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# **W. E. P.**

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## **Fragmentation in Property: Towards a General Model**

**Norbert Schulz, Francesco Parisi, and Ben Depoorter**

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# Fragmentation in Property: Towards a General Model

by

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This paper develops a general model of anticommons fragmentation in property. To this end, we differentiate between different forms of property fragmentation. With the use of several functionally related examples, we consider the equilibria obtained under different scenarios. The various illustrations are later utilized as building blocks for the development of a general model of fragmented property. The model reveals that the private incentives of excluders do not capture the external effects of their individual decisions. Specifically, our model suggests that the results of underutilization of joint property increase monotonically in both (a) the extent of fragmentation; and (b) the foregone synergies and complementarities between the property fragments. Within this context, we explore some of the important implications for the institutional responses to issues of property fragmentation. (JEL: K10, K11, K19, D62, D70)

## 1. Introduction

Recently, a new theory has become popular in the literature on common property. A number of scholars have pointed to the danger of excessive propertization in the context of what have been termed "anticommons" property regimes (Heller [1998]; Heller and Eisenberg [1999]; Buchanan and Yoon [2000]; Depoorter and Parisi [2000]).

When multiple owners hold rights to exclude others from a scarce resource and no one exercises an effective privilege of use, the resource might be prone to underuse: a problem known as the tragedy of the anticommons.<sup>2</sup>

In an Anticommons, a property regime in which multiple owners hold effective rights of exclusion in a scarce resource,<sup>3</sup> the coexistence of multiple exclusion rights creates conditions for sub optimal use of

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<sup>2</sup> The concept of the anticommons, first introduced by Michelmann [1982] and then made popular by Heller [1998 and 1999], refers to the reverse case of Hardin's [1968] well known tragedy of the commons.

<sup>3</sup> This definition of the anticommons employed by Heller [1998] provides a powerful tool for property theory. Heller [1998] recently revitalized the concept in an article on the transition to market institutions in contemporary Russia. He discusses the intriguing prevalence of empty storefronts in Moscow. Storefronts in Moscow are subject to underuse because there are too many owners (local, regional and federal government agencies, mafia, etc.) holding the right to exclude. Frank Michelman [1982] coined the term anticommons in an article on ethics, economics and the law of property. Michelman defined the anticommons as *a type of property in which everyone always has rights respecting the objects in the regime, and no one, consequently, is ever privileged to use any of them except as particularly authorized by others*. This definition has almost no counterpart in real-world property relations. The hypothetical example provided is that of a wilderness preserve that 'any person' has standing to enforce.

the common resource. If the common resource is subject to multiple exclusion rights held by two or more individuals, each co-owner will have incentives to withhold resources from other users to an inefficient level. In the presence of concurrent controls on entry exercised by individual co-owners acting under conditions of individualistic competition, exclusion rights will be exercised even when the use of the common resource by one party could yield net social benefits. The Tragedy of the Anticommons is the result of the fact that common resources will remain idle even in the economic region of positive marginal productivity. This is because the multiple holders of exclusion rights do not fully internalize the cost created by the enforcement of their right to exclude others.

The sources of externalities in an anticommons problem are twofold. First, there are static (or current) externalities, in that the **exercise of a right of exclusion** by one member reduces or eliminates the value of similar rights held by other individuals. In price theory terms one can think of this externality as the cross price effect of the various exclusion rights. Second, the withholding of productive resources may create dynamic (or future) externalities, because the underuse of productive inputs today bears consequences into the future, as standard growth theory suggests.<sup>4</sup>

In the following section, we build upon the definition of the anticommons and property fragmentation, which still lacks an accepted general formalization in the literature. In Section 3 we use our findings to formulate a few normative corollaries and to study the institutional responses to fragmentation, as found in various legal systems.

## *2. Fragmentation of Property Rights and Externalities: Some Examples and a General Result*

In this section we wish to illustrate the problem of property fragmentation with the use of several functionally related examples. We provide a model of unified property characterizing the essence of fragmentation problem as the result of a lack of conformity between use and exclusion rights. All our illustrations are intentionally simple and are used for the purpose of presenting the factual taxonomy of the problem. In all cases, we provide a formal description to clarify the intimate relationship between the issue of fragmentation of property rights and the issue of positive externalities. Despite the growing significance of the concept of the Anticommons<sup>4</sup> in both economic theory and law and economic scholarship, this notion still lacks a generalized formalization in the literature. The various illustrations are later utilized as

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<sup>4</sup> Elsewhere, we provide a dual model of property, where commons and anticommons problems are shown to result from symmetrical structural departures from a unified conception of property (Schulz, Parisi and Depoorter [2000]). In commons situations, the right to use stretches beyond the effective right (or power) to exclude others. Conversely, in anticommons situations, the co-owners' right of use is compressed, and potentially eliminated, by an overshadowing right of exclusion held by other co-owners. Put differently, in both commons and anticommons cases, rights of use and rights of exclusion have non-conforming boundaries. The lack of conformity causes a welfare loss from the forgone synergies between those complementary features of a unified property right. Welfare losses are produced by a discrepancy between the rights of use and the rights of exclusion held by the various owners.

examples for the development of a general model of fragmented property. While recognizing that reality may present situations that combine characteristics of the various categories presented here, we proceed by analyzing the two hypotheses (which two hypotheses???) separately. This facilitates our understanding of the different equilibrium results. We conclude with a discussion of some of the properties and the results of the general model.

*Example 1.* Let's begin our analysis considering a simple situation of property rights fragmentation. Suppose that, in the distant past, agent 1 purchased a parcel of land from agent 2 for the purpose of setting up an orchard. In the process of transferring title to the land, agent 2 retained the right to use the sold parcel for the purpose of letting his sheep graze on the land. As a result of this sale with a partial reservation of use, the unitary property right was fragmented, giving both agents a partial right of use and reciprocal exclusion privileges. The property right of agent 1 is constrained by the real interest retained by agent 2. Agent 1 holds a right to exclude any use of agent 2 other than letting his sheep graze on the land. Agent 2 conversely holds a right to exclude any use of the land by agent 1, which is in conflict with his grazing rights. In this sense the otherwise unitary proprietary interest over the land has been fragmented. Such fragmentation will remain mutually beneficial for the parties as long as the mixed use of the land as an orchard and a grazing parcel continues to remain the best use of the land for both parties.

