wage rate from (11) is the general equilibrium wage level, \( w = w(A(s^V), z(s^V)N) \). The safety level set by the confederation is denoted by \( s^V \). The optimality condition is (see app. A.4), using (9) again,

\[
\varepsilon_{VwL}\varepsilon_{wA}\varepsilon_{As} = \left[ \varepsilon_{VwL}\left[ 1 - \varepsilon_{wL} \right] + \varepsilon_{Vz} \right] \varepsilon_{zs},
\]

The optimality condition (14) again balances the positive and negative effects of a higher safety level. In contrast to the firm-level union, however, the union confederation does take the positive effect of an increased level of health on the labour supply into account,
the $\varepsilon_{wL}[1 - \varepsilon_{wL}]\varepsilon_{zs}$ term. In fact, condition (14) has more in common with the welfare-maximizing condition in (8) than with (13). Comparing (8) and (14) makes it clear that health per se has a similar impact on both conditions, the terms $\varepsilon_{Uz}\varepsilon_{zs}$ and $\varepsilon_{Vz}\varepsilon_{zs}$. However, the main difference resides in the fact that the union confederation is only interested in the workers’ income, $wL$, while the central planner considers the whole income, that is, the income of workers and of capital holders: $Y = wL + rK$.

5.2 An example

While intuitive, the first-order conditions of the planner, the unions or capital owners might not be satisfied. The positive effect of improved health could always be stronger than the negative effect of a lower TFP - or vice versa. The conditions also reveal little about the central determinants of health and safety levels. We therefore now look at a specific example in which a unique optimum can be easily identified and the conflict of interest in our economy can be studied.

- Functional forms

Assume a CES form for utility functions with arguments income and health. The household utility function in (2) and the firm-level union’s objective function in (4) are thus assumed to take the forms

$$u = \left\{ \mu c^\lambda + [1 - \mu] z(s)^\lambda \right\}^{1/\lambda},$$

$$v = \left\{ \gamma [wl]^\lambda + [1 - \gamma] z(s)^\lambda \right\}^{1/\lambda},$$

where $0 < \mu, \gamma < 1$ and $\lambda < 1$. The confederation’s utility in (5) and our example for the central planner’s objective (7) are

$$V = \left\{ \gamma [wz(s)N]^\lambda + [1 - \gamma] z(s)^\lambda \right\}^{1/\lambda},$$

$$U = \left\{ \mu Y(s)^\lambda + [1 - \mu] z(s)^\lambda \right\}^{1/\lambda}.$$

Let there be a CD production function at the firm level and therefore also on aggregate with $0 < \alpha < 1$,

$$y = A(s) k^\alpha l^{1-\alpha},$$

$$Y = A(s) K^\alpha [z(s)N]^{1-\alpha}.$$

Health is captured in all utility functions by $z(s)$ with a weight of $\mu$ for the households and the central planner and a corresponding weight $\gamma$ for unions. Unions might value health differently than “normal” households as all union members are subject to health effects from
working while households also include capital owners who are not exposed to health hazards. Likewise, income at the household or planner level is all income and can therefore be expressed by individual consumption $c$ or aggregate output $Y$. Income taken into account by unions is labour income only, i.e. $wl$ or $wL$. In all cases, the elasticity of substitution between income and health is given by $1/(1 - \lambda)$. For $\lambda \to 0$, the CES functions (15) to (18) become CD functions, e.g. $u = e^{\alpha} z(s)^{1-\mu}$ and $v = [wl]^\gamma z(s)^{1-\gamma}$ for (15) and (16).

Finally, let us choose functional forms for TFP and the share of time being healthy as related to OHS which have the properties discussed after (1) and (3),

$$A(s) = be^{-\phi s}, \quad z(s) = 1 - qe^{-\chi s},$$

(21)

where $b$, $\phi$ and $\chi$ are positive constants. When $s$ is very low, TFP is close to its maximum $b$ and the share of healthy hours is close to its minimum $1 - \bar{q}$. Restricting $\bar{q}$ to take values between zero and one, zero safety measures still imply that workers are on average healthy during $1 - \bar{q}$ percent of the time. The higher $s$ is, the closer TFP is to zero and the higher the average health $z(s)$ is.

- Optimal safety levels

The existence of optimal safety levels follows from computing first-order conditions and checking the sign of the first derivative to the left and right of the optimum in general equilibrium. A general equilibrium perspective has been taken for the maximization procedure by economy-wide institutions (the planner and the nation-wide union). Firm-level unions calculate their optimal safety level based on the firm’s labour demand function. We take these optimality conditions and replace firm variables (like the capital stock $k$) by aggregate variables adopting the standard symmetric equilibrium view with many identical unions.

Table 1 presents first-order conditions for CES utility functions (15) to (18) and corresponding CD results for $\lambda \to 0$, i.e. the safety levels for the welfare-maximizing and the output-maximizing planner and for both types of unions (see app. B.4).

The safety level $s^Y$ in (b) chosen by a planner who maximizes output only (i.e. $\mu = 1$ in (18)) is positive if the term in squared brackets is larger than one, $(1 + (1 - \alpha) \chi/\phi) \bar{q} > 1$. Given that $\bar{q}$ is the share of time spent sick, this expression is larger than one only for a sufficiently small $\alpha$ or $\phi$ or a large $\chi$. A small $\alpha$ implies a high output elasticity of labour. A planner will therefore provide more safety when this has a stronger positive effect on output. When $\phi$ is small, the cost of safety on TFP by (21) is not so strong and a planner will also provide more safety measures. Similarly with $\chi$: More safety measures, again by (21), increases health levels and labour supply strongly and the planner is induced to provide more safety. Let us assume that parameters are such that the planner indeed chooses a positive safety level $s^Y$. 