Suppose now that agent 1 sees an opportunity which would generate more value than the current use. Take, for example, the construction of a hotel on "his" land. This use is in conflict with the right of grazing held by agent 2. Agent 2 can withhold his consent to the transformation of the land and exercise his veto right impeding the transformation. As the opportunity is supposed to be more valuable than the current use, it would not be rational for agent 2 to object at any cost to the transformation. He would however rationally attempt to maximize his profit from the sale of his partial property right. Suppose agent 2 asks for some percentage of revenue created by the operation of the hotel. When deciding which share of the revenue to ask in exchange for his permission, agent 2 will consider that a revenue sharing agreement will likely affect the revenues of the hotel. The hotel revenues depend on how the hotel is run which in turn depends on the effort of agent 1. Optimal efforts for agent 1 will be affected by the sharing rule chosen by the parties. For convenience, let us restate our scenario in formal terms. This formalization will provide us with some illustrative results and serve as an example for the general model of anticommons presented at the end of this section.

Let  $1 - x_2$  be the percentage that agent 2 demands to grant his permission to transform the use of the land. Let the revenue be  $x_1$ , where  $x_1$  also denotes, for the purpose of simplicity, the effort of agent 1. Exerting

effort  $x_1$  generates costs of  $x_1^2/2$ . If  $v_1$  is the value of the land in its current orchard use and  $v_2$  is the additional value of using the land for grazing, the net value of operating the hotel would be

$$V_1(x_1, x_2) = x_2 x_1 - x_1^2/2 - v_1$$

while agent 2 would receive

$$V_2(x_2, x_1) = (1 - x_2) x_1 - v_2.$$

This situation can be described as a two-stage game where agent 2 chooses  $x_2$  in the first stage and agent 1 chooses  $x_1$  in the second stage. Given the simple structure it is easy to compute the equilibrium of this game:  $x_1^f = x_2^f = 1/2$  as long as  $v_1 < 1/8$  and  $v_2 < 1/4$ . Hence, the effort of agent 1 will be  $1/2$  in this fragmented situation.

If the property were held in sole ownership by agent 1, without encumbrances in favor of agent 2, the fragmentation problem would disappear and the chosen level of effort by agent 1 would be higher. Suppose agent 1 had not granted the grazing rights to agent 2, but had instead acquired exclusive title and use rights on the land. Then the net value accruing to him from the new opportunity would be

$$V(x_1, x_2) = x_1 - x_1^2/2 - v_1 - v_2.$$

Note that  $V = V_1 + V_2$ . We can easily verify that optimal level of effort for agent 1 would be equal to 1. Since the sum of the parties' value,  $V$ , represent the social surplus, it follows that in the fragmented case the land is underused, in the sense that the effort of agent 1 would be less than optimal and therefore the value of the land would not be properly maximized.

This result should not come as a surprise. The position of multiple property owners in the face a new opportunity which requires a reunification of their fragmented property rights creates a strategic problem similar to the typical hold-up problem to which we return in example 2. Sub-optimal final use of resources may result from such fragmentation. As mentioned above the example is not intended to produce this formal result per se but to clarify some language. The situation of fragmented property contemplated in example 1 represents a sequential case of anticommons: agent 2 –by demanding a "price" for his right – acts before agent 1 can run the hotel. Note that the right holders act on different levels of the value chain, as illustrated by the fact that agent 1 needs the right of agent 2 as an "input" for the production of the planned hotel. While sequential decision situations often arise in a vertical relationship, it is not true that all decisions in a vertical relationship are of the sequential type (see footnote 4 in example 2).

At this point, we should note that identical results would obtain if agent 1 and agent 2 were each owners of neighbouring parcels of land and the construction of the hotel necessitated the physical reunification of their lots. The results of example 1 do not strictly depend on the legal or physical nature of property fragmentation. In this context, property fragmentation merely indicates the existence of multiple rights held by different individuals to control or veto a change in the use of their land. As shown in our example, sub-optimal final allocations of resources may be the consequence of fragmented decision rights, even when such fragmentation concerns a unitary physical asset.<sup>5</sup> Even in the face of value enhancing opportunities, multiple right holders may face incentives to employ their veto power to maximize the private return from the joint enterprise. The combined effect of the various agents' strategies leads to an inefficient outcome.<sup>6</sup>

The outcome of the formal exercise in example 1 is perhaps most easily understood if it is further recognised that each agent exerts a positive externality on the other agent. A greater contribution to the joint project by one agent leads to greater benefits for the other agent, and vice-versa. The control variable  $x_2$  can, in fact, be interpreted as the degree to which agent 2 grants the right to use his property right to agent 1. Hence, the above result is consistent with conventional wisdom, according to which in situations of positive externalities the use of some resource is less than optimal.<sup>7</sup>

*Example 2* is a variant of the above example that closely resembles the familiar hold-up problem (see, e.g. Hart, 1995). Imagine that two agents share as co-owners a large commercial property, one utilizing his quarters for a food factory, and the other operating a music store. A new opportunity arises to lease some unused commonly owned space to a third party for the purpose of setting up an office of a well-established consulting firm. We assume that the rental of the common area to the consulting firm is the best use of the common resource. The income from the rental of the common space depends on the willingness of the co-owners to reduce the smoke and sound emissions from their respective commercial activities. The reduction of such emissions can be considered as “inputs” in the production of added value for the common rental space. The revenue from the rental thus depends on the extent to which agent 1 and agent 2 agree to reduce their respective emission levels. The overall value of the rental property depends on the inputs of both

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<sup>5</sup> With respect to the best use, rights may then turn out to be overly fragmented even though at the level of the objects themselves no fragmentation is visible.

<sup>6</sup> We should note here that exclusion rights – the lack of which is at the origin of the well-known commons problem – give origin to anticommons situations if simultaneously granted to multiple individuals.

<sup>7</sup> It may be helpful to note that the right of exclusion may imply an all or nothing decision but the price for which the right is transferred is not such a decision. A higher price may induce the new holder of the right to not fully exploit the value of the underlying object. [See examples 4 and 5.](#)

agent, given by the reduction of smoke level,  $x_1$ , and reduction of noise,  $x_2$ . The agents have exclusive control over their emissions, since they are the only ones to produce smoke or noise respectively.

The problem faced by the two agents closely describes the Anticommons scenario considered by Heller (1998). Let  $R(x_1, x_2)$  denote the rental revenues and let us assume that  $R$  increases in both arguments. Further assume that both inputs are complements to each other, in the sense that a combined reduction of smoke and noise emissions will best serve the production of environmental quality for the leased space. The two agents face different costs in the abatement of their respective emissions. The cost of reducing smoke emissions for the food factory  $x_1$  is  $C_1(x_1)$  and the cost for the reduction of the noise level for the music store,  $x_2$  is  $C_2(x_2)$ . Both cost functions are assumed to be convex. Following the convention in the hold-up literature, we let the two agents bargain on their respective share of the rental revenues. The Nash bargaining solution is then employed and this amounts here to give each agent one half of the revenues. This implies the following profits to agent 1:

$$V_1(x_1, x_2) = \frac{1}{2} R(x_1, x_2) - C_1(x_1)$$

Analogously agent 2 receives profits

$$V_2(x_2, x_1) = \frac{1}{2} R(x_1, x_2) - C_2(x_2)$$

Decision for the overall environmental quality offered for the rental space can then be described as the Nash equilibrium of a one-stage simultaneous move game. Note here that, if the entire commercial property is held by a single owner, such owner would rationally maximize the joint profit from his various uses of the commercial space at the net of his input costs

$$V(x_1, x_2) = R(x_1, x_2) - C_1(x_1) - C_2(x_2).$$

Comparing the equilibrium values of fragmented ownership to the profit maximising solution of unified ownership, we can show that, under some mild assumptions, the environmental quality level offered in the fragmented situation is less than the profit maximising choice that would be made in a unified property situation. We do not further elaborate on this result, since it is well known in the literature and represents a special case of our general result, presented later, at the end of this section. It should be noted at this point that also in example 2 the anticommons problem is induced by the fact that each agent's choice exerts a positive externality on the other agent.



Here again, the rights relating to the inputs necessary to produce the environmental quality for the leased space are fragmented in the non-integrated situation. That is to say, neither agent has control over the other agents reduction in emissions. Each co-owners has a full right to use his property as he pleases, with no internal mechanisms to induce the co-owner to choose the level of smoke or noise emissions that would maximize their respective share of income from the rental of the common space.

One difference between example 1 and example 2 is the fact that the use of the respective rights was modelled in a sequential fashion in example 1 and in a simultaneous fashion in example 2. We shall refer to situations of this type as "simultaneous anticommons". The results are quite similar, suggesting that the anticommons problem does not critically depend on the sequence of decisions between the various agents<sup>8</sup>. At the end of this section we will see to what extent this preliminary result can be generalized.

*Example 3* describes a multiple-agent version of the simultaneous anticommons problem. Consider a commonly owned commercial retail space, which comprises  $n$  retail spaces. Initially, each space is owned and controlled by a different owner. Each individual co-owner contributes through his own effort and investment to the décor of the retail space, including the quality of their own shop, the set up of the storefront windows and of the common areas. The overall quality of the commonly owned retail building thus depends on the uncoordinated investment in quality undertaken by each individual co-owner. Let the investment of owner  $i$  be denoted by  $x_i$ . The investment of some owner  $i$  does not only increase the utility derived from owning retail space  $i$  but also the utility of other co-owners who utilize neighbouring space. To be precise suppose the utility of owner  $i$  is of the linear quadratic type and  $c$  denotes the constant marginal cost of investing in quality  $x_i$ .

$$V_i(x_1, x_2, \dots, x_n) = a x_i - \frac{1}{2} \left( x_i^2 - \frac{2\theta}{n-1} x_i \sum_{j \neq i} x_j \right) - c x_i$$

Here  $0 < \theta < 1/2$  denotes the positive impact of the investment of the remaining owners on the utility of owner  $i$ . The decision to invest can again be described by the Nash equilibrium of a simultaneous move game. Solving the first order conditions yields the investment level

$$x_i^f = \frac{a-c}{1-\theta}$$

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<sup>8</sup> Note that in Hart's account of the hold-up problem agent 1 is an input supplier to agent 2. Hence the agents are related vertically along a value chain. The decisions are nevertheless simultaneous and not sequential.

Consider now the situation where the entire shopping mall is owned by a single owner. The single owner's total valuation of the ownership is given by the sum of the values of the individual stands:

$$V(x_1, x_2, \dots, x_n) = (a - c) \sum_i x_i - \frac{1}{2} \left( \sum_i \left( x_i^2 - \frac{2\theta}{n-1} x_i \sum_{j \neq i} x_j \right) \right)$$

Maximizing this expression yields

$$x_i^u = \frac{a - c}{1 - 2\theta}.$$

From this it follows immediately that in the case of the unified property investment is larger than in the fragmented case. However, more can be seen here. Firstly, the difference between the optimal level of per-unit investment in the unified case and the equilibrium level of per-unit investment in the fragmented case increases in  $\theta$ . Hence, the degree to which fragmented owners under-invest increases with the degree to which they are affected by the investment of others. Put in different terms, the stronger the externality of each owner's investment, the more serious the incentive problem becomes. Secondly, aggregate investment in the fragmented case differs more if the number of owners increases. Hence, if fragmentation increases, so does the under-investment problem.