16
Table 1 Optimal occupational health and safety levels for (19) to (21)

When looking at the signs of the first and second derivatives, one finds that $s^Y$ is indeed an optimum and one obtains an “inverted U” shape for $Y (s)$ from (20) as illustrated in fig. 1. To the right of $s^Y$, the positive effect of an increase in the safety level on health and thereby labour supply overcompensates the negative effect of lower TFP. This reverses to the left of $s^Y$.

Figure 1 Output and welfare as a function of occupational health and safety $s$
The other expressions in table 1 are implicit for the CES utility functions, as the elasticities \( \varepsilon (\cdot) \) are functions of the safety levels. We will return to these forms further below. For the CD case, we also obtain straightforward solutions which can be given similar interpretations with regard to the output-maximizing safety level. The additional factor in (a), (c) and (d) are the preference parameters \( \mu \) and \( \gamma \). When health is valued strongly, i.e. \( \mu \) and \( \gamma \) are low, the welfare, firm-level union or confederation safety levels, as expected, go up. Again, looking at the signs of the CD first and second derivatives shows that the optimal safety levels are indeed maxima.

- Conflict of interests?

Who wants what in our economy? Given the richness of channels visible in the CD-results of table 1, we make a weak assumption concerning parameters which allows us to focus on the most realistic conflicts of interest: \( \alpha < \gamma < \mu \). The output elasticity of capital, \( \alpha \), is around 1/3. When comparing this to \( \gamma \), the value attached by unions to labour income in (16) and (17), our assumption says that unions, even though they are health-oriented, attach a weight of at least 1/3 to labour income. The second part of the assumption says that unions value health more than society as a whole, \( \gamma < \mu \). This also appears plausible as members of unions are all subject to health risks while society also consists of capital owners who are not.

The planner, the unions and the capital owners all potentially desire different safety levels. The planner can appear either in its welfare or in its output-maximizing guise, unions and capital owners are both represented at the firm and the nation-wide level. With our assumption and CD results from table 1, we find (see app. C.2)

\[
s^\pi < s^v < s^R = s^Y < s^U < s^V.
\]

The output-maximizing planner and the capital owners agree on the safety level, \( s^R = s^Y \). What maximizes output, maximizes capital income \( rK \), clearly a property of the CD structure of output in (20). The welfare-maximizing planner wants a higher safety level than the output planner, \( s^Y < s^U \), see fn. 9.

Nation-wide unions desire a higher OHS level than the welfare planner due to \( \gamma < \mu \). If society and nation-wide unions had identical preferences (\( \gamma = \mu \)), unions could replace the central planner. They would internalize the production externality and would set the welfare-maximizing safety level.

When looking at capital and labour representatives at the firm level, we know already from the discussion after (11) that firms want the lowest possible safety level \( s^\pi \). Concerning unions, we find a surprising result: Firm-level unions want a lower safety level \( s^v \) than capital owners or a central planner who is purely interested in output maximization. The reason is that the central planner (and the capital owners) know about (and internalize) the benefits of improved health levels for labour supply. The firm-level union sees positive effects from higher OHS standards only in its pure health effect and neglects labour supply effects (in
fact, it looks at labour $l$ in its objective function as the labour demand by firms which falls as TFP falls as a result of higher safety levels).\footnote{Departing from our parameter assumption would imply that a firm-level union sets a higher safety level than a central output-planner if it only values health enough. App. C.2 shows that $s^v \lesssim s^Y \Leftrightarrow \alpha \lesssim \gamma$.}

Summarizing, the nation-wide union, given its “exaggerated” emphasis on health is in conflict with society as a whole which in turn wants higher OHS standards than output-maximizers and capital owners. The lowest safety providers are firm-owners and firm-level unions.\footnote{Again, departing from our assumption on parameters, one can show that for $\gamma = \alpha \mu$ the firm-level union would set the same safety level as a planner $s^v = s^U$ (see app. C.2).} Comparing union output and welfare with a laissez-faire economy is straightforward when using fig. 1. Unions are welfare or output increasing if the safety level that they set is to the left of $s^Y$ and $s^U$, respectively. If they “overdo things”, i.e. if the union safety level is too far to the right of $s^Y$ or $s^U$, they would still be beneficial to the economy if the negative effect on TFP is not too strong, i.e. if the decrease of output and welfare to the right of their maxima is modest. For illustration purposes, the ranking in (22) is also plotted in fig. 1.

5.3 OHS and development

Empirical analyses suggest a negative correlation between the development level of a country and the risk of injury while working (Hall and Leeson, 2007, Flanagan, 2006, pp. 44-7). Should this give rise to policy concerns or is this a feature of an efficient development process?

Using the implicit-function theorem on CES safety levels as presented in table 1 shows that the reaction depends on the elasticity of substitution between income ($w^l l^i$ for the firm-level union, $w^i z^i N^i$ for the union confederation and $Y^i$ for the planner) and health $z^i$ (see app. D),

$$\left\{ \begin{array}{ll}
\text{bad substitutes} & \Rightarrow \lambda \lesssim 0 \Rightarrow ds^{j,i} = 0 \\
\text{Cobb-Douglas (CD)} & \Rightarrow \frac{ds^{j,i}}{dK} > 0 \\
\text{good substitutes} & = 0 \\
\end{array} \right., \quad s^{j,i} \in \{ s^v,i, s^V,i, s^U,i \}. \quad (23)$$

Both the planner and the two types of unions would set a higher safety level if the elasticity of substitution between health and income is low. This can be understood by referring to the income and substitution effect. There is an income effect due to more capital which increases demand for health $z (s^i)$ and consumption, the two arguments in the planner’s utility function in (7). The price of health relative to consumption, however, rises the more capital there is and households tend to substitute health by income.