*Example 4* describes the problem of fragmented patents, as contemplated in Heller and Eisenberg (1998). Suppose that two firms each hold a patent in a particular technology. Assume also that the patents are complements. Any third party desiring use of the technology will need to obtain access to both patents. Suppose that there is a continuum of such third party firms where each firm is characterised by their willingness to pay for the use of the two patents, denoted  $w$ . Let  $w$  be uniformly distributed across  $[0, 1]$ . Suppose the patent holding firm  $i$  asks a price  $p_i$  for a license to use its patent. Hence the price to be paid to both patent holding firms is  $p_1 + p_2$ . All third party firms with a willingness to pay at least such amount will ask for a license from both firms. Given the assumption on the distribution of the potential licensees the demand for patents is  $1 - (p_1 + p_2)$ , patent holding firm 1 has a profit of

$$p_1 (1 - (p_1 + p_2)),$$

with an analogous expression for firm 2. The decision to set a price for a license can be modelled again as a Nash equilibrium of a simultaneous move game. The equilibrium value of both prices is  $1/3$  such that both licenses cost  $2/3$ .

Suppose now that both patents are in the hands of just one firm that demands a price of  $P$  for a license on both patents. Then the profit of this firm will be

$$P(1 - P),$$

which will be maximised at  $P = 1/2$ . Hence, fragmentation raises the price for both licences. This induces some firms not to employ the technology. Therefore fragmentation decreases the value created by the technology.

Note that this example again illustrates the simultaneous anticommons type as both holders of patent rights act simultaneously. The result is in line with oligopoly theory where it is conventional wisdom that oligopolists charge a higher price than a monopolist when the supplied commodities are complements. Formally this example is identical to the one contained at some length in Buchanan and Yoon (2000).

This example is different in that the decision variable of the agents is the price charged for use of the resource while the previous examples were phrased in terms of quantity (or activity) driven restrictions of use.

Example 4 demonstrates that the distinction between price- and quantity-driven restrictions is irrelevant to the anticommons problem. If one firm charges a high price to a third party this in effect decreases the potential for third party-use of the other firm's patent right.

*Example 5* describes a copyright law problem faced by an author in his dealings with a book publisher. Suppose that the author wants to sell his copyright to the book for price  $p_2$  per copy sold. The publisher expects the demand for the book to be  $1 - p_1$ , where  $p_1$  is the price the publisher charges. Aside from costs, the profit of the publisher is thus

$$(p_1 - p_2)(1 - p_1)$$

and the profit of the author is

$$p_2(1 - p_1).$$

If the author commits first to a price - a natural assumption in this context - the pricing decision can be described by the Nash equilibrium of a two stage game. Equilibrium values are  $1/2$  for  $p_2$  and  $3/4$  for  $p_1$ . If the author had the opportunity to market the book himself the price would be  $1/2$ . Once again the fragmentation

of rights results in a higher price and sub-optimal use of the intellectual property— in this example a decrease of potential readership.

Of course this result is well known. The problem that presents itself here can be considered as a variant of the problem of double marginalization. Example 5 resembles the sequential anticommons type to illustrate that the modelling strategy in terms of both activities/rights of use (example 1) and of prices is not critical to point out the vertical anticommons problem (as was previously demonstrated in the context of horizontal anticommons in example 4). In the context of double marginalization it is generally recognized that the severity of the problem increases with the length of the value chain. Similarly, an increase in fragmentation leads to a decrease in use of a resource.

All examples share the characteristic that agents exert a positive externality on the other agent in exercising their right of use of their property. This is apparent in examples 2 and 3. Example 1 exhibits a subtlety: here the positive externality of agent 2's actions do not relate to the use of his property (the right to let his sheep graze), but consists rather of granting a right of use to the other agent. In all examples 1 to 3  $x_i$  has one of these two interpretations. Where we state the general result below,  $x_i$  should be interpreted accordingly, depending on the particular characteristics of the case at hand.

The general results of under-provision in the presence of positive externalities constitute conventional wisdom. However, our general result is to be distinguished from pre-existing literature that emphasizes the departure from the first best allocation that results under decentralised allocation. In the context of this article we do not compare the first best allocation with the alternative of decentralised allocation. Our general result is obtained by comparison of fragmentation (decentralization) to unified property ("monopolization").<sup>9</sup> Also, existing literature that does compare the alternatives of fragmentation and unified property, such as hold-up and double marginalization problems, are highly context specific. Our examples are helpful to assess the importance of the more general anticommons problem and the intimately related externality problem. As we have seen it is sometimes reasonable to model the choices of the  $x_i$  (with the above interpretation) as a simultaneous move game and sometimes as a sequential move game. The latter seems more adequate in vertical relationships but not necessarily so. Therefore we establish two propositions. One relates to the simultaneous move game approach and one to the sequential move game approach. Both propositions set assumptions that imply the underprovision result that we have observed in our examples.

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<sup>9</sup> This strand of literature further distinguishes itself from our contribution in that "unified property" may not be first-best in a Paretian sense, although the resulting allocation could be closer a Pareto optimal allocation.

Let us first summarize a few assumptions relating to the properties of the value functions  $V_i (x_i, x_j)$  in

*Assumption 1:* (a)  $V_i$  and  $V := V_1 + V_2$  are strictly concave in  $x_i$  and twice continuously differentiable in both variables; (b)  $\partial V_i / \partial x_j > 0$  for  $i \neq j$ .

*Assumption 2:*  $\partial^2 V_i / \partial x_j \partial x_i$  are non-negative and smaller than the absolute value of  $\partial^2 V_i / \partial x_i^2$ .

Assumption 1 (b) reflects a positive externality. Assumption 2 in signing the second partial cross derivative reflects a complementarity between  $x_i$  and  $x_j$ . Indeed these variables are strategic complements under this assumption. The restriction concerning the absolute value will imply a unique equilibrium in simultaneous move games.

*Assumption 3:*  $x_i \in X_i \subset \Re$ .  $X_i$  is compact and convex.

In the following proposition we compare the Nash equilibrium  $x_i^f$  of the simultaneous move game (the fragmentation case), where  $x_i$  are interpreted as strategies and  $V_i$  as payoffs, and the values  $x_i^u$  maximise  $V$  (unified property).

*Proposition 1:* Under assumptions 1, 2, and 3 a Nash equilibrium  $x_i^f$  exists and is unique. Moreover, if  $(x_1^f, x_2^f)$  and  $(x_1^u, x_2^u)$  are in the interior of  $X_1 \times X_2$ , we have for  $i = 1, 2$ :

$$x_i^f < x_i^u$$

For the proof we refer to the appendix. The content of the proposition establishes the conventional view of externalities. The decentralised decisions (fragmentation) lead to decreased use of a resource if compared to a centralised decision (unified property).

The next proposition deals with the same question for sequential move games. Let agent 2 move first and agent 1 move after observing the move of agent 2. We summarise some special assumptions concerning  $V_i$  in

*Assumption 4:*  $\partial^2 V_1 / \partial x_1 \partial x_2$  and  $\partial^2 V / \partial x_1 \partial x_2$  are non-negative and

$$-\frac{\partial V_1}{\partial x_2} \frac{\partial^2 V_1}{\partial x_1^2} > \frac{\partial V_2}{\partial x_1} \frac{\partial^2 V_1}{\partial x_2 \partial x_1}.$$

*Proposition 2:* Under assumptions 1, 3, and 4 a Nash equilibrium exists. Moreover, if  $(x_1^f, x_2^f)$  and  $(x_1^u, x_2^u)$  are in the interior of  $X_1 \times X_2$ , we have for  $i = 1, 2$ :

$$x_i^f < x_i^u$$

Hence, in essence proposition 2 parallels proposition 1. The occurrence of the anticommons phenomenon is expected to arise regardless of whether the agents move simultaneously. This is not to say that the severity of the problem is identical in both cases. Indeed, as long as the strategies are strategic complements, the anticommons problem is more pronounced in the simultaneous move context than in the sequential move context. This is the result stated in the following proposition. In this proposition  $x_i^{f1}$  denotes the equilibrium value in the simultaneous move game (one stage game) and  $x_i^{f2}$  the equilibrium value in the sequential move game (two stage game).

*Proposition 3:* If assumptions 1, 2 and 3 are satisfied, if  $\partial^2 V_1 / \partial x_1 \partial x_2 > 0$ , and if the equilibrium pair of strategies are in the interior of  $X_1 \times X_2$ , then we have for  $i = 1, 2$ :

$$x_i^{f1} < x_i^{f2}$$

The intuition of this result is as follows: in a sequential move-game the first mover knows that the second mover will underuse his strategy because the positive externality is of no concern to him. But the second mover increases the use of his strategy if the first mover does (strategic complements). Therefore the first mover has an incentive to increase his use of the strategy and this induces the second mover to increase the use of his strategy as well.

To sum up, we have suggested a model of the anticommons problem which builds on a generalisation of the externality problem. The assumptions needed to assess the generality of the problem are sufficiently general to encompass the examples 1 through 3. Examples 4 and 5 could also be generalised by providing a pricing variant of propositions 1 and 2. For a symmetric context and for simultaneous move games we have established a version of proposition 1 in Schulz, Parisi and Depoorter [2000]. In the context of this paper it may suffice to build on the intuition suggested by examples 4 and 5.

In earlier versions, we have stressed the difference between "horizontal" versus "vertical" anticommons problems. In the horizontal case, the exclusion rights are often exercised simultaneously and independently by the various right holders. This may involve two agents who are linked in a horizontal relationship, such as multiple co-owners with cross-veto powers on other members' use of a common resource.<sup>10</sup> These situations are characterized by the aspect that both agents contribute rights on the same

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<sup>10</sup> More generally, one can think of multiple independent owners pursuing a project which requires the annexation of other individual's land or facing (with other individual owners in a similar position) the demand of a third party for their property rights.

level of a value chain.

In the vertical case, the exclusion rights are in a vertical relationship with one another.<sup>11</sup> The exclusion rights are often exercised sequentially by the various right holders. This may involve multiple parties in a hierarchy each of whom can exercise an exclusion or veto power over a given proposition. Real life examples can range from a bureaucracy-like situations where multiple permits need to be acquired in order to exercise a given activity, to a production process where a given producer purchases one essential input from a monopolistic seller.<sup>12</sup>

Both horizontal and vertical anticommons problems are the consequence of non-conformity between use and exclusion rights. As we have seen, the difference between horizontal and vertical anticommons problems is not an essential one, while the distinction of simultaneous and sequential situations has a systematic influence on the severity of the anticommons problem.

Hence, we have derived the quite general result that the uncoordinated exercise of exclusion rights leads to underutilization of a common resource.

The above model of “exclusion” rights has a parallel formulation where the two agents control the prices of their rights of use or exclusion,  $p_i, p_j$ , instead of their quantities,  $x_i, x_j$ . Such a dual version is analytically easier to represent if the two property rights are indivisible and strict complements.<sup>13</sup>

Our general treatment is helpful in at least two respects. First it allows, but does not require, strict complementarity. The two exclusionary rights may be partial (or less-than-perfect complements). The Buchanan and Yoon [2000] article assumes strict complementarity and thus represents a special case of our general model. Cases of partial exclusion rights are conceivable in real life property relations. One can think of several situations where the encumbrance of a third party exclusion right reduces, yet does not eliminate, the right of use (and the value) of the burdened property. More generally, one can think of various hypotheses of less-than-perfect complementarity between the two rights, as the above examples show.

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<sup>11</sup> This is meant to refer to a situation where one of the agents wants to pursue a project and needs to obtain the right of the other agent to do so. A classical textbook example of the double marginalization problem was first formalized by Spengler [1950], where the retailer needs the right to use an intermediate input of some producer. See also, Tirole [1993: 174].

<sup>12</sup> On property rights and transaction costs, see Miceli [1996]. One can think of various examples of administrative procedures (e.g. filings for building permits, etc.) with multiple administrative bodies (e.g., zoning, environmental, etc.) exercising control over a given proposal.

<sup>13</sup> Exclusivity of property rights are often seen as a prerequisite of selling the property right. With the transfer of the property the right to exclude is transferred as well. In this view a price of some property can also be seen as a price for the right to exclude others from using the property. This enables another variation of modeling the anticommons problem which is again a dual form of the first version.

### 3. Discussion

In the previous section, we showed that anticommons are the consequence of a dysfunctional fragmentation of a property right, where the nature of the fragmentation, as opposed to the mere extent of it, has a direct impact on the resulting deadweight loss.<sup>14</sup> The model predicts that Anticommons losses increase monotonically in both (a) the extent of fragmentation; and (b) the foregone synergies and complementarities between the property fragments.