In the CD case these effects cancel. Safety levels do not change in the course of the development of a country. This would be the “universal work standard” case advocated by some who postulate that all countries in the world, irrespective of their level of development, should have the same OHS standards. When substitution is easy, it is not clear which effect is
stronger. In this case, health standards could even decrease when a country becomes richer. The substitution effect would dominate the income effect.

The case that seems to be empirically more relevant is the one in which work standards are higher, the higher the development level of a country is (Hall and Leeson, 2007, Flanagan, 2006, pp. 44-7). This is the bad substitution case ($\lambda < 0$) in our model. When a society becomes richer, it can afford higher health standards and as income is a bad substitute for health, OHS standards go up accepting that this reduces TFP and therefore dampens the increase in income. Our view that the positive link between development and OHS standards is also due to unions is also shared by Kahn (1990, p.481) who writes that “union workers implicitly trade off wage and benefits growth for occupational safety improvements”.

Note that this empirical finding also points to the fact that real-world economies generally do not have a laissez-faire safety level of $s^*$ as introduced after (11). In a laissez-faire economy, more capital does not imply better OHS standards. Only if unions (or a benevolent central planner or related institutions) are present, can the safety level increase in the course of economic development.

6 Conclusion

The starting point of this paper was the belief that institutions like trade unions, which have been around for more than a century and are active in almost all countries in the world, are not just detrimental to economic production and welfare of a society. Studying activities of workers’ associations and trade unions beyond wage negotiation has shown that trade unions play a major role in providing workplace safety - at least in providing information about the necessity of measures that assure occupational health and safety (OHS). Trade unions did perform this role historically in what are now OECD countries and do play such a role today in certain industrializing economies.

Can these OHS activities of unions assign unions an output and welfare increasing role? Our analysis has shown that output and welfare effects of unions depend on union objectives and, more importantly, on the degree of centralization in an economy. Firm-level unions set lower OHS standards than economy-wide unions as the former neglect the positive labour supply effect of higher OHS. Firm-level unions are just as short-sighted (i.e. focused on this one firm) as firms and treat employment as the outcome of labour demand decisions by the firm. They provide OHS only as they value the health of their members per se. Economy-wide unions fully internalize the positive labour supply effect due to more OHS and therefore set higher safety standards. In fact, ruling out distributional effects from variations in the size of the labour force (i.e. assuming a Cobb-Douglas technology), economy-wide unions which attach the same importance to health as society as a whole set the social welfare-maximizing OHS standards. Even with a firm-level union, output and welfare increases compared to a laissez-faire economy.
Can other institutions play a similar role to unions? We have seen that capital owners - as opposed to individual atomistic firms - would also internalize economy-wide labour supply effects and value the health of workers. Capital owners trying to maximize their revenue would increase overall output and welfare of an economy as compared to a laissez-faire economy but never up to the social welfare-maximizing point. The incentives for capital owners to form a coalition and internalize the negative health externality, however, are much lower than for workers. Capital owners “do not feel health hazards”. It is only the workers who are directly confronted with risk at work. Hence, workers’ associations are the most probable institution to initially play this output and welfare increasing role. After some time, when general awareness in society about OHS standards or particular health issues has grown, the role of trade unions can be taken over by society as a whole, i.e. by some voting process through a government. This might be the reason why in the US, UK, Germany and many other OECD countries, governmental agencies take care of OHS standards nowadays and provide various types of work and health related insurances - and partly even make them compulsory.

The paper has various shortcomings which can be overcome in future work. Can unions play a welfare-increasing role in industrialized countries today where OHS standards are set by government agencies? One would have to start with an analysis where some firms or sectors in the North are unionized while others are not. A partial unionisation setup would also be useful to understand the effects of unions in the South better. Any increasing role would come gradually and unions would not become monopoly unions instantaneously. Second, the assumption of ignorance on the side of workers and perfect information of unions can be replaced by a Bayesian learning approach. One can expect that the relative degree of risk-aversion of workers (with respect to labour income relative to health effects) will determine whether “optimistic” workers (their prior predicts a higher expected share of time being healthy than a certain job actually implies) accept higher or lower wages than the perfect information compensating differential wage. One can then also precisely analyse the incentives for workers to join a union (thereby also reflecting the fact that no real-world economy is 100% unionized) and understand how joint learning increases welfare. Third, what happens if unions are allowed to set or negotiate wages? Is the traditional labour rationing distortion always overcompensated by the positive safety setting as portrayed here? All these extensions would make it possible to better understand the extent to which joint action and cooperative behaviour - as opposed to an individualistic view of society - is important for forming modern humane societies.