An important implication of such monotonicity is the fact that it is easier to fragment property than to rebundle it. Put it differently, at the limit, a single owner faces no strategic costs when deciding whether to fragment. But, once fragmentation takes place, reunification requires the involvement of at least two parties, with positive transaction costs; and, so on for greater number of parties. This creates a one-directional stickiness in the process of reallocating property among different levels of fragmentation.

#### 3.1. The nature of the Asymmetric Transaction Costs

In a world of zero transaction costs, an efficient allocation of resources occurs regardless of the initial allocation of legal entitlement and choice of remedies to protect them.<sup>15</sup> In our context, the Coase theorem suggests that if all rights are freely transferable and transaction costs are zero, an inefficient initial partitioning of property rights will not impede an efficient final use of the resources. In the event of inefficient fragmentation of property, voluntary agreements will reaggregate property into efficiently sized clusters, maximizing the total value of the resources.

Once the ideal conditions of the positive Coase theorem are relaxed, over-fragmentation poses an interesting situation of asymmetric transaction costs. The presence of such asymmetry is due to the fact that the reunification of fragmented rights usually involves transaction and strategic costs of a greater magnitude than those incurred for the original fragmentation of the right. As shown above, the intuition for such asymmetry is quite straightforward. A single owner faces no strategic costs when deciding how to partition his property. Conversely, as shown in Section 2, multiple non-conforming co-owners are faced with a strategic problem, given the interdependence of their decisions. The equilibrium pricing (or quantity supply) of anticommons owners will impede the optimal reunification of non-conforming fragments into a unified bundle.

In the context of the anticommons, the argument that it is often harder to regenerate separated bundles than to fragmentize them has been put forth by Heller [1998]. While intuitively correct, the

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<sup>14</sup> In the existing literature, the expression “partitioning of property rights” refers conjunctively to spatial and functional forms of fragmentation. See, e.g., Alchian [1977] describing situations when several people each possess some portion of the rights to use the land. He also provides examples of private land-use arrangements such as servitudes (e.g. the right to grow wheat on it, to dump ashes over it, etc.).

<sup>15</sup> Coase [1960]. See also on attenuation and partitioning of property rights, Eggertson [1990b, 38-39].



argument warrants some further explanation. In selecting the optimal level of fragmentation, a rational owner estimates the expected value of the alternative partitioning of his property and would rationally select the arrangement which yields the highest net present value. The owner's optimal choice would rest on the estimation of (a) the respective probability that each alternative partitioning may coincide with the desired final allocation, and (b) the respective ex post reallocation costs (if the chosen level of fragmentation proves to be ex post sub-optimal). This optimization process leads to the choice of an initial allocation that maximizes the present value of the property at the net of possible reallocation costs and resulting inefficiencies. In this respect, owners act efficiently taking full account of the available information and with full consideration of the asymmetric transaction costs induced by property fragmentation.

But, in spite of the perfect alignment of private and social incentives, anticommons problems remain. Owners aim at maximizing the value of their property, but - given some uncertainty on the optimal final use - they do so with some margin of error. Because of the one-directional stickiness in the fragmentation process (i.e., sub-optimal fragmentation can be easily corrected ex post, while excessive fragmentation is likely to be irreversible) the errors have cumulative, rather than offsetting, effects on society.<sup>16</sup>

The interesting point here is that, while anticommons fragmentation may be occasionally ex ante efficient (given the specific goals pursued by property owners), it may result in inefficient ex post allocations.

### 3.2. *Some Normative Corollaries*

Let's proceed to apply the formal analysis in Section 2 above to discuss the challenges of policymakers in the advent of anticommons deadweight losses. By revealing the economic mechanism operating within property fragmentation, as presented in the formal analysis above, we may achieve a better understanding of the concrete legal rules found in many areas of the law.

In the realm of non-conforming property arrangements, positive transaction costs often generate a

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<sup>16</sup> Parisi [2001] notes that this optimization process requires the assessment of the likelihood of different situations arising in the future, and the evaluation of the impact of asymmetric transaction costs, given the uncertainty over the optimal final allocations. In our case, in order to solve the allocation problem under uncertainty, owners would consider: (a) the relative cost of reallocating from the initial allocation  $i$  to the efficient allocation  $j$ , which the author denotes  $C_{ij}$ ; and (b) the probability  $P(i)$  that allocation  $i$  is the most efficient final allocation, among the  $n$  possible alternative allocations. The presence of asymmetric transaction costs renders the present optimization problem different from the usual optimal allocation problem of the normative Coase theorem. In the usual case with symmetric transaction costs,  $C_{ij} = C_{ji}$ , the optimization problem is simplified and reduces to an optimal choice driven by the respective probability of alternative efficient final allocations. The optimal initial allocation coincides with the most likely efficient final allocation (i.e.,  $i^* = \text{Max} \{P(i)\}$ ). In the presence of asymmetric transaction costs,  $C_{ij} \neq C_{ji}$ , the optimization problem should, instead, account for the asymmetry induced by the one-directional stickiness in the exchange process. The optimal initial allocation is derived considering the costs and benefits of alternative allocations, in light of such asymmetry.

one-directional stickiness in the transfer of legal entitlements. As discussed above, externalities and holdouts are two major impediments to transfers.

In this context our findings yield a normative proposition, a testable hypothesis and an explanatory proposition, along the following three points.

1. The normative proposition contains a cautionary echo in the current trend of “proptertization” of atypical entitlements, i.e., vesting individual property right protection in an increasingly broader scope of resources. This trend is particularly current in the domain of intellectual property law. Several areas of intellectual property are gradually shifting away from a commons regime toward a private property regime. Under the older commons regime, much of the knowledge was freely available in the public domain. Given the public good nature of those discoveries and information, research was publicly funded. Such information nowadays enjoys the increased protection of intellectual property laws. The protection provided by intellectual property laws has gradually shifted the balance towards privatization of research. Research is conducted on a competitive basis by research institutions and private firms. Whenever possible, the results of such research are generally patented and later licensed or traded in the marketplace. This, in turn, allows research firms and institutions to capture some of the value of their discoveries, with increased incentives for valuable research.