References


A. The first-order conditions for the general case

A.1 The welfare-maximizing safety level \((s^U)\)

Setting to zero the derivative of \(U(Y(s), z(s)) = U(Y(A(s), z(s)) N, z(s))\) with respect to \(s\) yields

\[
U_s = U_Y[Y_A A_s + Y_L z_s N] + U_z z_s = 0
\]

\[\iff \quad -U_Y Y_A A_s = [U_Y Y_L N + U_z] z_s
\]

\[\iff \quad -\frac{U_Y}{U} Y_A A_s = \left[\frac{U_Y}{U} Y_L N z_s + \frac{U_z}{U} z\right] z_s s
\]

\[\iff \quad -\frac{U_Y Y Y_A A A_s}{U} Y A_s = \left[\frac{U_Y}{Y} Y_L N + \frac{U_z}{Y} z\right] \frac{z_s s}{z}.
\]

Using (9) finally gives \(\varepsilon_{UY} \varepsilon_{Y A} \varepsilon_{A s} = [\varepsilon_{UY} \varepsilon_{Y L} + \varepsilon_{U z}] \varepsilon_{z s}.
\]

A.2 The output-maximizing safety level \((s^Y)\)

Setting to zero the derivative of \(Y(s) = Y(A(s), z(s)) N\) with respect to \(s\) yields

\[
Y_s = Y_A A_s + Y_L z_s N = 0 \iff -\frac{Y_A A_A s}{Y} A = \frac{Y_L L s z s}{Y} z.
\]

Using (9) finally gives \(\varepsilon_{Y A} \varepsilon_{A s} = \varepsilon_{Y L} \varepsilon_{z s}.
\)
A.3 The trade union’s maximization problem at the firm level \((s^v)\)

The problem can be reformulated as \(\max_s v = v (w l (A (s)) , z (s))\). The first-order condition is

\[
v_s = v_w l (A (s)) A_s + v_z z_s = 0 \iff \frac{v_w l (A (s)) A_s}{v} l = \frac{v_z z_s s}{z}.
\]

Using (9) yields \(\varepsilon_{v_w l} \varepsilon_{l A} \varepsilon_{A_s} = \varepsilon_{v_z z_s} \varepsilon_{z_s} \).

A.4 The trade union confederation’s maximization problem \((s^V)\)

The maximization problem is \(\max_V V = V (w (A (s)) , z (N), z (s) N, z (s))\). The first-order condition is

\[
V_s = V_w L [w_A A_s + w_L z_s N] z_N + w_z z_s + V_z z_s = 0 \iff -\frac{V_w L w_A A_s}{V} A = \left[\frac{V_w L}{V} \left[1 + \frac{w_L L}{w}\right] + \frac{V_z z_s}{z}\right] z_s.
\]

Using (9) yields \(\varepsilon_{V_w L} \varepsilon_{w A} \varepsilon_{A_s} = \varepsilon_{V_w L} [1 - \varepsilon_{w L}] + \varepsilon_{V_z z_s} \varepsilon_{z_s}.\)

A.5 The capital holders’ maximization problem at the country level \((s^R)\)

Setting to zero the derivative of \(r (s) K = r (A (s)) , z (s) N) K\) with respect to \(s\) yields

\[
r_s K = [r_A A_s + r_L z_s N] K = 0 \iff r_A A_s \frac{r_s}{r} = \frac{r_L z_s}{r}.
\]

Using (9) finally gives \(\varepsilon_{r A} \varepsilon_{A_s} = \varepsilon_{r L} \varepsilon_{z_s}.\)

B Explicit elasticities and first-order conditions

The following elasticities are necessary in order to be able to compute the first-order conditions reported on table 1 on the main text.

B.1 The elasticities for the Cobb-Douglas production function

The elasticities are computed for the CD production function in (19) and (20).

- TFP elasticities
For the CD production function in (19), the first-order conditions are

\[ w = A k^{\alpha} \left[ 1 - \alpha \right] l^{-\alpha} \iff l = \left[ \frac{A \left[ 1 - \alpha \right]}{w} \right]^{\frac{1}{\lambda}} k, \]  
(25)

\[ r = A \alpha k^{\alpha - 1} l^{1 - \alpha} \iff k = \left[ \frac{A \alpha}{r} \right]^{\frac{1}{\lambda - 1}} l. \]  
(26)

The TFP elasticity of labour can be computed with (25) as

\[ \varepsilon_{lA} \equiv l_A \frac{A}{l} = \frac{d \ln l}{d \ln A} = \frac{1}{\alpha}. \]  
(27)

The TFP elasticity of capital computed with (26) is \( \varepsilon_{kA} \equiv k_A \frac{A}{k} = \frac{1}{1 - \alpha}. \)

- The labour elasticities

The output elasticity of labour from (20) is \( 1 - \alpha \). The labour elasticity of the wage is \( \varepsilon_{wL} \equiv -w_L \frac{1}{w} = -\frac{d \ln w}{d \ln l} = \alpha \). The labour elasticity of the capital reward is \( \varepsilon_{rL} \equiv r_L \frac{L}{r} = 1 - \alpha \).

**B.2 The elasticities for the CES utility functions**

The elasticities are computed using (16), (17), and (18). To obtain the CD case, simply set \( \lambda = 0 \).

- The elasticities for the central planner

The output elasticity of welfare is

\[ \varepsilon_{UY} \equiv \frac{\partial U}{\partial Y} \frac{Y}{U} = \frac{1}{\lambda} \left\{ \mu Y^{\lambda} + [1 - \mu] z^{\lambda} \right\} \frac{1}{\lambda} - 1 \mu Y^{\lambda - 1} \frac{Y}{\left\{ \mu Y^{\lambda} + [1 - \mu] z^{\lambda} \right\}^{\frac{1}{\lambda}}} \]
\[ = \frac{\mu Y^{\lambda}}{\mu Y^{\lambda} + [1 - \mu] z^{\lambda}} = \frac{1}{1 + \frac{1 - \mu}{\mu} \left[ \frac{z}{Y} \right]^{\lambda}}. \]

The health elasticity of welfare is

\[ \varepsilon_{Uz} \equiv \frac{\partial U}{\partial z} \frac{z}{U} = \left\{ \mu Y^{\lambda} + [1 - \mu] z^{\lambda} \right\} \frac{1}{\lambda} - 1 \mu z^{\lambda} \frac{z}{\left\{ \mu Y^{\lambda} + [1 - \mu] z^{\lambda} \right\}^{\frac{1}{\lambda}}} \]
\[ = \frac{[1 - \mu] z^{\lambda}}{\mu Y^{\lambda} + [1 - \mu] z^{\lambda}} = \frac{1}{1 + \frac{1 - \mu}{\mu} \left[ \frac{Y}{z} \right]^{\lambda}}. \]

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• The elasticities for the firm-level trade union

The labour income elasticity of utility is

\[ \varepsilon_{vwl} = \frac{\partial v}{\partial w} \frac{wl}{v} = \frac{1}{\lambda} \left\{ \gamma [wl]^{\lambda} + [1 - \gamma] z^{\lambda} \right\}^{\frac{1}{\lambda} - 1} \gamma \lambda [wl]^{\lambda - 1} \frac{wl}{\left\{ \gamma [wl]^{\lambda} + [1 - \gamma] z^{\lambda} \right\}^{\frac{1}{\lambda}}} \]

\[ = \frac{\gamma [wl]^{\lambda}}{\gamma [wl]^{\lambda} + [1 - \gamma] z^{\lambda}} = \frac{1}{1 + \frac{1 - \gamma}{\gamma} \left[ \frac{z}{wl} \right]^{\lambda}}. \]

(28)

The health elasticity of utility is

\[ \varepsilon_{vz} = \frac{\partial v}{\partial z} \frac{z}{v} = \frac{1}{\lambda} \left\{ \gamma [wl]^{\lambda} + [1 - \gamma] z^{\lambda} \right\}^{\frac{1}{\lambda} - 1} [1 - \gamma] \lambda z^{\lambda - 1} \frac{z}{\left\{ \gamma [wl]^{\lambda} + [1 - \gamma] z^{\lambda} \right\}^{\frac{1}{\lambda}}} \]

\[ = \frac{[1 - \gamma] z^{\lambda}}{\gamma [wl]^{\lambda} + [1 - \gamma] z^{\lambda}} = \frac{1}{1 + \frac{1 - \gamma}{\lambda} \left[ \frac{z}{wl} \right]^{\lambda}}. \]

(29)

• The elasticities for the union confederation

The labour income elasticity of utility is

\[ \varepsilon_{V,wL} = \frac{\partial V}{\partial wL} \frac{wL}{V} = \frac{1}{\lambda} \left\{ \gamma [wL]^{\lambda} + [1 - \gamma] z^{\lambda} \right\}^{\frac{1}{\lambda} - 1} \gamma \lambda [wL]^{\lambda - 1} \frac{wL}{\left\{ \gamma [wL]^{\lambda} + [1 - \gamma] z^{\lambda} \right\}^{\frac{1}{\lambda}}} \]

\[ = \frac{\gamma [wL]^{\lambda}}{\gamma [wL]^{\lambda} + [1 - \gamma] z^{\lambda}} = \frac{1}{1 + \frac{1 - \gamma}{\lambda} \left[ \frac{wL}{z} \right]^{\lambda}}. \]

The health elasticity of utility is

\[ \varepsilon_{V,z} = \frac{\partial V}{\partial z} \frac{z}{V} = \frac{1}{\lambda} \left\{ \gamma [wL]^{\lambda} + [1 - \gamma] z^{\lambda} \right\}^{\frac{1}{\lambda} - 1} [1 - \gamma] \lambda z^{\lambda - 1} \frac{z}{\left\{ \gamma [wL]^{\lambda} + [1 - \gamma] z^{\lambda} \right\}^{\frac{1}{\lambda}}} \]

\[ = \frac{[1 - \gamma] z^{\lambda}}{\gamma [wL]^{\lambda} + [1 - \gamma] z^{\lambda}} = \frac{1}{1 + \frac{1 - \gamma}{\lambda} \left[ \frac{wL}{z} \right]^{\lambda}}. \]

B.3 The elasticities for the particular forms of \( A \) and \( z \)

The elasticities are computed using (21). The safety elasticity of TFP is

\[ \varepsilon_{As} = - \frac{\partial A}{\partial s} \frac{s}{A} = - \left[ \phi \right] \frac{be^{-\phi s}}{be^{-\phi s}} \frac{s}{be^{-\phi s}} = \phi s. \]

(30)
The safety elasticity of health is

\[ \varepsilon_{zs} \equiv \frac{\partial z}{\partial s} s^{\varepsilon_{zs}} = -[-\chi] q \exp(-\chi s) s = \frac{1 - z}{z} \chi s = [z(s)^{-1} - 1] \chi s. \]  

(31)

B.4 The first-order conditions for the particular case

The safety levels for our particular functions can be easily computed by inserting the above elasticities in the general first-order conditions from app. A.