The transition from commons to privatization, while greatly beneficial for the creation of private incentives for research, generates a gradual proliferation of exclusion rights with resulting anticommons problems.<sup>17</sup> As shown in Section 2 of this paper, anticommons problems are likely to be pervasive in the production of goods requiring highly complementary inputs supplied by independent parties. For example, the use of production technology protected by multiple third party patents often occasions a fragmentation that can be analogized to the cases of non-conforming property discussed above. In this context, Heller and Eisenberg [1999] apply the anticommons concept to patent technology. They argue that granting too many patent rights in biomedical research may delay the discovery and production of life-saving products. Product developers are often faced with a difficult decision problem. Before they can develop new products and bring them to the market, they need to solicit licenses from various patent holders. In the presence of positive externalities between upstream patent holders and downstream product developers, a situation parallel to the vertical anticommons setting described in Section 2 above, may emerge. The failure of parties to take into account these externalities may generate anticommons deadweight losses. In the context of intellectual property, privatization of research must, in this respect, be attentive to the need to promote

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<sup>17</sup> In response to the standard conclusions derived from the Tragedy of the Commons phenomenon, Carol Rose [1986] has described how in certain circumstances the answer to depletion or over-utilization of scarce resources is not found in granting private property rights. Rose illustrates this notion with striking examples of ‘public property’ where common law and statutory strategies have long provided the optimal resource management strategy.

research in the upstream market without delaying the discovery and implementation of downstream products.

More generally, “propertization” may be a good antidote for offsetting commons deadweight losses but, if carried out beyond measure, may lead to yet greater losses from anticommons problems.

2. The positive hypothesis suggests that courts and legislators, consciously or unconsciously, already account for the asymmetric effects of property fragmentation, when considering the optimal choice of rules and the optimal structure of remedies. Legal systems take into account the anticommons problems selecting rules designed to minimize the total deadweight losses of property fragmentation. One testable hypothesis, for example, is that legal systems would grant a less extensive property-type protection in favor of non-conforming property arrangements. Several rules and doctrines in the field of real property can be evaluated in light of this hypothesis. Take for instance, the body of mandatory rules in private land-use law that regulates the creation and enforcement of atypical easements and real covenants. Although the Anglo-American law of promises with respect to land is often described as nothing more than a historically evolved legal cobweb, close examination reveals that behind its technicalities lies a coherent economic logic. The attachment of promises to land creates user rights in a property resource and as such may be regarded as a partitioning of property rights. By treating land-related promises as enforceable contracts that bind the contracting parties rather than real rights that run with the land in perpetuity, doctrines such as touch and concern in common law, prediality<sup>18</sup> and the *numerus clausus*<sup>19</sup> principles in civil law, have served as instruments to limit the cases of dysfunctional fragmentation (Depoorter and Parisi [2000] and Parisi [2001b] evaluate the comparative and historical analysis of property rules concerning the creation and enforcement of atypical easements and real covenants in light of the positive hypothesis of transaction and strategic cost minimization)<sup>20</sup>.

Freedom of contract of the parties is left unrestrained in the domain of contractual and personal obligations. The creation of atypical property rights is, however, governed by categories and rules of contract law, with liability-type protection under most circumstances.<sup>21</sup> The dichotomous treatment of

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<sup>18</sup> The requirement of prediality (art. 637 C.C. in Belgium and France, art. 646 Louisiana code) holds that only land-promises which are of “real” nature may run with the land. Promises of personal nature are personal rights, not real rights, and as such they do not pertain the characteristics of a real right.

<sup>19</sup> The *numerus clausus* doctrine holds that there is a limited number of real property rights that the legal system recognizes and grants them property-type remedial protection.

<sup>20</sup> Rose [1999] and Merrill and Smith [2000] have used information-cost economics to explain the various legal efforts to avoid undue fragmentation. Their contributions concentrate on information costs and distinguish property from contract along the dimension of property law’s preoccupation with avoiding fragmentation. Most specifically, Merrill and Smith [2000] explain the “*numerus clausus*” doctrine as an attempt to constrain the proliferation of types of property rights leading to an increase in third-party information costs. For this reason property law enforces as property only those interests that conform to a limited number of standard forms.

<sup>21</sup> This apparent anomaly in the coordination of property and contract rules has been overlooked in the literature. Recent research suggests that Anglo-American courts intuitively responded to the dangers of unrestricted

typical and atypical property rights can be explained as an attempt to minimize the transaction and strategic costs resulting from dysfunctional property arrangements.

3. Furthermore, our formal analysis of the anticommons tragedy, as presented above in Section 2, may provide an alternative explanation for the long-lived heritage of default reunification mechanisms, commonplace in virtually all systems of Western legal tradition. Time limits, statutes of limitation, liberative prescription, rules of extinction for non-use, etc., can all be regarded as legal devices to facilitate the (otherwise costly and difficult) reunification of non-conforming fragments of a property right. These legal solutions can be analogized to a gravitational force, reunifying rights that, given their strict complementarity, would naturally be held by a single owner. This tendency towards reunification works to rebundle property rights in order to regenerate the natural conformity between use and exclusion rights (and, more generally, between any two complementary fragments of property). Along similar lines, a survey of American property law by Michael Heller [1999] reveals what he terms a ‘boundary principle’ which limits the right to subdivide private property into wasteful fragments. Property law responds to excessive fragmentation with the use of a variety of rules and doctrines such as the rule against perpetuity, zoning and subdivision restrictions, property taxes and registration fees, etc. (see, Heller [1999, 1173-1174], citing zoning and subdivision restrictions such as minimum lot sizes, floor areas and setbacks that prevent people from spatially fragmenting resources too much). Heller suggests that, by making the creation and maintenance of fragments more costly, for instance through annual disclosure expenses, excessive fragmentation into low-value fragments will be deterred and existing fragments will be abandoned so that the state can afterwards rebundle them.