- Welfare maximization

For example, the welfare-maximizing safety level \( s^U \) is described in (8) by the general first-order condition \( \varepsilon_{UY} \varepsilon_{YA} \varepsilon_{As} = [\varepsilon_{UY} \varepsilon_{YL} + \varepsilon_{Uz}] \varepsilon_{zs} \). Inserting the elasticities from app. B yields \( \varepsilon_{UY} \cdot 1 \cdot \phi s = [\varepsilon_{UY} (1 - \alpha) + \varepsilon_{Uz}] [z(s)^{-1} - 1] \chi s \). Plugging (21) and rearranging gives \( s^U \):

\[
\phi = \left[ 1 - \alpha + \frac{\varepsilon_{Uz}}{\varepsilon_{UY}} \right] \left[ \frac{1}{1 - \bar{q} \exp(-\chi s)} - 1 \right] \chi \iff \phi = \left[ 1 - \alpha + \frac{\varepsilon_{Uz}}{\varepsilon_{UY}} \right] \left[ \frac{\bar{q} \exp(-\chi s)}{1 - \bar{q} \exp(-\chi s)} \right] \chi \\
\iff \phi [1 - \bar{q} \exp(-\chi s)] = \left[ 1 - \alpha + \frac{\varepsilon_{Uz}}{\varepsilon_{UY}} \right] \bar{q} \exp(-\chi s) \chi \iff \left[ \frac{1 - \alpha + \frac{\varepsilon_{Uz}}{\varepsilon_{UY}} \chi + \phi}{\bar{q}} \right] e^\chi = e^\chi \\
\iff \ln \left[ \left[ 1 - \alpha + \frac{\varepsilon_{Uz}}{\varepsilon_{UY}} \chi + \phi \right] \bar{q} \right] = \chi s \\
\iff s^U = \chi^{-1} \ln \left[ 1 + \left[ \frac{\varepsilon_{Uz}}{\varepsilon_{UY}} (s^U) + 1 - \alpha \right] \chi \phi \right] \bar{q} \right].
\]

Other expressions can be derived in analogy.

C  OHS rankings

C.1 Rankings in the general case

Theorem 1 \( s^Y < s^U \).

**Proof.** \( s^U \) is determined by \( U_s = U_Y Y_s + U_z z_s = 0 \) and \( s^Y \) by \( Y_s = 0 \) which is equivalent to \( U_Y Y_s = 0 \). Now, for our model to make sense at all, we need to assume \( Y_s > 0 \) and \( Y_{ss} < 0 \). In other words, since \( U_Y > 0 \), the function \( U_Y Y_s \) falls monotonically in \( s \) until zero is reached and where \( s = s^Y \). Moreover, since \( U_z > 0 \) and \( z_s > 0 \), then \( U_z z_s > 0 \). The theorem follows since adding a positive term to \( U_Y Y_s \) implies that \( s^Y < s^U \).
C.2 Rankings in the Cobb-Douglas case

The following theorems allow us to compare the safety levels from Table 1.

**Theorem 2** \( s^U \leq s^V \iff \gamma \leq \mu \).

**Proof.** \( s^V > s^U \)

\[
\iff \chi^{-1} \ln \left[ \left( 1 + \frac{1 - \gamma}{\gamma} + 1 - \alpha \right) \frac{\chi}{\phi} \right] \bar{q} > \chi^{-1} \ln \left[ \left( 1 + \frac{1 - \mu}{\mu} + 1 - \alpha \right) \frac{\chi}{\phi} \right] \bar{q} \]
\[
\iff \mu > \gamma.
\]

The rest of the theorem can be proven in a similar fashion.

**Theorem 3** \( s^V > s^v \iff \alpha < 1, s^v \leq s^U \iff \mu \alpha \leq \gamma \)

**Proof.** Similar to the previous proof.

With these results, one can establish that \( \alpha < \gamma < \mu \) implies ranking (22) used in the main text.

D OHS and development

D.1 Capital and the welfare-maximizing safety level

**Theorem 4**

\[
\left\{ \begin{array}{l}
0 < \lambda < 1 \\
\lambda = 0 \\
\lambda < 0
\end{array} \right\} \Rightarrow \frac{ds^U}{dK} \left\{ \begin{array}{l}
? \\
= \\
> \end{array} \right\} 0.
\]

**Proof.** The proof has two parts.
(i) Using our results from app. B.2, we can compute

\[
\frac{\varepsilon_{Uz}}{\varepsilon_{UY}} = \frac{1 - \mu}{\mu} \left[ \frac{z(s^U)}{Y(s^U, K)} \right]^\lambda.
\]

For the Cobb-Douglas production function, we can compute

\[
\frac{z(s^U)}{Y(s^U, K)} = \frac{z(s^U)}{A(s^U) K^a z(s^U)^{1-a} N^{1-a}} = \frac{z(s^U)^a}{A(s^U) K^a N^{1-a}}.
\]
Since $z_s > 0$ and $A_s < 0$, we can conclude that $\frac{\partial(z/Y)}{\partial s_U} = 0$ and $\frac{\partial(z/Y)}{\partial K} < 0$, so that

$$
\frac{\partial \frac{\varepsilon_{Uz}}{\varepsilon_{UY}}}{\partial K} \left\{ \begin{array}{ll} < & 0 \iff \lambda \left\{ \begin{array}{ll} > & 0 \implies \frac{\partial \frac{\varepsilon_{Uz}}{\varepsilon_{UY}}}{\partial s_U} \left\{ \begin{array}{ll} > & 0. \end{array} \right. \end{array} \right. \end{array} \right.
$$

(ii) Plugging some of the elasticities from app. B into the general first-order condition giving $s_U$ and after rearranging, we can define

$$
M \equiv \phi - \left[ 1 - \alpha + \frac{\varepsilon_{Uz}}{\varepsilon_{UY}} (s_U, K) \right] \left[ \frac{1}{z(s_U)} - 1 \right] \chi = 0.
$$

With the aid of the implicit-function theorem, we can now compute

$$
\frac{ds_U}{dK} = -\frac{\partial M/\partial K}{\partial M/\partial s_U},
$$

where

$$
\frac{\partial M}{\partial K} = -\frac{\partial \frac{\varepsilon_{Uz}}{\varepsilon_{UY}}}{\partial s_U} \left[ \frac{1}{z} - 1 \right] \chi,
$$

and

$$
\frac{\partial M}{\partial s_U} = -\left[ \frac{\partial \frac{\varepsilon_{Uz}}{\varepsilon_{UY}}}{\partial s_U} \left[ \frac{1}{z} - 1 \right] + \left[ 1 - \alpha + \frac{\varepsilon_{Uz}}{\varepsilon_{UY}} \right] \left[ -1 \right] z^{-2} z_s \right] \chi
= \left[ \left[ 1 - \alpha + \frac{\varepsilon_{Uz}}{\varepsilon_{UY}} \right] \frac{z_s}{z^2} - \frac{\partial \frac{\varepsilon_{Uz}}{\varepsilon_{UY}}}{\partial s_U} \left[ \frac{1}{z} - 1 \right] \right] \chi.
$$

We can conclude with (32) that

$$
\frac{\partial M}{\partial s_U} \left\{ \begin{array}{ll} > & 0 \iff \lambda \left\{ \begin{array}{ll} > & 0 \implies \frac{\partial M}{\partial s_U} \left\{ \begin{array}{ll} > & 0. \end{array} \right. \end{array} \right. \end{array} \right. \end{array} \right.
$$

The derivative $\partial M/\partial s_U$ is positive for $\lambda > 0$ only if

$$
\left[ \left[ 1 - \alpha + \frac{\varepsilon_{Uz}}{\varepsilon_{UY}} \right] \frac{z_s}{z^2} - \frac{\partial \frac{\varepsilon_{Uz}}{\varepsilon_{UY}}}{\partial s_U} \left[ \frac{1}{z} - 1 \right] \right] \chi > 0.
$$

The theorem follows from (33) and (34). □
D.2 Capital and the union’s safety level at the firm level

Theorem 5

\[ \begin{align*}
0 < \lambda < 1 & \quad \Rightarrow \frac{d s^v}{d K} > 0 \\
\lambda = 0 & \\
\lambda < 0 & \Rightarrow \frac{\partial H}{\partial s^v} = 0.
\end{align*} \]

Proof. This proof is similar to that of Theorem 4. The proof has two parts.

(i) Using our results from app. B.2, we can compute

\[ \frac{\varepsilon_{vz}}{\varepsilon_{vwl}} = \frac{1 - \gamma}{\gamma} \left[ \frac{z(s^v)}{w l(A(s^v))} \right]^{\lambda} , \]

which after aggregation (we use here the symmetric equilibrium assumption and replace firm-level by aggregate variables) is given by

\[ \frac{\varepsilon_{vz}}{\varepsilon_{vwl}} = \frac{1 - \gamma}{\gamma} \left[ \frac{z(s^v)}{w(K, s^v) L(A(s^v), K)} \right]^{\lambda} . \]

As on the aggregate level labour demand equals labour supply, we use \( L(A(s^v), K) = z(s^v) N \). The wage rate that results from the Cobb-Douglas production function is \( w(K, s^v) = A(s^v) K^\alpha \left[ 1 - \alpha \right] z(s^v) N^{-\alpha} \). Total wage income is therefore \( wL = \left[ 1 - \alpha \right] A(s^v) K^\alpha \left[ z(s^v) N \right]^{1 - \alpha} = [1 - \alpha] Y(s^v) \). This allows us to compute

\[ \frac{z(s^v)}{w(K, s^v) L(A(s^v), K)} = \frac{z(s^v)^\alpha}{A(s^v) \left[ 1 - \alpha \right] K^\alpha N^{1 - \alpha}} , \]

Since \( z_s > 0 \) and \( A_s < 0 \), we can conclude that \( \frac{\partial z/wL}{\partial s} > 0 \) and \( \frac{\partial z/wL}{\partial K} < 0 \), so that

\[ \frac{\partial \varepsilon_{vz}}{\partial K} \begin{cases} < 0 \Rightarrow \lambda \begin{cases} > 0 \Rightarrow \frac{\partial \varepsilon_{vz}}{\partial s} \begin{cases} > 0 \Rightarrow \frac{\partial H}{\partial s^v} \begin{cases} = \end{cases} 0. \end{cases} \end{cases} \end{cases} \]

(ii) Plugging some of the elasticities from app. B into the general first-order condition giving \( s^v \) and after rearranging, we can define

\[ H = \frac{\phi}{\alpha} - \frac{\varepsilon_{vz}}{\varepsilon_{vwl}} (s^v, K) \left[ \frac{1}{z(s^v)} - 1 \right] \chi = 0. \]

With the aid of the implicit-function theorem, we can now compute

\[ \frac{d s^v}{d K} = -\frac{\partial H/\partial K}{\partial H/\partial s^v} . \]
where
\[
\frac{\partial H}{\partial K} = - \frac{\partial \varepsilon_{vz}}{\partial s^v} \left[ \frac{1}{z} \left( s^v \right) - 1 \right] \chi,
\]
and
\[
\frac{\partial H}{\partial s^v} = - \left[ \frac{\partial \varepsilon_{vz}}{\partial s^v} \left[ \frac{1}{z} - 1 \right] + \frac{\varepsilon_{vz} - 1}{\varepsilon_{vwl} z^2 s} \right] \chi
= \left[ \frac{\varepsilon_{vz} z}{\varepsilon_{vwl} z^2} - \frac{\partial \varepsilon_{vz}}{\partial s^v} \left[ \frac{1}{z} - 1 \right] \right] \chi.
\]