Interestingly, most of these reunification mechanisms do not apply with respect to typical property rights. Typical property rights already provide conforming boundaries of use and exclusion rights. This eliminates any reason to favor reunification over persisting fragmentation. Conversely, atypical property arrangements may justify the activation of reunification mechanisms to overcome entropy and persisting fragmentation (Parisi [2001b]).

#### *4. Conclusion*

In this paper we have developed a model of anticommons fragmentation in property. In both vertical and horizontal cases of property fragmentation, underutilization of a scarce resource results from the fact that the private incentives of excluders do not capture the external effects of their individual decisions. Our simple model suggests that the results of underutilization of joint property increase monotonically in both (a) the extent of fragmentation; and (b) the foregone synergies and complementarities between the property fragments. This monotonicity implies that it is easier to fragment

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fragmentation by obstructing the running of personal promises attached to land, in favor of objective arrangements intrinsic to the land in question. *See* Depoorter and Parisi [2000].

than to rebundle fragmented property. In the realm of non-conforming property arrangements, this generates a one-directional stickiness in the transfer of legal entitlements.

The recognition of such one directional transaction and strategic costs generates a promising research agenda for the study of laws and institutions designed to cope with such asymmetries. Along the lines of well known efficiency hypotheses of the common law, a positive hypothesis can be formulated according to which courts and legislators, consciously or unconsciously, account for the asymmetric effects of property fragmentation in designing default rules and remedies. This tendency may be reflected in the legal system's reluctance to grant extensive property-type protection in favor of non-conforming property arrangements and in the creation of default reunification mechanisms for atypical property right arrangements. As a general normative corollary we can anticipate a note of caution in the current trend of diffuse and fragmented "proptertization" and a revival of the notions of unified and absolute property. It is a point of future research to expose various institutional responses to potential problems of excessive property fragmentation.



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## Appendix

This appendix contains the proofs to propositions 1 to 3.

*Proposition 1:* Under assumptions 1, 2, and 3 a Nash equilibrium  $x_i^f$  exists and is unique. Moreover, if  $(x_1^f, x_2^f)$  and  $(x_1^u, x_2^u)$  are in the interior of  $X_1 \times X_2$ , we have for  $i = 1, 2$ :

$$x_i^f < x_i^u$$

*Proof:* The standard existence theorems of Nash equilibria imply that under assumptions 1 and 3 a Nash equilibrium exists. As  $X := X_1 \times X_2$  is compact and  $V$  is continuous, a pair of maximising strategies exists as well.

Assumption 2 enables us to use the uniqueness theorem contained e.g. in Friedman (????). This assumption also implies that best response functions,  $x_i(x_j)$ , are non-decreasing. Hence, the slope of the locus of all strategy pairs satisfying the first order conditions

$$(1) \quad \frac{\partial V_i}{\partial x_i}(x_i, x_j) = 0$$

is non-negative. The same is true for the locus of strategy pairs which satisfy:

$$(2) \quad \frac{\partial V}{\partial x_i}(x_i, x_j) = \frac{\partial V_i}{\partial x_i}(x_i, x_j) + \frac{\partial V_j}{\partial x_i}(x_j, x_i) = 0$$

Consider for fixed  $x_j$  the solution of (1). Because of assumption 1 the left hand side of (2) will be positive for a pair  $(x_i(x_j), x_j)$ . As  $V$  is concave in  $x_i$  by assumption 1 a solution of (2) must involve a larger value of  $x_i$  for a fixed value of  $x_j$ . This means that the locus of the solutions of (2) always involves higher values of  $x_i$  than the best response function. Given that both graphs are not decreasing anywhere this implies that the intersection point of (2) for  $i = 1, 2$  must also involve higher values than the intersection point of the two best response functions (1). This establishes proposition 1.

*Proposition 2:* Under assumptions 1, 3, and 4 a Nash equilibrium exists. Moreover, if  $(x_1^f, x_2^f)$  and  $(x_1^u, x_2^u)$  are in the interior of  $X_1 \times X_2$ , we have for  $i = 1, 2$ :

$$x_i^f < x_i^u$$

Proof: Under our assumptions best response functions are continuous. Therefore the locus the best response function of agent 1 is a compact set. In a sequential move game the equilibrium is determined by the value which maximises the value of agent 2 on this set. As the value function is continuous by assumption 1, this implies that an equilibrium exists.

Consider the locus of the solutions to (1) for  $i = 1$ , i.e. agent 1's best response function. Note first that as in the proof of proposition 1 the locus of the solutions to (2) always involve larger values. If we put  $x_1$  on the horizontal axis and  $x_2$  on the vertical axis this means that the solutions to (2) lie to the right of solutions to (1).

Consider now a point on agent 1's best response function which maximises agent 2's value function. If now the locus of solutions to (2) for  $i = 2$  intersects the best response function of agent 1 above such a point the claim of the proposition is shown to be true. Such a point is characterised by

$$(3) \quad \frac{\partial V_2}{\partial x_2}(x_2, x_1) + \frac{\partial V_2}{\partial x_1}(x_2, x_1) \frac{\partial x_1}{\partial x_2} = 0$$

while the locus of the solution to (2) for  $i = 2$  reads now:

$$(4) \quad \frac{\partial V_2}{\partial x_2}(x_2, x_1) + \frac{\partial V_1}{\partial x_2}(x_1, x_2) = 0$$

If the second term in (3) is smaller than the second term in (4) our claim is shown. But this condition is satisfied because of assumption 4.

*Proposition 3:* If assumptions 1, 2, and 3 are satisfied, if  $\partial^2 V_1 / \partial x_1 \partial x_2 > 0$ , and if the equilibrium pair of strategies are in the interior of  $X_1 \times X_2$ , then we have for  $i = 1, 2$ :

$$x_i^{f1} < x_i^{f2}$$

Proof: Both equilibria lie on the best response function of agent 1. This function is strictly increasing. Consider the intersection point with agent 2's best response function

$$(5) \quad \frac{\partial V_2}{\partial x_2}(x_2, x_1) = 0$$

and compare to (3) which characterises the two stage game solution. As the second term in (3) is positive, we can reiterate the arguments in the proof of proposition 1 to establish the claim.