We can conclude with (35) that
\[
\frac{\partial H}{\partial K} \begin{cases} > \\ < \end{cases} 0 \iff \lambda \begin{cases} > \\ = \end{cases} 0 \implies \frac{\partial H}{\partial s^v} \begin{cases} > \\ > \end{cases} 0.
\tag{37}
\]
The derivative \( \partial H/\partial s^v \) is positive for \( \lambda > 0 \) only if
\[
\left[ \frac{\varepsilon_{vz} z}{\varepsilon_{vwl} z^2} - \frac{\partial \varepsilon_{vz}}{\partial s^v} \left[ \frac{1}{z} - 1 \right] \right] \chi > 0.
\]
The theorem follows from (36) and (37). \( \blacksquare \)

D.3 Capital and the union confederation’s safety level

Theorem 6
\[
\begin{cases} 0 < \lambda < 1 \\ \lambda = 0 \\ \lambda < 0 \end{cases} \implies \frac{d s^V}{d K} \begin{cases} ? \\ = \end{cases} \frac{d s^V}{d k} \begin{cases} > \end{cases} 0.
\]

**Proof.** This proof is similar to that of Theorem 4. The proof has two parts.

(i) We know from Table 2 that
\[
\frac{\varepsilon_{Vz}}{\varepsilon_{VwL}} = \frac{1 - \gamma}{\gamma} \left[ \frac{z \left( s^V \right)}{w \left( s^V \right) z \left( s^V \right) N} \right]^\lambda.
\]
For the Cobb-Douglas production function, we can compute
\[
\frac{z \left( s^V \right)}{w \left( s^V \right) z \left( s^V \right) N} = \frac{1}{A \left( s^V \right) K^\alpha \left[ 1 - \alpha \right] z \left( s^V \right)^{-\alpha} N^{-\alpha} \cdot N} = \frac{z \left( s^V \right)^\alpha}{A \left( s^V \right) K^\alpha \left[ 1 - \alpha \right] N^{1-\alpha}}.
\]

33
Since \( z_s > 0 \) and \( A_s < 0 \), we can conclude that \( \frac{\partial z/wzN}{\partial s^V} > 0 \) and \( \frac{\partial z/wzN}{\partial K} < 0 \), so that

\[
\frac{\partial \frac{\varepsilon_{Vz}}{\varepsilon_{VwL}}}{\partial K} \begin{cases} < \\ > \end{cases} 0 \iff \lambda \begin{cases} > \\ < \end{cases} 0 \implies \frac{\partial \frac{\varepsilon_{Vz}}{\varepsilon_{VwL}}}{\partial s^V} \begin{cases} > \\ < \end{cases} 0. \tag{38}
\]

(ii) Plugging some of the elasticities from app. B into the general first-order condition giving \( s^V \) and after rearranging, we can define

\[
P \equiv \phi - \left[ 1 - \alpha + \frac{\varepsilon_{Vz}}{\varepsilon_{VwL}} (s^V, K) \right] \left[ \frac{1}{z(s^V)} - 1 \right] \chi = 0.
\]

With the aid of the implicit-function theorem, we can now compute

\[
\frac{ds^V}{dK} = -\frac{\partial P/\partial K}{\partial P/\partial s^V}, \tag{39}
\]

where

\[
\frac{\partial P}{\partial K} = -\frac{\partial \frac{\varepsilon_{Vz}}{\varepsilon_{VwL}}}{\partial K} \left[ \frac{1}{z(s^V)} - 1 \right] \chi
\]

and

\[
\frac{\partial P}{\partial s^V} = -\left[ \frac{\partial \frac{\varepsilon_{Vz}}{\varepsilon_{VwL}}}{\partial s^V} \left[ \frac{1}{z} - 1 \right] + \left[ 1 - \alpha + \frac{\varepsilon_{Vz}}{\varepsilon_{VwL}} \right] \frac{-1}{z^2} z_s \right] \chi
\]

\[
= \left[ \left[ 1 - \alpha + \frac{\varepsilon_{Vz}}{\varepsilon_{VwL}} \right] \frac{z_s}{z^2} - \frac{\partial \frac{\varepsilon_{Vz}}{\varepsilon_{VwL}}}{\partial s^V} \left[ \frac{1}{z} - 1 \right] \right] \chi.
\]

We can conclude with (38) that

\[
\frac{\partial P}{\partial K} \begin{cases} > \\ < \end{cases} 0 \iff \lambda \begin{cases} > \\ < \end{cases} 0 \implies \frac{\partial P}{\partial s^V} \begin{cases} ? \\ > \end{cases} 0. \tag{40}
\]

The derivative \( \partial P/\partial s^V \) is positive for \( \lambda > 0 \) only if

\[
\left[ \left[ 1 - \alpha + \frac{\varepsilon_{Vz}}{\varepsilon_{VwL}} \right] \frac{z_s}{z^2} - \frac{\partial \frac{\varepsilon_{Vz}}{\varepsilon_{VwL}}}{\partial s^V} \left[ \frac{1}{z} - 1 \right] \right] \chi > 0.
\]

The theorem follows from (39) and (40).